**Class-X Session 2022-23**

**Subject - Mathematics (Standard)**

**Sample Question Paper - 35**

**With Solution**

## BLUE PRINT

| Ch. No. | Chapter Name | Per Unit Marks | Section-A (1 Mark) | | Section-B (2 Marks) | Section-C (3 Marks) | Section-D (5 Marks) | Section-E (4 Marks) | Total Marks |
|---|---|---|---|---|---|---|---|---|---|
| | | | MCQ | A/R | VSA | SA | LA | Case-Study | |
| 1 | Real Number | 6 | 2(Q1,5) | 1(Q19) | | 1(Q26) | | | 6 |
| 2 | Polynomials | 20 | 1(Q4) | | | 1(Q27) | | | 4 |
| 3 | Pair of Linear Equations in Two Variables | | 1(Q3) | | 1(Q21) | 1(Q28) | | | 6 |
| 4 | Quadratic Equations | | 1(Q2) | | | | 1(Q34) | | 6 |
| 5 | Arithmetic Progression | | | | | | | 1(Q36) | 4 |
| 6 | Triangles | 15 | 2(Q6,8) | | 1(Q22) | | 1(Q32) | | 9 |
| 7 | Circles | | 1(Q13) | | 1(Q24) | 1(Q30) | | | 6 |
| 8 | Coordinate Geometry | 6 | 2(Q9,11) | | | | | 1(Q38) | 6 |
| 9 | Introduction to Trigonometry | 12 | 2(Q7,10) | 1(Q20) | 1(Q23) | 1(Q29) | | | 8 |
| 10 | Some Applications of Trigonometry | | | | | | | 1(Q37) | 4 |
| 11 | Areas Related to Circles | 10 | 2(Q12,14) | | 1(Q25) | | | | 4 |
| 12 | Surface Areas and Volumes | | 1(Q16) | | | | 1(Q33) | | 6 |
| 13 | Statistics | 11 | 2(Q15,17) | | | | 1(Q35) | | 7 |
| 14 | Probability | | 1(Q18) | | | 1(Q31) | | | 4 |
| Total Marks (Total Questions) | | 80 | 18(18) | 2(2) | 10(5) | 18(6) | 20(4) | 12(3) | 80(38) |

**Note :** The number given inside the bracket denotes question number, asked in the sample paper, while the number given outside the bracket are the number of questions from that particular chapter.

**Time : 3 Hours** **Max. Marks : 80**

## General Instructions

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 case based integrated units of assessment (4 marks each) with sub parts of values of 1, 1 and 2 marks each respectively.*

## SECTION-A (Multiple Choice Questions)

***Each question carries 1 mark.***

**1.** I. The L.C.M. of $x$ and 18 is 36.
II. The H.C.F. of $x$ and 18 is 2.
What is the number $x$ ?
(a) 1 (b) 2 (c) 3 (d) 4

**2.** If the sum of the roots of a quadratic equation is 6 and their product is 6, the equation is
(a) $x^2 - 6x + 6 = 0$ (b) $x^2 + 6x - 6 = 0$
(c) $x^2 - 6x - 6 = 0$ (d) $x^2 + 6x + 6 = 0$

**3.** For what values of $k$ will the following pair of linear equations have infinitely many solutions?
$kx + 3y - (k - 3) = 0$
$12x + ky - k = 0$
(a) $k = 4$ (b) $k = 3$ (c) $k = 6$ (d) $k = 2$

**4.** If the zeroes of the polynomial $f(x) = k^2x^2 - 17x + k + 2$, $(k > 0)$ are reciprocal of each other than value of $k$ is
(a) 2 (b) –1 (c) –2 (d) 1

**5.** Given that L.C.M. (91, 26) = 182, then H.C.F. (91, 26) is
(a) 13 (b) 26 (c) 17 (d) 9

**6.** Two triangles are similar if
(a) their corresponding angles are equal.
(b) their corresponding sides are equal.
(c) both are right triangle.
(d) None of the above

**7.** If $\tan^2\theta = 1 - a^2$, then the value of $\sec\theta + \tan^3\theta \operatorname{cosec}\theta$ is
(a) $(2 - a^2)$ (b) $(2 - a^2)^{1/2}$
(c) $(2 - a^2)^{2/3}$ (d) $(2 - a^2)^{3/2}$

**8.** If in two $\Delta DEF$ and $\Delta PQR$, $\angle D = \angle Q$ and $\angle R = \angle E$, then which of the following is not true?
(a) $\frac{EF}{PR} = \frac{DF}{PQ}$ (b) $\frac{DE}{PQ} = \frac{EF}{RP}$ (c) $\frac{DE}{QR} = \frac{DF}{PQ}$ (d) $\frac{EF}{RP} = \frac{DE}{QR}$

**9.** If P = (2, 5), Q = (x, –7) and PQ = 13, what is the value of 'x'?
(a) 5 (b) 3 (c) –3 (d) –5

**10.** If $b\tan\theta = a$, the value of $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$ is
(a) $\frac{a-b}{a^2+b^2}$ (b) $\frac{a+b}{a^2+b^2}$ (c) $\frac{a^2+b^2}{a^2-b^2}$ (d) $\frac{a^2-b^2}{a^2+b^2}$

**11.** In what ratio does the point (–2, 3) divide the line-segment joining the points (–3, 5) and (4, –9) ?
(a) 2 : 3 (b) 1 : 6 (c) 6 : 1 (d) 2 : 1

**(ASSERTION-REASON BASED QUESTIONS)**

***In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.***

(a) ***Both A and R are true and R is the correct explanation of A.***

(b) ***Both A and R are true but R is not the correct explanation of A.***

(c) ***A is true but R is false.***

(d) ***A is false but R is true.***

**19.** **Assertion :** $n^2 - n$ is divisible by 2 for every positive integer.

**Reason :** $\sqrt{2}$ is not a rational number.

**20.** **Assertion:** In a right angled triangle, if $\tan\theta = \frac{3}{4}$, the greatest side of the triangle is 5 units.

**Reason:** $(\text{greatest side})^2 = (\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$.

## SECTION-B

***This section comprises of very short answer type-questions (VSA) of 2 marks each.***

**21.** Which type of equations $x + 2y = 4$ and $2x + y = 5$ will be?

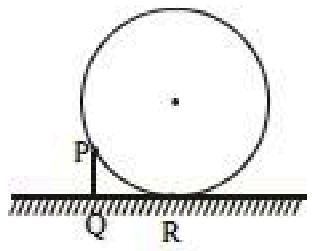
**OR**

For what value of k, the equations $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines?

**22.** Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of triangle PQR. Prove that $\Delta ABC \sim \Delta PQR$.

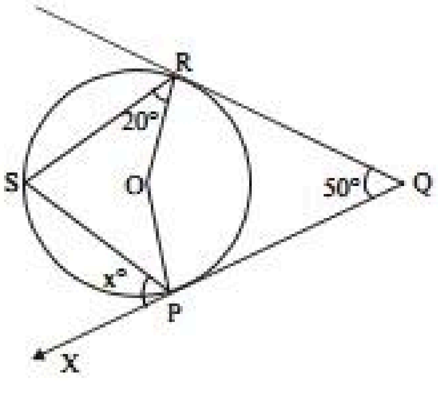
**23.** Prove that $\frac{\tan^2\theta}{\tan^2\theta - 1} + \frac{\operatorname{cosec}^2\theta}{\sec^2\theta - \operatorname{cosec}^2\theta} = \frac{1}{\sin^2\theta - \cos^2\theta}$

**24.** A ball is in the rest position against a step PQ. If PQ = 10 cm and QR = 15 cm, then the diameter of the ball is ____________.

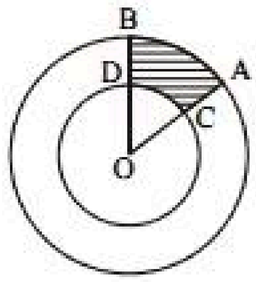



**OR**

In the diagram, PQ and QR are tangents to the circle centre O, at P and R respectively. Find the value of x.

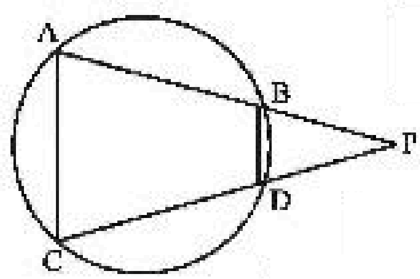


12. The figure shows two concentric circleswith centre O and radii 3.5 m and 7 m. If $\angle BOA = 40°$, find the area of the shaded region.




(a) $\frac{77}{6} cm^2$
(b) $\frac{76}{5}$
(c) $\frac{73}{6}$
(d) None of these

13. In the figure below (not to scale), AB = CD and $\overline{AB}$ and $\overline{CD}$ are produced to meet at the point $p$.




If $\angle BAC = 70°$, then $\angle P$ is
(a) 30°
(b) 40°
(c) 45°
(d) 50°

14. The area of a circular ring formed by two concentric circles whose radii are 5.7 cm and 4.3 cm respectively is (Take $\pi = 3.1416$)
(a) 43.98 sq.cm
(b) 53.67 sq. cm
(c) 47.24 sq.cm
(d) 38.54 sq.cm

15. The mean weight of a class of 35 students is 45kg. If the weight of a teacher be included, the mean weight increases by 500 grams. Find the weight of the teacher.
(a) 63 kg
(b) 61 kg
(c) 64 kg
(d) 70 kg

16. Volume of a spherical shell is given by
(a) $4\pi(R^2 - r^2)$
(b) $\pi(R^3 - r^3)$
(c) $4\pi(R^3 - r^3)$
(d) $\pi(R^3 - r^3)$

17. In the following distribution

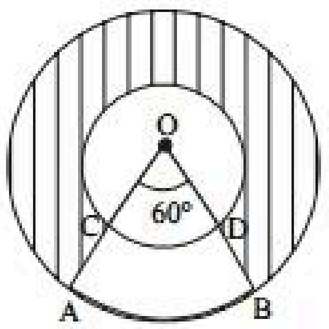
| Monthly income range (in ₹) | Number of families |
|---|---|
| Income more than ₹ 10,000 | 100 |
| Income more than ₹ 13,000 | 85 |
| Income more than ₹ 16,000 | 69 |
| Income more than ₹ 19,000 | 50 |
| Income more than ₹ 22,000 | 33 |
| Income more than ₹ 25,000 | 15 |

the number of families having income range (in ₹) 16000 – 19000 is
(a) 15
(b) 16
(c) 17
(d) 19

18. For an event E, P (E) + $P(\overline{E})$ = q, then
(a) $0 \le q < 1$
(b) $0 < q \le 1$
(c) $0 < q < 1$
(d) None of these

25. In figure, two concentric circles with centre O, have radii 21 cm and 42 cm. If $\angle AOB = 60°$, find the area of the shaded region.

[ Use $\pi = \frac{22}{7}$ ].




## SECTION-C

***This section comprises of short answer type questions (SA) of 3 marks each.***

26. Prove that $\sqrt{2} + \sqrt{5}$ is irrational.

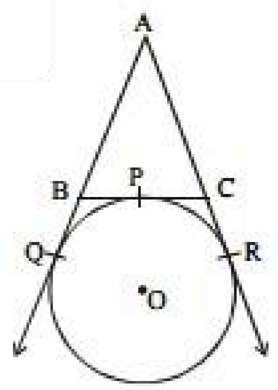
27. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as $\alpha$ and $\beta$. Now form a quadratic polynomial whose zeroes are $3\alpha$ and $3\beta$.

28. Solve the following system of equations :

$\frac{4}{x} + 5y = 7$ ; $\frac{3}{x} + 4y = 5$

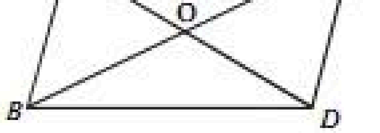
29. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$ ; $0° < A + B \leq 90°$; $A > B$, find A and B.

30. In fig. a circle touches the side BC of $\Delta ABC$ at P and touches AB and AC produced at Q and R respectively. If AQ = 5 cm, find the perimeter of $\Delta ABC$.




**OR**

In the given figure, $\Delta ABC$ and $\Delta DBC$ are on the same base $BC$. $AD$ and $BC$ intersect at $O$. Prove that $\frac{AE}{DF} = \frac{AO}{DO}$.



31. In a single throw of a pair of different dice, what is the probability of getting (i) a prime number on each dice ? (ii) a total of 9 or 11 ?

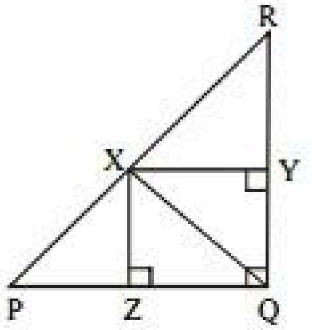
**OR**

Three different coins are tossed together. Find the probability of getting (i) exactly two heads (ii) at least two heads (iii) at least two tails.

## SECTION-D

***This section comprises of long answer-type questions (LA) of 5 marks each.***

32.




$\Delta PQR$ is right angled at $Q$. $QX \perp PR$, $XY \perp RQ$ and $XZ \perp PQ$ are drawn. Prove that $XZ^2 = PZ \times ZQ$.

33. A solid metallic right circular cone 20 cm high and whose vertical angle is 60°, is cut into two parts at the middle of its height by a plane parallel to its base. Find volume of small cone.

**OR**

From a solid cylinder of height 2.8 cm and diameter 4.2 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. [Take $\pi = \frac{22}{7}$].

34. Roots of the quadratic equation $36x^2 - 12ax + (a^2 - b^2) = 0$ are $\frac{a+b}{c}$ and $\frac{a-b}{c}$. Then, find the value of $c$.

**OR**

Find the real roots of the equation $x^{2/3} + x^{1/3} - 2 = 0$.

35. Find the median of the following data :

| **Height (in cm)** | Less than 120 | Less than 140 | Less than 160 | Less than 180 | Less than 200 |
|---|---|---|---|---|---|
| **Number of students** | 12 | 26 | 34 | 40 | 50 |

## SECTION-E

***This section comprises of 3 case study/passage - based questions of 4 marks each with three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively.***

36. **Case - Study 1:** Read the following passage and answer the questions given below.

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in $6^{th}$ year and 22600 in $9^{th}$ year.

Based on the above information, answer the following questions:

**(i)** Find the production during first year.

**(ii)** Find the production during $8^{th}$ year.

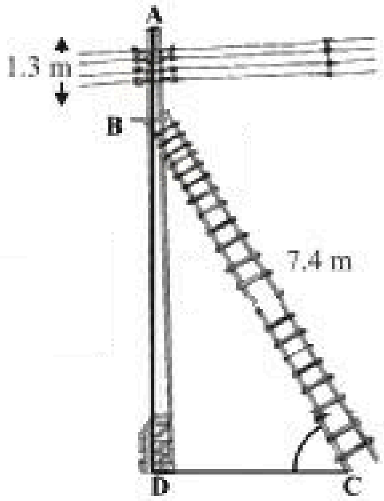
**(iii)** Find the production during first 3 years.

**OR**

In which year, the production is Rs 29,200.

**37. Case - Study 2:** Read the following passage and answer the questions given below.

An electrician has to repair an electric fault on a pole of height 5m. He needs to reach a point 1.3 m below the top of the pole to under take the repair work. He place the ladder of length 7.4 m at that position.




Then answer the following questions.

**(i)** Find measure of $\angle C$.

**(ii)** Find measure of $\angle B$.

**(iii)** Find distance between foot of ladder and pole.

**OR**

The value of $\sin^2 B + \sin^2 C$ is

**38. Case - Study 3:** Read the following passage and answer the questions given below.

A **hockey field** is the playing surface for the game of hockey. Historically, the game was played on natural turf (grass) but nowadays it is predominantly played on an artificial turf.

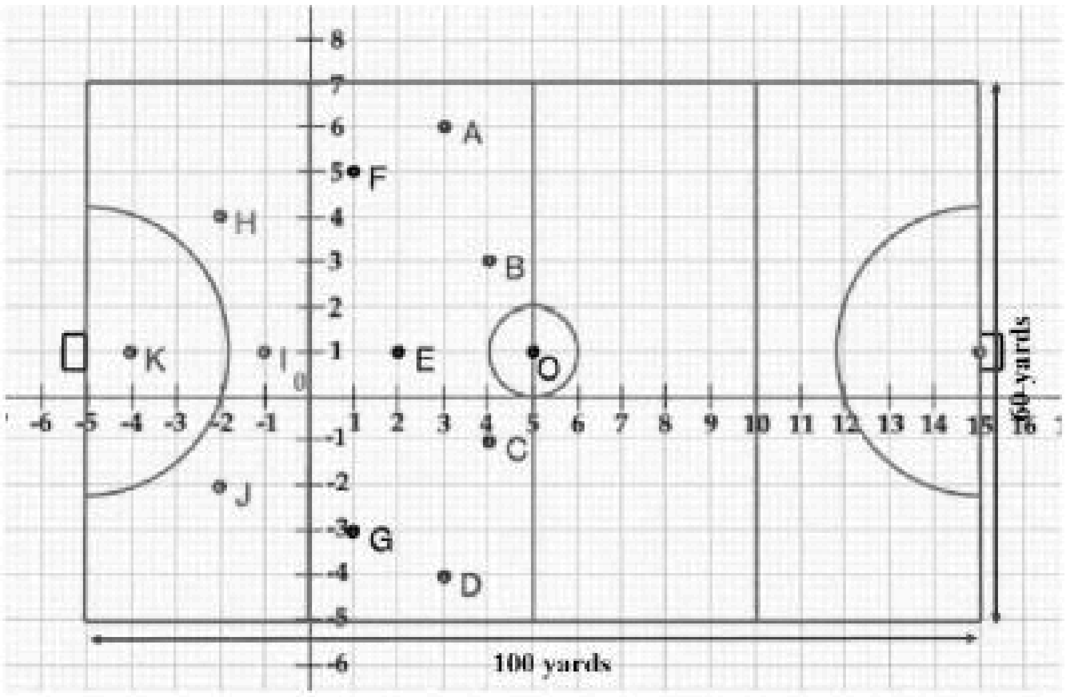
It is rectangular in shape - 100 yards by 60 yards. Goals consist of two upright posts placed equidistant from the centre of the backline, joined at the top by a horizontal crossbar. The inner edges of the posts must be 3.66 metres (4 yards) apart, and the lower edge of the crossbar must be 2.14 metres (7 feet) above the ground.

Each team plays with 11 players on the field during the game including the goalie.

Positions you might play include-

- **Forward :** As shown by players A, B, C and D.
- **Midfielders:** As shown by players E, F and G.
- **Fullbacks:** As shown by players H, I and J.

Goalie: As shown by players H, I and J.
Using the picture of a hockey field below, answer the questions that follow:




(i) Find the coordinates of the centroid of ΔEHJ.

(ii) If a player P needs to be at equal distances from A and G, such that A, P and G are in straight line, then find the position of P.

(iii) Find the point on x axis equidistant from I and E.

**OR**

What are the coordinates of the position of a player Q such that his distance from K is twice his distance from E and K, Q and E are collinear?

# SOLUTIONS

## SAMPLE PAPER-1

1. **(d)** L.C.M × H.C.F = First number × second number

Hence, required number = $\frac{36 \times 2}{18} = 4$.

2. **(a)** Required equation is $x^2 - 6x + 6 = 0$.

3. **(c)** Here, $\frac{a_1}{a_2} = \frac{k}{12}, \frac{b_1}{b_2} = \frac{3}{k}, \frac{c_1}{c_2} = \frac{k-3}{k}$

For a pair of linear equations to have infinitely many solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, we have $\frac{k}{12} = \frac{3}{k} = \frac{k-3}{k}$ or $\frac{k}{12} = \frac{3}{k}$ which gives $k^2 = 36$ i.e., $k = \pm 6$

Also, $\frac{3}{k} = \frac{k-3}{k}$ gives $3k = k^2 - 3k$, i.e., $6k = k^2$, which means $k = 0$ or $k = 6$.

Therefore, the value of $k$ that satisfies both the conditions, is $k = 6$. For this value, the pair of linear equations has infinitely many solutions.

4. **(a)** Since zeroes are reciprocal of each other, so product of the roots will be 1, so,

$k^2 - k - 2 = 0 \Rightarrow (k-2)(k+1) = 0$

$k = 2, k = -1$, Since $k > 0 \;\therefore\; k = 2$

5. **(a)** H.C.F. (91, 126) = $\frac{91 \times 126}{\text{L.C.M.}(91, 126)}$

$= \frac{91 \times 126}{182} = 13$

6. **(a)** (By definition of similar triangles).

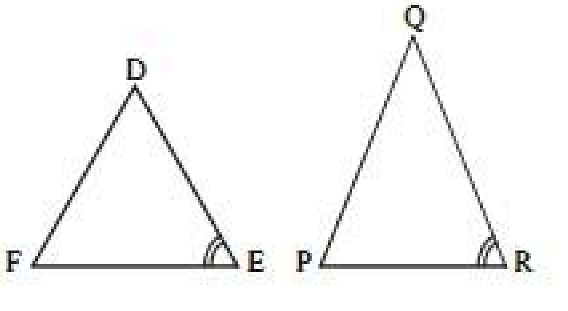
7. **(d)** $\sec\theta + \tan^3\theta \operatorname{cosec}\theta$

$= \sec\theta + \frac{\sin\theta}{\cos\theta}\tan^2\theta \operatorname{cosec}\theta = \sec\theta\,(1 + \tan^2\theta)$

$= (1 + \tan^2\theta)^{3/2} = [1 + (1 - a^2)]^{3/2}$

8. **(b)** Now, in $\Delta DEF$ and $\Delta PQR$,

$\angle D = \angle Q, \angle R = \angle E$




$\therefore\; \Delta DEF \sim \Delta QRP$ [by AAA similarity criterion]

$\Rightarrow\; \angle F = \angle P$ [corresponding angles]

$\therefore\; \frac{DF}{QP} = \frac{ED}{RQ} = \frac{FE}{PR}$ [corresponding sides]

9. **(c)** $PQ = 13 \Rightarrow PQ^2 = 169$

$\Rightarrow (x-2)^2 + (-7-5)^2 = 169$

$\Rightarrow x^2 - 4x + 4 + 144 = 169$

$\Rightarrow x^2 - 4x - 21 = 0 \Rightarrow x^2 - 7x + 3x - 21 = 0$

$\Rightarrow (x-7)(x+3) = 0 \Rightarrow x = 7, -3$

10. **(d)** Given, $\tan\theta = \frac{a}{b}$

$\therefore\; \frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a\tan\theta - b}{a\tan\theta + b} = \frac{a^2 - b^2}{a^2 + b^2}$

11. **(b)** Suppose the required ratio is $m_1 : m_2$

Then, using the section formula, we get

$-2 = \frac{m_1(4) + m_2(-3)}{m_1 + m_2}$

$\Rightarrow -2m_1 - 2m_2 = 4m_1 - 3m_2$

$\Rightarrow m_2 = 6m_1 \Rightarrow m_1 : m_2 = 1:6$

12. **(a)** Area of the shaded region

$= \frac{40°}{360°} \times \frac{22}{7} \times (7)^2 - \frac{40°}{360°} \times \frac{22}{7} \times (3.5)^2$

$= \frac{1}{9} \times \frac{22}{7} \times (7^2 - 3.5^2) = \frac{1}{9} \times \frac{22}{7} \times \left(49 - \frac{49}{4}\right)$

$= \frac{1}{9} \times \frac{22}{7} \times \frac{49}{4} \times 3 = \frac{77}{6}\text{cm}^2$

13. **(b)** Exterior angle of a cyclic quadrilateral is equal to its interior opposite angle.

$\angle BAC = \angle DCA$ and proceed.

14. **(a)** Let the radii of the outer and inner circles be $r_1$ and $r_2$ respectively; we have

Area $= \pi r_1^2 - \pi r_2^2 = \pi(r_1^2 - r_2^2)$

$= \pi(r_1 - r_2)(r_1 + r_2)$

$= \pi(5.7 - 4.3)(5.7 + 4.3) = \pi \times 1.4 \times 10$ sq. cm

$= 3.1416 \times 14$sq. cm. = 43.98 sq. cms.

15. **(a)** Let the mean weight of a class of 35 students be $\overline{x}_1$

and that of both students and a teacher be $\overline{x}_2$

Then $\overline{x}_1 = 45$ kg and

$\bar{x}_2 = 45 + \frac{500}{1000} = 45 + 0.5 = 45.5$ kg

$\bar{x}_1 = \frac{\Sigma x_1}{n_1}, \ \bar{x}_2 = \frac{\Sigma x_2}{n_1}$

$\Rightarrow 45 = \frac{\Sigma x_1}{35}, \ 45.5 = \frac{\Sigma x_2}{36}$

$\Rightarrow$ $\Sigma x_1 = 1575$ kg, $\Sigma x_2 = 1638$ kg

$\Rightarrow$ Total weight = weight of students + weight of teacher

$\therefore$ Weight of teacher = Total weight – weight of students

$\therefore$ Weight of the teacher = $\Sigma x_2 - \Sigma x_1$

= 1638 – 1575 = 63 kg

**16.** **(d)** Volume of spherical shell

$= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(R^3 - r^3)$

**17.** **(d)** Clearly, the number of families having income range (in ₹)

16000 – 19000 = 69 – 50 = 19.

**18.** **(d)** $\because \ P(E) + P(\bar{E}) = 1$

**19.** **(b)** Put $n = 1$ and $n = 2$.

**20.** **(a)** Both Assertion and Reason are correct and Reason is the correct explanation of the assertion.

greatest side = $\sqrt{(3)^2 + (4)^2} = 5$ units.

**21.** Comparing the given equations with $a_1x_1 + b_1y = c_1$ and $a_2x + b_2y = c_2$ we have

$\therefore \ \frac{a_1}{a_2} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{2}{1}; \frac{c_1}{c_2} = \frac{4}{5}$ [1 Mark]

Here, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ [½ Mark]

$\therefore$ The equations have consistent and unique olution. [½ Mark]

**OR**

Pair of lines are coincident if

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ...(i) [½ Mark]

Given lines,

$3x - y + 8 = 0$

and $6x - ky + 16 = 0$

Here, $a_1 = 3, b_1 = -1, c_1 = 8$

$a_2 = 6, b_2 = -k, c_2 = 16$ [½ Mark]

Using equation (i) we have

$\frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}$ [½ Mark]

$\Rightarrow \ \frac{1}{k} = \frac{1}{2}$

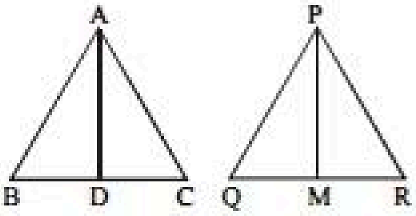
$\therefore$ k = 2 [½ Mark]

**22.** Given : $\Delta ABC$ and $\Delta PQR$, in which,

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

**To Prove :** $\Delta ABC \sim \Delta PQR$ [½ Mark]

**Proof :** $\frac{AB}{PQ} = \frac{BC}{QR}$ (Given)




$\frac{2BD}{2QM} = \frac{BC}{QR} \therefore \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

$\therefore$ $\Delta ABD \sim \Delta PQM$ (By SSS) [1 Mark]

$\therefore$ $\angle B = \angle Q$ (By corresponding angles of similar triangles)

Now, In $\Delta ABC$ and $\Delta PQR$, $\frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle B = \angle Q$

$\therefore$ **$\Delta$ABC ~ $\Delta$PQR (By SAS)** [½ Mark]

**23.** LHS $= \frac{\tan^2\theta}{\tan^2\theta - 1} + \frac{\operatorname{cosec}^2\theta}{\sec^2\theta - \operatorname{cosec}^2\theta}$

$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta} - 1} + \frac{\frac{1}{\sin^2\theta}}{\frac{1}{\cos^2\theta} - \frac{1}{\sin^2\theta}}$ [½ Mark]

$= \frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{\sin^2\theta - \cos^2\theta} + \frac{1}{\sin^2\theta} \times \frac{\sin^2\theta\cos^2\theta}{\sin^2\theta - \cos^2\theta}$ [½ Mark]

$= \frac{\sin^2\theta}{\sin^2\theta - \cos^2\theta} + \frac{\cos^2\theta}{\sin^2\theta - \cos^2\theta}$ [½ Mark]

$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta} = \frac{1}{\sin^2\theta - \cos^2\theta}$ = RHS [½ Mark]

**24.** In right $\Delta$ OSP,

$OP^2 = PS^2 + OS^2$ [1 Mark]

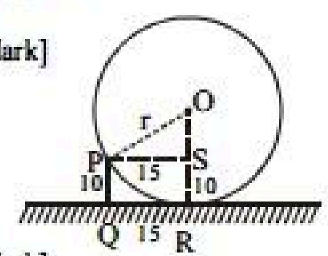
$\Rightarrow r^2 = 225 + (r - 10)^2$

$\Rightarrow r^2 = 225 + r^2 - 20r + 100$

$\Rightarrow 20r = 325$

$\Rightarrow 2r = 32.5$

Hence, diameter = 32.5 cm. [1 Mark]



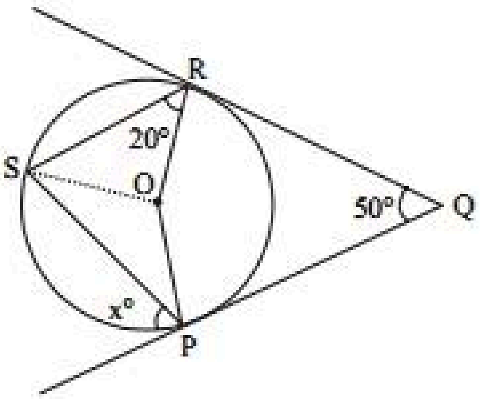

**Answer :** 32.5 cm

**OR**

$\angle POR + \angle PQR = 180°$

$\therefore \angle POR = 180° - 50° = 130°$

$\angle PSR = \frac{1}{2} \angle POR$ [½ Mark]




$\therefore \angle PSR = \frac{1}{2} \times 130° = 65°$ [½ Mark]

$\Rightarrow \angle PSR = \angle OSP + \angle OSR$

$\Rightarrow \angle PSR = \angle OSP + 20°$ [$\because \angle OSP = \angle OSR$]

$\Rightarrow 65° = \angle OSP = 20°$

$\Rightarrow \angle OSP = 45°$

$\Rightarrow \angle OPX = 90°$ [$\because$ PX is a tangent]

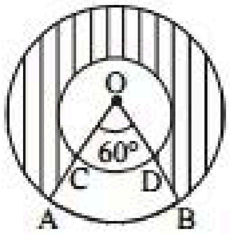
$\Rightarrow \angle SPX + \angle OPS = 90°$

$x° + \angle OSP = 90°$ [$\because \angle OPS = \angle OSP$]

$x° = 90° - 45° = 45°$

**Answer :** 45° [1 Mark]

**25.**




Radius of inner circle, r = 21 cm

Radius of outer circle, R = 42 cm

Area of (ABCD) = Area of sector (OAB) – Area of sector (OCD)

$= \frac{60}{360} \times \pi (42^2 - 21^2)$ [1 Mark]

$= \frac{1}{6} \times \frac{22}{7} \times (42+21)(42-21) = \frac{1}{6} \times \frac{22}{7} \times 63 \times 21$

$= 11 \times 21 \times 3 = 693\ cm^2$ [1 Mark]

Area of shaded region = Area of outer circle – Area of inner circle – Area (ABCD)

$= \pi(42)^2 - \pi(21)^2 - 693 = \frac{22}{7} \times 63 \times 21 - 693 = 4158 - 693$

$= 3465\ cm^2$ [1 Mark]

**26.** Let us assume on the contrary that $\sqrt{2} + \sqrt{5}$ is rational number. Then, there exist co-prime positive integers *a* and *b* such that

$\sqrt{2} + \sqrt{5} = \frac{a}{b} \Rightarrow \frac{a}{b} - \sqrt{2} = \sqrt{5}$ [½ Mark]

$\Rightarrow \left(\frac{a}{b} - \sqrt{2}\right)^2 = (\sqrt{5})^2$ [Squaring both sides] [½ Mark]

$\Rightarrow (a - \sqrt{2b})^2 = 5b^2 \Rightarrow a^2 + 2b^2 - 2ab\sqrt{2} = 5b^2$ [½ Mark]

$\Rightarrow a^2 + 2b^2 - 3b^2 = 2ab\sqrt{2} \Rightarrow \frac{a^2 - 3b^2}{2ab} = \sqrt{2}$ [½ Mark]

$\Rightarrow \sqrt{2}$ is a rational number

$\left[\because a, b \text{ are integers} \Rightarrow \frac{a^2 - 3b^2}{2ab} \text{ is rational}\right]$ [½ Mark]

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is wrong.

Hence, $\sqrt{2} + \sqrt{5}$ is irrational. [½ Mark]

**27.** If α and β are the zeroes of $2x^2 - 3x + 1$,

then $\alpha + \beta = \frac{-b}{a} = \frac{3}{2}$ and $\alpha\beta = \frac{c}{a} = \frac{1}{2}$ [1 Mark]

New quadratic polynomial whose zeroes are 3α and 3β is given by

$x^2$ – (Sum of the roots)x + Product of the roots

$= x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta$ [1 Mark]

$= x^2 - 3(\alpha + \beta)x + 9\alpha\beta = x^2 - 3\left(\frac{3}{2}\right)x + 9\left(\frac{1}{2}\right)$

$= x^2 - \frac{9}{2}x + \frac{9}{2} = \frac{1}{2}(2x^2 - 9x + 9)$ [1 Mark]

Hence, required quadratic polynomial is $\frac{1}{2}(2x^2 - 9x + 9)$.

**28.** $\frac{4}{x} + 5y = 7$ and $\frac{3}{x} + 4y = 5$

$\frac{4}{x} + 5y = 7$ .....(i)

$\frac{3}{x} + 4y = 5$ .....(ii)

[1 Mark]

Multiply equation (i) by (3) and equation (ii) by (4) we get,

$\frac{12}{x} + 15y = 21$ .....(iii)

$\frac{12}{x} + 16y = 20$ .....(iv)

Subtract (iii) and (iv), we get, $-y = 1 \Rightarrow y = -1$ [1 Mark]

Now, putting the value of y = – 1 in equation (i)

$\frac{4}{x} + 5(-1) = 7 \Rightarrow \frac{4}{x} = 7 + 5 \Rightarrow \frac{4}{x} = \frac{12}{1} \Rightarrow 12x$

$= 4 \Rightarrow x = \frac{4}{12} \Rightarrow x = \frac{1}{3}$ [1 Mark]

Hence $x = \frac{1}{3}$ and $y = -1$

**29.** $\tan(A + B) = \sqrt{3} \Rightarrow \tan(A + B) = \tan 60°$

$\left(\because \tan 60 = \sqrt{3}\right)$

$\Rightarrow A + B = 60°$ ......(i)

$\tan(A - B) = \frac{1}{\sqrt{3}} \Rightarrow \tan(A - B) = \tan 30°$ [1 Mark]

$\left(\because \tan 30° = \frac{1}{\sqrt{3}}\right)$

$\Rightarrow A - B = 30°$ ......(ii)

Adding (i) and (ii), we get; [1 Mark]

$2A = 90° \Rightarrow A = 45°$

Then from (i), $45° + B = 60° \Rightarrow B = 15°$. [1 Mark]

**30.** **To Find :** Perimeter of $\Delta ABC$

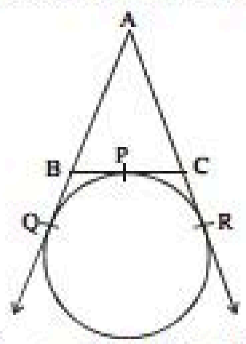
Let AQ = 5 cm and

$AQ = AR$ .....(i)

$BQ = BP$ .....(ii) [1 Mark]

$CP = CR$ .....(iii)

(Tangent drawn from an external points are equal)




[½ Mark]

Since perimeter of $\Delta ABC = AB + BC + CA$

$\Rightarrow$ Perimeter of $\Delta ABC = AB + BP + PC + CA$

$[\because BC = BP + PC]$ [1 Mark]

$= (AB + BQ) + (CR + CA)$ from (ii) and (iii)

$= AQ + AR$ $[\because AQ = AR$ from (i)$]$

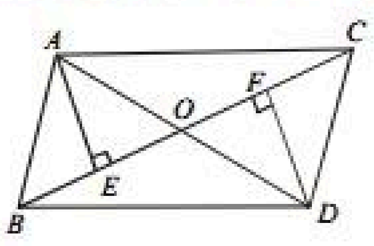
$= AQ + AQ = 2AQ = 2 \times 5 = 10$ cm [½ Mark]

$\therefore$ Perimeter of $\Delta ABC = 10$ cm.

**OR**

**To prove:** $\frac{AE}{DF} = \frac{AO}{DO}$

**Construction :** Draw $AE \perp BC$ and $DF \perp BC$.




[1 Mark]

**Proof :**

In $\Delta AOE$ and $\Delta DOF$,

$\angle AOE = \angle DOF$ (Vertically opposite angles)

$\angle AEO = \angle DFO = 90°$ (Construction)

$\Rightarrow \Delta AOE \sim \Delta DOF$ (By AA Similarity) [1 Mark]

$\therefore \frac{AO}{DO} = \frac{AE}{DF}$ ...(i) [1 Mark]

**31.** Total number of outcomes on throwing a pair of dice = $6 \times 6 = 36$

(i) Let E be the event of getting a prime number on each die.

So, favourable outcomes

= {(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5)}

Number of favourable outcomes = 9

Then, $P(E) = \frac{9}{36} = \frac{1}{4}$

Therefore, the probability of getting a prime number on each dice is $\frac{1}{4}$. [1½ Marks]

(ii) Let F be the event of getting a total of 9 or 11.

So, favourable outcomes

= {(3, 6), (4, 5), (5, 4), (6, 3), (5, 6), (6, 5)}.

Number of favourable outcomes = 6

Thus, $P(F) = \frac{6}{36} = \frac{1}{6}$

Hence, the probability of getting a total of 9 or 11 is $\frac{1}{6}$.

[1½ Marks]

**OR**

The possible outcomes of tossing three coins together :

{HHH, HHT, HTH, THH, HTT, THT, TTH ,TTT}

$\Rightarrow$ Total number of outcomes = 8

(i) Outcomes of getting exactly two heads

= {HHT, HTH and THH}

Favourable number of outcomes = 3

Probability (getting exactly two heads) $= \frac{3}{8}$

[1 Mark]

(ii) Outcomes of getting at least two heads

= {HHH, HHT, HTH and THH}

Favourable number of outcomes = 4

Probability (getting atleast two heads) $= \frac{4}{8} = \frac{1}{2}$

[1 Mark]

(iii) Outcomes of getting at least two tails

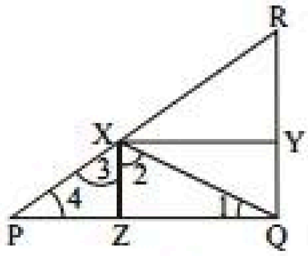
= {HTT, THT, TTH and TTT}

Favourable number of outcomes = 4

Probability (getting atleast two tails) $= \frac{4}{8} = \frac{1}{2}$

[1 Mark]

**32.**




$RQ \perp PQ, XZ \perp PQ$

$\Rightarrow XZ \parallel YQ$

$\therefore XY \parallel ZQ$ ...(i)

$XYQZ$ is a rectangle. [1½ Marks]

In $\Delta XZQ$, $\angle 1 + \angle 2 = 90°$ ...(ii)

In $\Delta PZX$, $\angle 3 + \angle 4 = 90°$ ...(iii)

$XQ \perp PR \Rightarrow \angle 2 + \angle 3 = 90°$

From eqs. (i) and (iii)

$\angle 1 = \angle 3$ [1½ Marks]

From eqs. (ii) and (iii)

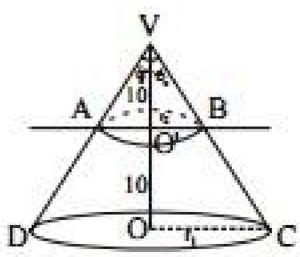
$\angle 2 = \angle 4$

$\Delta PZX \sim \Delta XZQ$ (*AA* similarity)

$$\frac{PZ}{XZ} = \frac{XZ}{ZQ}$$

$\Rightarrow XZ^2 = PZ \times ZQ$ [2 Marks]

**33.**




In $\Delta VO'A$ and $\Delta VOC$ [1 Mark]

$$\tan 30° = \frac{OC}{VO} \text{ and } \tan 30° = \frac{O'V}{VO'}$$

$$\frac{1}{\sqrt{3}} = \frac{r_1}{20} \text{ and } \frac{1}{\sqrt{3}} = \frac{r_2}{10}$$

$$r_1 = \frac{20}{\sqrt{3}} \text{ and } r_2 = \frac{10}{\sqrt{3}}$$ [2 Marks]

Volume of small cone

$$= \pi r_1 h = \frac{22}{7} \times \frac{10}{\sqrt{3}} \times 10$$

$$= \frac{2200}{7\sqrt{3}}$$ [2 Marks]

**OR**

Height of cylinder = 2.8 cm $= \frac{14}{5}$ cm

Radius of cylinder = 2.1 cm $= \frac{21}{10}$ cm

$l = \sqrt{(2.8)^2 + (2.1)^2} = 3.5$cm [2 Marks]

According to question a conical cavity of same height and diameter is hollowed out.

Now, T.S.A of remaining solid = C.S.A of cone + C.S.A of cylinder + Area of base of cylinder

$= 2\pi r l + 2\pi r h + \pi r^2$ [1 Mark]

$= \pi r [l + 2h + r]$

$$= \frac{22}{7} \times \frac{21}{10}\left[\frac{7}{2} + \frac{28}{5} + \frac{21}{10}\right] = \frac{22}{10} \times 3\left[\frac{35 + 56 + 21}{10}\right]$$

$$= \frac{22}{10} \times 3 \times \frac{112}{10} = 73.92\text{cm}^2$$ [2 Marks]

**34.** Given equation is $36x^2 - 12ax + (a^2 - b^2) = 0$

$D = (-12a)^2 - 4(36)(a^2 - b^2)$ [$\because D = b^2 - 4ac$] [1 Mark]

$= 144a^2 - 144(a^2 - b^2) = 144b^2$ [1 Mark]

Now, $= \frac{12a \pm 12b}{72} = \frac{a \pm b}{6}$ [2 Marks]

Hence, $c = 6$ [1 Mark]

**OR**

The given equation is $x^{2/3} + x^{1/3} - 2 = 0$

Put $x^{1/3} = y$, then $y^2 + y - 2 = 0$

$\Rightarrow y^2 + 2y - y - 2 = 0$ [1 Mark]

$\Rightarrow y(y+2) - 1(y+2) = 0 \Rightarrow (y-1)(y+2) = 0$

$\Rightarrow y = 1$ or $y = -2$ [1 Mark]

$\Rightarrow x^{1/3} = 1$ or $x^{1/3} = -2$ [1 Mark]

$\therefore x = (1)^3$ or $x = (-2)^3 = -8$ [1 Mark]

Hence, the real roots of the given equations are 1, –8. [1 Mark]

**35.**

| Height | Frequency | *c.f.* |
|---|---|---|
| 100–120 | 12 | 12 |
| 120–140 | 14 | 26 |
| 140–160 | 8 | 34 |
| 160–180 | 6 | 40 |
| 180–200 | 10 | 50 |
| Total | 50 | |

[1 Mark]

Here, $N = 50 \Rightarrow \frac{N}{2} = \frac{50}{2} = 25$ [1 Mark]

So, Median Class = 120 – 140

Here, $l = 120$, $h = 20$, $c.f. = 12$, $f = 14$ [1 Mark]

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h = 120 + \left(\frac{25 - 12}{14}\right) \times 20$$

$$= 120 + \frac{260}{14} = 120 + 18.57$$

Median = 138.57 [2 Marks]

**36.** **(i)** Given that

$a_6 = a + 5d = 16000$ .....(i)

$a_9 = a + 8d = 22600$ .....(ii)

$-3d = -6600 \Rightarrow d = 2200$

$\Rightarrow a = 5000$

$\therefore$ Production during first year = 5000 [1 Mark]

**(i)** Production during $8^{th}$ year is $(a + 7d)$

$= 5000 + 2(2200) = 20400$ [1 Mark]

**(ii)** Production during first 3 year

$= 5000 + 7200 + 9400 = 21600$ [2 Marks]

**OR**

$5000 + (n-1)2200 = 29200 \Rightarrow n = 12^{th}$ year [2 Marks]

**37.** $BD = 5 - 1.3 = 3.7m$

**(i)** $\sin C = \frac{BD}{BC} = \frac{3.7}{7.4} = \frac{1}{2}$

$\Rightarrow \sin C = \sin 30° \angle \angle C = 30°$ [1 Mark]

**(ii)** $\angle D + B + \angle C = 180°$

$\Rightarrow 90° + \angle B + 30° = 180°$

$\angle B = 60°$ [1 Mark]

**(iii)** $\tan 30° = \frac{BD}{CD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3.7}{CD}$

$\Rightarrow CD = 3.7\sqrt{3}m$ [2 Marks]

**OR**

$\sin^2 B + \sin^2 C = \sin^2 60° + \sin^2 30°$

$= \frac{3}{4} + \frac{1}{4} = 1$ [2 Marks]

**38.** **(i)** Centroid of $\Delta EHJ$ with E(2, 1), H(–2, 4) & J(–2, –2) is

Coordinates of centroid are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$\left(\frac{2 + (-2) + (-2)}{3}, \frac{1 + 4 + (-2)}{3}\right) = \left(\frac{-2}{3}, 1\right)$$ [1 Mark]

**(ii)** If P needs to be at equal distance from A(3, 6) and G(1, –3), such that A, P and G are collinear, then P will be the mid-point of AG.

So coordinates of P will be $\left(\frac{3+1}{2}, \frac{6+(-3)}{2}\right) = \left(2, \frac{3}{2}\right)$

[1 Mark]

**(iii)** Let the point on x-axis equidistant from I(–1, 1) and E(2, 1) be (x, 0) then $\sqrt{(x+1)^2 + (0-1)^2}$

$= \sqrt{(x-2)^2 + (0-1)^2}$

$x^2 + 1 + 2x + 1 = x^2 + 4 - 4x + 1$

$6x = 3$

So $x = \frac{1}{2}$

$\therefore$ the required point is $\left(\frac{1}{2}, 0\right)$ [2 Marks]

**OR**

Let the coordinates of the position of a player Q such that his distance from K(–4,1) is twice his distance from E(2,1) be Q(x, y)

Then KQ : QE = 2 : 1

$Q(x, y) = \left(\frac{2 \times 2 + 1 \times (-4)}{3}, \frac{2 \times 1 + 1 \times 1}{3}\right) = (0, 1)$ [2 Marks]