

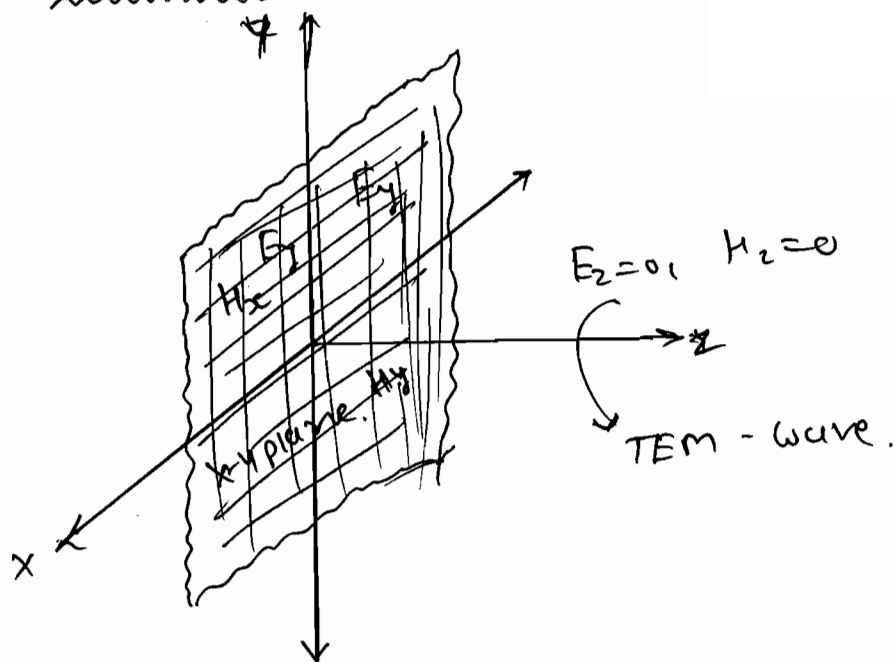
→ A constant can't be called as wave.

∴ We Choose $E_z = 0$

→ When wave is propagating in z direction the wave do not have any field component along the direction of propagation i.e along z-direction.

→ The possible non-zero components are E_x, E_y, H_x, H_y .

* Transverse Plane:-



→ In general,

$$\vec{E} = E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z$$

$$\vec{H} = H_x \hat{a}_x + H_y \hat{a}_y + H_z \hat{a}_z$$

⇒ Transverse plane:

→ It is that plane which exists normal to the direction of propagation.

→ When a wave propagating along z-direction the transverse plane would be xy-plane.

→ When wave is propagating along z-direction the possible non-zero field components are $E_x, E_y, H_x, & H_y$ lies in the xy-plane. i.e. in transverse plane.

→ The possible field components are lie in transverse plane and hence this wave is called transverse electromagnetic wave (or) TEM waves.

$$\therefore \frac{\partial^2}{\partial z^2} (E_x \hat{a}_x + E_y \hat{a}_y) = \mu \epsilon \frac{\partial^2}{\partial t^2} [E_x \hat{a}_x + E_y \hat{a}_y]$$

Compare $\hat{a}_x \Rightarrow \frac{\partial^2 E_x}{\partial z^2} = \mu \epsilon \frac{\partial^2 E_x}{\partial t^2}$ → Scalar wave eqs 2nd order

$$\hat{a}_y \Rightarrow \frac{\partial^2 E_y}{\partial z^2} = \mu \epsilon \frac{\partial^2 E_y}{\partial t^2}$$

Similarly,

$$\frac{\partial^2 H_x}{\partial z^2} = \mu \epsilon \frac{\partial^2 H_x}{\partial t^2}$$

$$\frac{\partial^2 H_y}{\partial z^2} = \mu \epsilon \frac{\partial^2 H_y}{\partial t^2}$$

→ Soln of these eqs are also said to be uniform plane waves.

→ Let us consider one of the equations.

$$\frac{\partial^2 E_y}{\partial z^2} = \mu \epsilon \frac{\partial^2 E_y}{\partial t^2}$$

→ By using a method of variable separable the function involved above can be thought of representing as a product of two independent functions:

$$E_y(z, t) \Rightarrow F_1(z) \cdot \underbrace{F_2(t)}_{e^{j\omega t}}$$

$F_1 \rightarrow$ Function of z -alone.

$F_2 \rightarrow$ Function of t -alone.

$$\rightarrow E_y(z, t) = \operatorname{Re} [E_y(z) \cdot e^{j\omega t}]$$

→ Thus, we approximate the time variation is $e^{j\omega t}$,

$$\begin{aligned} \therefore \frac{\partial^2}{\partial z^2} [\operatorname{Re} [E_y(z) \cdot e^{j\omega t}]] &= \mu \epsilon \frac{\partial^2}{\partial t^2} \{ \operatorname{Re} (E_y e^{j\omega t}) \} \\ &= + j^2 \omega^2 \mu \epsilon \{ \operatorname{Re} [E_y e^{j\omega t}] \} \\ &= -\omega^2 \mu \epsilon \{ \operatorname{Re} [E_y e^{j\omega t}] \} \end{aligned}$$

Suppress the time variation on both the sides.

$$\boxed{\frac{d^2 E_y}{dz^2} + \omega^2 \mu \epsilon E_y = 0}$$

This is a 2nd order simple D.E.

→ The above eqⁿ may be also called as harmonic eqⁿ.

→ Solⁿ of harmonic eqⁿ may take either sine (or) cosine (or) exponent form. We consider exponent form.

$$\therefore \frac{d^2 E_{ys}}{dz^2} + \beta^2 E_{ys} = 0. \quad \beta^2 = \omega^2 \mu \epsilon.$$

$$\beta = \omega \sqrt{\mu \epsilon}.$$

$$\therefore m^2 + \beta^2 = 0$$

$$\therefore m = \pm j\beta.$$

$$\therefore E_{ys} = C_1 e^{j\beta z} + C_2 e^{-j\beta z}.$$

$$\therefore E_y(z, t) = \text{Re} \left[C_1 e^{-j\omega z} \cdot e^{j\omega t} + C_2 e^{j\beta z} \cdot e^{j\omega t} \right].$$

$$= \text{Re} \left[\underbrace{C_1 e^{j(\omega t - \beta z)}}_{\text{Term - (1)}} + \underbrace{C_2 e^{j(\omega t + \beta z)}}_{\text{Term - (2)}} \right].$$

Term - (1)
→ (+z)

Term - (2)
→ (-z).

⇒ Term - (1)

$$\boxed{C_1} \cdot e^{j(\omega t - \beta z)} \rightarrow \text{Phase.}$$

Amplitude

$$\therefore \underbrace{\omega t^2 - \beta z^2}_{(+z)} = \text{const.}$$

$$\therefore \omega dt - \beta dz = 0.$$

$$\therefore \frac{dz}{dt} = \omega / \beta.$$

$$\therefore \boxed{v_0 = \omega / \beta \text{ m/s.}}$$

⇒ Term-②

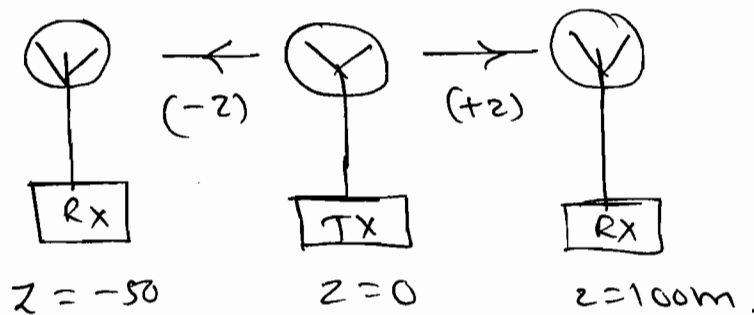
$$C_2 \cdot e^{j(\omega t + \beta z)} \rightarrow \text{phase.}$$

Amplitude $\uparrow$
 $\therefore \omega t + \beta z = \text{Const.}$
 $\downarrow$
 $\longrightarrow (-z).$

$\therefore \omega dt + \beta dz = 0.$

$\therefore \frac{dz}{dt} = -\omega/\beta.$

$\therefore \boxed{V_0 = -\omega/\beta \text{ m/s.}}$



Term-① in its mathematical form indicates that the wave is propagating along +ve z-direction only where as Term-② in its mathematical form indicates that the wave is propagating along -ve z-direction.

→ For the wave which are propagating along the +ve z direction we consider term ① only and we ignored term ②.

→ For the wave which are propagating along

-ve z direction we considered term ② only and we ignored term -①.

→ The velocity of wave is $= \frac{\omega}{\beta}$.

* Intrinsic Impedance: (η)

→ Unit is Ω .

- It is ratio of E to H.
- It will depend upon medium properties.

→ In a linear homogeneous isotropic nonconducting medium this is given by,

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\text{In free space } \eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{\frac{10^9}{36\pi}}}$$

$$\therefore \eta = \sqrt{4 \times 36 \times \pi^2 \times 100}$$

$$\therefore \eta = 2 \times 6 \times \pi \times 10$$
$$\boxed{\eta = 120\pi} \Omega$$

→ $\frac{E_z=0, H_z=0}{\partial/\partial x=0, \partial/\partial y=0} \rightarrow (+z)$

$$\therefore \boxed{\frac{E_x}{E_y} = \eta = -\frac{E_y}{E_x}}$$

- Impedance is not -ve, the ratio may be -ve.

$$\vec{E} = E_x \hat{a}_x + E_y \hat{a}_y + \boxed{E_z} \hat{a}_z$$

$$\vec{H} = H_x \hat{a}_x + H_y \hat{a}_y + \textcircled{H_z} \hat{a}_z \rightarrow 0$$

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2} = \eta \sqrt{H_x^2 + H_y^2}$$

$$\therefore |\vec{H}| = \sqrt{H_x^2 + H_y^2}$$

$$\therefore \left| \frac{\vec{E}}{\vec{H}} \right| = \frac{\eta \sqrt{H_y^2 + H_x^2}}{\sqrt{H_x^2 + H_y^2}}$$

$$\therefore \left| \frac{\vec{E}}{\vec{H}} \right| = \eta$$

$$\rightarrow \vec{E} \cdot \hat{a}_z = 0$$

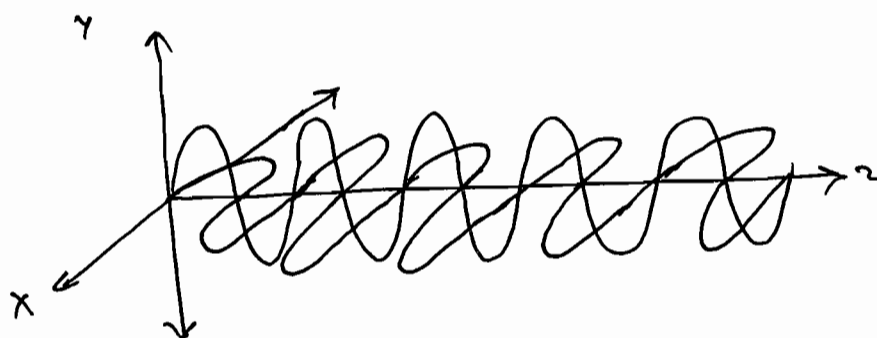
$$\vec{H} \cdot \hat{a}_z = 0$$

$$\vec{E} \cdot \vec{H} = E_x H_x + E_y H_y$$

$$= \eta H_y H_x - \eta H_x H_y$$

$$\vec{E} \cdot \vec{H} = 0$$

→ In a wave propagation, $\vec{E}$ & $\vec{H}$ and the vector correspond to direction of propagation are mutually orthogonal to each other.



①

$$\frac{\partial}{\partial y}()=0, \frac{\partial}{\partial z}=0$$

$$E_x=0, H_x=0 \rightarrow (+x)$$

$$\frac{E_y}{H_z} = \eta = -\frac{E_z}{H_y}$$

$$\frac{\partial}{\partial y}()=0, \frac{\partial}{\partial z}=0$$

$$E_x=0, H_x=0 \rightarrow (-x)$$

$$\frac{E_y}{H_z} = -\eta = -\frac{E_z}{H_y}$$

②

$$\frac{\partial}{\partial x}=0, \frac{\partial}{\partial z}=0$$

$$H_y=0, E_y=0 \rightarrow (+y)$$

$$\frac{E_z}{H_x} = \eta = -\frac{E_x}{H_z}$$

$$\frac{\partial}{\partial x}=0, \frac{\partial}{\partial z}=0$$

$$H_y=0, E_y=0 \rightarrow (-y)$$

$$\frac{E_z}{H_x} = -\eta = -\frac{E_x}{H_z}$$

③

$$\frac{\partial}{\partial x}=0, \frac{\partial}{\partial y}=0$$

$$H_z=0, E_z=0 \rightarrow (+z)$$

$$\frac{E_x}{H_y} = -\eta = \frac{E_y}{H_x}$$

$$\frac{\partial}{\partial x}=0, \frac{\partial}{\partial y}=0$$

$$H_z=0, E_z=0 \rightarrow (-z)$$

$$\frac{E_x}{H_y} = -\eta = -\frac{E_y}{H_x}$$

$$* \beta = \omega \sqrt{\mu \epsilon} \quad \left| \quad v_0 = \frac{1}{\sqrt{\mu \epsilon}} \right.$$

$$\beta = \frac{\omega}{v_0}$$

→ If 'f' is the freq of the wave and 'λ' is the corresponding wavelength, such that

$$f\lambda = v_0$$

$$\therefore \beta = \frac{2\pi f}{f\lambda} = \frac{2\pi}{\lambda} \text{ rad/m}$$

'β' is phase shift constant

$$'λ' \rightarrow '2\pi' \text{ rad.}$$

$$\beta = \frac{2\pi}{\lambda} \text{ rad/m.}$$

$$\underline{E}_x = \text{In air } \underline{E} = 50 \cos(10^8 t - \beta x) \hat{a}_y \text{ V/m}$$

Where β is a +ve real. Find $\underline{H}$, β , direction of propagation and also represents field in phasor form.

Ans: In air $\Rightarrow \mu_0 \neq \epsilon_0$. $\omega = 10^8 \text{ rad/s}$

$$\therefore \eta = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$\beta = \omega \sqrt{\mu_0 \epsilon_0} = 10^8 / 3 \times 10^8$$

$$\therefore \boxed{\eta = 120\pi}$$

$$\boxed{\beta = 1/3}$$

$$\therefore \underline{E} = 50 \cos(10^8 t - \beta x) \hat{a}_y$$

$\downarrow$
x-direction of propagation.

$$\frac{E_y}{H_z} = +\eta = -\frac{E_z}{H_y}$$

because $(\hat{a}_y)$.

$$\therefore \underline{H} = \frac{50}{\eta} \cos(10^8 t - \beta x) \hat{a}_z \rightarrow \because \underline{E} \text{ is in } \hat{a}_y$$

$$\therefore \boxed{\underline{H} = \frac{50}{120\pi} \cos(10^8 t - \beta x) \hat{a}_z}$$

$$\therefore \underline{E}_s = 50 e^{-j\beta x} \hat{a}_y$$

$$\underline{H}_s = \frac{50}{\eta} e^{-j\beta x} \hat{a}_z$$

$$\therefore \underline{E}_s = 50 e^{-jx/3} \hat{a}_y$$

$$\underline{H}_s = \frac{50}{120\pi} e^{-j\frac{x}{3}} \hat{a}_z$$

$$\left. \begin{aligned} \underline{E} \cdot \hat{a}_x &= 0 \\ \underline{H} \cdot \hat{a}_x &= 0 \\ \underline{E} \cdot \underline{H} &= 0 \end{aligned} \right\} \text{All are verified.}$$

Ex-2 In a non-magnetic medium. Let

$\vec{H} = 10 \cos(\omega t + 0.5z) \hat{a}_y$ Alm. Assume relative permittivity of medium is 4.0 find intrinsic impedance of the medium, freq. at which wave is propagating, β , direction propagation and also write the expression for $\vec{E}$.

Ans:

Non magnetic medium =

$$\mu = \mu_0, \quad \epsilon = \epsilon_0 \epsilon_r.$$

$$\therefore \eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}} = \sqrt{\frac{\mu_0}{4\epsilon_0}} = \frac{120\pi}{2}$$

$\eta = 60\pi \, \Omega$

$$\therefore \beta = 0.5$$

$$\therefore \beta = \omega \sqrt{\epsilon \mu}.$$

$$\therefore \beta = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_r}$$

$$\omega = \frac{\beta}{\sqrt{\epsilon_r} \cdot \sqrt{\mu_0 \epsilon_0}} = \frac{0.5 \times c}{\sqrt{\epsilon_r}}.$$

$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \frac{\text{Coul}}{\text{m/s}}$

$$\therefore \omega = \frac{0.5 \times 10^8 \times 3 \times 10^8}{2}$$

$$\therefore f = \frac{0.75 \times 10^8}{2\pi} \text{ Hz}$$

$$\therefore v_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}} = \frac{3 \times 10^8}{\sqrt{4}}$$

$v_0 = 1.5 \times 10^8 \text{ m/s.}$

$$\frac{E_x}{H_y} = -\eta = -\frac{E_y}{H_x}$$

$$\therefore \vec{E} = -10\eta \cos(\omega t + 0.5z) \hat{a}_{z0}$$

$$\left. \begin{aligned} \vec{E} \cdot \vec{H} &= 0 \\ \vec{E} \cdot (-\hat{a}_z) &= 0 \\ \vec{H} \cdot (-\hat{a}_z) &= 0 \end{aligned} \right\} \text{All are verified.}$$

Ex-3 In a Lossless medium the intrinsic impedance is 250Ω and given that

$$\vec{E} = 20 \cos(\omega t - \beta y) \hat{a}_x + 30 \sin(\omega t - \beta z) \hat{a}_z \text{ V/m.}$$

β is a positive real. Find direction of propagation and Expression for magnetic field intensity.

$$\text{Ans: } \vec{E} = \underbrace{20 \cos(\omega t - \beta y)}_{E_x} \underbrace{\hat{a}_x}_{+y} + \underbrace{30 \sin(\omega t - \beta z)}_{E_z} \hat{a}_z$$

$$\therefore \longrightarrow +y$$

$$\frac{E_z}{H_x} = +\eta = -\frac{E_x}{H_z}$$

$$\therefore H_x = \frac{E_z}{\eta}, \quad H_z = -\frac{E_x}{\eta}$$

$$\therefore \vec{H} = \frac{30 \sin(\omega t - \beta z)}{\eta} \hat{a}_x - \frac{20}{\eta} \cos(\omega t - \beta y) \hat{a}_z$$

→ In the above problem write the Electric field quantity in the phasor form.

$$\vec{E} = 20 \cos(\omega t - \beta y) \hat{a}_x + 30 \cos(\omega t - \beta y - \frac{\pi}{2}) \hat{a}_z \text{ V/m.}$$

$$\therefore \vec{E} = \text{Re} \left[20 e^{j(\omega t - \beta y)} \cdot \hat{a}_x + 30 e^{j(\omega t - \beta y - \frac{\pi}{2})} \cdot \hat{a}_z \right].$$

$$\therefore \vec{E}_s = 20 e^{-j\beta y} \hat{a}_x + 30 e^{-j(\beta y + \frac{\pi}{2})} \hat{a}_z.$$

Ex-3 Let, $\vec{E} = 30 \sin(10^8 t - \frac{\pi}{2}) \hat{a}_{yz}$ V/m. Find the direction of propagation and also find the expression for $\vec{H}$.

Ans: The given $\vec{E}$ doesn't represent a valid EM wave.

* Wave Propagation in a Conducting unbounded medium (or) Lossy Charge free medium.

→ Writing the Maxwell's eq's for the above assumed medium.

→ Inside the conductor the charge is zero. and also we are assuming charge free medium i.e. $\rho_v = 0$. indicates $\nabla \cdot \bar{E} = 0$.

$$\textcircled{1} \quad \nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}$$

$$\textcircled{2} \quad \nabla \times \bar{H} = \sigma \bar{E} + \epsilon \frac{\partial \bar{E}}{\partial t}$$

$$\textcircled{3} \quad \nabla \cdot \bar{D} = 0$$

$$\nabla \cdot \bar{E} = 0$$

$$\textcircled{4} \quad \nabla \cdot \bar{H} = 0$$

$$\nabla \cdot \bar{B} = 0$$

→ Taking curl on - ① both sides.

$$\nabla \times \nabla \times \bar{E} = -\mu \nabla \times \frac{\partial \bar{H}}{\partial t}$$

$$\therefore \nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \bar{H})$$

$$\therefore 0 - \nabla^2 \bar{E} = -\mu \sigma \frac{\partial \bar{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\therefore \boxed{\begin{aligned} \nabla^2 \bar{E} - \mu \sigma \frac{\partial \bar{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2} &= 0 \\ \nabla^2 \bar{H} - \mu \sigma \frac{\partial \bar{H}}{\partial t} - \mu \epsilon \frac{\partial^2 \bar{H}}{\partial t^2} &= 0 \end{aligned}}$$

Vector
Wave eq's.

very

→ w.r.t. the above in phasor form:

$$\nabla^2 \bar{E}_s - j\omega\mu\sigma \bar{E}_s - (j\omega)^2 \mu\epsilon \bar{E}_s = 0.$$

$$\therefore \cancel{\nabla^2 \bar{E}_s} \quad \boxed{\nabla^2 \bar{E}_s - \gamma^2 \bar{E}_s = 0.}$$

Similarly,

$$\boxed{\nabla^2 \bar{H}_s - \gamma^2 \bar{H}_s = 0.}$$

Wave eqⁿ in phasor form.

where, $\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon).$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\therefore \gamma = \alpha + j\beta.$$

γ : propagation constant.

α : Attenuation constant

(Np/m).

When base e
the neper will
come
in unit.

β : phase shift constant (rad/m).

$$* (\alpha + j\beta)^2 = j\omega\mu(\sigma + j\omega\epsilon).$$

$$\alpha^2 - \beta^2 = -\omega^2\mu\epsilon \quad (\text{real parts}) \quad \text{--- (A)}$$

$$2\alpha\beta = \omega\mu\sigma \quad (\text{Im. parts})$$

$$(\alpha^2 + \beta^2)^2 = (\alpha^2 - \beta^2)^2 + (2\alpha\beta)^2.$$

$$\therefore \alpha^2 + \beta^2 = \sqrt{\omega^4\mu^2\epsilon^2 + \omega^2\mu^2\sigma^2} \quad \text{--- (B)}$$

Using (A) and (B) find α' and β' .

Add A & B. $A + B.$

$$\therefore 2\alpha^2 = \omega^2 \mu^2 \sqrt{\omega^2 \epsilon^2 + \epsilon \frac{\sigma^2}{\omega^2}} - \omega^2 \mu \epsilon.$$

$$\therefore 2\alpha^2 = \omega^2 \mu \epsilon \left[\sqrt{\omega^2 \epsilon^2 + \sigma^2} - \omega \epsilon \right].$$

$$\therefore \alpha = \omega \sqrt{\frac{\mu \epsilon}{2} \left[\sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon^2}} - 1 \right]}$$

$$\therefore \beta = \omega \sqrt{\frac{\mu \epsilon}{2} \left[\sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon^2}} + 1 \right]}$$

$$\boxed{\begin{aligned} \alpha &= \omega \sqrt{\frac{\mu \epsilon}{2} \left[\sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon^2}} - 1 \right]} \\ \beta &= \omega \sqrt{\frac{\mu \epsilon}{2} \left[\sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon^2}} + 1 \right]} \end{aligned}}$$

* Expanding wave eqns in Cartesian coordinates.

→ For the simplicity let us assume that the wave is propagating along z-direction in the unbounded medium. Since there are ^{no} boundaries to meet along x & y directions.

We can conclude that partial variations of any field component with respect to x & y vanishing.

→ From $\nabla \cdot \vec{E} = 0$ and $\nabla \cdot \vec{H} = 0$ we can also show that E_z and H_z are zero. The vector wave eqn reduced to.

$$\left. \begin{aligned} \frac{\partial^2 \bar{E}_s}{\partial x^2} + \frac{\partial^2 \bar{E}_s}{\partial y^2} + \frac{\partial^2 \bar{E}_s}{\partial z^2} - \gamma^2 \bar{E}_s &= 0 \\ \frac{\partial^2 \bar{H}_s}{\partial x^2} + \frac{\partial^2 \bar{H}_s}{\partial y^2} + \frac{\partial^2 \bar{H}_s}{\partial z^2} - \gamma^2 \bar{H}_s &= 0 \end{aligned} \right\} \begin{array}{l} \text{These are} \\ \text{2nd order} \\ \text{3-dim} \\ \text{POE.} \end{array}$$

$$\therefore \frac{\partial/\partial x (1), \partial/\partial y (1) \rightarrow (2)}$$

$$\therefore \frac{\partial^2 \bar{E}}{\partial z^2} - \gamma^2 \bar{E} = 0.$$

$$\frac{d^2 \bar{H}}{dz^2} - \gamma^2 \bar{H} = 0$$

In general,

$$\bar{E} = E_x \hat{a}_x + E_y \hat{a}_y + \overset{=0}{E_z \hat{a}_z}$$

$$\bar{H} = H_x \hat{a}_x + H_y \hat{a}_y + \underset{\substack{|| \\ 0}}{H_z \hat{a}_z}$$

$$\rightarrow \left. \begin{aligned} \frac{d^2 E_{xs}}{dx^2} - \gamma^2 E_{xs} &= 0. \\ \frac{d^2 E_{ys}}{dy^2} - \gamma^2 E_{ys} &= 0. \\ \frac{d^2 H_{xs}}{dx^2} - \gamma^2 H_{xs} &= 0. \\ \frac{d^2 H_{ys}}{dy^2} - \gamma^2 H_{ys} &= 0. \end{aligned} \right\} \begin{array}{l} \text{2nd} \\ \text{order simple} \\ \text{ODEs} \end{array}$$

→ The above eqn is in the form of harmonic eqn. Soln of a harmonic eqn may take either sine or cosine or exponent.

Let, us Consider one of the eqs.

$$\therefore \frac{d^2 E_{ys}}{dz^2} - \gamma^2 E_{ys} = 0.$$

$$\therefore m^2 - \gamma^2 = 0$$

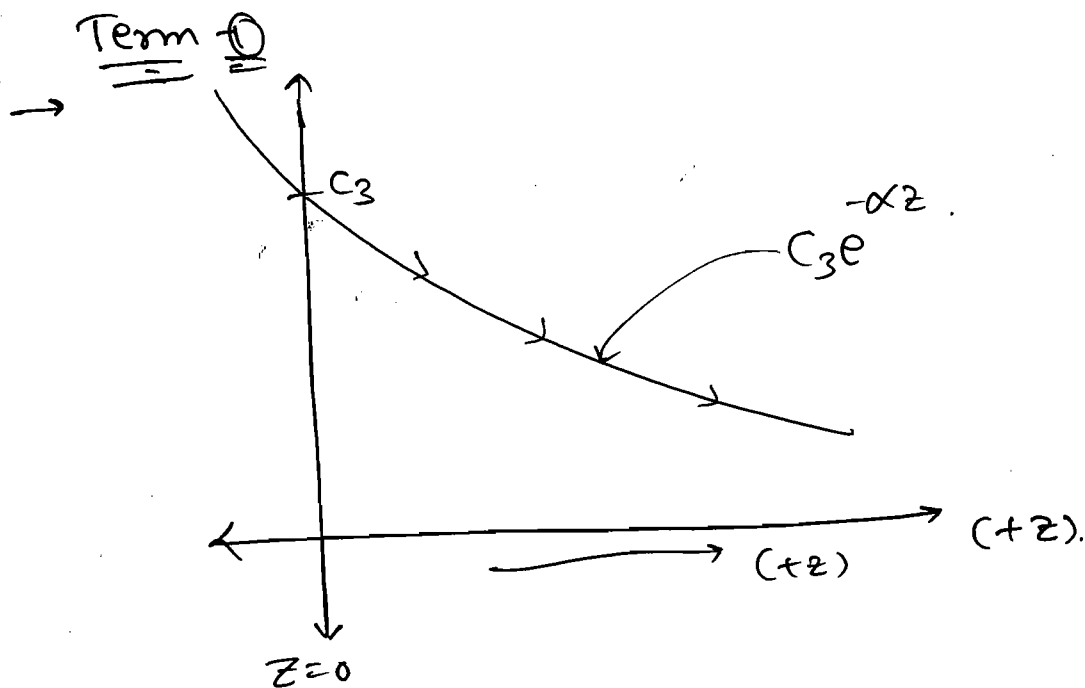
$$m = \pm \gamma.$$

$$\therefore E_{ys} = C_3 e^{-\gamma z} + C_4 e^{+\gamma z}.$$

$$\therefore E_y(z, t) = \text{Re} [E_{ys} e^{j\omega t}].$$

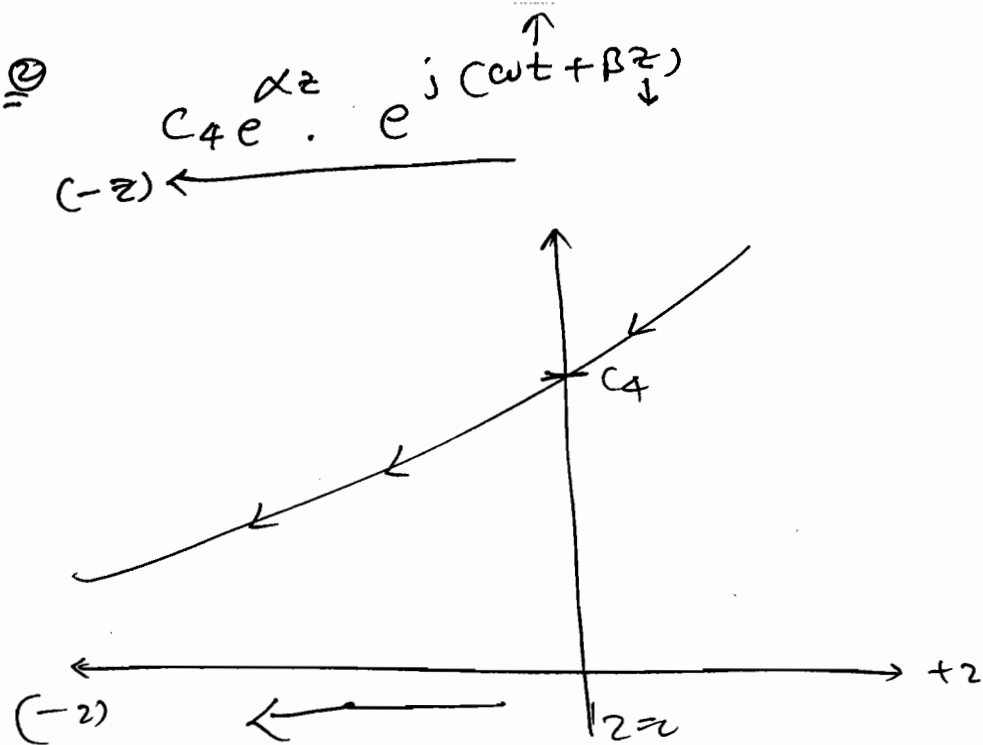
$$= \text{Re} [C_3 e^{-\gamma z} \cdot e^{j\omega t} + C_4 \cdot e^{+\gamma z} \cdot e^{j\omega t}].$$

$$\therefore E_y(z, t) = \text{Re} \left[\underbrace{C_3 e^{-\alpha z} \cdot e^{j(\omega t - \beta z)}}_{\text{term-①}} + \underbrace{C_4 \cdot e^{\alpha z} \cdot e^{j(\omega t + \beta z)}}_{\text{term-②}} \right].$$



→ $\boxed{C_4 e^{-\alpha z}} \cdot e^{j(\omega t - \beta z)}$

Term - ②



- Term - ① In its mathematical form indicates that wave is propagating along +z direction. While it is progressing its amplitude is decreasing exponentially.
- Term - ② in its mathematical form indicates that the wave is propagating along -ve z-direction. While it is propagating its amplitude is decreasing.
- For the waves which are progressing in along +ve z direction we consider term - ① only and ignoring term - ②
- For the waves which are progressing along -ve z-direction we consider term - ② only.

→ If the medium is perfect conductor,
 in perfect conductor $\sigma = \infty$ therefore, $\alpha = \infty$.
 and hence we conclude that the wave
 propagation through a perfect conductor is
 impossible. We can also conclude that
~~wave propagation~~ conducting medium is
 a lossy medium.

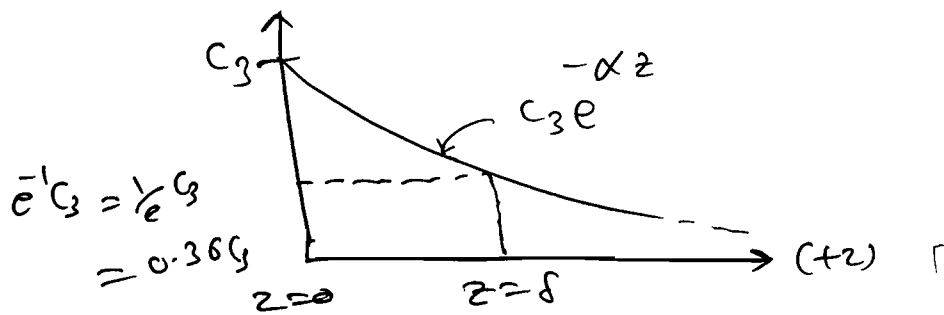
* Skin Depth ' δ '

→ It is also called depth of penetration.

→ It is that depth where the signal
 Amplitude becomes 36% of its original
 Value (or) $\frac{1}{e}$ times of its original value.

→ Let us assume that wave is propagating
 along +ve z -direction and further
 we assume that amplitude variation is
 given by $C_3 e^{-\alpha z}$.

→ Further we assume that medium starts
 from $z=0$.



$$C_3 e^{-\alpha z} = C_3 e^{-\alpha s} = C_3 e^{-1} = C_3 / e.$$

$$\therefore \text{ (or) } \alpha s = 1.$$

$$\therefore \boxed{s = 1/\alpha \text{ m}}$$

Skin depth is the reciprocal of attenuation constant.

→ Skin depth in a perfect conductor is zero. because wave can not penetrate into the perfect conductor.

→ Skin depth in a lossless medium is ∞ .

* Impedance of the medium:

→ It is the ratio of E to H .

→ It will solely depends upon medium properties.

$$\boxed{\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = |\eta| \angle \theta \text{ } \Omega.}$$

$|\eta|$ is the magnitude of η and θ is the phase angle.

$$\vec{J}_c = \sigma \vec{E} \quad \vec{J}_o = \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\vec{J}_{cs} = \sigma \vec{E}_s \quad \text{or } \vec{J}_{os} = j\omega\epsilon \vec{E}_s$$

$$\frac{\vec{J}_{cs}}{\vec{J}_{os}} = \frac{\sigma}{j\omega\epsilon} = \frac{\sigma}{\omega\epsilon} \angle -90^\circ$$

→ $\bar{\sigma}$ is larger $\bar{\sigma}$ is good.

→ $\frac{\sigma}{\omega\epsilon}$ is a bench mark to decide whether the given medium is a conductor (or) dielectric as follows:

If $\frac{\sigma}{\omega\epsilon} \gg 1 \Rightarrow$ Good Conductor
 $= \infty \Rightarrow$ perfect conductor
 $= 0 \Rightarrow$ perfect dielectric
 $\ll 1 \Rightarrow$ Good dielectric.

* In good conductor $\frac{\sigma}{\omega\epsilon} \gg 1$.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma(1 + j\frac{\omega\epsilon}{\sigma})}} \approx \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

(i.e) In a good conductor $\vec{E}$ leads $\vec{H}$ by 45° .

$$\begin{aligned} \sqrt{j\eta} &= \sqrt{\eta} \angle 45^\circ \\ &= \sqrt{\eta} e^{j\pi/4} \\ &= \sqrt{\eta} \cdot \sqrt{e^{j\pi/2}} \\ &= \sqrt{\eta} \cdot \angle 45^\circ \end{aligned}$$

Similarly $\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} \text{ Np/m.}$
 $\beta = \sqrt{\frac{\omega\mu\sigma}{2}} \text{ rad/m.}$

$\Rightarrow$ Skin depth

$$\therefore \delta = \sqrt{\frac{2}{\omega\mu\sigma}} \text{ m.}$$

$\frac{\sigma}{\omega\epsilon}$ is also called dissipation factor (or) loss tangent.

$$\phi = \tan^{-1} \left(\frac{\sigma}{\omega \epsilon} \right).$$

$$\tan \phi = \frac{\sigma}{\omega \epsilon} = \beta / \alpha.$$

It can be proved that $\phi = 2\theta_n$

Ex-1 In a lossy medium the Impedance of the medium is 200 at an angle of 30° and $\vec{H} = 10 e^{-\alpha x} \cos(\omega t - \frac{x}{2}) \hat{a}_z$ A/m.

Find attenuation constant α , β , δ direction of propagation. Expression for $\vec{E}$.

Ans:

$$Z = 200 \angle 30^\circ$$

$$\vec{H} = 10 e^{-\alpha x} \cos(\omega t - \frac{x}{2}) \hat{a}_z$$

$$\beta = +\frac{1}{2} \text{ rad/m}$$

$$\frac{E_y}{H_z} = Z = -\frac{E_z}{H_y}$$

$$\therefore E_y = Z H_z.$$

$$\therefore E_y = 2000 \angle 30^\circ \cdot e^{-\alpha x} \cos(\omega t - \frac{x}{2}) \hat{a}_z.$$

$$\therefore E_y = 2000 e^{-\alpha x} \cos(\omega t - \frac{x}{2} + \frac{\pi}{6}) \hat{a}_z.$$

$$\alpha = \omega \sqrt{\frac{\mu \epsilon}{2} \left[\sqrt{1 + \frac{\sigma^2}{\epsilon^2}} - 1 \right]}$$

$$\beta = \omega \sqrt{\frac{\mu \epsilon}{2} \left[\sqrt{1 + \frac{\sigma^2}{\epsilon^2}} + 1 \right]}$$

$$\Rightarrow \frac{\alpha}{\beta} =$$

$$\phi = 2\theta_n = 2(30^\circ) = 60^\circ$$

$$\therefore \tan \phi = \frac{\sigma}{\omega \epsilon}$$

$$\therefore \frac{\sigma}{\omega \epsilon} = \tan 60^\circ = \frac{1}{\sqrt{3}} \cdot \sqrt{3}$$

$$\Rightarrow \frac{\alpha}{\beta} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \alpha = \beta / \sqrt{3}$$

$$\therefore \boxed{\alpha = \frac{1}{2\sqrt{3}}} \text{ Nm/m}$$

$$\therefore \delta = \frac{1}{\alpha}$$

$$\therefore \boxed{\delta = 2\sqrt{3} \text{ m}}$$

~~Ex 2~~

Ex 2 In a good conductor attenuation constant α is given by 0.5 nep/m. Find phase shift constant and skin depth.

Ans: $\beta = 0.5$ $\alpha = \beta$ for good conductor.

$$\beta = 0.5 \text{ nep/m}$$

$$\therefore \delta = 2 \text{ m.}$$

If α is in the dB/m we have to convert it in nep/m.

★ Polarization:

- It is the one of the designed parameter of an antenna. and it is the of a wave.
- It is defined as orientation of electric field (or) It is the locus of resultant electric field as a function of time at a given location.
- If the Locus represents a straight line then it is called linear polarization.
- VP and HP are the special cases in the linear polarization.
- If the Locus represent a circle then it is called circular polarization.
- If the Locus represent an ellipse then it is called elliptical polarization.
- Right hand and Left hand are two different kinds in CP and in E.P.

→ For example we assumed that the wave is propagating along z direction. The possible non-zero transverse field components are $\hat{a}_x$ and $\hat{a}_y$

→ (z)

$$\vec{E}(z, t) = E_{x0} \cos(\omega t - \beta z) \hat{a}_x + E_{y0} \cos(\omega t - \beta z + \phi_p) \hat{a}_y$$

→ Phase shift b/w the transverse field component is ϕ_p .

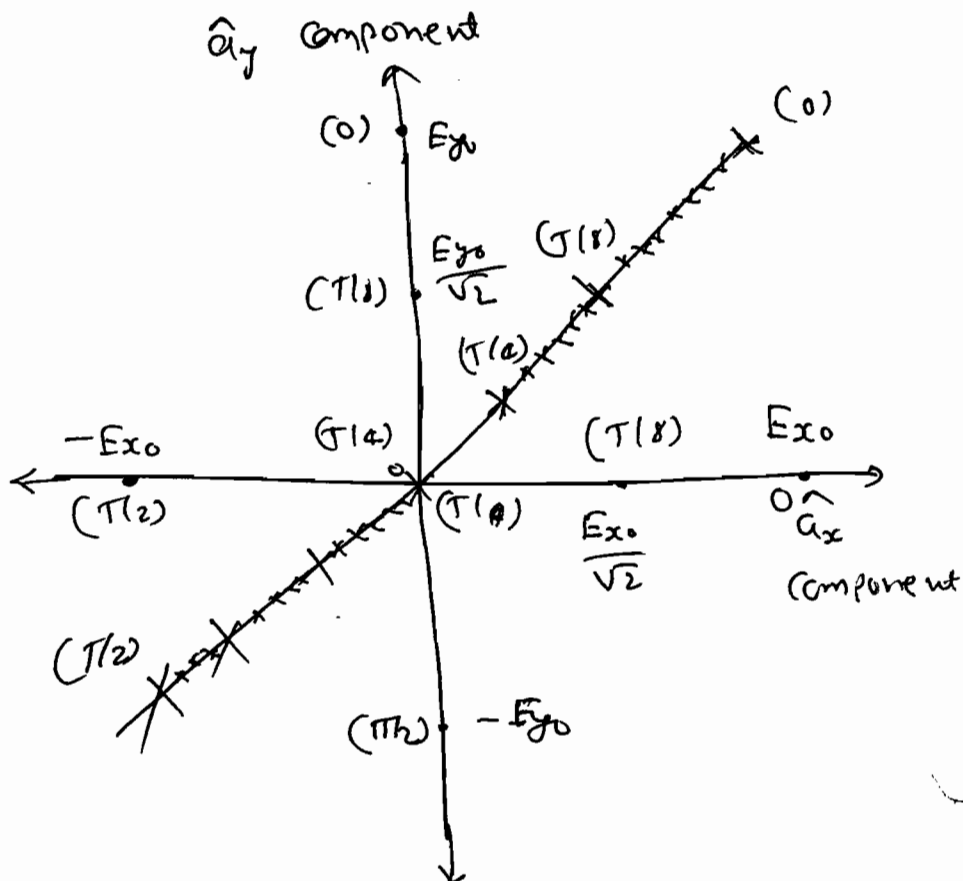
→ For convenience choose $z=0$.

$$\therefore \vec{E}(0, t) = E_{x0} \cos \omega t \hat{a}_x + E_{y0} \cos(\omega t + \phi_p) \hat{a}_y$$

$$\therefore \boxed{\text{Case - 1}} \quad \phi_p = 0$$

$$\therefore \vec{E} = E_{x0} \cos \omega t \hat{a}_x + E_{y0} \cos \omega t \hat{a}_y$$

t	$\omega t = \frac{2\pi}{T} t$
0	0
$\vdots$	
$\frac{T}{8}$	$\pi/4$
$\vdots$	
$\frac{T}{4}$	$\pi/2$
$\vdots$	
$\frac{T}{2}$	π
$\vdots$	
T	2π



→ Thus, the Locus represents a straight line and hence the wave is said to be linearly polarized.

→ If E_y Component is absent the locus lies along ~~vertical~~ horizontal axis that we may call wave is horizontally polarized.

→ If E_x Component is absent the locus lies along vertical axis that we may call wave is vertically polarized.

→ ✓ To achieve linear polarization the wave must have at least one transverse field components. If the wave has both the transverse field components then the phase difference betⁿ them must be equal to $\pm n\pi$ where $n = 0, 1, 2, \dots$

Case- 2:

$$\phi_p = 90^\circ, \quad E_{x0} = E_{y0} = E_0.$$

$$\vec{E} = E_0 \cos \omega t \hat{a}_x - E_0 \sin \omega t \hat{a}_y.$$

Waves must have equal components. They must have equal amplitudes and the phase difference betⁿ them must be equal to $\pm \frac{n\pi}{2}$ where 'n' is odd.

→ To achieve elliptical polarization the wave must have two transverse fields component, they must have unequal amplitudes and the phase difference betⁿ them must not equal to 0° or 180° .

Ex-1 Identify the polarization of the following.

1. $\vec{E} = 20 \cos(\omega t - \frac{x}{2}) \hat{a}_y$ V/m.

2. $\vec{E} = 45 \sin(\omega t - \beta x) \hat{a}_y + 45 \sin(\omega t - \beta x) \hat{a}_z$.

3. $\vec{E} = 25 \sin(\omega t - \beta z) \hat{a}_x + 25 \cos(\omega t - \beta z) \hat{a}_y$.

4. $\vec{E} = 35 \sin(\omega t - \beta y) \hat{a}_x + 45 \cos(\omega t - \beta y) \hat{a}_z$.

5. $\vec{E} = \text{Re} \left\{ [2\hat{a}_x + 3j\hat{a}_y] e^{j(\omega t - \beta z)} \right\}$.

6. $\vec{E} = 25 \cos(\omega t - \beta z) \hat{a}_y$ V/m.

7. $\vec{H} = 20 \sin(\omega t - \beta x) \hat{a}_z$

8. $\vec{E} = 20 \sin(\omega t - \beta x) \hat{a}_y + 20 \sin(\omega t - \beta x + \phi) \hat{a}_z$.

9. $\vec{E} = 20 \sin(\omega t - \beta z) \hat{a}_z$.

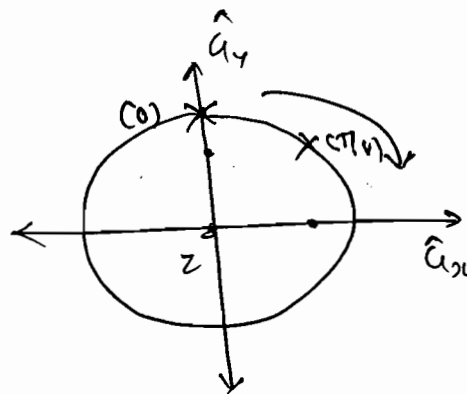
10. $\vec{E} = \text{Re} \left\{ [\hat{a}_y + j\hat{a}_z] e^{j(\omega t - \beta x)} \right\}$.

Ans:-1

→ The Wave is propagating along +x direction.
and it has one transverse fields
Component then for it is linearly polarized.
and it is polarized along y direction.
because the electric field has $\hat{a}_y$ component.

Ans:-2 X Wave is propagating along +x direction.
and phase difference is zero. so.
it is linearly polarized.

Ans:-3 Circular polarization. $\xrightarrow{\text{---}} +x$

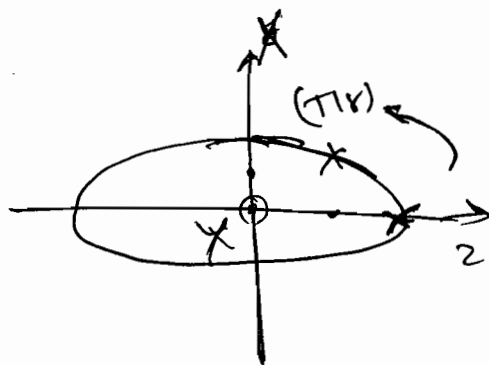


LHCP

Ans:-4 $\vec{E} = 35 \sin(\omega t - \beta z) \hat{a}_x + 45 \cos(\omega t - \beta z) \hat{a}_z$

$35 \neq 45$
so, Elliptical.

$\xrightarrow{\text{---}} +x$



RHCP

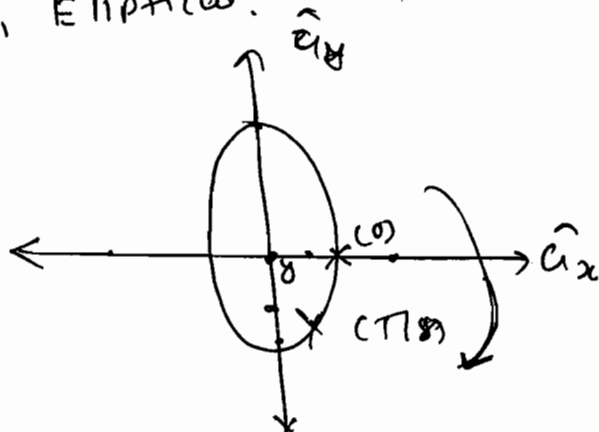
$$5. \quad E = \text{Re} \left\{ (2\hat{a}_x + 3j\hat{a}_y) \cdot (\cos(\omega t - \beta z) + j \sin(\omega t - \beta z)) \right\}$$

$$\therefore E = (2 \cos(\omega t - \beta z) \hat{a}_x - 3 \sin(\omega t - \beta z) \hat{a}_y)$$

$$2 \neq -3$$

$$= 2 \cos(\omega t - \beta z) \hat{a}_x + 3 \sin(\omega t - \beta z) \hat{a}_y$$

So, Elliptical.



$\frac{E_x}{E_y} = \frac{2}{3}$

LHEP.

$$6. \quad \vec{E} = 25 \cos(\omega t - \beta z) \hat{a}_\theta \text{ V/m.}$$

where, r, θ spherical coordinates.

→ ... $(+z)$

→ Linearly polarized. along θ direction.

→ this represents a wave which is propagating along radial direction i.e. $\hat{a}_r$ direction.

Therefore, the boundary plane would be

$r = \text{constant}$ plane (or) $\theta = \phi$ plane.

→ Therefore, the boundary field components are

$\hat{a}_\theta$ and $\hat{a}_\phi$. given electric field has

one boundary field components i.e. $\hat{a}_\theta$ component and hence wave is linearly polarized and is polarized along $\hat{a}_\theta$ direction.

7. $\vec{H} = 20 \sin(\omega t - \beta x) \hat{a}_z$.

$\xrightarrow{\dots} +x$

→ Linearly polarized.

$\vec{E} = 20 \sin(\omega t - \beta x) \hat{a}_y \rightarrow y\text{-direction.}$

→ This wave is linearly polarized and is polarized along y-direction because electric field have y-component.

8. $\vec{E} = 20 \sin(\omega t - \beta x) \hat{a}_y + 20 \sin(\omega t - \beta x + 40) \hat{a}_z$.

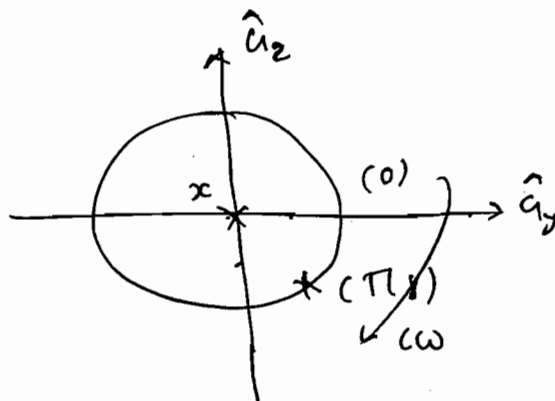
Ans: This wave may be valid em wave but it is not satisfying any polarization principle.

9. $\vec{E} = 20 \sin(\omega t - \beta z) \hat{a}_z$

→ Not valid EM wave

10. $\vec{E} = \text{Re} \left\{ [\hat{a}_y + j\hat{a}_z] \cdot \cos(\omega t - \beta x) + j(\sin \omega t - \beta x) \right\}$.

$\vec{E} = \cos(\omega t - \beta x) \hat{a}_y - \sin(\omega t - \beta x) \hat{a}_z$.



LHCP

★ POYNTING THEOREM / VECTOR.

→ This used to calculate average power.

→ The vector corresponding to this indicates direction of instantaneous energy flow ~~vector~~ simply direction of propagation.

→ If $\vec{E}$, $\vec{H}$ are electrical and magnetic field respectively then the instantaneous Poynting vector is given by ~~cross~~ $\vec{E} \times \vec{H}$.

$$\vec{P}_{\text{instantaneous}} = \vec{E} \times \vec{H}.$$

* The avg. Poynting vector is given by.

$$\vec{P}_{\text{avg}} = \frac{1}{2} \vec{E}_s \times \vec{H}_s^* \quad \text{W/m}^2$$

Complex
* $\rightarrow$ conjugate

→ $\vec{E}_s$, $\vec{H}_s$ are the fields in the phasor form. The avg. power crossing a cross sectional surface of 's' is given by,

$$\therefore W_{\text{avg}} = \int_s \vec{P}_{\text{avg}} \cdot d\vec{s} \quad \text{watts.}$$

$d\vec{s}$: vector differential surface element and will be projecting normal to the surface.

Ex-1 In a non-magnetic medium

$\vec{E} = 8 \cos (2\pi \times 10^7 t - 0.8x) \hat{a}_y$ V/m. Find the relative permittivity of the medium, intrinsic impedance of the medium, $\vec{H}$, Polarization and also calculate the amount of power crossing a 100 cm^2 area. define on the plane $x=1$.

Ans:

$$\vec{E} = 8 \cos (2\pi \times 10^7 t - 0.8x) \hat{a}_y$$

$$\rightarrow \omega = 2\pi \times 10^7 \text{ rad/s.}$$

$$\beta = 0.8$$

non-magnetic medium

$$\therefore \mu = \mu_0$$

$$\epsilon = \epsilon_0 \epsilon_r$$

$$\xrightarrow{(+x)}$$

$$\boxed{\frac{E_y}{H_z} = \eta = -\frac{E_z}{H_y}}$$

$$\therefore \eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\therefore \eta = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}}$$

$$\therefore \eta = \frac{120\pi}{\sqrt{\epsilon_r}}$$

$$\therefore \boxed{\eta = 98.61}$$

$$\therefore H_z = \frac{E_y}{\eta}$$

$$\therefore \boxed{\vec{H} = \frac{8}{98.61} \cos (2\pi \times 10^7 t - 0.8x) \hat{a}_z}$$

$$\vec{H} = 0.08113 \cos (2\pi \times 10^7 t - 0.8x) \hat{a}_z$$

$$\therefore \beta = \omega \sqrt{\mu \epsilon}$$

$$\therefore \beta = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_r}$$

$$\therefore \sqrt{\epsilon_r} = \frac{\beta}{\omega \sqrt{\mu_0 \epsilon_0}}$$

$$\therefore \sqrt{\epsilon_r} = \frac{0.8 \times 3 \times 10^8}{2\pi \times 10^7}$$

$$\therefore \boxed{\epsilon_r = 14.60}$$

$$\rightarrow \bar{P}_{avg} = \frac{1}{2} \bar{E}_s \times \bar{H}_s$$

$$\therefore \bar{E}_s = 8 \cdot e^{-j0.8x} \hat{a}_y$$

$$\bar{H}_s = \frac{8}{\eta} \cdot e^{+j0.8x} \hat{a}_z$$

$$\therefore \bar{P}_{avg} = \frac{1}{2} (\bar{E}_s \times \bar{H}_s)$$

$$= \frac{1}{2} \times \frac{32}{\eta} \times \hat{a}_x$$

$$\therefore \bar{P}_{avg} = \frac{32}{98.6} \times \hat{a}_x$$

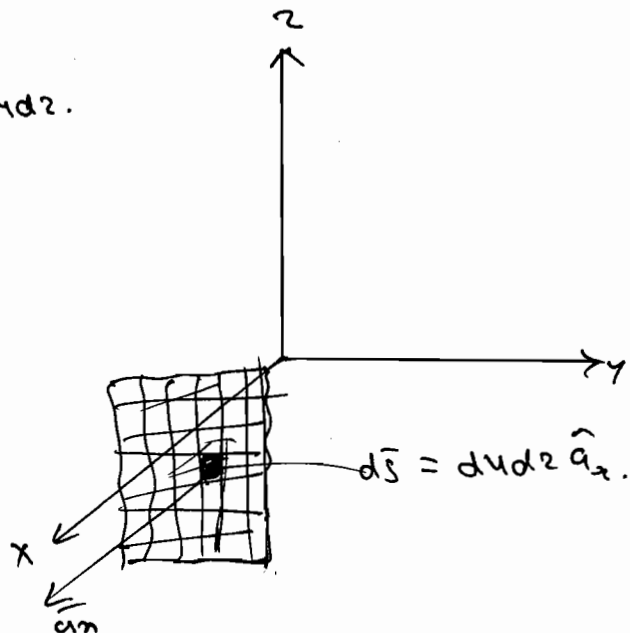
$$\therefore \boxed{\bar{P}_{avg} = 0.3245 \hat{a}_x}$$

$$\therefore \bar{W}_{avg} = \int \bar{P}_{avg} \cdot d\bar{S}$$

$$\therefore \bar{P}_{avg} \cdot d\bar{S} = 0.3245 \, dy \, dz$$

$$\therefore \bar{W}_{avg} = \int_S 0.3245 \, dy \, dz$$

$$= 0.3245 \int_S dy \, dz$$



$$d\bar{S} = dy \, dz \, \hat{a}_z$$

$$\begin{aligned} \int_S dy \, dz &= 100 \, \text{cm}^2 \\ &= \frac{100}{100 \times 100} \, \text{m}^2 \end{aligned}$$

$$\therefore \boxed{\bar{W}_{avg} = 3.245 \times 10^{-3} \, \text{Watts}}$$

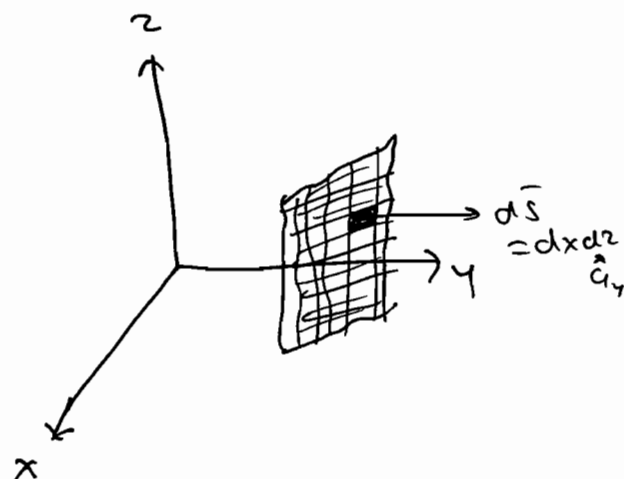
Ex-2 Repeat the above example to calculate avg. power crossing 100 cm^2 area define on the plane $y=1\text{m}$.

Ans: zero

$$d\vec{S} = dx dz \hat{a}_y$$

$$\therefore \bar{P}_{\text{avg}} \cdot d\vec{S} = 0.$$

$$\therefore \boxed{\bar{W}_{\text{avg}} = 0.}$$



Ex-2 Repeat the above problem to calculate the amount of power crossing through a 50 cm^2 area define on the plane $2x+3y=5$.

Ans: $2x+3y=5$

$$\therefore \hat{a}_n = \frac{2\hat{a}_x + 3\hat{a}_y}{\sqrt{13}}$$

$$\therefore d\vec{S} = dS \hat{a}_n$$

$$\therefore \bar{P}_{\text{avg}} \cdot d\vec{S} = 0.3245 \hat{a}_{20} \cdot \left(\frac{2\hat{a}_{20} + 3\hat{a}_{17}}{\sqrt{13}} \right) \\ = 0.18 dS.$$

$$\therefore W_{\text{avg}} = \int_S \bar{P}_{\text{avg}} \cdot d\vec{S}.$$

$$\therefore W_{\text{avg}} = \int 0.18 dS \\ = 0.18 \int dS$$

$$W_{avg} = 0.18 \times \frac{50 \times 100}{100 \times 100}$$

$$\therefore W_{avg} = 0.09 \text{ watts.}$$

Ex 3 Repeat the above problem to find the avg. power crossing on a circular disk of radius 2m define on the plane $x=2m$.

Ans: $\bar{P}_{avg} = 0.3245 \hat{a}_x$

$$\therefore d\bar{S} = dxdz \hat{a}_x$$

$$W_{avg} = \int_S \bar{P}_{avg} \cdot d\bar{S}$$

$$= \int_S 0.3245 dxdz$$

$$= 0.3245 \int dS$$

$$= 0.3245 \times \pi r^2$$

$$= 0.3245 \times \pi \times (2)^2$$

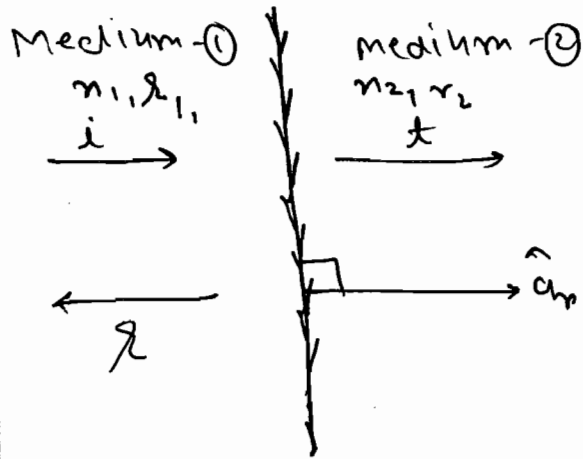
$$\therefore W_{avg} = 4.075 \text{ watts}$$

★ Reflection:

- When a wave is progressing from one medium to other medium impedance of a medium changes. Change in the impedance of the medium is said to be a impedance discontinuity (or) Impedance are irregular (or) not uniformly.
- Whenever there exist impedance discontinuity then part of the energy will be transmitted and the part of the energy will be reflected.
- If the vector corresponds to the direction of propagation is normal to the interface then it is called normal incidence.
- If the vector corresponds to direction of propagation makes an angle with the unit vector normal to the interface is called ~~normal incidence~~ oblique incidence.
- There are two types of oblique incidence.
 - ① Parallel Polarization.
 - ② Perpendicular Polarization.

Reflection

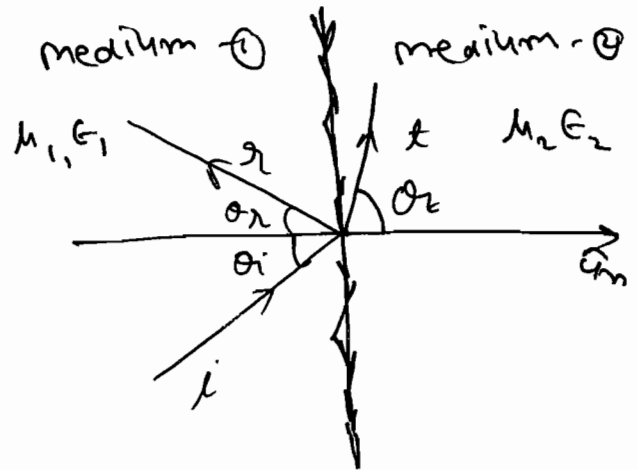
Normal incidence



interface.

$\rightarrow i$

oblique incidence

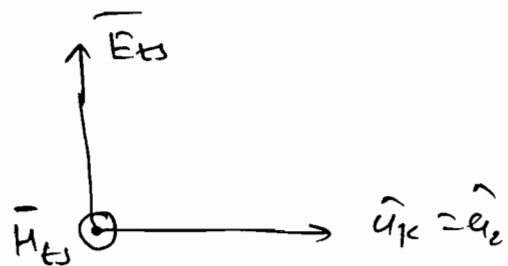
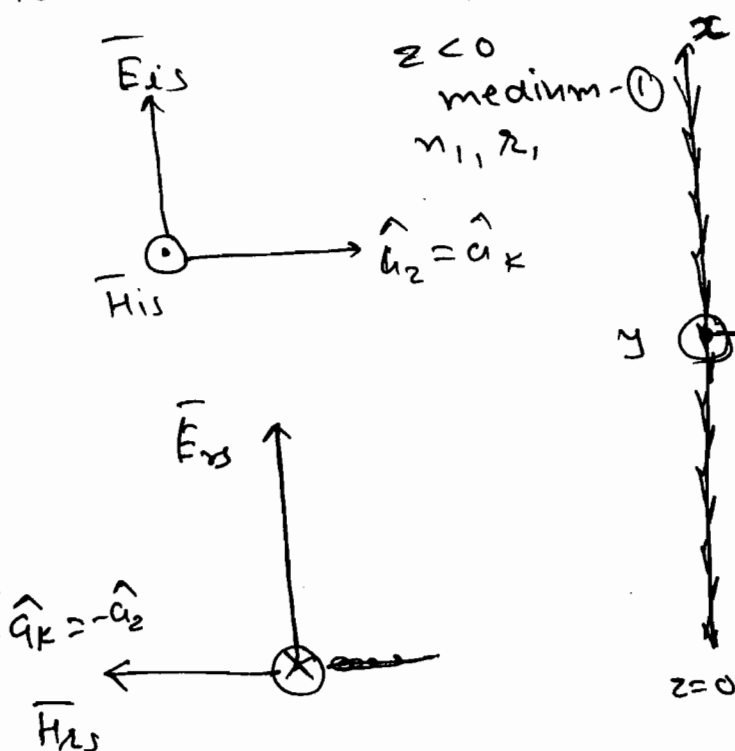


Snell's Law

$$\theta_i = \theta_r$$

$$\therefore \frac{\sin \theta_t}{\sin \theta_i} = \sqrt{\frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2}}$$

* Normal Incidence:



$z > 0$ medium-②

n_2, ϵ_2

→ i = incident
 r = reflected
 t = transmitted

$\hat{q}_k$: vector corresponding to direction of propagation.

→ For the incident $\hat{q}_k = +\hat{a}_z$

→ For the transmitted $\hat{q}_k = +\hat{a}_z$

→ For the reflected $\hat{q}_k = -\hat{a}_z$

⊙ Vector/axis coming out of the paper.

⊗ Vector/axis going into the paper.

→ Figure shows that an interface define by $z=0$.

→ $z < 0$ is medium - ① and is characterised by n_1, r_1 .

→ Medium - ② is define for $z > 0$ and is characterised by n_2, r_2 .

→ We assume that ~~there~~ wave is progressing from medium - ① to medium - ②.

→ Because of Impedence discontinuity part of the energy will be transmitted and part of the energy will be reflected.

* Incident $\rightarrow (+z)$

$$\vec{E}_i = E_{i0} e^{-\gamma_1 z} \hat{a}_x$$

$$\vec{H}_i = \frac{E_{i0}}{\eta_1} e^{-\gamma_1 z} \hat{a}_y$$

* Transmitted $\rightarrow (+z)$

$$\vec{E}_t = E_{t0} e^{-\gamma_2 z} \hat{a}_x$$

$$\vec{H}_t = H_{t0} e^{-\gamma_2 z} \hat{a}_y$$

$$\vec{H}_t = \frac{E_{t0}}{\eta_2} e^{-\gamma_2 z} \hat{a}_y$$

* Reflected $\rightarrow (-z)$

$$\therefore \vec{E}_r = \vec{E}_{r0} e^{+\gamma_1 z} \hat{a}_x$$

$$\vec{H}_r = -\frac{E_{r0}}{\eta_1} e^{+\gamma_1 z} \hat{a}_y$$

$\rightarrow$ In medium - ① across the interface there are two sets of fields (i) incidence (ii) Reflected.

$\rightarrow$ In medium - ② there exist one set of fields i.e. transmitted.

→ The electric and magnetic fields of this wave are projecting tangential to the interface.

→ We know that tangential components of electric field are continuous across a ~~current~~ the interface.

→ Similarly tangential components of magnetic field are also continuous across a current free interface.

→ As shown in the figure, the interface is defined by $z=0$.

→ $E_{i0} + E_{r0} = E_{t0}$ — (1) (continuity of E fields).

$\therefore \frac{E_{i0}}{n_1} + - \frac{E_{r0}}{n_1} = \frac{E_{t0}}{n_2}$ — (2) (continuity of H -fields).

→ defining reflection coefficient Γ as a ratio of reflected electric field to the incident electric field.

$$\therefore \Gamma = \frac{E_{r0}}{E_{i0}} = \frac{n_2 - n_1}{n_2 + n_1}$$

$$\therefore \frac{E_{i0} + E_{r0}}{E_{i0}} = \frac{E_{t0}}{E_{i0}}$$

$$+ \frac{n_2}{n_1} E_{i0} - \frac{n_2}{n_1} E_{r0} = E_{t0}$$

$$E_{i0} \left[\frac{n_1 - n_2}{n_1} \right] + E_{r0} \left[\frac{n_1 + n_2}{n_1} \right] = 0$$

$$\therefore E_{i0}(n_2 - n_1) = E_{r0}(n_2 + n_1).$$

$$\therefore \boxed{\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{n_2 - n_1}{n_2 + n_1}}$$

→ Defining transmission coefficient of T is a ratio of transmitted ~~Wave~~ $\vec{E}$ to the incident electric field.

$$T = \frac{E_{t0}}{E_{i0}} = \frac{2n_2}{n_1 + n_2} = 1 + \Gamma.$$

$$\rightarrow E_{i0} + E_{r0} = E_{t0}.$$

$$\rightarrow \frac{E_{i0}}{n_1} - \frac{E_{r0}}{n_1} = \frac{E_{t0}}{n_2}.$$

$$\therefore \begin{aligned} & E_{i0} + E_{r0} = E_{t0} \\ & + \quad E_{i0} - E_{r0} = \frac{n_1}{n_2} E_{t0} \\ \hline & 2E_{i0} = \left[\frac{n_1 + n_2}{n_2} \right] E_{t0}. \end{aligned}$$

$$\therefore \boxed{T = \frac{E_{t0}}{E_{i0}} = \frac{2n_2}{n_1 + n_2}}$$

→ If $n_2 = n_1 \Rightarrow$ ~~then~~ $\Gamma = 0$, $T = 1 \Rightarrow$ No Reflection.

$\Rightarrow$ No Impedance Continuity.

→ If $n_2 = 0 \Rightarrow$ (i.e) medium ② is perfect

conductor.

$$\text{hence } \underline{\Gamma = -1, T = 0}$$

$\Rightarrow$ Completely reflected, No transmission.

→ If Γ is reflection coefficient then $|\Gamma|^2$ is called power reflection coefficient.

→ $\% \text{ of Power reflected} = 100 |\Gamma|^2 \%$

→ $\% \text{ of Power transmitted} = \frac{100}{(1 - |\Gamma|^2)} \times 100 \%$

Ex-1 Assume normal incidence wave is progressing from free space to a medium whose permittivity is $4\epsilon_0$ and permeability is μ_0 . Find the reflected power, Γ , T , power reflection coefficient and also find $\% \text{ power transmitted}$.

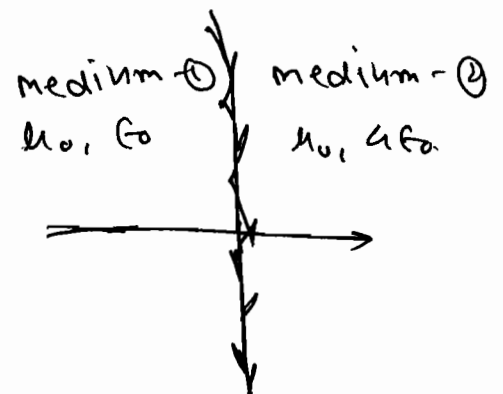
Ans:

$$n_1 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi$$

$$n_2 = \sqrt{\frac{\mu}{\epsilon}}$$

$$= \sqrt{\frac{\mu_0}{4\epsilon_0}} = \frac{120\pi}{2}$$

$$\therefore \boxed{n_2 = 60\pi}$$



$$\therefore \Gamma = \frac{n_2 - n_1}{n_2 + n_1} = \frac{60\pi - 120\pi}{60\pi + 120\pi} = \frac{-60\pi}{180\pi} = -\frac{1}{3}$$

$$\therefore \boxed{\Gamma = -\frac{1}{3}}$$

$$\therefore T = 1 + \Gamma = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\boxed{T = \frac{2}{3}}$$

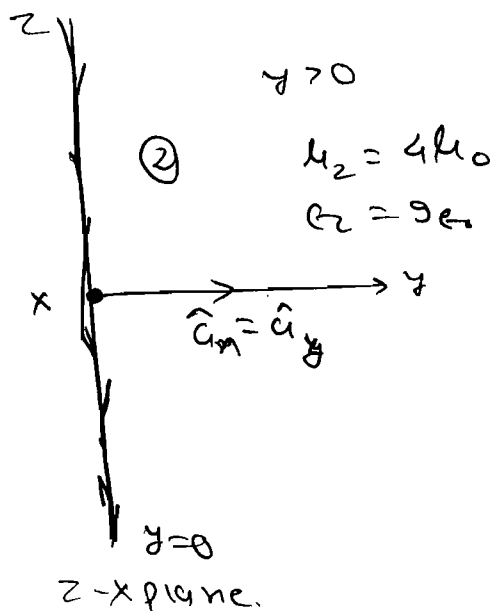
$|\Gamma|^2 = 1/9 \Rightarrow$ power reflection coefficient.

$\rightarrow$ % power reflected = $100 |\Gamma|^2 = \frac{100}{9} = 11.11\%$

Ex-2 Figure shows that an interface may be define by $y=0$, $y < 0$ is medium-① and is free space and $y > 0$ is medium-② and is characterised by $\mu_2 = 4\mu_0$ & $\epsilon_2 = 9\epsilon_0$ a wave is incident upon the interface whose electric field is given by $20 \sin(\omega t - \frac{y}{2}) \hat{a}_z$ V/m. Find the phase shift constant in medium-① and in medium-② also find Γ , T , power reflection coefficient, % power reflected, % power transmitted, $\bar{H}_i$, $\bar{E}_i$, $\bar{H}_r$, $\bar{E}_t$, $\bar{H}_t$ and also calculate Poynting Vector of incident, reflected and transmitted wave.

Ans:

$y < 0$
① free space
 μ_0
 ϵ_0
~~~~~ $\rightarrow$



$$\rightarrow E_i = 20 \sin \left( \omega t - \frac{\pi}{2} \right) \hat{a}_z \text{ V/m.}$$

$\rightarrow$  The unit vector normal to the interface is  $\hat{a}_y$ .

$\rightarrow$  From the given  $\vec{E}_i$  we can conclude that the wave is progressing along +y direction which is projecting normal to the interface. and hence this case is said to be normal incidence.

$$\rightarrow \beta_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

$$\therefore \beta_1 = \omega \sqrt{\epsilon_0 \mu_0}$$

$$\therefore \frac{1}{2} = \omega \sqrt{\mu_0 \epsilon_0}$$

But  $\boxed{\beta_1 = \frac{1}{2} \text{ rad/s}}$

$$\rightarrow \beta_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$\beta_2 = \omega \sqrt{9 \epsilon_0 \times \frac{4\mu_0}{4\mu_0}}$$

$$\therefore \beta_2 = 6 \omega \sqrt{\epsilon_0 \times \mu_0} = 6 \times \frac{1}{2} = 3$$

$$\therefore \boxed{\beta_2 = 3 \text{ rad/s}}$$

$$\therefore \eta_1 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi$$

$$\boxed{\eta_1 = 120\pi \Omega}$$

$$\boxed{\eta_2 = 80\pi \Omega}$$

$$\eta_2 = \sqrt{\mu/\epsilon}$$

$$\therefore \eta_2 = \sqrt{\frac{4\mu_0}{9\epsilon_0}} = \frac{40\pi}{120\pi \times 2} = 80\pi$$

$$\Gamma = \frac{n_2 - n_1}{n_2 + n_1}$$

$$\therefore \Gamma = \frac{80\pi - 120\pi}{80\pi + 120\pi} = -\frac{40\pi}{200} = -1/5$$

$$\therefore \boxed{\Gamma = -1/5}$$

$$\therefore T = 1 + \Gamma$$

$$= 1 - \frac{1}{5}$$

$$\therefore \boxed{T = \frac{4}{5}}$$

$$\therefore \vec{E}_i = 20 \sin(\omega t - y/2) \hat{a}_z$$

$\therefore$  Now.

—————→ (+y)

$$\therefore \boxed{\frac{E_z}{H_x} = +n_1 = -\frac{E_x}{H_z}}$$

$$\vec{H}_i = + \frac{\vec{E}_i}{n_1}$$

$$\therefore \boxed{\vec{H}_i = + \frac{20}{120\pi} \sin(\omega t - y/2) \hat{a}_{3L}}$$

$$\rightarrow \Gamma = \frac{E_r}{E_i}$$

$$\therefore \vec{E}_r = \Gamma \vec{E}_i$$

$$\therefore \vec{E}_r = -\frac{20}{5} \sin(\omega t + \beta_1 y) \hat{a}_z$$

$$\therefore \boxed{\vec{E}_r = -4 \sin(\omega t + \beta_1 y) \hat{a}_z}$$

$$\longrightarrow (-y)$$

$$\frac{E_z}{H_x} = -\eta_1 = -\frac{E_x}{H_z}$$

$$\therefore \overline{H}_x = -\frac{\overline{E}_z}{\eta_1}$$

$$\therefore \overline{H}_x = -\frac{1}{120\pi} \times (-4 \sin(\omega t + \beta y) \hat{a}_x)$$

$$\therefore \boxed{\overline{H}_x = \frac{1}{30\pi} \sin(\omega t + \beta y) \hat{a}_x}$$

$$\therefore T = \frac{\overline{E}_t}{\overline{E}_i}$$

$$\therefore \overline{E}_t = T \overline{E}_i$$

$$\therefore \overline{E}_t = \frac{4}{5} \times 4 \sin(\omega t + \beta_2 y) \hat{a}_2$$

$$\therefore \overline{E}_t = \frac{16}{5} \sin(\omega t + \beta_2 y) \hat{a}_2$$

$$\therefore \longrightarrow (+y)$$

$$\frac{E_z}{H_x} = -\eta_2 = \frac{E_x}{H_z}$$

$$\therefore \overline{H}_t = -\frac{\overline{E}_t}{\eta_2} = -\frac{16}{5 \times 80\pi} \sin(\omega t + \beta_2 y) \hat{a}_2$$

$$\therefore \boxed{\overline{H}_t = -\frac{1}{25\pi} \sin(\omega t + \beta_2 y) \hat{a}_2}$$

$$\rightarrow |\Gamma|^2 = \frac{1}{25}$$

$$\therefore \% \text{ of power reflected} = 100 \times \frac{1}{25} \% = 4 \%$$

$\therefore$  % of power transmitted = 96-1.

$$\therefore \textcircled{1} \quad \begin{aligned} \overline{E}_{1s} &= 20 e^{-j\beta_1 y} \hat{a}_z \\ \overline{H}_{1s}^* &= -\frac{1}{6\pi} e^{+j\beta_1 y} \hat{a}_x \end{aligned}$$

$$\therefore \overline{P}_{avg} = \frac{1}{2} \overline{E}_{1s} \times \overline{H}_{1s}^*$$

$$= \frac{1}{2} \times 20 \times -\frac{1}{6\pi} \times \hat{a}_y$$

$$\therefore \boxed{\overline{P}_{avg} = -\frac{5}{3\pi} \hat{a}_y} \text{ W/m}^2$$

$$\textcircled{2} \quad \begin{aligned} \overline{E}_{2s} &= -4 e^{j\beta_1 y} \hat{a}_z \\ \overline{H}_{2s}^* &= \frac{1}{30\pi} e^{j\beta_1 y} \hat{a}_x \end{aligned}$$

$$\therefore \overline{P}_{avg} = \frac{1}{2} \overline{E}_{2s} \times \overline{H}_{2s}^*$$

$$\therefore \overline{P}_{avg} = \frac{1}{2} \times -4 \times \frac{1}{30\pi} \times \hat{a}_y$$

$$\therefore \boxed{\overline{P}_{avg} = -\frac{1}{15\pi} \hat{a}_y} \text{ W/m}^2$$

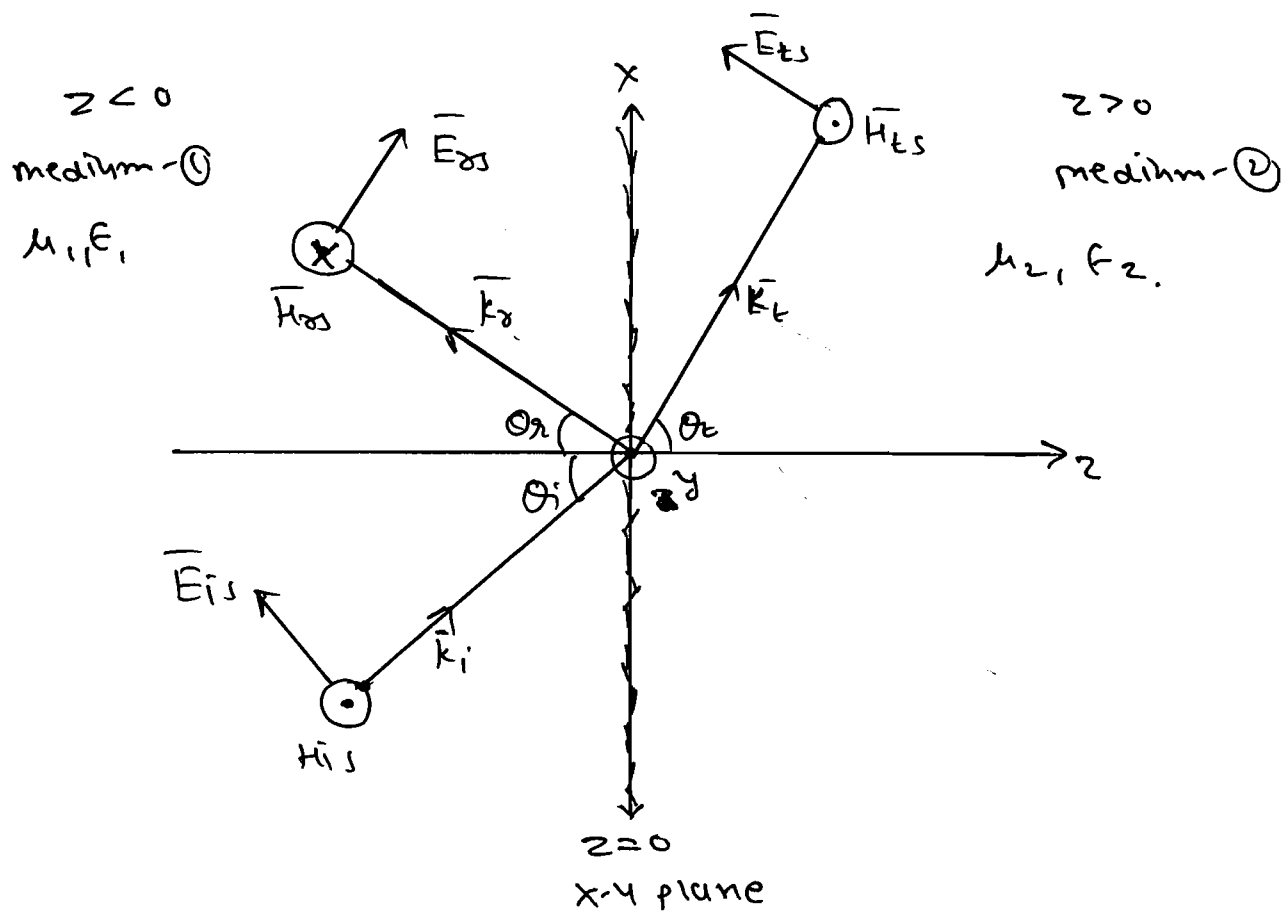
$$\textcircled{3} \quad \begin{aligned} \overline{E}_{3s} &= \frac{16}{5} e^{-j\beta_2 y} \hat{a}_z \\ \overline{H}_{3s} &= -\frac{1}{25\pi} e^{-j\beta_2 y} \hat{a}_x \end{aligned}$$

$$\overline{P}_{avg} = \frac{1}{2} \overline{E}_{3s} \times \overline{H}_{3s}^* = \frac{1}{2} \times \frac{16}{5} \times -\frac{1}{25\pi} \hat{a}_y$$

$$\therefore \boxed{\overline{P}_{avg} = -\frac{8}{25\pi} \hat{a}_y} \text{ W/m}^2$$



\* Oblique Incidence : Parallel Polarization.



→ By Snell's Law,

$$\theta_i = \theta_r.$$

$$\therefore \frac{\sin \theta_t}{\sin \theta_i} = \sqrt{\frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2}}$$

→ Figure shows that an interface is defined by  $z=0$ ,  $z < 0$  is medium-① and  $z > 0$  is medium-②.

→ The wave is progressing from medium-① to medium-②.

→ If the vector corresponds to the direction of the propagation makes an angle with the unit vector normal to the interface is

oblique incidence.

→  $\vec{k}_i$ ,  $\vec{k}_r$  and  $\vec{k}_t$  are the vectors corresponds to direction of propagation of incident, reflected and transmitted waves. These are making angle  $\theta_i$ ,  $\theta_r$  and  $\theta_t$  respectively.

\* Plane of Incidence:

→ It is that plane on which  $\vec{k}_i$ ,  $\vec{k}_r$  and  $\vec{k}_t$  and the unit vector normal to the interface ( $\hat{a}_n$ ) are lies on this plane.

→ As shown in figure all these are lying on z-x plane. Therefore z-x plane is called plane of incidence.

→ The electric field vector is  $\parallel$  to plane of incidence i.e.  $\parallel$  to z-x plane and hence this case is said to be parallel Polarization.

(If  $\theta_i = 0$  then the case is said to be normal incidence).

$$\rightarrow \Gamma_{11} = \frac{n_2 \cos \theta_2 - n_1 \cos \theta_1}{n_2 \cos \theta_2 + n_1 \cos \theta_1}$$

$$\rightarrow T_{11} = 1 + \Gamma_{11}$$

\* Brewster Angle: ( $\theta_{B11}$ )

→ It is that particular angle of incidence for which no reflection takes place.

→ ~~It is that angle of incidence for which no reflection takes place.~~

$$\therefore \sin \theta_{11} = \frac{1 - \frac{\mu_2 \epsilon_1}{\mu_1 \epsilon_2}}{\sqrt{1 - \left(\frac{\epsilon_1}{\epsilon_2}\right)^2}}$$

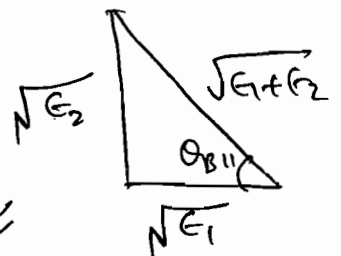
for non-magnetic media  $\mu_1 = \mu_2 = \mu_0$ .

$$\therefore \sin \theta_{11} = \frac{1 - \frac{\epsilon_1}{\epsilon_2}}{\sqrt{1 - \left(\frac{\epsilon_1}{\epsilon_2}\right)^2}}$$

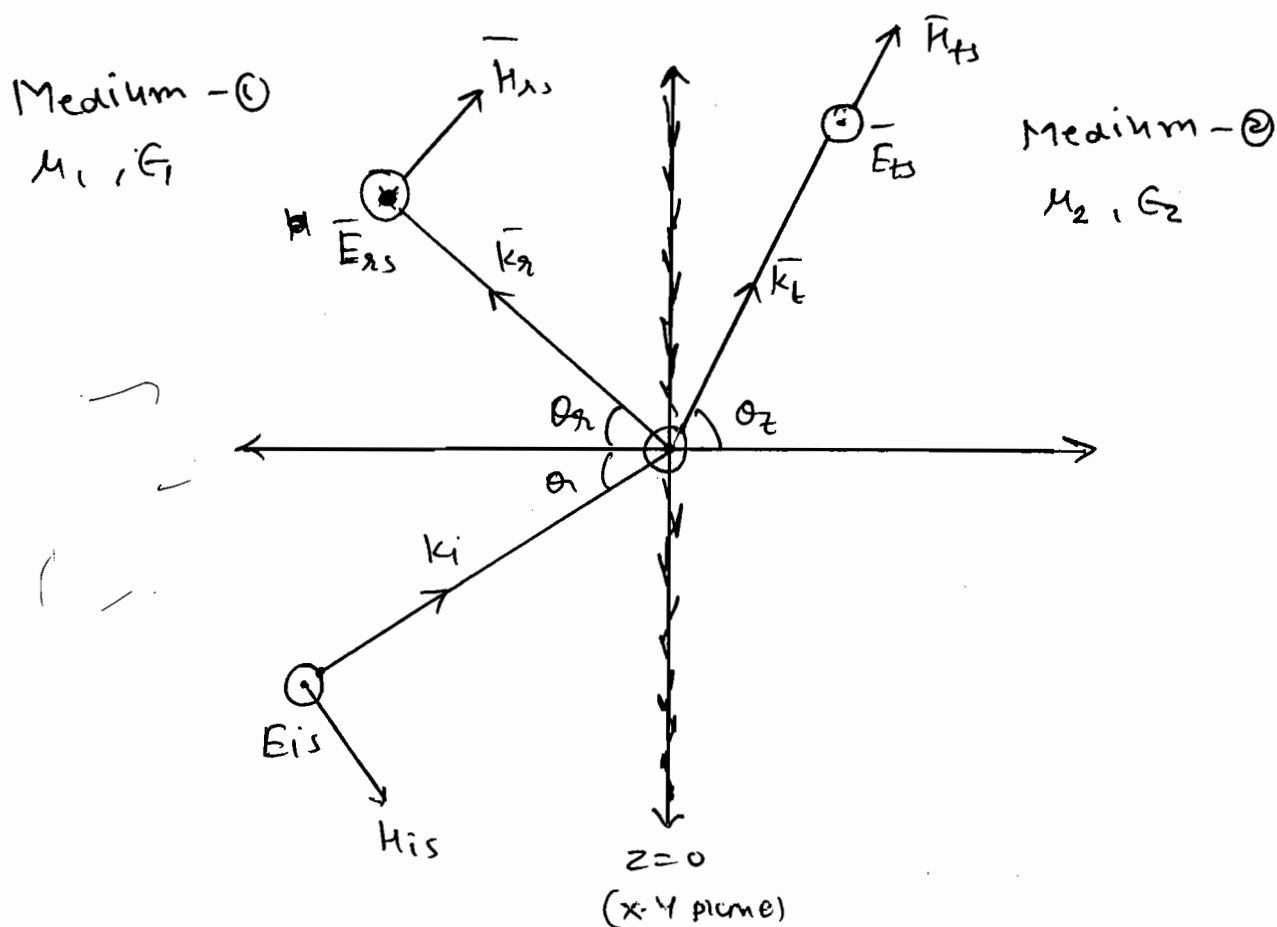
$$\therefore \sin \theta_{11} = \sqrt{\frac{\epsilon_2 - \epsilon_1}{(\epsilon_2 + \epsilon_1)^2} \times \epsilon_2}$$

$$\therefore \sin \theta_{11} = \sqrt{\frac{\epsilon_2}{\epsilon_1 + \epsilon_2}}$$

(or)  $\tan \theta_{B11} = \sqrt{\epsilon_2 / \epsilon_1}$



\* Oblique Incidence: (Perpendicular Polarization)



→ 
$$\Gamma_{\perp} = \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_2 \cos \theta_i + n_1 \cos \theta_t}$$

$$T_{\perp} = 1 + \Gamma_{\perp}$$

→ Electric field is perpendicular to the plane of incidence.

\* Brewster Angle: ' $\theta_{BS}$ '

∴ 
$$\sin \theta_{B\perp} = \sqrt{\frac{1 - \frac{\mu_1 \epsilon_2}{\mu_2 \epsilon_1}}{1 - \left(\frac{\mu_1}{\mu_2}\right)^2}}$$
 ✓

→ For non-magnetic media  $\mu_1 = \mu_2 = \mu_0$

$\sin \theta_1 \rightarrow \infty \Rightarrow$  which is impossible.

$\therefore \theta_B$  does not occur for non-magnetic media for  $1^{st}$  Polarization.

→ In general an EM wave can be represented as

$$\vec{E} = \text{Re} \left[ \vec{E}_0 e^{j(\omega t - \vec{k} \cdot \vec{r})} \right]$$

→  $\vec{r}$  = Radius Vector (or) position vector.

$$\vec{r} = x \hat{a}_x + y \hat{a}_y + z \hat{a}_z.$$

$\vec{k}$  = Propagation Vector (or) wave number vector.

→  $|\vec{k}|$  is related to  $\beta$ .

$$\beta = \omega \sqrt{\mu \epsilon}.$$

$$|\vec{k}| = \sqrt{k_x^2 + k_y^2 + k_z^2} = \beta.$$

\*  $\vec{E}$ ,  $\vec{H}$  and  $\vec{k}$  are mutually orthogonal to each other.

\*  $\vec{E}$ ,  $\vec{H}$  lies in a plane defined by  $\vec{k}$  &  $\vec{r}$

$$\vec{k} \cdot \vec{r} = \text{constant}.$$

$$\left. \begin{aligned} \vec{k} \cdot \vec{E} &= 0 \\ \vec{k} \cdot \vec{H} &= 0 \\ \vec{E} \cdot \vec{H} &= 0 \end{aligned} \right\} \text{mutually orthogonal.}$$

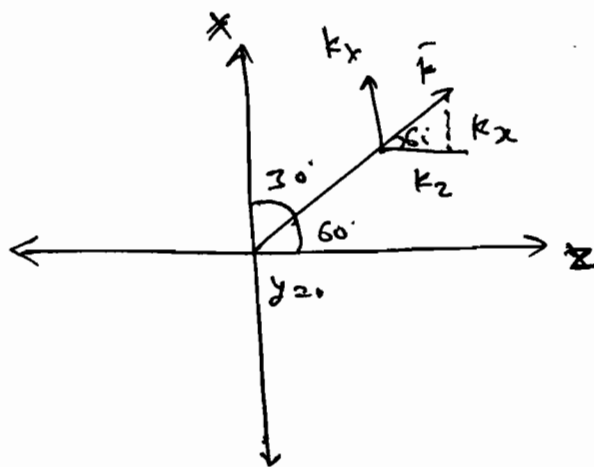
→ Further we write

$$\vec{k} \times \vec{E} = \omega \mu \vec{H}$$

$$\vec{k} \times \vec{H} = \omega \epsilon \vec{E}.$$

Ex-1 An EM wave is propagating in the free space making an angle  $30^\circ$  into +ve x-axis and  $60^\circ$  with the +ve z-axis. and go with the +ve y-axis. Find the vector corresponds to the direction of propagation, assume  $\beta = \frac{2\pi}{\lambda}$  and also give general representation for find  $\vec{E}$ .

Ans:



$$\rightarrow \vec{k} = k_x \hat{a}_x + k_y \hat{a}_y.$$

$$\therefore |\vec{k}| = \sqrt{k_x^2 + k_y^2} = \beta.$$

$$k_x = |\vec{k}| \sin 60^\circ$$

$$\therefore k_x = \beta \times \sqrt{3}/2 = \sqrt{3}/2 \times \frac{2\pi}{\lambda} = \frac{\sqrt{3}\pi}{\lambda}$$

$$\therefore k_y = \beta \cos 60^\circ = \beta \times \frac{1}{2} = \beta/2 = \frac{\lambda}{2\pi} \times \pi = \lambda/2$$

$$\therefore \boxed{\vec{k} = \frac{\sqrt{3}\pi}{\lambda} \hat{a}_x + \frac{\pi}{\lambda} \hat{a}_z}$$

$$\therefore \vec{E} = \text{Re} \left[ e^{-j(\omega t - \vec{k} \cdot \vec{r})} \right]$$

$$\therefore \boxed{\vec{E} = \text{Re} \left[ e^{-j \left( \omega t - \frac{\sqrt{3}\pi x}{\lambda} - \frac{\pi}{\lambda} z \right)} \right]}$$

Ex 2 Interface is defined by  $z=0$ ,  $z < 0$  is free space ( $\mu_0, \epsilon_0$ ) and  $z > 0$  is medium - ②

and is characterised by  $\mu_2 = \mu_0$ ,  $\epsilon_2 = 4\epsilon_0$ .

A wave incidence upon the interface whose electric field is given by

$8 \cos(\omega t - 3x - 4z) \hat{a}_y$  V/m. Find angle of incidence, angle of reflection and angle of transmission, type of polarization,  $\Gamma$ ,  $T$ ,  $\vec{k}_i$ ,  $\vec{k}_r$ ,  $\vec{k}_t$ ,  $\vec{E}_i$ ,  $\vec{E}_r$ .

Ans:

$$\vec{E}_i = 8 \cos(\omega t - 3x - 4z) \hat{a}_y$$

$$\therefore k_x = 3, \quad k_y = 4$$

$$\therefore \beta_1 = \sqrt{k_x^2 + k_y^2}$$

$$\therefore \beta_i = \sqrt{9 + 16} = 5.$$

$$\therefore \beta_i = \omega \sqrt{\mu \epsilon}.$$

$$\therefore \beta_i = \omega \sqrt{\mu \epsilon}.$$

$$\therefore n_i = \sqrt{\frac{\mu}{\epsilon}}.$$

$$\therefore n_i = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$\therefore \boxed{n_i = 12 \cdot \pi} \Omega$$

$$\therefore \boxed{\beta_t = 10}$$

$$n_t = \sqrt{\frac{\mu_0}{4 \epsilon_0}}$$

$$n_t = \frac{120\pi}{2}$$

$$\boxed{n_t = 60\pi} \Omega$$

$$\beta_i = \omega \sqrt{\mu_0 \epsilon_0}$$

$$\therefore \beta_t = \omega \sqrt{\mu_2 \epsilon_2}$$

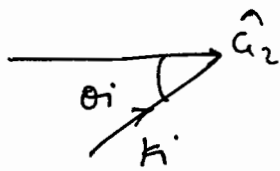
$$\therefore \beta_t = \omega \sqrt{\mu_0 4 \epsilon_0}$$

$$\beta_t = 2 \beta_i$$

→ The unit vector normal to the interface  $\hat{a}_n = \hat{a}_z$ , the vector corresponds to direction of propagation of incident wave is  $\vec{k}_i = 3\hat{a}_x + 4\hat{a}_z$  and lies on z-x plane.

→ The unit vector normal to the interface and  $\vec{k}_i$  lies in z-x plane and hence z-x plane is said to be a plane of incidence given electric field has  $\hat{a}_y$  component and hence the wave is said to be ~~very~~ perpendicular.





$$\therefore \text{By } \vec{k}_i \cdot \hat{a}_2 = |\vec{k}_i| \cdot |\hat{a}_2| \cos \theta_i$$

$$\therefore \cos \theta_i = \frac{4}{5 \times 1}$$

$$\therefore \theta_i = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\therefore \boxed{\theta_i = 36.87^\circ}$$

$\therefore$  By Snell's Law:

$$\therefore \boxed{\theta_i = \theta_r = 36.87^\circ}$$

$$\therefore \frac{\sin \theta_r}{\sin \theta_i} = \sqrt{\frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2}}$$

$$\therefore \sin \theta_r = \frac{1}{2} \times \sin \theta_i$$

$$\therefore \sin \theta_r = 0.3$$

$$\therefore \boxed{\theta_r = 17.46^\circ}$$

$$\therefore \vec{k}_r = -k_x \hat{a}_x + k_z \hat{a}_z$$

$$\therefore k_x = \beta_i \sin \theta_i$$

$$\therefore k_x = 5 \sin 36.87^\circ = 3$$

$$\therefore k_z = 5 \cos 36.87^\circ$$

$$\therefore \boxed{k_z = 4}$$

$$\therefore \vec{k}_t = k_{tz} \hat{a}_z + k_{tx} \hat{a}_x$$

$$\therefore \vec{k}_t = \beta_2 \cos \theta_t \hat{a}_z + \beta_2 \sin \theta_t \hat{a}_x$$

$$\vec{k}_r = -4 \hat{a}_z + 3 \hat{a}_x$$

$$\therefore \vec{E} = \text{max}$$

$$\boxed{\vec{E}_t = 9.5 \hat{a}_2 + 3 \hat{a}_x}$$

$$\therefore \Gamma_{\perp} = \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_2 \cos \theta_i + n_1 \cos \theta_t}$$

$$\begin{aligned} \therefore \Gamma_{\perp} &= \frac{60\pi \cos 36.87^\circ - 120\pi \cos 17.46^\circ}{60\pi \cos 36.87^\circ + 120\pi \cos 17.46^\circ} \\ &= \frac{150 - 359.44}{150 + 359.44} \end{aligned}$$

$$\therefore \boxed{\Gamma_{\perp} = -0.411}$$

$$T_{\perp} = 1 + \Gamma_{\perp}$$

$$\boxed{T_{\perp} = 1.411}$$

$$\therefore E_{r0} = \Gamma_{\perp} E_{i0}$$

$$\therefore E_{r0} = -0.411 E_{i0} = -3.3.$$

$$\therefore \boxed{\vec{E}_r = -3.3 \cos(\omega t + 4x - 3x) \hat{a}_y \text{ V/m.}}$$

$$\therefore E_{t0} = T_{\perp} E_{i0} = 11.3.$$

$$\therefore \boxed{\vec{E}_t = 11.3 \cos(\omega t - 9.5 \hat{a}_2 - 3 \hat{a}_x) \hat{a}_y \text{ V/m.}}$$

## ★ Wave guides:

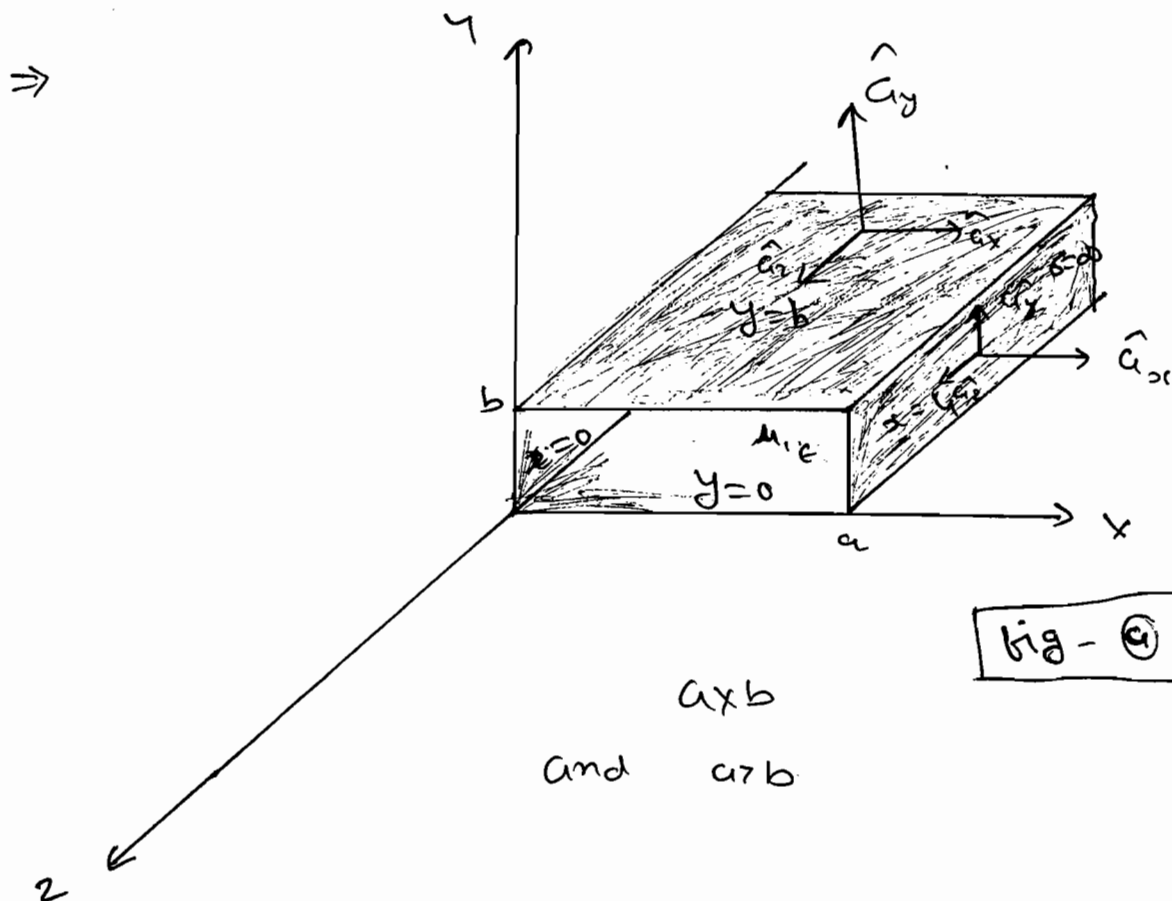
- Waveguides (or) transmission lines which are used at microwave frequencies. These are cylindrical in structure. The preferred cross section of the waveguides are Rectangular, circular or elliptical. No other cross sections are preferred because there are no advantages bound with the other cross sections.
- Waveguides are the examples for the wave propagation through a bounded medium.
- The ~~medium~~ Wave guides has conductor boundaries.
- The medium bet<sup>n</sup> the conductor boundaries is non-conducting. It may be filled with dielectric material.
- For example Air filled waveguide.
- The energy progresses along the length of the waveguide. To investigate electromagnetic field behaviour inside the wave-guide, Maxwell's eq<sup>n</sup>s and the wave eq<sup>n</sup>s are solved subjected to the boundary

Conditions (a<sub>2</sub>) electrical & magnetic fields across the conductor boundaries.

→ We know that tangential component of electric field and normal component of magnetic fields vanishes across a conductor boundaries.

→ We consider rectangular waveguides.

→ Rectangular waveguides means its cross section is Rectangular.



→ In general,

$$\vec{E} = E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z.$$

$$\vec{H} = H_x \hat{a}_x + H_y \hat{a}_y + H_z \hat{a}_z.$$

→ Fig. Shows a rectangular waveguide with the cross section ~~dimension~~ <sup>dimension</sup>  $a \times b$  and  $a \times b$ .

→ There exist four conducting plane which are located at  $x=0$ ,  $x=a$ ,  $y=0$ ,  $y=b$ .

→ This conducting planes are assumed to have infinite conductivity & medium bet<sup>n</sup> the conducting plane is linear, homogenous, isotropic, Charge free, Non-Conducting. For example in a air filled waveguide.

→ There exist conductor interfaces which are located at  $x=0$ ,  $x=a$ ,  $y=0$ ,  $y=b$ .

→ Tangential component of electric fields & normal component of magnetic field vanishes across the conductor interfaces.

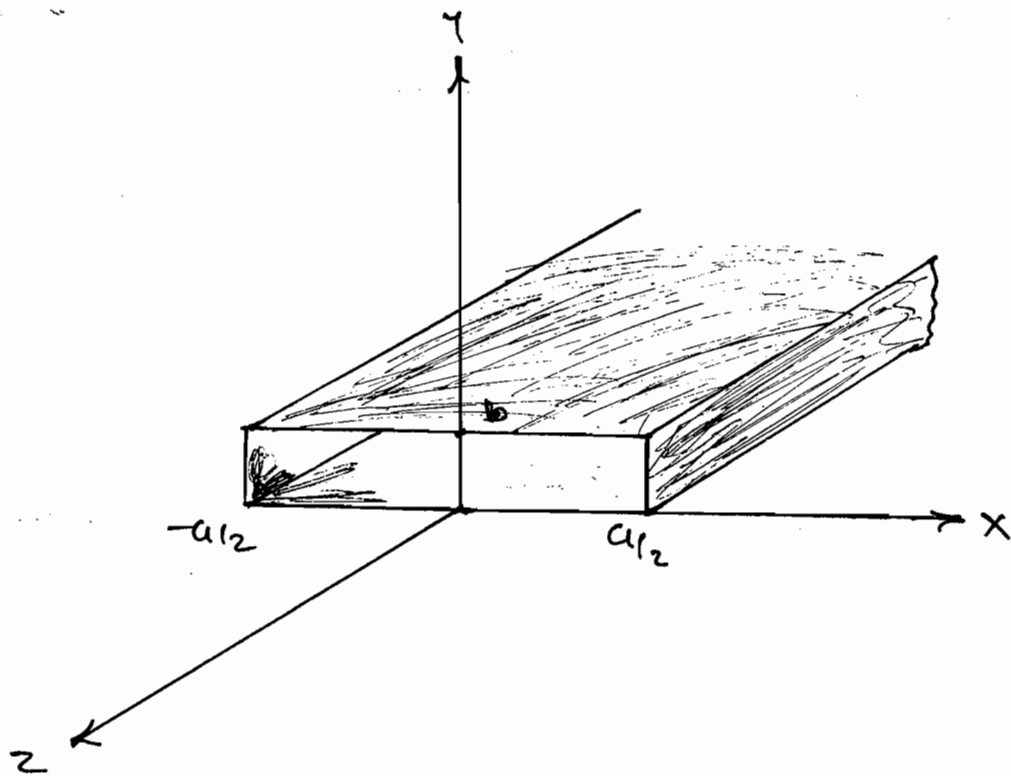
→ at  $x=0$ ,  $x=a$

$$E_y = 0, E_z = 0, H_x = 0.$$

→ at  $y=0$ ,  $y=b$ .

$$E_x = 0, E_z = 0, H_y = 0.$$

Fig-⑥



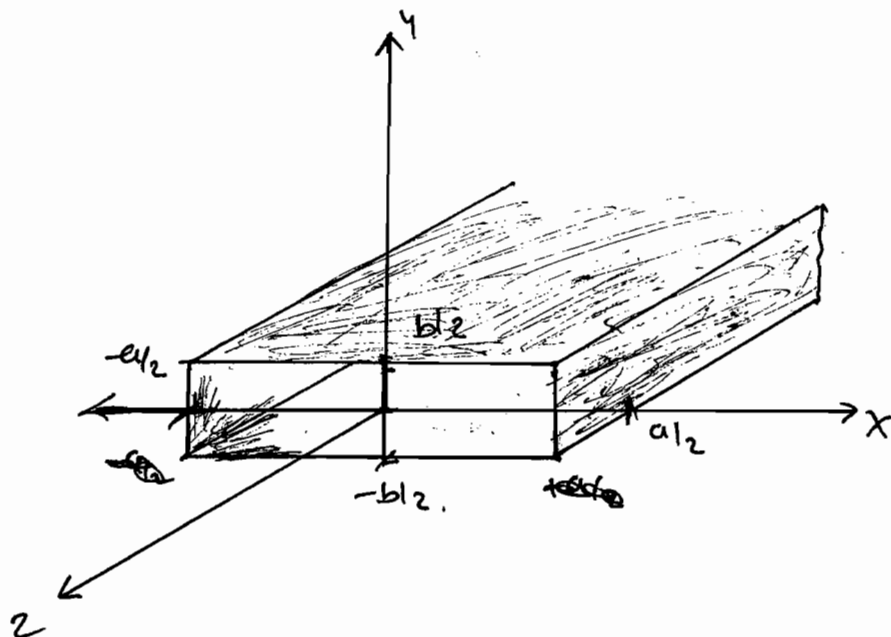
→ at  $x = \pm a/2$ , ~~y~~

$$E_y = 0, E_z \neq 0, H_x = 0$$

→ at  $y = 0$  &  $y = b$ .

$$\therefore E_x = 0, E_z = 0, H_y = 0.$$

Fig-⑦



→ At  $x = \pm a/2$ .

$$\therefore E_y = 0, E_z = 0, H_x = 0$$

→ At  $y = \pm b/2$ .

$$\therefore E_x = 0, E_z = 0, H_y = 0.$$

→ We consider fig-(a) for our analysis.

→ As shown in the figure -(a) there exist 4 - Conducting planes. which are located at  $x=0, x=a, y=0, y=b$ .

→ This conducting planes are assume to have infinite conductivity and the medium bet<sup>n</sup> conducting planes is linear, homogenous, isotropic, charge free and non-conducting.

→ Writing the Maxwell's eq<sup>n</sup>s for the medium assumed bet<sup>n</sup> the conducting planes.

→ ①  $\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$

②  $\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t}$  (Non conducting medium is assumed  $\sigma=0$ ).

③  $\nabla \cdot \vec{D} = 0$  ( $\because$  charge free medium is assumed  $\rho_v=0$ ).

$$\rightarrow \epsilon \nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{E} = 0 \quad (\text{homo genim } \text{medim}).$$

$$\textcircled{4} \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{H} = 0 \quad (\text{homo genim medim}).$$

$\rightarrow$  Taking curl on eqn-① both the side,

$$\nabla \times \nabla \times \vec{E} = -\mu \nabla \times \frac{\partial \vec{E}}{\partial t}.$$

$$\therefore \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \vec{E}).$$

$\swarrow$   $\searrow$   
 $0$   $\nabla \times \frac{\partial \vec{E}}{\partial t}$

$$\therefore \nabla^2 \vec{E} = \mu \epsilon \frac{\partial \vec{E}}{\partial t}.$$

Her

$$\nabla^2 \vec{H} = \mu \epsilon \frac{\partial \vec{H}}{\partial t}.$$

Vector wave eqn

$\rightarrow$  Expanding wave eqn in the cartesian co-ordinates because the geometry is well suited for cartesian co-ordinates.

$$\frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}.$$

$$\frac{\partial^2 \vec{H}}{\partial x^2} + \frac{\partial^2 \vec{H}}{\partial y^2} + \frac{\partial^2 \vec{H}}{\partial z^2} = \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}.$$

$$\rightarrow \vec{E} = \text{Re} [\vec{E}_0 e^{j\omega t}].$$

Writing the above eqn in phasor form.



$$\frac{\partial^2 \bar{E}_x}{\partial x^2} + \frac{\partial^2 \bar{E}_x}{\partial y^2} + \frac{\partial^2 \bar{E}_x}{\partial z^2} = -\omega^2 \mu \epsilon \bar{E}_x$$

similarly  $\frac{\partial^2 \bar{H}_x}{\partial x^2} + \frac{\partial^2 \bar{H}_x}{\partial y^2} + \frac{\partial^2 \bar{H}_x}{\partial z^2} = -\omega^2 \mu \epsilon \bar{H}_x$

→ When a wave is propagating along z-direction in the unbounded medium, we have conclude that the partial variations of any field component w.r.t.  $x$  &  $y$  vanishing.

→ Where as in the case of fig-(a) if we assume the energy is being progressing along the length of the wave guide i.e. along z-direction, since there are conducting walls which are located at  $x=0$ ,  $x=a$ ,  $y=0$  and at  $y=b$ , then it is not possible to assume partial variation of any field component w.r.t. to  $x$  &  $y$  to be zero.

NOTE:

→ We have assumed that energy is being progressing along z-direction the variations along  $z$  can be approximated as  $e^{-\gamma z}$

where  $\bar{r} = \bar{\alpha} + j\bar{\beta}$

→  $\bar{\gamma}$  is not a vector quantity to distinguish ~~with~~  $\bar{\gamma}$  of the waves are propagating in waveguide  $\bar{\gamma}$  is used.

$$\rightarrow \bar{E}_s(x, y, z) = \bar{E}_s(x, y) \cdot e^{-\bar{\gamma}z}$$

$$\frac{\partial \bar{E}_s}{\partial z} = -\bar{\gamma} \bar{E}_s$$

$$\frac{\partial^2 \bar{E}_s}{\partial z^2} = \bar{\gamma}^2 \bar{E}_s$$

→ The wave eqs. can be written as

$$\left. \begin{aligned} \frac{\partial^2 \bar{E}_s}{\partial x^2} + \frac{\partial^2 \bar{E}_s}{\partial y^2} + \bar{\gamma}^2 \bar{E}_s + \omega^2 \mu \epsilon \bar{E}_s &= 0 \\ \text{or} \quad \frac{\partial^2 \bar{H}_s}{\partial x^2} + \frac{\partial^2 \bar{H}_s}{\partial y^2} + h^2 \bar{H}_s &= 0 \end{aligned} \right\} \begin{array}{l} \text{These are} \\ \text{2nd order} \\ \text{2-DIM} \\ \text{PDEs.} \end{array}$$

$$\text{where, } h^2 = \bar{\gamma}^2 + \omega^2 \mu \epsilon$$

→ The above second order PDE can be split into two simple 2<sup>nd</sup> order differential eqs. They will be in the form of harmonic eqs. Sol<sup>n</sup> of a harmonic eq<sup>n</sup> may take either sine or cosine or exponent. We consider sine and cosine forms.

→ First two Maxwell eq<sup>n</sup> in phasor form:

$$\nabla \times \bar{E}_s = -j\omega\mu\bar{H}_s$$

&

$$\nabla \times \bar{H}_s = j\omega\epsilon\bar{E}_s$$

$$\begin{aligned} \frac{\partial E_{zs}}{\partial y} - \left( \frac{\partial E_{ys}}{\partial z} \right) &= -j\omega\mu H_{xs} & \text{set ②} & \rightarrow \frac{\partial H_{zs}}{\partial y} - \left( \frac{\partial H_{ys}}{\partial z} \right) = j\omega\epsilon E_{xs} & \text{set ①} \\ -\gamma E_{xs} \left( \frac{\partial E_{xs}}{\partial z} \right) - \frac{\partial E_{zs}}{\partial x} &= -j\omega\mu H_{ys} & & \rightarrow \left( \frac{\partial H_{xs}}{\partial z} \right) - \frac{\partial H_{zs}}{\partial x} = j\omega\epsilon E_{ys} \\ \frac{\partial E_{ys}}{\partial x} - \frac{\partial E_{xs}}{\partial y} &= -j\omega\mu H_{zs} & & \frac{\partial H_{ys}}{\partial x} - \frac{\partial H_{xs}}{\partial y} = j\omega\epsilon E_{zs} \end{aligned}$$

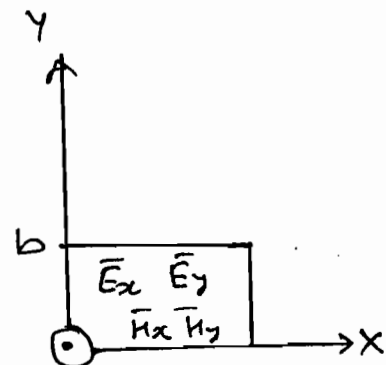
→ Using set ①  $E_{xs} \rightarrow$  can be written in terms of  $E_{zs}, H_{zs}$ .  
 $H_{ys} \rightarrow$  can be written in terms of  $E_{zs}, H_{zs}$ .

→ Using set ②  $H_{xs} \rightarrow$  can be written in terms of  $E_{zs}, H_{zs}$ .  
 $H_{zs} \rightarrow$  can be written in terms of  $E_{zs}, H_{zs}$ .

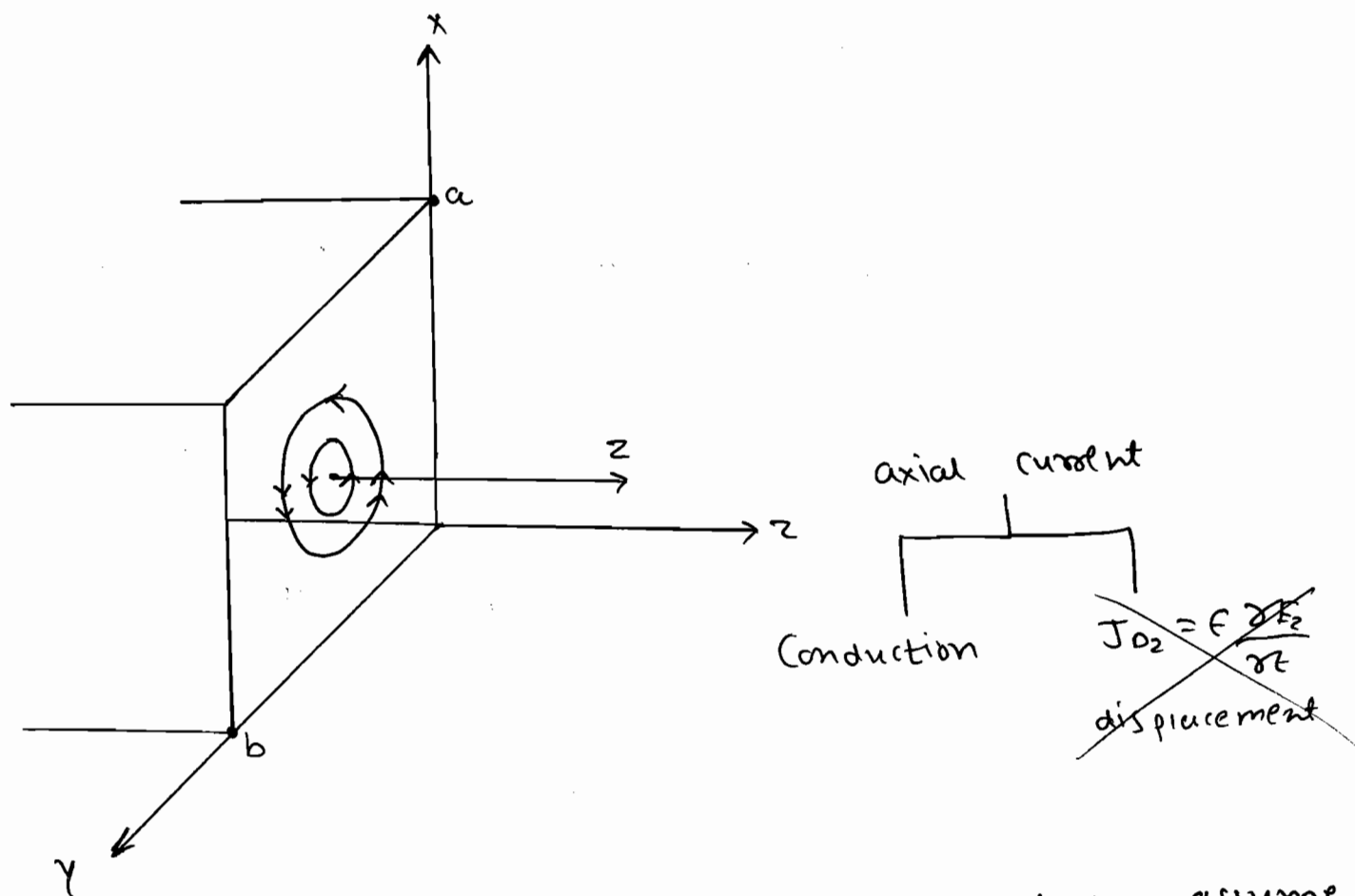
$$\bar{E}_s = \bar{E}_{s0}(x, y) e^{-\gamma z}$$

$$\bar{E}_{ys} = E_{ys0}(x, y) e^{-\gamma z}$$

$$\frac{\partial E_{ys}}{\partial z} = -\gamma E_{ys}$$



~~TEM to z~~  
 $E_z = 0, H_z = 0$



→ With reference to fig-①, we have assumed that the energy is been progressing along  $z$  direction the transverse plane would be  $xy$  plane. The field component which lies in the transverse plane are  $E_x, E_y, H_x, H_y$

→ The field component along the direction of propagation of energy are  $E_z$  &  $H_z$ .

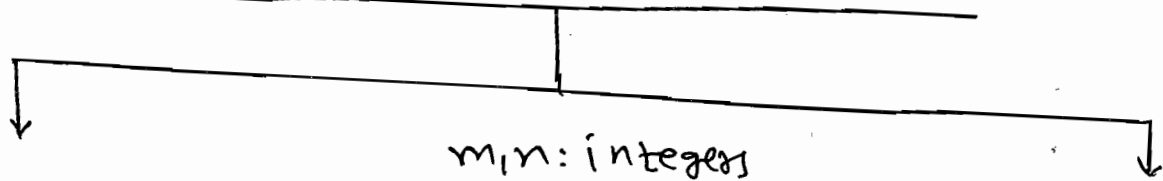
→ As shown above the transverse field components have been represented in terms of the field components along the directions of propagation of energy (i.e.)  $E_x, E_y, H_x, H_y$  have been represented in terms of

$E_z \neq H_z$ . To have a TEM wave propagation to  $z$ , then  $E_z$  and  $H_z$  must be zero.

→ If these are zero, no field component is existing inside the waveguide.

✓ → We can conclude that TEM wave propagation is impossible to exist through any cylindrical waveguide system of any cross section with no central conductor.

### Waves in the Waveguide



→  $TM_{mn}$  waves

→ Transverse magnetic waves.

→  $H_z = 0$

→ (i.e) magnetic field lies entirely in the transverse plane.

→ These are called E-waves

→  $TE_{mn}$  waves

→ Transverse Electric waves

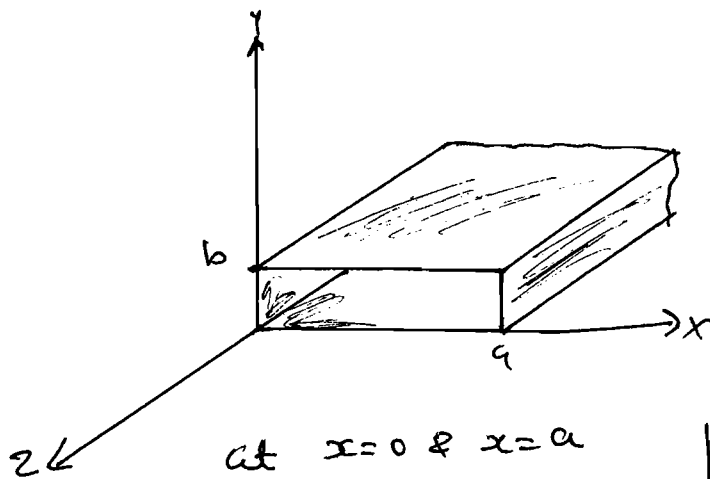
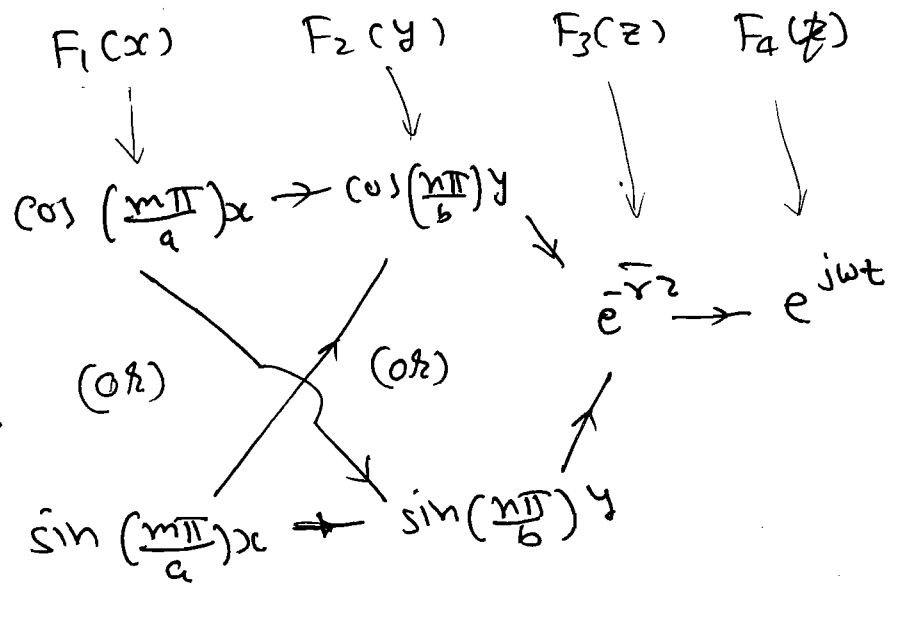
→  $E_z = 0$

(i.e) Electric field lies entirely in the transverse plane.

→ These are also called H-waves.

→

Any field  
Components is  
a product of  
four independent  
functions



at  $x=0$  &  $x=a$   
 $E_y=0, E_z=0$   
 $H_x=0$

at  $y=0, y=b$   
 $E_x=0, E_z=0$   
 $H_y=0$

\* TM<sub>mn</sub> Waves:

→  $E_{xs} = E_{x0} \cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-\gamma z}$  ,  $H_{xs} = -\frac{E_{xs}}{\eta_{TMmn}}$

→  $E_{ys} = E_{y0} \sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{-\gamma z}$  ,  $H_{ys} = \frac{E_{ys}}{\eta_{TMmn}}$

→  $E_{zs} = E_{z0} \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-\gamma z}$  ,  $H_{zs} = 0$

→  $\frac{E_x}{H_y} = \eta_{TMmn} = -\frac{E_y}{H_x}$

→ TM<sub>mn</sub>

→  $\eta_{TM_{mn}}$  is the characteristic wave impedance of  $TM_{mn}$  waves.

\* TE<sub>mn</sub> Waves.

$$\rightarrow E_{xs} = E_{x0} \cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-\gamma z}, \quad H_{xs} = -\frac{E_{ys}}{\eta_{TE_{mn}}}$$

$$\rightarrow E_{ys} = E_{y0} \sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{-\gamma z}, \quad H_{ys} = \frac{E_{xs}}{\eta_{TE_{mn}}}$$

$$E_{zs} = 0, \quad H_{zs} = H_{z0} \cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{-\gamma z}$$

→ (+z)

$$\frac{E_x}{H_y} = \eta_{TE_{mn}} = -\frac{E_y}{H_x}$$

→  $\eta_{TE_{mn}}$  is the characteristic wave impedance of  $TE_{mn}$  waves.

\* Characteristics of TE<sub>mn</sub> waves & TM<sub>mn</sub> waves.

$$\gamma^2 + \omega^2 \mu \epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\therefore \boxed{\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon}}$$

We have known →  $\boxed{\gamma = \alpha + j\beta}$

$m, n$ : integers (mode).

$a, b$ : cross sectional dimensions.

$\mu, \epsilon$ : medium properties (Linear, homogeneous, isotropic).

$\omega = 2\pi f$  ← freq. of the wave.

$$\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon}$$

→ If  $\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 > \omega^2 \mu \epsilon \Rightarrow \bar{\gamma}$  is purely real  
 $\Rightarrow \alpha = 0 \Rightarrow$  No propagation takes place

→ If  $\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 < \omega^2 \mu \epsilon \Rightarrow \bar{\gamma}$  is purely imaginary  
 $\Rightarrow \alpha \neq 0 \Rightarrow$  ~~No~~ propagation takes place.

→ For a given waveguide,

→ From the above we can conclude that for the low freq  $\bar{\gamma}$  is purely real therefore propagation through the waveguide is not possible.

→ At high freq.  $\bar{\gamma}$  is purely imaginary therefore propagation is allowed through the waveguide.



→ we defined a limiting freq. called cut-off freq. i.e. at  $f = f_c$  (or)  $\omega = \omega_c \Rightarrow \boxed{\bar{r} = 0}$

$$\omega_c^2 \mu \epsilon = \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2.$$

$$\therefore \omega_c = \sqrt{\frac{1}{\mu \epsilon} \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right]}$$

$$(or) \quad f_c = \frac{1}{2\pi \sqrt{\mu \epsilon}} \times \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right]^{\frac{1}{2}}$$

→ Corresponds to  $f_c$  we define the cut-off wavelength such that

$$f_c \lambda_c = \frac{1}{\sqrt{\mu \epsilon}} = v_0.$$

$$(or) \quad \lambda_c = \frac{1}{f_c \times \sqrt{\mu \epsilon}}$$

$$= \frac{2\pi}{\left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right]^{\frac{1}{2}}}$$

$$\therefore \lambda_c = \frac{2}{\left[ \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 \right]^{\frac{1}{2}}}$$

$f_c$  or  $\lambda_c$ : They purely depend upon the medium properties i.e.  $m, n, a, \& b$ .

(or) physical properties of the waveguide.

→ If  $f'$  is the freq. of the wave and  $\lambda$  is the wavelength of the wave such that  $f\lambda = \frac{1}{\sqrt{\mu\epsilon}} = v_0$ .

→ If  $f > f_c$  (or)  $\lambda < \lambda_c \Rightarrow$  propagation is allowed through the waveguide.

→ If  $f < f_c$  (or)  $\lambda > \lambda_c \Rightarrow$  propagation don't allow through the wr.

→ Thus, the Waveguide is simulating the action of a high pass filter.

→ For  $f > f_c$ ,

$$\bar{\gamma} = j\beta.$$

$$\therefore \beta = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \text{ rad/s.}$$

$$\therefore \bar{v} = \frac{\omega}{\beta} \text{ m/s.}$$

$$\therefore \bar{\lambda} = \frac{2\pi}{\beta};$$

$\bar{\lambda} =$  guide wave length. (or)

wavelength measured inside the waveguide.

$$\eta_{TE} = \frac{\eta}{\sqrt{1 - (f_c/f)^2}} \rightarrow \infty$$

$$\eta_{TM} = \eta \sqrt{1 - (f_c/f)^2} \rightarrow 0$$

Where  $\eta = \sqrt{\frac{\mu}{\epsilon}}$  ||  $\eta_{TE} \cdot \eta_{TM} = \eta^2$

At  $f = f_c \Rightarrow \bar{\beta} \rightarrow 0$

$$\bar{v} \rightarrow \infty$$

$$\bar{\lambda} \rightarrow \infty$$

$$\eta_{TE} \rightarrow \infty$$

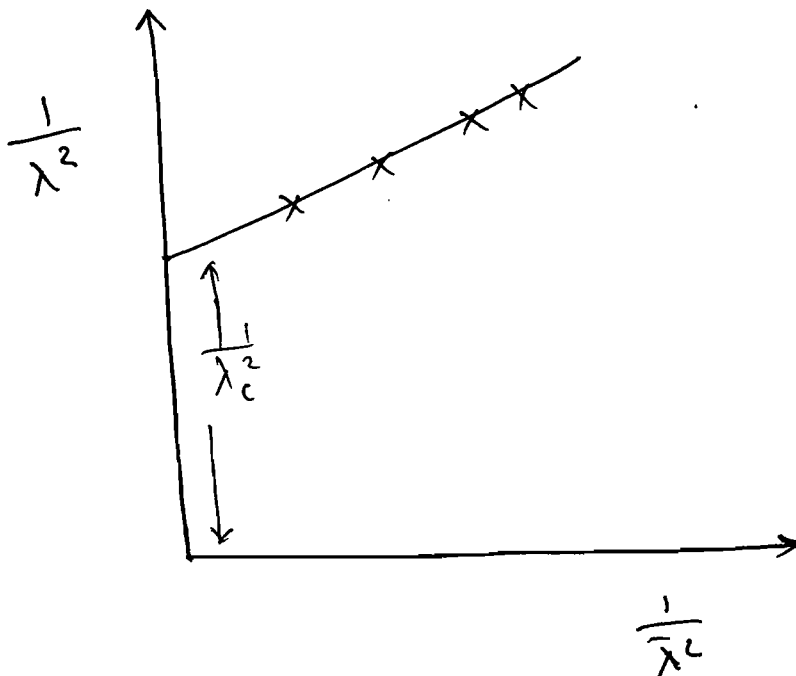
$$\eta_{TM} \rightarrow 0$$

→ The relation b/w  $\lambda, \lambda_c, \bar{\lambda}$

$$\frac{1}{\bar{\lambda}^2} = \frac{\bar{\beta}^2}{(2\pi)^2} = \frac{(\omega^2 \mu \epsilon)}{(2\pi)^2} - \frac{(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2}{(2\pi)^2}$$

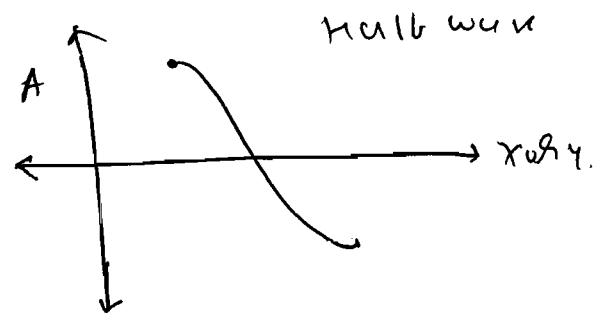
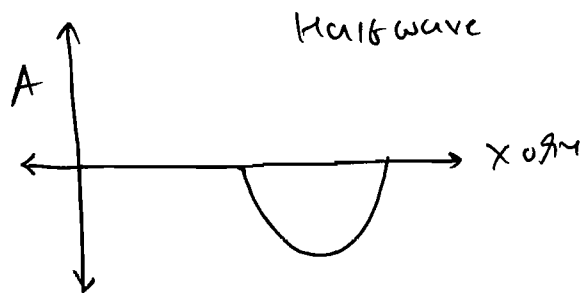
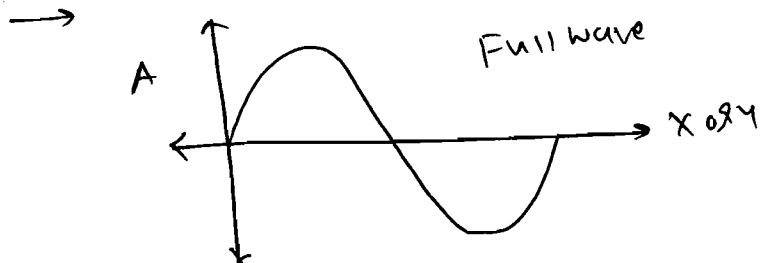
$$\Rightarrow \frac{1}{\bar{\lambda}^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}$$

(or)  $\frac{1}{\lambda^2} = \frac{1}{\bar{\lambda}^2} + \frac{1}{\lambda_c^2}$



# \* Significance of $m, n$ :

→  $m$ : no. of half field variations along  $x$ .  
 $n$ : no. of half field variations along  $y$ .



→  $m=1, n=0$ .

TE<sub>10</sub>

$$\boxed{f > f_c}$$

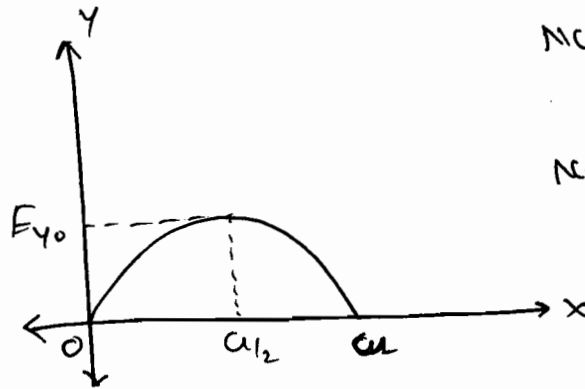
○  $E_{xs} = 0$ .

○  $E_{ys} = E_{y0} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}$ .

○  $E_{zs} = 0$

$$|\bar{E}_3| = |E_{y3}| = E_{y0} \sin\left(\frac{\pi x}{a}\right).$$

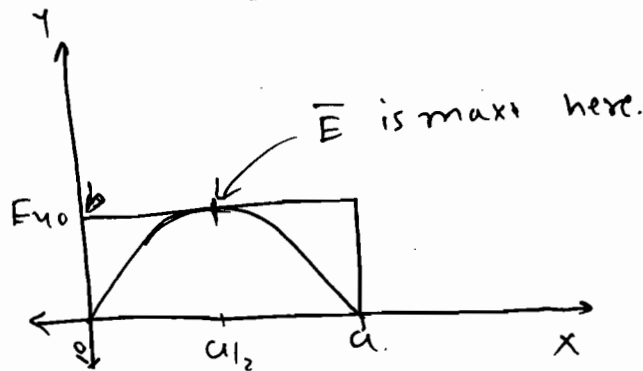
(say at  $z=0$ )



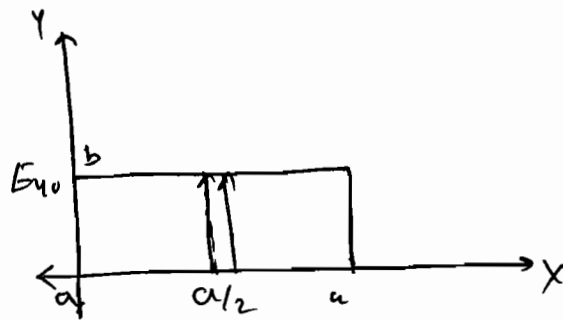
No. of half field along  $x=a$

No. of half field along  $y=0$ .

↓



(or)



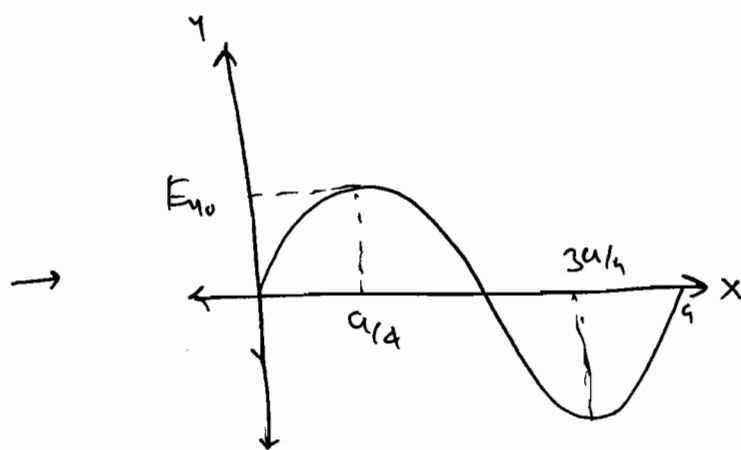
→ Let us take  $m=2$ ,  $n=0$ .

TE<sub>20</sub>,  $f > f_c$

$$E_{y3} = E_{y0} \sin\left(\frac{2\pi x}{a}\right) e^{-j\beta z}$$

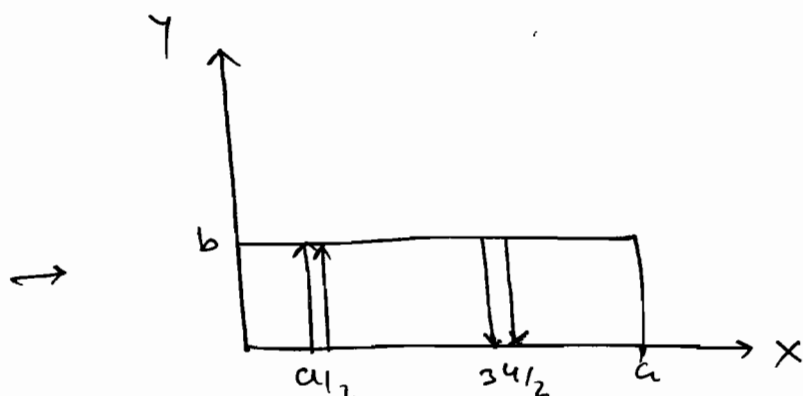
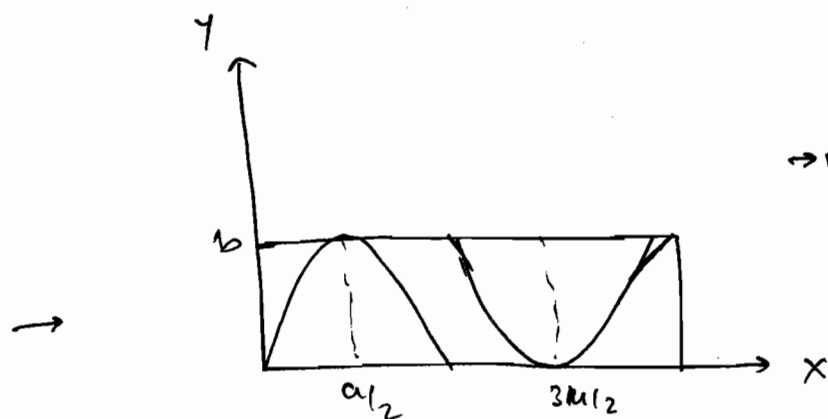
$$|\bar{E}_3| = |E_{y3}| = E_{y0} \sin\left(\frac{2\pi x}{a}\right).$$

(say at  $z=0$ )



→ No. of half fields  
along  $x = 2$ .

→ No. of half fields  
along  $y = 0$ .



\* Let us take  $m=0, n=1$ :

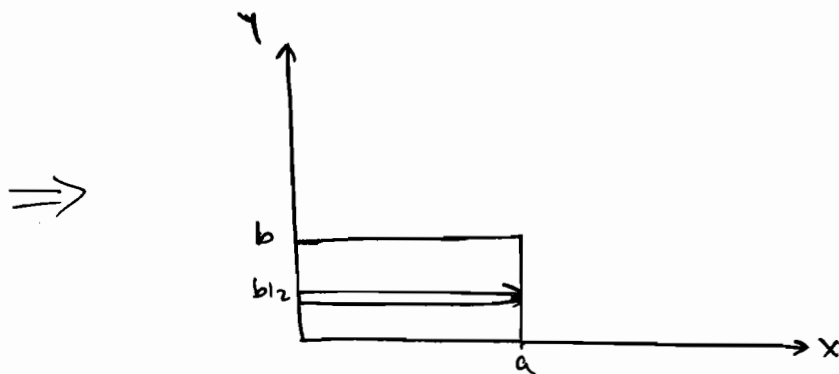
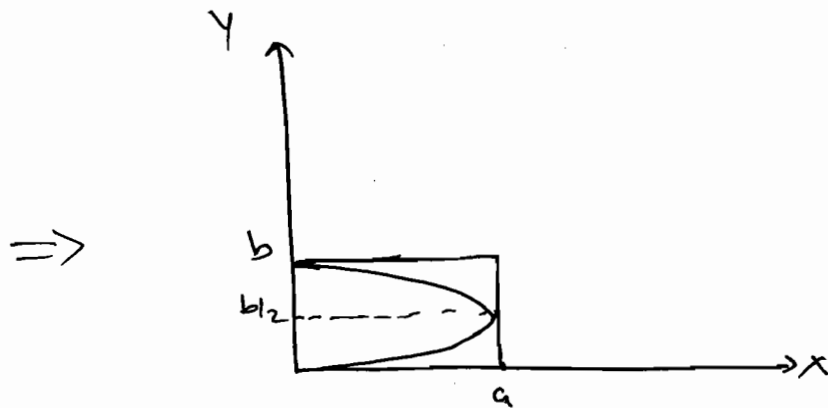
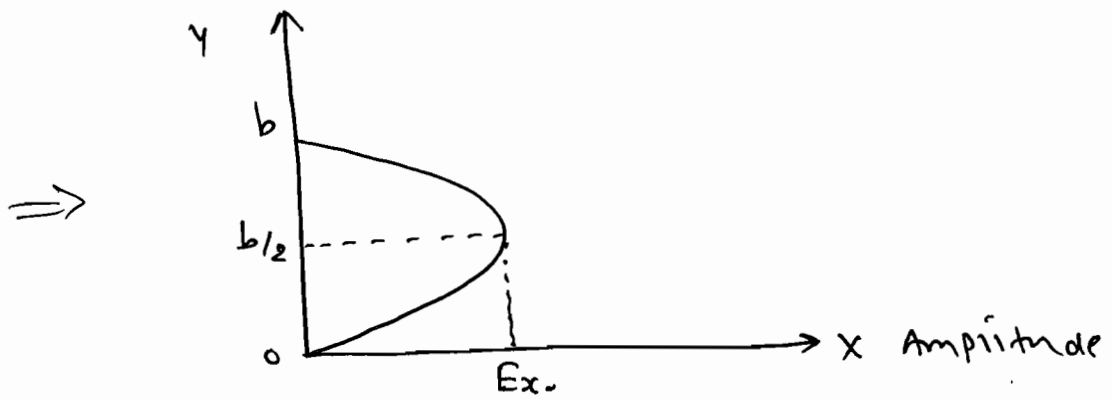
→  $TE_{01}$ ,  $f > f_c$ .

$$\therefore E_y, \bar{E}_x = E_{x0} \sin \frac{\pi}{b} y \cdot e^{-j\beta z}$$

$$\therefore |\bar{E}_s| = |\bar{E}_x| = E_{x0} \sin \frac{\pi}{b} y$$

No. of half fields along  $y = 1$

No. of half-fields along  $x = 0$ .



### \* Dominant Mode:

$\rightarrow$  It is the lowest possible Propagating mode and must have lowest cut-off frequency.  $\Rightarrow$  In the dominant mode it is possible to transfer maximum energy from source to the load.

$\rightarrow$  In the case of the rectangular waveguide

$TE_{10}$  is the dominant mode.

→ In the case of TM<sub>mn</sub> the lowest possible propagating mode is TM<sub>11</sub> i.e. minimum value of m, n are at least 1, 1 respectively.

Therefore, the lowest possible mode in the case of TM<sub>mn</sub> is TM<sub>11</sub>. This is not called dominant mode.

→ Ex-1 A Rectangular waveguide with a cross-section dimensions 4 cm x 7 cm is air filled. The waveguide is intended to operate at the following frequencies. (i) 3000 MCPS. (ii) 6000 MCPS. What are the different modes that can be propagated through this WR at the above freq.

Ans: To have propagation  $f > f_c$  (or)  $\lambda_c < \lambda$ .

$$\therefore f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}} \times \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right]^{1/2} < f.$$

$\therefore$  air filled so  $\mu = \mu_0$ ,  $\epsilon = \epsilon_0$

$$\therefore f_c = \frac{1 \times \pi}{2\pi\sqrt{\mu_0\epsilon_0}} \times \left[ \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 \right]^{1/2} < f$$

$$\therefore \frac{3 \times 10^8}{2} \times \left[ \left( \frac{m}{0.04} \right)^2 + \left( \frac{n}{0.07} \right)^2 \right]^{1/2} < f.$$

$$\therefore \boxed{16m^2 + 49n^2 < 31.4}$$



| Values of $m$ | Values of $n$ | Satisfaction. | possible mode |
|---------------|---------------|---------------|---------------|
| 1             | 0             | Yes.          | $TE_{10}$ .   |
| 0             | 1             | No            |               |
| 1             | 1             | No            |               |

(ii)  $f = 6000 \text{ MCPS.}$

$f = 6 \times 10^9 \text{ Hz.}$

$a = 0.07 \text{ m, } b = 0.04 \text{ m.}$

$\rightarrow 16m^2 + 9n^2 < 125.6$

| Values of $m$ | Values of $n$ | Satisfaction        |
|---------------|---------------|---------------------|
| 0             | 1             | yes                 |
| 1             | 0             | yes.                |
| 1             | 1             | yes.                |
| 2             | 0             | yes.                |
| 2             | 1             | yes.                |
| 2             | 2             | <del>yes.</del> No  |
| 0             | 2             | <del>yes.</del> NO. |

$\rightarrow$  possible modes.

$TE_{10}, TE_{01}, \textcircled{TE_{11}}, TE_{20}, \textcircled{TE_{21}}, TE_{22}, TE_{32},$   
 $\textcircled{TM_{11}}, \textcircled{TM_{21}},$

### \* Degenerate modes:

→ If different modes are having same cut-off freq. then those modes are said to be degenerate modes.

→ In the above example  $TE_{11}$ ,  $TM_{11}$  and  $TE_{21}$ ,  $TM_{21}$  are degenerate modes respectively.

### \* Evanescent wave.

→ Evanescent waves are the modes which can not be propagated through the waveguide.

Ex-1 A Rectangular wave guide with a cross-section dimension  $0.9'' \times 0.4''$  is air filled and is operating in the dominant mode. find cut-off freq.?  
decide a  $10 \text{ GHz}$  wave can be propagated or not? If it is propagated calculate

Ans:  $\beta$ ,  $\lambda$ ,  $n_{TE_{10}}$  and  $\bar{v}$ .

→ The expression for the avg. power transport through a rectangular waveguide in the dominant mode is given by

$$W_{avg} = \frac{E_0^2}{4\eta_{TE_{10}}} ab \text{ Watts.}$$

$E_0$ : Peak Electric field.

$a, b$ : dimension of c/s.

$\eta_{TE_{10}}$ : Characteristic wave impedance.

Ex-1 A Rectangular waveguide with c/s dimensions  $2\text{cm} \times 1\text{cm}$  is operating in the dominant mode at the freq.  $30\text{ MHz}$ . it transport energy at the rate of  $0.5\text{ HP} = 373\text{ Watts}$  ( $1\text{ HP} = 746\text{ Watts}$ ). What is the peak value of electric field occurring inside the waveguide.

Ans:  $f = 30\text{ MHz} = 3 \times 10^7\text{ Hz}$ .

$a = 2\text{cm} = 0.02\text{m}$

$b = 1\text{cm} = 0.01\text{m}$ .

$W_{avg} = 0.5\text{ HP} = 373\text{ Watts}$ .

$$\lambda_c = \frac{2}{\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{1/2}} \quad m=1, n=0$$

$$\therefore \lambda_c = 2a.$$

$$\boxed{\lambda_c = 0.04 \text{ m.}}$$

$$\therefore \lambda = \frac{v_0}{f} = \frac{3 \times 10^8}{3 \times 10^{10}} = 10^{-2} = 0.01 \text{ m.}$$

$$\therefore \boxed{\lambda_c > \lambda} \text{ so, @ TE}_{10} \text{ mode is propagated.}$$

$$\text{Now, } \eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \, \Omega$$

$$\begin{aligned} \therefore \eta_{\text{TE}_{10}} &= \frac{\eta}{\sqrt{1 - (f_c/f)^2}} \\ &= \frac{\eta}{\sqrt{1 - (\lambda_0/\lambda_c)^2}} \\ &= \frac{120\pi}{\sqrt{1 - \left(\frac{0.01}{0.04}\right)^2}} \end{aligned}$$

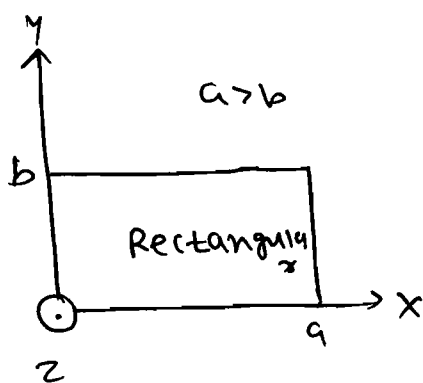
$$\therefore \eta_{\text{TE}_{10}} = \frac{120 \times 2\pi}{\sqrt{3}}.$$

$$\boxed{\eta_{\text{TE}_{10}} = 389 \, \Omega.}$$

$$\rightarrow E_{y0} = \sqrt{\frac{W_{\text{avg}} 4 \eta_{\text{TE}_{10}}}{ab}}$$

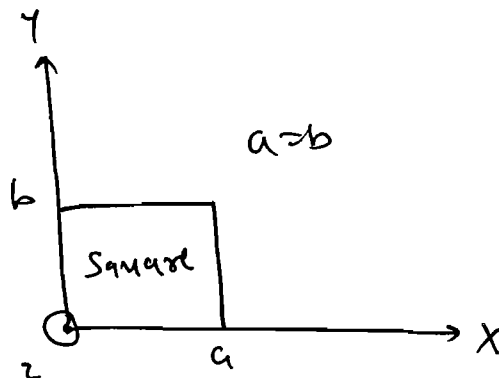
$$\therefore \boxed{E_{y0} = 53 \text{ kV/m.}}$$

\*



$$\lambda_{c10} = 2a$$

$\boxed{TE_{10}}$



$$\lambda_{c10} = 2a$$

$\boxed{TE_{01}}$

$$\lambda_{c01} = 2b$$

$$\lambda_{c01} = 2b = 2a$$

→ The mode properties of  $TE_{10}$  and  $TE_{01}$  are identically same in the case of a square waveguide whereas these are different in the rectangular waveguide. therefore different modes are not possible with square waveguides.

→ The wave propagation through the waveguide is by means of total internal reflection bet<sup>n</sup> the walls.

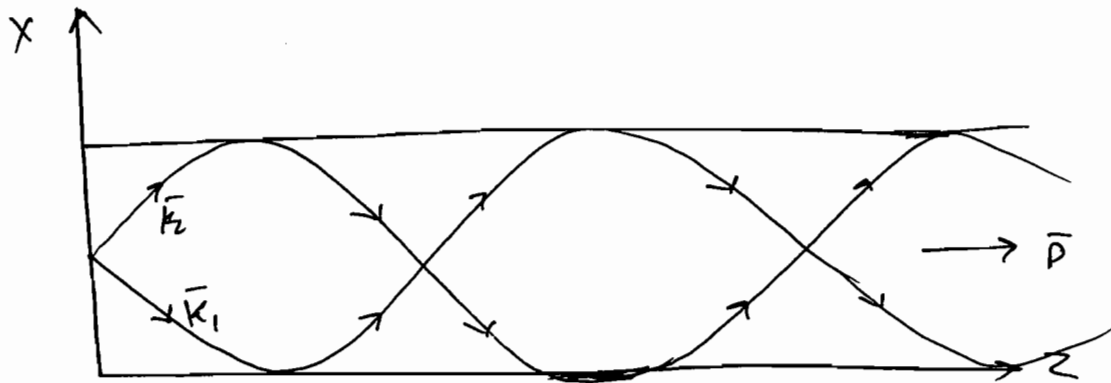
$$\begin{aligned} \rightarrow E_y &= E_{y0} \sin \frac{\pi x}{a} \cdot e^{-j\beta z} \\ &= E_{y0} \frac{e^{j\pi x/a} - e^{-j\pi x/a}}{2j} \cdot e^{-j\beta z} \end{aligned}$$

$$\therefore E_{ys} = E_{y0}' \left[ e^{j(\frac{\pi x}{a} - \bar{\beta} z)} - e^{-j(\frac{\pi x}{a} + \bar{\beta} z)} \right]$$

$$\therefore E_y = \text{Re} \left[ E_{y0}' \left\{ e^{j(\omega t + \frac{\pi x}{a} - \bar{\beta} z)} - e^{j(\omega t - \frac{\pi x}{a} - \bar{\beta} z)} \right\} \right]$$

$$\therefore E_y = \text{Re} \left[ E_{y0}' \left[ e^{j(\omega t - \bar{K}_1 \cdot \bar{r})} - e^{j(\omega t - \bar{K}_2 \cdot \bar{r})} \right] \right]$$

$$\bar{K}_1 = -\frac{\pi}{a} \hat{a}_x + \bar{\beta} \hat{a}_z \quad , \quad \bar{K}_2 = \frac{\pi}{a} \hat{a}_x + \bar{\beta} \hat{a}_z$$



→ Energy transfer takes place along the length of the waveguide.

→  $\bar{v}$

$v_0$

$v_g \leftarrow$  group velocity. at which energy propagates.

$$\bar{v} \cdot v_g = v_0^2$$

# ★ TWO WIRE TRANSMISSION LINE:

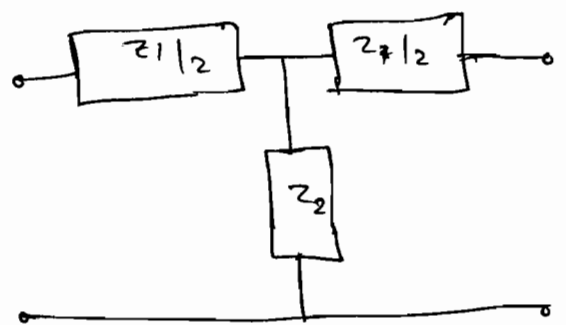
→ Even if the ports are interchange if the electric properties of the network are unaltered rather not disturb then a network is said to be symmetrical.



inp → out

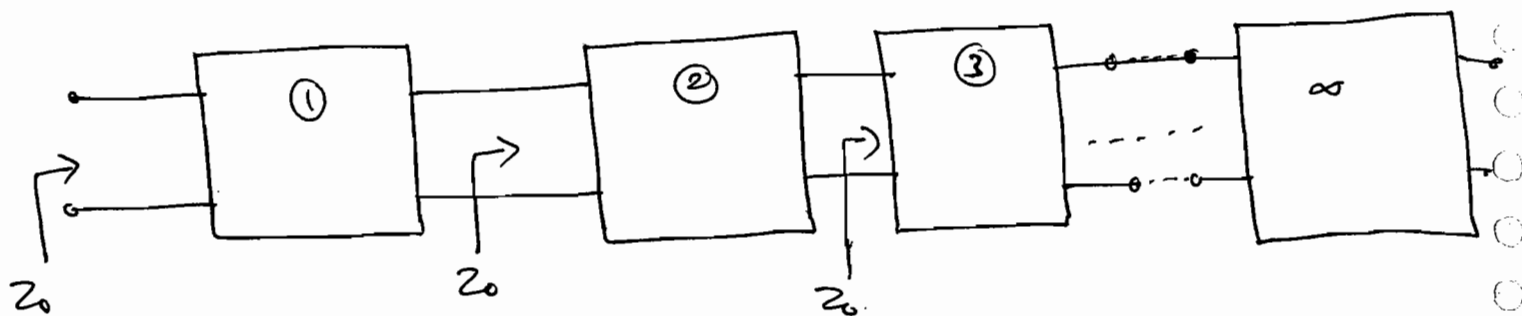
out ← inp

(Symm. N/w)



(Symm. N/w)

⇒ Def<sup>n</sup> of Characteristic Impedence  $Z_0$ :



→ Infinite no. of identical symm. N/w are connected in cascade. The impedance seen at the inp of the ① N/w is defined as  $Z_0$ .

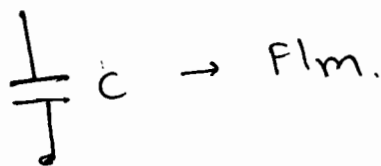
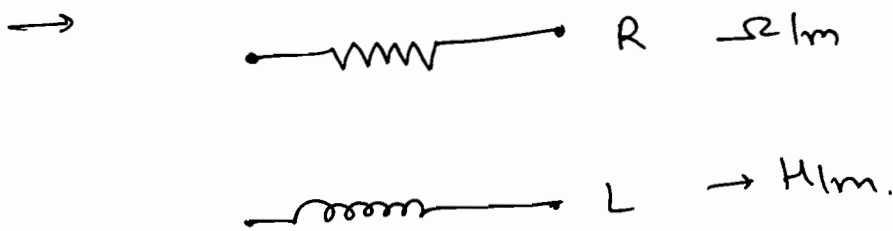
# \* Alternate Def<sup>n</sup>:

→ When a symmetrical N/w is terminated by  $Z_0$  then the impedance seen at the input of the network is also equal to  $Z_0$ . ~~These~~

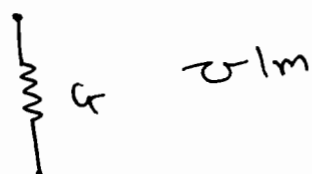


input end  
Sending end  
Source end  
Transmitting end

Output end  
Receiving end  
Load end  
terminating end.



Charges built on wire.  
distributed N/w.



→ The purpose of a transmission line is to transport energy from source to the load.



→ When a voltage is applied across this two wires the current passes through them. When a current is passing through a conductor there exists voltage drop between the conductors. Significant voltage drop indicates that the line is having series resistance  $R$ .

→ When a current is passing through a conductor there exists magnetic field around the conductor. Significant magnetic field indicates that the line is having series inductance  $L$ .

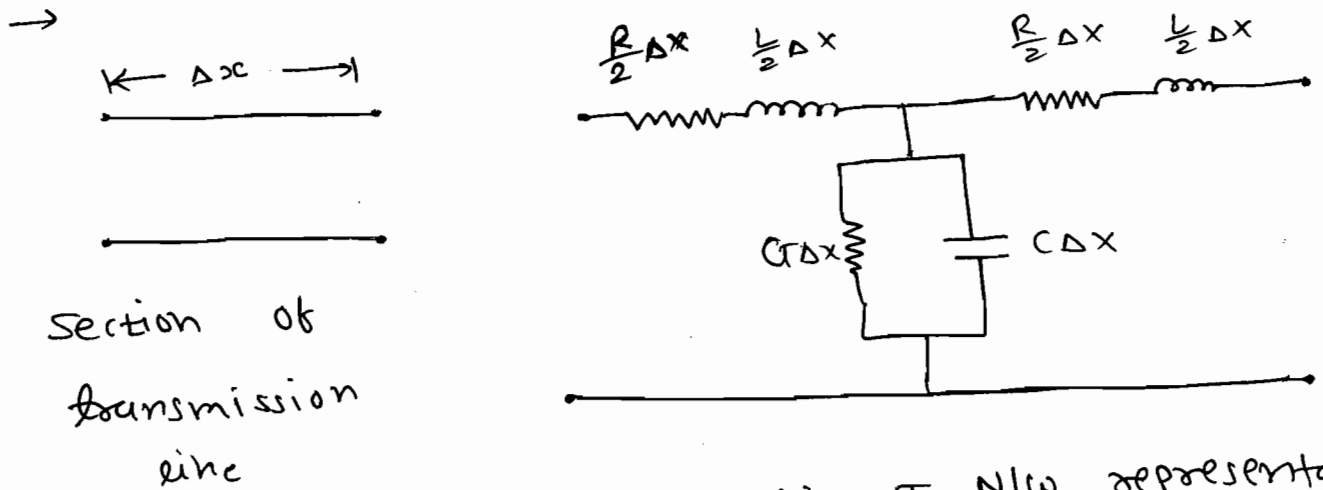
→ When a voltage is applied across this two wires the charges build up on the wires. Significant charge on this indicates that the line is having shunt capacitance  $C$ .

→ A capacitance never becomes an ideal one. It has leakage conductance  $G$ .

→  $R, L, C, G$  are not seen physically on the transmission line. They are distributed throughout the transmission line. Therefore a transmission line is said to be an example for a distributed network. And these are indicated

per unit length. If these are distributed uniformly then the transmission line is said to be uniform transmission line.

→ A section of a transmission line can be thought an example for symmetrical network.



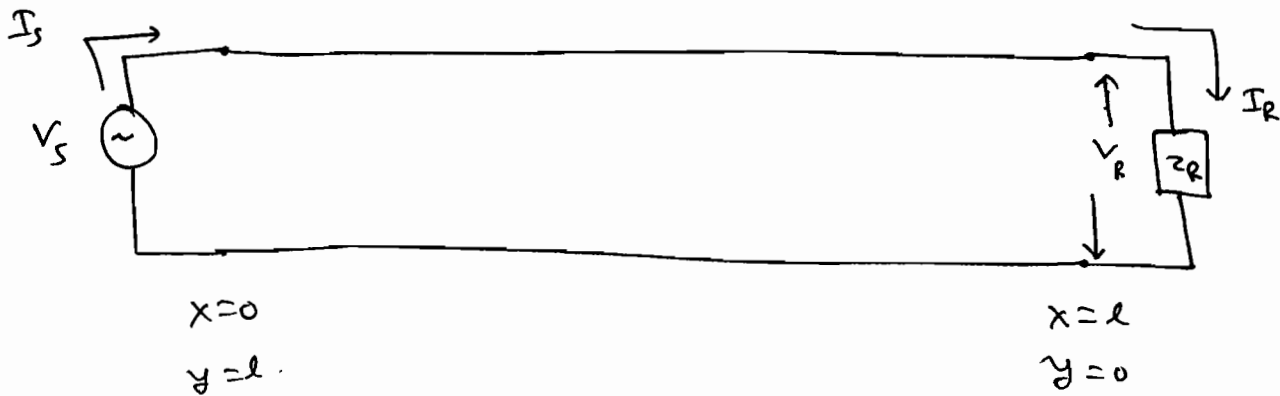
→ Symmetric T N/w representation.

→ Series Impedence  $Z_s = (R + j\omega L) \Delta x / m$ .

→ Shunt Impedence  $Y = (G + j\omega C) \Delta x / m$ .

# \* Analysis of Transmission Line:

→ Transmission line analysis means finding out voltage and current at any point on the transmission line.



S: Sending

at  $x=0$

$$V = V_s$$

$$I = I_s$$

$x$ : distance measured from sending end.

$y$ : distance measured from receiving end

R: Received.

at  $x=l$

$$V = V_R$$

$$I = I_R$$

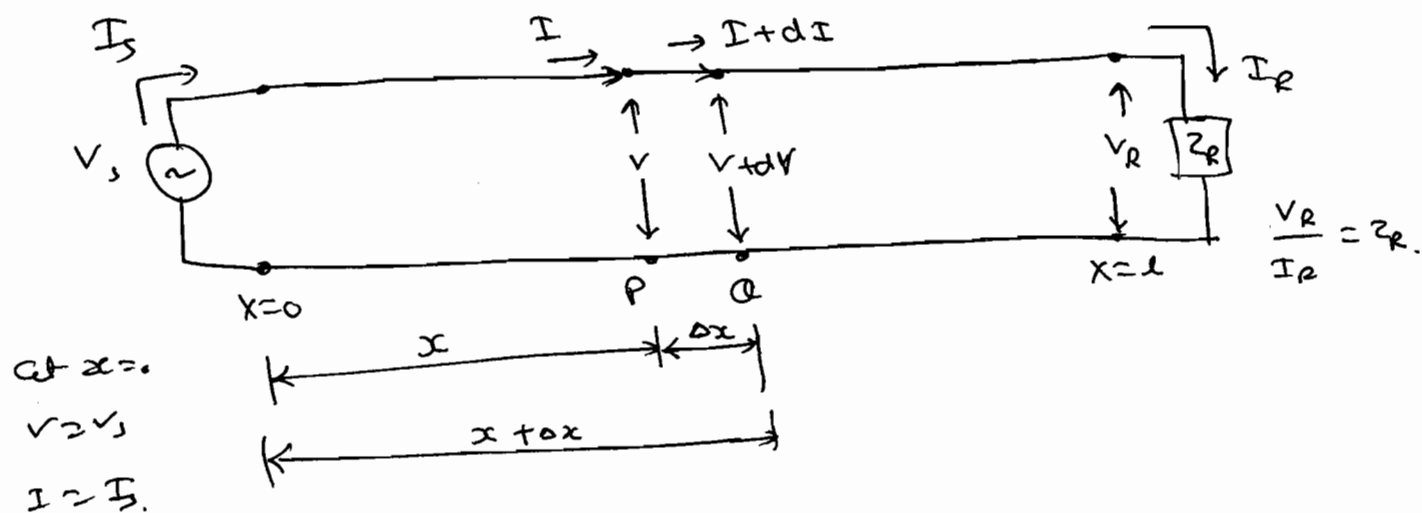
$x=0$  is corresponds to  $y=l$   
 $y=0$  is corresponds to  $x=l$ .

→  $V$ : Voltage,  $I$ : current at any point on the line.

$\left. \begin{matrix} V \\ I \end{matrix} \right\} \Rightarrow \left\{ \begin{array}{l} \text{They can be represented in terms of } V_s, I_s \text{ and } x \\ \text{They can be represented in terms of } V_R, I_R \text{ and } y. \end{array} \right.$

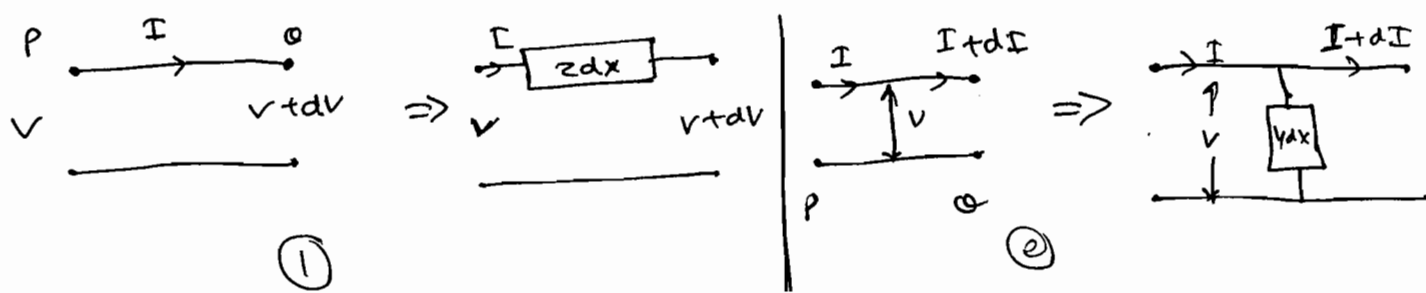
\* Expression for  $V$  &  $I$  at any point on the transmission line in terms of  $V_s, I_s$  &  $x$ :

$\Rightarrow$



$\rightarrow$  To calculate Change in the Voltage from P to Q we assume the current is constant over a small length  $dx$  and vice versa.

$\rightarrow$  The change in the voltage from P to Q is because of the current  $I$  passing through the series impedance  $z \cdot dx$ . similarly Change in the current from P to Q is because of the voltage applied across the shunt impedance  $y \cdot dx$ .



→ KVL at ①

$$V - (V + dv) = IZ dx.$$

$$-dv = IZ dx$$

$$\therefore \boxed{\frac{dV}{dx} = -IZ}$$

differentiate again.

$$\therefore \frac{d^2V}{dx^2} = -Z \frac{dI}{dx}$$

$$- \frac{d^2V}{dx^2} = +IZ$$

$$\therefore \boxed{\frac{d^2V}{dx^2} - Y^2 V = 0}$$

KCL - ②

$$I - (I + dI) = VY dx.$$

$$\therefore -dI = VY dx$$

$$\therefore \boxed{\frac{dI}{dx} = -VY}$$

$$\frac{d^2I}{dx^2} = -Y \frac{dV}{dx}$$

$$\boxed{\frac{d^2I}{dx^2} - Y^2 I = 0}$$

$$\rightarrow Y^2 = \sqrt{(R + j\omega L)(G + j\omega C)}.$$

$$\therefore \begin{aligned} \alpha &= R + j\omega L \\ \beta &= G + j\omega C. \end{aligned}$$

$$Y^2 = \alpha + j\beta.$$

$\alpha$   $\gamma$ : propagation const.

$\alpha$  = Attenuation const. (Np/m)

$\beta$  = Phase shift const. (rad/m).

→ The above eqns in the form of harmonic eqn. The soln of a harmonic eqn may take either sine (or) cosine (or) exponent.

$$\frac{d^2V}{dx^2} - Y^2 V = 0,$$

$$m = \pm Y$$

$$\frac{d^2I}{dx^2} - Y^2 I = 0$$

$$m = \pm Y$$

$$\therefore V = C_1 e^{-rx} + C_2 e^{+rx}$$

$$\therefore I = C_3 e^{-rx} + C_4 e^{+rx}$$

$$\therefore \text{from (1)} \quad I = -\frac{1}{Z} \frac{dV}{dx}$$

$$\therefore I = -\frac{1}{Z} [-r C_1 e^{-rx} + r C_2 e^{+rx}]$$

$$\therefore I = r/2 [C_1 e^{-rx} - C_2 e^{+rx}]$$

$$\therefore I = \frac{1}{Z_0} [C_1 e^{-rx} - C_2 e^{+rx}]$$

$$\frac{r}{2} = \frac{\sqrt{2Y}}{2}$$

$$= \sqrt{\frac{2Y}{2^2}}$$

$$\frac{r}{2} = \sqrt{\frac{Y}{2}} = \frac{1}{Z_0}$$

$$Z_0 = \sqrt{\frac{2}{Y}}$$

→  $C_1$  &  $C_2$  are evaluated by using the conditions (i.e.) at  $x=0$   $V=V_s$  &  $I=I_s$ .  
by simplifying we get.

$$\therefore V = V_s \cosh rx - I_s Z_0 \sinh rx$$

$$I = I_s \cosh rx - \frac{V_s}{Z_0} \sinh rx$$

$$\therefore \begin{aligned} V &= C_1 e^{-rx} + C_2 e^{rx} \\ I &= \frac{1}{Z_0} [C_1 e^{-rx} - C_2 e^{rx}] \end{aligned}$$

\* Infinite line and the definition of Characteristic Impedance.

- Infinite line means line length is  $\infty$ .
- An infinite line can be thought of <sup>infinite no. of</sup> small section or <sup>sub-section</sup> connected in cascade.

→ we know that each subsection is equal to a Symmetrical N/w. Therefore an infinite line can be thought of infinite no. of identical Symmetrical N/w. which are connected in cascade. Therefore, we define input impedance of an infinite line is equal to Characteristic impedance.

$$\rightarrow V_{\text{at } x=l} = V_R = V_S \cosh rl - I_S Z_0 \sinh rl.$$

$$I_{\text{at } x=l} = I_R = I_S \cosh rl - \frac{V_S}{Z_0} \sinh rl.$$

$$\therefore \frac{V_R}{I_R} = Z_0 = \frac{V_S \cosh rl - I_S Z_0 \sinh rl}{I_S \cosh rl - \frac{V_S}{Z_0} \sinh rl}.$$

2. Cross multiply

we get  $\frac{V_S}{I_S} = Z_0.$

$$\therefore Z_0 I_S \cosh rl - V_S \sinh rl = V_S \cosh rl - I_S Z_0 \sinh rl.$$

$$\therefore Z_0 I_S [\cosh rl + \sinh rl] = V_S [\cosh rl + \sinh rl]$$

$$\therefore \boxed{\frac{V_S}{I_S} = Z_0.}$$

→ An infinite line is equal to a finite line when the finite line is terminated by  $Z_0$ .

→ i.e. When a finite line transmission line is terminated by  $Z_0$  then the impedance seen at the input of the transmission line is also equal to  $Z_0$ .

→ When  $Z_R = Z_0 \Rightarrow V_s = I_s Z_0$ .

$$V = V_s \cosh \gamma x - I_s \sinh \gamma x.$$

$$\therefore \begin{matrix} V_s & V_0 \\ \text{very} & \end{matrix} \quad \boxed{\begin{matrix} V = V_s e^{-\gamma x} \\ I = I_s e^{-\gamma x} \end{matrix}}$$

These are valid when  $Z_R = Z_0$  only.

→ If 'l' is called physical length of the line, the ' $\beta l$ ' is called 'electrical length'.

$$l \rightarrow \text{m}$$

$$\beta \rightarrow \text{rad/m}$$

$$\beta l \rightarrow \frac{\text{rad}}{\text{m}} \cdot \text{m} = \text{rad (or) degree}$$

Ex-1: The physical length of the transmission line is  $\lambda/8$ . Find the electrical length.

Ans:  $l: \lambda/8$

$$\therefore \beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$$

$$\therefore \boxed{\text{Electrical length} = \frac{\pi}{4}}$$



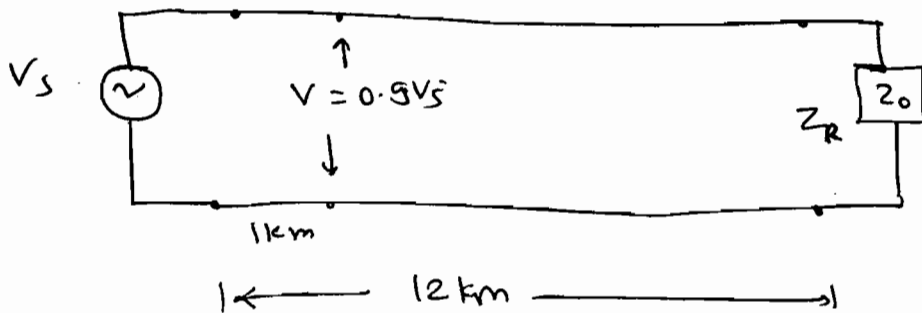
→ Ex<sup>2</sup>  
 A 12 km long line is terminated by  $Z_0$ .

The Voltage at 1 km from the sending end is 10% below than at the sending end.

Find Voltage across the Load impedance in terms of the v.o of the sending end voltage.

Ans: 28%.

Ans:



$$\begin{aligned} & 10\% \text{ below} \\ & = V_s - 0.1 V_s \\ & = 0.9 V_s. \end{aligned}$$

→ at 1 km  $V = 0.9 V_s$ .

∴ the line is terminated in  $Z_0$ .

$$\therefore V = V_s e^{-rx}$$

$$\text{At } x = 1 \text{ km} \Rightarrow V = 0.9 V_s = V_s e^{-r(1)}$$

$$\therefore \boxed{0.9 = e^{-r}}$$

$$\text{at } x = 12 \text{ km} \quad V = V_s e^{-r(12)}$$

$$\therefore V = V_s (0.9)^{12} = 0.28 V_s$$

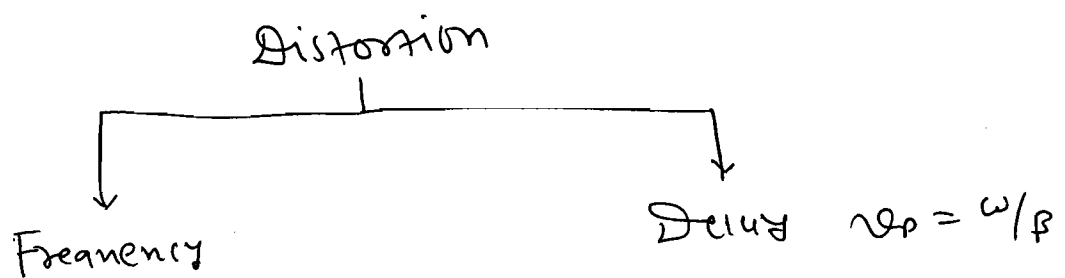
$$\therefore \boxed{V(x=12 \text{ km}) = 28\% \text{ of } V_s}$$

$r, Z_0$  } Secondary Constant

$R, \alpha, L$  → primary Constant.

## \* Distortion:

- The purpose of a transmission line is to transport band of freq.
- $\alpha$  is a f<sup>n</sup> of freq. therefore different freq. components undergoes different attenuation levels. This is said to be freq. Distortion.
- Undergoing different freq. components with the different phase velocity is called delay distortion.  $v_p = \omega/\beta$ .



$$\alpha = \text{real}(\gamma)$$

$$= \text{Real} \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\therefore = f(R, L, G, C, \omega).$$

$$v_p = \frac{\omega}{\beta} = \frac{d\omega}{d\beta}$$

- when the line is free from delay distortion and freq distortion then it is said to be distortion less transmission line.

\* Distortion less condition:

$$V = \sqrt{(R + j\omega L) (G + j\omega C)}$$

$\therefore \cancel{V} = \cancel{R} \cancel{L} + j\omega \cancel{L} \cancel{C}$

$$\therefore V = \sqrt{L \left[ \frac{R}{L} + j\omega \right] \cdot C \left[ \frac{G}{C} + j\omega \right]}$$

if we choose  $\frac{R}{L} = \frac{G}{C}$ .

$$\therefore V = \sqrt{LC \left[ \frac{R}{L} + j\omega \right]^2} \quad \text{or} \quad V = \sqrt{LC \left[ \frac{G}{C} + j\omega \right]^2}$$

$$\therefore V = \sqrt{LC} \left[ \frac{R}{L} + j\omega \right] \quad (\text{or}) \quad V = \sqrt{LC} \left[ \frac{G}{C} + j\omega \right]$$

$$\therefore V = R \sqrt{\frac{C}{L}} + j\omega \sqrt{LC} \quad \text{or} \quad V = G \sqrt{\frac{L}{C}} + j\omega \sqrt{LC}$$

$\therefore \cancel{V} = \cancel{L} \cancel{C} \quad \alpha = R \sqrt{\frac{C}{L}} \quad (\text{or}) \quad G \sqrt{\frac{L}{C}} \quad \text{Np/m.}$

$\therefore \quad \beta = \omega \sqrt{LC} \quad \text{m/s.} \Rightarrow V_p = \frac{1}{\sqrt{LC}}$

$\rightarrow \alpha$  and  $v_p$  are independent of frequency.

$\therefore \boxed{\frac{R}{L} = \frac{G}{C} : \text{Distortionless condition.}}$

The same condition is also valid for min. attenuation (or) Low loss.

$$\therefore Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L \left( \frac{R}{L} + j\omega \right)}{C \left( \frac{G}{C} + j\omega \right)}}$$

$\therefore \boxed{Z_0 = \sqrt{\frac{L}{C}} \quad \Omega}$

→ Delay in the transmission line is the geometric mean of  $L$  &  $C$ . (or)

$$\therefore \boxed{\text{Delay} = \sqrt{LC} \text{ s/m.}}$$

$$\beta = \omega \sqrt{LC}$$

$$\therefore v_p = \frac{\omega}{\beta}$$

$$\therefore \frac{\omega}{\beta}$$

Ex-1 A distortionless transmission line has an attenuation constant of  $20 \text{ m Np/m}$ . The phase velocity on the line is  $0.6$  of the velocity of light assume characteristic impedance of the line is  $50 \Omega$ . Find primary constants. it

Ans:

$$\alpha = 20 \text{ m Np/m.}$$

$$\alpha = 20 \times 10^{-3} \text{ Np/m.}$$

$$\therefore v_p = \omega / \beta = 0.6 \times 3 \times 10^8 = 1.8 \times 10^8.$$

$$\therefore \frac{\omega}{\beta} = 1.8 \times 10^8 \text{ m/s.}$$

$$\therefore Z_0 = 50 \Omega = \sqrt{\frac{L}{C}}$$

$$\therefore \alpha = R \sqrt{\frac{C}{L}}, \quad (\text{or}) \quad \alpha = G \sqrt{\frac{L}{C}}$$

$$\therefore \alpha = G Z_0$$

$$\therefore G = \frac{20 \times 10^{-3}}{50}$$

$$\boxed{G = 0.4 \times 10^{-3} \text{ S/m}}$$

$$\therefore \alpha R = \alpha / Z_0 \quad \Omega / m.$$

$$\therefore \alpha R = \frac{20 \times 10^{-3}}{50} \quad \text{or} \quad 0.4 \times 10^{-4}$$

$$\therefore \boxed{R = 0.4 \times 10^{-3} \quad \Omega / m.}$$

$$\therefore \text{Good } \rho = \alpha \cdot Z_0.$$

$$\therefore \rho = 20 \times 10^{-3} \times 50 = 1000 \times 10^{-3}$$

$$\therefore \boxed{\rho = 1 \quad \Omega / m}$$

$$\therefore L = \frac{Z_0}{v_p}$$

$$L = \frac{50 \times 1}{0.6 \times 10^8 \times 3} = \frac{50 \times 10^{-8}}{1.8}$$

$$\therefore \boxed{L = 27.6 \times 10^{-9} \quad H / m.}$$

$$\therefore C = \frac{1}{Z_0 v_p} \quad F / m.$$

$$C = \frac{1}{50 \times 3 \times 0.5 \times 10^8}$$

$$C = \frac{100}{75} \times 10^{-8} \times 10^{-2}$$

$$\therefore C = 1.3 \times 10^{-10} \quad F.$$

$$\boxed{C = 133.33 \text{ pF}}$$

\* For a lossless line  $\boxed{\alpha = 0.}$

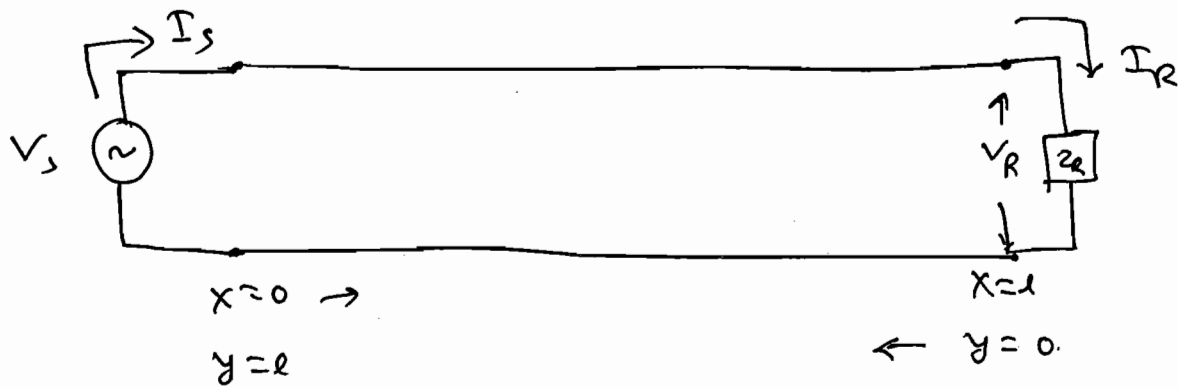
$$V = jB$$

$$\sinh \gamma x = \sinh(j\beta x).$$

$$= j \sinh(\beta x).$$

$$\cosh \gamma x = \cosh(j\beta x) = \cos \beta x.$$

\* Expression for  $V$  &  $I$  at any point on the transmission line in terms of  $V_R, I_R, y$ .



At  $y=0$ ,  $V=V_R$   
 $I=I_R$ .

$$\therefore V = V_s \cosh \gamma x - I_s z_0 \sinh \gamma x.$$

$$I = I_s \cosh \gamma x - \frac{V_s}{z_0} \sinh \gamma x.$$

$\Downarrow$

Replace

$$x \rightarrow -y$$

$$V_s \rightarrow V_R.$$

$$I_s \rightarrow I_R.$$

$$\therefore V = V_R \cosh \gamma y + I_R z_0 \sinh \gamma y.$$

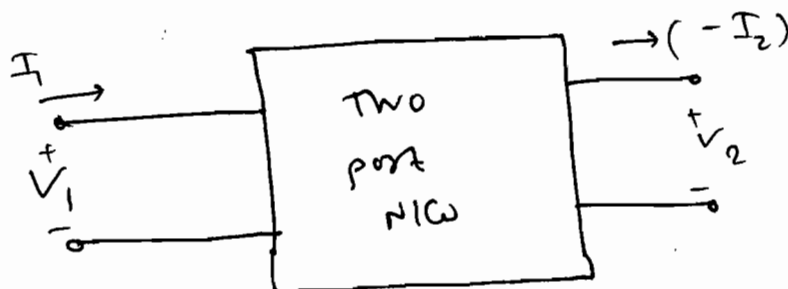
$$\therefore I = I_R \cosh \gamma y + \frac{V_R}{z_0} \sinh \gamma y.$$

at  $y=l \Rightarrow \boxed{V \approx V_s, I \approx I_s}$   
input end

$$\therefore \begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \boxed{\cosh \gamma l} & \boxed{Z_0 \sinh \gamma l} \\ \boxed{\frac{1}{Z_0} \sinh \gamma l} & \boxed{\cosh \gamma l} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

→ For a lossless line  $\alpha=0 \Rightarrow \gamma=j\beta$ .

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \boxed{\cos \beta l} & \boxed{j Z_0 \sin \beta l} \\ \boxed{j \sin \beta l} & \boxed{\cos \beta l} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$



$$V_1 = A V_2 - B I_2$$

$$I_1 = C V_2 - D I_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\rightarrow A = \left. \frac{V_1}{V_2} \right|_{I_2=0} \quad \rightarrow B = \left. -\frac{V_1}{I_2} \right|_{V_2=0}$$

$$\rightarrow C = \left. \frac{I_1}{V_2} \right|_{I_2=0} \quad \rightarrow D = \left. -\frac{I_1}{I_2} \right|_{V_2=0}$$

→ As Shown above a section of a transmission line is satisfying reciprocity condition, ~~of~~ Symmetry condition of ABCD parameters. We can also prove that a section of a transmission line is an example for Symmetrical and reciprocal

Ex-2 For a loss-less transmission line of a length  $\lambda/8$ . Find ABCD parameters.

Ans: Assume characteristic impedance of the is  $50 \Omega$ .

$$\begin{aligned} \rightarrow \text{as loss-less} & \quad \gamma = \alpha + j\beta \\ & \quad \downarrow \\ & \quad \alpha = 0 \end{aligned} \quad \therefore \Rightarrow \gamma = j\beta$$

$$\therefore \underline{Z_0 = 50 \Omega}$$

For a loss-less line  $\alpha = 0 \Rightarrow \gamma = j\beta$ .

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \cos \frac{2\pi}{\lambda} \times \frac{\lambda}{8} & j 50 \sin \frac{2\pi}{\lambda} \times \frac{\lambda}{8} \\ \frac{j}{50} \sin \frac{2\pi}{\lambda} \times \frac{\lambda}{8} & \cos \frac{2\pi}{\lambda} \times \frac{\lambda}{8} \end{bmatrix} \begin{bmatrix} V_R \\ -I_R \end{bmatrix}$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & j \frac{50}{\sqrt{2}} \\ \frac{j}{50\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} V_R \\ -I_R \end{bmatrix}$$



## ★ Impedance:

→ Impedance at any point on the transmission line looking towards the load is the ratio of Voltage to the current at that point  $Z_y$  is the Impedance at a distance  $y$  from a load end.

$$Z_y = \frac{V_R \cosh ry + I_R Z_0 \sinh ry}{I_R \cosh ry + \frac{V_R}{Z_0} \sinh ry}$$

→ divide the numerator and denominator by  $I_R \cosh ry$ . and use  $\frac{V_R}{I_R} = Z_R$  and simplifying.

$$\rightarrow Z_y = Z_0 \left[ \frac{Z_R + Z_0 \tanh ry}{Z_0 + Z_R \tanh ry} \right]$$

→ For a lossless line  $\alpha = 0$ .

$$\therefore Z_y = Z_0 \cdot \frac{Z_R + Z_0 \tanh \beta y}{Z_0 + Z_R \tanh \beta y}$$

→ When  $\gamma = 0 \Rightarrow$  input and  $Z_L$  is denoted as  $Z_m$

(or) 
$$Z_{in} = \frac{V_s}{I_s}$$

→ For a lossy line,

$$Z_{in} = Z_0 \cdot \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$

→ For a loss-less line,

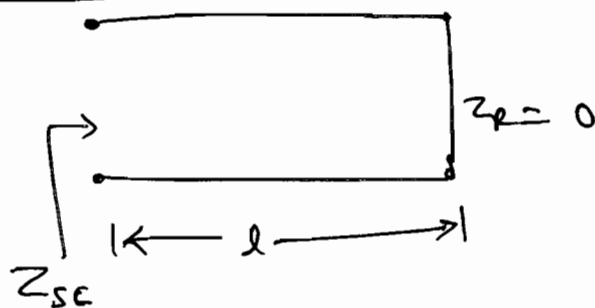
$$Z_{in} = Z_0 \cdot \frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l}$$

→ When a transmission line is terminated by  $Z_0$ , then, the impedance seen at any point on the transmission line and also the input impedance is same as  $Z_0$  whether the line is lossy (or) lossless.

→ when a finite length transmission line is open circuited at the terminating <sup>end</sup> ~~at~~ is called OC line then the impedance seen at input of the transmission line is designated by  $Z_{oc}$ .

→ When a finite length Tx line is short cktd at the terminating end is called an SC line then the impedance seen at the input of the transmission line is designated by  $Z_{sc}$ .

→ ① sc line

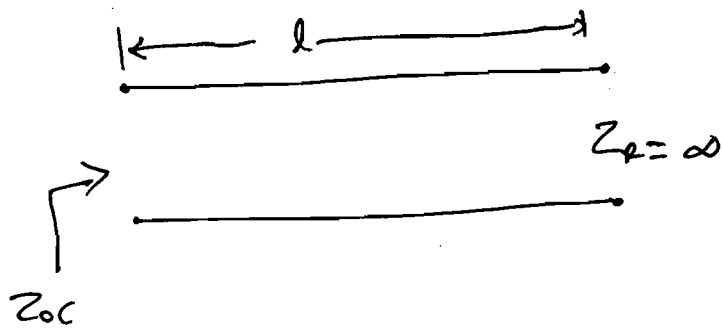


$$\therefore Z_{sc} = Z_0 \tanh \gamma l \approx$$

For a loss less line.

$$Z_{sc} = j Z_0 \tan \beta l \approx$$

② oc line.



$$Z_{oc} = Z_0 \coth \gamma l \quad \Omega.$$

For a loss less line.

$$Z_{oc} = -j Z_0 \frac{1}{\cot \beta l} \quad \Omega.$$

$$Z_{oc} = -j Z_0 \cot \beta l \quad \Omega.$$

→ Whether the line is lossy (or) lossless,

$$\boxed{Z_{oc} \cdot Z_{sc} = Z_0^2}$$

→  $Z_0$ : Char. impedance  $50 \Omega$  line,  $75 \Omega$  line etc.

$\beta l$ : Electrical length.

→  $\tan \beta l$  } They assume all possible values ranging from  $-\infty$  to  $\infty$ , depending upon the value  $\beta l$ .

→ For a loss less line.

$$Z_{sc} = j (Z_0 \tan \beta l) \quad \Omega$$

$$Z_{oc} = -j (Z_0 \cot \beta l) \quad \Omega. \quad \text{Reactance.}$$

$$\rightarrow Y_{sc} = -\frac{j}{Z_0} \cot \beta l$$

$$Y_{oc} = \frac{j}{Z_0} \tan \beta l$$

Susceptance.

→ Admittance

Impedences

$$\frac{1}{j\omega L} = -j\beta_L$$

$$j\omega C = +j\beta_C$$

Susceptance

$$j\omega L \Omega = jX_L \Omega$$

$$\frac{1}{j\omega C} \Omega = -jX_C \Omega$$

→ We conclude that a section of a lossless transmission line either it is open circuited (or) SC can act as circuit reactive element (or) circuit susceptive element. Desire Reactance (or) Susceptance can be achieved by properly choosing length of the transmission line this sections are named as stubs. and are used in the impedance matching techniques hence the name stub matching.

Ex-1 A lossless  $\lambda/8$  transmission line is SC. What type of reactance the line indicates. at 50 kHz find the value of equivalent passive component.  
assume  $Z_0 = 50 \Omega$ .

Ans: as S.C.

$$Z_{sc} = j Z_0 \tan \beta l \quad \Omega.$$

$$\therefore l = \lambda/8, \quad \beta = \frac{2\pi}{\lambda}.$$

$$\therefore Z_{sc} = j 50 \tan\left(\frac{\pi}{4}\right).$$

$$Z_{sc} = j 50 \Omega$$

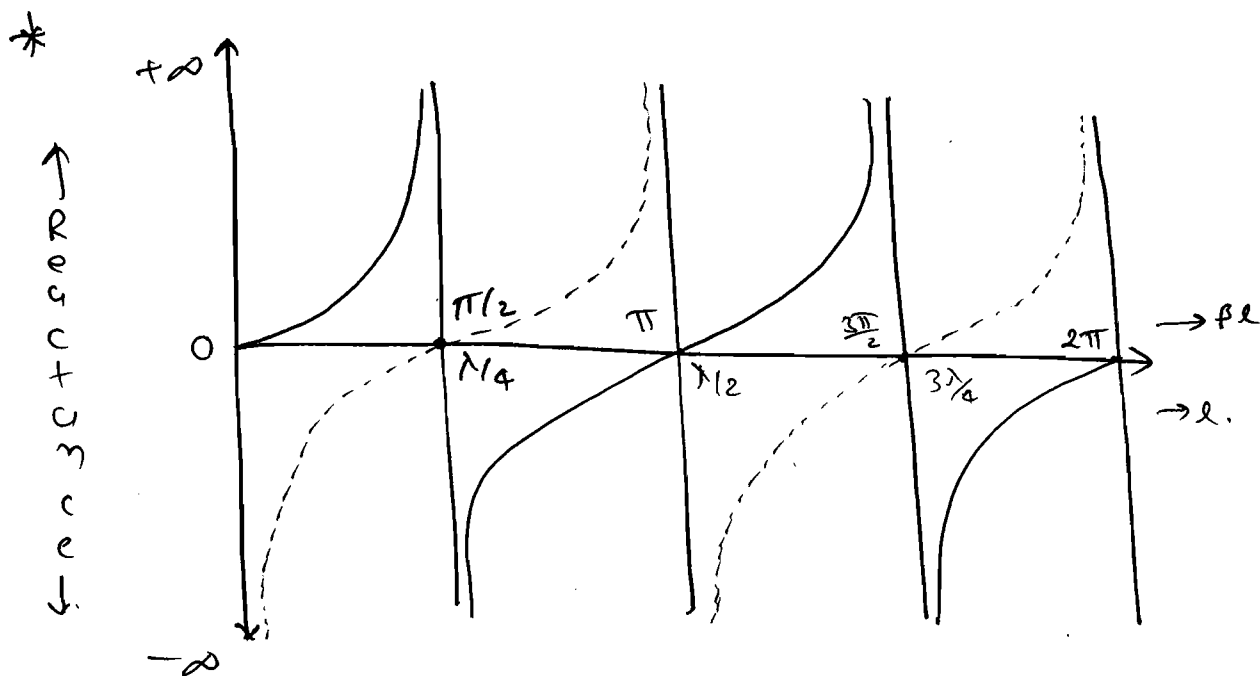
$\therefore$  line offers inductive reactance.

$$j 50 = j \omega L$$

$$2\pi f L = 50$$

$$\therefore L = \frac{50}{2 \times \pi \times 50 \times 10^3}$$

$$\therefore \boxed{L = 0.159 \text{ mH}}$$



$$Z_{sc} = jZ_0 \tan \beta l.$$

$$\therefore Z_{oc} = -jZ_0 \cot \beta l.$$

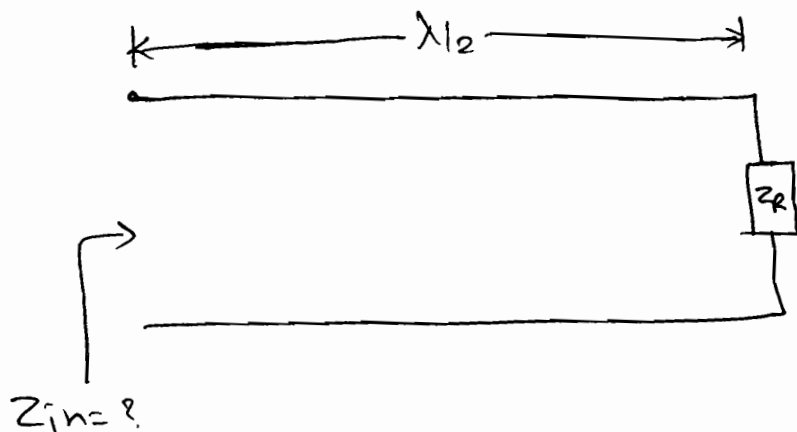
→ figure shows the variation of  $Z_{oc}$  and  $Z_{sc}$  as a function of physical length (or) electrical length. When the line length is varied from 0 to  $\lambda/2$ ,  $Z_{oc}$  and  $Z_{sc}$  assumes all possible reactances ranging from  $-\infty$  to  $+\infty$  that desired reactance can be achieved by properly choosing length of the transmission line.

→ when the line length is varied from 0 to  $\lambda/4$  sc line would offer inductive reactance and oc line would offer capacitive reactance.

Ex 1/2

\* Properties of  $\lambda/2$  and  $\lambda/4$  lines.

→ Consider a lossless line

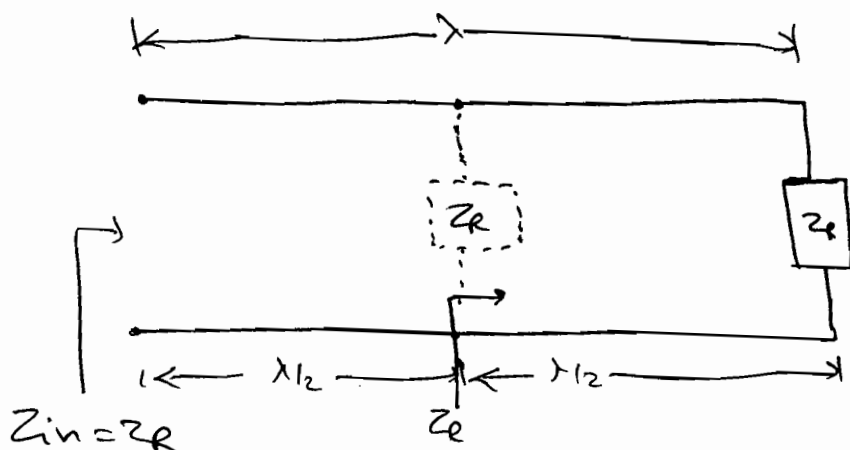


→  $l = \lambda/2$   
 $\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$

$\tan \beta l = \tan \pi = 0.$

$\therefore \boxed{Z_{in} = Z_L}$

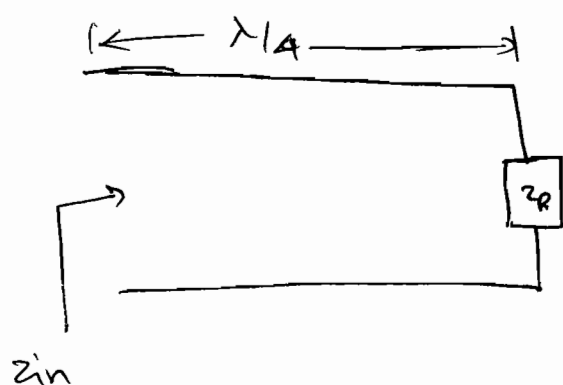
( $\because Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l}$ )



→ The input impedance of a  $\frac{n\lambda}{2}$  line is same as  $\lambda/2$  line and is equal to load impedance.



⇒  $\lambda/4$  line.



$$l = \lambda/4.$$

$$\therefore \beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}.$$

$$\tan \beta l = \tan \frac{\pi}{2} = \infty.$$

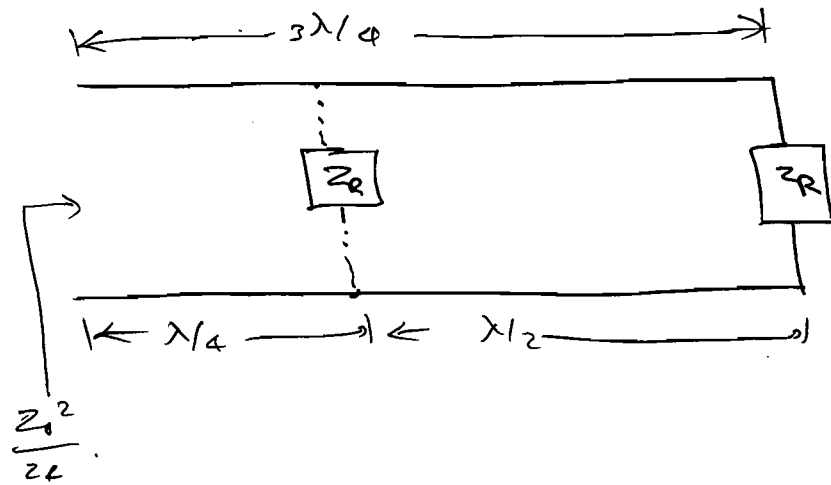
$$\therefore Z_{in} = Z_0 \cdot \frac{\frac{Z_R}{\tan \beta l} + j Z_0}{\frac{Z_0}{\tan \beta l} + j Z_R}$$

$$\therefore Z_{in} = \frac{Z_0^2}{Z_R}.$$

| When $Z_R =$           | $Z_{in} = ?$           |
|------------------------|------------------------|
| $0 \Omega$ (S.C.)      | $\infty \Omega$ (O.C.) |
| $\infty \Omega$ (O.C.) | $0 \Omega$ (S.C.)      |

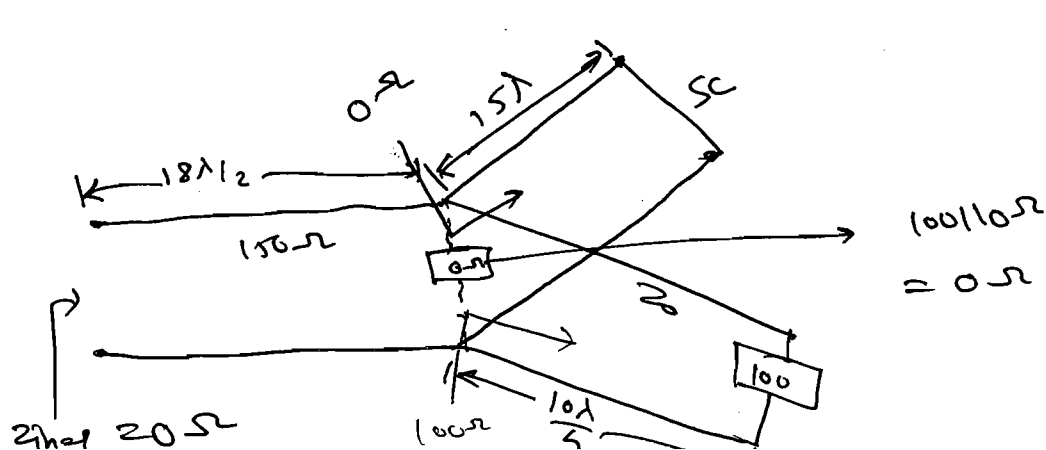
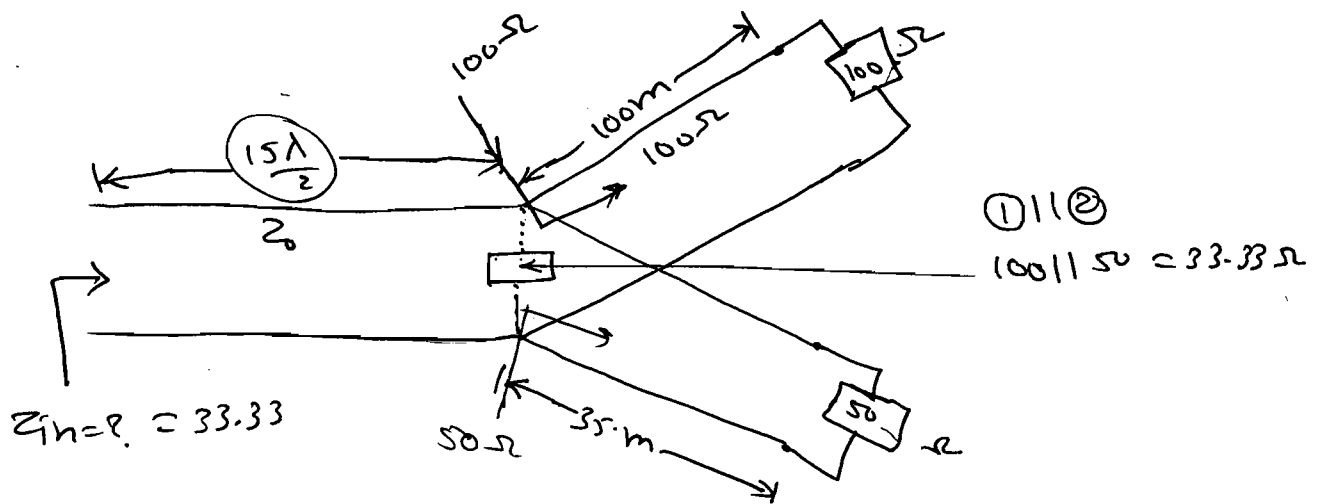
→ A Quarterwave line is also called as

✓ Impedance transformer. because it transforms high impedance to low impedance and vice versa.

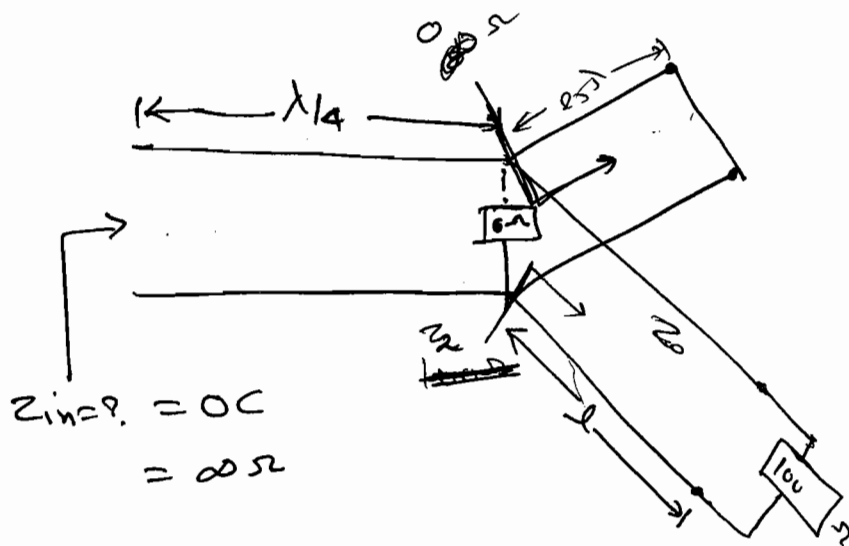


→ The input impedance of a  $\boxed{\frac{n\lambda}{4}}$  line is same as  $\lambda/4$  line and is equal to  $\boxed{\frac{Z_0^2}{Z_R}}$  where  $\boxed{n \text{ is odd.}}$

Ex-1 = Find the input impedance of the following transmission line. Assume Lossless line.



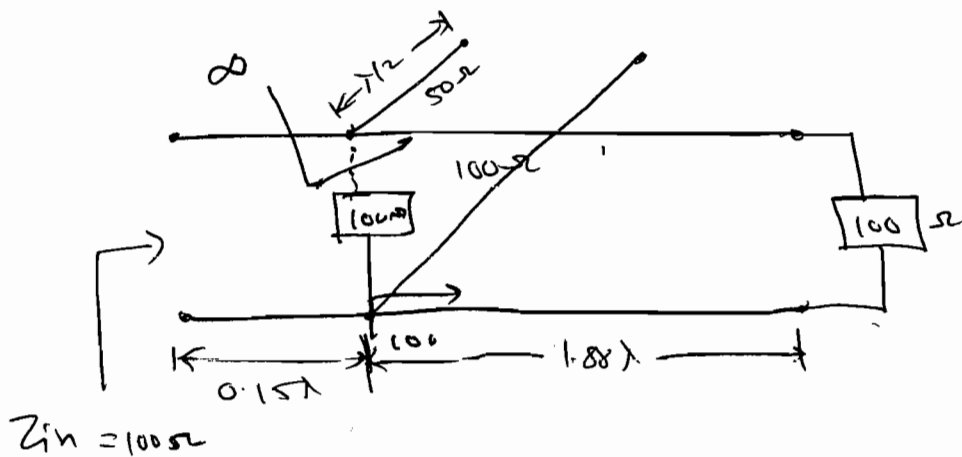
3



$$0 \parallel 100 = 0$$

$$Z_2 \parallel 0\Omega = 0\Omega$$

4



$$\infty \parallel 100 = 100$$

$$100\Omega \parallel 0 = 100\Omega$$

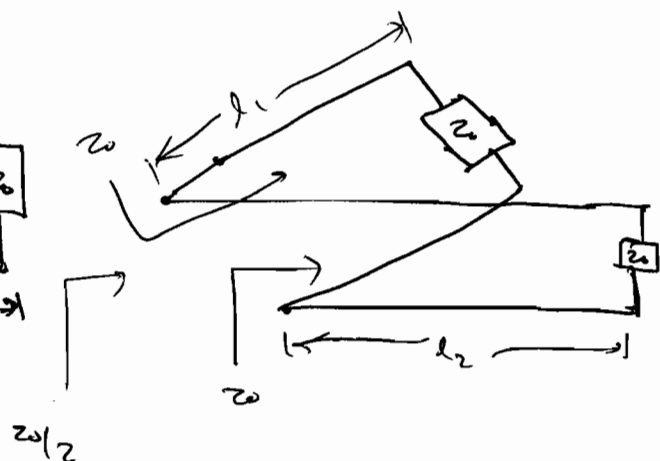
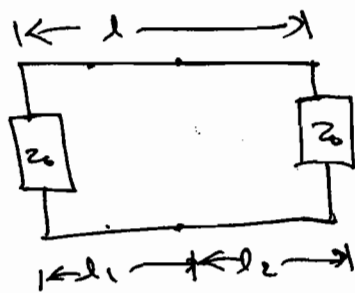
5 The two ends of a lossless trans. line is terminating  $Z_0$  where  $Z_0$  is the characteristic impedance of the line. Find the impedance at the middle point and at any point on the line.

(a)  $Z_0, Z_0$

(b)  $\frac{Z_0}{2}, Z_0$

(c)  $Z_0, Z_0/2$

(d)  $\frac{Z_0}{2}, \frac{Z_0}{2}$



## \* Reflection

→  $Z_y$  is the impedance at any point on the transmission line looking towards the load and is given by

$$Z_y = Z_0 \frac{Z_L + Z_0 \tan \beta l}{Z_0 + Z_L \tan \beta l}$$

→ When  $Z_L = Z_0 \Rightarrow Z_y = Z_0$

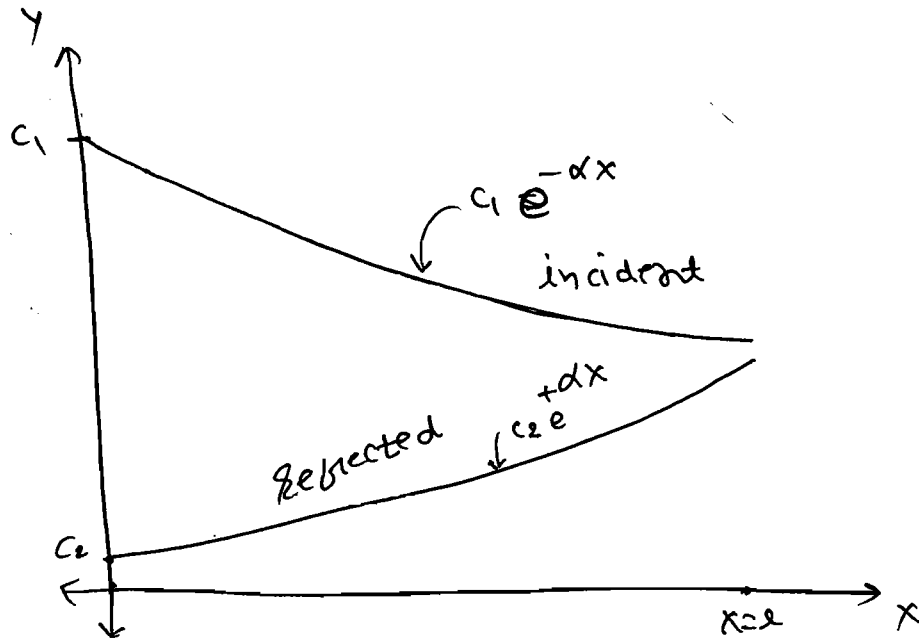
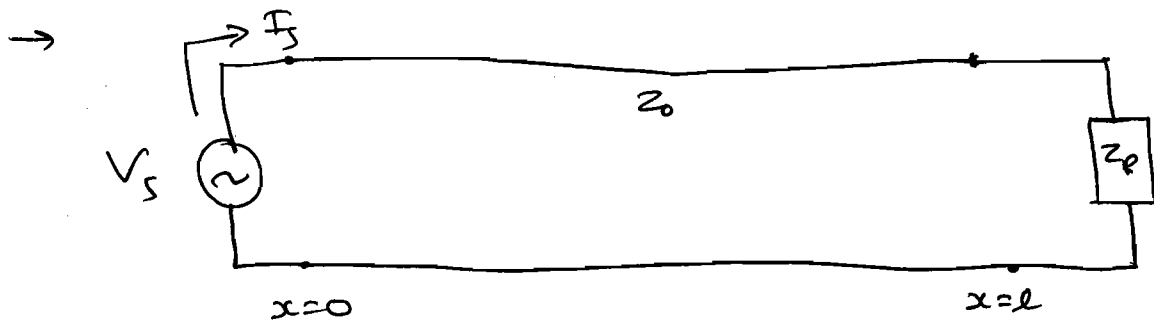
$\Rightarrow$  Impedance is matched (or)  
Uniform (or)  
Regular (or)  
Continuous

→ When  $Z_L \neq Z_0$

$\Rightarrow Z_y$  changes its value from point to point on the line.

$\Rightarrow$  Impedance is mismatched (or)  
discontinuous (or)  
Irregular (or)  
Non-uniform.

→ Whenever there exist impedance discontinuity (or) irregularity (or) mismatched (or) nonuniformity there exist reflection. This reflection will travel from load to the source



→ The Voltage at any point on the line is given by

$$V = c_1 e^{-\alpha x} + c_2 e^{+\alpha x}$$

$$= V = \underbrace{c_1 e^{-\alpha x} \cdot e^{-j\beta x}}_{\text{Term (1) incident}} + \underbrace{c_2 e^{+\alpha x} \cdot e^{j\beta x}}_{\text{Term (2) Reflected.}}$$

→ Term - (1)

$$| c_1 e^{-\alpha x} \cdot e^{-j\beta x} | = c_1 e^{-\alpha x}$$

→ Term - (2)

$$| c_2 e^{+\alpha x} \cdot e^{j\beta x} | = c_2 e^{+\alpha x}$$

When  $Z_L = Z_0$ .

$$V = V_s \cdot e^{-\gamma x} + \text{No second term (reflection is zero).}$$

→ Term - (1) in its mathematical form indicates an incident wave which is propagating from source to the load while it is propagating its amplitude is decreasing exponentially.

→ The Term - (2) in its mathematical form indicates reflected wave which is propagating from load to the source while it is propagating its amplitude is decreasing exponentially.

→ When  $Z_L = Z_0$  Reflections are zero. i.e. there exist only one wave i.e. incident wave.

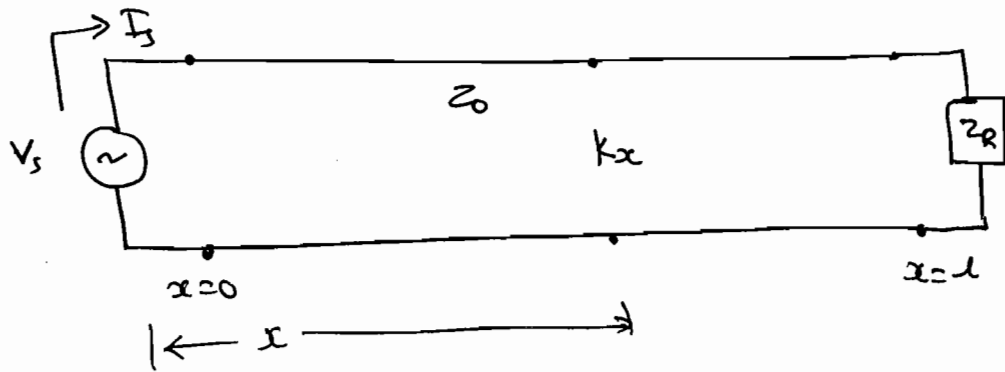
$$\rightarrow V = C_1 e^{-\gamma x} + C_2 e^{+\gamma x}$$

$$\therefore V(x, t) = \text{Re} \left[ \underbrace{C_1 e^{-\alpha x} \cdot e^{j(\omega t - \beta x)}}_{\text{incident wave}} + \underbrace{C_2 e^{+\alpha x} \cdot e^{j(\omega t + \beta x)}}_{\text{Reflected wave}} \right]$$

→ Reflection Coefficient :

→ It is a ratio of reflected wave voltage to the incident wave voltage. ( $K_r$ ).

→  $K_r$  is the reflection coefficient is defined in a distance  $x$  from the sending end.



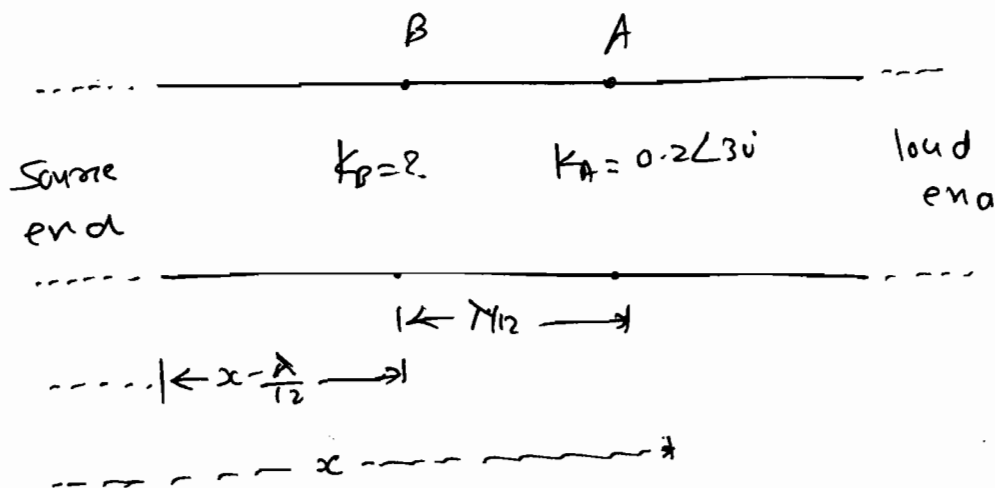
$$\rightarrow K_x = \frac{C_2 e^{Vx}}{C_1 e^{-Vx}} = \frac{C_2}{C_1} e^{2Vx} = \frac{C_2}{C_1} e^{2\alpha x} e^{2j\beta x}$$

For a lossless line

$$K_x = \frac{C_2}{C_1} \cdot e^{2j\beta x}$$

Ex-1 A finite length transmission line is lossless and terminated by an unknown load impedance. Reflection coefficient measured at point A is given by  $0.2 \angle 30^\circ$ . What is its value at a distance of  $\frac{\lambda}{12}$  from the above point towards the sending end.

Ans:



$$K_A = \frac{C_2}{C_1} \cdot e^{2j\beta x}$$

$$K_B = \frac{C_2}{C_1} \cdot e^{2j\beta(x - \frac{\lambda}{2})} = \frac{C_2}{C_1} e^{2j\beta x} \cdot e^{-j2(\frac{2\pi}{\lambda}) \cdot (\frac{\lambda}{2})}$$

$$= \frac{C_2}{C_1} \cdot e^{2j\beta x} \cdot e^{-j\pi(2)}$$

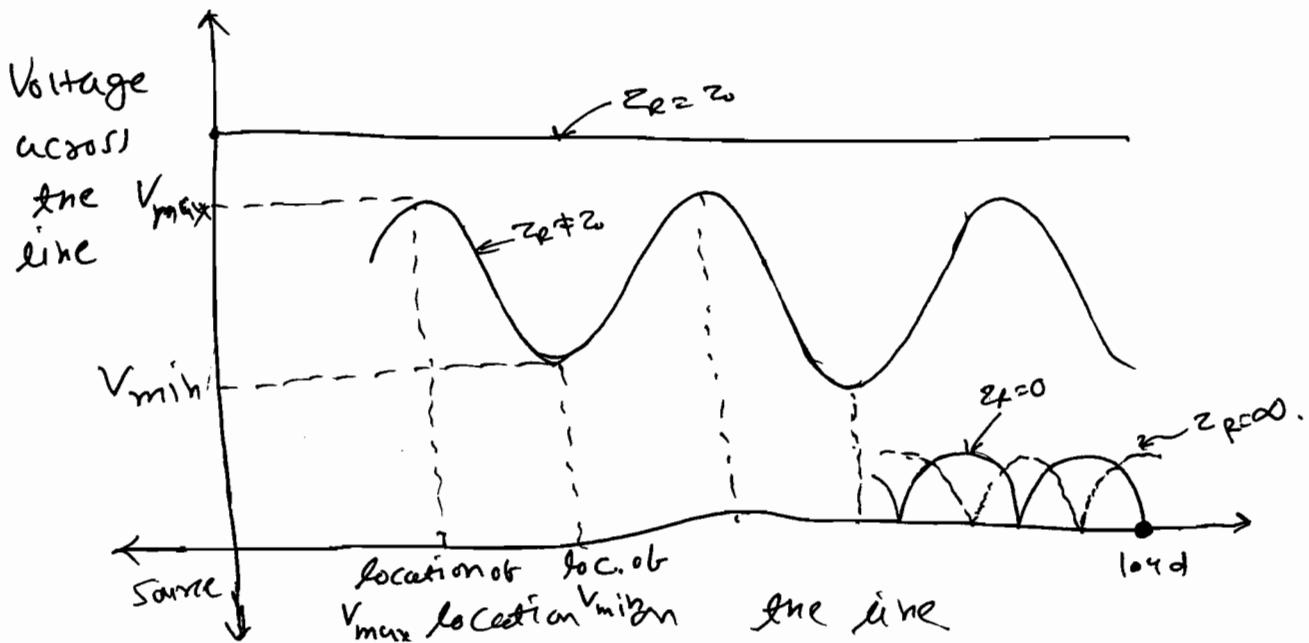
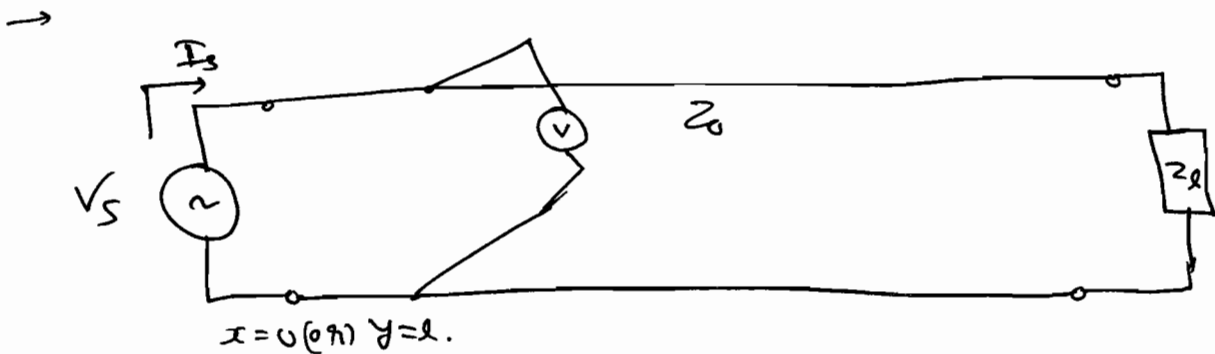
$$\therefore \frac{K_B}{K_A} = \frac{\frac{C_2}{C_1} \cdot e^{2j\beta x} \cdot e^{-j\pi(2)}}{\frac{C_2}{C_1} \cdot e^{2j\beta x}}$$

$$\therefore \frac{K_B}{K_A} = 1 \angle -0^\circ$$

$$\therefore K_B = (0.2 \angle 30^\circ) (1 \angle -90^\circ)$$

$$\therefore K_B = 0.2 \angle -30^\circ$$

\* Voltage Standing Waves and Standing Wave ratio:





$$V_{\max} = |V_i| + |V_r| \quad \checkmark$$

$$\therefore V_{\min} = |V_i| - |V_r| \quad \checkmark$$

$i$ : incident

$r$ : reflected.

→ The Voltage at any point on the transmission line is the vectorial sum of incident wave voltage and reflected wave voltage at same location on the transmission line. This two voltages may add in phase. When they add in phase a voltage maximum is observed on the transmission line. When they adding in the out of phase a voltage minimum is observed on the transmission line. therefore, the voltage across a transmission line swings bet<sup>n</sup> maximum voltage to the minimum voltage and vice versa.

Fig. shows

→ Voltage Standing Waves for different loads

→ The successive distance bet<sup>n</sup> minima to maxima (or) maxima to minima is quarter wavelength. ( $\lambda/4$ ).

→ The successive distance bet<sup>n</sup> two minimas (or) two maximas is half wave length ( $\lambda/2$ ).

→ location of  $V_{max} \longleftrightarrow I_{min} \longleftrightarrow Z_{max}$ .  
 $V_{min} \longleftrightarrow I_{max} \longleftrightarrow Z_{min}$ .

\* VSWR (S):

→ 
$$S = \frac{V_{max}}{V_{min}}$$

$$\therefore = \frac{|V_i| + |V_r|}{|V_i| - |V_r|}$$

$$= \frac{1 + \left| \frac{V_r}{V_i} \right|}{1 - \left| \frac{V_r}{V_i} \right|}$$

$$\therefore S = \frac{1 + |k|}{1 - |k|}$$

Where,  $k = \frac{Z_L - Z_0}{Z_L + Z_0}$

$$k = |k| \angle \phi$$

→ 'k' is the reflection coefficient at the load.

→ |k| is the magnitude of 'k'.

$\phi$  is the phase of 'k'.

| when $Z_L =$         | $k$                          | $ k $ | $S$      |
|----------------------|------------------------------|-------|----------|
| $Z_0$                | 0                            | 0     | 1        |
| $0 \Omega$ (SC)      | $-1$<br>$1 \angle 180^\circ$ | 1     | $\infty$ |
| $\infty \Omega$ (OC) | $1$<br>$= 1 \angle 0^\circ$  | 1     | $\infty$ |

$$|k|_{\max} = 1, \quad |k|_{\min} = 0.$$

$$S_{\max} = \infty, \quad S_{\min} = 1.$$

→ It can be proved that

$$Z_{\max} = S Z_0 \checkmark$$

$$Z_{\min} = \frac{Z_0}{S} \checkmark$$

→ If 'k' is the reflection coefficient at the load then  $|k|^2$  is called power refl. coeff.

→ % of power reflected =  $100 |k|^2 \%$ .

% of power delivered =  $(1 - |k|^2) 100 \%$ .

→ Return loss =  $20 \log |k|$  dB.

= decibal value of  $|k|^2$

$$\therefore \text{Return loss} = 10 \log_{10} |k|^2.$$

$$= 20 \log_{10} |k|.$$

\* Location of  $V_{\max}$

$$2\beta z_{\max} - \phi = 2n\pi$$

$$n=0 \rightarrow 1^{\text{st}} \text{ max.}$$

$$n=1 \rightarrow 2^{\text{nd}} \text{ max.}$$

$\vdots$

Location of  $V_{\min}$

$$2\beta z_{\min} - \phi = (2n+1)\pi$$

$$n=0 \rightarrow 1^{\text{st}} \text{ min.}$$

$$n=1 \rightarrow 2^{\text{nd}} \text{ min.}$$

$\vdots$

Ex-1 A transmission line is terminated by ~~pure~~ reactance. Assume characteristic impedance of the line is  $Z_0 \Omega$  (pure real). Find magnitude of the reflection coefficient and Voltage standing wave ratio.

Ans:

$$K = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{jX - Z_0}{jX + Z_0}$$

$$\therefore |K| = 1.$$

$$S = \frac{1+|K|}{1-|K|} = \frac{1+1}{1-1}$$

$$\therefore \boxed{S = \infty}$$

Power-reflection coefficient  $|K|^2 = 1.$

Ex 1 A transmission line is terminated by  $100\Omega$

Assume Characteristic impedance of the line is  $50\Omega$ . the incidence power is  $10W$ .

Find the power delivered to the load, the amount of power Reflected. Assume the line is lossless.

Ans:  $Z_R = 100\Omega$ ,  $Z_0 = 50\Omega$

$$\therefore K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{100 - 50}{100 + 50}$$

$$\therefore K = \frac{50}{150} = \frac{1}{3}$$

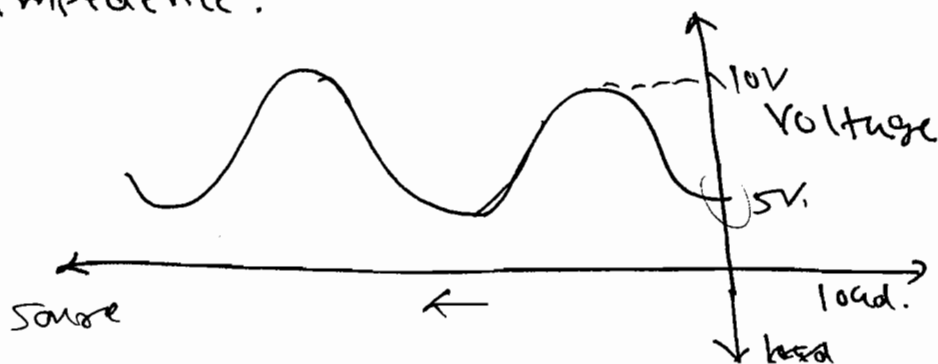
$$\therefore |K|^2 = \frac{1}{9}$$

power incident =  $10W$ .

power reflected =  $\frac{1}{9}(10) = 1.11W$ .

power delivered =  $(1 - \frac{1}{9})10 = 8.89 \text{ Watts}$ .

Ex 2 figure shows that A Voltage standing wave pattern of a loss-less transmission line. Assume characteristic impedance of the line  $100\Omega$ . Find  $S$ ,  $K$  and load impedance.



$$V_{\max} = 10 \text{ V.}$$

$$V_{\min} = 5 \text{ V.}$$

$$\therefore S = \left| \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}} \right|$$

$$\therefore S = \frac{10}{5} = 2$$

$$\boxed{S = 2}$$

$$\therefore |k| = \frac{S-1}{S+1} = \frac{2-1}{2+1} = \frac{1}{3}$$

$$\therefore k = |k| \angle \phi$$

$$\therefore k = \frac{1}{3} \angle \phi$$

$$\therefore 2\beta y_{\min 1} - \phi = (2n+1)\pi \quad \leftarrow n=0 \text{ for } 1^{\text{st}} \text{ min.}$$

$$\therefore 2\left(\frac{2\pi}{\lambda}\right)(0) - \phi = \pi$$

$$\uparrow y_{\min 1} = 0 \quad (\text{i.e.}) \text{ at the load itself.}$$

$$\phi = \pi$$

$\therefore$

$$\therefore k = \frac{1}{3} \angle 180^\circ = -\frac{1}{3}$$

$$\therefore \boxed{k = -\frac{1}{3}}$$

$$\therefore \frac{Z_R}{Z_0} = \frac{1+k}{1-k} = \frac{1-\frac{1}{3}}{1+\frac{1}{3}}$$

$$\therefore Z_R = \frac{1}{2} \times Z_0$$

$$\therefore Z_R = \frac{100}{2} \Rightarrow \boxed{Z_R = 50 \Omega}$$

Ex 3 In the above problem find min impedance and max impedance in transmission line.

Ans:

$$Z_{\max} = S Z_0$$

$$Z_{\min} = Z_0 / S$$

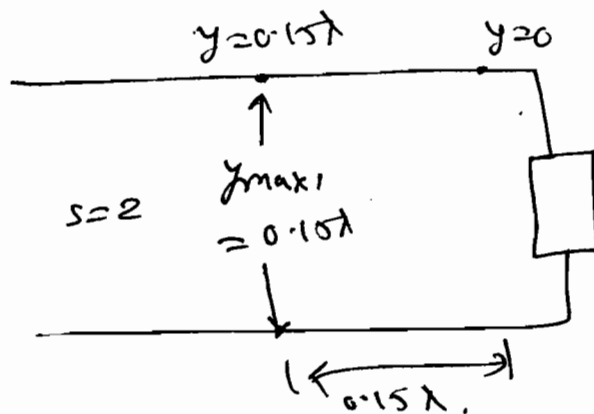
$$\therefore Z_{\max} = 2(100) = 200 \Omega.$$

$$Z_{\min} = \frac{100}{2} = 50 \Omega.$$

Ex 4 A transmission line is terminated by an unknown load impedance. The voltage maximum is observed in a distance of  $0.15\lambda$  from the load. The VSWR measured on the line is ~~to be~~ 2.0. Find the normalized load impedance.

(Hint: If any impedance is divided with  $Z_0$  then that impedance is known as normalized impedance)

Ans:



$$\therefore |K| = \frac{S-1}{S+1} = \frac{2-1}{2+1} = 1/3.$$

$$\therefore 2\beta\gamma_{\max} - \phi = 2n\pi. \quad n=0 \text{ for 1st max.}$$

$$\therefore 2 \times \frac{2\pi}{\lambda} \times 0.15\lambda - \phi = 0$$

$$\therefore \boxed{\phi = 108^\circ}$$

$$\therefore \boxed{k = 0.333 \angle 108^\circ}$$

$$\therefore \frac{Z_R}{Z_0} = \text{Normalized impedance} = \frac{1+k}{1-k}$$

$$\therefore \boxed{\frac{Z_R}{Z_0} = \frac{1 + 0.333 \angle 108^\circ}{1 - 0.333 \angle 108^\circ}} =$$

### ★ Impedance Matching.

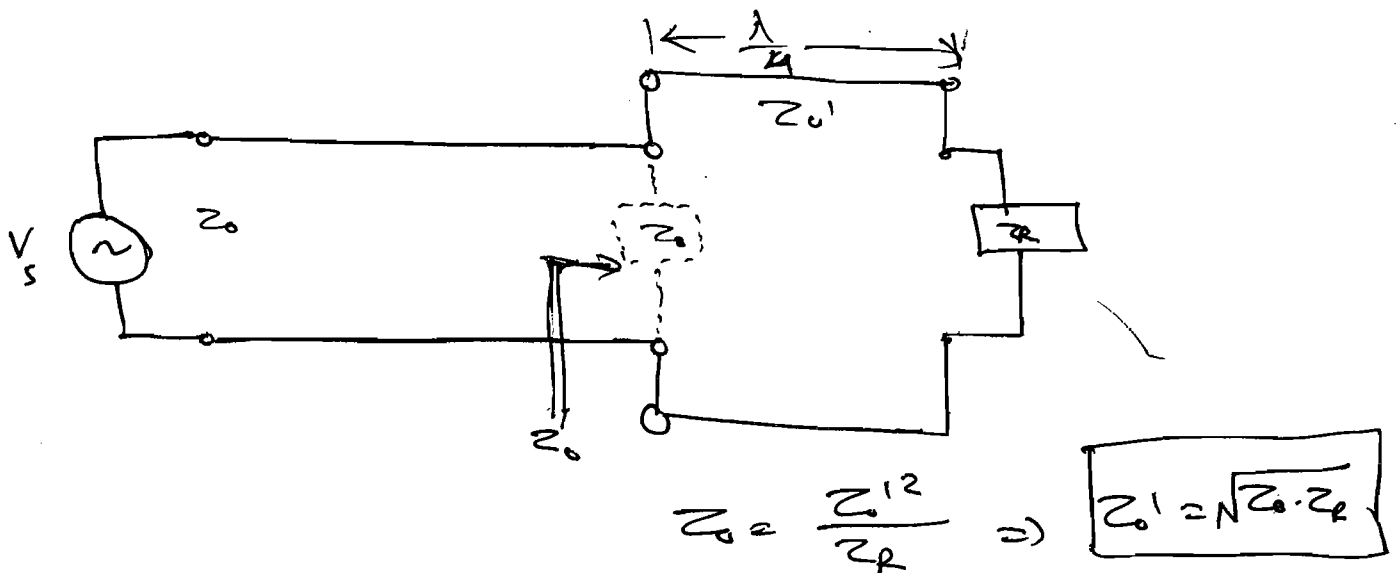
→ When  $Z_R \neq Z_0$  reflection would result. This reflection will travel towards the source. The purpose of a source is to deliver the energy because of the reflection the source is bound to accept the reflection. Due to this some source tends to unstable (or) they may lead to damage. Impedance matching technique is employed for protecting the source from the reflections.



### (1) Quarter Wave Transformer:

→ Connecting a Quarter wave line between the main transmission line and the load impedance is shown in the figure.

Characteristic Impedance of this Quarter wave line is the geometric mean of  $Z_0$  and  $Z_R$

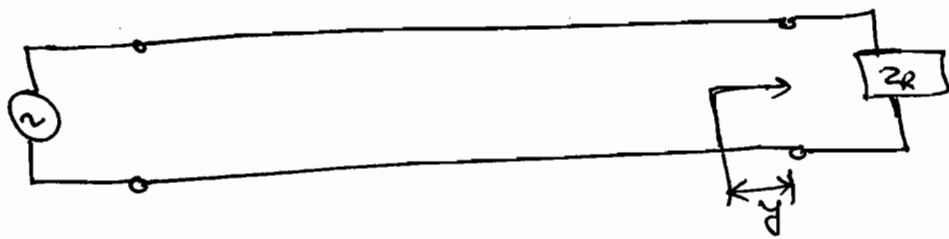


→ This technique is very simple. It has disadvantage or whenever the freq. or operation change length of the quarter wave line has to be adjusted by disconnecting from the main transmission line.

## ② Stub matching:

→ A section of a lossless transmission line either it is open circuited (or) short circuited can act as a circuit reactive elements (or) circuit susceptive elements desired reactance (or) susceptance can be achieved by properly choosing length of the transmission line. This section are named as stubs and are used in the impedance matching techniques. Hence, the name stub matching.

⇒



$$Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l} \Omega$$

Normalized Imp  $Z_{in} = \frac{Z_{in}}{Z_0}$

$$\therefore Z_{in} = \frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l}$$

$\therefore$  Normalized admittance  $Y_{in} = \frac{1}{Z_{in}} = \frac{Z_0 + j Z_L \tan \beta l}{Z_L + j Z_0 \tan \beta l}$

If  $\boxed{Z_{in} = 1} \Rightarrow Z_L = Z_0 \Rightarrow \text{line is matched.}$

If  $\boxed{Y_{in} = 1}$

$$\Rightarrow Y_{in} = \frac{Z_0 + j Z_0 \tan \beta \gamma}{Z_0 + j Z_0 \tan \beta \gamma}$$

By rationalizing, the real and im. parts can be separated.

$\therefore Y_{in} = \text{real part} + j \text{Im-part.}$

→ real part → function of  $(\underline{Z_L}, \underline{Z_0}, \underline{\beta}, \underline{\gamma}) = 1$   
Im. part → function of  $(\underline{Z_L}, \underline{Z_0}, \underline{\beta}, \underline{\gamma}) = ?$

Q1. Can we find at what  $\gamma$ , the real part becomes unity?

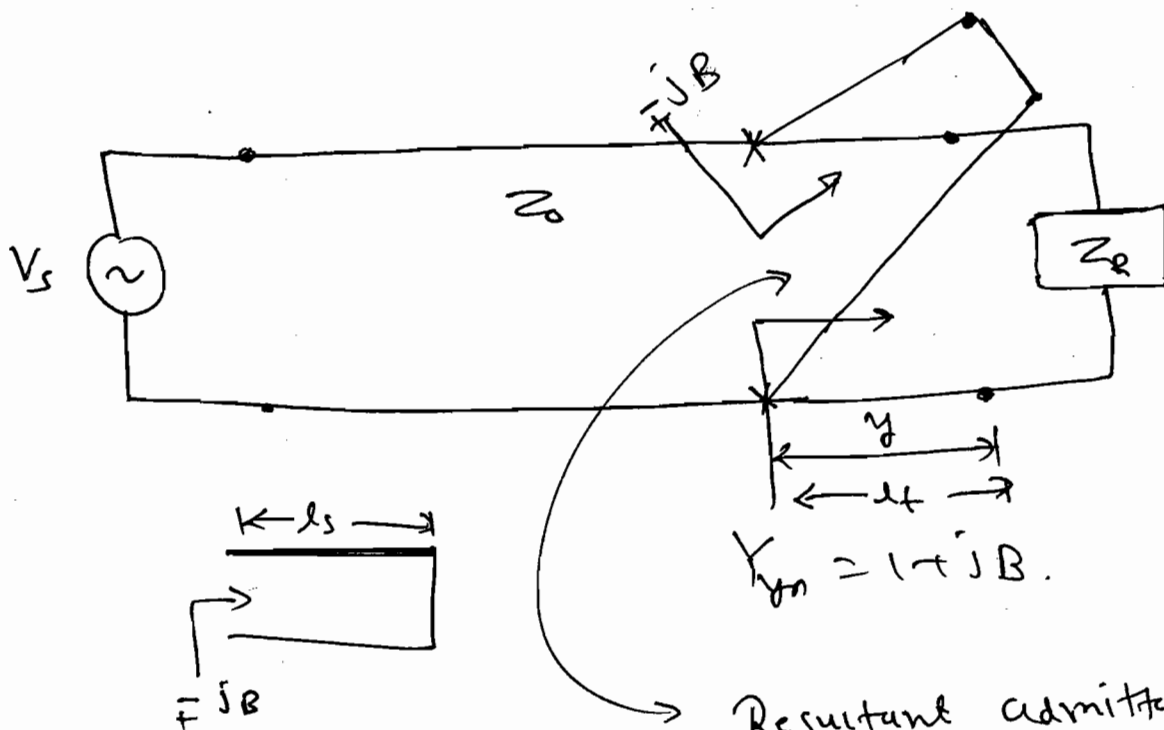
Ans: Yes.

Q2: ~~form~~ At the above  $\gamma$ , can we find the value of Im. part?

Ans: Yes.

$\Rightarrow$  Somehow we are calculating above  $\gamma$ . At that  $\gamma$  Normalized admittance

$$\therefore \boxed{Y_{in} = 1 + jB} \Rightarrow B = \text{Susceptance.}$$



Resultant admittance

$$Y_{\text{in}} = 1 + jB + jB = 1.$$

$$\therefore Y_{\text{in}} = 1$$

$$\therefore Y_{\text{in}}$$

line is matched.

→ We have to find  $l_s$  and  $l_t$ .

$$\therefore l_s = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{S}}{S-1}.$$

$$l_t = \frac{\phi + \pi - \cos^{-1} |K|}{2\beta}.$$

$l_s$ : length the S.C. stub.

$l_t$ : location of the stub from the load.

## \* SMITH CHART:

→ It is a graphical solution provider for any transmission line problems.

It is a family of circles.

$$\rightarrow K = \frac{Z_R \bar{Z}_0}{Z_R + Z_0}$$

$$\therefore \frac{Z_R}{Z_0} = \frac{1+K}{1-K}$$

$$\text{let } Z_R = \frac{Z_R}{Z_0} = \frac{1+K}{1-K}, \quad Z_R = \text{no units.}$$
$$= R + jX$$

'R': Real part of  $Z_R$ .

'X': Im. part of  $Z_R$ .

$$\rightarrow K = |K| \angle \theta.$$

$$K = K_R + jK_X.$$

$K_R$ : real (K).

$K_X$ : Imag. (K).

$$R + jX = \frac{(1 + K_R + jK_X)(1 - K_R + jK_X)}{(1 - K_R - jK_X)(1 - K_R + jK_X)}.$$

→ By rationalizing, one can separate real and Im. parts.

$$R = \frac{1 - K_R^2 - K_X^2}{(1 - K_R)^2 + K_X^2} \quad \text{--- (1)}$$

$$\therefore X = \frac{2K_x}{(1-K_x)^2 + K_x^2} \quad (2)$$

→ Eqn (1) & (2) represents a family of circles.

Consider eqn (1).

$$R = \frac{1 - K_x^2 - K_x^2}{(1-K_x)^2 + K_x^2}$$

Cross multiply.

$$\therefore RK_x^2 - 2RK_x + R + RK_x^2 = 1 - K_x^2 - K_x^2$$

$$\therefore K_x^2(1+R) - 2K_xR + K_x^2(1+R) = 1-R$$

divide by  $(1+R)$ .

$$\therefore K_x^2 - \frac{2R}{(1+R)} K_x + K_x^2 = \frac{1-R}{1+R}$$

add  $\left(\frac{R}{1+R}\right)^2$  on both sides.

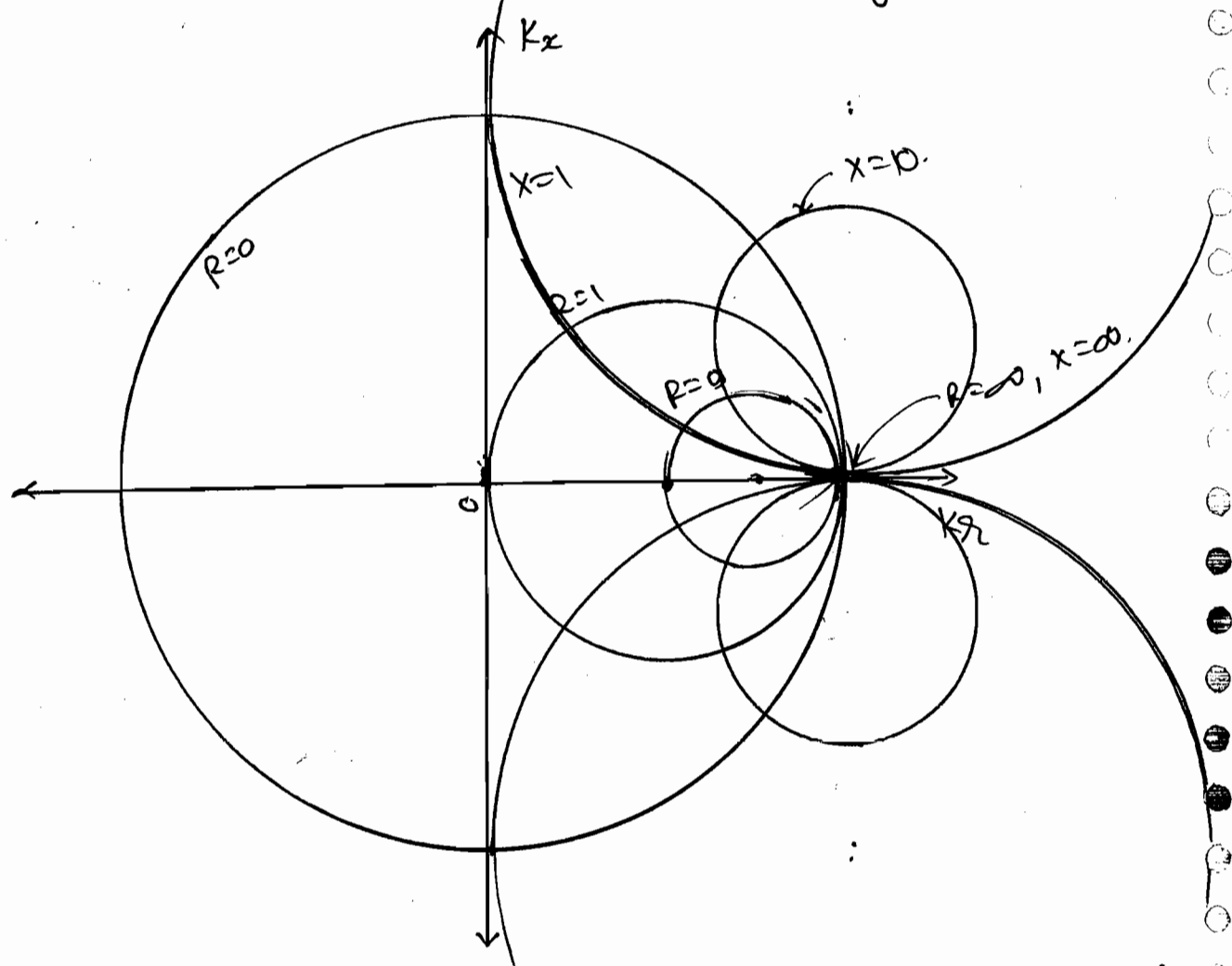
$$\left(K_x - \frac{R}{1+R}\right)^2 + (K_x - 0)^2 = \left(\frac{1}{1+R}\right)^2$$

→ The above eqn represents eqn of a circle with centre  $\left(\frac{R}{1+R}, 0\right)$  and radius  $\left(\frac{1}{1+R}\right)$  on  $(K_x, K_x)$  axes.

→ R can assume all possible values ranging from 0 to  $\infty$ . Therefore, one (and no

Infinite number of circles

→ As a value of  $R$  is increasing the radius of a circle is decreasing.



\* All constant  $R$  circles having following properties.

- (i) All the circles are passing through  $(1,0)$
- (ii) All the circles are having their centres bet<sup>n</sup>  $(0,1)$  on  $K_x$  axis.
- (iii) All the circles are neither concentric nor cutting each other.

→ consider eqn ②

$$X = \frac{2k_x}{(1-k_x)^2 + k_x^2}$$

$$\therefore k_x^2 x - 2x k_x + x + k_x^2 = 2k_x$$

divide with  $X$ .

$$\therefore k_x^2 - 2k_x + 1 + \frac{k_x^2}{x} = \frac{2k_x}{x}$$

add  $(\frac{1}{x})^2$  on both sides.

$$(k_x - 1)^2 + (k_x - \frac{1}{x})^2 = (\frac{1}{x})^2$$

→ The above eqn represents eqn of a circle with centre  $(1, \frac{1}{x})$  and radius  $\frac{1}{x}$  and radius  $\frac{1}{x}$  on  $k_x, k_y$  axes.

→  $X$  can assume all possible values ranging from  $(-\infty, +\infty)$  therefore infinite no. of circles can be drawn

→  $X \rightarrow -\infty$  to  $+\infty$ .

$$X=1 \\ (1, 1), 1$$

$$X=10 \\ (1, 0.1), 0.1$$

$$X=-1 \\ (1, -1), 1$$

$$X=0 \\ (1, \infty), \infty$$



$\Rightarrow$  As  $x$  increases  $\Rightarrow$  Reactance is decreasing:

$\rightarrow$  All constant  $x$  circles are having following properties

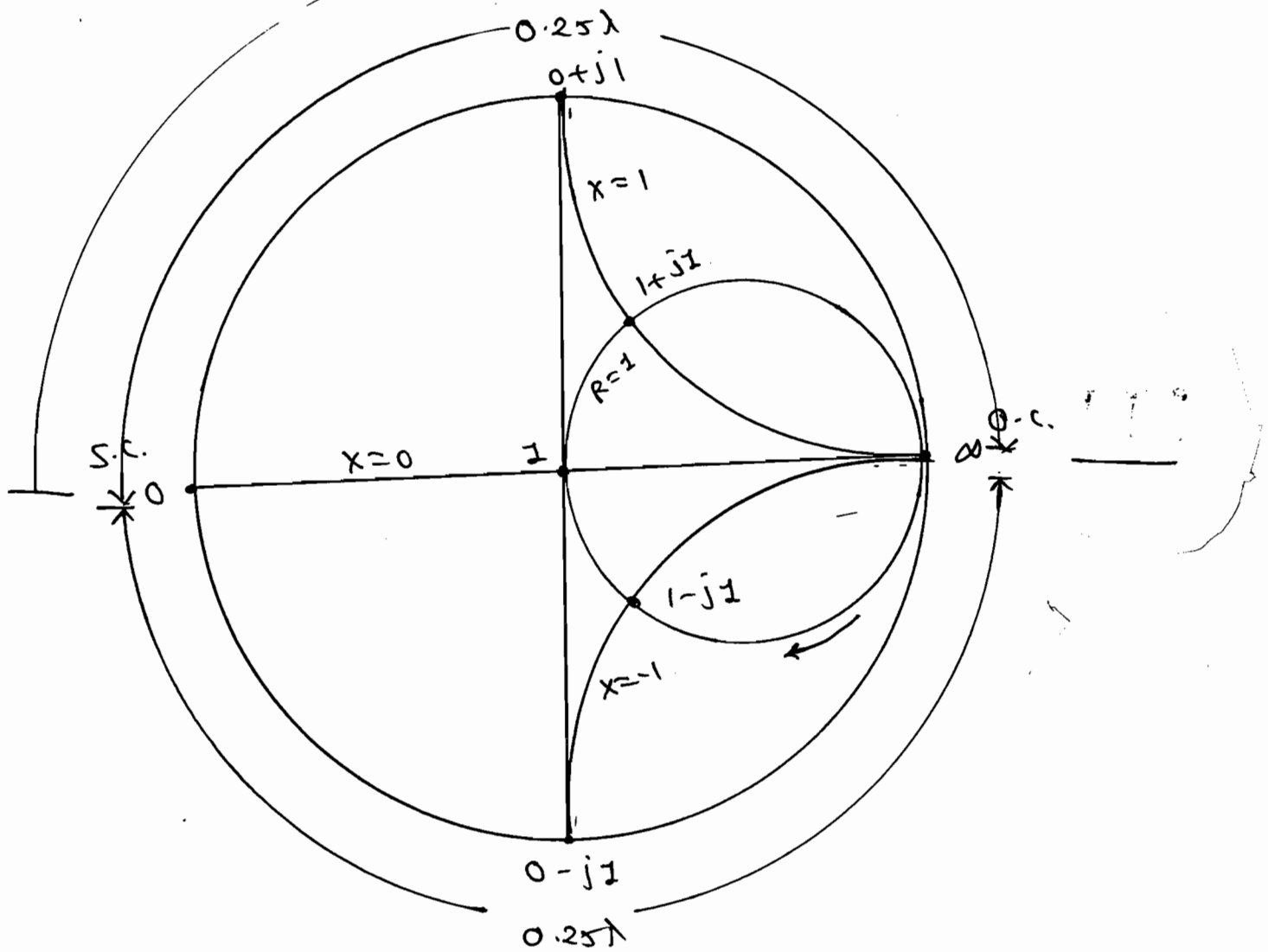
(1) All the circles are having their centres on  $k_R = 1$  line

(2) All the circles are passing through  $(1, 0)$ .

(3) The circles above the horizontal axis represents locus of +ve reactance whereas below the horizontal axis represents locus of (-ve) Reactance.

(4) All the circles are neither concentric nor cutting each other.

(5) The locus of constant  $-R$  circles and constant  $x$ -circles together is called Smith Chart



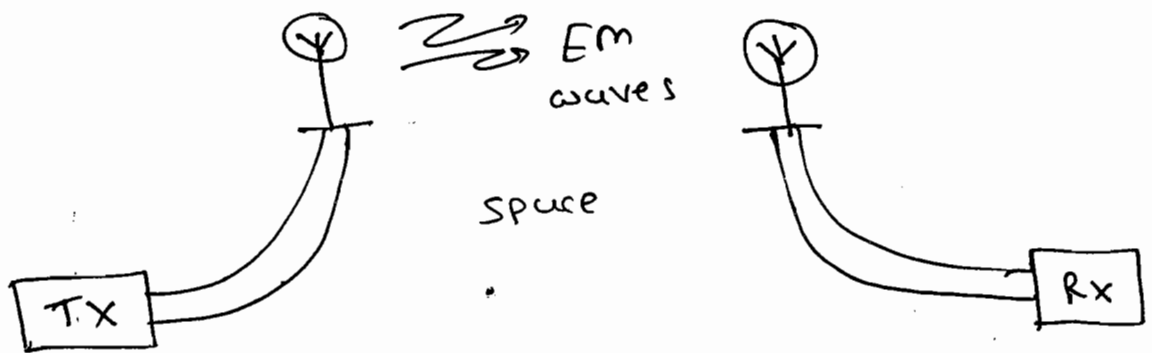
→ one complete revolution of the Smith Chart indicates a distance of  $\lambda/2$ .

→ ~~this~~ on a constant R circle if we are moving in the clockwise direction we are adding an inductive reactance in series to the impedance.

✓

# ★ Basics of Antenna:

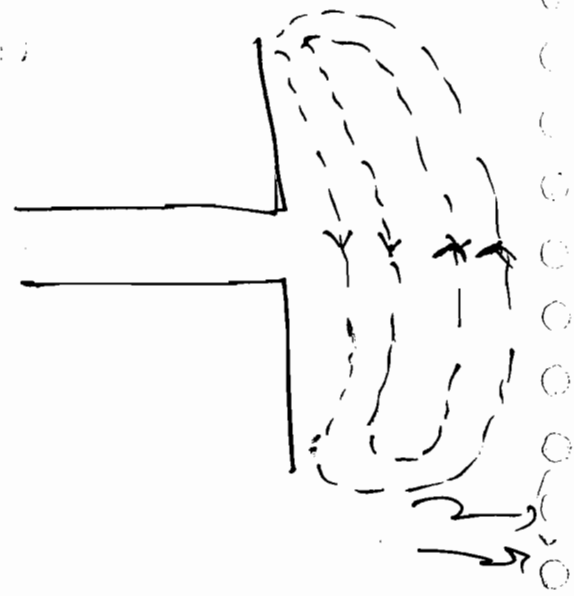
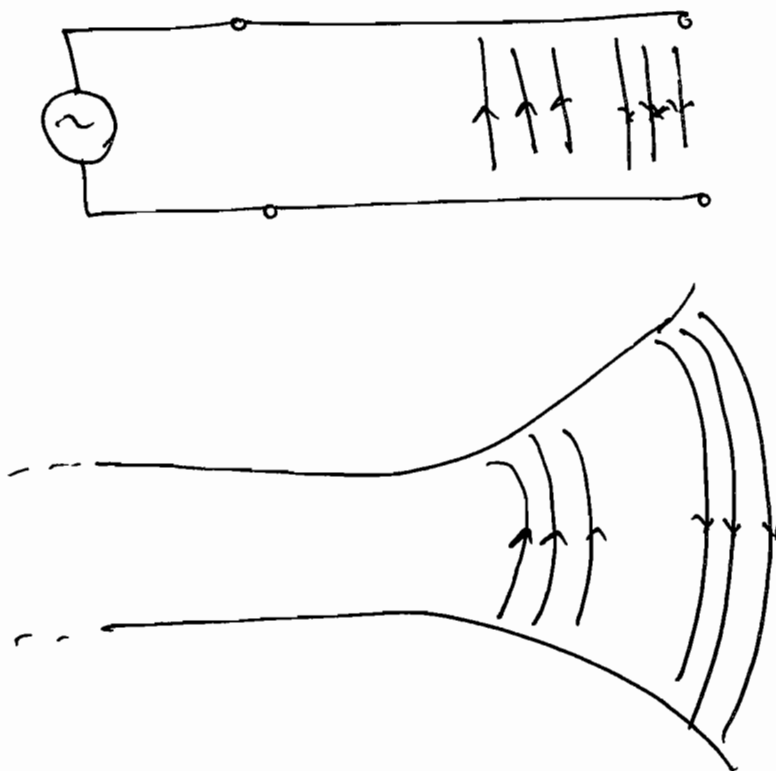
Symbol:



Antenna:

- Coupling devices (B/w Tx to Space & Space to Rx).
- It radiates / Receives EM Waves.
- It is tuned.
- Passive.

\* Radiation mechanism:

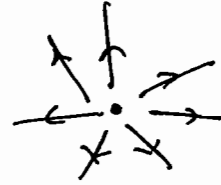


### \* Isootropic Radiator:

→ It is an impractical antenna (fictitious).

It is capable of radiating, receiving uniformly in all directions.

e.g. (i) point source.



### \* Directional Radiators:

→ All practical antennas are Directional radiators. They are capable of radiating receiving EM waves through some particular directions.

### \* Omnidirectional Radiators:

→ This is a special kind of directional radiator capable of radiating uniformly in the azimuth plane and having non-uniform radiation in the elevation plane.

e.g. dipole antenna.

⇒ By Reciprocity theorem it is proved that radiation properties of an antenna are identically same whether the antenna use for transmission purpose or reception purpose.

→ Radiation Properties include \* radiation pattern.

\* Average Radiation density.

\* Average Radiation intensity.

\* Average Radiation Power.

\* Directive gain.

\* Directivity.

\* Power gain.

\* maximum power gain.

\* Total efficiency of Antenna.

\* effective aperture area.

\* Antenna polarization. etc.

\* ⇒ These are the major things of antenna.

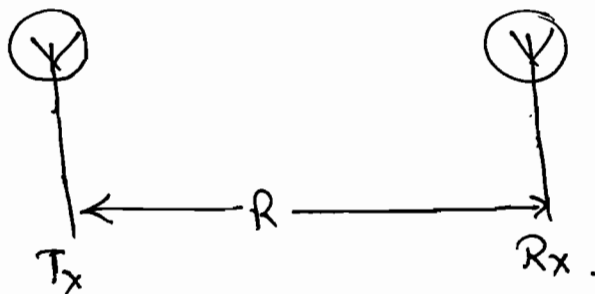
⇒ These are the antenna parameters.

### (1) Radiation Pattern:

→ It is the locus of received field strength (or) power at a fixed far distance as a function of Space Co-ordinates if the received quantity is field strength then it is called field strength pattern.

→ If the received quantity is power then it is called power pattern.

⇒



## \* Frumhofers Region of field:

→ If  $R > \frac{2D^2}{\lambda}$  ⇒ Frumhofers far field zone  
 ⇒ The fields in this zone are

$E \propto \frac{1}{r}$  Called active fields.

⇒ They are useful for radiation purpose.

→ If  $R < \frac{2D^2}{\lambda}$  ⇒ Frumhofers near field zone.  
 ⇒ The fields in this zone are

$E \propto \frac{1}{r^2}$  called reactive fields.

⇒ They are not useful for radiation purpose.

D: max. dimension of the antenna.

$\lambda$ : operating wavelength.

→ All the antennas are intended to be operated in the Frumhofers far field zone only.

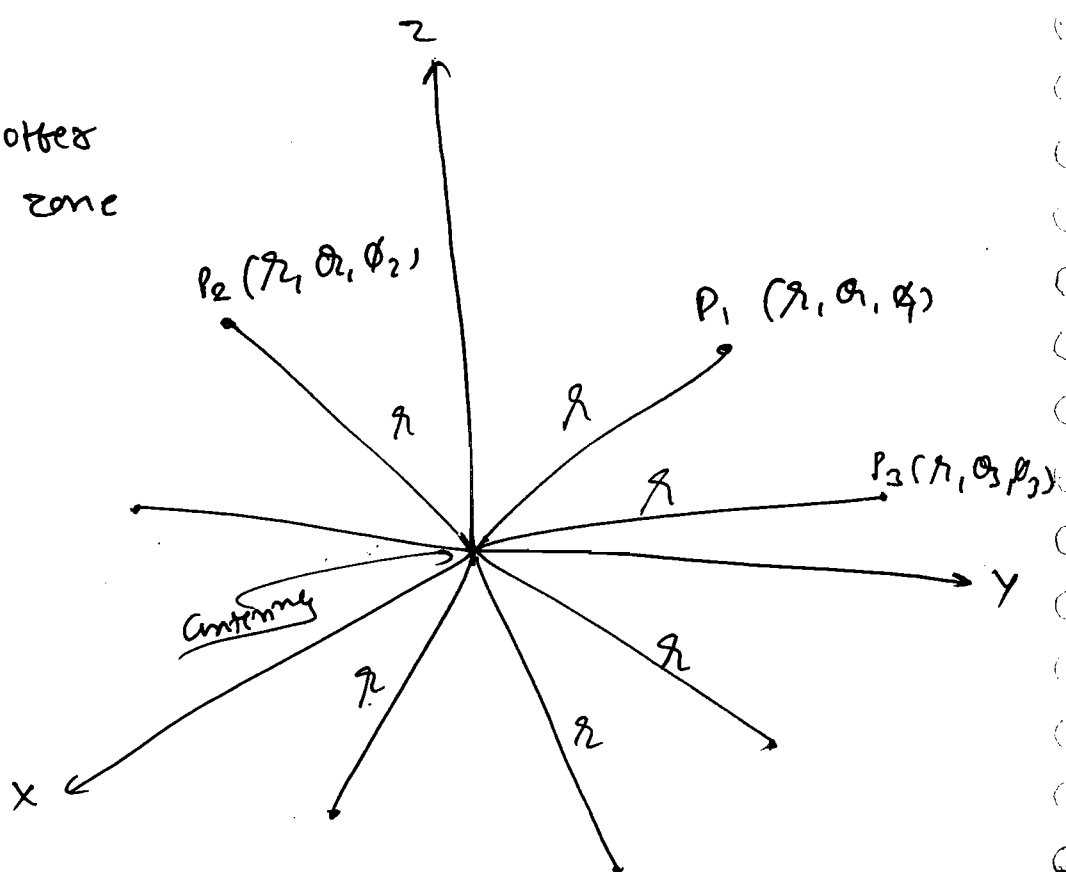
\*

Satisfies far field zone

$$r = \text{constant}$$

$$r = \text{const.}$$

$$\begin{aligned} \theta &\Rightarrow 0 \text{ to } 180 \\ \phi &\Rightarrow 0 \text{ to } 360 \end{aligned}$$



→ We assume that, the antenna is positioned at the origin. Around the antenna at a fixed distance  $r$  one can have infinite no. of points. All those points are having equal distance from the origin and are distinguishable.

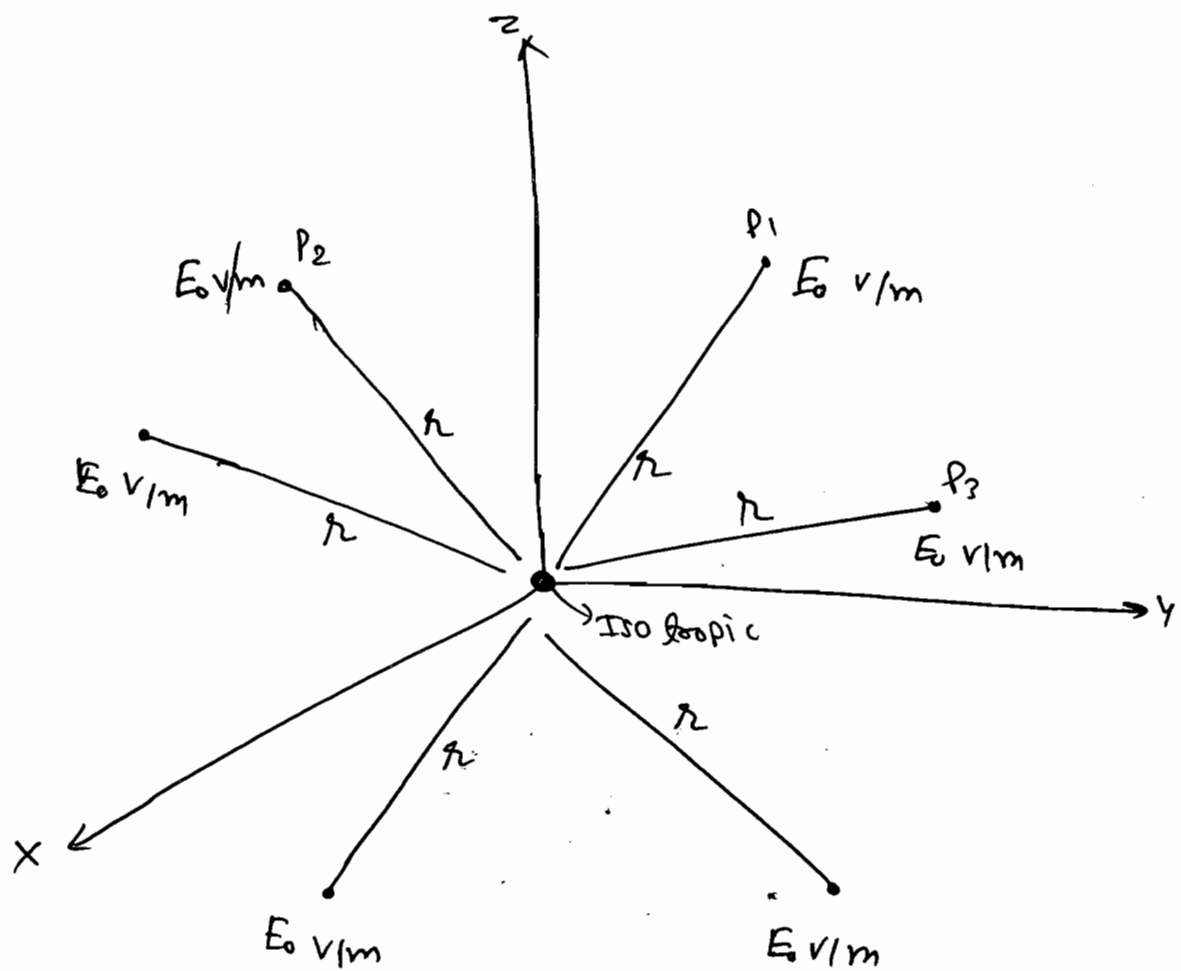
\* Radiation pattern of an Isotropic Radiator:

→ we assume that isotropic radiator is positioned at the origin. At a fixed distance in the far field zone we observe  $E_{\theta}$  v/m at various points. i.e.

$$r = \text{const.}$$

$$\theta = 0 \text{ to } \pi$$

$$\phi = 0 \text{ to } 2\pi$$



→ The three dimensional radiation pattern of an Isotropic antenna appears to like a spherical cell because it radiates uniformly in all directions.

→ The radiation pattern is a three dimensional view. Radiation pattern can also be investigated in the following two principal plane:

1. Azimuthal plane patterns ( $\theta = 90^\circ$ ).
2. Elevation plane patterns ( $\phi = \text{const}$ ).

→ we consider  $\phi = 0^\circ$  and  $\phi = 180^\circ$ .



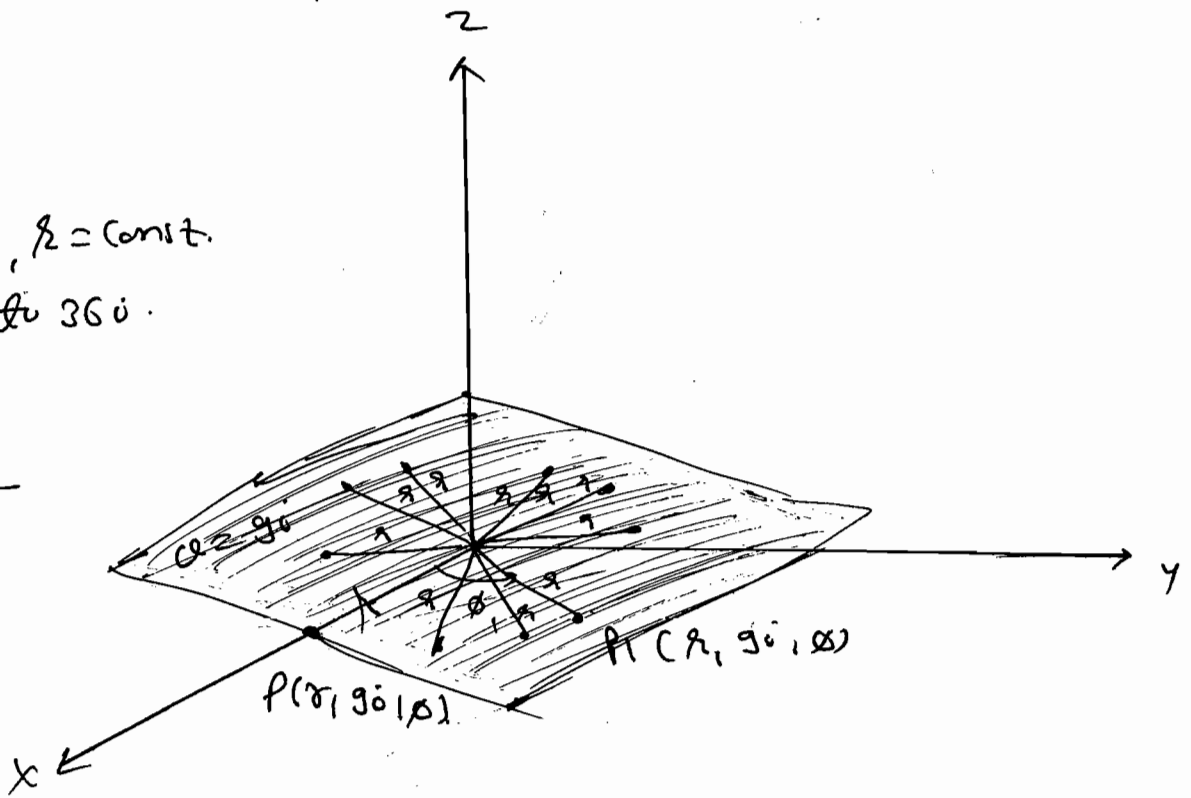
→ Azimuthal plane pattern of a Radiator.

\* Azimuthal Plane Pattern of an Isotropic Radiator.

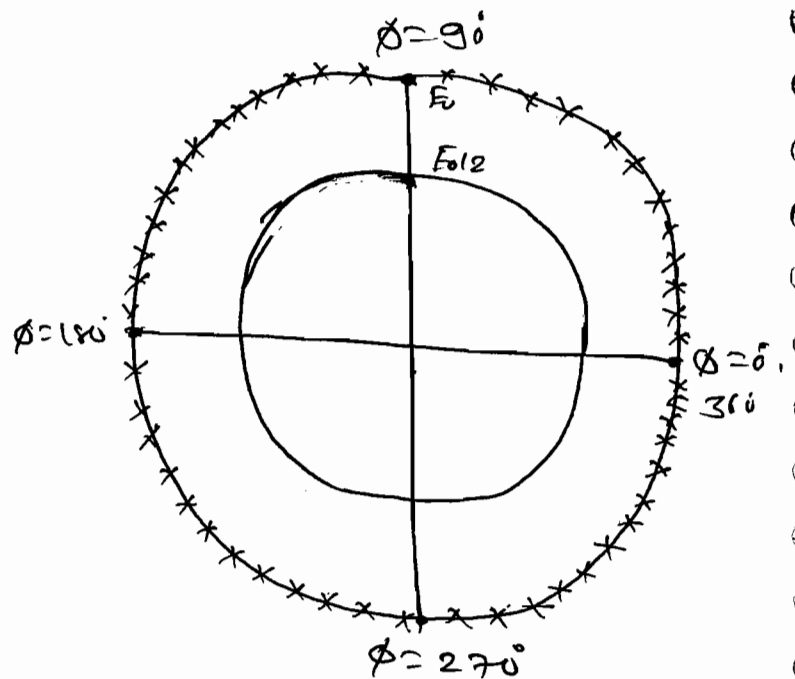
$$\theta = 90^\circ$$

$$\theta = 90^\circ, r = \text{const.}$$

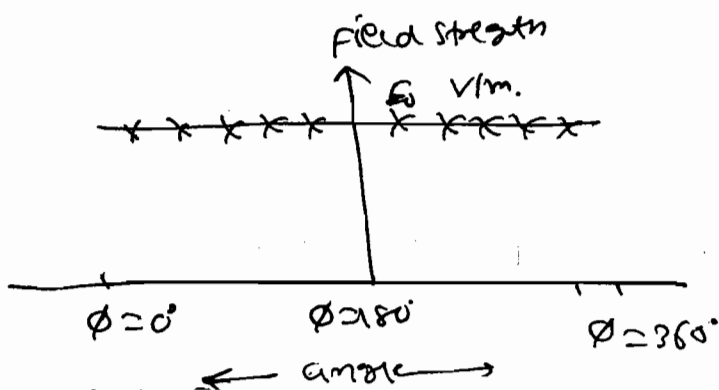
$$\phi \rightarrow 0 \text{ to } 360^\circ.$$



| $\phi =$    | Field strength    |
|-------------|-------------------|
| $0^\circ$   | $E_0 \text{ V/m}$ |
| $1^\circ$   | $E_0 \text{ V/m}$ |
| $2^\circ$   | $E_0 \text{ V/m}$ |
| $\vdots$    | $\vdots$          |
| $20^\circ$  | $E_0 \text{ V/m}$ |
| $\vdots$    | $\vdots$          |
| $360^\circ$ | $E_0 \text{ V/m}$ |



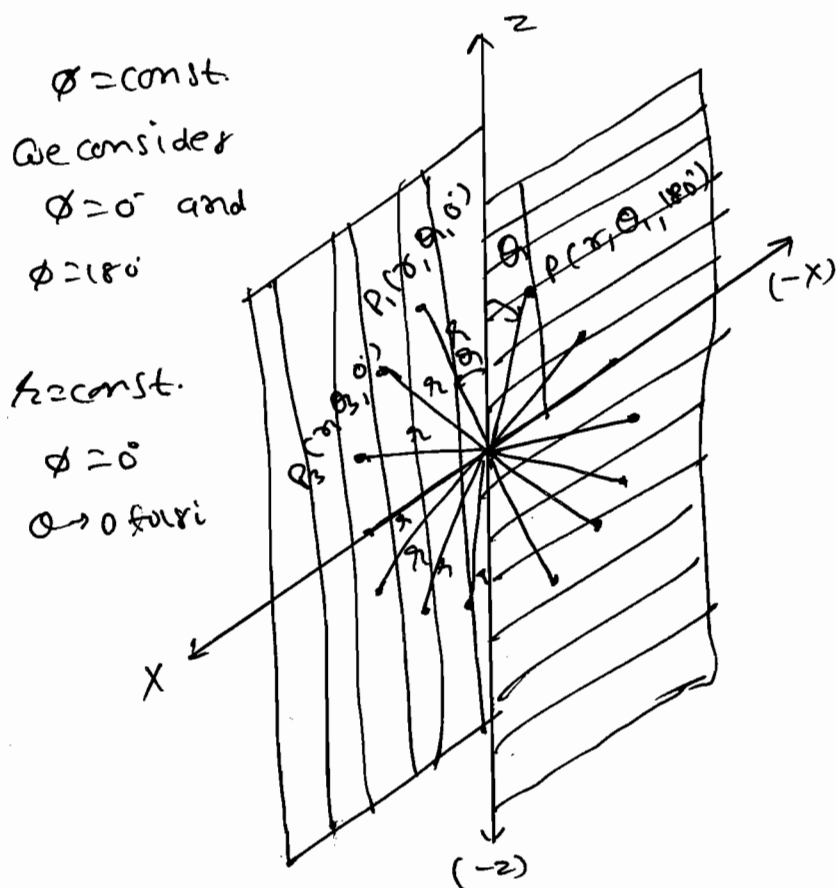
\* Linear plot:



Polar plot

→ circular locus on the Polar plot and horizontal straight line locus on the linear plot indicates that Isotropic radiator radiates uniformly in all directions in the azimuthal plane.

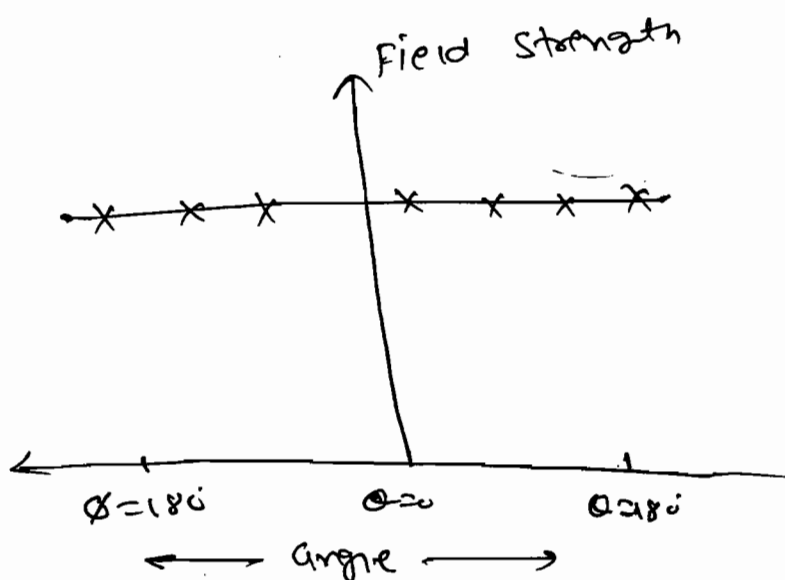
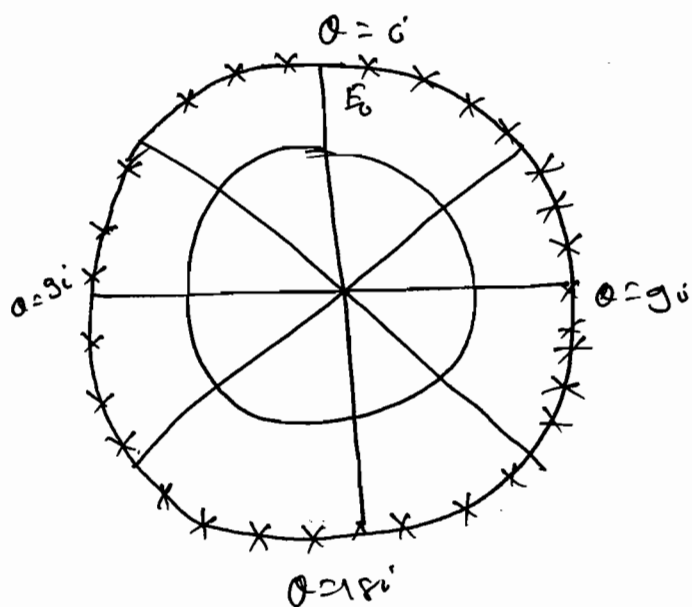
### \* Elevation plane pattern of a Isotropic radiator.



| $\phi$      | Field strength   |                     |
|-------------|------------------|---------------------|
| $0^\circ$   | $E_0 \sqrt{1/n}$ | $\phi = 0$          |
| $1^\circ$   | $E_0 \sqrt{1/n}$ | $r = \text{const.}$ |
| $2^\circ$   | $E \sqrt{1/n}$   |                     |
| $\vdots$    | $\vdots$         |                     |
| $180^\circ$ | $E_0 \sqrt{1/n}$ |                     |

| $\phi =$    | Field strength |                     |
|-------------|----------------|---------------------|
| $0^\circ$   | $E_0$          | $\phi = 180^\circ$  |
| $1^\circ$   | $E_0$          | $r = \text{const.}$ |
| $2^\circ$   | $\vdots$       |                     |
| $\vdots$    | $\vdots$       |                     |
| $180^\circ$ | $E$            |                     |

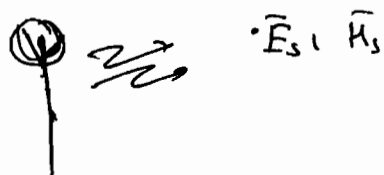
### Polar plot



→ circular locus on the polar plot and horizontal straight line locus on the linear plot indicates, that isotropic radiator radiates uniformly in all directions in the elevation plane.

## (2) Average Radiation Density:

→ It is defined as the avg. power radiated per unit area



→ If  $\vec{E}_s, \vec{H}_s$  are radiated fields in the Fraunhofer far field zone.

$$\therefore \boxed{\bar{P}_{\text{rad}} = \frac{1}{2} \vec{E}_s \times \vec{H}_s^*} \quad \text{W/m}^2$$

Complex Conjugated.

Avg power Radiated

$$\boxed{W_{\text{rad}} = \int_S \bar{P}_{\text{avg}} \cdot d\vec{s} \quad \text{Watts.}}$$

$d\vec{s}$  : Vector diff. surface element.

$$d\vec{s} = r^2 \sin\theta \, d\alpha \, d\beta \, \hat{r}$$

Ex 1  $\Rightarrow$  The avg. radiation density of an Antenna in the radial direction is given by

$$\bar{P}_{\text{rad}} = \frac{A_0 \sin \theta}{r^2} \hat{a}_r \text{ W/m}^2.$$

Where,  $A_0 = \text{Constant}$

$r, \theta$  : Spherical Coordinates

Find avg. radiated power.

Ans: The given  $\bar{P}_{\text{rad}}$  will belong to an Antenna. which is omni directional. It has uniform radiation in the azimuthal plane and non-uniform radiations in the elevation plane.

$$\begin{aligned} \bar{P}_{\text{rad}} \cdot d\bar{s} &= \frac{A_0 \sin \theta}{r^2} \times r^2 \sin \theta \cdot d\alpha d\theta. \\ &= A_0 \sin^2 \theta \cdot d\alpha d\theta. \end{aligned}$$

$$\therefore W_{\text{rad}} = \int_{(\alpha)} \int_{(\theta)} \bar{P}_{\text{rad}} \cdot d\bar{s}$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{360^\circ} A_0 \sin^2 \theta \cdot d\alpha d\theta.$$

$$= A_0 \times (2\pi) \times \left[ \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{2\pi}.$$

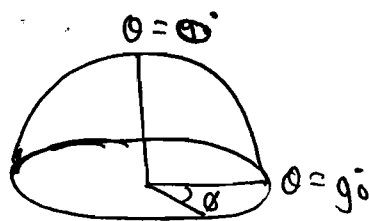
$$= A_0 \times 2\pi \times \left[ \frac{\pi}{2} \right]$$

$$\therefore \boxed{W_{\text{rad}} = \pi^2 A_0 \text{ Watts}}$$

Ex 2  
 $\rightarrow$  Repeat the above example to calculate avg. power radiated in the upper hemisphere.

Ans: Upper hemisphere

$\theta = 0$  to  $90^\circ$   
 $\phi = 0$  to  $360^\circ$ .



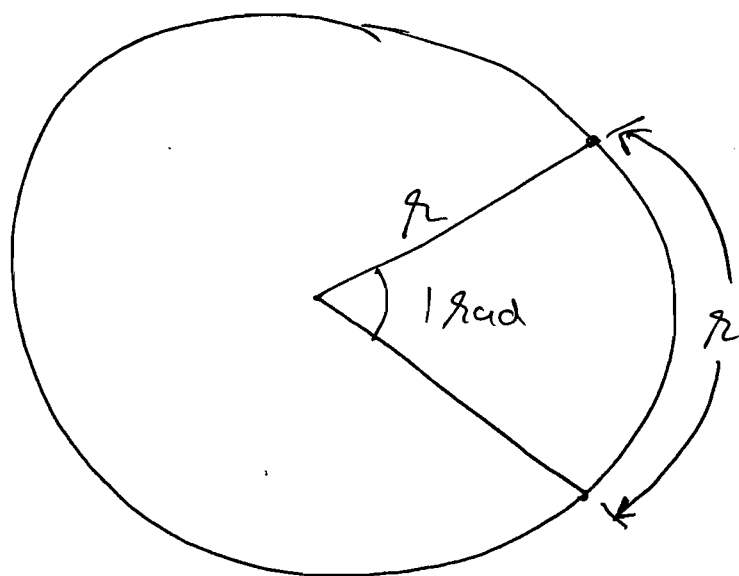
$$\therefore W_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi/2} A_0 \left[ \frac{1 - \cos 2\theta}{2} \right] d\theta d\phi.$$

$$= A_0 \times [2\pi] \times \left[ \frac{\theta}{2} - \frac{\sin 2\theta}{2} \right]_0^{\pi/2}.$$

$$= A_0 \times 2\pi \times \left[ \frac{\pi}{4} \right]$$

$$\therefore W_{\text{rad}} = \frac{1}{2} \pi^2 A_0 \text{ Watts.}$$

\* Radian:



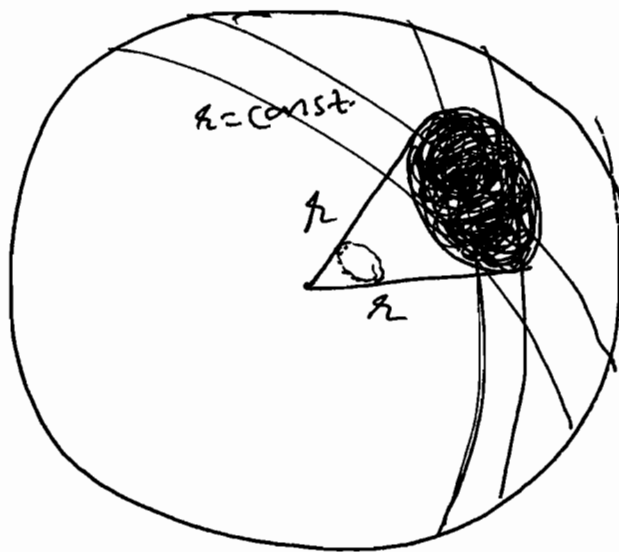
$$r = 1 \text{ rad.}$$

$$2\pi r = \frac{2\pi r}{r} \times 1$$

$$= 2\pi \text{ radian}$$

The total angle subtended at the centre of a circle is  $2\pi$  rad.

## \* Steradian:



$$r^2 \rightarrow 1 \text{ steradian}$$

$$4\pi r^2 \rightarrow \frac{4\pi r^2}{r^2} \times 1 = 4\pi \text{ st.}$$

→ The total solid angle subtended at the centre of a sphere =  $4\pi \text{ st.}$

(3) Avg. Radiation Intensity :  $\bar{U}$

→ It is defined as average power radiated per unit solid angle.

$$\checkmark \quad \boxed{\bar{U} = r^2 \bar{P}_{\text{rad}} \text{ W/st.}}$$

Ex 1 The average radiation intensity of an antenna in the radial direction is proportional to  $\cos^2 \theta$ . assume the proportionality constant is unity. find the average radiated power.

Ans:  $\bar{U} = r^2 \bar{P}_{\text{rad}} \text{ W/st.}$

$$|\bar{U}| \propto \cos^2 \theta.$$

$$|\bar{U}| = \cos^4 \theta \cdot 0 \Rightarrow \bar{U} = \cos^4 \theta \hat{a}_n \text{ W/m}^2$$

$$\therefore \bar{P}_{\text{rad}} = \frac{\bar{U}}{r^2}$$

$$\therefore \bar{P}_{\text{rad}} = \frac{\cos^4 \theta}{r^2} \hat{a}_n$$

$$\therefore d\bar{S} = r^2 \sin \theta \cdot d\theta d\phi \hat{a}_n$$

$$W_{\text{rad}} = \oint_S \bar{P}_{\text{rad}} \cdot d\bar{S}$$

$$= \int_0^{2\pi} \int_0^{\pi} \cos^4 \theta \cdot \sin \theta \cdot d\theta d\phi$$

$$= \left[ \frac{t^5}{5} \right]_{-1}^{+1} [2\pi]$$

$$= \left[ \frac{t^5}{5} \right]_{-1}^{+1} [2\pi]$$

$$= \frac{(1+2)}{5} 2\pi$$

$$\therefore \boxed{W_{\text{rad}} = \frac{4\pi}{5} \cdot W_{\text{utts}}}$$

(4) Directive gain ( $D_g$ ):

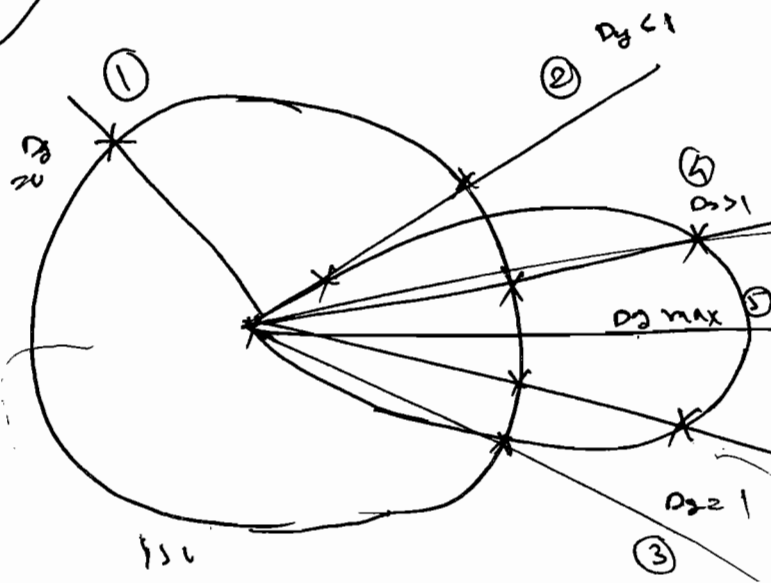
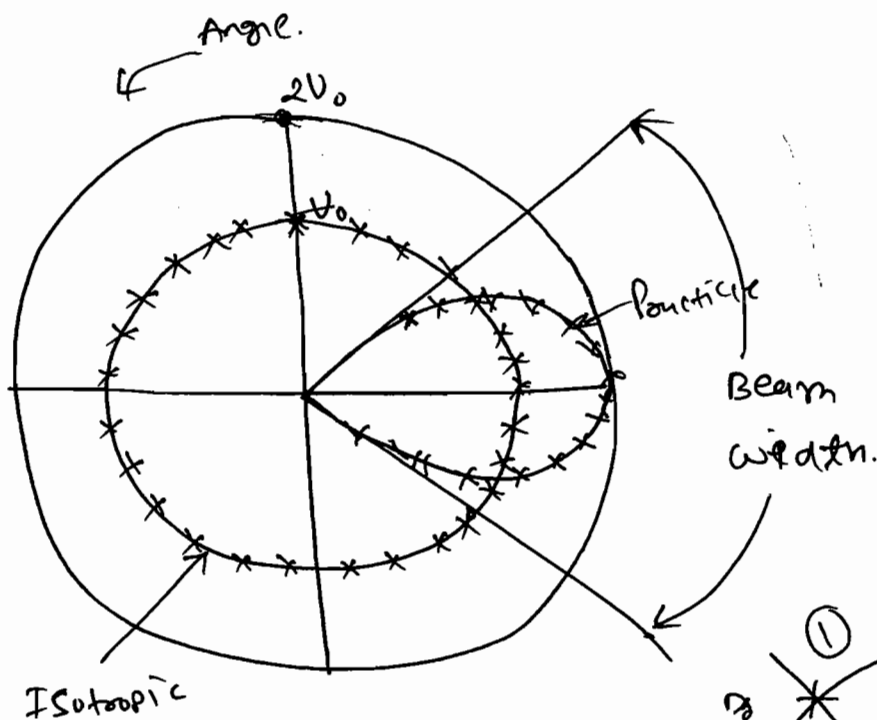
$\Rightarrow$

$$\boxed{D_g = \frac{U}{U_0}}$$

$U$ : Practical antenna

→ Directive gain in a given direction is defined as the ratio of radiation intensity of the practical antenna whose directive gain you want to calculate to the radiation intensity of the reference antenna. The reference antenna is chosen to be Isotropic radiator.

→ The above definition is valid under the condition that both antennas are assumed to be radiating same amount of avg. radiated powers.



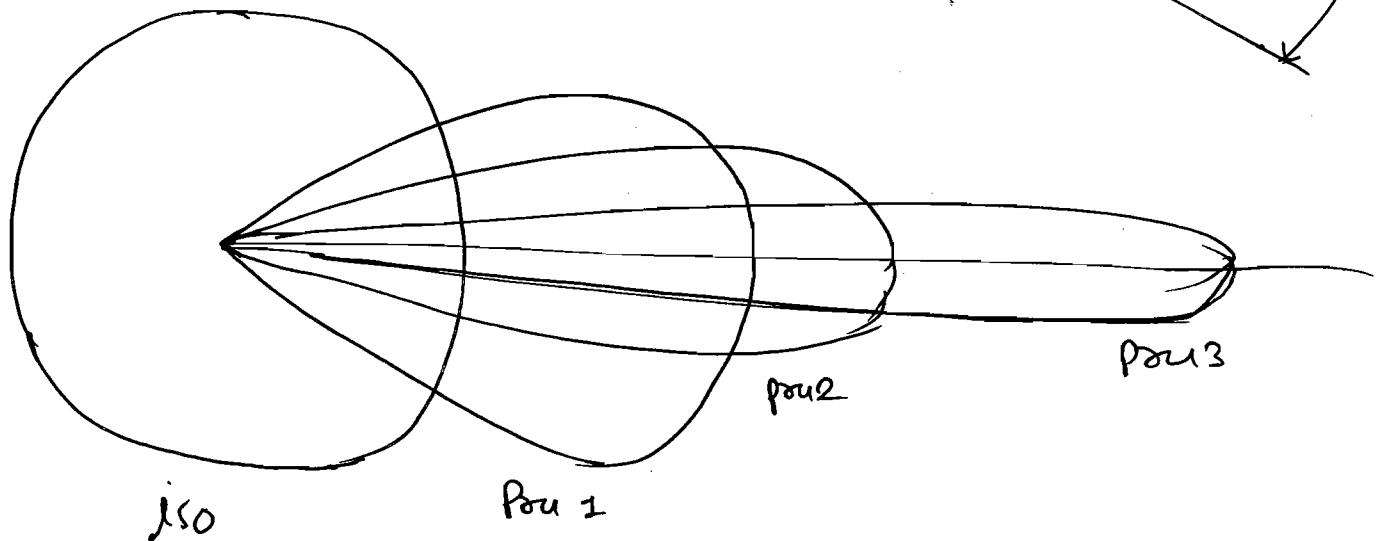
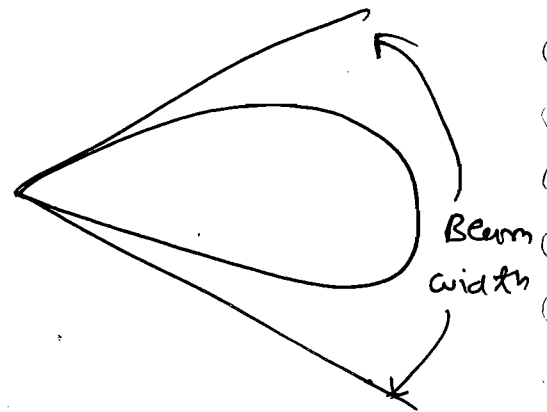


$$W_{\text{rad}} = W_{\text{rad}}(\text{iso})$$

(Pou)

$$M_{\text{ax}}(D_0) = D_0$$

'D<sub>0</sub>' : Directivity.



$$W_{\text{rad}}(\text{iso}) = W_{\text{rad}}(\text{Pou1}) = W_{\text{rad}}(\text{Pou2}) = W_{\text{rad}}(\text{Pou2}) = W_{\text{rad}}(\text{Pou3})$$

$$(D_0)_3 > (D_0)_2 > (D_0)_1 > (D_0)_1$$

$$D_{0\text{iso}} = \frac{U_0}{U_0} = 1$$

→ As the directivity increases Beam width ✓ decreases.

→ For point to point communication the antenna and the antenna system must have high directivity. for ~~th~~

→ For the Broadcasting purposes the antenna and the antenna system must have low directivity.

$$\rightarrow \bar{U}_{iso} = U_0 \hat{q}_n.$$

$$\bar{P}_{rad}(iso) = \frac{U_0}{r^2} \hat{q}_n.$$

$$\therefore W_{rad}(iso) = \int_{(\theta=0)}^{\pi} \int_{(\phi=0)}^{2\pi} \frac{U_0}{r^2} \cdot \hat{q}_n \cdot r^2 \sin\theta \cdot d\theta d\phi \hat{q}_n = 4\pi U_0.$$

$$\therefore U_0 = \frac{W_{rad}(iso)}{4\pi} = \frac{W_{rad}(pow)}{4\pi} = \frac{W_{rad}}{4\pi}.$$

$$\therefore D_g = \frac{U}{(W_{rad}/4\pi)} \quad D_g$$

$$\therefore D_g = 4\pi \times \frac{U}{W_{rad}}.$$

$$\text{Let, } \bar{P}_{rad} = \frac{A_0 \sin\theta}{r^2} \hat{q}_n.$$

$$D_0 = 4\pi \times \frac{U_{max}}{W_{rad}}$$

$$\therefore \bar{U} = r^2 \bar{P}_{rad} = A_0 \sin\theta \hat{q}_n.$$

$$\therefore U_{max} = A_0.$$

Find  $D_0$ .

$$\therefore W_{rad} = \pi^2 A_0 \text{ Watts.}$$

$$\therefore D_0 = 4\pi \cdot \frac{U_{max}}{W_{rad}}.$$

$$\therefore D_0 = 4\pi \cdot \frac{A_0}{\pi^2 A_0}.$$

$$\checkmark \therefore D_0 = \frac{4}{\pi}.$$

$$D_0 (\text{in dB}) = 10 \log_{10} \left( \frac{4}{\pi} \right) \text{ dB.}$$

Ex-1

→ The radiation density of an antenna in a radial direction is proportional to the  $\cos^1 \theta$   
assume the antenna has radiation in the upper hemisphere. Find the directivity.

Ans:

$$|\bar{U}| \propto \cos^1 \theta.$$

$$\therefore |\bar{U}| = A_0 \cos^1 \theta.$$

$$\bar{U} = A_0 \cos^1 \theta \hat{q}_n. \quad \text{Where } A_0 = \text{const.}$$

$$\therefore \bar{P}_{\text{rad}} = \frac{A_0 \cos^1 \theta}{r^2} \hat{q}_n \text{ W/m}^2.$$

$$\therefore W_{\text{rad}} = \int_{\alpha=0}^{\pi/2} \int_{\theta=0}^{2\pi} \frac{A_0 \cos^1 \theta}{r^2} \cdot r^2 \sin \alpha \cdot d\alpha d\theta.$$

$$\therefore W_{\text{rad}} = \int_{\alpha=0}^{\pi/2} \int_0^{2\pi} A_0 \sin \alpha \cdot \cos^1 \alpha \cdot d\alpha d\theta.$$

$$= 2\pi \times A_0 \times \frac{9 \times 7 \times 5 \times 3 \times 1}{10 \times 9 \times 7 \times 5 \times 3 \times 1} \quad (1).$$

$$\therefore \boxed{W_{\text{rad}} = \frac{2\pi A_0}{10} \text{ Watts.}}$$

$$\therefore D_0 = 4\pi \frac{U_{\text{max}}}{W_{\text{rad}}}.$$

$$= 4\pi \frac{A_0}{2\pi A_0} \times 11$$

$$D_0 = 22$$

$$\therefore D_0 \text{ (in dB)} = 10 \log_{10} (22) \text{ dB.}$$

Ex-2  
→

Radiation density of an antenna in the radial direction is given by  $\sin \theta \cdot \cos^2 \phi \hat{r}$ .  
Guthrie / student assume the antenna is radiating  
for  $0 \leq \theta < \pi/2$ ,  $0 < \phi < \pi/2$ . Find Directivity.

90.2

(5) Power Gain:

$$\Rightarrow C_{FP} = 4\pi \frac{U}{\omega_{in}}$$

$$\rightarrow \omega_{in} = \omega_{rad} + \omega_{loss}$$

$$\rightarrow \text{Max. Power Gain } C_{to} = 4\pi \frac{U_{max}}{\omega_{in}}$$

(6) Total Efficiency of an Antenna:

$$\Rightarrow \rho_t = \frac{\omega_{rad}}{\omega_{in}}$$

$$\rho_t = \frac{C_{to}}{D_o}$$

(7) Effective Aperture Area ( $A_e$ )

$\Rightarrow$  It refers to physical size of the antenna.

$\rightarrow$  Larger antennas will have larger aperture area and vice versa.

$\rightarrow$  If the antenna dimension are larger than  $\lambda$  then they are called large antennas and vice versa.

→ It is defined as average power receive to the average power density of the incident wave.

→ By expression,

$$A_e = \frac{\lambda^2}{4\pi} \cdot D_g.$$

$$\therefore A_{\text{max}} = \frac{\lambda^2}{4\pi} \cdot D_o.$$

$\uparrow A_e \Rightarrow \uparrow D_o \Rightarrow \downarrow \text{Beamwidth (narrower)}$

$\downarrow A_e \Rightarrow \downarrow D_o \Rightarrow \uparrow \text{Beamwidth (wider)}$

### (8) Antenna Polarization:

→ Antenna Polarization and the wave polarization are identically same because the antenna radiates EM waves and also receives EM waves.

## \* Magnetic Vector Potential ( $\vec{A}$ ).

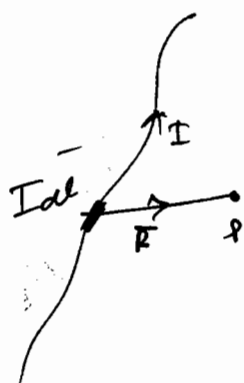
→ A divergence <sup>less</sup> ~~space~~ Vector may be a curl of some other vector  
 $\nabla \cdot \vec{B} = 0$

$$\therefore \vec{B} = \nabla \times \vec{A}$$

$\vec{A}$  is named as magnetic vector potential.

$\vec{A}$ : Magnetic Vector Potential

$$\vec{A} \rightarrow \text{T.m (or) wbm.}$$



$$d\vec{A} = \frac{\mu_0 I d\vec{l}}{4\pi r^2}$$

$$\vec{A} = \int d\vec{A}$$

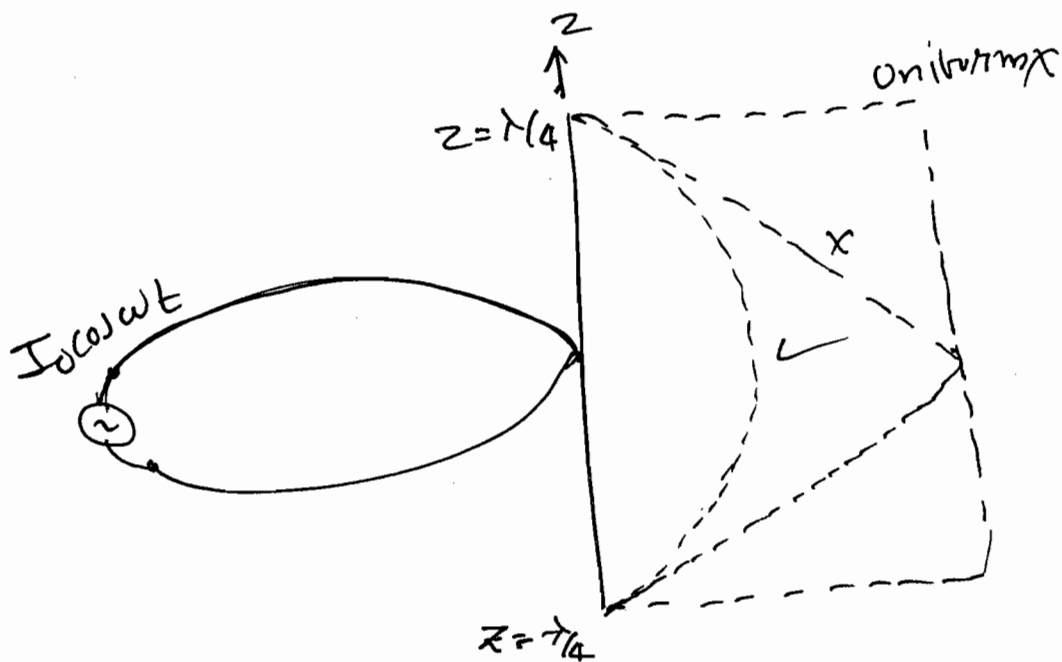
→ Source of magnetic potential is current element which is a vector quantity therefore ~~this~~ magnetic potential is named as magnetic vector potential.

## \* Current distribution:

→ when an antenna is excited by a transmission line (or) by some means distribution of currents takes place on the antenna geometry. ~~as a function of~~

for e.g.

→ The Suitable Current distribution a half-wave dipole antenna is sinusoidal i.e. at a centre it is maximum and adjacent is minimum. This current distribution is used in the process of antenna analysis



→ on a halfwave dipole antenna:

$$[I] = I_0 \cos \omega t \cdot \cos \beta z.$$

$$I_z = I_0 \cos \beta z.$$

$$\text{at } z = \pm \lambda/4, \quad \beta z = \pm \pi/2 \Rightarrow \cos \beta z = 0.$$

$$\text{At } z = 0, \quad \beta z = 0 \Rightarrow \cos \beta z = 1.$$

$$z = 0, \quad \beta z = 0, \quad \Rightarrow \cos \beta z = 1.$$



## \* Current Distribution on the wire Antennas:

⇒



$\lambda/2$

Half wave  
Dipole



$\lambda$



$1.5\lambda$



$2\lambda$



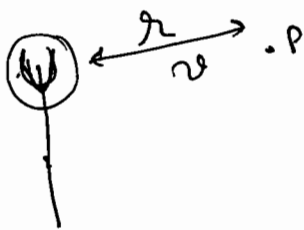
$\lambda/4$   
Quarter  
wave  
mono  
pole.

→ A Quarter Wave monopole is backed by an infinite ground plane.

## \* Antenna analysis Procedure:

Step-1: Assume a suitable current distribution

Step-2: Find  $\bar{A}$  at a distant point



→ This potential is also named as retarded potential. It is retarded by  $r/v$  seconds.

Step-3: Find  $\bar{A}$

$$\bar{B} = \nabla \times \bar{A}$$

$$\text{and } \bar{H} = \frac{1}{\mu} \nabla \times \bar{A}$$

Step-4:

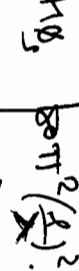
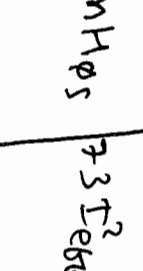
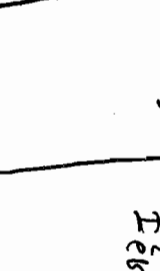
find  $\bar{E}$ .

$$\nabla \times \bar{H} = \epsilon \frac{\partial \bar{E}}{\partial t}$$

(or)  $\bar{E} = \frac{1}{\epsilon} \int (\nabla \times \bar{H}) dt.$

Step-5:

Investigate all the antenna parameters and then by one can decide the performance of an antenna.

| Sr No. | Name of the antenna   | Length (l)      | Geometry & current distr.                                                             | $H_\theta$                                                                           | $E_\theta$                                                 | Rad (currents)                                       | Rad (SR)                                   | $D_\theta$                                              | $D_\phi$ |
|--------|-----------------------|-----------------|---------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|------------------------------------------------------------|------------------------------------------------------|--------------------------------------------|---------------------------------------------------------|----------|
| 1      | Hertzian dipole       | $l \ll \lambda$ |  | $\frac{j I_0 B l}{4 \pi r^2} \sin \theta e^{-j \beta r}$                             | $n H_\theta$                                               | $8 \pi^2 \left(\frac{l}{\lambda}\right)^2 I_{eff}^2$ | $8 \pi^2 \left(\frac{l}{\lambda}\right)^2$ | $1.5 \sin^2 \theta$                                     | 1.5      |
| 2      | Half wave dipole      | $l = \lambda/2$ |    | $\frac{j I_0 \cos(\frac{\pi}{2} \cos \theta)}{2 \pi r^2 \sin \theta} e^{-j \beta r}$ | $n H_\theta$                                               | $7.3 I_{eff}^2$                                      | 7.3                                        | $\frac{1.64 (\omega^2 I_0 \cos \theta)}{\sin^2 \theta}$ | 1.64     |
| 3      | Quarter wave monopole | $l = \lambda/4$ |    | $\frac{j I_0 \cos(\frac{\pi}{2} \cos \theta)}{4 \pi r^2 \sin \theta} e^{-j \beta r}$ | $n H_\theta$                                               | $36.5 I_{eff}^2$                                     | 36.5                                       | $\frac{2.28 (\omega^2 I_0 \cos \theta)}{\sin^2 \theta}$ | 2.28     |
|        |                       |                 |                                                                                       |                                                                                      | $I_{eff} = I_0 / \sqrt{2}$<br>$n = \frac{P_{avg}}{P_{in}}$ |                                                      |                                            |                                                         |          |

→ Hertzian dipole also called infinitesimal dipole.  
It is an impractical antenna. It is used as a basic building block for analysing finite length antennas.

→ A Quarter wave Monopole is backed by an infinite ground plane. Therefore it radiates in the upper hemisphere only.

→ In the above analysis we have considered inverse distance terms only. Other terms we have ignored.

→ The above radiated fields represents the waves which are propagating in the radial direction i.e.  $\hat{a}_r$  direction.

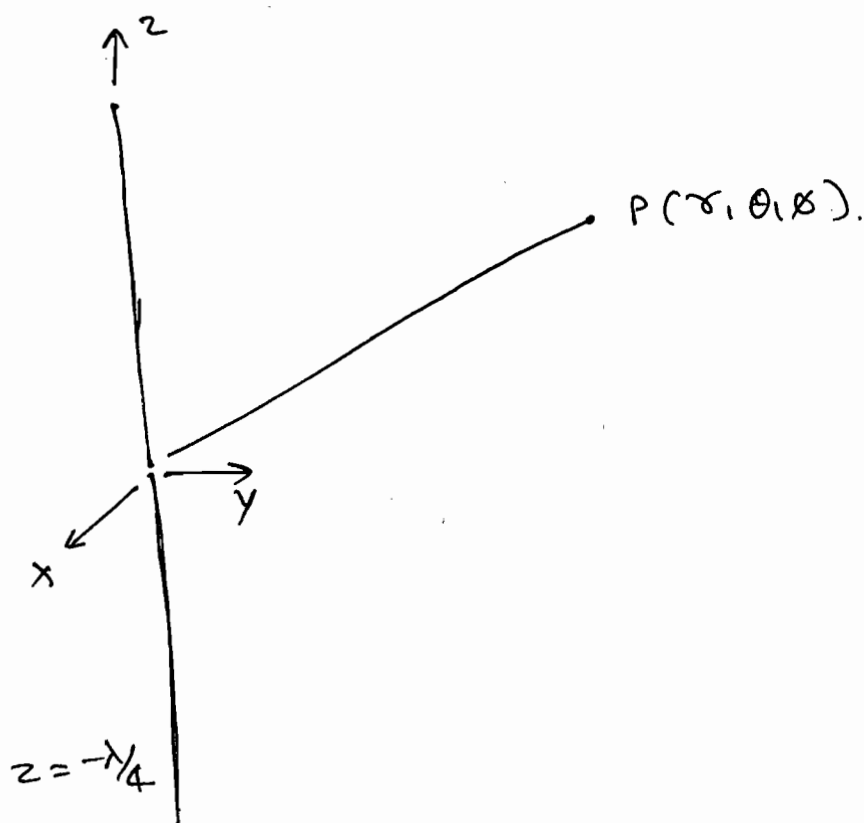
$$H_{\theta s} = \frac{j I_0 \beta r \sin \theta}{4 \pi r} e^{-j \beta r}$$

$$H_{\theta} = \operatorname{Re} [C e^{j(\omega t - \beta r)}]$$

$$H_{\theta s} = C e^{-j \beta r} \rightarrow (r)$$

$$\frac{E_{\theta}}{H_{\theta}} = \eta$$

\*



$$\rightarrow E_{\theta r} = \left[ \frac{j\eta I_0}{2\pi r} \cdot e^{-j\beta r} \right] \cdot \frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta}$$

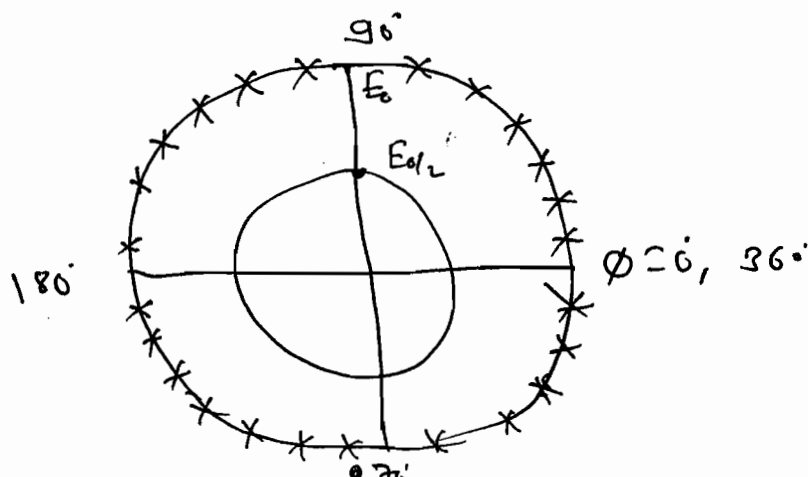
[fixed for distance  $r = \text{const.}$ ]

$$E_{\theta r} = E_0 \cdot \frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta}$$

\* Azimuthal plane pattern:  
 $\theta = 90^\circ, \quad \phi \rightarrow 0 \text{ to } 360^\circ$

$$|E_{\theta r}| = E_0 \cdot \left| \frac{\cos\left(\frac{\pi}{2} \cos 90^\circ\right)}{\sin 90^\circ} \right|$$

$$\therefore |E_{\theta r}| = E_0$$



→ Circular locus on the polar plot indicates that halfwave dipole antenna radiates uniformly in the azimuthal plane.

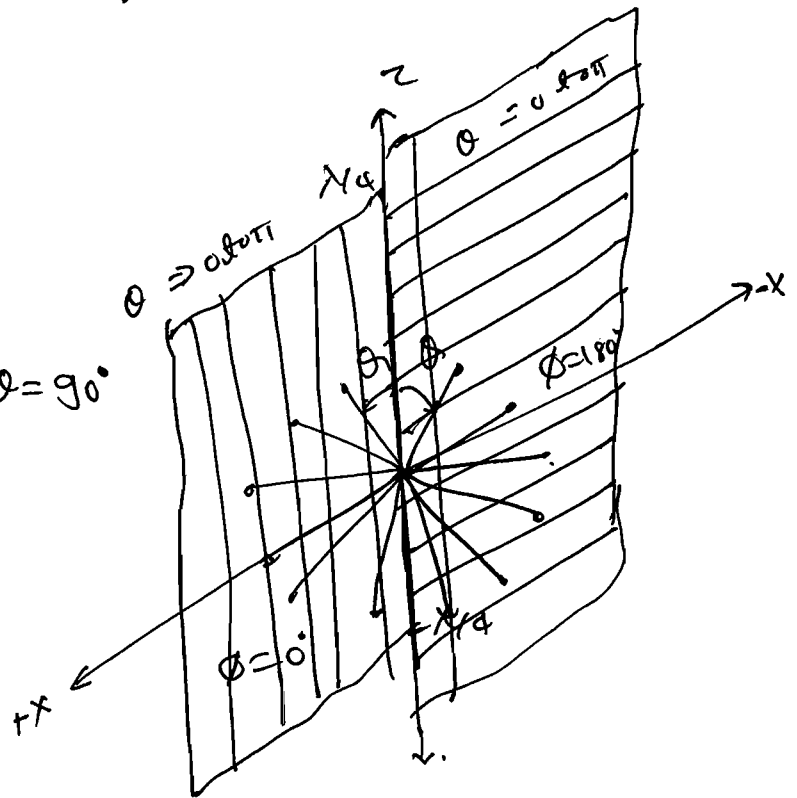
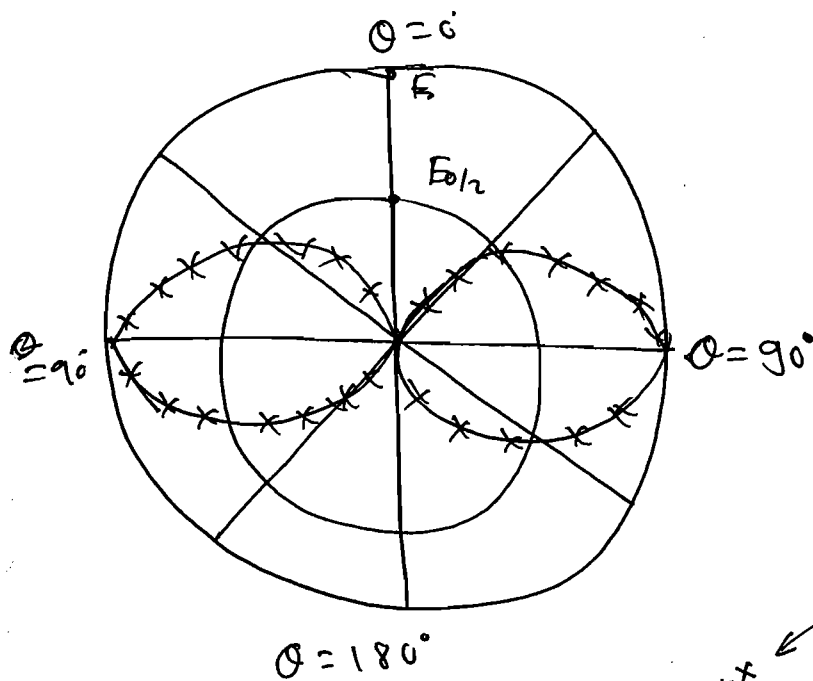
### \* Elevation plane: Pattern:

→  $\phi = \text{Const.}$

We consider  $\phi = 0^\circ$  and  $\phi = 180^\circ$ .

$\theta \rightarrow 0$  to  $180^\circ$ .

$$|\bar{E}_z| = \left| E_0 \cdot \frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta} \right|$$



→ Locus on the Polar Plot indicates that in the elevation plane half wave dipole antenna radiates non-uniformly. Further we can also say that along the line of the antenna it has zero radiation and normal to the line of the antenna it has maximum radiation.

→ An antenna which is having uniform radiations in the azimuthal plane and having non-uniform radiation in the elevation plane than that kind of antenna is said to be omnidirectional antenna. Therefore dipole antenna is omnidirectional.

→ Quarterwave Monopole doesn't radiate for  $90^\circ \leq \theta \leq 180^\circ$ . because it is backed by an infinite ground plane. ~~It can~~

Ex:- It is desired to observe a magnetic field strength of  $5 \mu A/m$  at  $r = 2 km$ ,  $\theta = 90^\circ$ . How much power that antenna do with to radiate if it is a

- Hertzian dipole of length  $\lambda/20$ .
- Halfwave dipole
- Quarterwave monopoles.

Ans: ① Hertzian dipole

$$l = \lambda/20, \quad r = 2 \times 10^3 m, \quad \theta = 90^\circ$$

$$|H_\phi| = |H_{\phi s}| e$$

$$= 5 \times 10^{-6} A/m.$$

$$|H_{\phi s}| = \left| \frac{j \frac{I_0 l}{4\pi r} \sin\theta \cdot e^{-j\beta r}}{1} \right|$$

$$= \frac{I_0 \beta l}{4\pi r} \cdot \sin \alpha \quad \left| \quad \beta = \frac{2\pi}{\lambda} \right.$$

$$\therefore I_0 =$$

$$\rightarrow I_{eff} = I_0 / \sqrt{2}$$

$$\begin{aligned} \rightarrow W_{rad} &= R_{rad} \cdot I_{eff}^2 \\ &= 80\pi^2 \left(\frac{l}{\lambda}\right)^2 I_{eff}^2 \quad \therefore \end{aligned}$$

$$\begin{aligned} \rightarrow \textcircled{2} |F_r| &= |H_{\theta}| = \left| \frac{j I_0}{2\pi r} \frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} \cdot e^{-j\beta r} \right| \\ &= \frac{I_0}{2\pi r} \cdot \frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} \end{aligned}$$

$$I_0 =$$

$$I_{eff} = I_0 / \sqrt{2}$$

$$\therefore W_{rad} = I_{eff}^2 \cdot R_{rad} = I_{eff}^2 (73) \text{ Watts}$$

③  $I_{eff}$  is same as halfwave dipole.

$$W_{rad} = I_{eff}^2 (36.5) \text{ Watts.}$$



## ★ Scattering Parameters:

→ Conventional two port parameters such as  $z, y, h, T, g$ , and  $T'$  are less useful for the freq. beyond 1 GHz because of the following reasons.

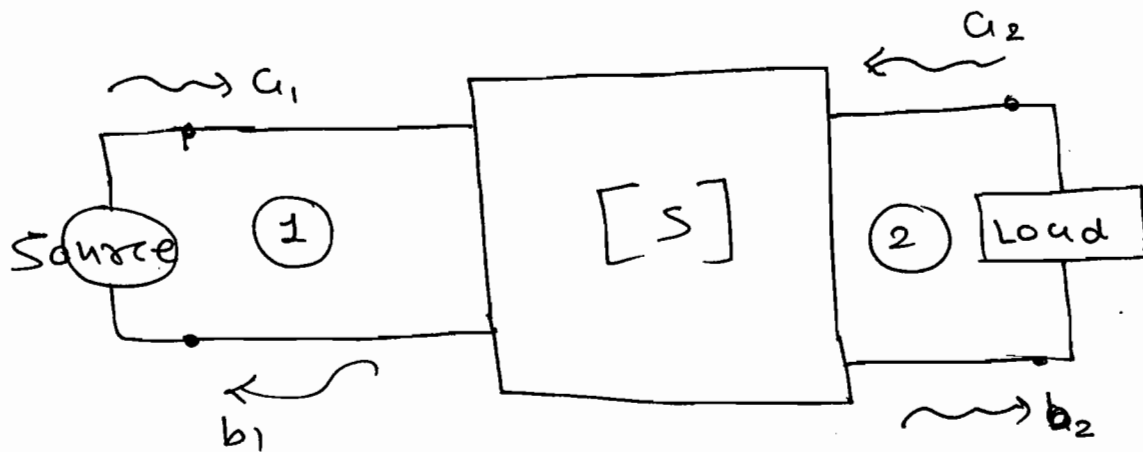
① Equipment is not readily available for the measurement of port voltages and currents.

② Active devices like Power Transistors, Tunnel Diodes becomes frequently unstable due to open and short circuit (Conventional two port parameters are define as a ratio of  $V-V$  or  $V-I$  (or)  $I-V$  (or)  $I-I$  and by open circuiting (or) short circuits one of the ports)

→ ③ It is impractical to realized an ideal open circuit at microwave frequencies. Due to the above reasons

$S$  parameters ~~are~~ (or) Scattering Parameters are used for analysing the circuits beyond 1 GHz.

# \* S-parameters:



→ All  $a_i$  are entering the port.

→ All  $b_i$  are leaving the port.

→ They are related by

$$\begin{aligned} b_1 &= S_{11} a_1 + S_{12} a_2 \\ b_2 &= S_{21} a_1 + S_{22} a_2 \end{aligned}$$

$$\Rightarrow \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0}$$

$$S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0}$$

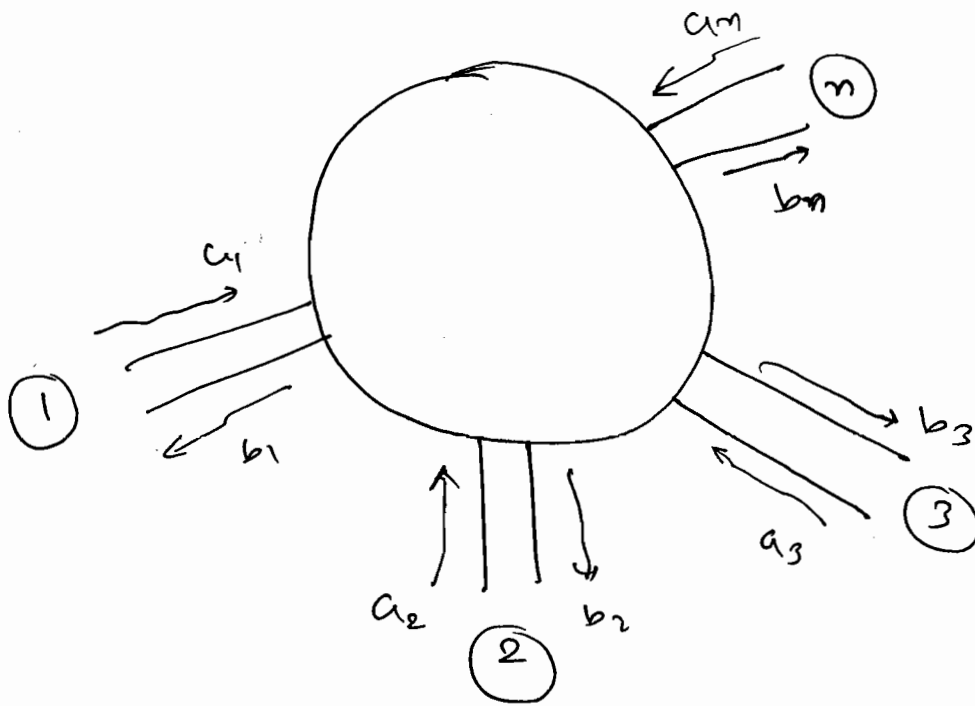
$$S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0}$$

$$\Rightarrow [b] = [S][a]$$

$\Rightarrow [S]$ : Scattering matrix

→ Coefficient in  $S$  are called Scattering coefficient.

→ In general microwave junction may have 'n' ports.



$$\rightarrow b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + \dots + S_{1n}a_n$$

$$b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3 + \dots + S_{2n}a_n$$

$\vdots$

$$b_n = S_{n1}a_1 + S_{n2}a_2 + S_{n3}a_3 + \dots + S_{nn}a_n$$

$$\therefore \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \dots & S_{1n} \\ S_{21} & S_{22} & S_{23} & \dots & S_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ S_{n1} & S_{n2} & S_{n3} & \dots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

Reflection coefficient

→ In wave junction will have  $n^2$  scattering coefficient.

→ All scattering coefficients are dimensionless quantities

→  $S_{ij}$ : Reflection coeff. of  $i^{\text{th}}$  port, if  $i=j$ , with all other ports are matched.

→  $S_{ij}$ : Forward transmission coeff. of  $j^{\text{th}}$  port if  $i>j$ , with all other ports are matched.

→  $S_{ij}$ : ~~Forward~~ Reverse transmission coeff. of  $j^{\text{th}}$  order if  $i<j$ , with all other ports are matched.

→ The main diagonal elements of S-matrix are reflection coefficients.

→ The elements below the main diagonal ~~(~~are~~)~~ are forward transmission coefficients.

and

→ The elements above the main diagonal are Reverse transmission coefficients.

## \* Properties of Scattering Parameters:

① If all the main diagonal elements are zero then all the ports of the N/w are matched.

$$S_{11} = S_{22} = S_{33} = \dots = S_{nn} = 0.$$

### ② Symmetry property

(or)  $S_{ij} = S_{ji}$ .

→ If the microwave junction do not have any non-reciprocal components <sup>(or)</sup> ~~are not~~ active devices and the junction obeys reciprocity then  $[S] = [S]^T$

### ③ Unity Property.

→ If the microwave junction is lossless then Sum of the product of the complex conjugate of the elements in a Row (or) column is equal to unity.

$$\rightarrow S_{11} S_{11}^* + S_{12} S_{12}^* = 1 \quad (\text{Row 1}) \checkmark$$

$$\rightarrow S_{21} S_{21}^* + S_{22} S_{22}^* = 1 \quad (\text{Row 2}) \checkmark$$

$$\rightarrow S_{11} S_{11}^* + S_{21} S_{21}^* = 1 \quad (\text{Column 1}) \checkmark$$

$$\rightarrow S_{12} S_{12}^* + S_{22} S_{22}^* = 1 \quad (\text{Column 2}) \checkmark$$

#### ④ Zero Property:

→ Sum of the product of the Complex Conjugate of the elements of the other row (or) other column = 0. (Row to Row, Column to Column).

$$\therefore S_{11} S_{21}^* + S_{12} S_{22}^* = 0 \quad (\text{Row 1 \& 2})$$

$$S_{21} S_{12}^* + S_{22} S_{11}^* = 0 \quad (\text{Col 1 \& 2}).$$

#### \* Relation b/w, $[S]$ , $[z]$ , $[Y]$ :

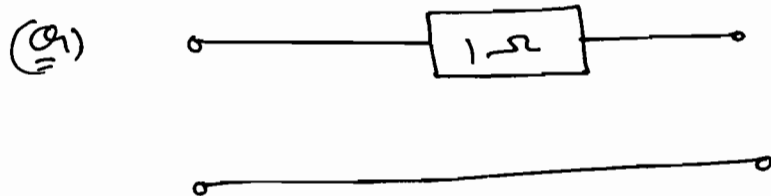
$$\rightarrow [S] = [z - U] [z + U]^{-1}$$

$$\rightarrow [S] = [U - Y] [U + Y]^{-1}$$

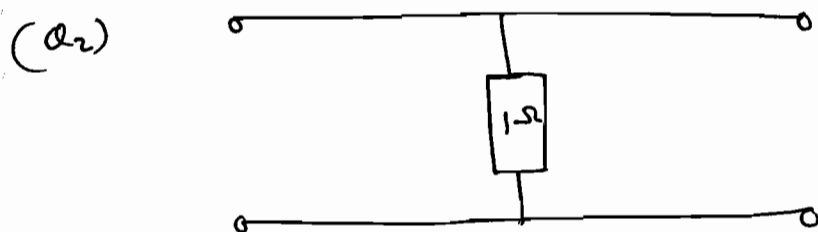
$$\rightarrow [Y] = [U] - 2[S] [U + S]^{-1}$$

$$[z] = [U] + 2[S] [U - S]^{-1}$$

Ex-1 Find  $[S]$  Parameters:



Step: 1 find  $[Y]$   
Step: 2 find  $[S]$



Step 1: find  $[z]$   
Step-2:  $[S]$ .

Ex-2

$$[S] = \begin{bmatrix} 0.1 \angle 40^\circ & 0.88 \angle 30^\circ \\ 0.88 \angle 30^\circ & 0.1 \angle 40^\circ \end{bmatrix}$$

The N/w is

- (a) Lossy and Reciprocal.  
(b) Lossy and Non Reciprocal.  
(c) Non-lossy and Reciprocal.  
(d) Non-lossy and Non-Reciprocal

$$\rightarrow [S] = [S]^T \rightarrow \text{Reciprocal.}$$

$$S_{11} \neq S_{12}.$$

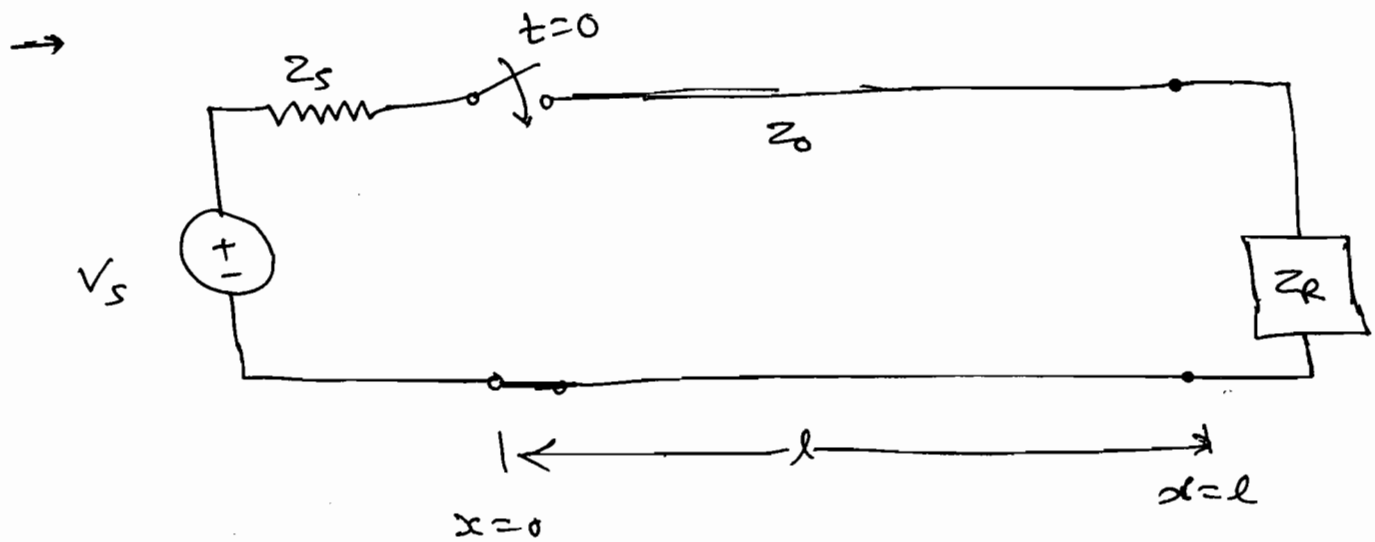
Ans: The above matrix satisfying Symmetry condition therefore reciprocal and it is not satisfying unity property. Therefore it is lossy.

\*

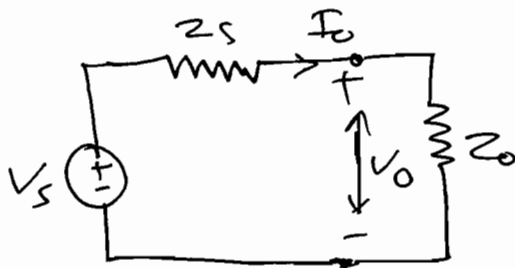
$\rightarrow$  When a DC source is connected to a transmission line and is switched on it takes some time for the voltage and current on the line to reach a steady value. means Transition period is called Transient.

$\rightarrow$  Fig. Shows a transmission line driven by

a Dc Voltage Source.



→ Equivalent ckt at  $t=0^+$  &  $x=0$ ,



→ at  $t=0$ , the switch is closed, then the source current 'sees' only  $Z_s$  and  $Z_0$ .

$$I \text{ (at } x=0, t=0^+) = \frac{V_s}{Z_s + Z_0}$$

$$\therefore V \text{ (at } x=0, t=0^+) = \frac{Z_0}{Z_0 + Z_s} V_s$$

$$= I_0 Z_0$$

$$= V_0$$

→ After the switch is closed  $I_i = I_0$ ,  $V_i = V_0$  propagates towards the load with a speed of  $\frac{1}{\sqrt{LC}}$  m/s ( $= v_p$ ) ( $i = \text{incident}$ ).



transient time  $t_1 = \frac{l}{v_p}$ .

[The wave could take certain time to reach the load].

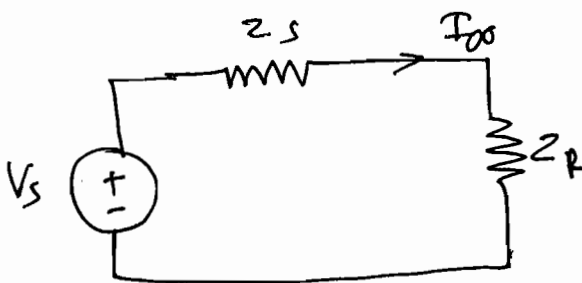
→ After ' $t_1$ ' seconds the wave reach the load. The  $V$  and  $I$  at the load is the sum of incident and reflected.

$$V(l, t_1) = V_i + V_r = V_0 + kV_0 \\ = V_0(1+k).$$

$$\therefore k = \frac{Z_L - Z_0}{Z_L + Z_0}.$$

$$I(l, t_1) = I_i + I_r \\ = I_0 - kI_0 \\ = I_0(1-k)$$

∴ Eavt ckt at  $t = \infty$



( $I_\infty$ : Steady state current)

$$\therefore \boxed{I_\infty = \frac{V_s}{Z_s + Z_L}}$$

Ex-1 Let  $V_s = 30V$ ,  $Z_s = 0$ ,  $Z_0 = 50 \Omega$ .

$$t_1 = 400 \mu s.$$

$$V \text{ at } l, t_1 = 400 \mu s = 40V.$$

Find  $k$ ,  $Z_L$  and  $I_\infty$ .

$$\therefore V_{\text{out}} x=1, (F400\mu\text{H}) = 40\text{V} = V_i + V_R$$

$$= V_o (1+k).$$

$$(\because Z_s = 0) \\ V_o = V_s).$$

$$\therefore 40 = 30(1+k).$$

$$\therefore \boxed{k = 0.33.}$$

$$\therefore k = 0.33.$$

$$\rightarrow \frac{Z_R - Z_o}{Z_R + Z_o} = k.$$

$$\therefore \frac{Z_R}{Z_o} = \frac{1+k}{1-k}.$$

$$\therefore Z_R = Z_o(2) = 100 \Omega.$$

$$\therefore I_{\infty} = \frac{V_s}{Z_s + Z_R} = \frac{30}{100}.$$

$$\therefore \boxed{I_{\infty} = 0.3 \text{ A}}$$

Ex 1 A Transmission line is terminated by a load impedance of  $50 + j50 \Omega$ . Assume characteristics impedance of the line is  $50 \Omega$ .

(i) Locate Normalized load impedance, on the Smith Chart.

Ans:  $Z_L = 50 + j50$

$$Z_0 = 50$$

$$Z_n = \frac{Z_L}{Z_0}$$

$$\therefore \boxed{Z_n = 1 + j1}$$

→ Point A on the Smith Chart indicates Normalized load impedance.

(ii) Using Smith Chart find normalized admittance.

$$Y_n = \frac{1}{Z_n} = \frac{1}{1+j} = \frac{1-j}{(1+j)^2} = \frac{1}{2} \angle -90^\circ = 0.5 - j0.5$$

→ The value at B indicates normalized load admittance  $\boxed{Y_n = 0.5 - j0.5}$

(iii) find VSWR

→ The value at C and value at D also indicates VSWR and the value is 2.6.

(iv) Find <sup>max</sup> voltage reflection coefficient

$$|K| = \frac{S-1}{S+1} = \frac{2.6-1}{2.6+1}$$

$$|F| = 0.44.$$

The value at E represent mag. of the reflection coefficient and is 0.45

(V) Also find phase angle of the reflection coefficient.

Ans: The value at F represent the phase angle of the <sup>voltage</sup> reflection coefficient and value is  $65^\circ$ .

(vi) Power reflection coefficient = ?

Ans: The value at G indicates the power reflection coefficient and the value is 0.2.

(vii) Find maximum impedance and the minimum impedance on the transmission line?

Ans: The value at D indicates normalized

$$\frac{Z_{max}}{Z_0} =$$

$$\therefore \frac{Z_{max}}{Z_0} = 5Z_0.$$

$$Z_{min} = Z_0/5.$$

$$\frac{Z_{max}}{Z_0} = 5.$$

→ The value at D indicates normalized maximum impedance on the transmission line and is the value at H indicated minimum impedance on the transmission line.

$$\therefore Z_{\max} = 2.6 \times 50.$$

$$Z_{\min} = \frac{50}{0.4}.$$

(viii) find impedance on the transmission line at a distance of  $0.1\lambda$  from the load.

Ans: The value at J indicated normalized impedance and the value is  $z = 2.6 - j0.4$ .

$$\therefore Z = (2.6 - j0.4) 50 \Omega$$

(10) Find the impedance on the transmission line at distance.  ~~$\lambda = 0.3\lambda$~~   $\lambda = 0.3\lambda$  from load.

→ The point A itself is the impedance at a distance of  $0.5\lambda$  from the load. Which is load impedance.

(11) find the impedance on the transmission line at a distance of  $0.25\lambda$  from the load.

Ans: The point B indicates the answer of the question.

as line KL is  $0.25\lambda$ .

→ The point B indicates normalized load impedance at a distance  $0.25\lambda$  from the load.

NOTE: With  $O$  as centre  $OA$  as radius we draw a circle. That circle indicates Locus of the impedance on the transmission line.

- ⑪ Find the distance bet<sup>n</sup> load and the first voltage max.

→ The line km indicates the distance bet<sup>n</sup> the load and the first voltage max.

$$0.25\lambda - 0.16\lambda = 0.09\lambda$$

- ⑫ Find the distance bet<sup>n</sup> load and first voltage minimum.

$$0.5 - 0.16 = 0.34\lambda$$

∴

(oh)  $0.09 + 0.25 = 0.34\lambda$ .

The ~~km~~ line k indicates the distance bet<sup>n</sup> the load and first voltage minimum the ~~km~~ distance is  $0.34\lambda$

- ⑬ how many maxima and how many minima occur on the transmission line if the line length is  $2.02\lambda$ .

Ans:

| length        | cumulative length | $V_{max}$ | $V_{min}$ |
|---------------|-------------------|-----------|-----------|
| $0.5\lambda$  | $0.5$             |           |           |
| $0.5\lambda$  | $1.0\lambda$      |           |           |
| $0.5\lambda$  | $1.5\lambda$      |           |           |
| $0.5\lambda$  | $2.0\lambda$      |           |           |
| $0.02\lambda$ | $2.02\lambda$     | <hr/>     | <hr/>     |
|               |                   | 4         | 4         |

4-maxima &

4-minima.