Chapter 14

Data Interpretation

CHAPTER HIGHLIGHTS

- *Methods of Presenting Data*
- *Data table*
- *Pie Charts*
- *Two-Dimensional Graphs*

Introduction

Not a day passes without our coming across figures and statistics. Study and manipulation of such data leads us to an important area: Data Interpretation. Data can be organized in a number of ways so that larger volume of data can be presented in a more compact and precise form. Data thus presented have to be deciphered correctly by the user of the data. This process of deciphering the data from its compactly presented form is called Data Interpretation.

Methods of Presenting Data

Numerical data can be presented in one or more of the following ways:

1. Data Tables
2. Pie Charts
3. 2-Dimensional Graphs
4. Bar Charts
5. 3-Dimensional Graphs
6. Venn Diagrams
7. Geometrical Diagrams
8. Pert Charts
9. Others

The 'Others' category covers miscellaneous forms like descriptive case format customized for the situation. Data can also be presented by using a combination of two or more of the aforementioned forms.

While some data can be presented in many different forms, some other data may be amenable to be presented only in a few ways. In real life situations, the style of data presentation is based on the end-objective. In certain situations, data have to be presented as a combination of two or more forms of data presentation.

Let us understand each of the above forms of data presentation with an example.

Data Table

Here, data are presented in the form of simple table. While any type of data can be presented in tabular form, that too in a very accurate manner, interpreting the data in tabular form is more difficult and time consuming than the other modes, all of which are basically pictorial or graphical in presentation.

Data tables can be of a number of types. They can be of a single-table variety or combination of tables. Some examples of tables are given below.

Table 1 Movement of goods by different modes of transport

(in 000's of metric-ton-kms)

Year	Road	Rail	Air	Water	Total
1985	1000	1500	120	20	2640
1986	1600	2000	129	24	3753
1987	2907	3090	139	28	6164
1988	4625	5200	152	27	10004
1989	6346	7540	174	33	14093
1990	7920	10250	212	40	18422
1991	9540	13780	266	50	23636

From the table, we can deduce the following:

1. Rate of growth by each mode of transport in successive years as well as cumulative annual growth.
2. Rate of growth of total haulage by all modes of transport together in any year.
3. Contribution by each mode of transport to the total haulage in any given year.
4. Trends of growth over time for various modes of transport.
5. Given the cost of transportation for each mode, we can calculate the total annual cost of transportation over the years for various modes of transport as well make a cost comparison.
6. Finding out the mode of transportation in any given year that forms the largest percentage of total haulage.
7. For a given mode of transport, finding out the year in which the percentage increase in haulage over the previous year was the highest.

Pie Charts

This is probably the simplest of all pictorial forms of data presentation. Here, total quantity to be shown is distributed over one complete circle or 360 degrees. In pie-charts, data is essentially presented with respect to only one parameter (unlike in 2- and 3-dimensional graphs described later). This form essentially presents shares of various elements as proportion or percentage of the total quantity. Each element or group in the pie-chart is represented in terms of quantity (or value, as the case may be) or as the angle made by the sector representing the elements or as a proportion of the total or as a percentage of the total.

Figure 1 gives distribution of the population in different geographical zones.

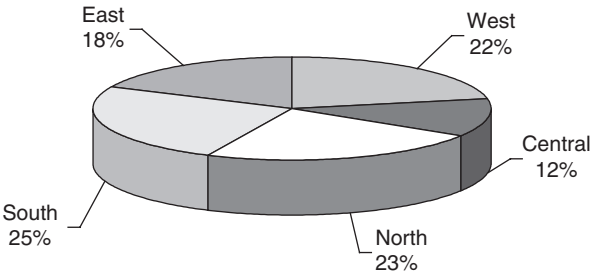


Figure 1 Distribution of population in geographical zones

From the above-mentioned pie chart, we can calculate the following:

1. Population in any zone given the total population.
2. Population of any zone as a percentage of that of another zone.
3. Percentage increase in the total population given the percentage increase in the population of one or more zones.

Pie Charts are also very frequently used in combination with other forms of data or along with other pie charts.

Two-Dimensional Graphs

This is essentially used for continuous data but can also be used for depicting discrete data provided we understand the limitation. Also known as Cartesian Graphs, they represent variation of one parameter with respect to another parameter each shown on a different axis. These types of graphs are useful in studying the rate of change or understanding the trends through extrapolations.

These graphs can be of various types, and a few of them are shown below (Figures 2 and 3):

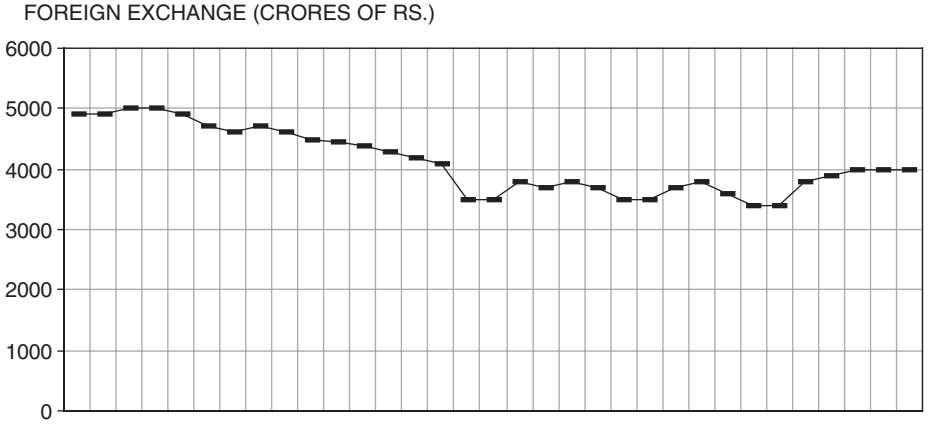


Figure 2 Foreign exchange reserves of India

The graph in Figure 2 shows the changes in the foreign exchange reserves of our country during a period of time. One can find out trends and the growth rates of foreign exchange reserves.

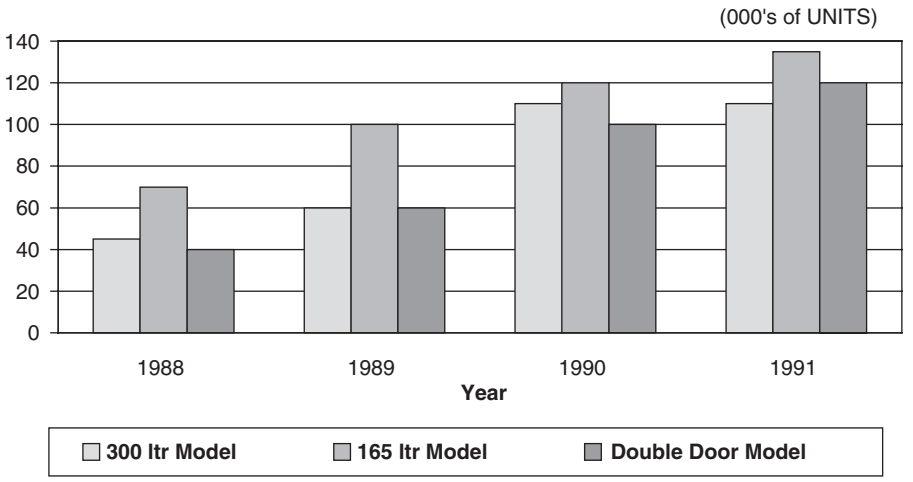


Figure 3 Refrigerator sales of company ABC

Chart 3 shows model wise sales of refrigerators during four years. From this graph, we can obtain the following:

1. Percentage contribution of each model to the company's total sales for four years.
2. Relative increase or decrease in the share of each model.
3. Sales trend of various models.

Using this bar chart, one can carry out a detailed performance evaluation of the company with respect to the sales of the four year period 1988 to 1991 for any given model. These bar charts can also be depicted horizontally. Another variation could be showing each product at one place (rather than each year at one place).

Example: These questions are based on the following table, which gives the details of the sports which students in all the classes of a school like.

The table gives the number of students in each class and the percentage of students in it who like cricket, volleyball, basketball, and football.

Class	Number of students	Cricket	Volleyball	Basketball	Football
6	120	60%	70%	50%	60%
7	140	50%	60%	60%	50%
8	160	40%	65%	55%	45%
9	180	65%	75%	65%	55%
10	240	70%	80%	75%	45%

Solved Examples

Example 1

How many students in the school like cricket?

(A) 436 (B) 432 (C) 491 (D) 511

Solution

Number of students who like cricket

$$= \frac{60}{100}(120) + \frac{50}{100}(140) + \frac{40}{100}(160) + \frac{65}{100}(180) + \frac{70}{100}(240) = 491$$

Example 2

By what percentage is the number of students who like volleyball in class 6 more/less than those who like basketball in class 10?

(A) 40% (B) 50%
(C) 53.33% (D) 56.67%

Solution

Number of students who like volleyball in class 6

$$= \frac{70}{100}(120) = 84$$

Number of students who like basketball in class 10

$$= \frac{75}{100}(240) = 180$$

84 is less than 180 by $\frac{180-84}{180}(100) = 53.33\%$.

Example 3

The number of students who like cricket in class 7 is what percentage of the number of students who like football in class 8?

(A) 88% (B) 93.5%
(C) 95.6% (D) 97.2%

Solution

Number of students who like cricket in class 7 = $\frac{50}{100}(140) = 70$

Number of students who like football in class 8 = $\frac{45}{100}(160) = 72$

Required percentage = $\frac{70}{72} \times 100 = 97.2\%$

Example 4

In how many of the given classes can more than 90 students like all the three games?
(A) 2 (B) 3 (C) 1 (D) 0

Solution

In any class, the maximum value of the number of students who like all the three games would be the number of students who like the game liked by the least number of students.

In class 6, the percentage of students who like a game is the least for basketball. Number of those who like basketball = $\frac{50}{100}(120) = 60 < 90$.

In class 7, the percentage of students who like a game is the least for cricket and football. Number of students who like cricket

$$= \frac{50}{100}(140) = 70 < 90$$

In class 8, the number of students who like a game is the least for cricket. Number of students who like cricket = $\frac{40}{100}(160) = 64 < 90$.

In class 9, the percentage of students who like a game is the least for football. Number of those who like football

$$= \frac{55}{100}(180) = 99 > 90$$

In class 10, the percentage of students who like a game is the least for football. Number of students who like football

$$= \frac{45}{100}(240) = 108 > 90$$

∴ In two classes, more than 90 students can like all the games.

Example 5

What can be the maximum percentage of students in class 6 who do not like any of the given games?
(A) 40% (B) 10% (C) 50% (D) 30%

Solution

In class 6, the maximum percentage of students who like a game = Percentage of students who like volleyball, i.e. 70%. Percentage of students who like at least one game would be minimum when all students who like other games are the ones who like volleyball.

∴ Maximum percentage required
= 100 – 70 = 30%.

EXERCISES

Direction for question 1: Select the correct alternative from the given choices.

1. The table shows the total marks of four students P, Q, R, and S in all their subjects for the two years 2012 and 2013.

Students	2012	2013
P	997	1295
Q	664	876
R	585	732
S	480	689

How many students had a percentage Increase in their total marks of more than 35% from 2012 to 2013?
(A) 1 (B) 2 (C) 3 (D) 4

Direction for questions 2 to 4: These questions are based on the following data which give some details of new states joining the United States of America across time.

State	Capital	Joined the union	Union rank	Population	Number of representatives in the house of representatives
Washington	Olympia	Nov 11, 1889	42	62,87,759	9
Texas	Austin	Dec 29, 1845	28	2,28,59,968	32
Delaware	Dover	Dec 7, 1781	1	8,43,524	1
Virginia	Raleigh	Nov 21, 1789	12	86,83,242	13

(*Continued*)

State	Capital	Joined the union	Union rank	Population	Number of representatives in the house of representatives
Minnesota	St. Paul	May 11, 1832	32	51,32,799	8
Kansas	Topeka	Jan 29, 1861	34	27,44,687	4
Illinois	Springfield	Dec 3, 1818	21	1,27,63,371	19
New Hampshire	Concord	June 21, 1788	9	13,09,940	2
Arizona	Phoenix	Feb 14, 1912	48	59,39,292	8
Hawai	Honolulu	Aug 21, 1959	50	12,75,194	2
Indiana	Indianapolis	Dec 11, 1816	19	62,71,973	9
Vermont	Montpelier	March 14, 1791	14	6,23,050	1
Nebraska	Lincoln	March 1, 1867	37	17,58,787	3
Georgia	Atlanta	Jan 2, 1788	4	9,07,256	13

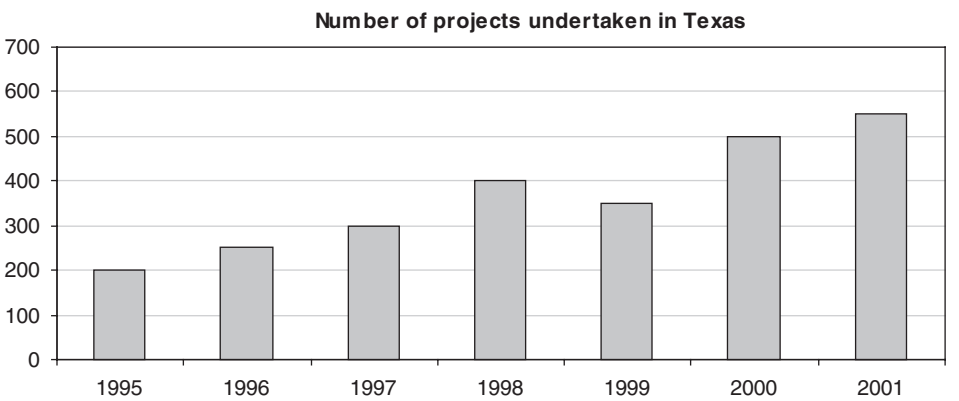
Union rank is the chronological order in which the states joined the Union.

2. How many states joined the Union from March 1, 1867, to Feb 14, 1912?
(A) 11 (B) 12
(C) 13 (D) 14

3. If it is known that the House of Representatives of USA has a strength of 535 members, then the number of representatives in the House of Representatives of the given states will form what approximate percentage of the total strength of the House of Representatives?
(A) 16 (B) 19 (C) 21 (D) 23

4. In how many of the given states is the population less than 15 million but the number of representatives is not less than six?
(A) 4 (B) 5 (C) 6 (D) 7

Direction for questions 5 to 7: These questions are based on the following graphs.



5. In how many years was the number of projects undertaken in Mexico greater than that in Texas?
(A) 3 (B) 6 (C) 5 (D) 4

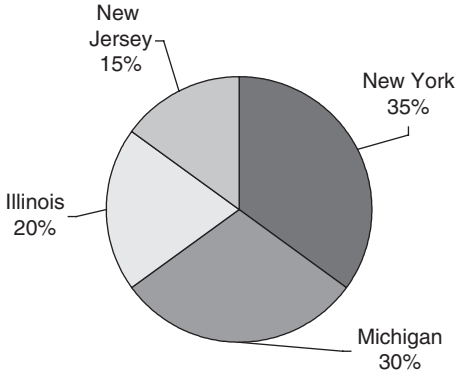
6. How many projects were undertaken in the year 1998 in both places together?
(A) 1000 (B) 900 (C) 750 (D) 500

7. In which of the following years was the average (arithmetic mean) number of projects undertaken in both places the highest?
(A) 2000 (B) 1996 (C) 1998 (D) 1999

Direction for questions 14 to 16: The table below shows the percentages of colleges offering the courses mentioned in medicine in four states, New York, New Jersey, Illinois, and Michigan, in a certain year.

Sl. No.	Course	New York	New Jersey	Illinois	Michigan
1.	Biochemistry	86	80	74	68
2.	Biophysics	74	92	88	64
3.	Biomechanics	59	82	84	68
4.	Biostatistics	56	84	86	70

The total number of colleges offering courses in medicine in the four states is 2000. The percentage-wise distribution of the number of colleges in the four states is as shown below.



8. The number of colleges offering the Biochemistry course is more than 230 in
(A) All the four states. (B) Exactly three states.
(C) Exactly two states. (D) Exactly one state.

9. What percentage of the colleges in the four states do not offer Biophysics as well as Biochemistry?
(A) 41%
(B) 36%
(C) 34%
(D) Cannot the determined

10. What is the total number of colleges offering Biostatistics in all the four states?
(A) 1392 (B) 1408 (C) 1432 (D) 1476

Direction for questions 11 and 12: Study the given table and answer the questions that follow.

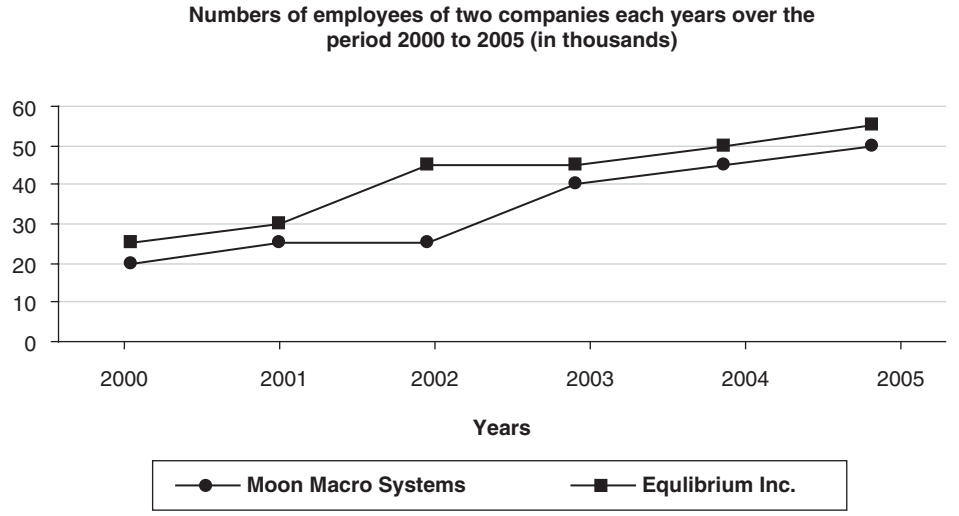
Percentage break-up of the workers working in six different factories *J, K, L, M, N*, and *P*.

Factory	Total no. of workers	Percentage		
		Men	Women	Boys
J	4800	50	37.5	12.5
K	8750	40	36	24
L	5250	24	56	20
M	12000	35	25	40
N	8500	38	30	32
P	2700	45	40	15

11. By what percent is the number of women working in factory *P* is more than the number of boys working in factory *J*?
(A) 20% (B) $44\frac{4}{9}\%$
(C) 80% (D) 180%

12. What is the ratio of the number of men working in factory *M* to the number of women working in factory *L*?
(A) 7 : 10 (B) 10 : 7 (C) 7 : 5 (D) 5 : 7

Direction for questions 13 to 15: These questions are based on the following line graph.

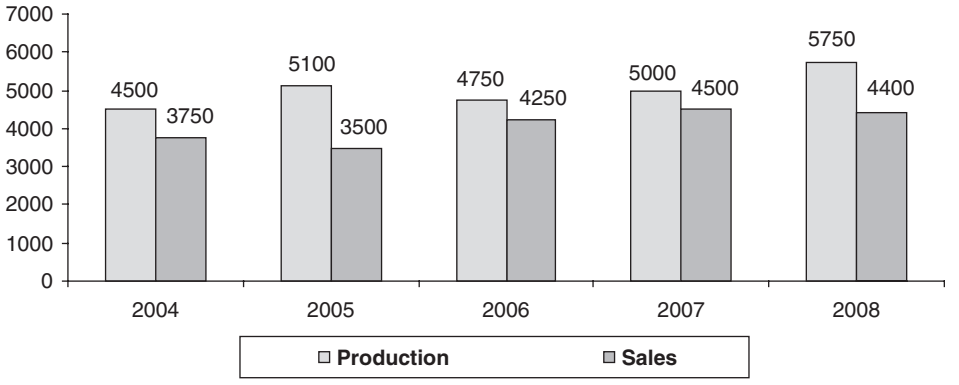


13. What is the ratio of the average number of employees of company Moon Macro Systems for the period 2001, 2002 and 2003 and the average number of employees of company Equilibrium Inc. for the same period?
(A) 2 : 5 (B) 3 : 5 (C) 3 : 4 (D) 4 : 3

14. During which of the following years was the percentage increase in the number of employees of company Equilibrium Inc. over that in the previous year the highest?
(A) 2000
(B) 2001
(C) 2002
(D) 2004

15. What is the approximate percentage increase in the total number of employees of the two companies from 2004 to 2005?
(A) 8.5% (B) 9%
(C) 9.5% (D) 10.5%

Direction for questions 16 and 17: These questions are based on the following bar graph that gives the production and sales of a company across five years from 2004 to 2008.



16. In the given period, what percentage was the average production more than the average sales?
(A) 20% (B) 23%
(C) 25% (D) 28%

17. The percentage increase/decrease in the total sales of the company in a given year with respect to that in the previous year was highest in which of the following years?
(A) 2005 (B) 2006 (C) 2007 (D) 2008

Previous Years' Questions

1. If $\left(z+\frac{1}{z}\right)^2 = 98$, compute $\left(z^2+\frac{1}{z^2}\right)$. **[GATE, 2014]**

2. The roots of $ax^2 + bx + c = 0$ are real and positive. a, b and c are real. Then $ax^2 + b|x| + c = 0$ has **[GATE, 2014]**
(A) no roots (B) 2 real roots
(C) 3 real roots (D) 4 real roots

3. Round–trip tickets to a tourist destination are eligible for a discount of 10% on the total fare. In addition, groups of 4 or more get a discount of 5% on the total fare. If the one way signle person fare is ₹100, a group of 5 tourists purchasing round-trip tickets will be charged ₹_____. **[GATE, 2014]**

4. In a survey, 300 respondents were asked whether they own a vehicle or not. If yes, they were further asked to mention whether they own a car or scooter or both. Their responses are tabulated below. What percent of respondents do not own a scooter? **[GATE, 2014]**

		Men	Women
Own vehicle	Car	40	34
	Scooter	30	20
	Both	60	46
Do not own vehicle		20	50

5. When a point inside of a tetrahedron (a solid with four triangular surfaces) is connected by straight lines to its corners, how many (new) internal planes are created with these lines? _____. **[GATE, 2014]**

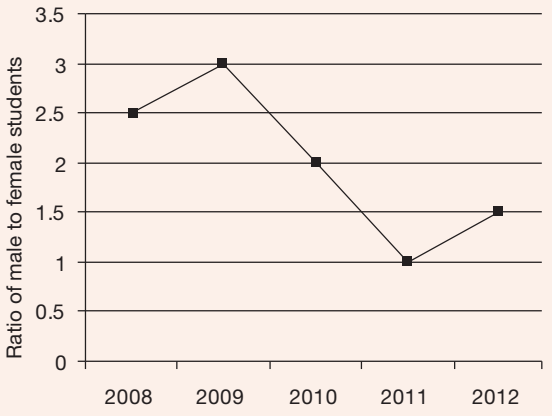
6. What is the average of all multiples of 10 from 2 to 198? **[GATE, 2014]**
(A) 90 (B) 100
(C) 110 (D) 120

7. The value of $\sqrt{12+\sqrt{12+\sqrt{12+\cdots}}}$ is **[GATE, 2014]**
(A) 3.464 (B) 3.932
(C) 4.000 (D) 4.444

8. If x is real and $|x^2 - 2x + 3| = 11$, then possible values of $|-x^3 + x^2 - x|$ include **[GATE, 2014]**

(A) 2, 4 (B) 2, 14
(C) 4, 52 (D) 14, 52

9. The ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students doubled in 2009, by what percent did the number of male students increase in 2009? **[GATE, 2014]**



(A) 1 : 1 (B) 2 : 1
(C) 1.5 : 1 (D) 2.5 : 1

10. The table below has question-wise data on the performance of students in an examination. The marks for each questions are also listed. There is no negative or partial marking in the examination.

Q.No.	Marks	Answered correctly	Answered wrongly	Not Attempted
1	2	21	17	6
2	3	15	27	2
3	2	23	18	3

What is the average of the marks obtained by the class in the examination? **[GATE, 2014]**

(A) 1.34 (B) 1.74
(C) 3.02 (D) 3.91

11. The Gross Domestic Product (GDP) in Rupees grew at 7% during 2012–2013. For international comparison, the GDP is compared in US Dollars (USD) after conversion based on the market exchange rate. During the period 2012–2013 the exchange rate for the USD increased from ₹50/USD to ₹60/USD. India's GDP in USD during the period 2012–2013 **[GATE, 2014]**

(A) Increased by 5%
(B) Decreased by 13%
(C) Decreased by 20%
(D) Decreased by 11%

12. Consider the equation: $(7256)_8 - (Y)_8 = (4364)_8$, where $(X)_N$ stands for X to the base N. Find Y. **[GATE, 2014]**

(A) 1634 (B) 1737
(C) 3142 (D) 3162

13. What will be the maximum sum of 44, 42, 40, …? **[GATE, 2013]**

(A) 502 (B) 504
(C) 506 (D) 500

14. A tourist covers half of his journey by train at 60 km/h, half of the remainder by bus at 30 km/h and the rest by cycle at 10 km/h. The average speed of the tourist in km/h during his entire journey is **[GATE, 2013]**

(A) 36 (B) 30
(C) 24 (D) 18

15. The current erection cost of a structure is ₹13,200. If the labour wages per day increase by $\frac{1}{5}$ of the current wages and the working hours decrease by $\frac{1}{24}$ of the current period, then the new cost of erection in ₹ is **[GATE, 2013]**

(A) 16,500 (B) 15,180
(C) 11,000 (D) 10,120

16. Out of all the 2-digit integers between 1 and 100, a 2-digit number has to be selected at random. What is the probability that the selected number is not divisible by 7? **[GATE, 2013]**

(A) $\frac{13}{90}$ (B) $\frac{12}{90}$
(C) $\frac{78}{90}$ (D) $\frac{77}{90}$

17. Find the sum of the expression

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{80}+\sqrt{81}}$$

[GATE, 2013]

(A) 7 (B) 8
(C) 9 (D) 10

18. The cost function for a product in a firm is given by $5q^2$, where q is the amount of production. The firm can sell the product at a market price of ₹50 per unit. The number of units to be produced by the firm such that the profit is maximized is **[GATE, 2012]**

(A) 5 (B) 10
(C) 15 (D) 25

19. Which of the following assertions are **CORRECT**?

P: Adding 7 to each entry in a list adds 7 to the mean of the list.

Q: Adding 7 to each entry in a list adds 7 to the standard deviation of the list.

R: Doubling each entry in a list doubles the mean of the list.

S: Doubling each entry in a list leaves the standard deviation of the list unchanged. **[GATE, 2012]**

(A) P, Q (B) Q, R
(C) P, R (D) R, S

20. A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation $y = 2x - 0.1x^2$ where y is the height of the arch in meters. The maximum possible height of the arch is **[GATE, 2012]**

(A) 8 meters
(B) 10 meters
(C) 12 meters
(D) 14 meters

21. An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable.

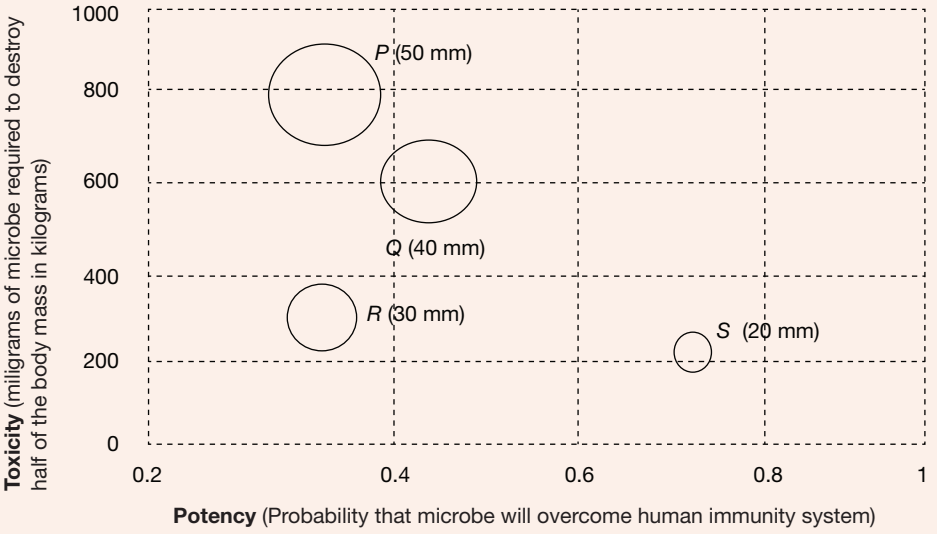
The probability that a randomly chosen shock absorber, which is found to be reliable, is made by Y is **[GATE, 2012]**

(A) 0.288 (B) 0.334
(C) 0.667 (D) 0.720

22. If $\text{Log}(P) = \left(\frac{1}{2}\right)\text{Log}(Q) - \left(\frac{1}{3}\right)\text{Log}(R)$ then which of the following options is TRUE? **[GATE, 2011]**

(A) $P^2 = Q^3R^2$ (B) $Q^2 = PR$
(C) $Q^2 = R^3P$ (D) $R = P^3Q^2$

23. P, Q, R and S are four types of dangerous microbes recently found in a human habitat. The area of each circle with its diameter printed in brackets represents the growth of a single microbe surviving human immunity system within 24 hours of entering the body. The danger to human beings varies proportionately with the toxicity, potency and growth attributed to a microbe shown in the figure below:



A pharmaceutical company is contemplating the development of a vaccine against the most dangerous microbe. Which microbe should the company target in its first attempt? **[GATE, 2011]**

(A) P (B) Q
(C) R (D) S

24. The variable cost (V) of manufacturing a product varies according to the equation $V = 4q$, where q is the quantity produced. The fixed cost (F) of production of same product reduces with q according to the equation $F = \frac{100}{q}$. How many units should be produced to minimize the total cost ($V + F$)? **[GATE, 2011]**

(A) 5 (B) 4
(C) 7 (D) 6

25. A transporter receives the same number of orders each day. Currently, he has some pending orders (backlog) to be shipped. If he uses 7 trucks, then at the end of the 4th day he can clear all the orders. Alternatively, if he uses only 3 trucks, then all the orders are cleared at the end of the 10th day. What is the minimum number of trucks required so that there will be no pending order at the end of the 5th day? **[GATE, 2011]**

(A) 4 (B) 5
(C) 6 (D) 7

26. A container originally contains 10 litres of pure spirit. From this container 1 litre of spirit is replaced with 1

litre of water. Subsequently 1 litre of the mixture is again replaced with 1 litre of water and this process is repeated one more time. How much spirit is now left in the container? **[GATE, 2011]**

(A) 7.58 litres (B) 7.84 litres
(C) 7 litres (D) 7.29 litres

27. If 137 + 276 = 435, how much is 731+672? **[GATE, 2010]**

(A) 534 (B) 1403
(C) 1623 (D) 1513

28. 5 skilled workers can build a wall in 20 days, 8 semi-skilled workers can build the wall in 25 days, 10 unskilled workers can build the wall in 30 days. If a team has 2 skilled, 6 semi-skilled and 5 unskilled workers, how long will it take to build the wall? **[GATE, 2010]**

(A) 20 (B) 18
(C) 16 (D) 15

29. From the digits 2, 2, 3, 3, 3, 4, 4, 4, 4, how many distinct 4-digit numbers greater than 3000 can be formed? **[GATE, 2010]**

(A) 50 (B) 51
(C) 52 (D) 54

Answer Keys

Exercises

1. A	**2.** B	**3.** D	**4.** D	**5.** A	**6.** C	**7.** A	**8.** A	**9.** D	**10.** B
11. C	**12.** B	**13.** 0	**14.** 0	**15.** D	**16.** B	**17.** B			

Previous Years' Questions

1. 96	**2.** D	**3.** 850	**4.** 48%	**5.** 6	**6.** B	**7.** C	**8.** D	**9.** C	**10.** C
11. D	**12.** C	**13.** C	**14.** C	**15.** B	**16.** D	**17.** B	**18.** A	**19.** C	**20.** B
21. B	**22.** B	**23.** D	**24.** A	**25.** C	**26.** D	**27.** C	**28.** D	**29.** B	