Chapter 5 Quadratic Functions

Ex 5.6

Answer 1e.

Let $f(x) = a_n x^n + \ldots + a_1 x + a_0$ be a polynomial function having integer coefficients.

The Rational Zero Theorem states that every rational zero of f will have the form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coeffeicient } a_n}.$$

Therefore, if a polynomial function has integer coefficients, then every rational zero of the function has the form $\frac{p}{q}$, where p is a factor of the **constant term** and q is a factor of the **leading coefficient**.

Answer 1gp.

The rational zero theorem states that if $f(x) = a_n x^n + \ldots + a_1 x + a_0$ has integer coefficients, then every rational zero of f will have the form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coeffeicient } a_n}.$$

We need to find the factors of the constant term as well as the factors of the leading coefficient to find the list of possible rational zeros.

The constant term in the given function is 15, and the leading coefficient is 1. Find the positive and negative factors of these two numbers.

Factors of the constant term: $\pm 1, \pm 3, \pm 5, \pm 15$

Factors of the leading coefficient: ± 1

Now, divide the factors of constant term by the factors of the leading coefficient to get the list of possible rational zeros.

Possible rational zeros : $\pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{5}{1}, \pm\frac{15}{1}$

The list can be simplified.
Simplified list of possible zeros : $\pm 1, \pm 3, \pm 5, \pm 15$

Therefore, the possible rational zeros of the given function are $\pm 1, \pm 3, \pm 5$, and ± 15.

Answer 1q.

The monomial 2 is common among the terms in the given polynomial.

We can take out 2 from each term.
$2x^3 - 54 = 2(x^3 - 27)$

The expression $x^3 - 27$ can be identified as the difference of two cubes.

Factor using the pattern $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

$$2\left(x^3 - 27\right) = 2\left(x^3 - 3^3\right)$$
$$= 2\left(x - 3\right)\left(x^2 + 3x + 9\right)$$

Therefore, the given polynomial in completely factored form is
$2(x - 3)(x^2 + 3x + 9)$.

Answer 2e

We need to describe a method we can use to shorten the list of possible rational zero when using the rational zero theorem.

To find the possible rational zeros of f we use the following Rational Zero Theorem:
If
$f(x) = a_n x^n + \ldots + a_1 x + a_0$
has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

The above Rational Zero Theorem list only the possible values.
In order to find the actual zeros of a polynomial function f, we must test values from the list of possible zeros.
When the leading coefficient is not 1, the list of possible rational zeros can increase dramatically.
In such cases the search can be shortened by sketching the function's graph.

Answer 2gp.

We need to list the possible rational zeros of f using the rational zero theorem.
$f(x) = 2x^3 + 3x^2 - 11x - 6$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.
The Rational Zero Theorem: If
$f(x) = a_n x^n + \ldots + a_1 x + a_0$
has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:

$f(x)=2x^3+3x^2-11x-6$

Factors of constant term: $+1, \pm 2, \pm 3, \pm 6$

Factors of leading coefficient: $\pm 1, \pm 2$

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{3}{2}, \pm\frac{6}{2}$

Simplified list of possible zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}$

Thus, the list of possible rational zeros of f is $\boxed{\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}}$

Answer 2q.

We need to factor the polynomial

$f(x)=x^3-3x^2+2x-6$

completely.

Consider the given polynomial

$$\begin{aligned} f(x) &= x^3-3x^2+2x-6 \\ &= x^2(x-3)+2(x-3) && \text{[Factor by grouping]} \\ &= (x^2+2)(x-3) && \text{[Distributive property]} \end{aligned}$$

Thus, the factorization of

$f(x)=x^3-3x^2+2x-6$

is

$\boxed{(x^2+2)(x-3)}$

Answer 3e.

The Rational Zero Theorem states that if $f(x)=a_nx^n+\ldots+a_1x+a_0$ has integer coefficients, then every rational zero of f will have the form:

$$\frac{p}{q}=\frac{\text{factor of constant term } a_0}{\text{factor of leading coeffeicient } a_n}.$$

We need to find the factors of the constant term as well as factors of the leading coefficient to find the list of possible rational zeros.

The constant term in the given function is 28, and the leading coefficient is 1. Find the positive and negative factors of these two numbers.

Factors of the constant term: $\pm1, \pm2, \pm4, \pm7, \pm14, \pm28$

Factors of the leading coefficient: ±1

Now, divide the factors of constant term by the factors of the leading coefficient to get the list of possible rational zeros.

Possible rational zeros : $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{7}{1}, \pm\frac{14}{1}, \pm\frac{28}{1}$

The list can be simplified.
Simplified list of possible zeros : $\pm1, \pm2, \pm4, \pm7, \pm14, \pm28$

Therefore, the possible rational zeros of the given function are $\pm1, \pm2, \pm4, \pm7, \pm14$, and ±28.

Answer 3gp.

STEP 1 **List** the possible rational zeros.
The leading coefficient of the given function is 1 and the constant term is 18. Divide each factor of the constant term by each factor of the leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}, \pm\frac{9}{1}, \pm\frac{18}{1}$$

The possible rational zeros of the function are $\pm1, \pm2, \pm3, \pm6, \pm9$, and ±18.

STEP 2 **Test** these zeros using synthetic division.

Test $x = -1$

$$\begin{array}{r|rrrr} -1 & 1 & -4 & -15 & 18 \\ & & -1 & -5 & -20 \\ \hline & 1 & -5 & -20 & -2 \end{array}$$

Test $x = -3$

$$\begin{array}{r|rrrr} -3 & 1 & -4 & -15 & 18 \\ & & -3 & 21 & -18 \\ \hline & 1 & -7 & 6 & 0 \end{array}$$

The remainder is 0 in the case of -3, which means that -3 is a zero of $f(x)$. Thus, we have $x + 3$ as one of the factors of the function. We can use this result to express $f(x)$ as a product of factors.

$f(x) = (x + 3)(x^2 - 7x + 6)$

STEP 3 **Factor** the trinomial in $f(x)$ and use the factor theorem.
$f(x) = (x + 3)(x - 6)(x - 1)$

The other two factors of $f(x)$ are $x - 6$ and $x - 1$, from which we get the two zeros 6 and 1.

Therefore, the zeros of f are -3, 1, and 6.

Answer 3q.

Apply the pattern for factoring by grouping.

Take out the common monomial x^2 from the first two terms in the given polynomial.
$x^3 + x^2 + x + 1 = x^2(x + 1) + (x + 1)$

Now, use the distributive property.
$x^2(x + 1) + (x + 1) = (x^2 + 1)(x + 1)$

Therefore, the given polynomial in completely factored form is $(x^2 + 1)(x + 1)$.

Answer 4e.

We need to list the possible rational zeros of f using the rational zero theorem.
$f(x) = x^3 - 4x^2 + x - 10$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.
The Rational Zero Theorem: If
$f(x) = a_n x^n + \ldots + a_1 x + a_0$
has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:
$f(x) = x^3 - 4x^2 + x - 10$
Factors of constant term: $\pm 1, \pm 2, \pm 5, \pm 10$
Factors of leading coefficient: ± 1
Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{5}{1}, \pm\frac{10}{1}$
Simplified list of possible zeros: $\pm 1, \pm 2, \pm 5, \pm 10$

Thus, the list of possible rational zeros of f is $\boxed{\pm 1, \pm 2, \pm 5, \pm 10}$

Answer 4gp.

We need to find all real zeros of

$f(x) = x^3 - 8x^2 + 5x + 14$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.

The Rational Zero Theorem: If

$f(x) = a_n x^n + \ldots + a_1 x + a_0$

has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:

$f(x) = x^3 - 8x^2 + 5x + 14$

Let us list the possible rational zeros.

Factors of constant term: $+1, \pm 2, \pm 7, \pm 14$

Factors of leading coefficient: ± 1

Possible rational zeros: $\pm 1, \pm 2, \pm 7, \pm 14$

Let us test these zeros using synthetic division.

Test $x = 1$:

$$\begin{array}{r|rrrr} 1 & 1 & -8 & 5 & 14 \\ & & 1 & -8 & -3 \\ \hline & 1 & -8 & -3 & 11 \end{array}$$

Here the remainder is 11 and hence 1 is not a zero.

Test $x = -1$:

$$\begin{array}{r|rrrr} -1 & 1 & -8 & 5 & 14 \\ & & -1 & 9 & -14 \\ \hline & 1 & -9 & 14 & 0 \end{array}$$

Here the remainder is 0 and hence -1 is a zero.

Because -1 is a zero, we can write

$f(x) = (x+1)(x^2 - 9x + 14)$

Let us factor the trinomial in $f(x)$ and use the Factor Theorem.

$$\begin{aligned} f(x) &= (x+1)(x^2 - 9x + 14) \\ &= (x+1)(x^2 - 7x - 2x + 14) \\ &= (x+1)(x-7)(x-2) \end{aligned}$$

Thus, the zeros of $f(x)$ are $\boxed{-1, 7 \text{ and } 2}$

Answer 4q.

We need to factor the polynomial

$f(x) = 6x^5 - 150x$

completely.

Consider the given polynomial

$$\begin{aligned} f(x) &= 6x^5 - 150x \\ &= 6x\left(x^4 - 25\right) \text{ [Factor out } 6x\text{]} \\ &= 6x\left(\left(x^2\right)^2 - 5^2\right) \end{aligned}$$

Let us use the following identity:

$a^2 - b^2 = (a-b)(a+b)$

Here, $a = x^2, b = 5$

Therefore,

$$\begin{aligned} f(x) &= 6x\left(\left(x^2\right)^2 - 5^2\right) \\ &= 6x\left(\left(x^2\right) - 5\right)\left(\left(x^2\right) + 5\right) \end{aligned}$$

Thus, the factorization of

$f(x) = 6x^5 - 150x$

is

$$\boxed{6x\left(\left(x^2\right) - 5\right)\left(\left(x^2\right) + 5\right)}$$

Answer 5e.

The Rational Zero Theorem states that if $f(x) = a_n x^n + \ldots + a_1 x + a_0$ has integer coefficients, then every rational zero of f will have the form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coeffeicient } a_n}.$$

We need to find the factors of the constant term as well as factors of the leading coefficient to find the list of possible rational zeros.

The constant term in the given function is 9, and the leading coefficient is 2. Find the positive and negative factors of these two numbers.

Factors of the constant term: $\pm 1, \pm 3, \pm 9$

Factors of the leading coefficient: $\pm 1, \pm 2$

Now, divide the factors of the constant term by the factors of leading coefficient to get the list of possible rational zeros.

$$\text{Possible rational zeros} = \frac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 2}$$
$$= \pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{9}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$$

The list can be simplified.

$$\text{Simplified list of possible zeros}: \pm 1, \pm 3, \pm 9, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$$

Therefore, the possible rational zeros of the given function are

$\pm 1, \pm 3, \pm 9, \pm\frac{1}{2}, \pm\frac{3}{2}$, and $\pm\frac{9}{2}$.

Answer 5gp.

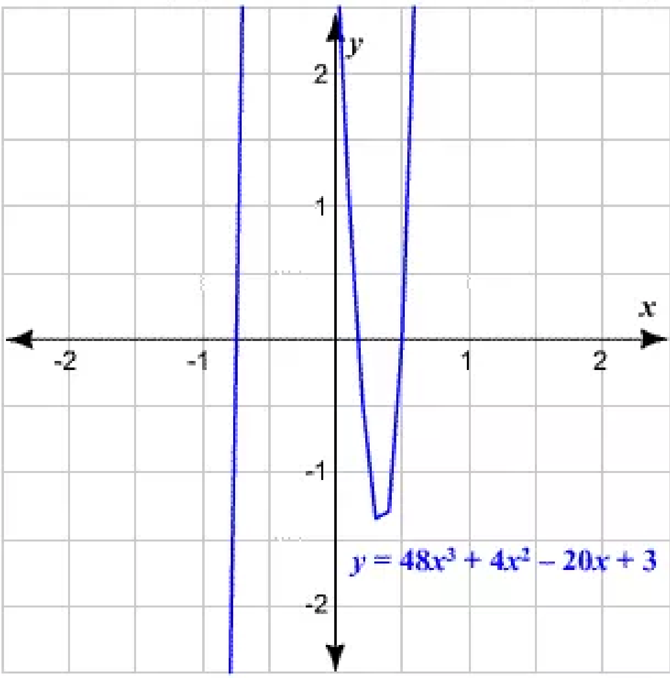
STEP 1 **List** the possible rational zeros.
The leading coefficient of the given function is 3 and the constant term is 48. Divide each factor of the constant term by each factor of the leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{3}{3}, \pm\frac{3}{4}, \pm\frac{3}{6}, \pm\frac{3}{8}, \pm\frac{3}{12}, \pm\frac{3}{16}, \pm\frac{3}{24}, \pm\frac{3}{48},$$
$$\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{8}, \pm\frac{1}{12}, \pm\frac{1}{16}, \pm\frac{1}{24}, \pm\frac{1}{48}$$

The possible rational zeros of the function are

$\pm 3, \pm\frac{3}{2}, \pm 1, \pm\frac{3}{4}, \pm\frac{1}{2}, \pm\frac{3}{8}, \pm\frac{1}{4}, \pm\frac{3}{16}, \pm\frac{1}{8}, \pm\frac{1}{16}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm\frac{1}{8}, \pm\frac{1}{12}$, $\pm\frac{1}{16}, \pm\frac{1}{24}$, and $\pm\frac{1}{48}$.

STEP 2 Sketch the graph of the function using any graphing utility.



Among the possible zeros found in step 1, $x = -\frac{3}{4}$ and $x = \frac{1}{2}$ seem to be reasonable based on the graph.

STEP 3 **Test** the reasonable zeros found in step 2 using synthetic division.

Test $x = -\frac{3}{4}$

$$\begin{array}{r|rrrr} -\frac{3}{4} & 48 & 4 & -20 & 3 \\ & & -36 & 24 & -3 \\ \hline & 48 & -32 & 4 & 0 \end{array}$$

The remainder is 0 in the case of $-\frac{3}{4}$, which means that $-\frac{3}{4}$ is a zero of $f(x)$. Thus, we have $x + \frac{3}{4}$ as one of the factors of the function. We can use this result to express $f(x)$ as a product of factors.

$$f(x) = \left(x + \frac{3}{4}\right)\left(48x^2 - 32x + 4\right)$$

STEP 4 **Factor** the trinomial in $f(x)$ and use the factor theorem.

Take out 4 from the second factor.

$$f(x) = \left(x + \frac{3}{4}\right)4\left(12x^2 - 8x + 1\right)$$

Multiply the first factor by 4.

$f(x) = (4x + 3)(12x^2 - 8x + 1)$

Now, factor the trinomial $12x^2 - 8x + 1$.

$f(x) = (4x + 3)(6x - 1)(2x - 1)$

On solving $6x - 1 = 0$ and $2x - 1 = 0$, we get the other two zeros as $\frac{1}{6}$ and $\frac{1}{2}$.

Therefore, the real zeros of f are $-\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{6}$.

Answer 5q.

The given expression contains a common monomial 3.

$3x^4 - 24x^2 + 48 = 3(x^4 - 8x^2 + 16)$

Now, rewrite the expression within the parentheses using the replacement $u = x^2$.

$3(u^2 - 8u + 16)$

Factor the perfect-square trinomial $u^2 - 8u + 16$.

$3(u^2 - 8u + 16) = 3(u - 4)^2$

Now, replace u with x^2.

$3(u - 4)^2 = 3(x^2 - 4)^2$

We can factor $x^2 - 4$ using the pattern for the difference of two squares.

$3(x^2 - 4)^2 = 3[(x + 2)(x - 2)]^2$

Therefore, the given expression in completely factored form is $3[(x + 2)(x - 2)]^2$.

Answer 6e.

We need to list the possible rational zeros of f using the rational zero theorem.

$h(x) = 2x^3 + x^2 - x - 18$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.

The Rational Zero Theorem: If

$f(x) = a_n x^n + ... + a_1 x + a_0$

has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:

$h(x) = 2x^3 + x^2 - x - 18$

Factors of constant term: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

Factors of leading coefficient: $\pm 1, \pm 2$

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}, \pm\frac{9}{1}, \pm\frac{18}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{3}{2}, \pm\frac{6}{2}, \pm\frac{9}{2}, \pm\frac{18}{2}$

Simplified list of possible zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$

Thus, the list of possible rational zeros of h is $\boxed{\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}}$

Answer 6gp.

We need to find all real zeros of

$f(x) = 2x^4 + 5x^3 - 18x^2 - 19x + 42$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.

The Rational Zero Theorem: If

$f(x) = a_n x^n + ... + a_1 x + a_0$

has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:

$f(x)=2x^4+5x^3-18x^2-19x+42$

Let us list the possible rational zeros.

Factors of constant term: $\pm1,\pm2,\pm3,\pm6,\pm7,+14,\pm21,42$

Factors of leading coefficient: $\pm1,\pm2$

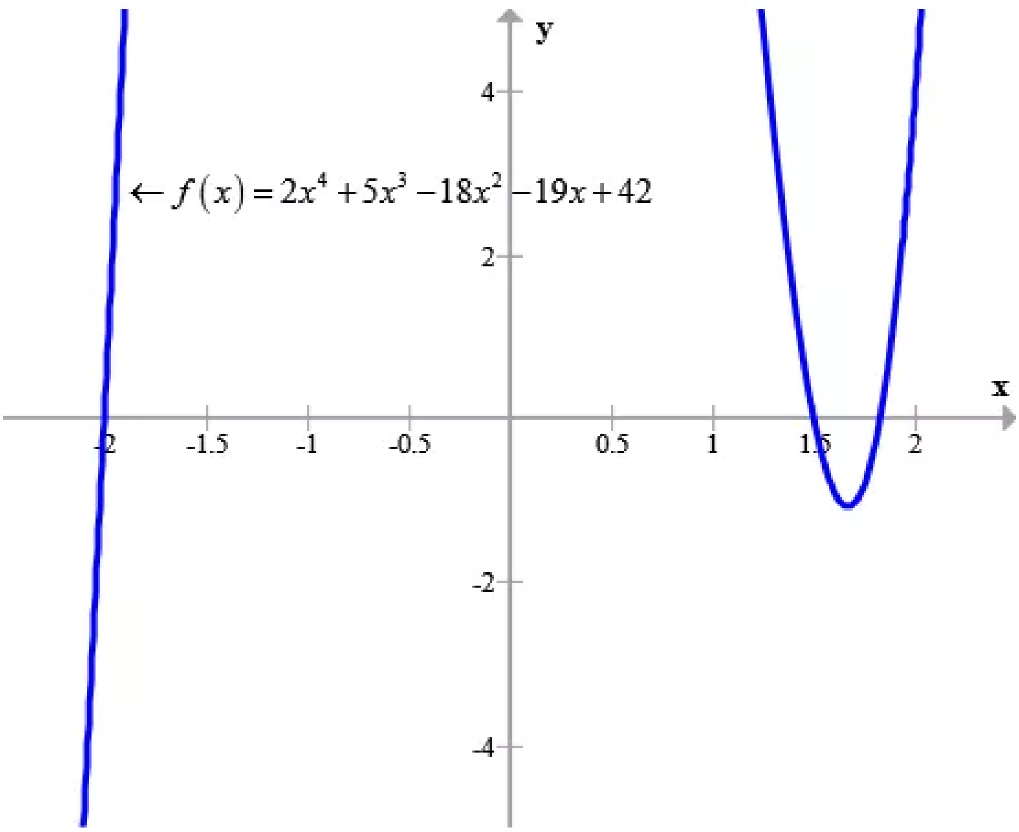
Possible rational zeros:

$\pm\frac{1}{1},\pm\frac{2}{1},\pm\frac{3}{1},\pm\frac{6}{1},\pm\frac{7}{1},+\frac{14}{1},\pm\frac{21}{1},\frac{42}{1},\pm\frac{1}{2},\pm\frac{2}{2},\pm\frac{3}{2},\pm\frac{6}{2},\pm\frac{7}{2},+\frac{14}{2},\pm\frac{21}{2},\pm\frac{42}{2}$

That is the possible rational roots are

$\pm1,\pm2,\pm3,\pm6,\pm7,\pm14,\pm21,\pm42,\pm\frac{1}{2},\pm\frac{3}{2},\pm\frac{7}{2},\pm\frac{21}{2}$

Choose the reasonable values from the above list using the graph function.
Let us observe following graph.



Thus the values $x=-2, x=1.5, x=1.83$ are reasonable based on the graph.

Let us test these zeros using synthetic division.

Test $x=-2$:

$$\begin{array}{r|rrrrr} -2 & 2 & 5 & -18 & -19 & 42 \\ & & -4 & -2 & 40 & -42 \\ \hline & 2 & 1 & -20 & 21 & 0 \end{array}$$

Here the remainder is 0 and hence $x=-2$ is a zero

Factor out a binomial using the result of synthetic division.

Because -2 is a zero, we can write

$$f(x)=(x+2)(2x^3+x^2-20x+21)$$

Now consider,

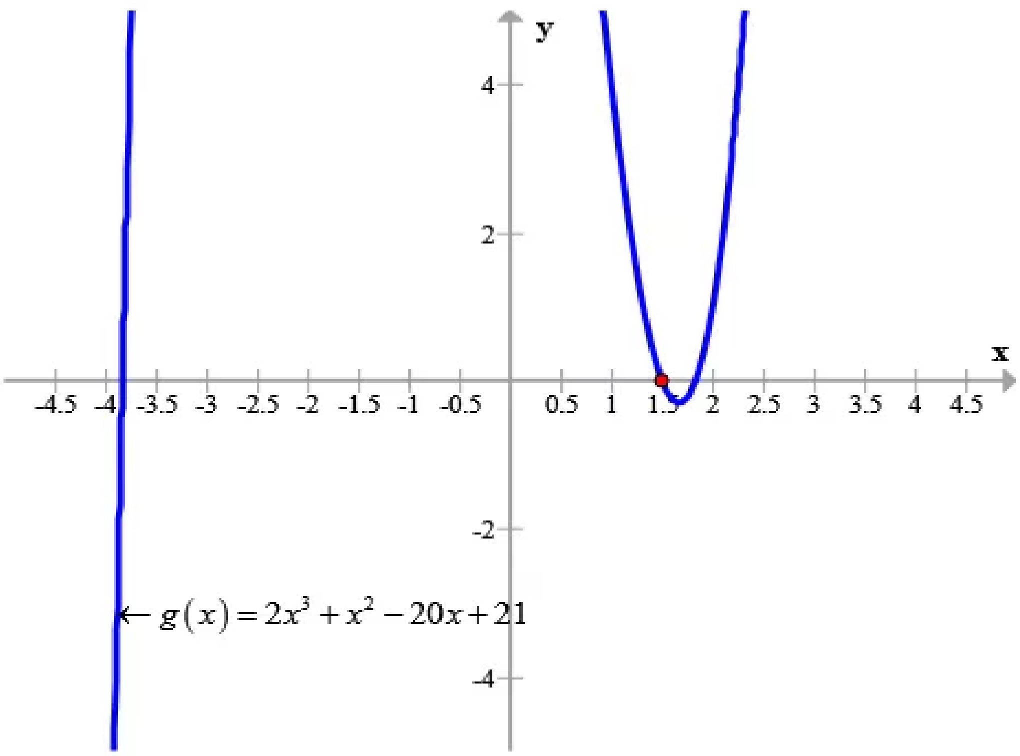
$$g(x)=2x^3+x^2-20x+21$$

Any zero of $g(x)=2x^3+x^2-20x+21$ will also be a zero of $f(x)$.

The possible rational zeros of $g(x)$ are:

$$\pm1,\pm3,\pm7,\pm21,\pm\frac{1}{2},\pm\frac{3}{2},\pm\frac{7}{2},\pm\frac{21}{2}$$

Let us observe the following graph of $g(x)$:



The graph of g shows that $x=\frac{3}{2}$ may be a zero. Synthetic division shows that $x=\frac{3}{2}$ is a zero of $g(x)$ and

$$\begin{aligned} f(x)&=(x+2)g(x) \\ &=(x+2)(2x-3)(2x^2+4x-14) \end{aligned}$$

Let us find the remaining zeros of $f(x)$ by solving $2x^2+4x-14=0$

$$x=\frac{-4\pm\sqrt{16-4(2)(-14)}}{2\times 2}$$
$$=\frac{-4\pm\sqrt{128}}{4}$$
$$=-1\pm 8\sqrt{2}$$

Thus, the zerors of $f(x)$ are $\boxed{-2,\frac{3}{2},-1+8\sqrt{2} \text{ and } -1-8\sqrt{2}}$

Answer 6q.

We need to factor the polynomial

$f(x)=2x^3-3x^2-12x+18$

completely.

Consider the given polynomial

$$f(x)=2x^3-3x^2-12x+18$$
$$=x^2(2x-3)-6(2x-3) \quad \text{[Factor by grouping]}$$
$$=(x^2-6)(2x-3) \quad \text{[Distributive property]}$$

Thus, the factorization of

$f(x)=2x^3-3x^2-12x+18$

is

$\boxed{(x^2-6)(2x-3)}$

Answer 7e.

The Rational Zero Theorem states that if $f(x)=a_nx^n+\ldots+a_1x+a_0$ has integer coefficients, then every rational zero of f will have the form:

$$\frac{p}{q}=\frac{\text{factor of constant term } a_0}{\text{factor of leading coeffeicient } a_n}.$$

We need to find the factors of the constant term as well as factors of the leading coefficient to find the list of possible rational zeros.

The constant term in the given function is -14, and the leading coefficient is 4. Find the positive and negative factors of these two numbers.

Factors of the constant term: $\pm1, \pm2, \pm7, \pm14$

Factors of the leading coefficient: $\pm1, \pm2, \pm4$

Now, divide the factors of the constant term by the factors of leading coefficient to get the list of possible rational zeros.

$$\text{Possible rational zeros} = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{7}{1}, \pm\frac{14}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{7}{2}, \pm\frac{14}{2}, \pm\frac{1}{4}, \pm\frac{2}{4}, \pm\frac{7}{4}, \pm\frac{14}{4}$$

The list can be simplified.

$$\text{Simplified list of possible zeros}: \pm1,\ \pm2,\ \pm7,\ \pm14,\ \pm\frac{1}{2},\ \pm\frac{7}{2},\ \pm\frac{1}{4},\ \pm\frac{7}{4}$$

Therefore, the possible rational zeros of the given function are

$\pm1,\ \pm2,\ \pm7,\ \pm14,\ \pm\frac{1}{2},\ \pm\frac{7}{2},\ \pm\frac{1}{4}$, and $\pm\frac{7}{4}$.

Answer 7gp.

STEP 1 **Write** an equation for the volume of the ice sculpture.
Use the fact that the volume of a pyramid is given by one-third the product of the area of its base and height.

$$\underset{\text{(cubic feet)}}{\text{Volume}} = \frac{1}{3} \cdot \underset{\text{(square feet)}}{\text{Area of base}} \cdot \underset{\text{(feet)}}{\text{Height}}$$

$$\Downarrow \qquad\qquad \Downarrow \qquad\qquad \Downarrow$$

$$6 = \frac{1}{3} \cdot (x+1)^2 \cdot x$$

We get the equation $6 = \frac{1}{3}(x+1)^2 x$.

Multiply both the sides by 3 and simplify.

$18 = (x+1)^2 x$

$18 = x^3 + 2x^2 + x$

Now, rewrite the equation in standard form.

$0 = x^3 + 2x^2 + x - 18$

STEP 2 **List** the possible rational solutions.
The leading coefficient of the function is 1 and the constant term is 18.

Thus, the possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9$, and ± 18.

STEP 3 **Test** possible solutions. Only positive x-values make sense.

Test $x = 1$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & 1 & -18 \\ & & 1 & 3 & 4 \\ \hline & 1 & 3 & 4 & -14 \end{array}$$

Test $x = 2$

$$\begin{array}{r|rrrr} 2 & 1 & 2 & 1 & -18 \\ & & 2 & 8 & 18 \\ \hline & 1 & 4 & 9 & 0 \end{array}$$

The remainder being 0 in the case of 2, means that 2 is a zero or solution of the function. We can use this result to express the function as a product of factors.
$0 = (x - 2)(x^2 + 4x + 9)$

STEP 4 **Check** for other solutions.
For this, solve $x^2 + 4x + 9 = 0$ using the quadratic formula. On solving, we get the other two solutions satisfying the equation as $-2 \pm i\sqrt{5}$, which can be discarded because they are imaginary numbers.

The only reasonable solution is thus $x = 2$. Since the height of the sculpture is 2 feet, the length of each side will be 2 + 1 or 3 feet.

Therefore, the dimensions of the mold are 2 feet by 3 feet.

Answer 7q.

When a polynomial $f(x)$ is divided by a divisor $d(x)$, we obtain a quotient $q(x)$ and a remainder $r(x)$ such that $\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$.

In polynomial long division, include 0 as the coefficient of the missing terms in the dividend. The x term is missing in the given dividend. Use 0 as its coefficient.

$$x^2 + 5x - 2 \overline{\big) x^4 + x^3 - 8x^2 + 5x + 5}$$

Then, divide the term with highest power in what is left of the dividend at each stage by the first term of the divisor.

Multiply the divisor by $\frac{x^4}{x^2}$, or x^2.

$$\begin{array}{r} x^2 \\ x^2 + 5x - 2 \overline{\big) x^4 + x^3 - 8x^2 + 5x + 5} \\ \underline{x^4 + 5x^3 - 2x^2} \end{array}$$

Subtract the corresponding terms and bring down the next term.

$$
\begin{array}{r}
x^2 \phantom{{}+5x+5} \\
x^2+5x-2\,\big)\overline{x^4+x^3-8x^2+5x+5} \\
\underline{x^4+5x^3-2x^2} \phantom{{}+5x+5} \\
-4x^3-6x^2+5x \phantom{{}+5}
\end{array}
$$

Now, multiply the divisor by $\frac{-4x^3}{x^2}$, or $-4x$.

$$
\begin{array}{r}
x^2-4x \phantom{{}+5} \\
x^2+5x-2\,\big)\overline{x^4+x^3-8x^2+5x+5} \\
\underline{x^4+5x^3-2x^2} \phantom{{}+5x+5} \\
-4x^3-6x^2+5x \phantom{{}+5} \\
-4x^3-20x^2-8x \phantom{{}+5}
\end{array}
$$

Subtract the terms and bring down the next term.

$$
\begin{array}{r}
x^2-4x \phantom{{}+5} \\
x^2+5x-2\,\big)\overline{x^4+x^3-8x^2+5x+5} \\
\underline{x^4+5x^3-2x^2} \phantom{{}+5x+5} \\
-4x^3-6x^2+5x \phantom{{}+5} \\
\underline{-4x^3-20x^2+8x} \phantom{{}+5} \\
14x^2-3x+5
\end{array}
$$

Repeat the process by multiplying the divisor by $\frac{14x^2}{x^2}$, or 14.

$$
\begin{array}{r}
x^2-4x+14 \\
x^2+5x-2\,\big)\overline{x^4+x^3-8x^2+5x+5} \\
\underline{x^4+5x^3-2x^2} \phantom{{}+5x+5} \\
-4x^3-6x^2+5x \phantom{{}+5} \\
\underline{-4x^3-20x^2+8x} \phantom{{}+5} \\
14x^2-3x+5 \\
\underline{14x^2+70x-28} \\
-73x+33
\end{array}
$$

Thus, $\dfrac{x^4+x^3-8x^2+5x+5}{x^2+5x-2} = x^2-4x+14+\dfrac{-73x+33}{x^2+5x-2}$.

Check the result by multiplying the quotient by the divisor and adding the remainder. The result should be the dividend.

$$\left(x^2 - 4x + 14\right)\left(x^2 + 5x - 2\right) + \left(-73x + 33\right)$$
$$= x^2\left(x^2 + 5x - 2\right) - 4x\left(x^2 + 5x - 2\right) + 14\left(x^2 + 5x - 2\right) - 73x + 33$$
$$= x^4 + 5x^3 - 2x^2 - 4x^3 - 20x^2 + 8x + 14x^2 + 70x - 28 - 73x + 33$$
$$= x^4 + x^3 - 8x^2 + 5x + 5$$

The solution checks.

Answer 8e.

We need to list the possible rational zeros of f using the rational zero theorem.

$$f(x) = 3x^4 + 5x^3 - 3x + 42$$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.

The Rational Zero Theorem: If

$$f(x) = a_n x^n + \ldots + a_1 x + a_0$$

has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Let us list the possible rational zeros.

Factors of constant term: $\pm1, \pm2, \pm3, \pm6, \pm7, +14, \pm21, 42$

Factors of leading coefficient: $\pm1, \pm3$

Possible rational zeros:

$$\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}, \pm\frac{7}{1}, +\frac{14}{1}, \pm\frac{21}{1}, \frac{42}{1}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{3}{3}, \pm\frac{6}{3}, \pm\frac{7}{3}, +\frac{14}{3}, \pm\frac{21}{3}, \pm\frac{42}{3}$$

That is the possible rational roots are

$$\pm1, \pm2, \pm3, \pm6, \pm7, +14, \pm21, \pm42, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{7}{3}, +\frac{14}{3}$$

Thus, the list of possible rational zeros of

f is $\boxed{\pm1, \pm2, \pm3, \pm6, \pm7, +14, \pm21, \pm42, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{7}{3}, +\frac{14}{3}}$

Answer 8q.

We need to divide the polynomial

$$4x^4 + 27x^2 + 3x + 64$$

by the polynomial

$$x + 7$$

using polynomial long division or synthetic division

Let us do the division by polynomial long division.

$$\begin{array}{r|l} & 4x^3-28x^2+223x-1558 \\ \hline x+7 & 4x^4+0x^3+27x^2+3x+64 \\ & \underline{4x^4+28x^3} \\ & \quad -28x^3+27x^2 \\ & \quad \underline{-28x^3-196x^2} \\ & \qquad 223x^2+3x \\ & \qquad \underline{223x^2+1561x} \\ & \qquad\quad -1558x+64 \\ & \qquad\quad \underline{-1558x-10906} \\ & \qquad\qquad 10970 \end{array}$$

Thus, $\dfrac{4x^4+27x^2+3x+64}{x+7}=4x^3-28x^2+223x-1558+\dfrac{10970}{x+7}$

Therefore,

$$\boxed{\begin{array}{l}\text{Quotient}=4x^3-28x^2+223x-1558 \\ \text{Remainder}=10970\end{array}}$$

Answer 9e.

The Rational Zero Theorem states that if $f(x)=a_nx^n+\ldots+a_1x+a_0$ has integer coefficients, then every rational zero of f will have the form:

$$\frac{p}{q}=\frac{\text{factor of constant term } a_0}{\text{factor of leading coeffeicient } a_n}.$$

We need to find the factors of the constant term as well as factors of the leading coefficient to find the list of possible rational zeros.

The constant term in the given function is 15, and the leading coefficient is 8. Find the positive and negative factors of these two numbers.

Factors of the constant term: $\pm1, \pm3, \pm5, \pm15$

Factors of the leading coefficient: $\pm1, \pm2, \pm4, \pm8$

Now, divide the factors of the constant term by the factors of leading coefficient to get the list of possible rational zeros.

$$\text{Possible rational zeros}=\pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{5}{1}, \pm\frac{15}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{15}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{5}{8}, \pm\frac{15}{8}$$

The list can be simplified.

Simplified list of possible zeros: $\pm 1,\ \pm 3,\ \pm 5,\ \pm 15,\ \pm\frac{1}{2},\ \pm\frac{3}{2},\ \pm\frac{5}{2},\ \pm\frac{15}{2},\ \pm\frac{1}{4},\ \pm\frac{3}{4},$

$$\pm\frac{5}{4},\ \pm\frac{15}{4},\ \pm\frac{1}{8},\ \pm\frac{3}{8},\ \pm\frac{5}{8},\ \pm\frac{15}{8}$$

Therefore, the possible rational zeros of the given function are $\pm 1,\ \pm 3,\ \pm 5,\ \pm 15,$ $\pm\frac{1}{2},\ \pm\frac{3}{2},\ \pm\frac{5}{2},\ \pm\frac{15}{2},\ \pm\frac{1}{4},\ \pm\frac{3}{4},\ \pm\frac{5}{4},\ \pm\frac{15}{4},\ \pm\frac{1}{8},\ \pm\frac{3}{8},\ \pm\frac{5}{8},$ and $\pm\frac{15}{8}$.

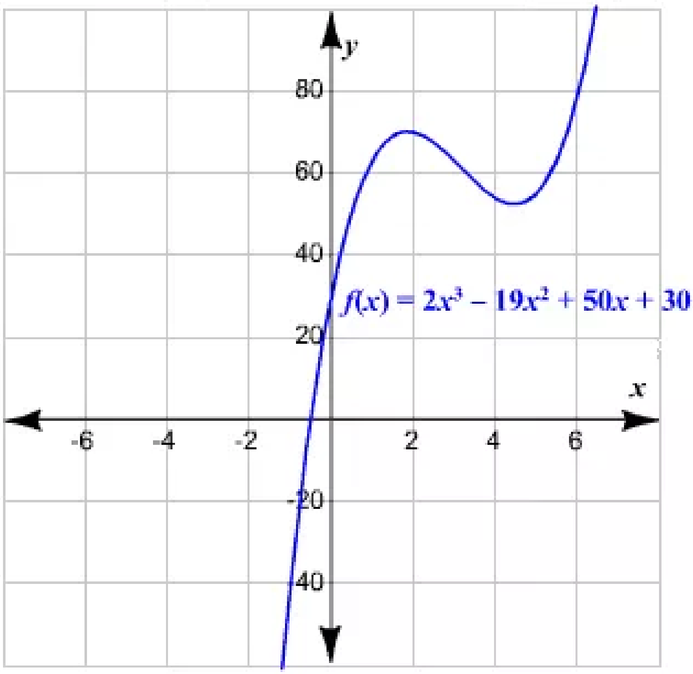
Answer 9q.

STEP 1 **List** the possible rational zeros.
The leading coefficient of the given function is 2 and the constant term is 30. Divide each factor of the constant term by each factor of the leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{5}{1}, \pm\frac{6}{1}, \pm\frac{10}{1}, \pm\frac{15}{1}, \pm\frac{30}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{6}{2}, \pm\frac{10}{2}, \pm\frac{15}{2}, \pm\frac{30}{2}$$

The possible rational zeros of the function are
$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2},$ and $\pm\frac{15}{2}$.

STEP 2 Sketch the graph of the function using any graphing utility.



Among the possible zeros found in step 1, $x = -\frac{1}{2}$ seem to be reasonable based on the graph.

STEP 3 **Test** the reasonable zero found in step 2 using synthetic division

Test $\boldsymbol{x = -\frac{1}{2}}$

$$
\begin{array}{r|rrrr}
-\frac{1}{2} & 2 & -19 & 50 & 30 \\
 & & -1 & 10 & -30 \\
\hline
 & 2 & -20 & 60 & 0
\end{array}
$$

The remainder is 0 which means that $-\frac{1}{2}$ is a zero of $f(x)$. As a result, $x + \frac{1}{2}$ is one of the factors of the function. We can use this result to express $f(x)$ as a product of factors.

$$f(x) = \left(x + \frac{1}{2}\right)\left(2x^2 - 20x + 60\right)$$

STEP 3 **Factor** the trinomial in $f(x)$ and use the factor theorem.

$$f(x) = \left(x + \frac{1}{2}\right)\left(2x^2 - 20x + 60\right)$$

Take out 2 from the second factor.

$$f(x) = \left(x + \frac{1}{2}\right)2\left(x^2 - 10x + 30\right)$$

Multiply the first factor by 2.
$f(x) = (2x + 1)(x^2 - 10x + 30)$

We can find the remaining zeros of f by solving $x^2 - 10x + 30 = 0$ using the quadratic formula. Replace a with 1, b with -10, and c with 30.

$$
\begin{aligned}
x &= \frac{10 \pm \sqrt{10^2 - 4(1)(30)}}{2(2)} \\
&= \frac{10 \pm \sqrt{-20}}{4} \\
&= \frac{10 \pm 2i\sqrt{5}}{4} \\
&= \frac{5 \pm i\sqrt{5}}{2}
\end{aligned}
$$

As we seek only the real zeros of f, $\frac{5 \pm i\sqrt{5}}{2}$ cannot be considered.

Therefore, the only zero of f is $-\frac{1}{2}$.

Answer 10e.

We need to list the possible rational zeros of f using the rational zero theorem.

$h(x)=6x^3-3x^2+12$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.

The Rational Zero Theorem: If

$f(x)=a_n x^n+...+a_1 x+a_0$

has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q}=\frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Let us list the possible rational zeros.

Factors of constant term: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

Factors of leading coefficient: $\pm1, \pm2, \pm3, \pm6$

Possible rational zeros:

$$\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{3}{2}, \pm\frac{4}{2}, \pm\frac{6}{2}, \pm\frac{12}{2},$$
$$\pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{3}{3}, \pm\frac{4}{3}, \pm\frac{6}{3}, \pm\frac{12}{3}, \pm\frac{1}{6}, \pm\frac{2}{6}, \pm\frac{3}{6}, \pm\frac{4}{6}, \pm\frac{6}{6}, \pm\frac{12}{6}$$
$$\pm\frac{1}{12}, \pm\frac{2}{12}, \pm\frac{3}{12}, \pm\frac{4}{12}, \pm\frac{6}{12}, \pm\frac{12}{12}$$

That is the possible rational roots are

$$\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{1}{6}, \pm\frac{1}{4}$$

Thus, the list of possible rational zeros of

h is $\boxed{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{1}{6}, \pm\frac{1}{4}}$

Answer 10q.

We need to all real zeros of the function

$f(x)=x^3-4x^2-25x-56$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.

The Rational Zero Theorem: If

$f(x)=a_n x^n+...+a_1 x+a_0$

has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q}=\frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:

$f(x) = x^3 - 4x^2 - 25x - 56$

Let us list the possible rational zeros.

Factors of constant term: $\pm1, \pm2, \pm4, \pm7, \pm8, \pm14, \pm28, \pm56$

Factors of leading coefficient: ±1

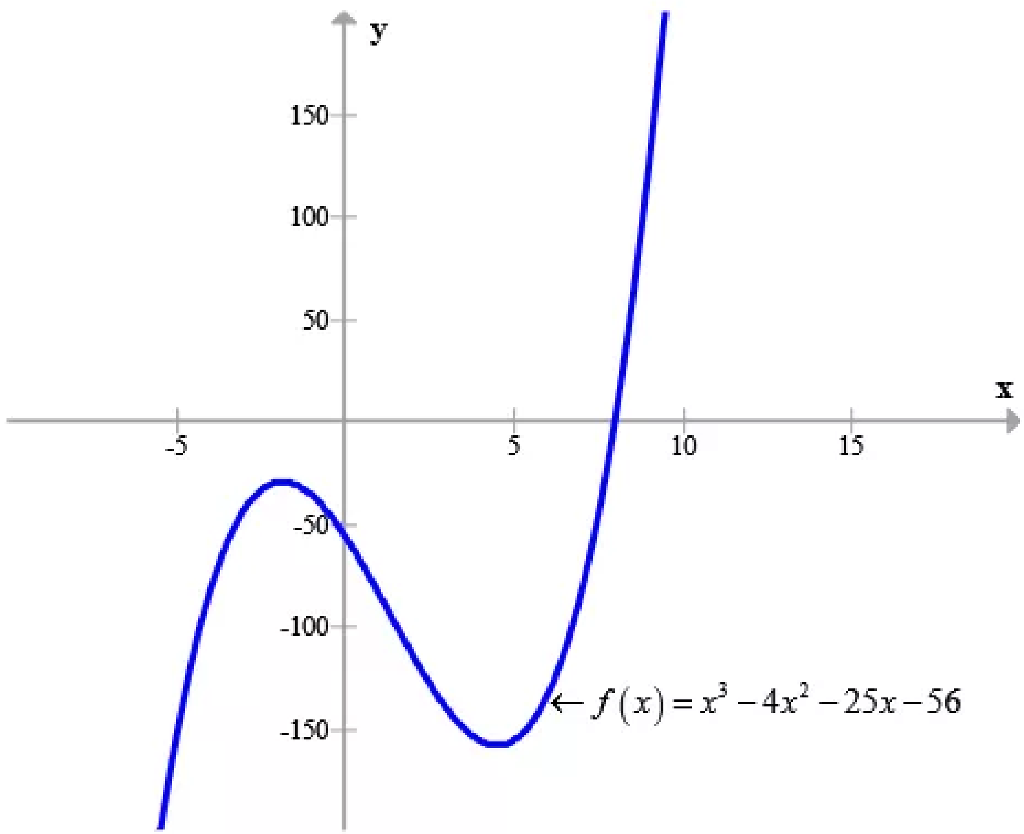
Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{7}{1}, \pm\frac{8}{1}, \pm\frac{14}{1}, \pm\frac{28}{1}, \pm\frac{56}{1}$

That is the possible rational roots are

$\pm1, \pm2, \pm4, \pm7, \pm8, \pm14, \pm28, \pm56$

Choose the reasonable values from the above list using the graph function.

Let us observe following graph.



Thus the value $x = 8$ is reasonable based on the graph.

Let us test this zero using synthetic division.

Test $x = 8$:

$$\begin{array}{c|cccc} 8 & 1 & -4 & -25 & -56 \\ & & 8 & 32 & 56 \\ \hline & 1 & 4 & 7 & 0 \end{array}$$

Here the remainder is 0 and hence $x = 8$ is a zero

Factor out a binomial using the result of synthetic division.
Because -2 is a zero, we can write

$$f(x)=(x-8)(x^2+4x+7)$$

Now consider,

$$g(x)=x^2+4x+7$$

Any zero of $g(x)=x^2+4x+7$ will also be a zero of $f(x)$.

Compute the zeros using quadratic formula.

$$x=\frac{-4\pm\sqrt{16-4(1)(7)}}{2}$$

$$=\frac{-4\pm\sqrt{16-28}}{2}$$

$$=\frac{-4\pm\sqrt{-12}}{2}$$

Thus both the zeros of $g(x)=x^2+4x+7$ are imaginary.

Thus, the zero of $f(x)$ is $\boxed{8}$

Answer 11e.

STEP 1 **List** the possible rational zeros.
The leading coefficient of the given function is 1 and the constant term is -24. Divide each factor of the constant term by each factor of leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{8}{1}, \pm\frac{12}{1}, \pm\frac{24}{1}$$

The possible rational zeros of the function are
$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12$, and ±24.

STEP 2 **Test** these zeros using synthetic division.

Test $x = 1$

$$\begin{array}{r|rrrr} 1 & 1 & -12 & 35 & -24 \\ & & 1 & -11 & 24 \\ \hline & 1 & -11 & 24 & 0 \end{array}$$

The remainder is 0 which means that 1 is a zero of $f(x)$. Thus, we have $x-1$ as one of the factors of the function. We can use this result to express $f(x)$ as a product of factors.

$f(x) = (x-1)(x^2-11x+24)$

STEP 3	**Factor** the trinomial in $f(x)$ and use the factor theorem.

$f(x) = (x - 1)(x - 8)(x - 3)$

The other two factors of $f(x)$ are $x - 8$ and $x - 3$, from which we get the two zeros 3 and 8.

Therefore, the zeros of f are 1, 3, and 8.

Answer 11q.

STEP 1 **List** the possible rational zeros.

The leading coefficient of the given function is 2 and the constant term is 4. Divide each factor of the constant term by each factor of the leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}$$

The possible rational zeros of the function are $\pm1, \pm2, \pm3, \pm4, \pm6$ and ±12.

STEP 2 **Test** these zeros using synthetic division.

Test $x = 1$

$$\begin{array}{r|rrrrr} 1 & 1 & 4 & -13 & -4 & 12 \\ & & 1 & 5 & -8 & -12 \\ \hline & 1 & 5 & -8 & -12 & 0 \end{array}$$

The remainder being 0 in the case of 1, means that 1 is a zero of $f(x)$. As a result, $x - 1$ is one of the factors of the function. We can use this result to express $f(x)$ as a product of factors.

$f(x) = (x - 1)(x^3 + 5x^2 - 8x - 12)$

STEP 3 **Factor** the polynomial in $f(x)$ and use the factor theorem.

We can repeat the steps above for $g(x) = x^3 + 5x^2 - 8x - 12$ and find its factors.

The possible rational zeros of g are again $\pm1, \pm2, \pm3, \pm4, \pm6$ and ±12.

Now, we can test these zeros using synthetic division. Take -1 for instance.

Test $x = -1$

$$\begin{array}{r|rrrr} -1 & 1 & 5 & -8 & -12 \\ & & -1 & -4 & 12 \\ \hline & 1 & 4 & -12 & 0 \end{array}$$

Since the remainder is 0, we have $x + 1$ as one of the factors of the function. Use this result to express $g(x)$ as a product of factors.

$g(x) = (x + 1)(x^2 + 4x - 12)$

Replace $x^3 + 5x^2 - 8x - 12$ with $(x + 1)(x^2 + 4x - 12)$ in $f(x)$.
$f(x) = (x - 1)(x + 1)(x^2 + 4x - 12)$

Factor $x^2 + 4x - 12$.
$f(x) = (x - 1)(x + 1)(x + 6)(x - 2)$

Since any zero of g is also a zero of f, we get the other real zeros of the function as -6, -1, and 2

Therefore, the zeros of f are -6, -1, 1, and 2.

Answer 12e.

We need to find all real zeros of
$f(x) = x^3 - 5x^2 - 22x + 56$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.
The Rational Zero Theorem: If
$f(x) = a_n x^n + \ldots + a_1 x + a_0$
has integer coefficients, then every rational zero of f has the following form:
$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:
$f(x) = x^3 - 5x^2 - 22x + 56$
Let us list the possible rational zeros.
Factors of constant term: $\pm1, \pm2, \pm4, \pm7, \pm8, \pm14, \pm28, \pm56$
Factors of leading coefficient: ±1
Possible rational zeros: $\pm1, \pm2, \pm4, \pm7, \pm8, \pm14, \pm28, \pm56$

Let us test these zeros using synthetic division.
Test $x = 1$:

1	1	-5	-22	56
		1	-4	-26
	1	-4	-26	30

Here the remainder is 30 and hence 1 is not a zero.

Test $x = -4$:

-4	1	-5	-22	56
		-4	36	-56
	1	-9	14	0

Here the remainder is 0 and hence -4 is a zero.

Because -4 is a zero, we can write

$f(x)=(x+4)(x^2-9x+14)$

Let us factor the trinomial in $f(x)$ and use the Factor Theorem.

$$\begin{aligned} f(x) &= (x+4)(x^2-9x+14) \\ &= (x+4)(x^2-7x-2x+14) \\ &= (x+4)(x-7)(x-2) \end{aligned}$$

Thus, the zeros of $f(x)$ are $\boxed{-4, 7 \text{ and } 2}$

Answer 12q.

We need to all real zeros of the function

$f(x)=4x^4-5x^2+42x-20$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.

The Rational Zero Theorem: If

$f(x)=a_n x^n+\ldots+a_1 x+a_0$

has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q}=\frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:

$f(x)=4x^4-5x^2+42x-20$

Let us list the possible rational zeros.

Factors of constant term: $\pm1, \pm2, \pm4, \pm5, \pm10, \pm20$

Factors of leading coefficient: $\pm1, \pm2, \pm4$

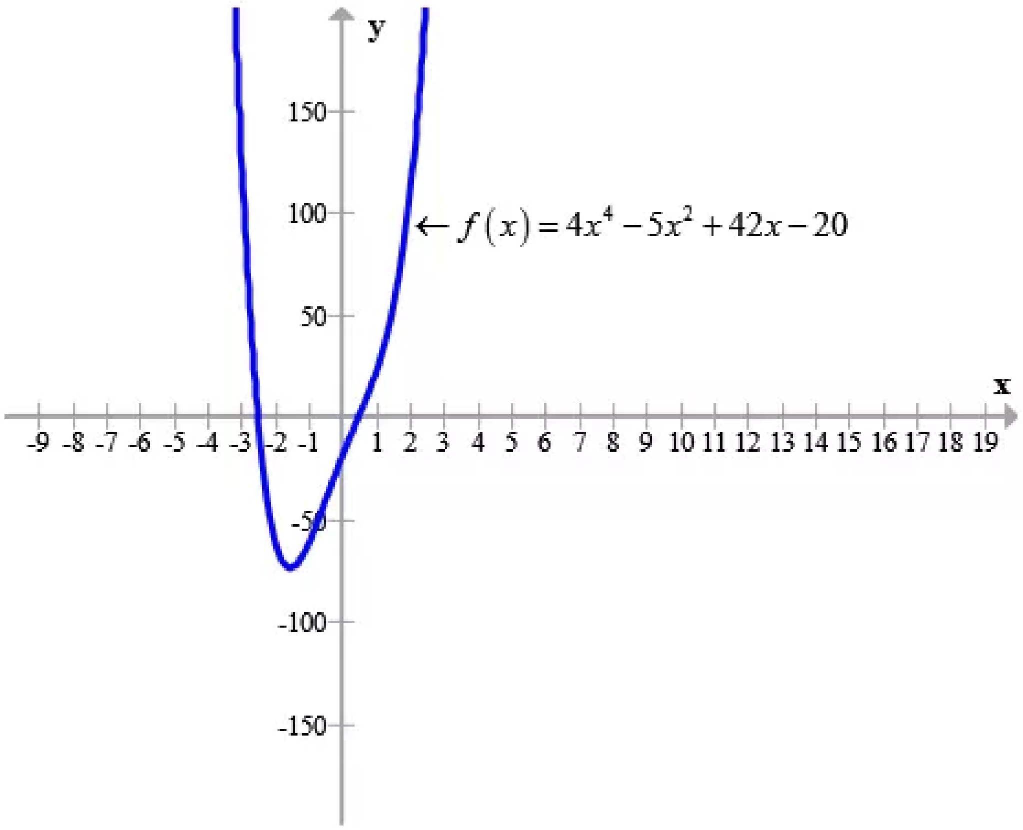
Possible rational zeros:

$$\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{5}{1}, \pm\frac{10}{1}, \pm\frac{20}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{4}{2}, \pm\frac{5}{2}, \pm\frac{10}{2}, \pm\frac{20}{2}, \pm\frac{1}{4}, \pm\frac{2}{4}, \pm\frac{4}{4}, \pm\frac{5}{4}, \pm\frac{10}{4}, \pm\frac{20}{4}$$

That is the possible rational roots are

$$\pm1, \pm2, \pm4, \pm5, \pm10, \pm20, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{4}, \pm\frac{5}{4}$$

Choose the reasonable values from the above list using the graph function.
Let us observe following graph.



Thus the values $x = \frac{1}{2}$ and $x = -\frac{5}{2}$ are reasonable based on the graph.

Let us test this zero using synthetic division.

Test $x = \frac{1}{2}$:

$\frac{1}{2}$	4	0	−5	42	−20
		2	1	−2	20
	4	2	−4	40	0

Here the remainder is 0 and hence $x = \frac{1}{2}$ is a zero

Factor out a binomial using the result of synthetic division.

Because $\frac{1}{2}$ is a zero, we can write

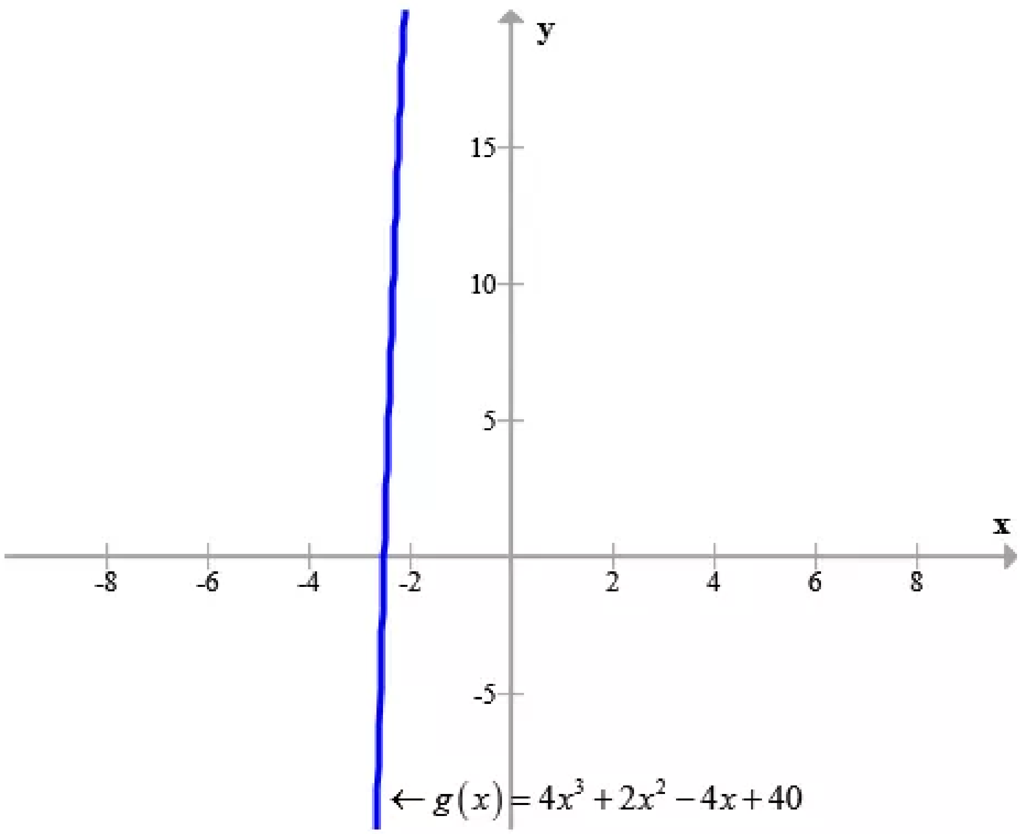
$f(x)=(2x-1)\left(4x^3+2x^2-4x+40\right)$

Now consider,

$g(x)=4x^3+2x^2-4x+40$

Any zero of $g(x)=4x^3+2x^2-4x+40$ will also be a zero of $f(x)$.

Let us observe the graph of $g(x)=4x^3+2x^2-4x+40$



Thus the value $x=-\frac{5}{2}$ is reasonable based on the graph.

Let us test this zero using synthetic division.

Test $x=-\frac{5}{2}$:

$\frac{-5}{2}$	4	2	−4	40
		−10	20	−40
	4	−8	16	0

Here the remainder is 0 and hence $x=-\frac{5}{2}$ is a zero.

Factor out a binomial using the result of synthetic division.

Because $-\frac{5}{2}$ is a zero, we can write

$$f(x)=(2x-1)(2x+5)(4x^2-8x+16)$$

Let us find the zeros of the quadratic

$4x^2-8x+16$

using quadratic formula:

$$x=\frac{8\pm\sqrt{64-4(4)(16)}}{2(4)}$$

$$=\frac{8\pm\sqrt{64-256}}{8}$$

$$=\frac{8\pm\sqrt{-192}}{8}$$

Thus, both the roots of the quadratic are imaginary.

Thus, the zeros of $f(x)$ are $\boxed{x=\frac{1}{2},\frac{-5}{2}}$

Answer 13e.

STEP 1 **List** the possible rational zeros.

The leading coefficient of the given function is 1, and the constant term is -30. Divide each factor of the constant term by each factor of leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{5}{1}, \pm\frac{6}{1}, \pm\frac{10}{1}, \pm\frac{15}{1}, \pm\frac{30}{1}$$

The possible rational zeros of the function are $\pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15,$ and ±30.

STEP 2 **Test** these zeros using synthetic division.

Test $x = 1$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -31 & -30 \\ & & 1 & 1 & -30 \\ \hline & 1 & 1 & -30 & -60 \end{array}$$

Test $x = -1$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -31 & -30 \\ & & -1 & 1 & 30 \\ \hline & 1 & -1 & -30 & 0 \end{array}$$

The remainder 0 in the case of -1 means that -1 is a zero of $g(x)$. Thus, we have $x + 1$ as one of the factors of the function. We can use this result to express $g(x)$ as a product of factors.

$g(x) = (x + 1)(x^2 - x - 30)$

STEP 3 **Factor** the trinomial in $g(x)$ and use the factor theorem.

$g(x) = (x + 1)(x - 6)(x + 5)$

The other two factors of $g(x)$ are $x - 6$ and $x + 5$, from which we get the two zeros 6 and -5.

Therefore, the zeros of g are -5, -1, and 6.

Answer 13q.

STEP 1 **Write** an equation for the volume of the square patio.
We use the fact that the volume of a square is given by the product of its area and thickness.

$$\underset{\text{(cubic feet)}}{\text{Volume}} = \underset{\text{(square feet)}}{\text{Area of square}} \cdot \underset{\text{(feet)}}{\text{Thickness}}$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$128 = x^2 \cdot (x - 15.5)$$

We get the equation $128 = x^2(x - 15.5)$.

Remove the parentheses using the distributive property.
$128 = x^3 - 15.5x^2$

Now, rewrite the equation in standard form.
$0 = x^3 - 15.5x^2 - 128$

STEP 2 **List** the possible rational solutions.
The leading coefficient of the function is 1 and the constant term is 128.

Thus, some of the possible rational zeros are
$\pm 1, \pm 2, \pm 4, \pm 6, \pm 16, \pm 32, \pm 64,$ and ± 128.

STEP 3 **Test** possible solutions. Only positive x-values make sense.

Test $x = 16$

$$\begin{array}{r|rrrr} 16 & 1 & -15.5 & 0 & -128 \\ & & 16 & 8 & 128 \\ \hline & 1 & 0.5 & 8 & 0 \end{array}$$

The remainder being 0 in the case of 16, means that 16 is a zero or solution of the function. We can use this result to express the function as a product of factors.
$0 = (x - 16)(x^2 - 0.5x + 8)$

STEP 4 **Check** for other solutions.

For this, solve $x^2 - 0.5x + 8 = 0$ using the quadratic formula. On solving, we get two values for x which are 3.05 and −2.55. Discard −2.55 because it is negative.

If $x = 16$, the thickness of the patio will then be $16 - 15.5$ or 0.5 feet. And, if $x = 3.05$, the thickness will be $3.05 - 15.5$ or -12.45 feet.

The only reasonable solution is $x = 16$. Thus, the dimensions of the square patio are 16 feet by 0.5 feet.

Answer 14e.

We need to find all real zeros of

$h(x) = x^3 + 8x^2 - 9x - 72$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.

The Rational Zero Theorem: If

$f(x) = a_n x^n + \ldots + a_1 x + a_0$

has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:

$h(x) = x^3 + 8x^2 - 9x - 72$

Let us list the possible rational zeros.

Factors of constant term: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm9, \pm12, \pm18, \pm24, \pm36, \pm72$

Factors of leading coefficient: ±1

Possible rational zeros: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm9, \pm12, \pm18, \pm24, \pm36, \pm72$

Let us test these zeros using synthetic division.

Test $x = 1$:

$$\begin{array}{r|rrrr} 1 & 1 & 8 & -9 & -72 \\ & & 1 & 9 & 0 \\ \hline & 1 & 9 & 0 & -72 \end{array}$$

Here the remainder is -72 and hence 1 is not a zero.

Test $x = 3$:

$$\begin{array}{r|rrrr} 3 & 1 & 8 & -9 & -72 \\ & & 3 & 33 & 72 \\ \hline & 1 & 11 & 24 & 0 \end{array}$$

Here the remainder is 0 and hence 3 is a zero.

Because 3 is a zero, we can write

$h(x)=(x-3)(x^2+11x+24)$

Let us factor the trinomial in $f(x)$ and use the Factor Theorem.

$$
\begin{aligned}
h(x) &= (x-3)(x^2+11x+24) \\
&= (x-3)(x^2+8x+3x+24) \\
&= (x-3)(x+8)(x+3)
\end{aligned}
$$

Thus, the zeros of $h(x)$ are $\boxed{3, -3 \text{ and } -8}$

Answer 15e.

STEP 1 **List** the possible rational zeros.

The leading coefficient of the given function is 1 and the constant term is 24. Divide each factor of the constant term by each factor of the leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{8}{1}, \pm\frac{12}{1}, \pm\frac{24}{1}$$

The possible rational zeros of the function are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12$, and ± 24.

STEP 2 **Test** these zeros using synthetic division.

Test $x = 1$

$$
\begin{array}{r|rrrrr}
1 & 1 & 7 & 26 & 44 & 24 \\
 & & 1 & 8 & 34 & 78 \\
\hline
 & 1 & 8 & 34 & 78 & 102
\end{array}
$$

Test $x = -1$

$$
\begin{array}{r|rrrrr}
-1 & 1 & 7 & 26 & 44 & 24 \\
 & & -1 & -6 & -20 & -24 \\
\hline
 & 1 & 6 & 20 & 24 & 0
\end{array}
$$

The remainder being 0 means that -1 is a zero of $h(x)$. Thus, we have $x + 1$ as one of the factors of the function. We can use this result to express $h(x)$ as a product of factors.

$h(x) = (x + 1)(x^3 + 6x^2 + 20x + 24)$

Now, we can test these zeros using synthetic division. Take 2 and -2 for instance.

Test $x = 2$

$$
\begin{array}{r|rrrr}
2 & 1 & 6 & 20 & 24 \\
 & & 2 & 16 & 72 \\
\hline
 & 1 & 8 & 36 & 96
\end{array}
$$

Test $x = -2$

$$
\begin{array}{r|rrrr}
-2 & 1 & 6 & 20 & 24 \\
 & & -2 & -8 & -24 \\
\hline
 & 1 & 4 & 12 & 0
\end{array}
$$

Since the remainder is 0 in the case of -2, we have $x + 2$ as one of the factors of the function. Use this result to express $g(x)$ as a product of factors.

$g(x) = (x + 2)(x^2 + 4x + 12)$

Replace $x^3 + 6x^2 + 20x + 24$ with $(x + 2)(x^2 + 4x + 12)$ in $h(x)$.
$h(x) = (x + 1)(x + 2)(x^2 + 4x + 12)$

Since any zero of g is also a zero of h, we get the two real zeros of the function as -2 and -1.

Therefore, the zeros of h are -2 and -1.

Answer 16e.

We need to find all real zeros of
$f(x) = x^4 - 2x^3 - 9x^2 + 10x - 24$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.
The Rational Zero Theorem: If
$f(x) = a_n x^n + ... + a_1 x + a_0$
has integer coefficients, then every rational zero of f has the following form:
$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Let us test these zeros using synthetic division.
Test $x = 4$:

4	1	−2	−9	10	−24
		4	8	−4	24
	1	2	−1	6	0

Here the remainder is 0 and hence $x = 4$ is a zero

Factor out a binomial using the result of synthetic division.
Because 4 is a zero, we can write
$f(x) = (x - 4)(x^3 + 2x^2 - x + 6)$

Now consider,
$g(x) = x^3 + 2x^2 - x + 6$
Any zero of $g(x) = x^3 + 2x^2 - x + 6$ will also be a zero of $f(x)$.
The possible rational zeros of $g(x)$ are: $\pm1, \pm2, \pm3, \pm6$

Let us find the remaining zeros of $f(x)$ by solving $x^2 - x + 2 = 0$
$$x = \frac{1 \pm \sqrt{1 - 4(1)(1)}}{2}$$
$$= \frac{1 \pm i\sqrt{3}}{2}$$
These two roots of $f(x)$ are imaginary.

Answer 17e.

STEP 1 **List** the possible rational zeros.

The leading coefficient of the given function is 1 and the constant term is 8. Divide each factor of the constant term by each factor of the leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{8}{1}$$

The possible rational zeros of the function are $\pm 1, \pm 2, \pm 4,$ and ± 8.

STEP 2 **Test** these zeros using synthetic division. Take 1 first.

Test $x = 1$

$$\begin{array}{r|rrrrr} 1 & 1 & 2 & -9 & -2 & 8 \\ & & 1 & 3 & -6 & -8 \\ \hline & 1 & 3 & -6 & -8 & 0 \end{array}$$

The remainder being 0 means that 1 is a zero of $f(x)$. Thus, we have $x - 1$ as one of the factors of the function. We can use this result to express $f(x)$ as a product of factors.

$f(x) = (x - 1)(x^3 + 3x^2 - 6x - 8)$

STEP 3 **Factor** the polynomial in $f(x)$ and use the factor theorem.

We can repeat the steps above for $g(x) = x^3 + 3x^2 - 6x - 8$ and find its factors.

The possible rational zeros of g are $\pm 1, \pm 2, \pm 4,$ and ± 8.

Now, we can test these zeros using synthetic division.

Take −1 for instance.

Test $x = -1$

$$\begin{array}{r|rrrr} -1 & 1 & 3 & -6 & -8 \\ & & -1 & -2 & 8 \\ \hline & 1 & 2 & -8 & 0 \end{array}$$

Since the remainder is 0, we have $x + 1$ as one of the factors of the function. Use this result to express $g(x)$ as a product of factors.

$g(x) = (x + 1)(x^2 + 2x - 8)$

Answer 18e.

We need to find all real zeros of

$g(x) = x^4 - 16x^2 - 40x - 25$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.

The Rational Zero Theorem: If

$f(x)=a_nx^n+...+a_1x+a_0$

has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q}=\frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:

$g(x)=x^4-16x^2-40x-25$

Let us list the possible rational zeros.

Factors of constant term: $\pm1,\pm5,\pm25$

Factors of leading coefficient: ±1

Possible rational zeros: $\pm1,\pm5,\pm25$

Factor out a binomial using the result of synthetic division.

Because 5 is a zero, we can write

$g(x)=(x-5)(x^3+5x^2+9x+5)$

Let us test this zero using synthetic division.

Test $x=-1$:

$$\begin{array}{r|rrrr} -1 & 1 & 5 & 9 & 5 \\ & & -1 & -4 & -5 \\ \hline & 1 & 4 & 5 & 0 \end{array}$$

Here the remainder is 0 and hence $x=-1$ is a zero

Synthetic division shows that $x=-1$ is a zero of $h(x)$ and

$$\begin{aligned} g(x)&=(x-5)h(x) \\ &=(x-5)(x+1)(x^2+4x+5) \end{aligned}$$

Thus, the real zeros of $g(x)$ are $\boxed{5 \text{ and } -1}$

Answer 19e.

STEP 1 **List** the possible rational zeros of f.

Divide each factor of the constant term 4 by each factor of the leading coefficient 16 to get the list of possible rational zeros.

$$x=\pm\frac{1}{1},\pm\frac{2}{1},\pm\frac{4}{1},\pm\frac{8}{1},\pm\frac{16}{1},\pm\frac{1}{2},\pm\frac{2}{2},\pm\frac{4}{2},\pm\frac{8}{2},\pm\frac{16}{2},$$
$$\pm\frac{1}{4},\pm\frac{2}{4},\pm\frac{4}{4},\pm\frac{8}{4},\pm\frac{16}{4}$$

The possible rational zeros of f are $\pm1, \pm2, \pm4, \pm8, \pm16, \pm\frac{1}{2}$, and $\pm\frac{1}{4}$.

STEP 2 **Choose** reasonable values from the list above, to check using the graph of the function.

Based on the given graph, the only value which seems to be reasonable for *f* is 1.

STEP 3 **Check** the values using synt6hetic division until a zero is found.

Test $x = 1$

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -20 & 16 \\ & & 4 & 4 & -16 \\ \hline & 4 & 4 & -16 & 0 \end{array}$$

We get the remainder as 0, which means that 1 is a zero of *f*.

STEP 4 **Factor** out a binomial using the result of the synthetic division.

Since 1 is a zero, $x - 1$ is one of the factors of the function. We can use this result to express $f(x)$ as a product of factors.

$f(x) = (x - 1)(4x^2 + 4x - 16)$

Take out 4 of the second factor.

$f(x) = (x - 1)4(x^2 + x - 4)$

Now, multiply the first factor by 4.

$f(x) = (4x - 4)(x^2 + x - 4)$

STEP 5 **Repeat** the steps above for $g(x) = x^2 + x - 4$. Any zero of *g* will also be a zero of *f*.

Since we cannot find any other real zero by using synthetic division, we will skip this step.

STEP 6 **Find** the remaining zeros of *f* by solving $x^2 + x - 4 = 0$.

Use the quadratic formula by replacing *a* and *b* with 1 and *c* with –4.

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-4)}}{2(1)}$$

Simplify.

$$x = \frac{-1 \pm \sqrt{1 + 16}}{2}$$
$$= \frac{-1 \pm \sqrt{17}}{2}$$

Therefore, the real zeros of *f* are 1 and $\frac{-1 \pm \sqrt{17}}{2}$.

Answer 20e.

We need to find all real zeros of

$f(x)=4x^3-12x^2-x+15$

Use the graph to shorten the list of possible rational zeros of the function.

Let us use the Rational Zero Theorem to find the possible rational zeros of f.

The Rational Zero Theorem: If

$f(x)=a_n x^n+...+a_1 x+a_0$

has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q}=\frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:

$f(x)=4x^3-12x^2-x+15$

Let us list the possible rational zeros.

Factors of constant term: $\pm1,\pm3,\pm5,\pm15$

Factors of leading coefficient: $\pm1,\pm2,\pm4$

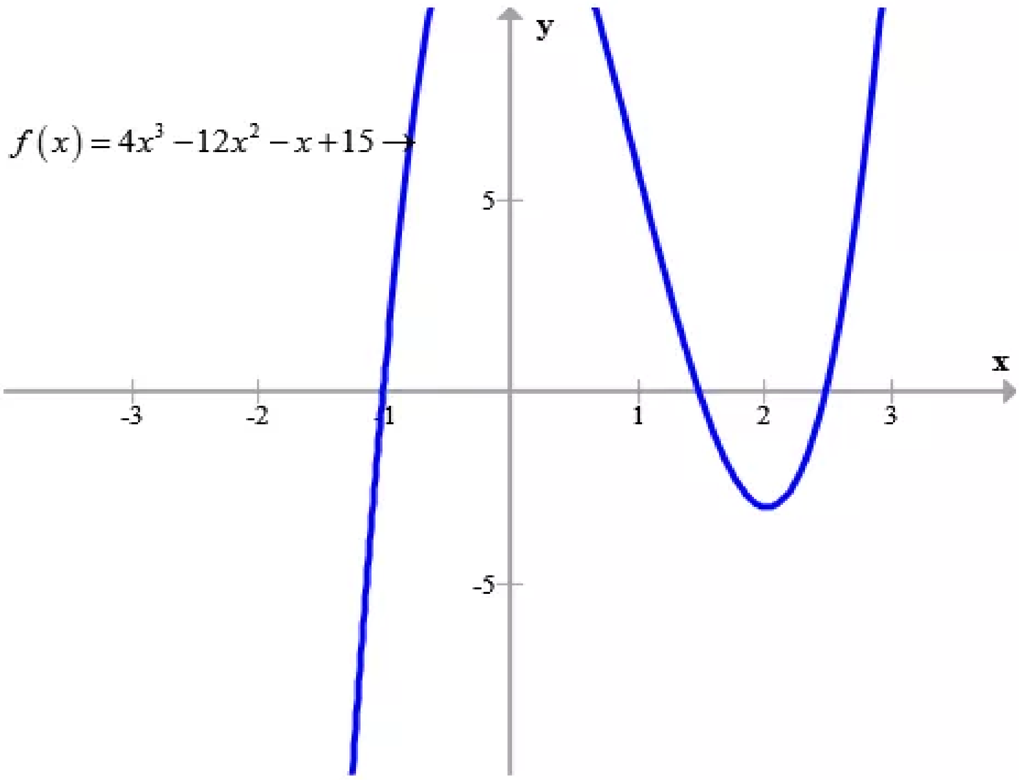
Possible rational zeros: $\pm\frac{1}{1},\pm\frac{3}{1},\pm\frac{5}{1},\pm\frac{15}{1},\pm\frac{1}{2},\pm\frac{3}{2},\pm\frac{5}{2},\pm\frac{15}{2},\pm\frac{1}{4},\pm\frac{3}{4},\pm\frac{5}{4},\pm\frac{15}{4}$

That is the possible rational roots are

$\pm1,\pm3,\pm5,\pm15,\pm\frac{1}{2},\pm\frac{3}{2},\pm\frac{5}{2},\pm\frac{15}{2},\pm\frac{1}{4},\pm\frac{3}{4},\pm\frac{5}{4},\pm\frac{15}{4}$

Choose the reasonable values from the above list using the graph function.

Let us observe following graph.



Thus the values $x=-1, x=1.5, x=2.5$ are reasonable based on the graph.

Let us test these zeros using synthetic division.

Test $x = -1$:

$$\begin{array}{r|rrrr} -1 & 4 & -12 & -1 & 15 \\ & & -4 & 16 & -15 \\ \hline & 4 & -16 & 15 & 0 \end{array}$$

Here the remainder is 0 and hence $x = -1$ is a zero

Factor out a binomial using the result of synthetic division.

Because -1 is a zero, we can write

$f(x) = (x+1)(4x^2 - 16x + 15)$

Now consider,

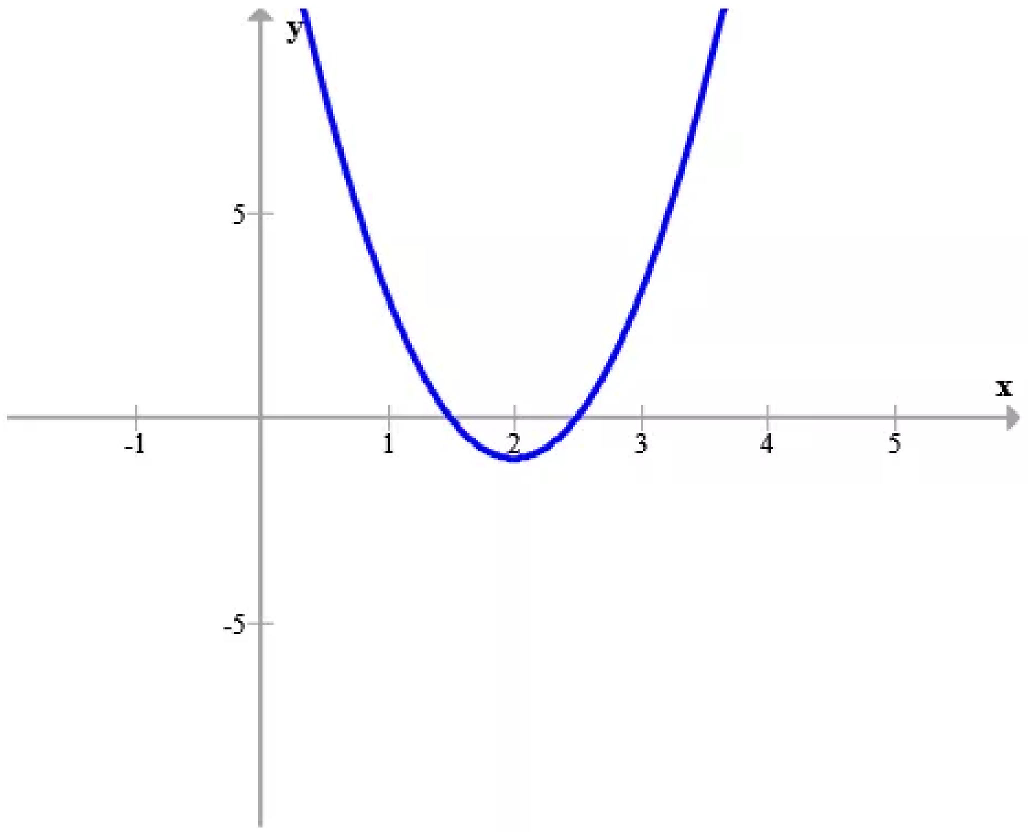
$g(x) = 4x^2 - 16x + 15$

Any zero of $g(x) = 4x^2 - 16x + 15$ will also be a zero of $f(x)$.

The possible rational zeros of $g(x)$ are:

$\pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{5}{1}, \pm\frac{15}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{15}{4}$

Let us observe the following graph of $g(x)$:



The graph of g shows that $x = \frac{3}{2}$ may be a zero. Synthetic division shows that $x = \frac{3}{2}$ is a zero of $g(x)$ and

$$\begin{aligned} f(x) &= (x+1)(4x^2 - 16x + 15) \\ &= (x+1)(2x-3)(2x-5) \end{aligned}$$

Thus, the zeros of $f(x)$ are $\boxed{-1, \frac{3}{2} \text{ and } \frac{5}{2}}$

Answer 21e.

STEP 1 **List** the possible rational zeros of *f*.

Divide each factor of the constant term 15 by each factors of the leading coefficient 6 to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{5}{1}, \pm\frac{15}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{3}, \pm\frac{3}{3},$$
$$\pm\frac{5}{3}, \pm\frac{15}{3}, \pm\frac{1}{6}, \pm\frac{3}{6}, \pm\frac{5}{6}, \pm\frac{15}{6}$$

The possible rational zeros of *f* are

$$\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}, \text{ and } \pm\frac{5}{6}.$$

STEP 2 **Choose** reasonable values from the list above, to check using the graph of the function.

Based on the given graph, the values which seem to be reasonable for *f* are $-3, -\frac{5}{3}$, and $\frac{1}{3}$.

STEP 3 **Check** the values using synthetic division until a zero is found.

Test $x = -3$

−3	6	25	16	−15
		−18	−21	15
	6	7	−5	0

We get the remainder as 0, which means that −3 is a zero of *f*.

STEP 4 **Factor** out a binomial using the result of the synthetic division.

Since −3 is a zero, $x - (-3)$ or $x + 3$ is one of the factors of the function.

We can use this result to express $f(x)$ as a product of factors.

$f(x) = (x + 3)(6x^2 + 7x - 5)$

Factor the second factor.

$f(x) = (x + 3)(2x - 1)(3x + 5)$

On applying the factor theorem, we get the other zeros of the function as $-\frac{5}{3}$ and $\frac{1}{2}$.

STEP 5, STEP 6 These steps can be skipped since we have already found the possible rational zeros of the function in the steps above.

Therefore, the real zeros of f are $-3, -\frac{5}{3},$ and $\frac{1}{2}$.

Answer 22e.

We need to find all real zeros of

$$f(x) = -3x^3 + 20x^2 - 36x + 16$$

Use the graph to shorten the list of possible rational zeros of the function.

Let us use the Rational Zero Theorem to find the possible rational zeros of f.

The Rational Zero Theorem: If

$$f(x) = a_n x^n + \ldots + a_1 x + a_0$$

has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:

$$f(x) = -3x^3 + 20x^2 - 36x + 16$$

Let us list the possible rational zeros.

Factors of constant term: $\pm1, \pm2, \pm4, \pm16$

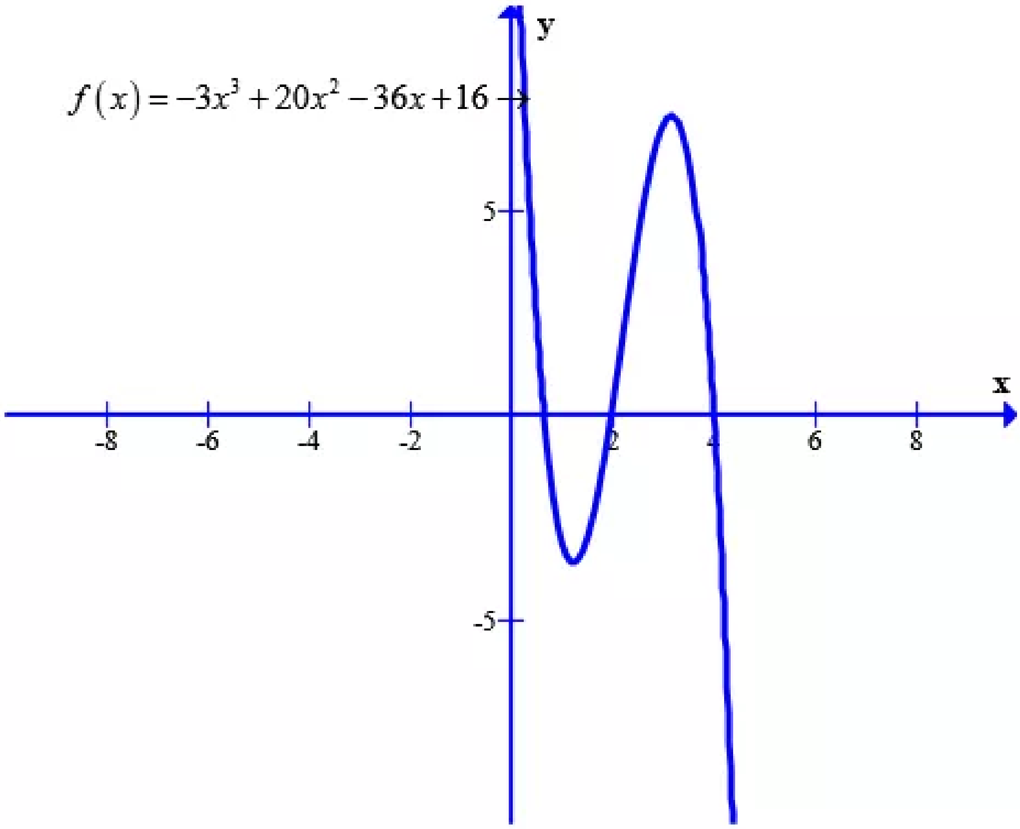
Factors of leading coefficient: $\pm1, \pm3$

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{16}{1}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{16}{3}$

That is the possible rational roots are

$$\pm1, \pm2, \pm4, \pm16, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{16}{3}$$

Choose the reasonable values from the above list using the graph function.
Let us observe following graph.



Thus the values $x = 2, x = 4$ are reasonable based on the graph.

Let us test these zeros using synthetic division.

Test $x = 2$:

2	-3	20	-36	16
		-6	28	-16
	-3	14	-8	0

Here the remainder is 0 and hence $x = 2$ is a zero

Factor out a binomial using the result of synthetic division.
Because 2 is a zero, we can write

$$f(x) = (x-2)(-3x^2 + 14x - 8)$$

Now consider,

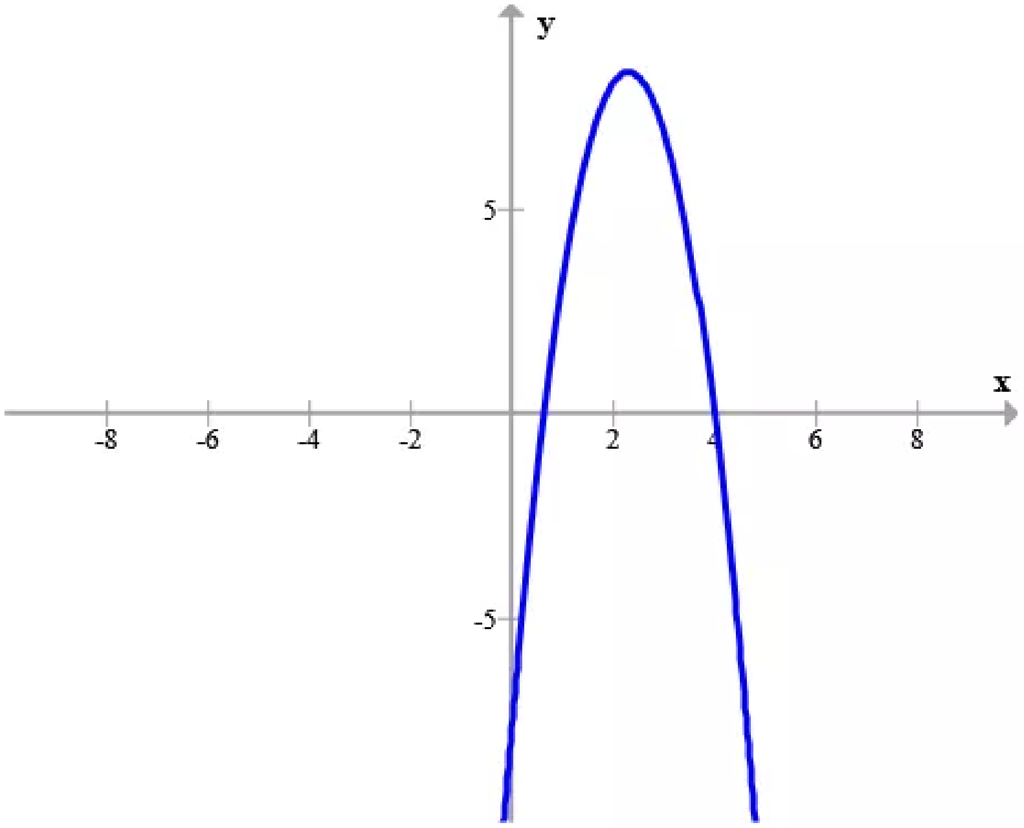
$$g(x) = -3x^2 + 14x - 8$$

Any zero of $g(x) = -3x^2 + 14x - 8$ will also be a zero of $f(x)$.

The possible rational zeros of $g(x)$ are:

$$\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{8}{1}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$$

Let us observe the following graph of $g(x)$:



The graph of g shows that $x=\frac{2}{3}$ may be a zero. Synthetic division shows that $x=\frac{2}{3}$ is a zero of $g(x)$ and

$$f(x)=(x-2)(-3x^2+14x-8)$$
$$=(x-2)(3x-2)(-3x+12)$$

Thus, the zeros of $f(x)$ are $\boxed{2, \frac{2}{3} \text{ and } 4}$

Answer 23e.

The Rational Zero Theorem states that if $f(x) = a_n x^n + \ldots + a_1 x + a_0$ has integer coefficients, then every rational zero of f will have the form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coeffeicient } a_n}.$$

The constant term in the given function is 9, and the leading coefficient is 2.

Factors of the constant term: $\pm1, \pm3, \pm9$

Factors of the leading coefficient: $\pm1, \pm2$

Divide the factors of the constant term by the factors of leading coefficient to get the list of possible rational zeros.

$$\text{Possible rational zeros} = \pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{9}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$$
$$= \pm 1, \pm 3, \pm 9, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$$

By comparing the given choices with the above list, it can be seen that $\frac{5}{2}$ cannot be a possible zero of f.

Therefore, choice **C** does not represent a possible rational zero of the function.

Answer 24e.

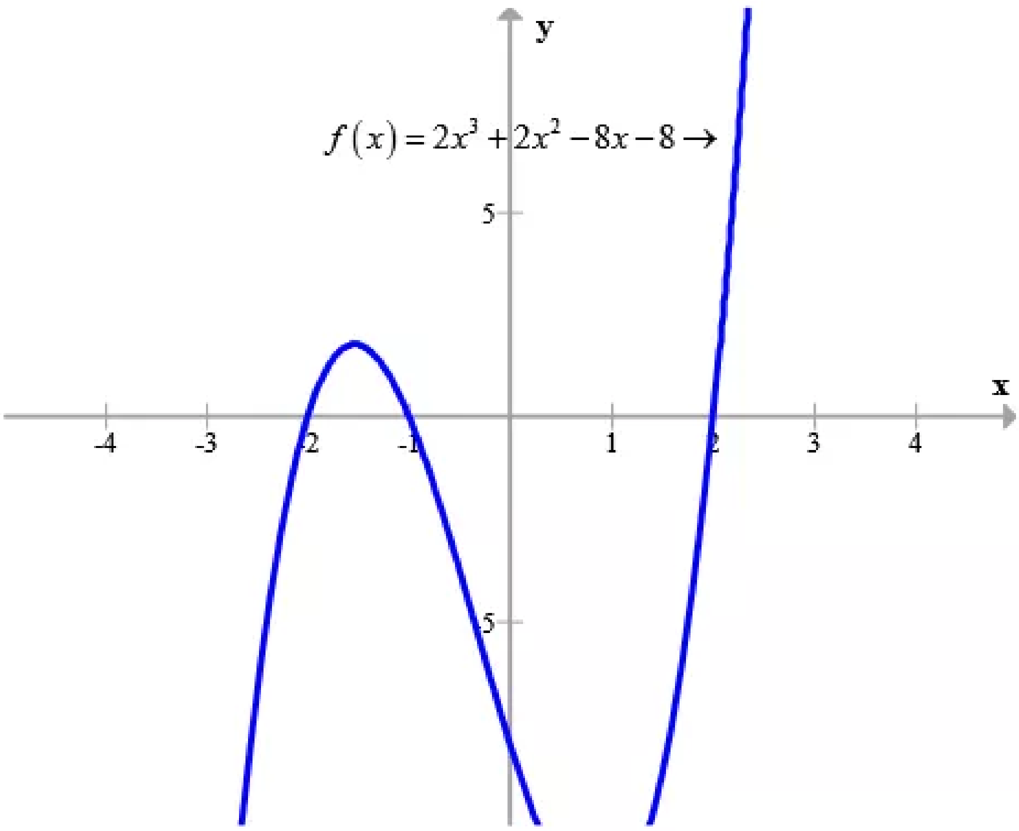
We need to find all real zeros of
$f(x) = 2x^3 + 2x^2 - 8x - 8$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.
The Rational Zero Theorem: If
$f(x) = a_n x^n + \ldots + a_1 x + a_0$
has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:
$f(x) = 2x^3 + 2x^2 - 8x - 8$
Let us list the possible rational zeros.
Factors of constant term: $\pm 1, \pm 2, \pm 4, \pm 8$
Factors of leading coefficient: $\pm 1, \pm 2$
Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{8}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{4}{2}, \pm\frac{8}{2}$
That is the possible rational roots are
$\pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{2}$

Choose the reasonable values from the above list using the graph function.
Let us observe following graph.



Thus the values $x = -2, x = -1, x = 2$ are reasonable based on the graph.

Let us test these zeros using synthetic division.
Test $x = -2$:

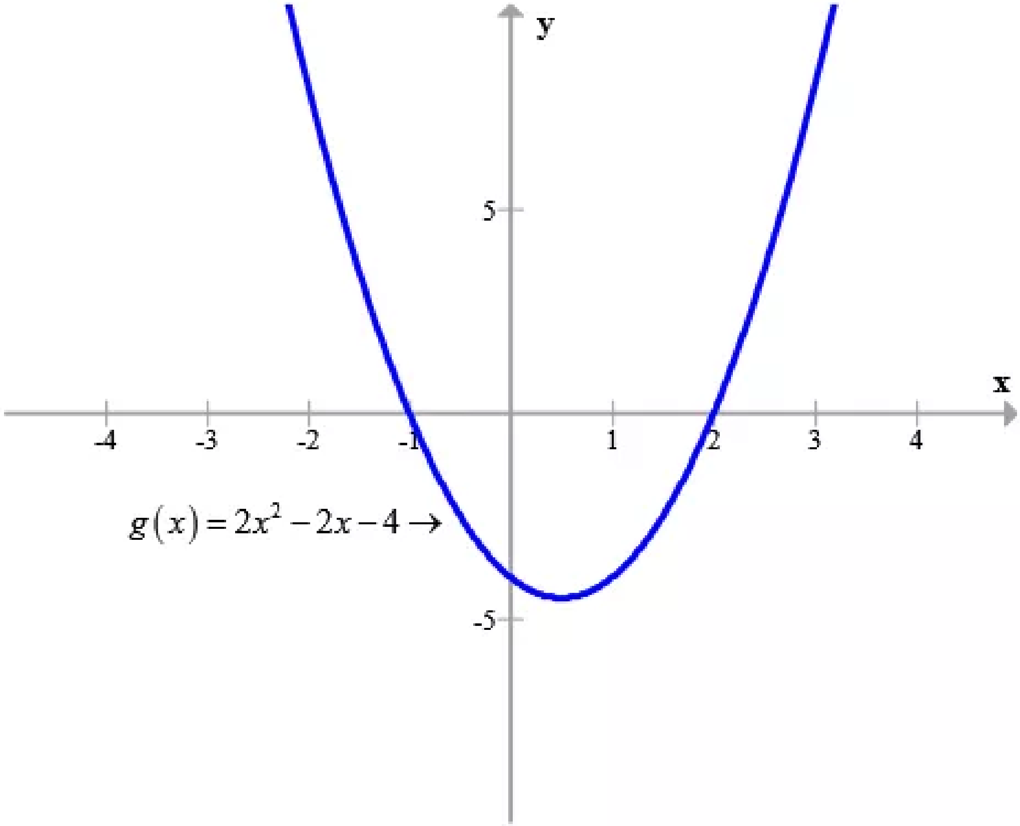
−2	2	2	−8	−8
		−4	4	8
	2	−2	−4	0

Here the remainder is 0 and hence $x = -2$ is a zero

Factor out a binomial using the result of synthetic division.
Because -2 is a zero, we can write
$f(x) = (x+2)(2x^2 - 2x - 4)$

Now consider,
$g(x) = 2x^2 - 2x - 4$
Any zero of $g(x) = 2x^2 - 2x - 4$ will also be a zero of $f(x)$.
The possible rational zeros of $g(x)$ are:
$\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$

Let us observe the following graph of $g(x)$:



The graph of g shows that $x=-1$ may be a zero. Synthetic division shows that $x=-1$ is a zero of $g(x)$ and

$$\begin{aligned} f(x) &= (x+2)g(x) \\ &= (x+2)(x+1)(x-2) \end{aligned}$$

Thus, the zeros of $f(x)$ are $\boxed{-2, 2 \text{ and } -1}$

Answer 25e.

STEP 1 **List** the possible rational zeros.
The leading coefficient of the given function is 2 and the constant term is 9. Divide each factor of the constant term by each factor of the leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{9}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$$

The possible rational zeros of the function are

$\pm1, \pm3, \pm9, \pm\frac{1}{2}, \pm\frac{3}{2}$, and $\pm\frac{9}{2}$.

STEP 2 Test these zeros using synthetic division.

Test $x = 1$

1	2	−7	0	9
		2	−5	−5
	2	−5	−5	4

Test $x = -1$

−1	2	−7	0	9
		−2	9	−9
	2	−9	9	0

The remainder is 0 in the case of -1, which means that -1 is a zero of $g(x)$. Thus, we have $x - (-1)$ or $x + 1$ as one of the factors of the function. We can use this result to express $g(x)$ as a product of factors.
$g(x) = (x + 1)(2x^2 - 9x + 9)$

STEP 3 **Factor** the trinomial in $g(x)$ and use the factor theorem.
$g(x) = (x + 1)(2x - 3)(x - 3)$

On solving $2x - 3 = 0$ and $x - 3 = 0$, we get two other zeros $\frac{3}{2}$ and 3.

Therefore, the zeros of g are $-1, \frac{3}{2}$, and 3.

Answer 26e.

We need to find all real zeros of
$h(x) = 2x^3 - 3x^2 - 14x + 15$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.
The Rational Zero Theorem: If
$f(x) = a_n x^n + ... + a_1 x + a_0$
has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:
$h(x) = 2x^3 - 3x^2 - 14x + 15$

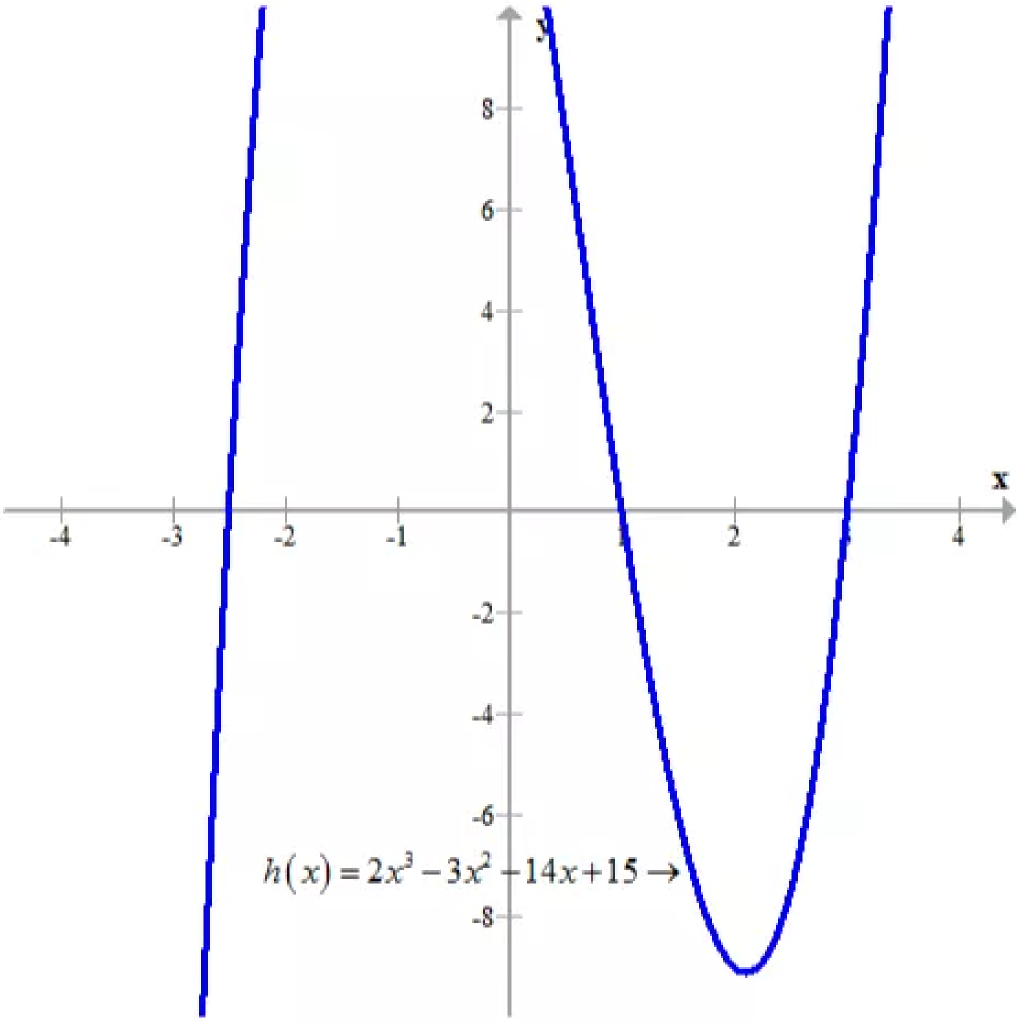
Let us list the possible rational zeros.
Factors of constant term: $\pm 1, \pm 3, \pm 5, \pm 15$
Factors of leading coefficient: $\pm 1, \pm 2$

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{5}{1}, \pm\frac{15}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

That is the possible rational roots are

$\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

Choose the reasonable values from the above list using the graph function.
Let us observe following graph.



Thus the values $x = -\frac{5}{2}, x = 1, x = 3$ are reasonable based on the graph.

Let us test these zeros using synthetic division.

Test $x = 1$:

$$\begin{array}{c|cccc} 1 & 2 & -3 & -14 & 15 \\ & & 2 & -1 & -15 \\ \hline & 2 & -1 & -15 & 0 \end{array}$$

Here the remainder is 0 and hence $x = 1$ is a zero

Factor out a binomial using the result of synthetic division.
Because 1 is a zero, we can write

$$h(x) = (x-1)(2x^2 - x - 15)$$

Now consider,

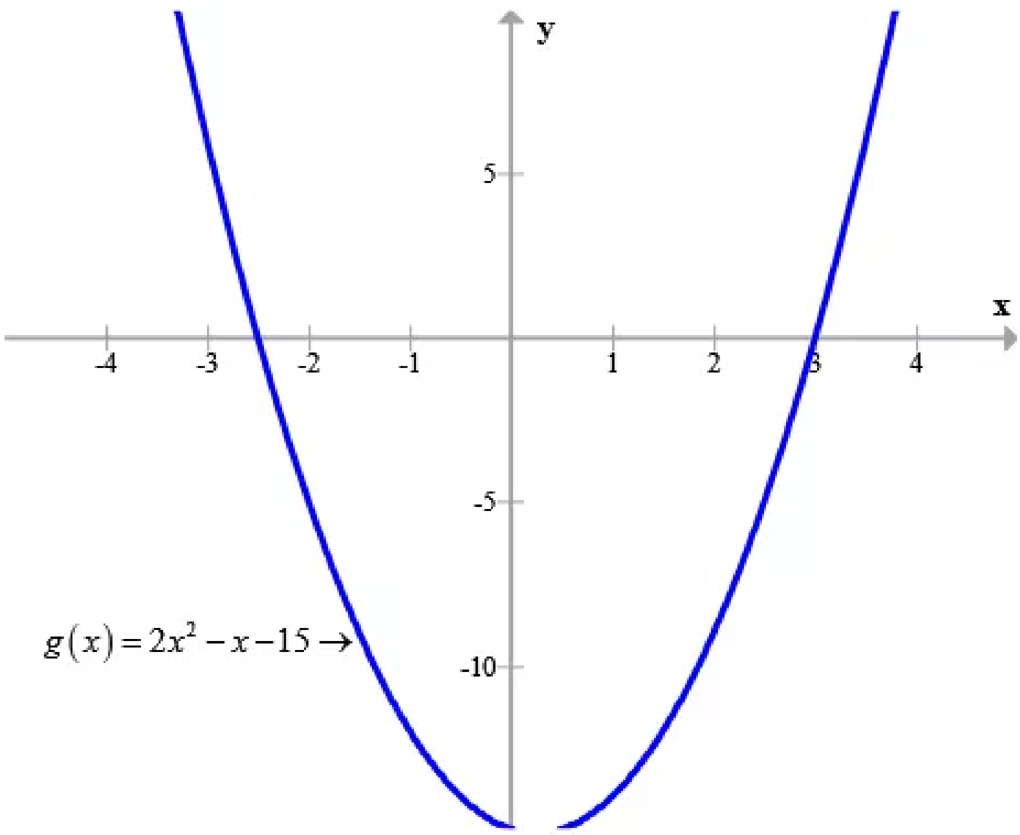
$g(x) = 2x^2 - x - 15$

Any zero of $g(x) = 2x^2 - x - 15$ will also be a zero of $h(x)$.

The possible rational zeros of $g(x)$ are:

$\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$

Let us observe the following graph of $g(x)$:



The graph of g shows that $x = 3$ may be a zero. Synthetic division shows that $x = 3$ is a zero of $g(x)$ and

$$h(x) = (x-1)\,g(x)$$
$$= (x-1)(x-3)(5x+2)$$

Thus, the zeros of $h(x)$ are $\boxed{1, 3 \text{ and } -\frac{5}{2}}$

Answer 27e.

STEP 1 **List** the possible rational zeros.
The leading coefficient of the given function is 3 and the constant term is −12. Divide each factor of the constant term by each factor of the leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{3}{3}, \pm\frac{4}{3}, \pm\frac{6}{3}, \pm\frac{12}{3}$$

The possible rational zeros of the function are
$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{3}, \pm\frac{2}{3}$, and $\pm\frac{4}{3}$.

STEP 2 **Test** these zeros using synthetic division. Let us test the fractions first.

Test $x = \frac{1}{3}$

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & 4 & -35 & -12 \\ & & 1 & \frac{5}{3} & -\frac{100}{3} \\ \hline & 3 & 5 & -\frac{100}{3} & -\frac{136}{3} \end{array}$$

Test $x = -\frac{1}{3}$

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & 4 & -35 & -12 \\ & & -1 & -1 & 12 \\ \hline & 3 & 3 & -36 & 0 \end{array}$$

The remainder is 0 in the case of $-\frac{1}{3}$, which means that $-\frac{1}{3}$ is a zero of $f(x)$. Thus, we have $x + \frac{1}{3}$ as one of the factors of the function. We can use this result to express $f(x)$ as a product of factors.

$$f(x) = \left(x + \frac{1}{3}\right)\left(3x^2 + 3x - 36\right)$$

STEP 3 **Factor** the trinomial in $f(x)$ and use the factor theorem.
Take out 3 from the second factor.

$$f(x) = \left(x + \frac{1}{3}\right)3\left(x^2 + x - 12\right)$$

Multiply the first factor by 3.
$f(x) = (3x + 1)(x^2 + x - 12)$

Now, factor the trinomial $x^2 + x - 12$.
$f(x) = (3x + 1)(x + 4)(x - 3)$

On solving $x + 4 = 0$ and $x - 3 = 0$, we get two other zeros -4 and 3.

Therefore, the zeros of f are $-4, -\frac{1}{3}$, and 3.

Answer 28e.

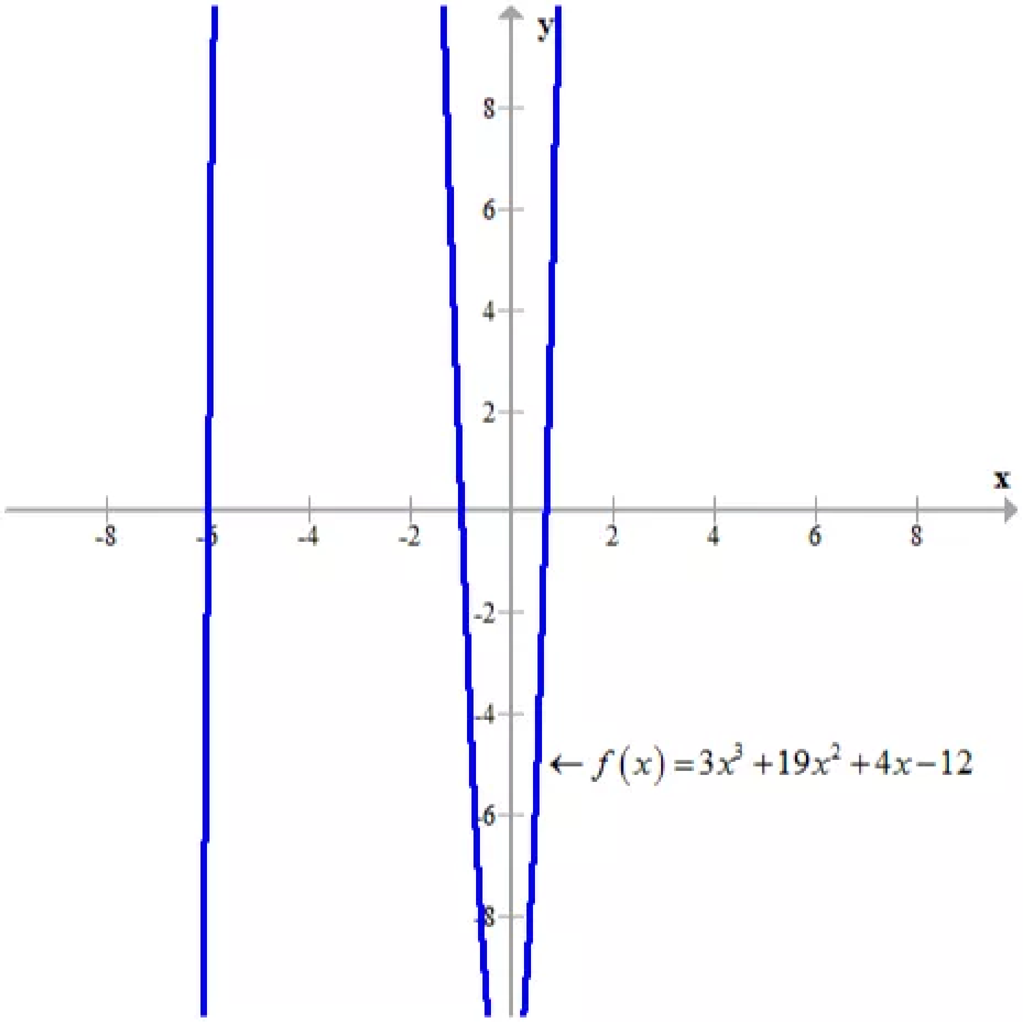
We need to find all real zeros of
$f(x) = 3x^3 + 19x^2 + 4x - 12$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.
The Rational Zero Theorem: If
$f(x) = a_n x^n + \ldots + a_1 x + a_0$
has integer coefficients, then every rational zero of f has the following form:
$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:
$f(x) = 3x^3 + 19x^2 + 4x - 12$
Let us list the possible rational zeros.
Factors of constant term: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$
Factors of leading coefficient: $\pm1, \pm3$
Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{3}{3}, \pm\frac{4}{3}, \pm\frac{6}{3}, \pm\frac{12}{3}$

That is the possible rational roots are
$\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

Choose the reasonable values from the above list using the graph function.
Let us observe following graph.



Thus the values $x=-6, x=-1, x=0.66667$ are reasonable based on the graph.

Let us test these zeros using synthetic division.
Test $x=1$:

$$\begin{array}{r|rrrr} -1 & 3 & 19 & 4 & -12 \\ & & -3 & -16 & 12 \\ \hline & 3 & 16 & -12 & 0 \end{array}$$

Here the remainder is 0 and hence $x=-1$ is a zero

Factor out a binomial using the result of synthetic division.
Because 1 is a zero, we can write
$f(x)=(x+1)(3x^2+16x-12)$

Now consider,

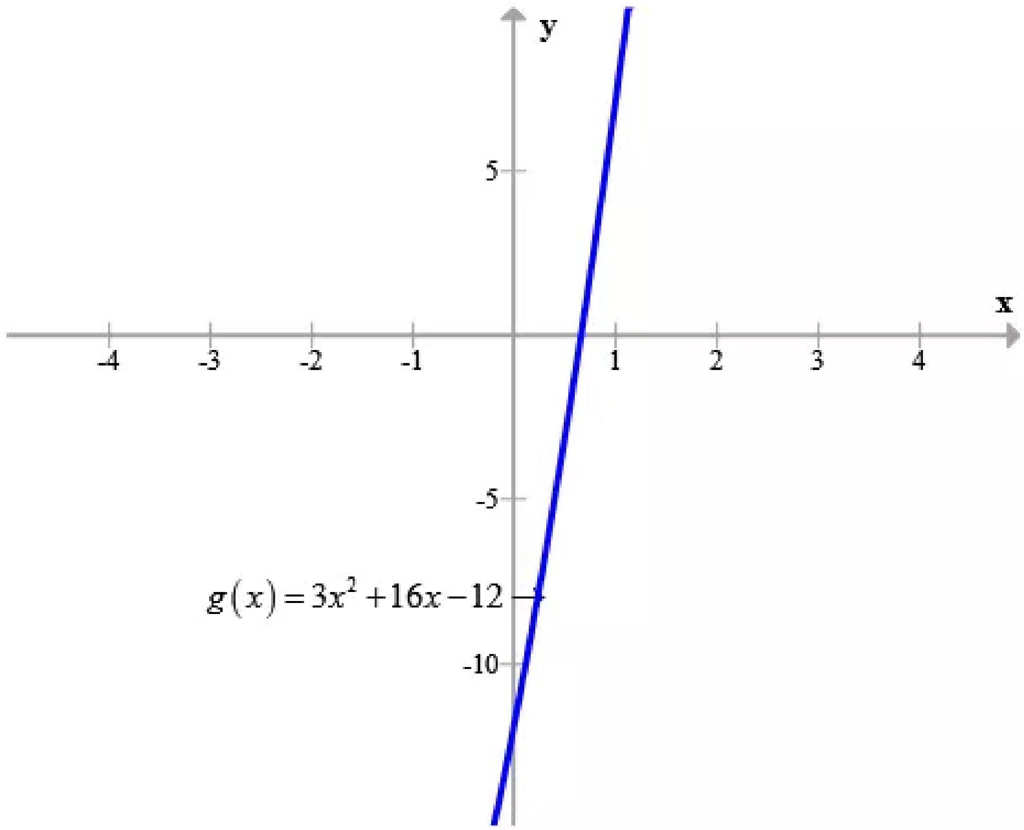
$g(x)=3x^2+16x-12$

Any zero of $g(x)=3x^2+16x-12$ will also be a zero of $f(x)$.

The possible rational zeros of $g(x)$ are:

$\pm1,\pm2,\pm3,\pm4,\pm6,\pm12,\pm\frac{1}{3},\pm\frac{2}{3},\pm\frac{4}{3}$

Let us observe the following graph of $g(x)$:



The graph of g shows that $x=\frac{2}{3}$ may be a zero. Synthetic division shows that $x=\frac{2}{3}$ is a zero of $g(x)$ and

$$f(x)=(x+1)g(x)$$
$$=(x+1)(3x-2)(3x+18)$$

Thus, the zerors of $f(x)$ are $\boxed{-1,\frac{2}{3}\text{ and }-6}$

Answer 29e.

STEP 1 **List** the possible rational zeros.
The leading coefficient of the given function is 3 and the constant term is −14. Divide each factor of the constant term by each factor of the leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{7}{1}, \pm\frac{14}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{7}{2}, \pm\frac{14}{2}$$

The possible rational zeros of the function are $\pm1, \pm2, \pm7, \pm14, \pm\frac{1}{2}, \pm\frac{7}{2}$.

STEP 2 **Test** these zeros using synthetic division. Let us test the last fraction first.

Test $x = \frac{7}{2}$

$$\begin{array}{c|cccc} \frac{7}{2} & 2 & 5 & -11 & -14 \\ & & 7 & 42 & \frac{217}{2} \\ \hline & 2 & 12 & 31 & \frac{189}{2} \end{array}$$

Test $x = -\frac{7}{2}$

$$\begin{array}{c|cccc} -\frac{7}{2} & 2 & 5 & -11 & -14 \\ & & -7 & 7 & 14 \\ \hline & 2 & -2 & -4 & 0 \end{array}$$

The remainder is 0 in the case of $-\frac{7}{2}$, which means that $-\frac{7}{2}$ is a zero of $g(x)$. Thus, we have $x + \frac{7}{2}$ as one of the factors of the function. We can use this result to express $f(x)$ as a product of factors.

$$g(x) = \left(x + \frac{7}{2}\right)\left(2x^2 - 2x - 4\right)$$

STEP 3 **Factor** the trinomial in $g(x)$ and use the factor theorem.
Take out 3 from the second factor.

$$g(x) = \left(x + \frac{7}{2}\right)2\left(x^2 - x - 2\right)$$

Multiply the first factor by 2.
$g(x) = (2x + 7)(x^2 + x - 2)$

Now, factor the trinomial $x^2 + x - 2$.
$g(x) = (2x + 7)(x + 2)(x - 1)$

On solving $x + 2 = 0$ and $x - 1 = 0$, we get the other two zeros -2 and 1.

Therefore, the real zeros of f are $-\frac{7}{2}, -2,$ and 1.

Answer 30e.

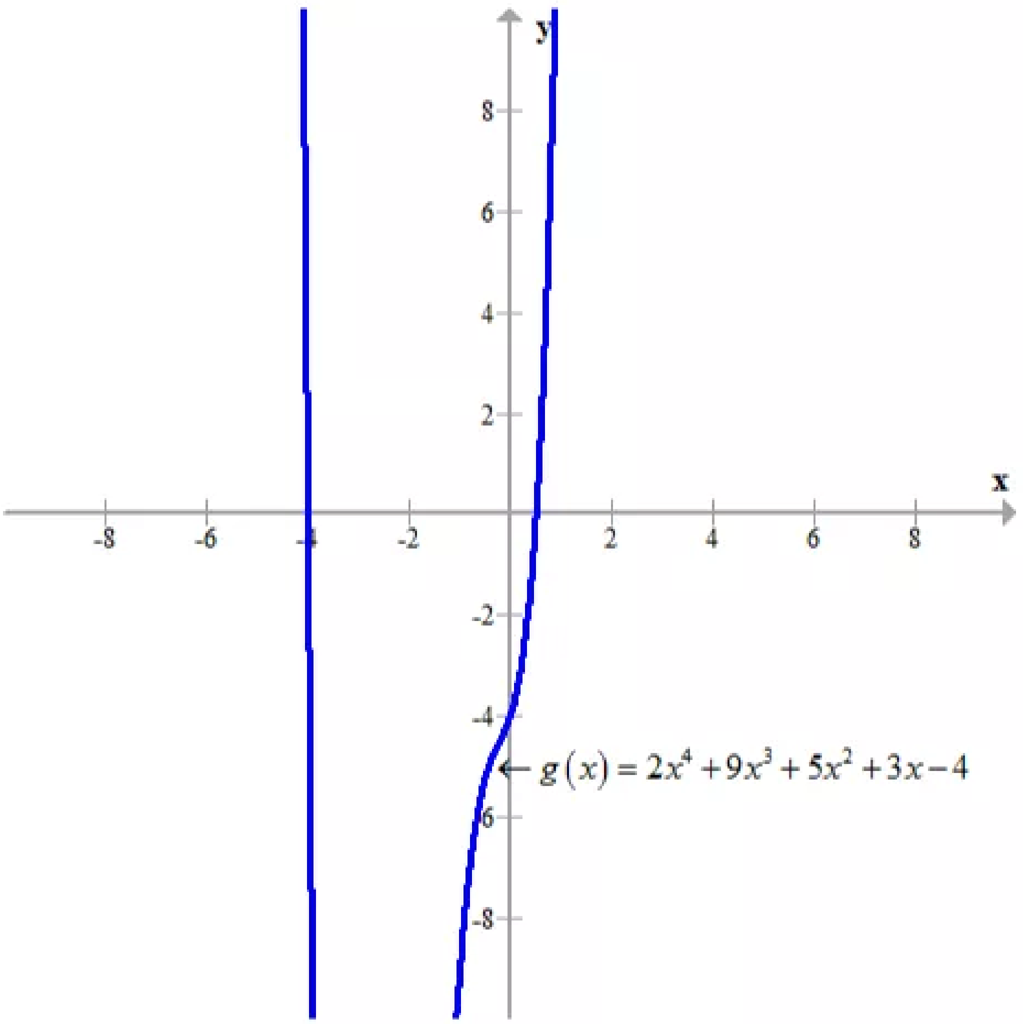
We need to find all real zeros of
$g(x) = 2x^4 + 9x^3 + +5x^2 + 3x - 4$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.
The Rational Zero Theorem: If
$f(x) = a_n x^n + \ldots + a_1 x + a_0$
has integer coefficients, then every rational zero of f has the following form:
$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:
$g(x) = 2x^4 + 9x^3 + 5x^2 + 3x - 4$
Let us list the possible rational zeros.
Factors of constant term: $\pm1, \pm2, \pm4$
Factors of leading coefficient: $\pm1, \pm2$
Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{4}{2}$

That is the possible rational roots are
$\pm1, \pm2, \pm4, \pm\frac{1}{2}$

Choose the reasonable values from the above list using the graph function.
Let us observe following graph.



Thus the values $x=-4, x=0.5$ are reasonable based on the graph.

Let us test these zeros using synthetic division.
Test $x=-4$:

-4	2	9	5	3	-4
		-8	-4	-4	4
	2	1	1	-1	0

Here the remainder is 0 and hence $x=-4$ is a zero

Factor out a binomial using the result of synthetic division.
Because -4 is a zero, we can write
$g(x)=(x+4)(2x^3+x^2+x-1)$

Now consider,

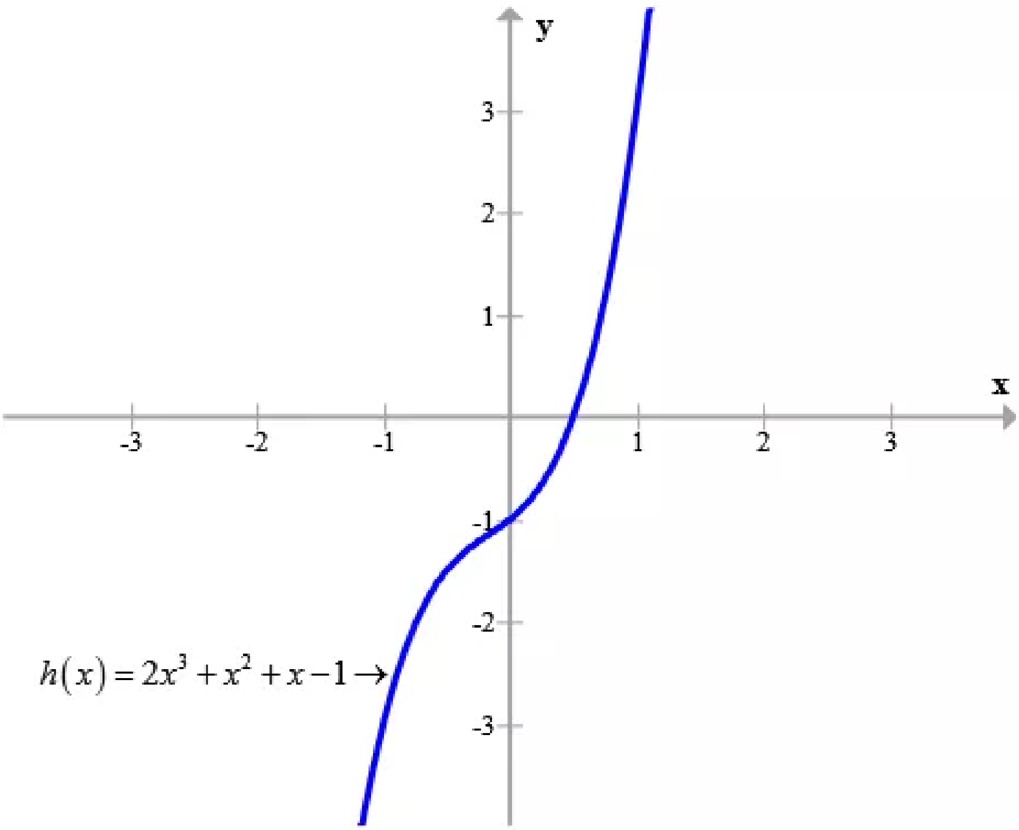
$h(x) = 2x^3 + x^2 + x - 1$

Any zero of $h(x) = 2x^3 + x^2 + x - 1$ will also be a zero of $g(x)$.

The possible rational zeros of $h(x)$ are:

$\pm 1, \pm \frac{1}{2}$

Let us observe the following graph of $h(x)$:



The graph of g shows that $x = \frac{1}{2}$ may be a zero.

Synthetic division shows that $x = \frac{1}{2}$ is a zero of $h(x)$ and

$$\begin{aligned} g(x) &= (x+4)h(x) \\ &= (x+4)(2x-1)(2x^2+2x+2) \end{aligned}$$

The zeros of the equation $f(x) = (2x^2 + 2x + 2)$ are imaginary.

Thus, the zeros of $f(x)$ are $\boxed{-4 \text{ and } \frac{1}{2}}$

Answer 31e.

STEP 1 **List** the possible rational zeros.

The leading coefficient of the given function is 2 and the constant term is 4. Divide each factor of the constant term by each factor of the leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{4}{2}$$

The possible rational zeros of the function are $\pm 1, \pm 2, \pm 4$, and $\pm\frac{1}{2}$.

STEP 2 **Test** these zeros using synthetic division.

Test $x = 1$

$$\begin{array}{r|rrrrr} 1 & 2 & -1 & -7 & 4 & -4 \\ & & 2 & 1 & -6 & -2 \\ \hline & 2 & 1 & -6 & -2 & -6 \end{array}$$

Test $x = -2$

$$\begin{array}{r|rrrrr} -2 & 2 & -1 & -7 & 4 & -4 \\ & & -4 & 10 & -6 & 4 \\ \hline & 2 & -5 & 3 & -2 & 0 \end{array}$$

The remainder being 0 in the case of -2, means that -2 is a zero of $h(x)$. Thus, we have $x - (-2)$ or $x + 2$ as one of the factors of the function. We can use this result to express $h(x)$ as a product of factors.

$h(x) = (x + 2)(2x^3 - 5x^2 + 3x - 2)$

STEP 3 **Factor** the polynomial in $h(x)$ and use the factor theorem.

We can repeat the steps above for $g(x) = 2x^3 - 5x^2 + 3x - 2$ and find its factors.

The possible rational zeros of g are ± 1 and ± 2.

Now, we can test these zeros using synthetic division. Take 2 for instance.

Test $x = 2$

$$\begin{array}{r|rrrr} 2 & 2 & -5 & 3 & -2 \\ & & 4 & -2 & 2 \\ \hline & 2 & -1 & 1 & 0 \end{array}$$

Since the remainder is 0, we have $x - 2$ as one of the factors of the function. Use this result to express $g(x)$ as a product of factors.

$g(x) = (x - 2)(2x^2 - x + 1)$

Replace $2x^3 - 5x^2 + 3x - 2$ with $(x - 2)(2x^2 - x + 1)$ in $h(x)$.
$h(x) = (x + 2)(x - 2)(2x^2 - x + 1)$

Another zero of h is 2. We can find the remaining zeros of f by solving $2x^2 - x + 1 = 0$ using the quadratic formula.
Replacing a with 2, b with -1, and c with 1.

$$x = \frac{1 \pm \sqrt{1^2 - 4(2)(1)}}{2(2)}$$
$$= \frac{1 \pm \sqrt{-7}}{4}$$
$$= \frac{1 \pm i\sqrt{7}}{4}$$

We seek only the real zeros of h, and hence $\frac{1 \pm i\sqrt{7}}{4}$ cannot be considered.

Therefore, the zeros of h are -2 and 2.

Answer 32e.

We need to find all real zeros of
$h(x) = 3x^4 - 6x^3 - 32x^2 + 35x - 12$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.
The Rational Zero Theorem: If
$f(x) = a_n x^n + \ldots + a_1 x + a_0$
has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

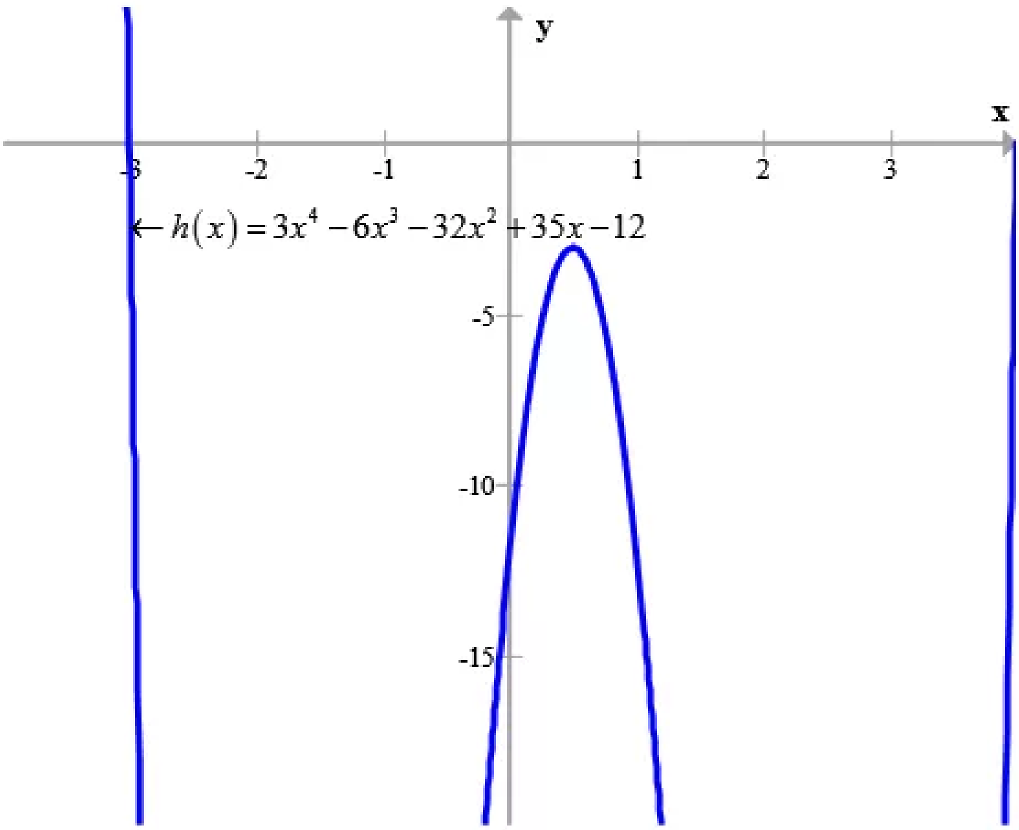
Consider the function:
$h(x) = 3x^4 - 6x^3 - 32x^2 + 35x - 12$
Let us list the possible rational zeros.
Factors of constant term: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
Factors of leading coefficient: $\pm 1, \pm 3$

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{3}{3}, \pm\frac{4}{3}, \pm\frac{6}{3}, \pm\frac{12}{3}$

That is the possible rational roots are

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

Choose the reasonable values from the above list using the graph function.
Let us observe following graph.



Thus the value $x=-3$ is reasonable based on the graph.

Let us test this zero using synthetic division.

Test $x=-3$:

-3	3	-6	-32	35	-12
		-9	45	-39	12
	3	-15	13	-4	0

Here the remainder is 0 and hence $x=-3$ is a zero

Factor out a binomial using the result of synthetic division.

Because -3 is a zero, we can write

$$h(x)=(x+3)(3x^3-15x^2+13x-4)$$

Now consider,

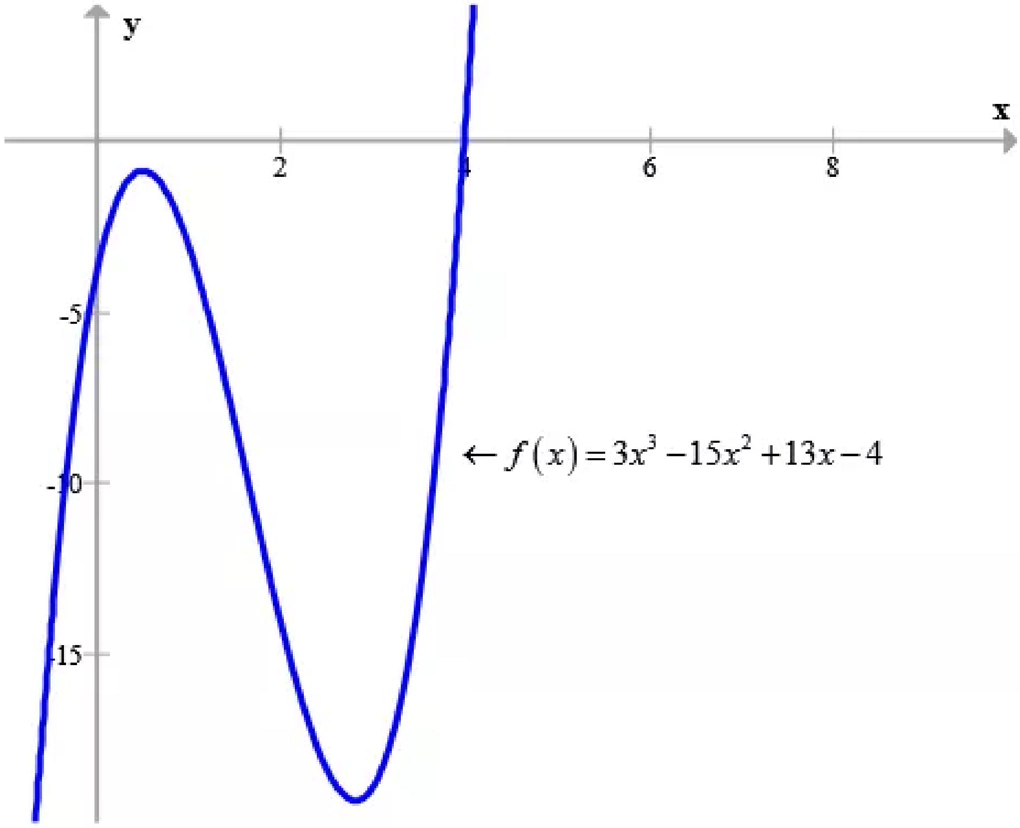
$f(x)=3x^3-15x^2+13x-4$

Any zero of $f(x)=3x^3-15x^2+13x-4$ will also be a zero of $h(x)$.

The possible rational zeros of $f(x)$ are:

$\pm1,\pm2,\pm4,\pm\frac{1}{3},\pm\frac{2}{3},\pm\frac{4}{3}$

Let us observe the following graph of $f(x)$:



The graph of f shows that $x=4$ may be a zero.

Synthetic division shows that $x=4$ is a zero of $f(x)$ and

$$h(x)=(x+3)f(x)$$
$$=(x+3)(x-4)(3x^2-3x+1)$$

The zeros of the equation $p(x)=3x^2-3x+1$ are imaginary.

Thus, the real zeros of $h(x)$ are $\boxed{-3 \text{ and } 4}$

Answer 33e.

STEP 1 **List** the possible rational zeros.

The leading coefficient of the given function is 2 and the constant term is −30. Divide each factor of the constant term by each factor of the leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{5}{1}, \pm\frac{6}{1}, \pm\frac{10}{1}, \pm\frac{15}{1}, \pm\frac{30}{1}, \pm\frac{1}{2},$$
$$\pm\frac{2}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{6}{2}, \pm\frac{10}{2}, \pm\frac{15}{2}, \pm\frac{30}{2}$$

The possible rational zeros of the function are

$\pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}$, and $\pm\frac{15}{2}$.

STEP 2 **Test** these zeros using synthetic division.

Test $x = 2$

2	2	−9	0	37	−30
		4	−10	−20	34
	2	−5	−10	17	4

Test $x = -2$

−2	2	−9	0	37	−30
		−4	26	−52	30
	2	−13	26	−15	0

The remainder being 0 in the case of −2, means that −2 is a zero of $f(x)$. Thus, we have $x - (-2)$ or $x + 2$ as one of the factors of the function. We can use this result to express $f(x)$ as a product of factors.

$f(x) = (x + 2)(2x^3 - 13x^2 + 26x - 15)$

STEP 3 **Factor** the polynomial in $h(x)$ and use the factor theorem.

We can repeat the steps above for $g(x) = 2x^3 - 13x^2 + 26x - 15$ and find its factors.

The possible rational zeros of g are $\pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}$, and $\pm\frac{5}{2}$.

Now, we can test these zeros using synthetic division. Take $\frac{5}{2}$ for instance.

Test $x = \frac{5}{2}$

$\frac{5}{2}$	2	−13	26	−15
		5	−20	15
	2	−8	6	0

Since the remainder is 0, we have $x - \frac{5}{2}$ as one of the factors of the function. Use this result to express $g(x)$ as a product of factors.

$$g(x) = \left(x - \frac{5}{2}\right)(2x^2 - 8x + 6)$$

Take out 2 from the second factor.

$$g(x) = \left(x - \frac{5}{2}\right)2\left(x^2 - 4x + 3\right)$$

Now, multiply the first factor by 2.

$g(x) = (2x - 5)(x^2 - 4x + 3)$

Factor $x^2 - 4x + 3$.

$g(x) = (2x - 5)(x - 3)(x - 1)$

Replace $2x^3 - 13x^2 + 26x - 15$ with $(2x - 5)(x - 3)(x - 1)$ in $f(x)$.

$f(x) = (x + 2)(2x - 5)(x - 3)(x - 1)$

Since any zero of g is also a zero of f, we get the other real zeros of the function as 3 and 1.

Therefore, the zeros of f are $-2, 1, \frac{5}{2}$, and 3.

Answer 34e.

We need to find all real zeros of

$$f(x) = x^5 - 3x^4 - 5x^3 + 15x^2 + 4x - 12$$

Let us use the Rational Zero Theorem to find the possible rational zeros of f.

The Rational Zero Theorem: If

$$f(x) = a_n x^n + \ldots + a_1 x + a_0$$

has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:

$$f(x) = x^5 - 3x^4 - 5x^3 + 15x^2 + 4x - 12$$

Let us list the possible rational zeros.

Factors of constant term: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

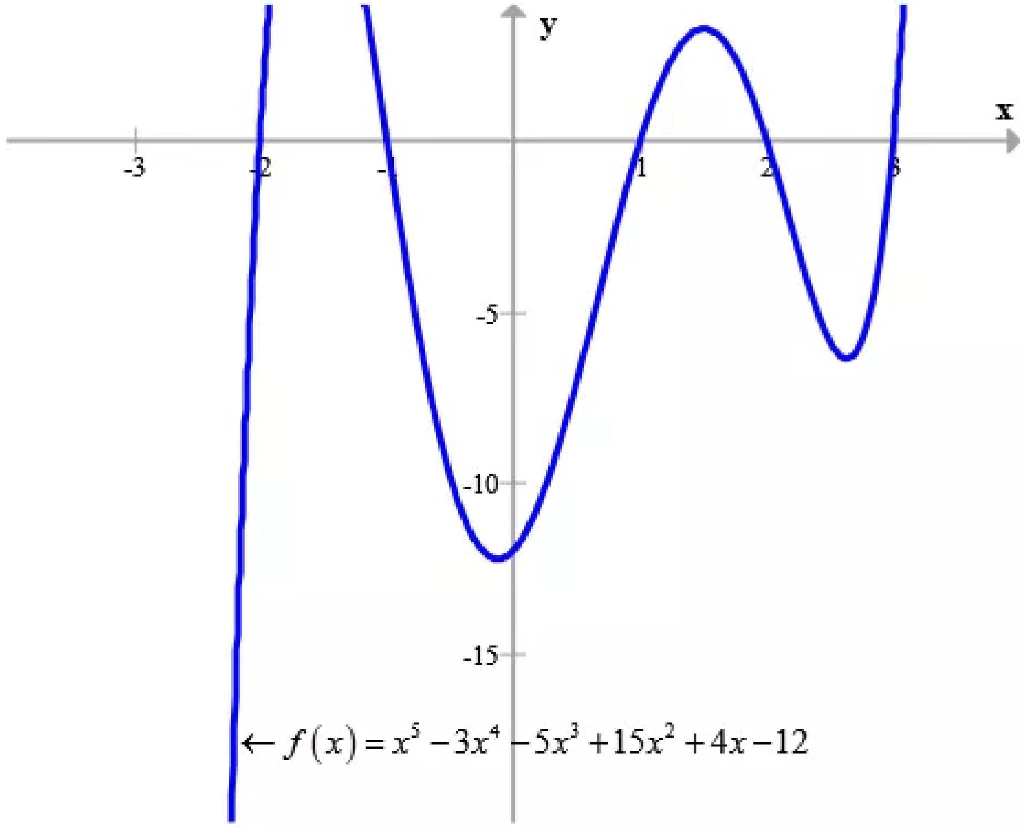
Factors of leading coefficient: ±1

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}$

That is the possible rational roots are

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

Choose the reasonable values from the above list using the graph function.
Let us observe following graph.



Thus the valuex $x=-2, x=1, x=2, x=3$ are reasonable based on the graph.

Let us test this zero using synthetic division.

Test $x=-2$:

−2	1	−3	−5	15	4	−12
		−2	10	−10	−10	12
	1	−5	5	5	−6	0

Here the remainder is 0 and hence $x=-2$ is a zero

Factor out a binomial using the result of synthetic division.

Because -2 is a zero, we can write

$f(x)=(x+2)(x^4-5x^3+5x^2+5x-6)$

Now consider,

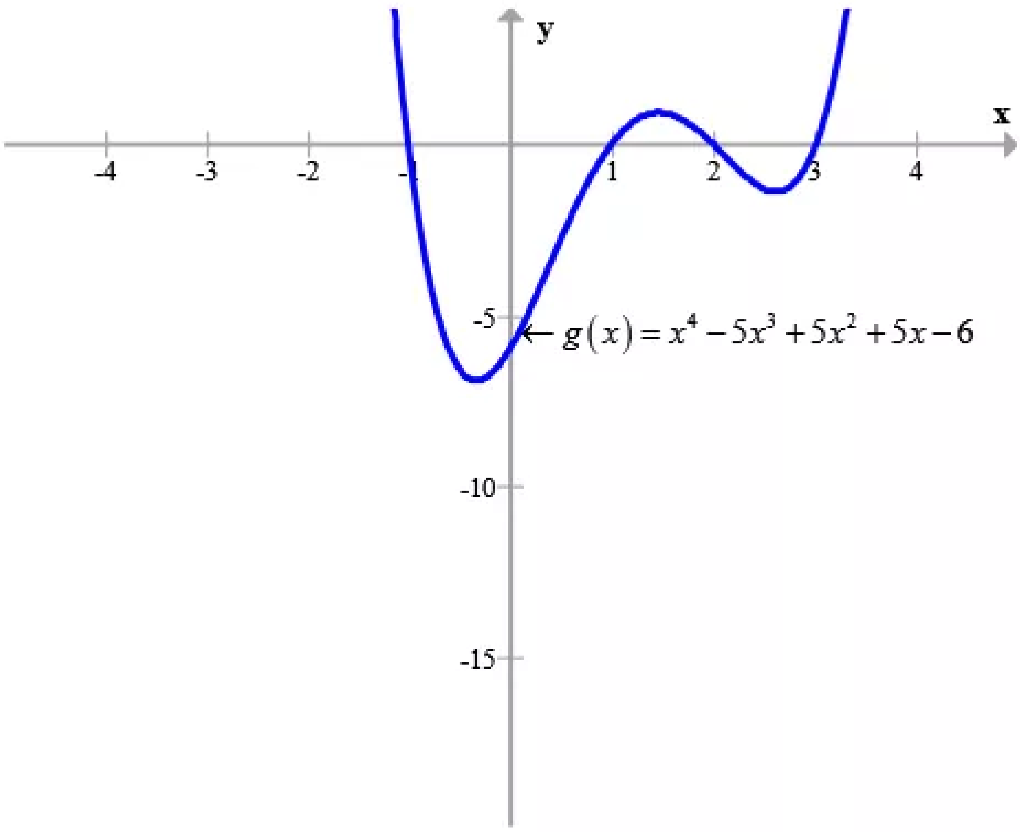
$g(x) = x^4 - 5x^3 + 5x^2 + 5x - 6$

Any zero of $g(x) = x^4 - 5x^3 + 5x^2 + 5x - 6$ will also be a zero of $f(x)$.

The possible rational zeros of $g(x)$ are:

$\pm 1, \pm 2, \pm 3, \pm 6$

Let us observe the following graph of $g(x)$:



The graph of g shows that $x = 1$ may be a zero.

Let us test this zero using synthetic division.

Test $x = 1$:

1	1	−5	5	5	−6
		1	−4	1	6
	1	−4	1	6	0

Here the remainder is 0 and hence $x = 1$ is a zero.

Synthetic division shows that $x=1$ is a zero of $g(x)$ and

$$f(x)=(x+2)g(x)$$
$$=(x+2)(x-1)(x^3-4x^2+x+6)$$

Let us consider

$$h(x)=x^3-4x^2+x+6$$

Thus,

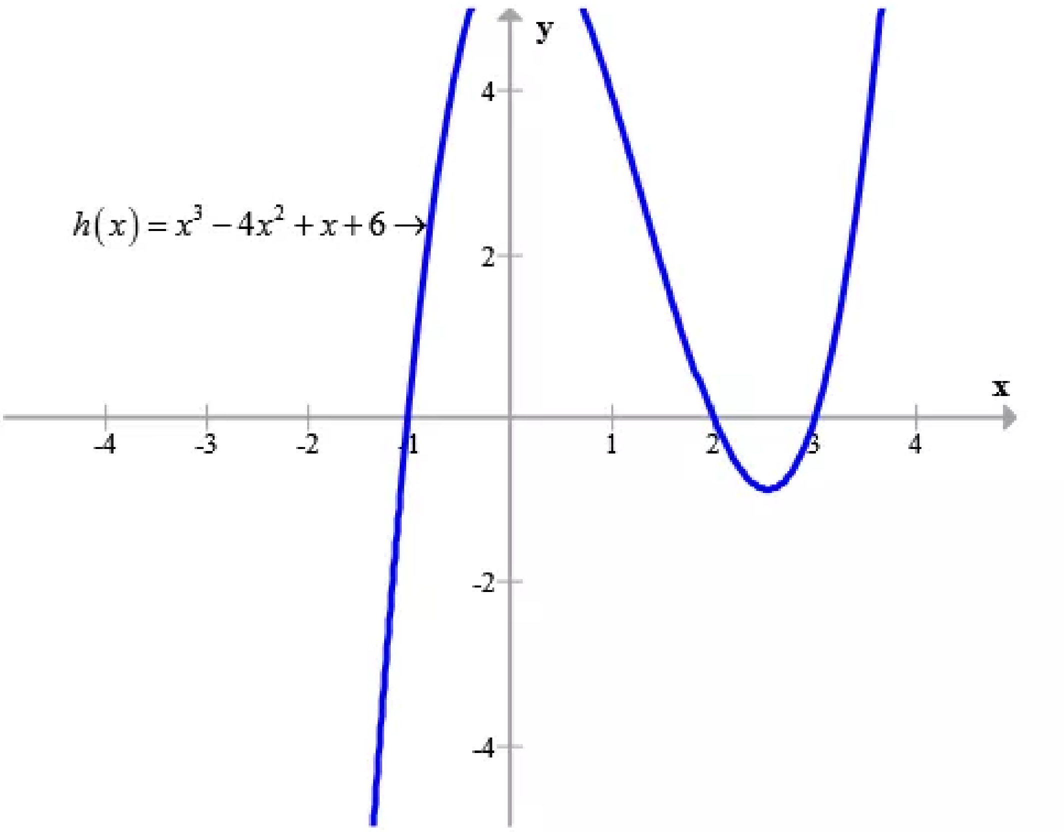
$$f(x)=(x+2)(x-1)h(x)$$

Any zero of $h(x)=x^3-4x^2+x+6$ will also be a zero of $f(x)$.

The possible rational zeros of $h(x)$ are:

$\pm1, \pm2, \pm3, \pm6$

Let us observe the following graph of $h(x)$:



The graph of h shows that $x=-1$ may be a zero.

Let us test this zero using synthetic division.

Test $x = -1$:

-1	1	-4	1	6
		-1	5	-6
	1	-5	6	0

Here the remainder is 0 and hence $x = -1$ is a zero.

Synthetic division shows that $x = -1$ is a zero of $h(x)$ and

$$f(x) = (x+2)(x-1)h(x)$$
$$= (x+2)(x-1)(x+1)(x^2 - 5x + 6)$$

That is,

$$f(x) = (x+2)(x-1)(x+1)(x-2)(x-3)$$

Thus, the real zeros of $f(x)$ are $\boxed{\pm 1, \pm 2 \text{ and } 3}$

Answer 35e.

STEP 1 **List** the possible rational zeros.

The leading coefficient of the given function is 2 and the constant term is 3. Divide each factor of the constant term by each factor of the leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}$$

The possible rational zeros of the function are ± 1, ± 3, $\pm\frac{1}{2}$, and $\pm\frac{3}{2}$.

STEP 2 **Test** these zeros using synthetic division.

Test $x = \frac{1}{2}$

$\frac{1}{2}$	2	5	−3	−2	−5	3
		1	3	0	−1	−3
	2	6	0	−2	−6	0

The remainder 0 means that $\frac{1}{2}$ is a zero of $h(x)$. Thus, we have $x - \frac{1}{2}$ as one of the factors of the function. We can use this result to express $h(x)$ as a product of factors.

$$h(x) = \left(x - \frac{1}{2}\right)\left(2x^4 + 6x^3 - 2x - 6\right)$$

STEP 3 **Factor** the polynomial in $h(x)$ and use the factor theorem.

$$h(x) = \left(x - \frac{1}{2}\right)\left(2x^3 - 2\right)(x + 3)$$

Take out 2 from the second factor.

$$h(x) = \left(x - \frac{1}{2}\right)2\left(x^3 - 1\right)(x + 3)$$

Now, multiply the first factor by 2.

$h(x) = (2x - 1)(x^3 - 1)(x + 3)$

On solving $x^3 - 1 = 0$ and $x + 3 = 0$, we get two other zeros 1 and 3.

Therefore, the real zeros of h are -3, $\frac{1}{2}$, and 1.

Answer 36e.

We need to describe and correct the error in listing the possible rational zeros of the function $f(x) = x^3 + 7x^2 + 2x + 14$

Possible zeros:

1, 2, 7, 14

Let us use the Rational Zero Theorem to find solve this problem

The Rational Zero Theorem: If

$f(x) = a_n x^n + \ldots + a_1 x + a_0$

has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:

$f(x) = x^3 + 7x^2 + 2x + 14$

Factors of constant term: $\pm1, \pm2, \pm7, \pm14$

Factors of leading coefficient: ±1

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{7}{1}, \pm\frac{14}{1}$

That is the possible rational roots are

$\pm1, \pm2, \pm7, \pm14$

Given possible rational zeros are: 1, 2, 7, 14

Error in listing the possible rational zeros is that we had neglected the negative zeros of the given function.

We need to consider both the positive and negative rational zeros of the function.

Thus, the possible rational zeros are $\boxed{\pm1, \pm2, \pm7, \pm14}$

Answer 37e.

The Rational Zero Theorem states that if $f(x) = a_n x^n + \ldots + a_1 x + a_0$ has integer coefficients, then every rational zero of f will have the form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coeffeicient } a_n}.$$

The constant term in the given function is 5, and the leading coefficient is 6.

Factors of the constant term: $\pm1, \pm5$

Factors of the leading coefficient: $\pm1, \pm2, \pm3, \pm6$

Divide the factors of the constant term by the factors of the leading coefficient to get the list of possible rational zeros.

$$\text{Possible rational zeros} = \pm\frac{1}{1}, \pm\frac{5}{1}, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$$

$$= \pm1, \pm5, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$$

The list of real zeros of f are $\pm1, \pm5, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}$, and $\pm\frac{5}{6}$, where p represents the factors of the constant term 5 and q represents the factors of the leading coefficient 6.

In the given list, p has the factors of 6 and q has the factors of 5, which is not correct.

Therefore, the correct list of possible rational zeros is

$\pm1, \pm5, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}$, and $\pm\frac{5}{6}$.

Answer 38e.

We need to write a polynomial function f that has a leading coefficient of 4 and has 12 possible rational zeros according to the rational zero theorem.

Let us use the Rational Zero Theorem to find solve this problem
The Rational Zero Theorem: If
$f(x)=a_nx^n+...+a_1x+a_0$
has integer coefficients, then every rational zero of f has the following form:
$$\frac{p}{q}=\frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

The factors of the leading coefficient, 4, are: $\pm1, \pm2, \pm4$
By Rational Zero Theorem, a rational zero can be of the form
$$\frac{p}{q}=\frac{\text{factor of constant term}}{\text{factor of leading coefficient}}$$
That is, the possible rational zeros are
$$\frac{\text{factor of constant term}}{\pm1}, \frac{\text{factor of constant term}}{\pm2}, \frac{\text{factor of constant term}}{\pm4}$$
This shows that the number of possible rational zeros is 6.
Since the given number of possible rational zeros is 12, the factors of the constant term should be $\frac{12}{6}=2$.
The factors of a number will be 2 for a prime number.
Thus the constant term should be a prime number.

Since the constant term is a prime number, the factors of a prime numbers are 1 and the number itself.
Thus, it generates 12 unique zeros.
A polynomial of nth degree has precisely n distinct zeros.
Thus, the degree of the polynomial is 12.

Thus, the required polynomial is $\boxed{f(x)=4x^{12}-p}$ where p is a prime number.

Answer 39e.

We need to check which of the following not a zero of the function is

$f(x)=40x^5-42x^4-107x^3+107x^2+33x-36$

(A) $-\frac{3}{2}$

(B) $-\frac{3}{8}$

(C) $\frac{3}{4}$

(D) $\frac{4}{5}$

Consider the function:

$f(x)=40x^5-42x^4-107x^3+107x^2+33x-36$

Let us list the possible rational zeros.

Factors of constant term: $\pm1,\pm2,\pm3,\pm4,\pm6,\pm9,\pm12,\pm18,\pm36$

Factors of leading coefficient: $\pm1,\pm2,\pm4,\pm5,\pm8,\pm10,\pm20,\pm40$

Possible rational zeros:

$\pm\frac{1}{1},\pm\frac{2}{1},\pm\frac{3}{1},\pm\frac{4}{1},\pm\frac{6}{1},\pm\frac{9}{1},\pm\frac{12}{1},\pm\frac{18}{1},\pm\frac{36}{1},$

$\pm\frac{1}{2},\pm\frac{2}{2},\pm\frac{3}{2},\pm\frac{4}{2},\pm\frac{6}{2},\pm\frac{9}{2},\pm\frac{12}{2},\pm\frac{18}{2},\pm\frac{36}{2},$

$\pm\frac{1}{4},\pm\frac{2}{4},\pm\frac{3}{4},\pm\frac{4}{4},\pm\frac{6}{4},\pm\frac{9}{4},\pm\frac{12}{4},\pm\frac{18}{4},\pm\frac{36}{4},$

$\pm\frac{1}{5},\pm\frac{2}{5},\pm\frac{3}{5},\pm\frac{4}{5},\pm\frac{6}{5},\pm\frac{9}{5},\pm\frac{12}{5},\pm\frac{18}{5},\pm\frac{36}{5},$

$\pm\frac{1}{8},\pm\frac{2}{8},\pm\frac{3}{8},\pm\frac{4}{8},\pm\frac{6}{8},\pm\frac{9}{8},\pm\frac{12}{8},\pm\frac{18}{8},\pm\frac{36}{8},$

$\pm\frac{1}{10},\pm\frac{2}{10},\pm\frac{3}{10},\pm\frac{4}{10},\pm\frac{6}{10},\pm\frac{9}{10},\pm\frac{12}{10},\pm\frac{18}{10},\pm\frac{36}{10},$

$\pm\frac{1}{20},\pm\frac{2}{20},\pm\frac{3}{20},\pm\frac{4}{20},\pm\frac{6}{20},\pm\frac{9}{20},\pm\frac{12}{20},\pm\frac{18}{20},\pm\frac{36}{20},$

$\pm\frac{1}{40},\pm\frac{2}{40},\pm\frac{3}{40},\pm\frac{4}{40},\pm\frac{6}{40},\pm\frac{9}{40},\pm\frac{12}{40},\pm\frac{18}{40},\pm\frac{36}{40}$

That is the possible zeros are

$\pm1,\pm2,\pm3,\pm4,\pm6,\pm9,\pm12,\pm18,\pm36,$

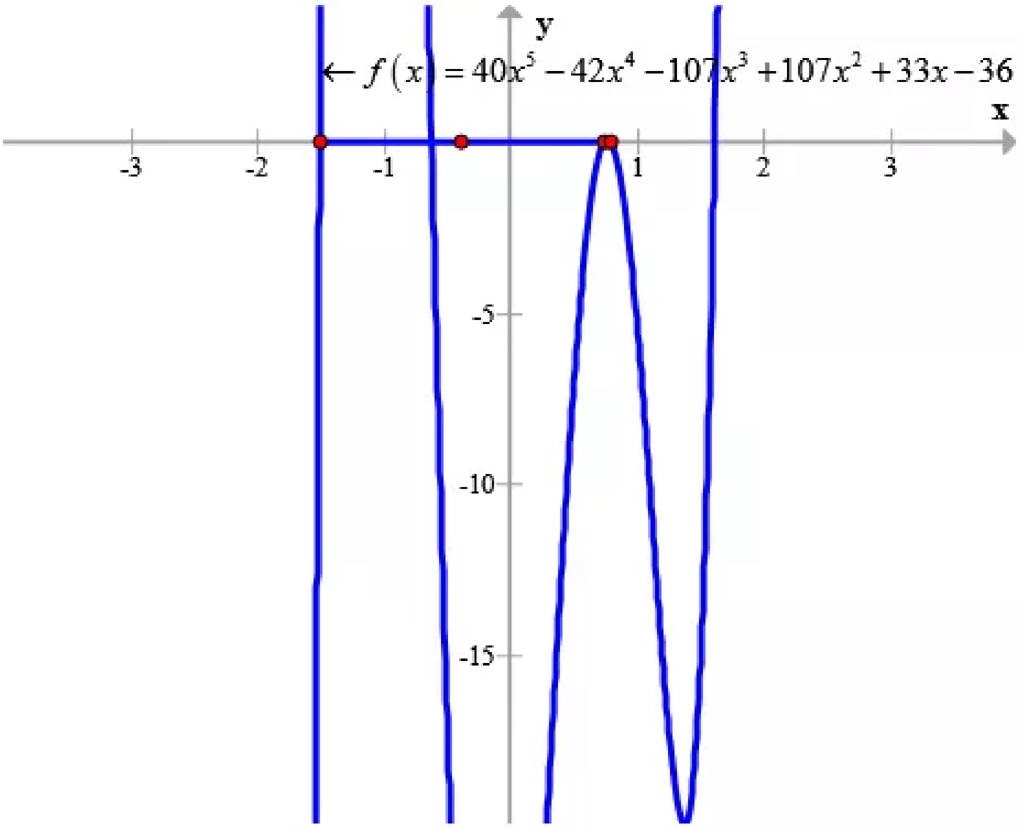
$\pm\frac{1}{2},\pm\frac{3}{2},\pm\frac{9}{2},\pm\frac{1}{4},\pm\frac{3}{4},\pm\frac{9}{4},$

$\pm\frac{1}{5},\pm\frac{2}{5},\pm\frac{3}{5},\pm\frac{4}{5},\pm\frac{6}{5},\pm\frac{9}{5},\pm\frac{12}{5},\pm\frac{18}{5},\pm\frac{36}{5},$

$\pm\frac{1}{8},\pm\frac{3}{8},\pm\frac{9}{8},\pm\frac{1}{10},\pm\frac{3}{10},\pm\frac{9}{10},$

$\pm\frac{1}{20},\pm\frac{3}{20},\pm\frac{9}{20},\pm\frac{1}{40},\pm\frac{3}{40},\pm\frac{9}{40}$

Let us observe following graph.



Thus the values $x = -\frac{3}{2}, x = \frac{3}{4}$ and $x = \frac{4}{5}$ are reasonable based on the graph.

Therefore,

$$\boxed{x = \frac{-3}{8}}$$

is not a zero of the given function.

Answer 40e.

Let us consider the function:

$f(x) = a_n x^n + ... + a_1 x + a_0$

Let a_n has r factors and a_0 has s factors

We need to find the largest number of possible rational zeros of f that can be generated by the rational zero theorem.

Let us use the Rational Zero Theorem to solve this problem

The Rational Zero Theorem: If

$f(x) = a_n x^n + \ldots + a_1 x + a_0$

has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Since the constant term a_0 has s factors, the number of factors of constant term is s.

Since the leading coefficient a_n has r factors, the number of factors of leading coefficient a_n is s.

Thus, the largest possible number of rational roots are $\boxed{r \times s}$

Answer 41e.

Let us use the step-by-step method to find the real zeros of the given function.

STEP 1 **List** the possible rational zeros.

The leading coefficient of the given function is 3 and the constant term is 2. Divide each factor of the constant term by each factor of the leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}$$

The possible rational zeros of the function are ± 1 and ± 2.

STEP 2 **Test** these zeros using synthetic division.

Test $x = 1$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -1 & 2 \\ & & 1 & -1 & -2 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

The remainder is 0 which means that 1 is a zero of $f(x)$. Thus, we have $x - 1$ as one of the factors of the function. We can use this result to express $f(x)$ as a product of factors.

$f(x) = (x - 1)(x^2 - x - 2)$

STEP 3 **Factor** the trinomial in $f(x)$ and use the factor theorem.
$f(x) = (x - 1)(x - 2)(x + 1)$

On solving $x - 2 = 0$ and $x + 1 = 0$, we get two other zeros 2 and -1.

Thus, the zeros of f are -1, 1, and 2.

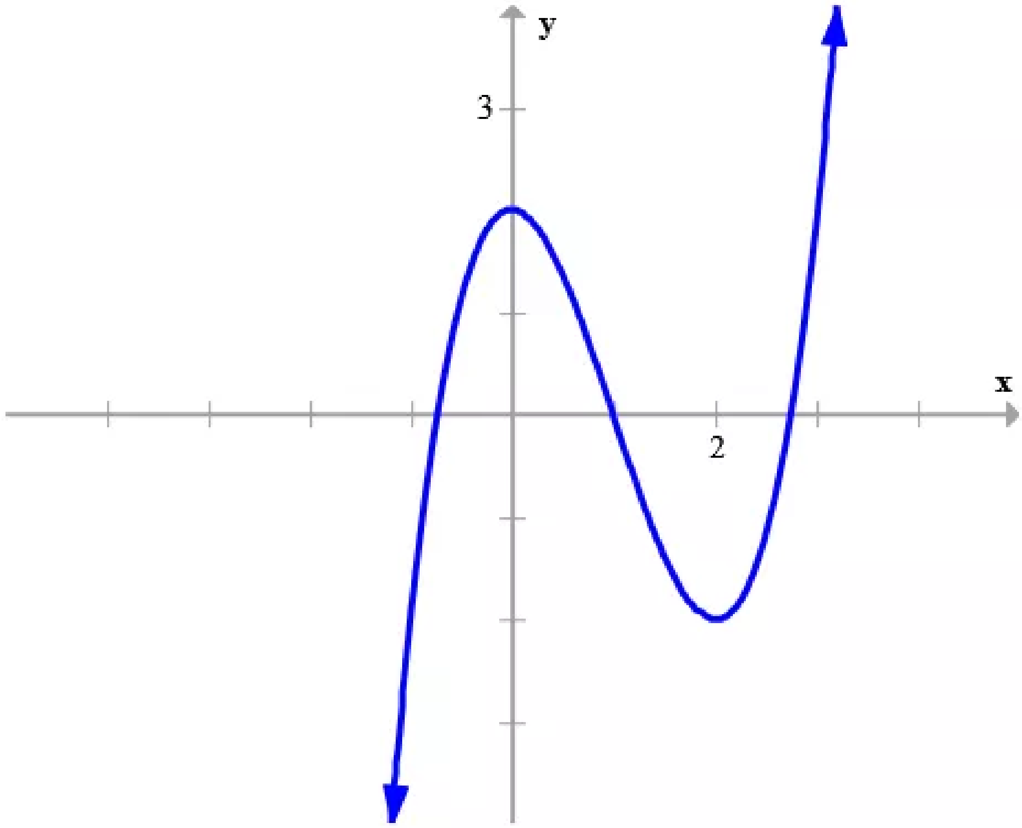
The corresponding graph of the function will have x-intercepts at the points -1, 1, and 2.

By observing the given graphs, it can be seen that the one given in choice **B** has x-intercepts at -1, 1, and 2. Therefore, the graph in choice **B** matches with the given function.

Answer 42e.

We need to find all zeros of the function
$g(x) = x^3 - 3x^2 + 2$
Let us match the function with the graph.

Thus, the graph of the given function is matched with ©.



Let us use the Rational Zero Theorem to find the possible rational zeros of f.

The Rational Zero Theorem: If

$f(x)=a_nx^n+...+a_1x+a_0$

has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q}=\frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Consider the function:

$g(x)=x^3-3x^2+2$

Let us list the possible rational zeros.

Factors of constant term: $\pm1,\pm2$

Factors of leading coefficient: ±1

Possible rational zeros: $\pm1,\pm2$

Let us test these zeros using synthetic division.

Test $x=1$:

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 0 & 2 \\ & & 1 & -2 & -2 \\ \hline & 1 & -2 & -2 & 0 \end{array}$$

Here the remainder is 0 and hence $x=1$ is a zero.

Factor out a binomial using the result of synthetic division.

Because 1 is a zero, we can write

$f(x)=(x-1)(x^2-2x-2)$

Now consider,

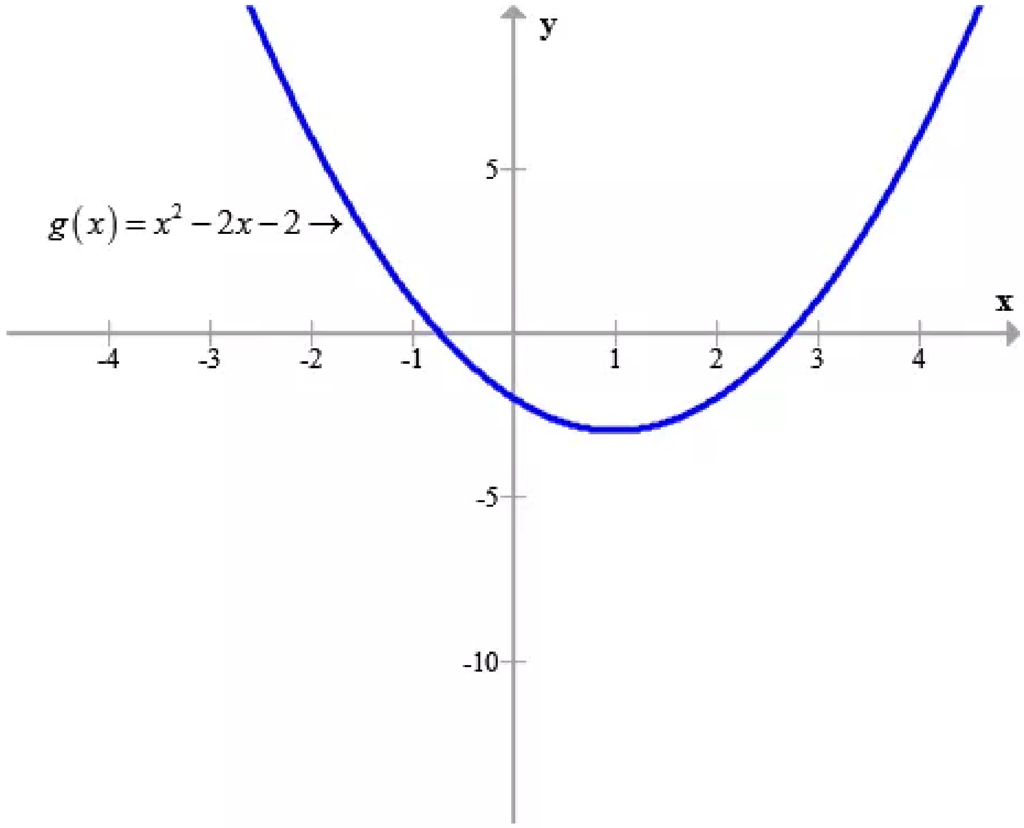
$g(x)=x^2-2x-2$

Any zero of $g(x)=x^2-2x-2$ will also be a zero of $f(x)$.

The possible rational zeros of $g(x)$ are:

$\pm1,\pm2$

Let us observe the following graph of $g(x)$:



$$f(x)=(x-1)(x^2-2x-2)$$
$$=(x-1)g(x)$$

Solve the quadratic to get the values,

$$x=\frac{2\pm\sqrt{4-4(1)(-2)}}{2}$$
$$=\frac{2\pm\sqrt{12}}{2}$$
$$=1\pm\sqrt{3}$$

Thus, the zeros of $f(x)$ are $\boxed{1, 1+\sqrt{3} \text{ and } 1-\sqrt{3}}$

Answer 43e.

Let us use the step-by-step method to find the real zeros of the given function.

STEP 1 **List** the possible rational zeros.
The leading coefficient of the given function is 3 and the constant term is 2. Divide each factor of the constant term by each factor of the leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}$$

The possible rational zeros of the function are ± 1 and ± 2.

STEP 2 **Test** these zeros using synthetic division.

Test $x = 2$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -1 & 2 \\ & & 2 & 6 & 10 \\ \hline & 1 & 3 & 5 & 12 \end{array}$$

Test $x = -2$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -1 & 2 \\ & & -2 & 2 & -2 \\ \hline & 1 & -1 & 1 & 0 \end{array}$$

The remainder is 0 in the case of -2, which means that -2 is a zero of $f(x)$. Thus, we have $x + 2$ as one of the factors of the function. We can use this result to express $h(x)$ as a product of factors.
$h(x) = (x + 2)(x^2 - x + 1)$

STEP 3 **Find** the remaining zeros of h by solving $x^2 - x + 1 = 0$.
Use the quadratic formula by replacing a and c with 1 and b with -1.

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

Simplify.

$$x = \frac{1 \pm \sqrt{-3}}{2}$$
$$= \frac{1 \pm i\sqrt{3}}{2}$$

Thus, the only real zeros of f is -2.

The corresponding graph of the function will have an x-intercept at the point -2.

By observing the given graphs, it can be seen that the one given in choice **A** has an x-intercept at -2. Therefore, the graph in choice **A** matches with the given function.

Answer 44e.

We need to explain whether it is possible for a cubic function to have more than 3 real zeros.
We shall also explain whether is it possible for a cubic function to have no real zeros.

A polynomial of nth degree has precisely n distinct zeros.
A cubic polynomial is of degree 3.
Thus, it has exactly 3 distinct zeros and it cannot have more than 3 real zeros.

Imaginary roots occur in pairs.
Since a cubic polynomial is having 3 zeros, at least one root out of 3 should be real.
Thus, it is not possible for a cubic function to have no real zeros.

Answer 45e.

STEP 1 **Write** an equation for the volume of the ice sculpture.
Use the fact that the volume of a rectangle is given by the product of its length, width, and height.

Volume (cubic inches)	=	Length (inches)	·	Width (inches)	·	Height (inches)
⇓				⇓		⇓
63	=	x	·	x	·	$(x+4)$

We get the equation $63 = x^2(x+4)$.

Remove the parentheses using the distributive property.
$63 = x^3 + 4x^2$

Now, rewrite the equation in standard form.
$0 = x^3 + 4x^2 - 63$

STEP 2 **List** the possible rational solutions.
The leading coefficient of the function is 1 and the constant term is 63.

Thus, the possible rational zeros are $\pm1, \pm3, \pm7, \pm9, \pm21$, and ±63.

STEP 3 **Test** possible solutions. Only positive x-values make sense.

Test $x = 1$

$$\begin{array}{r|rrrr} 1 & 1 & 4 & 0 & -63 \\ & & 1 & 5 & 5 \\ \hline & 1 & 5 & 5 & -58 \end{array}$$

Test $x = 3$

$$\begin{array}{r|rrrr} 3 & 1 & 4 & 0 & -63 \\ & & 3 & 21 & 63 \\ \hline & 1 & 7 & 21 & 0 \end{array}$$

The remainder being 0 in the case of 3, means that 3 is a zero or solution of the function. We can use this result to express the function as a product of factors.
$0 = (x - 3)(x^2 + 7x + 21)$

STEP 4 **Check** for other solutions.
For this, solve $x^2 + 7x + 21 = 0$ using the quadratic formula. On solving, we get the other two solutions satisfying the equation as $\dfrac{-7 \pm i\sqrt{35}}{2}$, which can be discarded because they are imaginary numbers.

The only reasonable solution is thus $x = 3$. Since the length of each side of the prism is 3 inches, the height will be $3 + 4$ or 7 inches

Therefore, the dimensions of the mold are 3 inches by 3 inches by 7 inches.

Answer 46e.

The width of the pool is 5 feet more than the depth and length is 35 feet more than the depth. The pool holds 2000 cubic feet of water.
We need to find the dimensions of the pool.

Let d, w and l be the depth, width and length of the pool respectively
Given that,
$w = d + 5$ and $l = d + 35$
The volume or capacity of the pool is given by
$V = lwd$
Since the pool holds 2000 cubic feet,
$2000 = lwd$
Substitute the value of width and length in terms of depth, we have

$$2000 = (d + 35)(d + 5)d$$

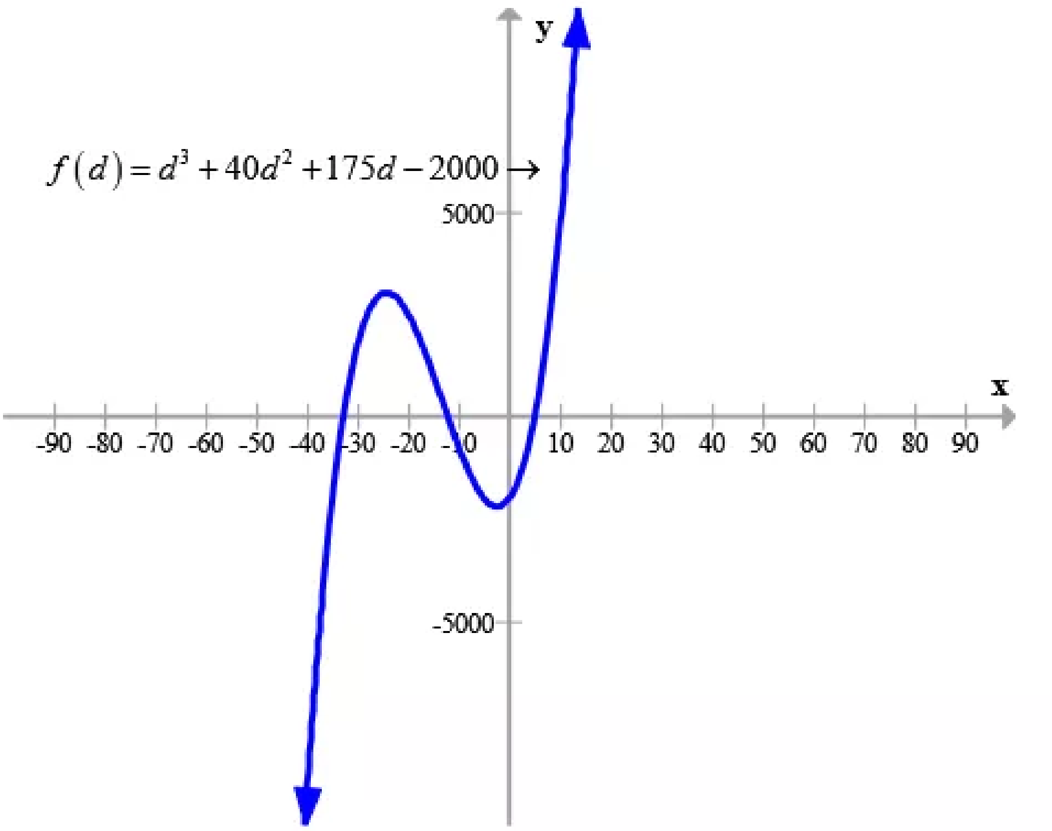
$$2000 = (d + 35)(d^2 + 5d)$$

$$2000 = d^3 + 5d^2 + 35d^2 + 175d$$

$$d^3 + 40d^2 + 175d = 2000$$

$$d^3 + 40d^2 + 175d - 2000 = 0 \qquad ......(1)$$

We need to solve the cubic equation to find the dimensions of the pool.
Let us graph the equation (1).



The graph suggests that $d=5$ is a zero of the function.

Let us test this zeros using synthetic division.
Test $d=5$:

5	1	40	175	−2000
		5	225	2000
	1	45	400	0

Here the remainder is 0 and hence $d=5$ is a zero.

Thus,
$f(d)=(d-5)g(d)$,
where
$g(d)=d^2+45d+400$
Let us solve the quadratic equation
$g(d)=d^2+45d+400$
Thus,

$$d=\frac{-45\pm\sqrt{45^2-4\times 400}}{2}$$
$$=\frac{-45\pm\sqrt{2025-1600}}{2}$$
$$=\frac{-45\pm\sqrt{425}}{2}$$
$$=\frac{-45\pm 5\sqrt{17}}{2}$$

Thus, both the values $\frac{-45+5\sqrt{17}}{2}$ and $\frac{-45-5\sqrt{17}}{2}$ are negative.

Since depth cannot be negative, let us discard the values, $\frac{-45+5\sqrt{17}}{2}$ and $\frac{-45-5\sqrt{17}}{2}$.

Therefore, $d = 5$ feet

Therefore, the dimensions of the pool are $\boxed{d = 5 \text{ feet, } w = 10 \text{ feet and } l = 40 \text{ feet}}$

Answer 47e.

We know that the volume of a rectangle is given by the product of its length, width, and height.

It is given that the length, width and height of the rectangular prism are x, $x - 1$, and $x - 2$ respectively.

$$\begin{array}{ccccccc} \text{Volume} & = & \text{Length} & \cdot & \text{Width} & \cdot & \text{Height} \\ \Downarrow & & & & \Downarrow & & \Downarrow \\ 24 & = & x & \cdot & (x-1) & \cdot & (x-2) \end{array}$$

We get the equation $24 = x(x - 1)(x - 2)$.

Remove the parentheses using the distributive property.

$$24 = x\left(x^2 - 3x + 2\right)$$
$$24 = x^3 - 3x^2 + 2x$$

Now, rewrite the equation in standard form.

$$0 = x^3 - 3x^2 + 2x - 24$$

The equation that models the situation is $x^3 - 3x^2 + 2x - 24 = 0$.

The leading coefficient of the function is 1 and the constant term is 24.

Divide each factor of constant term by each factor of the leading coefficient to get the list of possible rational solutions of the equation.

$$\begin{aligned} x &= \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{8}{1}, \pm\frac{12}{1}, \pm\frac{24}{1} \\ &= \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24 \end{aligned}$$

Thus, the possible rational solutions are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12$, and ± 24.

Answer 48e.

A pyramid has a square base with sides of length x, a height of $2x-5$, and a volume of 3.

We need to write a polynomial equation to model the situation. Let us also list the possible rational solutions of the equation.

The volume of a pyramid is given by the formula:

$$\text{Volume} = \text{Area of the base} \times \text{Height} \times \frac{1}{3}$$

Area of the square base $= x^2$

Height of the Pyramid is $h = 2x - 5$

Thus,

$$\text{Volume} = x^2 \times (2x-5) \times \frac{1}{3}$$

$$3 = x^2 \times (2x-5) \times \frac{1}{3}$$

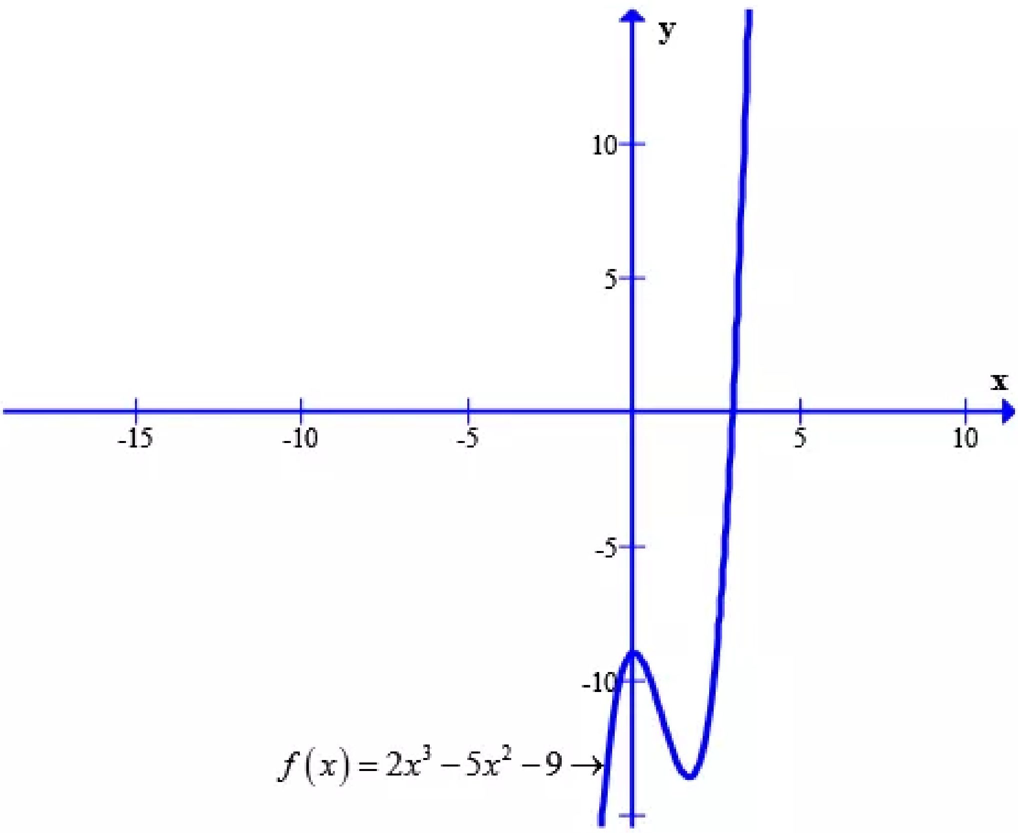
$$9 = 2x^3 - 5x^2$$

$$2x^3 - 5x^2 - 9 = 0$$

Let us solve the cubic equation

$2x^3 - 5x^2 - 9 = 0$

The graph of the equation $f(x) = 2x^3 - 5x^2 - 9$ is as follows:



The graph suggests that $x = 3$ is a zero of the curve $f(x) = 2x^3 - 5x^2 - 9$.

Let us test this zero using synthetic division.

Test $x = 3$:

$$\begin{array}{c|cccc} 3 & 2 & -5 & 0 & -9 \\ & & 6 & 3 & 9 \\ \hline & 2 & 1 & 3 & 0 \end{array}$$

Here the remainder is 0 and hence $x = 3$ is a zero

Factor out a binomial using the result of synthetic division.

Because 3 is a zero, we can write

$f(x) = (x-3)(2x^2 + x + 3)$

Now consider,

$g(x) = 2x^2 + x + 3$

Any zero of $g(x) = 2x^2 + x + 3$ will also be a zero of $f(x)$.

The roots of $g(x) = 2x^2 + x + 3$ are imaginary.

Thus the only real zero of the function $f(x)$ is $\boxed{x = 3}$

Answer 49e.

a. It is given that the total amount of athletic equipments sold was about \$20,300 millions.

Substitute 20,300 for $E(t)$ in the given model.

$20{,}300 = -10t^3 + 140t^2 - 20t + 18{,}150$

Now, rewrite the equation in standard form.

For this, subtract 20,300 from each side.

$0 = -10t^3 + 140t^2 - 20t - 2{,}150$

The polynomial equation that can be used to find the answer to the given situation is $-10t^3 + 140t^2 - 20t - 2{,}150 = 0$.

b. The leading coefficient of the function is -10 and the constant term is -2150.

We are required to find the rational zeros that are less than 10. This can be accomplished by dividing each factor of the constant term by each factor of the leading coefficient. Take factors less than 10 in either case.

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{5}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{5}{2}, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{5}{5}$$

$$= \pm 1, \pm 2, \pm 5, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{5}, \pm\frac{2}{5}$$

Out of the several possibilities, we choose only ± 1, ± 2, and ± 5 since we seek only whole-number solutions.

c. Let us check the possible rational zeros. Check the positive values first.

Test $x = 1$

$$\begin{array}{r|rrrr} 1 & -10 & 140 & -20 & -2150 \\ & & -10 & 130 & 110 \\ \hline & -10 & 130 & 110 & -2040 \end{array}$$

Test $x = 2$

$$\begin{array}{r|rrrr} 2 & -10 & 140 & -20 & -2150 \\ & & -20 & 120 & 200 \\ \hline & -10 & 120 & 100 & -1950 \end{array}$$

Test $x = 5$

$$\begin{array}{r|rrrr} 5 & -10 & 140 & -20 & -2150 \\ & & -50 & 450 & 2150 \\ \hline & -10 & 90 & 430 & 0 \end{array}$$

The remainder is 0 in the case of 5, which means that 5 is a zero of the function. As a result, $x - 5$ is one of the factors of the function.

$0 = (x-5)(-10x^2 + 90x + 430)$

Take -10 out from the second factor.

$0 = (x-5)(-10)(10x^2 - 9x + 43)$

Multiply the first factor by -10.

$0 = (-10x + 50)(10x^2 - 9x + 43)$

Now, find the remaining factors of the function by solving $10x^2 - 9x + 43 = 0$. Use the quadratic formula by replacing a with 10, b with -9 and c with 43.

$$x = \frac{9 \pm \sqrt{(-9)^2 - 4(10)(43)}}{2(10)}$$

Simplify.

$$x = \frac{9 \pm \sqrt{-1639}}{20}$$

$$= \frac{9 \pm i\sqrt{1639}}{20}$$

The only real solution is 5. The year that is five years from 1994 is 1999. Therefore, the amount of athletic equipments sold will be about \$20,300 in the year 1999.

Answer 50e.

Since 1990, the number of U.S. travelers to foreign countries F (in thousands) can be modeled by

$F(t) = 12t^4 - 264t^3 + 2028t^2 - 3924t + 43916$

where t is the number of years since 1990.

We need to use the following steps to find the year when there were about 56,300,000 travelers.

(a) Write a polynomial equation that can be used to find the answer

(b) List the possible whole-number solutions of the equation in part(a) that are less than or equal to 10

(c) Use synthetic division to determine which of the possible solutions in part (b) is an actual solution.

(d) Graph the function $F(t)$ and explain why there are no other reasonable solutions. Then calculate the year which corresponds to the solution.

(a)

Consider the model

$F(t) = 12t^4 - 264t^3 + 2028t^2 - 3924t + 43916$

where $F(t)$ is the number of U.S. travelers to foreign countries in thousands and t is the number of years since 1990.

When there were about 56,300,000 travelers,

$56300 = 12t^4 - 264t^3 + 2028t^2 - 3924t + 43916$

Thus, the required polynomial equation is

$\boxed{F(t) = 12t^4 - 264t^3 + 2028t^2 - 3924t - 12384}$ ......(1)

(b)

When $t = 8$,

$F(8) = 12(8)^4 - 264(8)^3 + 2028(8)^2 - 3924(8) - 12384$

$= 49152 - 135168 + 129792 - 31392 - 12384$

$= -178944 + 178944$

$= 0$

Thus, $\boxed{t = 8 \text{ years}}$ is the only whole number solution of the equation (1) which is less than or equal to 10.

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Let us test these zeros using synthetic division.

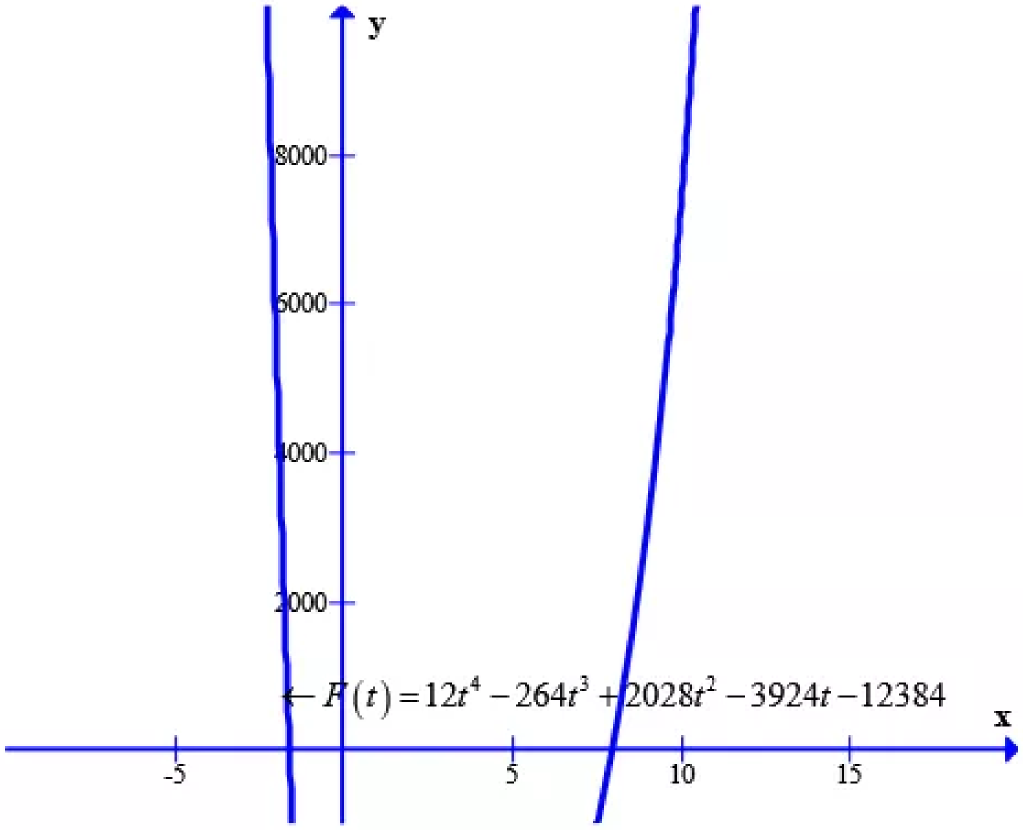
Test $t = 8$:

8	12	−264	2028	−3924	−12384
		96	−1344	5472	12384
	12	−168	684	1548	0

Here the remainder is 0 and hence $t = 8$ is a zero of the equation (1)

(d)

Consider the graph of the equation (1) as follows:



Let us observe the above graph.

The graph crosses the x-axis at two points when $t=8$ and $t=-1.6$

The number of years cannot be negative, and hence there is only one solution, $t=8$.

The variable, t, represents the number of years after 1990.

Since $t=8$, there were about 56,300,000 U.S.travelers to Foreign countries in the year $\boxed{1998}$.

Answer 51e.

The left ramp is twice as long as the right ramp. If 150 cubic feet of concrete are used to build the two ramps, we need to find the dimensions of each ramp.

The total length of the two ramps is $21x+6-3x$

That is, the length of the two ramps is $18x+6$

Since, the left ramp is twice as long as the right ramp, the ratio of the length of the left ramp to the length of the right ramp is $\frac{2(18x+6)}{3}:\frac{(18x+6)}{3}=12x+4:6x+2$

The height of the ramp is x

Thus, the area of the front triangular region is

$$A=\frac{1}{2}(12x+4)x+\frac{1}{2}(6x+2)x \qquad [\text{Area}=\frac{1}{2}\times\text{base}\times\text{height}]$$

Volume of the two ramps is

$$V=A\times 3x$$

$$150=\left[\frac{1}{2}(12x+4)x+\frac{1}{2}(6x+2)x\right]3x$$

$$150=\left[(12x+4)+(6x+2)\right]\frac{3x^2}{2}$$

$$150=[18x+6]\frac{3x^2}{2}$$

$$300=54x^3+18x^2$$

$$54x^3+18x^2-300=0$$

$$18x^3+6x^2-100=0$$

We need to solve the cubic equation

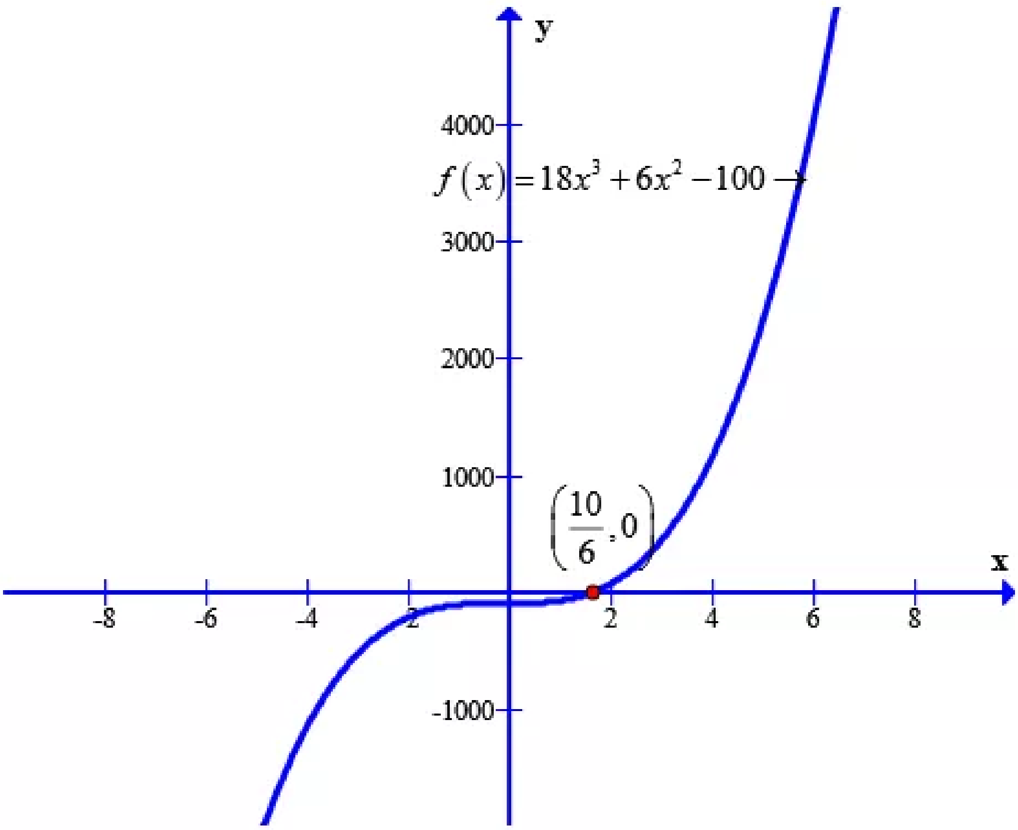
$18x^3+6x^2-100=0$

to find the dimensions of the ramp.

Consider

$f(x)=18x^3+6x^2-100$

Let us observe the following graph:



Thus, the value $x=\frac{5}{3}$ is reasonable based on the graph.

Let us test this zero using synthetic division.

$\frac{5}{3}$	18	6	0	-100
		30	60	100
	18	36	60	0

Since the remainder is 0, $x=\frac{5}{3}$ is a zero of the function.

Because $x=\frac{5}{3}$ is a zero, we can write

$$f(x)=\left(x-\frac{5}{3}\right)\left(18x^2+36x+60\right)$$

Now consider,

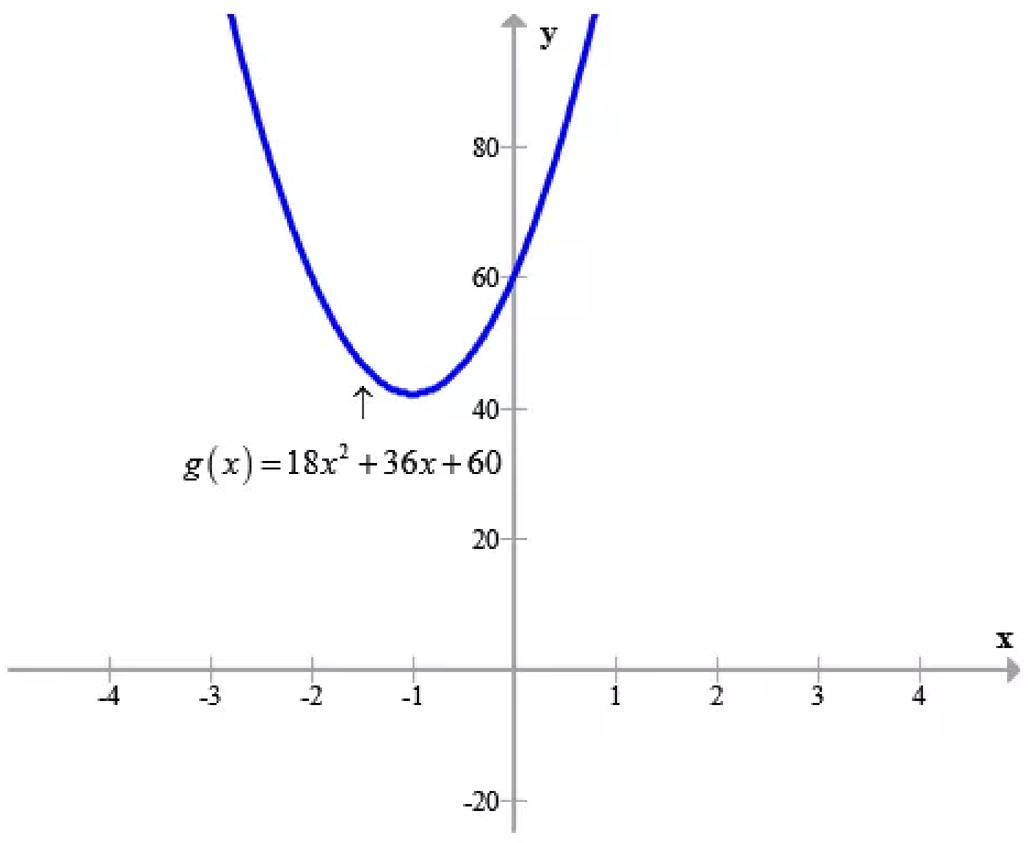
$g(x)=18x^2+36x+60$

Any zero of $g(x)=18x^2+36x+60$ will also be a zero of $f(x)$.

The possible rational zeros of $g(x)$ are:

$\pm1,\pm2,\pm3,\pm4,\pm5,\pm6,\pm10,\pm12,\pm15,\pm20,\pm30,\pm60$

Let us observe the following graph of $g(x)$:



The graph of g does not crosses the x-axis.

Thus, the roots of the quadratic

$g(x)=18x^2+36x+60$

are imaginary.

Thus, the only real zero of $f(x)$ is $x=\frac{5}{3}$

Thus, Length, width and height of the left ramp are

$l=12\left(\frac{5}{3}\right)+4, w=\left(\frac{5}{3}\right)3$ and $h=\frac{5}{3}$ respectively.

The , Length, width and height of the left ramp are

$l=6\left(\frac{5}{3}\right)+2, w=\left(\frac{5}{3}\right)3$ and $h=\frac{5}{3}$ respectively.

Thus, the dimensions (length, width and height) of the left ramp are

$\boxed{20 \text{ feet, } 5 \text{ feet and } \frac{5}{3} \text{ feet}}$

Thus, the dimensions (length, width and height) of the right ramp are

$\boxed{10 \text{ feet, } 5 \text{ feet and } \frac{5}{3} \text{ feet}}$

Answer 52e.

We need to solve the equation

$4x-6=18$

Consider the given equation

$$4x-6=18$$

$$4x-6+6=18+6 \quad \text{[Add 6 to both sides]}$$

$$4x+0=24 \quad \text{[Do the addition]}$$

$$4x=24 \quad \text{[0 is additive identity]}$$

$$\frac{4x}{4}=\frac{24}{4} \quad \text{[Divide both sides by 4]}$$

$$x=6 \quad \text{[Do the division]}$$

Thus, $\boxed{x=6}$

Answer 53e.

Subtract 7 from both sides of the given equation.

$$3y+7-7=-14-7$$

$$3y=-21$$

Divide both the sides by 3.

$$\frac{3y}{3}=\frac{-21}{3}$$

$$y=-7$$

The solution is −7.

CHECK

Substitute −7 for y in the original equation.

$$\begin{aligned} 3y + 7 &= -14 \\ 3(-7) + 7 &\overset{?}{=} -14 \\ -21 + 7 &\overset{?}{=} -14 \\ -14 &= -14 \checkmark \end{aligned}$$

The solution checks.

Answer 54e.

We need to solve the equation

$$|2p+5| = 15$$

Consider the given equation

$$|2p+5| = 15$$

$$2p+5 = 15 \quad \text{or} \quad -2p-5 = 15$$

Consider

$$\begin{aligned} 2p+5 &= 15 \\ 2p+5-5 &= 15-5 && \text{[Add } -5 \text{ to both sides of the equation]} \\ 2p+0 &= 10 && \text{[Do the addition]} \\ 2p &= 10 && \text{[0 is the additive identity]} \\ \frac{2p}{2} &= \frac{10}{2} && \text{[Divide both sides of the equation by 2]} \\ p &= 5 \end{aligned}$$

Now consider,

$$\begin{aligned} -2p-5 &= 15 \\ -2p-5+5 &= 15+5 && \text{[Add 5 to both sides]} \\ -2p+0 &= 20 && \text{[Do the addition]} \\ -2p &= 20 && \text{[0 is the additive identity]} \\ \frac{-2p}{-2} &= \frac{20}{-2} && \text{[Divide both sides of the equation by } -2\text{]} \\ p &= -10 && \text{[Do the division]} \end{aligned}$$

Thus, $\boxed{p = 5, p = -10}$

Answer 55e.

The first step in factoring an expression is to check whether there is any common monomial, other than 1.

There is no common factor, other than 1, in the expression on the left side.

The expression $49z^2 - 14z + 1$ is of the form $ax^2 + bx + c$. This could be a perfect square trinomial since the first term, $49z^2$ or $(7z)^2$, and the last term, 1 or 1^2, are perfect squares.

Check whether the expression satisfies the second condition. The required middle term is $-14z$. Since $2(7z)(1)$ is $14z$, the expression is a perfect square trinomial.

Factor using special factoring patterns.
$(7z - 1)^2 = 0$

Take the square root of both the sides.

$$\sqrt{(7z-1)^2} = \sqrt{0}$$
$$7z - 1 = 0$$

Solve for z.

$$7z = 1$$
$$z = \frac{1}{7}$$

Therefore, the solution of the given equation is $\frac{1}{7}$.

Answer 56e.

We need to solve the equation

$$8x^2 - 30x + 7 = 0$$

Consider the given equation

$$8x^2 - 30x + 7 = 0$$
$$8x^2 - 2x - 28x + 7 = 0$$
$$2x(4x-1) - 7(4x-1) = 0$$
$$(2x-7)(4x-1) = 0$$
$$(2x-7) = 0 \text{ or } (4x-1) = 0$$
$$2x = 7 \text{ or } 4x = 1$$
$$x = \frac{7}{2} \text{ or } x = \frac{1}{4}$$

Thus, $\boxed{x = \frac{7}{2} \text{ or } x = \frac{1}{4}}$

Answer 57e.

First, divide each side of the equation by -3.

$$\frac{-3(q+2)^2}{-3} = \frac{-18}{-3}$$
$$(q+2)^2 = 6$$

Take the square root on each side.

$$\sqrt{(q+2)^2} = \sqrt{6}$$
$$q+2 = \pm\sqrt{6}$$

Now, subtract 2 from each side.

$$q+2-2 = \pm\sqrt{6}-2$$
$$q = -2\pm\sqrt{6}$$

CHECK

Substitute the solutions for x in the original equation and evaluate.

Let $q = -2+\sqrt{6}$

$$-3(q+2)^2 = -18$$
$$-3\left(-2+\sqrt{6}+2\right)^2 \stackrel{?}{=} -18$$
$$-3(6) \stackrel{?}{=} -18$$
$$-18 = -18 \checkmark$$

Let $q = -2-\sqrt{6}$

$$-3(q+2)^2 = -18$$
$$-3\left(-2-\sqrt{6}+2\right)^2 \stackrel{?}{=} -18$$
$$-3(6) \stackrel{?}{=} 8$$
$$-18 = -18 \checkmark$$

Therefore, the solutions are $-2+\sqrt{6}$ and $-2-\sqrt{6}$.

Answer 58e.

We need to solve the matrix equation

$$\begin{bmatrix} 1 & 5 \\ -2 & -1 \end{bmatrix} X = \begin{bmatrix} -3 & 5 \\ 6 & -1 \end{bmatrix}$$

Consider the given equation

$$\begin{bmatrix} 1 & 5 \\ -2 & -1 \end{bmatrix} X = \begin{bmatrix} -3 & 5 \\ 6 & -1 \end{bmatrix}$$

Let

$$A = \begin{bmatrix} 1 & 5 \\ -2 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 5 \\ 6 & -1 \end{bmatrix}$$

Thus, the given equation is

$$AX = B$$

$A^{-1}AX = A^{-1}B$ [Multiply by A^{-1} to both sides of the equation]

$IX = A^{-1}B$ [$A^{-1}A = I$]

$X = A^{-1}B$ [I is the identity matrix, so $IX = X$]

Hence we need to find the inverse of A and multiply with B.

To find the inverse of a 2×2 matrix:

Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then

$$A^{-1}=\frac{1}{(ad-bc)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Thus, if

$$A=\begin{bmatrix} 1 & 5 \\ -2 & -1 \end{bmatrix}$$

Then

$$A^{-1}=\frac{1}{(-1-(-10))}\begin{bmatrix} -1 & -5 \\ 2 & 1 \end{bmatrix}$$

$$=\frac{1}{9}\begin{bmatrix} -1 & -5 \\ 2 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} \frac{-1}{9} & \frac{-5}{9} \\ \frac{2}{9} & \frac{1}{9} \end{bmatrix}$$

We have,

$$X=A^{-1}B$$

$$A^{-1}=\begin{bmatrix} \frac{-1}{9} & \frac{-5}{9} \\ \frac{2}{9} & \frac{1}{9} \end{bmatrix} \text{ and } B=\begin{bmatrix} -3 & 5 \\ 6 & -1 \end{bmatrix}$$

Thus,

$$A^{-1}B=\begin{bmatrix} \frac{-1}{9} & \frac{-5}{9} \\ \frac{2}{9} & \frac{1}{9} \end{bmatrix}\begin{bmatrix} -3 & 5 \\ 6 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} \left(\frac{-1}{9}\right)(-3)+\left(\frac{-5}{9}\right)(6) & \left(\frac{-1}{9}\right)(5)+\left(\frac{-5}{9}\right)(-1) \\ \left(\frac{2}{9}\right)(-3)+\left(\frac{1}{9}\right)(6) & \left(\frac{2}{9}\right)(5)+\left(\frac{1}{9}\right)(-1) \end{bmatrix}$$

$$=\begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus, $\boxed{X = \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix}}$

Answer 59e.

The given equation has the form $AX = B$. Thus,

$A = \begin{bmatrix} -2 & 1 \\ 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 0 \\ -1 & 10 \end{bmatrix}$.

In order to solve the matrix equation, first multiply both sides of the equation $AX = B$ by A^{-1} on the left.

$A^{-1}AX = A^{-1}B$

The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Find the determinant of A first. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{aligned} |A| &= \begin{vmatrix} -2 & 1 \\ 4 & 0 \end{vmatrix} \\ &= (-2)(0) - (1)(4) \\ &= -2 - 4 \\ &= -6 \end{aligned}$$

Now, find A^{-1}.

$$A^{-1} = \frac{1}{-6}\begin{bmatrix} 0 & -1 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{6} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

Substitute the known values in $A^{-1}AX = A^{-1}B$.

$$\begin{bmatrix} 0 & \frac{1}{6} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 4 & 0 \end{bmatrix}X = \begin{bmatrix} 0 & \frac{1}{6} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}\begin{bmatrix} 6 & 0 \\ -1 & 10 \end{bmatrix}$$

Next, multiply the matrices. We know that $AA^{-1} = I$.
To find the element in the ith row and jth column of the product matrix $A^{-1}B$, multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, then add the products.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 0(6) + \frac{1}{6}(-1) & 0(0) + \frac{1}{6}(10) \\ \frac{2}{3}(6) + \frac{1}{3}(-1) & \frac{2}{3}(0) + \frac{1}{3}(10) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{6} & \frac{5}{3} \\ \frac{11}{3} & \frac{10}{3} \end{bmatrix}$$

The equation is now in the form $IX = A^{-1}B$.

The matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an identity matrix. We know that $IX = X$.

Thus, $X = A^{-1}B$.

The matrix for X is $\begin{bmatrix} -\frac{1}{6} & \frac{5}{3} \\ \frac{11}{3} & \frac{10}{3} \end{bmatrix}$.

Answer 60e.

We need to solve the matrix equation

$$\begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix} X = \begin{bmatrix} -3 & 1 & 2 \\ 0 & -4 & -1 \end{bmatrix}$$

Consider the given equation

$$\begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix} X = \begin{bmatrix} -3 & 1 & 2 \\ 0 & -4 & -1 \end{bmatrix}$$

Let

$$A = \begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 1 & 2 \\ 0 & -4 & -1 \end{bmatrix}$$

Thus, the given equation is

$$AX = B$$

$A^{-1}AX = A^{-1}B$ [Multiply by A^{-1} to both sides of the equation]

$IX = A^{-1}B$ [$A^{-1}A = I$]

$X = A^{-1}B$ [I is the identity matrix, so $IX = X$]

Hence we need to find the inverse of A and multiply with B.

To find the inverse of a 2×2 matrix:

Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then

$$A^{-1}=\frac{1}{(ad-bc)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Thus, if

$$A=\begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}$$

Then

$$A^{-1}=\frac{1}{(10-12)}\begin{bmatrix} 2 & -3 \\ -4 & 5 \end{bmatrix}$$

$$=\frac{1}{-2}\begin{bmatrix} 2 & -3 \\ -4 & 5 \end{bmatrix}$$

$$=\begin{bmatrix} -1 & \frac{3}{2} \\ 2 & \frac{-5}{2} \end{bmatrix}$$

We have,

$$X=A^{-1}B$$

$$A^{-1}=\begin{bmatrix} -1 & \frac{3}{2} \\ 2 & \frac{-5}{2} \end{bmatrix} \text{ and } B=\begin{bmatrix} -3 & 1 & 2 \\ 0 & -4 & -1 \end{bmatrix}$$

Thus,

$$A^{-1}B=\begin{bmatrix} -1 & \frac{3}{2} \\ 2 & \frac{-5}{2} \end{bmatrix}\begin{bmatrix} -3 & 1 & 2 \\ 0 & -4 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} (-1)(-3)+\left(\frac{3}{2}\right)(0) & (-1)(1)+\left(\frac{3}{2}\right)(-4) & (-1)(2)+\left(\frac{3}{2}\right)(-1) \\ (2)(-3)+\left(\frac{-5}{2}\right)(0) & (2)(1)+\left(\frac{-5}{2}\right)(-4) & (2)(2)+\left(\frac{-5}{2}\right)(-1) \end{bmatrix}$$

$$=\begin{bmatrix} 3 & -7 & \frac{-7}{2} \\ -6 & 12 & \frac{13}{2} \end{bmatrix}$$

Thus, $X = \begin{bmatrix} 3 & -7 & \frac{-7}{2} \\ -6 & 12 & \frac{13}{2} \end{bmatrix}$

Answer 61e.

The given equation has the form $AX = B$. Thus,

$A = \begin{bmatrix} 2 & -8 \\ 3 & -7 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & -3 \end{bmatrix}$.

In order to solve the matrix equation, first multiply both the sides of the equation $AX = B$ by A^{-1} on the left.

$A^{-1}AX = A^{-1}B$

The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Find the determinant of A first. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{aligned} |A| &= \begin{vmatrix} 2 & -8 \\ 3 & -7 \end{vmatrix} \\ &= (2)(-7) - (-8)(3) \\ &= -14 - (-24) \\ &= 10 \end{aligned}$$

Now, find A^{-1}.

$$\begin{aligned} A^{-1} &= \frac{1}{10}\begin{bmatrix} -7 & 8 \\ -3 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -\frac{7}{10} & \frac{4}{5} \\ -\frac{3}{10} & \frac{1}{5} \end{bmatrix} \end{aligned}$$

Substitute the known values in $A^{-1}AX = A^{-1}B$.

$$\begin{bmatrix} -\frac{7}{10} & \frac{4}{5} \\ -\frac{3}{10} & \frac{1}{5} \end{bmatrix}\begin{bmatrix} 2 & -8 \\ 3 & -7 \end{bmatrix}X = \begin{bmatrix} -\frac{7}{10} & \frac{4}{5} \\ -\frac{3}{10} & \frac{1}{5} \end{bmatrix}\begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & -3 \end{bmatrix}$$

Next, multiply the matrices. We know that $AA^{-1} = I$.
To find the element in the ith row and jth column of the product matrix $A^{-1}B$, multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, then add the products.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} -\frac{7}{10} & \frac{4}{5} \\ -\frac{3}{10} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{7}{10}(-1) + \frac{4}{5}(3) & -\frac{7}{10}(4) + \frac{4}{5}(0) & -\frac{7}{10}(2) + \frac{4}{5}(-3) \\ -\frac{3}{10}(-1) + \frac{1}{5}(3) & -\frac{3}{10}(4) + \frac{1}{5}(0) & -\frac{3}{10}(2) + \frac{1}{5}(-3) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{10} + \frac{12}{5} & -\frac{14}{5} & -\frac{7}{5} - \frac{12}{5} \\ \frac{3}{10} + \frac{3}{5} & -\frac{6}{5} & -\frac{3}{5} - \frac{3}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{31}{10} & -\frac{14}{5} & -\frac{19}{5} \\ \frac{9}{10} & -\frac{6}{5} & -\frac{6}{5} \end{bmatrix}$$

The equation is now in the form $IX = A^{-1}B$.

The matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an identity matrix.. We know that $IX = X$.

Thus, $X = A^{-1}B$.

The matrix for X is $\begin{bmatrix} \frac{31}{10} & -\frac{14}{5} & -\frac{19}{5} \\ \frac{9}{10} & -\frac{6}{5} & -\frac{6}{5} \end{bmatrix}$.

Answer 62e.

We need to find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

$x^2 - 4x + 11 = 0$

Consider the given equation

$x^2 - 4x + 11 = 0$

Let us compare the given equation with the standard equation

$ax^2 + bx + c = 0$

The discriminant of the quadratic equation is given by

$D = b^2 - 4ac$

The discriminant of the given equation is

$$\begin{aligned} D &= (-4)^2 - 4(1)(11) \\ &= 16 - 44 \\ &= -28 \\ &< 0 \end{aligned}$$

Since the discriminant is less than zero, both the roots of the quadratic equation are imaginary.

Answer 63e.

In the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression $b^2 - 4ac$ is called the discriminant.

Substitute 1 for a, -4 for b, and 49 for c in $b^2 - 4ac$ and evaluate.

$$\begin{aligned} b^2 - 4ac &= (-4)^2 - 4(1)(49) \\ &= 16 - 196 \\ &= -180 \end{aligned}$$

The discriminant of the given quadratic equation is less than 0.
Therefore, the equation has two imaginary solutions.

Answer 64e.

We need to find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

$3t^2 - 8t - 5 = 0$

Consider the given equation

$3t^2 - 8t - 5 = 0$

Let us compare the given equation with the standard equation

$at^2 + bt + c = 0$

The discriminant of the quadratic equation is given by

$D = b^2 - 4ac$

The discriminant of the given equation is

$$\begin{aligned} D &= (-8)^2 - 4(3)(-5) \\ &= 64 + 60 \\ &= 124 \\ &> 0 \end{aligned}$$

Since the discriminant is greater than zero, both the roots of the quadratic equation are distinct and real.

Answer 65e.

In the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression $b^2 - 4ac$ is called the discriminant.

Substitute -2 for a, -5 for b, and -3 for c in $b^2 - 4ac$ and evaluate.

$$\begin{aligned} b^2 - 4ac &= (-5)^2 - 4(-2)(-3) \\ &= 25 - 24 \\ &= 1 \end{aligned}$$

The discriminant of the given quadratic equation is greater than 0.
Therefore, the equation has two real solutions.

Answer 66e.

We need to find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

$81p^2 + 18p + 1 = 0$

Consider the given equation

$81p^2 + 18p + 1 = 0$

Let us compare the given equation with the standard equation

$ap^2 + bp + c = 0$

The discriminant of the quadratic equation is given by

$D = b^2 - 4ac$

The discriminant of the given equation is

$$\begin{aligned} D &= (18)^2 - 4(81)(1) \\ &= 324 - 324 \\ &= 0 \end{aligned}$$

Since the discriminant is equal to zero, both the roots of the quadratic equation are real and equal.

Answer 67e.

In the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression $b^2 - 4ac$ is called the discriminant.

Substitute 7 for a, 0 for b, and 5 for c in $b^2 - 4ac$ and evaluate.

$$\begin{aligned} b^2 - 4ac &= 0^2 - 4(7)(5) \\ &= 0 - 140 \\ &= -140 \end{aligned}$$

The discriminant of the given quadratic equation is less than 0.
Therefore, the equation has two imaginary solutions.