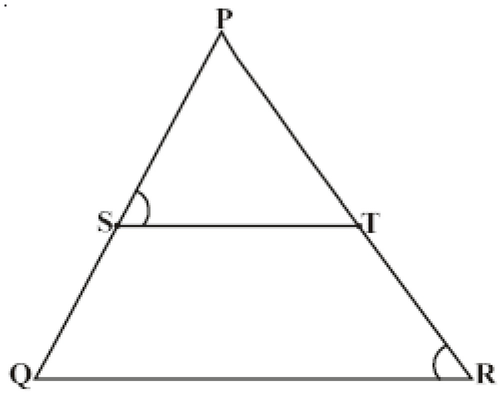
Similar Triangles

Exercise 8.1

Q. 1. In Δ PQR, ST is a line such that $\frac{PS}{SQ} = \frac{PT}{TR}$ and also ∠PST = ∠PRQ. Prove that ΔPQR, is an isosceles triangle.



Answer :

Given,

$\frac{PS}{SQ} = \frac{PT}{TR}$

Also, ∠PST = ∠PRQ

⇒ Need to prove that PQR is an isosceles triangle.

⇒ If a line divides any two sides of a triangles in the same ratio then the same line is parallel to the third side.

⇒ Since, ST || QR

⇒ ∠PST = ∠PQR …………eq(1)

(Corresponding angles)

Also given ∠PST = ∠PRQ ……………eq(2)

From eq(1) and eq(2) we get

⇒ ∠PQR = ∠PRQ

Since, sides opposite to equal angles are equal

⇒ PR = PQ

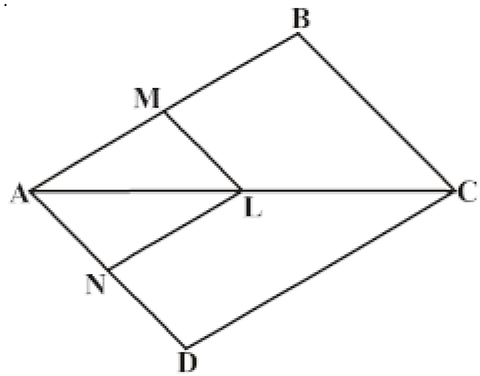
∴ two sides of ΔPQR is equal

PQR is an isosceles triangle

Hence proved.

Q. 2. In the given figure, LM || CB and LN || CD Prove that

$$\frac{AM}{AB} = \frac{AN}{AD}$$



Answer : Given, LM||CB and LN||CD

Need to prove

$$\frac{AM}{AB} = \frac{AN}{AD}$$

⇒ In Δ ACB, LM || CB

⇒ now, we know that line drawn parallel to one side of the triangle, intersects the other two sides in distinct points, then it divides the other 2 side in same ratio.

$\therefore \frac{AL}{LC} = \frac{AM}{MB}$ ....eq(1)

And in Δ ACD, LN||CD

$\Rightarrow \frac{AL}{LC} = \frac{AN}{ND}$ ....eq(2)

From eq(1) and eq(2), we get

$\Rightarrow \frac{AM}{MB} = \frac{AN}{ND}$

$\Rightarrow \frac{MB}{AM} = \frac{ND}{AN}$

Adding 1 on both sides we get

$\Rightarrow \frac{MB}{AM} + 1 = \frac{ND}{AN} + 1$

$\Rightarrow \frac{MB + AM}{AM} = \frac{ND + AN}{AN}$

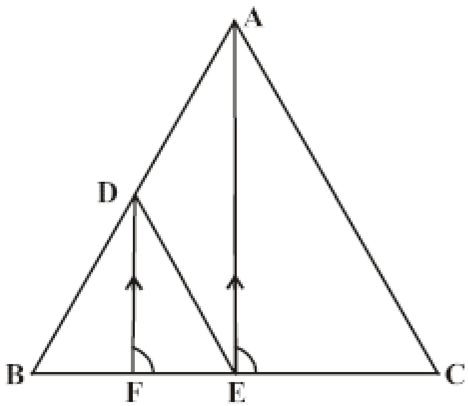
$\Rightarrow \frac{AB}{AM} = \frac{AD}{AN}$

$\Rightarrow \frac{AM}{AB} = \frac{AN}{AD}$

Hence, proved.

Q. 3. In the given figure, DE||AC and DF||AE Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}.$$



Answer :

Given, DE || AC and DF || AE

Need to prove

$$\frac{BF}{FE} = \frac{BE}{EC}$$

⇒ In Δ ABC, DE || AC

⇒ now, we know line drawn parallel to one side of triangle, intersects the other two sides in distinct points, then it divides the other 2 side in same ratio.

$$\Rightarrow \frac{BE}{EC} = \frac{BD}{DA} \text{.........eq(1)}$$

⇒ In ΔAEB, DF || AE

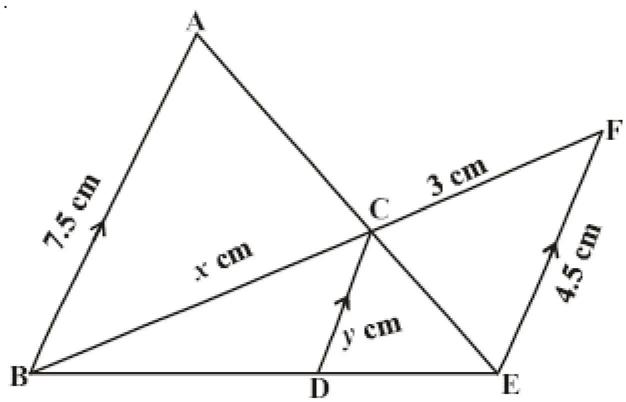
$$\Rightarrow \frac{BF}{FE} = \frac{BD}{DA} \text{.........eq(2)}$$

From (1) and (2)

$\Rightarrow \frac{BF}{FE} = \frac{BE}{EC}$

Hence proved.

Q. 4. In the given figure, AB||CD||EF. given AB = 7.5 cm, DC = ycm, EF = 4.5cm, BC = x cm. Calculate the values of x and y.



Answer : Given, Ab = 7.5cm, DC = ycm, EF = 4.5cm and BC = xcm

Need to calculate values of x and y

⇒ Let us consider Δ ACB and Δ CEF

⇒ Both are similar triangles

$\therefore \frac{AB}{EF} = \frac{X}{3}$

⇒ x = 5cm

⇒ let us consider Δ BCD and Δ BFE

⇒ from basic proportionality theorem we have we know that line drawn parallel to one side of the triangle, intersects the other two sides in distinct points, then it divides the other 2 side in same ratio.

$\Rightarrow \frac{x}{y} = \frac{x+3}{4.5}$

$\Rightarrow y = \frac{5 \times 4.5}{8}$

$\Rightarrow Y = \frac{45}{16}$

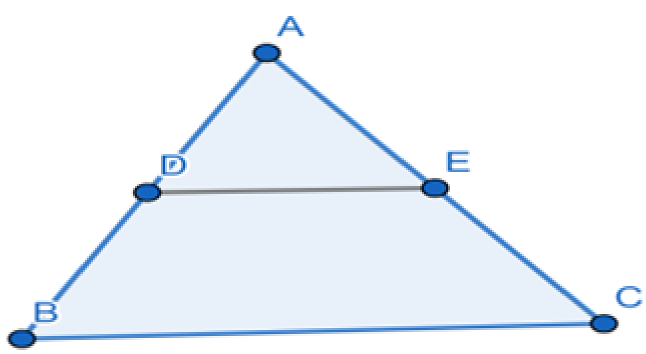
$\Rightarrow Y = 2\frac{13}{16}$

Hence, the value of X is 5cm and Y is $2\frac{13}{16}$

Q. 5. Prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side (Using basic proportionality theorem).

Answer : To prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side

⇒ Let us assume Δ ABC where DE is parallel to BC and D is the midpoint of AB.



<u>Proof:</u>

In Δ ABC, DE||BC

∴ AD = DB

Since, D is the midpoint of AB

$\Rightarrow \frac{AD}{DB} = 1$ ……..eq(1)

⇒ now we know that basic proportionality theorem if a line drawn to one side of a triangle intersects the other two sides in distinct points, then it divides the other 2 side in the same ratio.

$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow 1 = \frac{AE}{EC}$ ....eq(2)

From eq(1) and eq(2)

⇒ EC = AE

⇒ E is the midpoint of AC

Hence proved.

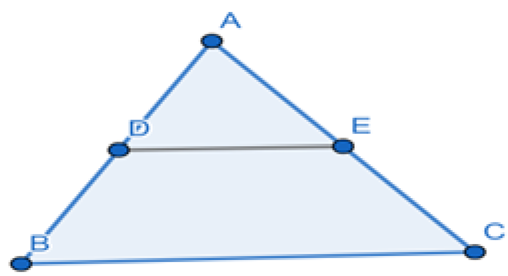
Q. 6. Prove that a line joining the midpoints of any two sides of a triangle is parallel to the third side. (Using converse of basic proportionality theorem)

Answer : To prove that a line joining the midpoints of any two sides of a triangle is parallel to the third side.

⇒ now we know the converse of a basic proportionality theorem is if a line divides any two sides of a triangle in the same ratio then the line must be parallel to the third side.

⇒ Let us assume Δ ABC in which D and E are the mid points of AB and Ac respectively such that

⇒ AD = BD and AE = EC.



⇒ To prove that DE || BC

⇒ D is the midpoint of AB

∴ AD = DB

$\Rightarrow \frac{AD}{DB} = 1$ ....eq(1)

Also, E is the midpoint of AC

∴ AE = EC

⇒ $\frac{AE}{EC} = 1$ ……..eq(2)

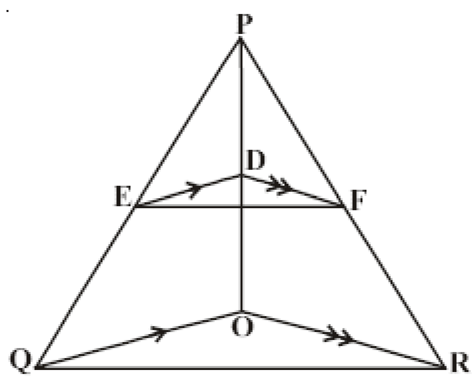
From equation (1) and (2) we get

⇒ $\frac{AD}{BD} = \frac{AE}{EC}$

∴ DE || BC by converse of proportionality thereom

Hence, the line joining the mid points of any two sides of a triangle is parallel to three sides.

Q. 7. In the given figure, DE||OQ and DF||OR. Show that EF||QR.



Answer : Given, DE ||OQ and DF || OR

Need to prove that EF || QR

⇒ Let us consider Δ POQ

⇒ By Basic proportionality theorem we have if a line drawn to one side of a triangle intersects the other two sides in distinct points, then it divides the other 2 side in the same ratio.

$\therefore \frac{PE}{EQ} = \frac{PD}{DO}$ ....eq(1)

⇒ Consider Δ POR

⇒ DF || OR

$\therefore \frac{PF}{FR} = \frac{PD}{DO}$ ....eq(2)

⇒ From eq(1) and eq(2) we have

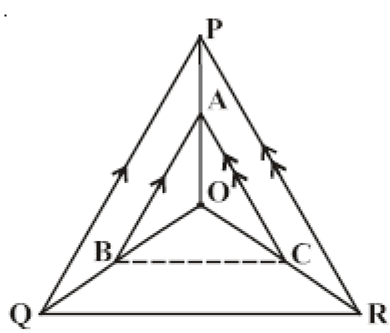
$\Rightarrow \frac{PE}{EQ} = \frac{PF}{FR}$

∴ By converse of basic proportionality theorem we have if a line divides any two sides of a triangle in the same ratio then the line must be parallel to the third side.

∴ EF || QR

Hence proved

Q. 8. In the adjacent figure, A, B, and C are points on OP, OQ and Or respectively such that AB||PQ and AC||PR. Show that BC||QR.



Answer : Given, AB || PQ and AC||PR

Need to prove BC || QR

⇒ In ΔOPQ, AB || PQ

⇒ Since, line drawn parallel to one side of triangle, intersects the other two sided in distinct point, then it divides the other 2 sides in same ratio.

$$\Rightarrow \frac{OA}{AP} = \frac{OB}{OQ} \text{...............eq(1)}$$

⇒ In Δ OPR, AC || PR

$$\Rightarrow \frac{OC}{CR} = \frac{OA}{AP} \text{...............eq(2)}$$

From eq(1) and (2)

$$\Rightarrow \frac{OC}{CR} = \frac{OB}{OQ}$$

Thus in Δ OQR, $\frac{OC}{CR} = \frac{OB}{OQ}$

⇒ Line BC divides the triangle OQR in the same ratio

⇒ We know that if a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

∴ BC || QR

Hence proved.

Q. 9. ABCD is a trapezium in which AB||DC and its diagonals intersect each other at point 'O'. Show that

$$\frac{AO}{BO} = \frac{CO}{DO}.$$

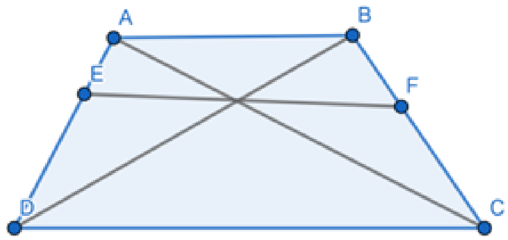
Answer : Given, ABCD is a trapezium where AB||DC

And diagonals intersect at each other ‘O’.

Need to prove

$\frac{AO}{BO} = \frac{CO}{DO}$

⇒ Let us draw a line EF||DC passing through point O.



⇒ Now, in Δ ADC, EO || DC

(because EF || DC)

So, $\frac{AE}{DE} = \frac{AO}{CO}$ ....eq (1)

Now, Line drawn parallel to one side of a triangle intersects the other two sides in distinct points, and then it divides the other 2 sides.

Similarly, in Δ DBA, EO || AB

(Because EF || AB)

⇒ $\frac{AE}{DE} = \frac{BO}{DO}$ ....eq(2)

⇒ From eq(1) and eq(2)

⇒ $\frac{AO}{CO} = \frac{BO}{DO}$

⇒ $\frac{AO}{BO} = \frac{CO}{DO}$

Q. 10. Draw a line segment of length 7.2 cm and divide it in the ratio 5 : 3. Measure the two parts.

Answer : Need to draw a line segment with length 7.2cm

Also, to divide it 5:3 parts. Measure them.

⇒ Let m = 5 and n = 3

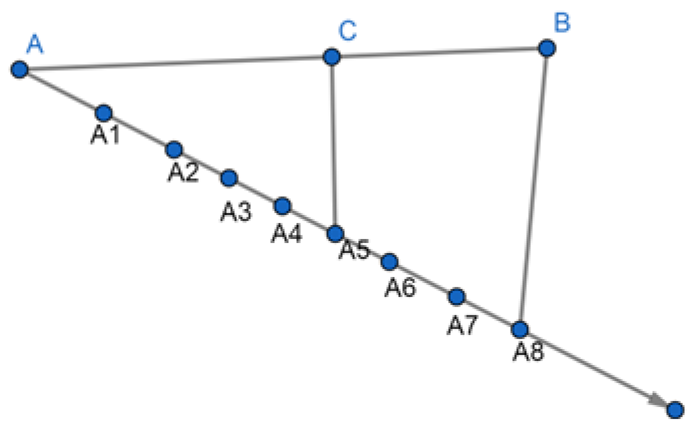
<u>Construction steps:</u>

1) Draw a ray AX, making an acute angle with AB.

2) Locate 8 = m + n points A1,A2,A3,A4,A5,A6,A7,A8 on AX such that AA1 = A1A2 = A2A3 = A3A4 = A4A5 = A5A6 = A6A7 = A7A8

3) Join BA8

4) Through the point A5 (m = 5) draw a line parallel to A8B at A5 intersecting AB at the point C. Then AC:CB = 5:3



⇒ Let the ratio be 5x:3x

⇒ 5x + 3X = 7.2cm

⇒ 8x = 7.2cm.

⇒ $x = \frac{7.2}{8}$

⇒ x = 0.9cm.

Substituting ‘x’ value in 5x,3x we get

$\Rightarrow 5x = 5 \times 0.9$

$= 4.5cm$

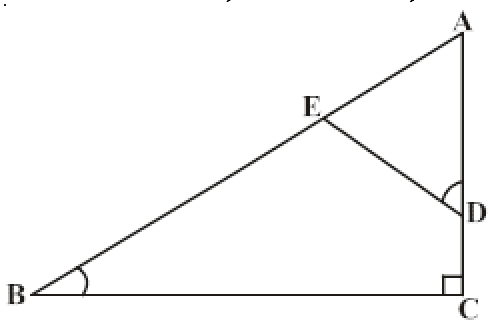
$\Rightarrow 3x = 3 \times 0.9$

$= 2.7cm.$

Exercise 8.2

Q. 1. In the given figure, ∠ADE = ∠B

i. Show that ΔABC ~ ΔADE
ii. If AD = 3.8cm, AE = 3.6 cm, BE = 2.1cm, BC = 4.2cm. find DE.



Answer :

(i) Given, ∠ADE = ∠B = θ,∠C = 90°

⇒ ∠A = 90° -θ

⇒ if ∠A = 90°- θ, ∠B = θ

⇒ ∠ AED = 90°

⇒ now, comparing Δ ABC with Δ AED we have

∠ A common in both triangles

∠ C = ∠ AED = 90°

∠ ADE = ∠ B

∴ By AAA property we have in two triangles if the angles are equal, then sides opposite to the equal angles are in the same ratio (or proportional) and hence the triangles are similar.

Hence, Δ ABC and Δ ADE are similar triangles.

(ii) Given, Ad = 3.8cm, AE = 3.6cm, BE = 2.1cm, BC = 4.2cm

Need to find DE.

As Δ ABC and Δ ADE are similar triangles we have

$$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

$$\Rightarrow \frac{AB}{3.8} = \frac{4.2}{DE} = \frac{AC}{3.6}$$

$$\Rightarrow AE + BE = 3.6 + 2.1 = 5.7$$

$$\Rightarrow \frac{5.7}{3.8} = \frac{4.2}{DE}$$

$$\Rightarrow DE = 4.2 \times \frac{2}{3}$$

= 2.8cm

Hence, the value of DE is 2.8cm

Q. 2. The perimeters of two similar triangles are 30 cm and 20 cm respectively. If one side of the first triangle is 12 cm, determine the corresponding side of the second triangle.

Answer : Given, perimeters of two similar triangles are 30cm and 20cm

And one side of the triangle is 12cm.

Need to find out the side of the second triangle.

⇒ since, the triangle are similar

$$\Rightarrow \frac{Perimeter\ of\ 1st\ triangle}{Perimeter\ of\ 2nd\ triangle} = \frac{Any\ side\ of\ 1st\ triangle}{Corresponding\ side\ of\ 2nd\ triangle}$$

$$\Rightarrow \frac{30}{20} = \frac{12}{x}$$

$$\Rightarrow x = \frac{20 \times 12}{30}$$

⇒ x = 8cm.

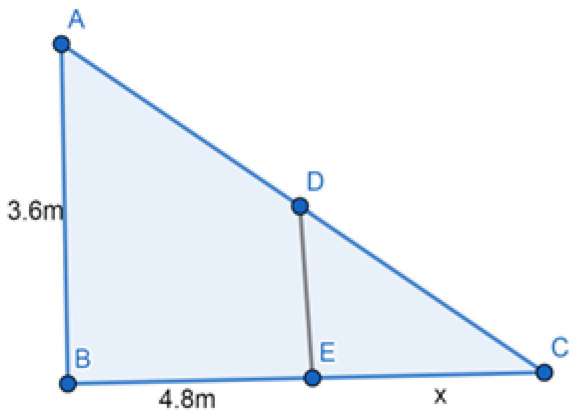
Hence, the corresponding side of the second triangle is 8cm

Q. 3. A girl of height 90 cm is walking away from the base of a lamp post at a speed of 1.2m/sec. If the lamp post is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Answer : Given, a girl with height 90cm and speed 1.2m/sec

And the lamp post is 3.6m above

Need to find the length of her shadow after 4sec



Consider Δ ABC and Δ DEC

⇒ ∠ ABC = ∠ DEC = 90°

⇒ ∠ ACB = ∠ DCE

⇒ Δ ABC ~ Δ DEC

⇒ By converse proportionality theorem if a line divides any two sides of a triangle in the same ratio then the line must be parallel to the third side.

$\Rightarrow$ we have $\frac{DE}{AB} = \frac{CE}{BC}$

$\Rightarrow \frac{0.9}{3.6} = \frac{CE}{CE + EB}$

$\Rightarrow \frac{1}{4} = \frac{x}{x + 4.8}$

$\Rightarrow$ x + 4.8 = 4x

$\Rightarrow$ 4x – x = 4.8

$\Rightarrow$ 3x = 4.8

$\Rightarrow X = \frac{4.8}{3}$

$\Rightarrow$ X = 1.6m

CE = 1.6m

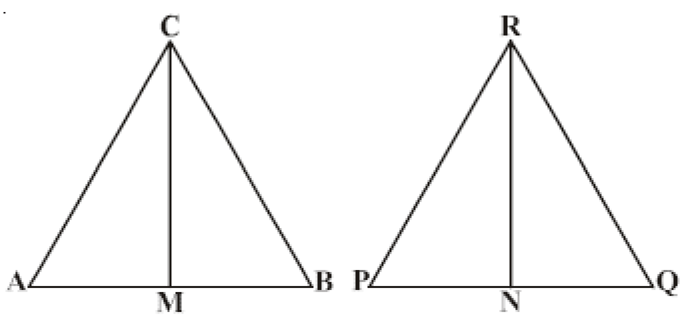
Hence, the length of her shadow after 4 sec is 1.6m

Q. 4. Given that Δ ABC ~ Δ PQR, CM and RN are respectively the medians of Δ ABC and Δ PQR Prove that

i. Δ AMC ~ Δ PNR

ii. $\frac{CM}{RN} = \frac{AB}{PQ}$

iii. ΔCMB ~ ΔRNQ



Answer :

(i) Given, Δ ABC ~ Δ PQR

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ ....eq(1)

And ∠ A = ∠ P, ∠ B = ∠ Q and ∠ C = ∠ R ……..eq(2)

⇒ As CM and RN are medians

⇒ AB = 2AM and PQ = 2PN

From eq(1) we have

$\Rightarrow \frac{2AM}{2PN} = \frac{CA}{RP}$

i.e., $\frac{AM}{PN} = \frac{CA}{RP}$ ....eq(3)

Also, from eq(2) Δ MAC = Δ NPR ……eq(4)

⇒ From eq(3) and eq(4) we have

⇒ Δ AMC ~ Δ PNR …..eq(5)

⇒ By SAS similarity if one angle of a triangle is equal to another angle of a triangle and the including sides of the these angles are proportional, then the two triangles are similar.

(ii) From eq(5) we have $\frac{CM}{RN} = \frac{CA}{RP}$ ....eq(6)

⇒ From eq(1) we have,

$\Rightarrow \frac{CA}{RP} = \frac{AB}{PQ}$ ....eq(7)

⇒ From eq(6) and eq(7) we have

$\Rightarrow \frac{CM}{RN} = \frac{AB}{PQ}$ .....eq(8)

(iii) Again from eq(1) we have

$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR}$

⇒ From eq(8) we have

$\Rightarrow \frac{CM}{RN} = \frac{BC}{QR}$

$\Rightarrow \frac{CM}{RN} = \frac{AB}{PQ} = \frac{2BM}{2QN}$

i.e., $\frac{CM}{RN} = \frac{BM}{QN}$ .....eq(10)

⇒ From eq(9) and eq(10) we have

$\Rightarrow \frac{CM}{RN} = \frac{BC}{QR} = \frac{BM}{QN}$

∴ Δ CMB ~ Δ RNQ

Q. 5. Diagonals AC and BD of a trapezium ABCD with AB||DC intersect each other at the point 'O'. Using the criterion of similarity for two triangles, show that

$$\frac{OA}{OC} = \frac{OB}{OD}.$$

Answer : Given, trapezium ABCD and diagonals AC and BD intersect each other.

Need to prove

$$\frac{OA}{OC}=\frac{OB}{OD}.$$

⇒ Let us consider Δ AOB and Δ DOC

⇒ ∠ AOB = ∠ DOC (vertically opposite angles)

⇒ by alternative interior angles we have

⇒ ∠ OAB = ∠ OCD

⇒ ∠ OBA = ∠ ODC

⇒ By AAA similarity we have in two triangles if the angles are equal, then sides opposite to the equal angles are in the same ratio (or proportional) and hence the triangles are similar.

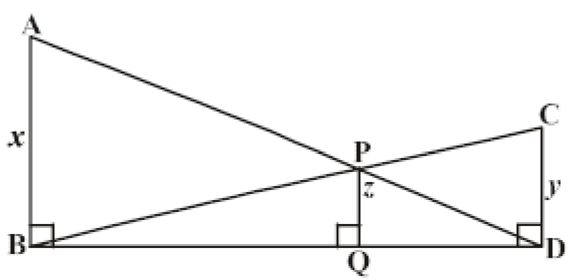
⇒ Δ Aob ~ Δ DOC

⇒ $\frac{OA}{OC} = \frac{OB}{OD}$

Hence proved.

Q. 6. AB, CD, PQ are perpendicular to BD. AB = x, CD = y and PQ = z prove that

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{z}.$$



Answer : Given, in Δ BCD, PQ || CD

$\frac{BQ}{BD} = \frac{PQ}{CD}$ ....eq(1)

And in Δ ABD, PQ||AB

$\Rightarrow \frac{QD}{BD} = \frac{PQ}{AB}$ ....eq(2)

Need to prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

⇒ From eq(1) and eq(2) we have

$\Rightarrow 1 - \frac{BQ}{BD} = \frac{PQ}{AB}$

$\Rightarrow$ 1-$\frac{PQ}{CD} = \frac{PQ}{AB}$ [FROM EQ(1)]

$\Rightarrow$ 1 = PQ($\frac{1}{CD} + \frac{1}{AB}$)

$\Rightarrow \frac{1}{PQ} = \left(\frac{1}{CD} + \frac{1}{AB}\right)$ .....eq(3)

Since, from the question we know that AB = X CD = Y and PQ = z

Substituting those values in eq(3) we get

$\Rightarrow \frac{1}{z} = \frac{1}{y} + \frac{1}{x}$

$\therefore \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

Hence proved

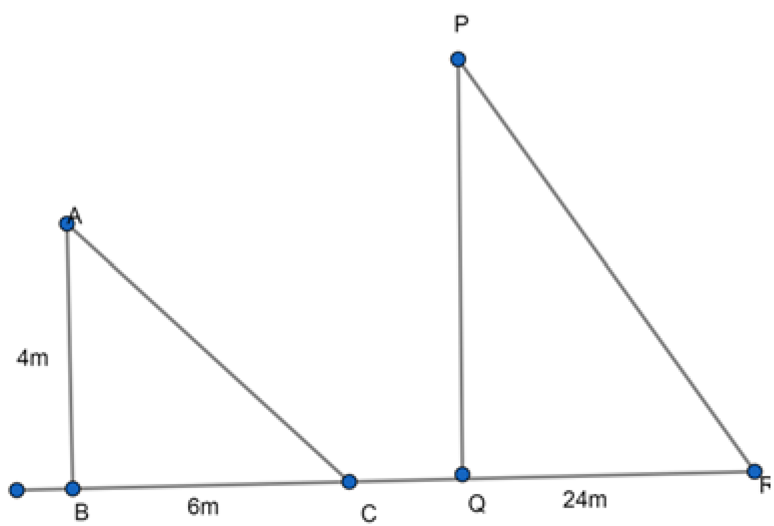
Q. 7. A flag pole 4 m tall casts a 6 m., shadow. At the same time, a nearby building casts a shadow of 24m. How tall is the building?

Answer : Given, a flag pole 4m tall with shadow 6m

And a building shadow 24m

Need to calculate the building

In Δ ABC and Δ PQR



⇒ ∠ B = ∠ Q = 90°

⇒ ∠ C = ∠ R

At any instance all sun rays are parallel, AC || PR

⇒ by AAA similarity we have in two triangles if the angles are equal, then sides opposite to the equal angles are in the same ratio (or proportional) and hence the triangles are similar.

⇒ Δ ABC ~ Δ PQR

⇒ By Converse proportionality theorem we have

if a line divides any two sides of a triangle in the same ratio then the line must be parallel to the third side.

$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR}$

$\Rightarrow \frac{4}{PQ} = \frac{6}{24}$

$\Rightarrow PQ = \frac{4 \times 24}{6}$

= 16m

Hence the height of a building is 16m

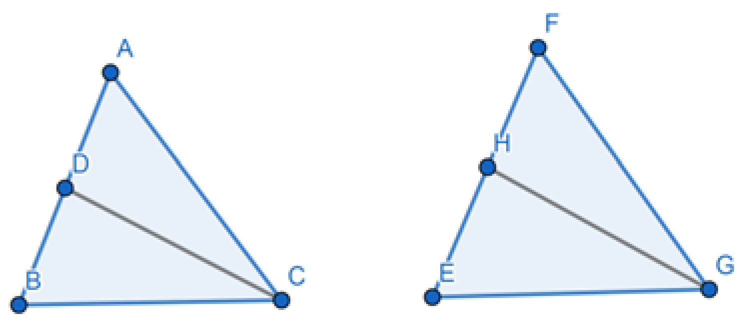
Q. 8. CD and GH are respectively the bisectors of ∠ACE and ∠EGF such that D and H lie on sides AB and FE of Δ ABC and Δ FEG respectively. If Δ ABC ~ Δ FEG then show that

i. $\frac{CD}{GH} = \frac{AC}{FG}$

ii. Δ DCB ~ ΔHGE

iii. Δ DCA ~ ΔHGF

Answer :



Given, Δ ABC ~ Δ FEG …..eq(1)

⇒ corresponding angles of similar triangles

⇒ ∠ BAC = ∠ EFG ….eq(2)

And ∠ ABC = ∠ FEG …….eq(3)

⇒ ∠ ACB = ∠ FGE

⇒ $\frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE$

⇒ ∠ ACD = ∠ FGH and ∠ BCD = ∠ EGH ……eq(4)

Consider Δ ACD and Δ FGH

⇒ From eq(2) we have

⇒ ∠ DAC = ∠ HFG

⇒ From eq(4) we have

⇒ ∠ ACD = ∠ EGH

Also, ∠ ADC = ∠ FGH

⇒ If the 2 angle of triangle are equal to the 2 angle of another triangle, then by angle sum property of triangle 3^{rd} angle will also be equal.

⇒ by AAA similarity we have in two triangles if the angles are equal, then sides opposite to the equal angles are in the same ratio (or proportional) and hence the triangles are similar.

∴ Δ ADC ~ Δ FHG

⇒ By Converse proportionality theorem

⇒ $\frac{CD}{GH} = \frac{AC}{FG}$

Consider Δ DCB and Δ HGE

From eq(3) we have

⇒ ∠ DBC = ∠ HEG

⇒ From eq(4) we have

⇒ ∠ BCD = ∠ FGH

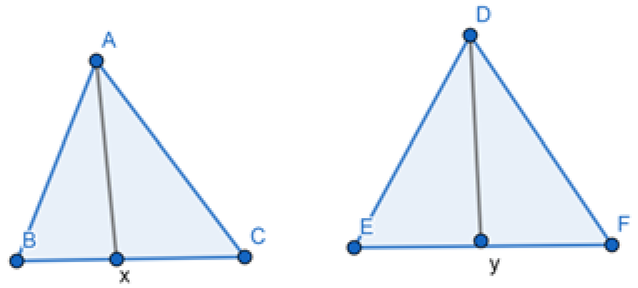
Also, ∠ BDC = ∠ EHG

∴ Δ DCB ~ ΔHGE

Hence proved.

Q. 9. AX and DY are altitudes of two similar Δ ABC and ΔDEF. Prove that AX : DY = AB : DE.

Answer :



Given, Δ ABC ~ Δ DEF

⇒ ∠ ABC = ∠ DEF

⇒ consider Δ ABX and Δ DEY

⇒ ∠ ABX = ∠ DEY

⇒ ∠ AXB = ∠ DYE = 90°

⇒ ∠ BAX = ∠ EDY

⇒ By AAA property we have in two triangles if the angles are equal, then sides opposite to the equal angles are in the same ratio (or proportional) and hence the triangles

⇒ Δ ABX ~ Δ DEY

⇒ By Converse proportionality theorem we have

if a line divides any two sides of a triangle in the same ratio then the line must be parallel to the third side.

⇒ $\frac{AX}{DY} = \frac{AB}{DE}$

∴ AX:DY = AB:DE

Hence proved

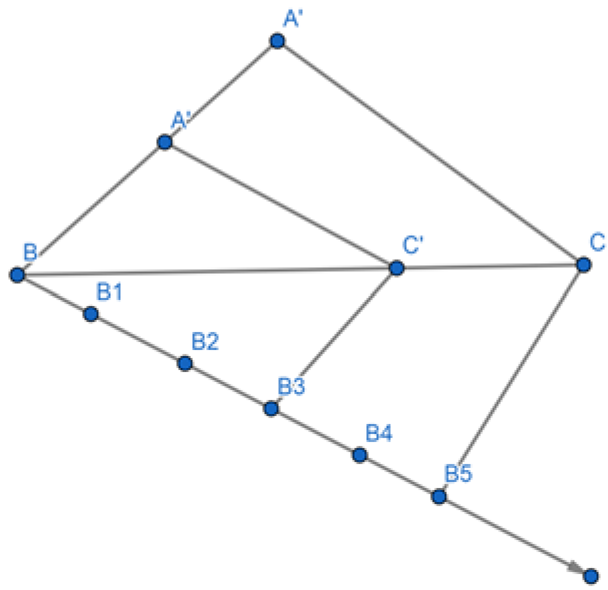
Q. 10. Construct a triangle shadow similar to the given ΔABC, with its sides equal to $\frac{5}{3}$ of the corresponding sides of the triangle ABC.

Answer : Given, a Δ ABC, we are required to construct a triangle whose side are of the corresponding sides of Δ ABC

Construction Steps:

1) Draw any ray BX making an acute angel with BC on the sides opposite to the vertex A.

2) Locate the points B1, B2, B3, B4, B5 on BX such that BB1 = B1B2 = B2B3 = B3B4 = B4B5.



3) Join B3 to C as 3 being smaller and through B5 draw a line parallel to B3C, intersecting the extended line segment BC at C'.

4) Draw a line through C parallel to CA intersecting the extended line segment BA at A'.

Then A'BC' is the required triangle.

Justification:

Note Δ ABC ~ Δ A'BC'

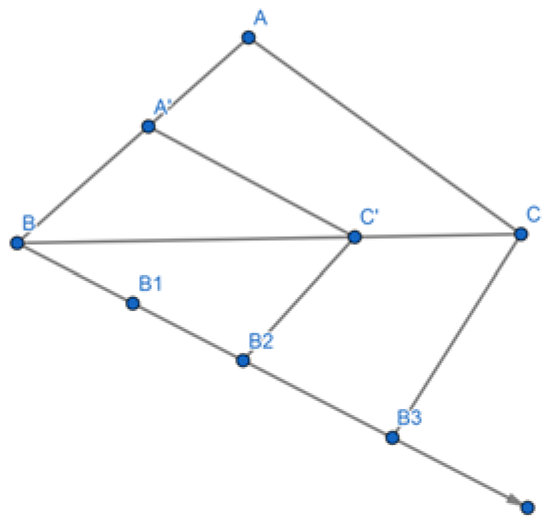
$\therefore \frac{AB}{A'B} = \frac{AC}{A'C'} = \frac{BC}{BC'}$

So, $\frac{BC}{BC'} = \frac{5}{3}$

$\therefore \frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{5}{3}$

Q. 11. Construct a triangle of sides 4 cm, 5 cm and 6 cm. Then, construct a triangle similar to it, whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

Answer :



Given, sides AB = 4cm, BC = 5cm, CA = 6cm.

Need to construct a triangle whose sides are $\frac{2}{3}$ of the corresponding sides of Δ ABC.

Construction steps:

1) Making an angle of 30° draw any ray BX with the base BC of Δ ABC on the opposite side of the vertex A

2) Locate three points B1,B2,B3 on BX so that BB1 = BB2 = BB3, the number of points should be greater of m and n in the scale factor $\frac{m}{n}$

3) Join B2 to C' and draw a line through B3 parallel to B2C intersecting the line segment BC at C.

4) Draw a line through C parallel to C'A intersecting the line segment BA at A'. Then A'B'C is the required triangle.

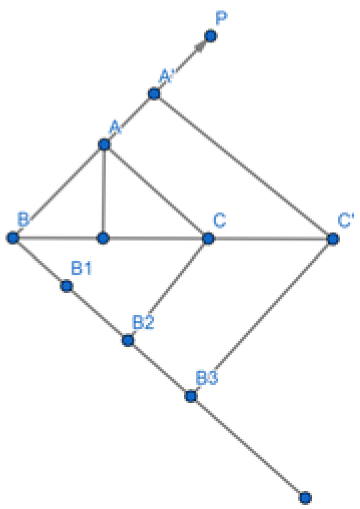
Hence proved

Q. 12. Construct an Isosceles triangle whose base is 8 cm and altitude is 4 cm. Then, draw another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

Answer : Given, base of a triangle as 8cm and altitude as 4cm

Need to draw an isosceles triangle

Construction steps:



1) Draw a line segment BC = 8cm

2) Draw a perpendicular bisector AD of BC

3) Join AB and AC we get an isosceles triangle Δ ABC

4) Construct an acute angle ∠ CBX downwards.

5) On BX make three equal arts.

6) Join C to B2 and draw a line through B3 parallel to B2C intersecting the line extended line segment BC at C'

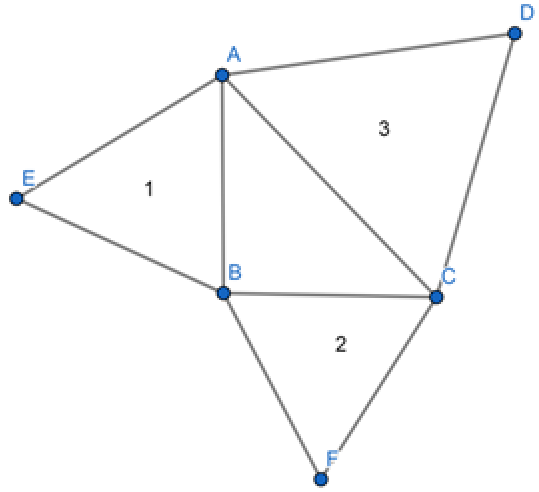
7) Again draw a parallel line C'A' to AC cutting BP at A'

8) Δ A'BC' is the required triangle.

Exercise 8.3

Q. 1. Equilateral triangles are drawn on the three sides of a right angled triangle. Show that the area of the triangle on the hypotenuse is equal to the sum of the areas of triangles on the other two sides.

Answer : Given, right angled triangle ABC with AC as hypotenuse



⇒ Let AB = a, BC = b, AC = c

⇒ we have $a^2 + b^2 = c^2$ ....(1)

⇒ We know that area of equilateral triangle = $\frac{\sqrt{3}}{4} \times side^2$

⇒ Area of ACD = $\frac{\sqrt{3}}{4} \times c^2$

⇒ Area of BCF = $\frac{\sqrt{3}}{4} \times b^2$

⇒ Area of AEB + Area of BCF = $\frac{\sqrt{3}}{4}[a^2 + b^2]$

⇒ From eq(1) we have $a^2 + b^2 = c^2$

⇒ Area of AEB + Area of BCF = $\frac{\sqrt{3}}{4}c^2$ = Area of ACD

Hence, the area of the triangle on the hypotenuse is equal to the sum of the areas of triangles on the other two sides.

Q. 2. Prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangles described on its diagonal.

Answer : Need to prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangles described on its diagonal

⇒ Let us take a square with side 'a'

⇒ Then the diagonal of square will be a√ 2

⇒ Area of equilateral triangle with side 'a' is $\frac{\sqrt{3}}{4}a^2$

⇒ Area of equilateral triangle with side a√2 is $\frac{\sqrt{3}}{4}(a\sqrt{2})^2$

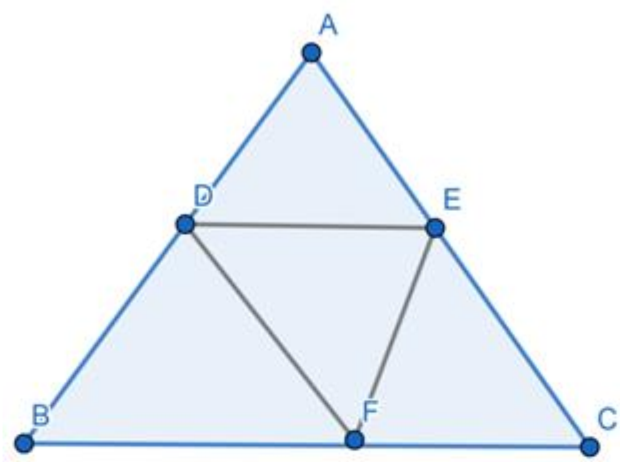
⇒ Ratio of two areas can be given as follows

$$\Rightarrow \frac{\frac{\sqrt{3}}{4}a^2}{\frac{\sqrt{3}}{4}2a^2} = \frac{1}{2}$$

Hence proved

Q. 3. D, E, F are mid points of sides BC, CA, AB to Δ ABC. Find the ratio of areas of ΔDEF and Δ ABC.

Answer :



⇒ Given, D, E, F are mid points of BC, CA, AB

⇒ Need to find the ratios of Δ DEF and Δ ABC

⇒ DE || AF or DF || BE

⇒ similarly EF || AB or EF || DB

⇒ AFED is a parallelogram as both pair of opposite sides are parallel

⇒ By the property of parallelogram

⇒ ∠ DBE = ∠ DFE

Or ∠ DFE = ∠ ABC ………..eq(1)

⇒ Similarly ∠ FEB = ∠ ACB …..eq(2)

⇒ In Δ DEF and Δ ABC from eq(1) and eq(2) we have

⇒ Δ DEF ~ Δ CAB

⇒ $\frac{ar(\Delta\ DEF)}{ar(\Delta ABC)} = \frac{DE^2}{2DE^2} = \frac{1}{4}$

⇒ ar(Δ DEF) : ar(Δ ABC) = 1:4

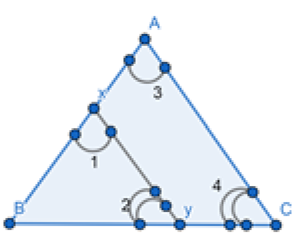
Hence proved

Q. 4. In Δ ABC, XY || AC and XY divides the triangle into two parts of equal area. Find the ratio of

$\frac{AX}{XB}$.

Answer : Given, XY || AC

Need to find the ratio of AX:XB



⇒ ∠1 = ∠3 and ∠2 = ∠4 [corresponding angles]

$\Rightarrow \Delta$ BXY ~ Δ BAC

$\Rightarrow \frac{ar(\Delta\ BXY)}{ar(\Delta BAC)} = \frac{BX^2}{BA^2}$ ....eq(1)

Also, we are given that

$\Rightarrow ar(\Delta\ BXY) = \frac{1}{2}\ ar(\Delta\ BAC)$

$\Rightarrow \frac{ar(\Delta BXY)}{ar(\Delta BAC)} = \frac{1}{2}$

From (1) and (2)

$\Rightarrow \frac{BX^2}{BA^2} = \frac{1}{2}$

$\Rightarrow \frac{BX}{BA} = \frac{1}{\sqrt{2}}$

$\Rightarrow$ Now, $\frac{AX}{XB} = \frac{AB-BX}{XB} = \frac{AB}{BX} - 1 = \frac{\sqrt{2}}{1} - 1$

$\Rightarrow \frac{AX}{XB} = \frac{\sqrt{2}-1}{1}$

$\therefore$ AX:XB = $\sqrt{2}$ -1 : 1

Q. 5. Prove that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer : Need to prove that the ratio of area of two similar triangles is equal to the square of the ratio of their corresponding medians

$\Rightarrow$ In case of two similar triangles ABC and PQR we have

$\Rightarrow \frac{ar(\Delta\ ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2}$

$= \frac{AC^2}{PR^2}$

⇒ Let us assume AD and PM are the medians of these two triangles

Then

$\Rightarrow \frac{AX}{XB} = \frac{\sqrt{2}-1}{1}$

Hence, ar(Δ ABC) : ar(Δ PQR) = $AD^2 : PM^2$

Q. 6. Δ ABC ~ Δ DEF. BC = 3 cm EF = 4 cm and area of ΔABC = $54cm^2$. Determine the area of ΔDEF.

Answer : Given, BC = 3 cm EF = 4 cm

Also, area of Δ ABC = $54cm^2$

Need to find area of Δ DEF

⇒ since, Δ ABC ~ Δ DEF

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$

$\Rightarrow \frac{54}{ar(\Delta DEF)} = \frac{3^2}{4^2}$

$\Rightarrow \frac{54}{ar(\Delta DEF)} = \frac{9}{16}$

$\Rightarrow \frac{54 \times 16}{9}$ = ar(Δ DEF)

⇒ ar(Δ DEF) = 96 cm^2

Hence, the area of Δ DEF is $96cm^2$

Q. 7.ABC is a triangle and PQ is a straight line meeting AB and P and AC in Q. If AP = 1 cm. and BP = 3 cm, AQ = 1.5 cm., CQ = 4.5 cm.

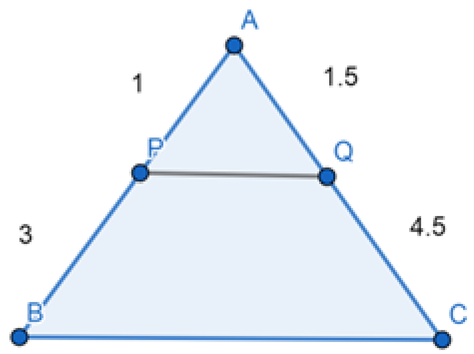
Prove that

$$\text{(area of } \Delta APQ) = \frac{1}{16}(\text{area of } \Delta ABC)$$

Answer :

Given, AP = 1cm, BP = 3cm and AQ = 1.5cm,CQ = 4.5cm

Need to prove (area of Δ APQ) = $\frac{1}{16}$ area of Δ ABC



⇒ it is evident that Δ ABC ~ Δ APQ we know that

$$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \left[\frac{AP}{AP + PB}\right]^2$$

$$= \left[\frac{AQ}{AQ + QC}\right]^2 = \frac{1}{16}$$

$$\Rightarrow \text{Area of } \Delta\ APQ = \frac{1}{16} \text{ Area of } \Delta\ ABC$$

Hence proved

Q. 8. The areas of two similar triangles are 81cm^2 and 49cm^2 respectively. If the attitude of the bigger triangle is 4.5 cm. Find the corresponding attitude of the smaller triangle.

Answer : Given, area of two similar triangles as 81cm^2 and 49cm^2

Altitude of the bigger triangle is 4.5cm

Need to find out the corresponding altitude of the smaller triangle

⇒ Δ ABC = Δ DEF

⇒ AP and DQ are corresponding altitude of triangle

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left[\frac{AP}{DQ}\right]^2$$

$$\Rightarrow \frac{81}{49} = \left(\frac{4.5}{DQ}\right)^2$$

$$\Rightarrow \left(\frac{4.5}{DQ}\right)^2 = \frac{81}{49}$$

$$\Rightarrow \frac{4.5}{DQ} = \sqrt{\frac{81}{49}}$$

$$\Rightarrow \frac{4.5}{DQ} = \frac{9}{7}$$

$$\Rightarrow \frac{4.5 \times 7}{9} = DQ$$

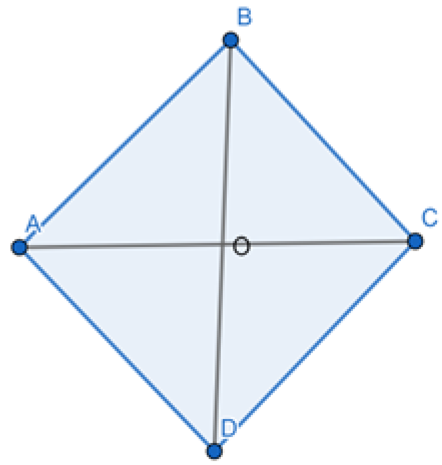
⇒ DQ = 3.5cm

Hence, the altitude of similar triangle is 3.5cm

Exercise 8.4

Q. 1. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Answer : Need to prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals



ABCD is a rhombus in which diagonals AC and BD intersect at point O.

We need to prove $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + DB^2$

⇒ In Δ AOB; $AB^2 = AO^2 + BO^2$

⇒ In Δ BOC; $BC^2 = CO^2 + BO^2$

⇒ In Δ COD; $CD^2 = DO^2 + CO^2$

⇒ In Δ AOD; $AD^2 = DO^2 + AO^2$

⇒ Adding the above 4 equations we get

⇒ $AB^2 + BC^2 + CD^2 + DA^2 = AO^2 + BO^2 + CO^2 + BO^2 + DO^2 + CO^2 + DO^2 + AO^2$

⇒ $= 2(AO^2 + BO^2 + CO^2 + DO^2)$

Since, $AO^2 = CO^2$ and $BO^2 = DO^2$

$= 2(2\ AO^2 + 2\ BO^2)$

$= 4(AO^2 + BO^2)$ ……eq(1)

Now, let us take the sum of squares of diagonals

⇒ $AC^2 + DB^2 = (AO + CO)^2 + (DO+ BO)^2$

$= (2AO)^2 + (2DO)^2$

$= 4\ AO^2 + 4\ BO^2$ ……eq(2)

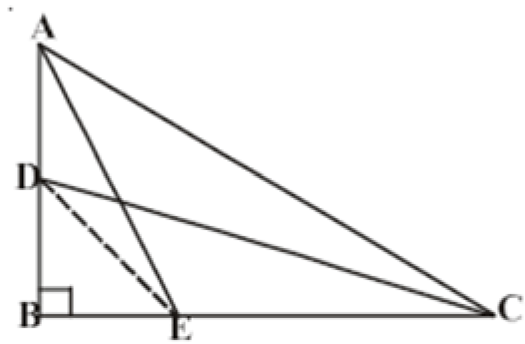
From eq(1) and eq(2) we get

⇒ $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + DB^2$

Hence, proved

Q. 2. ABC is a right triangle right angled at B. Let D and E be any points on AB and BC respectively.

Prove that $AE^2 + CD^2 = AC^2 + DE^2$.



Answer : Given, ABC as a right angled triangle

Need to prove that $AE^2 + CD^2 = AC^2 + DE^2$

⇒ In right angled triangle ABC and DBC, we have

⇒ $AE^2 = AB^2 + BE^2$ ………..eq(1)

⇒ $DC^2 = DB^2 + BC^2$ ……….eq(2)

⇒ Adding equation 1 and 2 we have

⇒ $AE^2 + DC^2 = AB^2 + BE^2 + DB^2 + BC^2$

$= (AB^2 + BC^2) + (BE^2 + DB^2)$

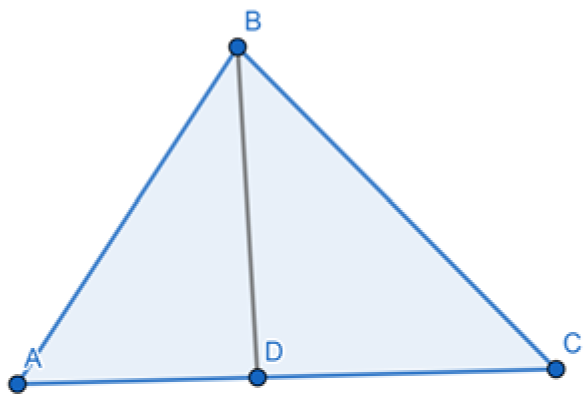
⇒ Since $AB^2 + BC^2 = AC^2$ in right angled triangle ABC

∴ $AC^2 + DE^2$

Hence proved

Q. 3. Prove that three times the square of any side of an equilateral triangle is equal to four times the square of the altitude.

Answer :



Given, an equilateral triangle ABC, in which AD perpendicular BC

Need to prove that 3 $AB^2 = 4AD^2$

⇒ Let AB = BC = CA = a

⇒ In Δ ABD and Δ ACD

⇒ AB = AC, AD = AD and ∠ ADB = ∠ ADC

∴ Δ ABD ≅ Δ ACD

∴ BD = CD = $\frac{a}{2}$

⇒ Now, in Δ ABD, ∠ D = 90°

∴ $AB^2 = BD^2 + AD^2$

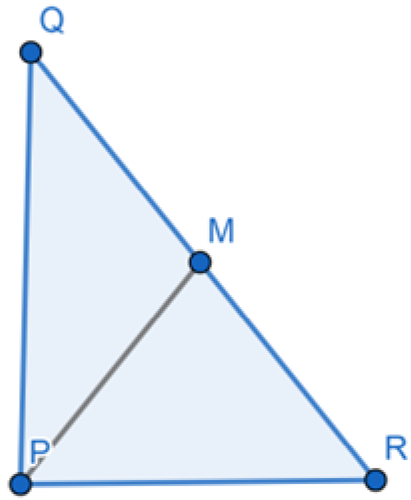
$$\Rightarrow AB^2 = \left[\frac{CD}{2}\right]^2 + AD^2$$

$$= \left[\frac{AB}{2}\right]^2 + AD^2$$

$$3AB^2 = 4\ AD^2$$

Q. 4. PQR is a triangle right angled at P and M is a point on QR such that PM ⊥QR. Show that PM^2 = QM.MR.

Answer :



⇒ Let ∠MPR = x

⇒ In Δ MPR, ∠MRP = 180-90-x

⇒ ∠MRP = 90-x

Similarly in Δ MPQ,

∠MPQ = 90-∠MPR = 90-x

⇒ ∠MQP = 180-90-(90-x)

⇒ ∠MQP = x

In Δ QMP and Δ PMR

⇒ ∠MPQ = ∠MRP

⇒ ∠PMQ = ∠RMP

⇒ ∠MQP = ∠MPR

⇒ Δ QMP ~ Δ PMR

$\Rightarrow \frac{QM}{PM} = \frac{MP}{MR}$

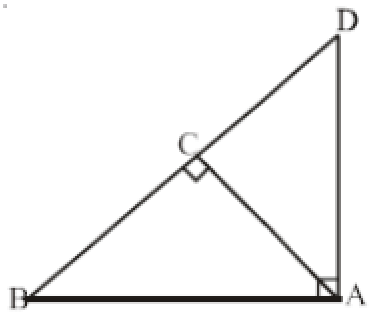
⇒ PM^2 = MR × QM

Hence proved

Q. 5. ABD is a triangle right angled at A and AC ⊥ BD

Show that

i. $AB^2 = BC \cdot BD$
ii. $AC^2 = BC \cdot DC$
iii. $AD^2 = BD \cdot CD$



Answer : Given, ABCD is a right angled triangle and AC is perpendicular to BD

(i) consider two triangles ACB and DAB

⇒ We have ∠ ABC = ∠ DBC

⇒ ∠ ACB = ∠ DAB

⇒ ∠ CAB = ∠ ADB

∴ they are similar and corresponding sides must be proportional

i.e, ∠ ADC = ∠ ADB

⇒ $\frac{AC}{DA} = \frac{CB}{AB} = \frac{AB}{DB}$

∴ $AB^2 = BC \times CD$

(ii) ∠ BDA = ∠ BDC = 90°

⇒ ∠ 3 = ∠ 2 = 90° ∠ 1

⇒ ∠ 2 + ∠ 4 = 90° ∠ 2

⇒ From AAA criterion of similarity we have in two triangles if the angles are equal, then sides opposite to the equal angles are in the same ratio (or proportional) and hence the triangles are similar

Their corresponding sides must be proportional

$\Rightarrow \frac{DC}{AC} = \frac{CA}{CB} = \frac{DA}{AB}$

$\Rightarrow \frac{DC}{AC} = \frac{CA}{CB}$

$\Rightarrow CA^2 = BC \times DC$

(iii) In two triangles ADB and ABC we have

∠ ADC = ∠ ADB

⇒ ∠ DCA = ∠ DAB

⇒ ∠ DAC = ∠ DBA

⇒ ∠ DCA = ∠ DAB

⇒ Triangle ADB and ABC are similar and so their corresponding sides must be proportion.

$\Rightarrow \frac{DC}{AC} = \frac{CA}{CB} = \frac{DA}{AB}$

$\Rightarrow \frac{DC}{AC} = \frac{CA}{CB}$

⇒ $AD^2 = DB \times DC$

Hence proved

Q. 6. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Answer : Since the triangle is right angled at C

∴ the side AB is hypotenuse.

⇒ Let the base of the triangle be AC and the altitude be BC.

⇒ Applying the Pythagorean theorem

⇒ $HYP^2 = Base^2 + Alt^2$

⇒ $AB^2 = AC^2 + BC^2$

Since the triangle is isosceles triangle two of the sides shall be equal

∴ AC = BC

Thus $AB^2 = AC^2 + BC^2$

$AB^2 = 2AC^2$

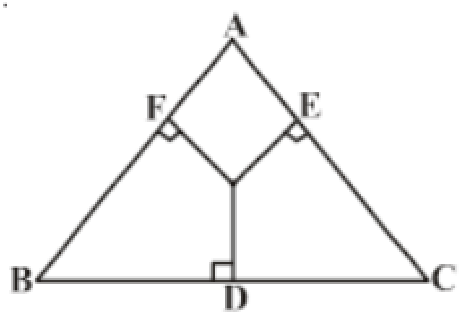
Hence, proved

Q. 7. 'O' is any point in the interior of a triangle ABC.

OD ⊥ BC, OE ⊥ AC and OF ⊥ AB, show that

i. $OA^2 + OB^2 + OC^2 – OD^2 – OE^2 – OF^2 = AF^2 + BD^2 + CE^2$

ii. $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.



Answer : Given, Δ ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB,

Need to prove $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

⇒ Join point O to A,B and C

(i) ∠AFO = 90°

$AO^2 = AF^2 + OF^2$

⇒ $AF^2 = AO^2 - OF^2$ …….eq(1)

Similarly $BD^2 = BO^2 - OD^2$ …..eq(2)

⇒ $CE^2 = CO^2 - OE^2$ …..eq(3)

Adding eq(1), (2) and (3) we get

⇒ $AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

(ii) $AF^2 + BD^2 + CE^2 = (AO^2 - OE^2) + (BO^2 - OF^2) + (CO^2 - OD^2)$

$= AE^2 + CD^2 + BF^2$

Hence, proved

Q. 8. A wire attached to vertically pole of height 18m is24m long and has a stake attached to other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Answer : Given, height of a pole is 18 and wire attached is 24m

Need to find the distance from the base to keep wire taut

⇒ Let AB be a wire and pole be BC

⇒ to keep the wire taut let it be fixed at A

⇒ $AB^2 = AC^2 + BC^2$

⇒ $24^2 = AC^2 + 18^2$

⇒ $AC^2 = 24^2 + 18^2$

⇒ $AC^2 = 576 - 324$

⇒ = 252

⇒ $AC = \sqrt{252}$

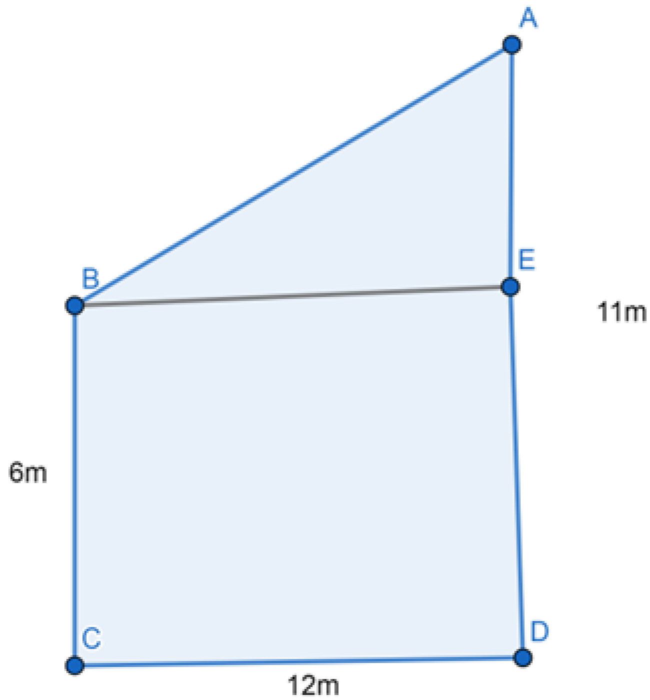
$= \sqrt{(36 \times 7)}$

$= 6\sqrt{7}$

Hence, the stake may be placed at a distance of $6\sqrt{7}$m the base of pole

Q. 9. Two poles of heights 6m and 11m stand on a plane ground. If the distance between the feet of the pole is 12m find the distance between their tops.

Answer :



Given, BC = 6m, AD = 11m, BC = ED

And AE = AD-ED = 11-6 = 5m

BE = CD = 12m

Need to find AB

⇒ Now, In Δ ABE, ∠E = 90°

⇒ $AB^2 = AE^2 + BE^2$

⇒ $AB^2 = 5^2 + 12^2 = 169$

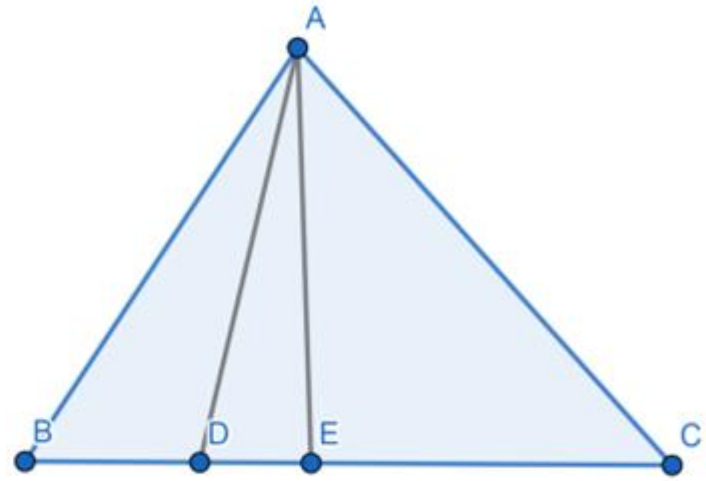
⇒ $AB^2 = 169$

⇒ AB = 13m

The distance between their tops is 13m

Q. 10. In an equilateral triangle ABC, D is on a side BC such that $BD = \frac{1}{3}BC.$ **Prove that** $9AD^2 = 7AB^2$**.**

Answer :



Given, ABC is a equilateral triangle where AB = BC = AC and BD = $\frac{1}{3}$BC

Draw AE perpendicular BC

$\Rightarrow \Delta$ ABE $\cong \Delta$ ACE

$\therefore$ BE = EC = $\frac{BC}{2}$

$\Rightarrow$ Now in Δ ABE, $AB^2 = BE^2 + AE^2$

$\Rightarrow$ also $AD^2 = AE^2 + DE^2$

$\therefore AB^2 - AD^2 = BE^2 - DE^2$

$= BE^2 - (BE-BD)^2$

$= (\frac{BC}{2})^2 - \left[\frac{BC}{2} - \frac{BC}{3}\right]^2$

$= (\frac{AB}{2})^2 - \left[\frac{AB}{2} - \frac{AB}{3}\right]^2$

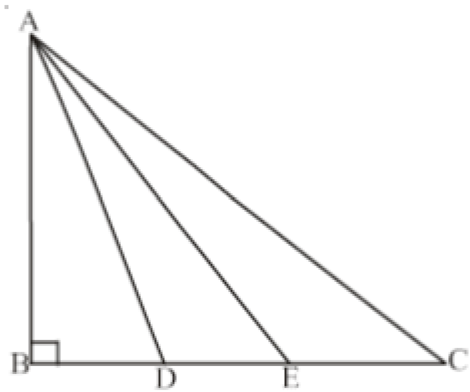
$AB^2 - AD^2 = 2\frac{AB^2}{9}$

Or $7 AB^2 = 9 AD^2$

Hence, proved

Q. 11. In the given figure, ABC is a triangle right angled at B. D and E are points on BC trisect it.

Prove that $8AE^2 = 3AC^2 + 5AD^2$.



Answer : Given, ABC triangle

Need to prove $8AE^2 = 3AC^2 + 5AD^2$

⇒ In Δ ABD, ∠B = 90°

∴ $AC^2 = AB^2 + BC^2$ ….eq(1)

⇒ Similarly, $AE^2 = AB^2 + BE^2$ …….eq(2)

⇒ And $AD^2 = AB^2 + BD^2$ …….eq(3)

⇒ Form eq(1)

⇒ $3AC^2 = 3AB^2 + 3\ BC^2$ …eq(4)

⇒ From eq(2)

⇒ $5AD^2 = 5AB^2 + 5BD^2$ ……eq(5)

Adding equation (4) and (5)

$3AC^2 + 5AD^2 = 8AB^2 + 3\ BC^2 + 5BD^2$

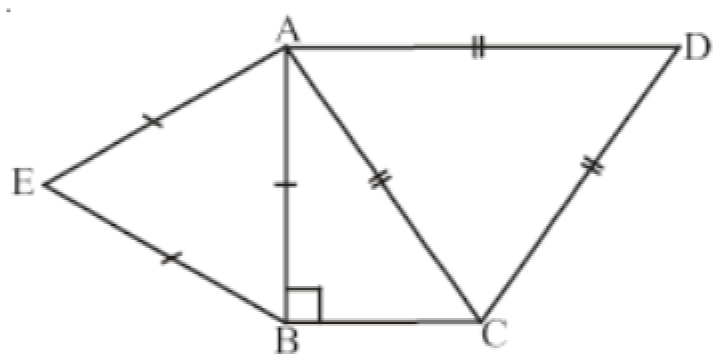
$= 8AB^2 + 3\left[\frac{3}{2}BE\right]^2 + 5\left[\frac{BE}{2}\right]^2$

$= 8(AB^2 + BE^2)$

= $8AE^2$

Hence, proved

Q. 12. ABC is an isosceles triangle right angled at B. Similar triangles ACD and ABE are constructed on sides AC and AB. Find the ratio between the areas of ΔABE and ΔACD.



Answer : Given, ABC is an isosceles triangle in which ∠B = 90°

Need to find the ratio between the areas of Δ ABE and Δ ACD

⇒ AB = BC

⇒ By Pythagoras theorem, we have $AC^2 = AB^2 + BC^2$

⇒ since AB = BC

⇒ $AC^2 = AB^2 + AB^2$

⇒ $AC^2 = 2\ AB^2$ …..eq(1)

⇒ it is also given that Δ ABE ~ Δ ACD

(ratio of areas of similar triangles is equal to ratio of squares of their corresponding sides)

$$\Rightarrow \frac{\text{ar}(\Delta\ \text{ABC})}{\text{ar}(\Delta\text{ACD})} = \left[\frac{AB}{AC}\right]^2$$

$$\Rightarrow \frac{\text{ar}(\Delta\ \text{ABC})}{\text{ar}(\Delta\text{ACD})} = \frac{AB^2}{2AB^2} \text{ from 1}$$

$$\Rightarrow \frac{\text{ar}(\Delta\ \text{ABC})}{\text{ar}(\Delta\text{ACD})} = \frac{1}{2}$$

∴ ar(Δ ABC):ar(Δ ACD) = 1:2

Hence the ratio is 1:2