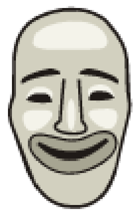
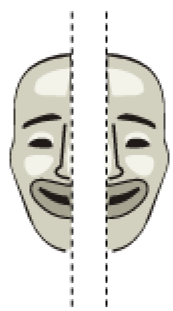
Symmetry

Symmetrical Figures and Lines of Symmetry

Let us consider the following mask.



If we cut this mask exactly from middle, we obtain two halves of the mask as shown below.



We can observe that both left and right half faces are exact copies of each other.

Similarly, if we fold the following pictures from the middle, then the left half of the picture will exactly overlap over the right half of the picture.

India gate

Taj Mahal

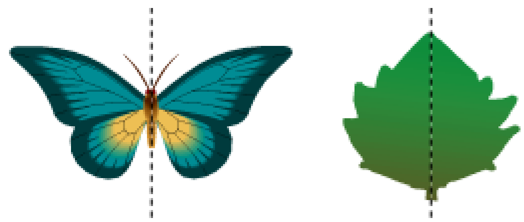
Gateway of India

Such figures are known as **symmetrical figures** or **symmetric.**

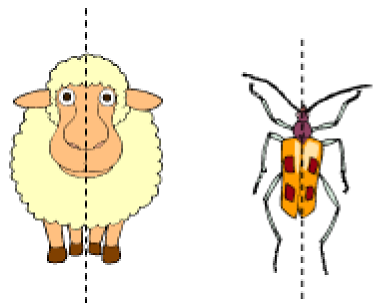
If we place a mirror on the line where we folded these pictures, then we will find that the image of one half of the figure is exactly the same as the other half.

Symmetry is something that we observe in many places in our daily lives without even noticing it. It is easily noticeable in various arts, buildings, and monuments.

Nature uses symmetry to make things beautiful. For example, consider the pictures of the butterfly and the leaf drawn below.



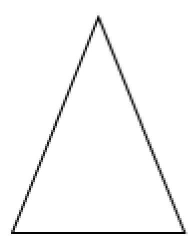
Let us also consider the following pictures.



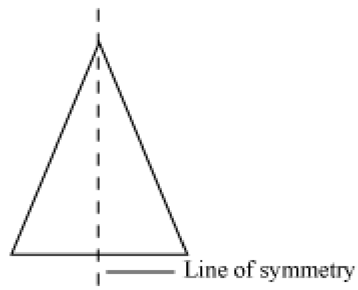
In each of these pictures, the dotted line divides the picture into two parts such that the left half of the picture is exactly same as the right half. This dotted line is known as the **line of symmetry** or **mirror line**. It can be defined as:

The line through which the figure can be folded to form two identical figures is called line of symmetry or axis of symmetry or mirror line.

Let us consider the following isosceles triangle.



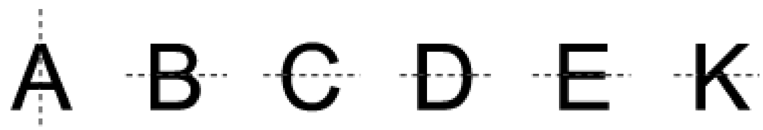
If we draw a dotted line at the middle of the triangle, then we get two parts such that the left part is exactly same as right part.



Here, the dotted line is the line of symmetry. We cannot draw more lines of symmetry for this triangle. Therefore, we can say that an isosceles triangle has only **one line of symmetry**.

Some letters of the English alphabet have only one line of symmetry.

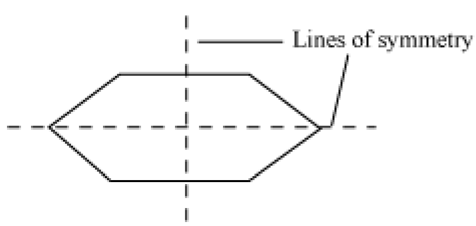
For example, A, B, C, D, E, K, etc.



Here, letter A has a vertical line of symmetry, while each of the letters B, C, D, E, and K has a horizontal line of symmetry.

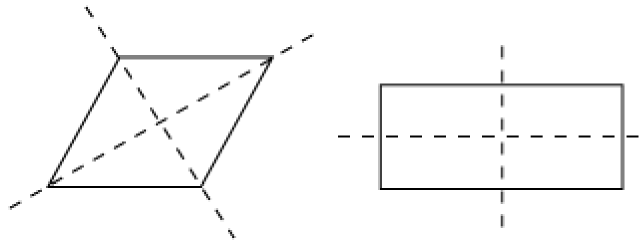
However, it is not necessary that a figure has only one line of symmetry. A figure can have more than one line of symmetry. Let us consider the following figure.

We can draw the lines of symmetry as shown below.



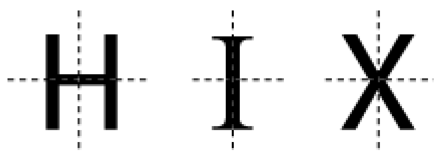
For this figure, we cannot draw more lines of symmetry except these two. Therefore, we can say that this figure has **two lines of symmetry.**

Some more figures with two lines of symmetry are shown below.



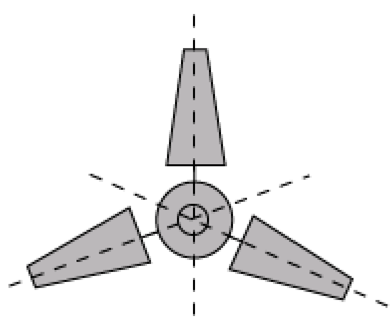
Some letters of the English alphabet contains two lines of symmetry.

For example, H, I, X, etc.

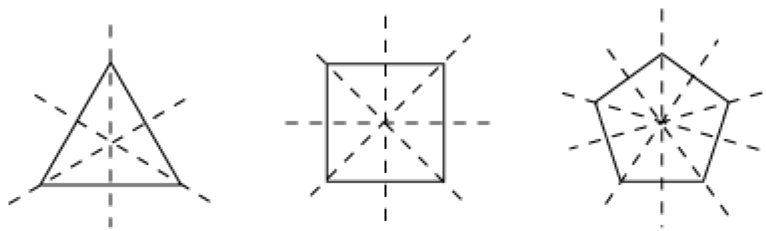


Note that each of the letters H, I, and X has both vertical and horizontal lines of symmetry.

Let us look at the fan drawn in the following figure. It has **three lines of symmetry**. When a figure has more than two lines of symmetry, we say that it has **multiple lines of symmetry**. Thus, we can also say that the fan drawn below has multiple lines of symmetry.



Some geometric figures with multiple lines of symmetry are shown below.

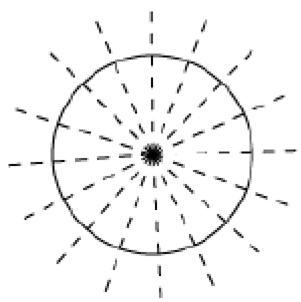


The figures drawn above represent

- an equilateral triangle with three lines of symmetry
- a square with four lines of symmetry
- a regular pentagon with five lines of symmetry

Using the concept of line of symmetry, can we tell how many lines of symmetry are there in a circle?

A circle has infinite number of lines of symmetry as shown in the figure below.

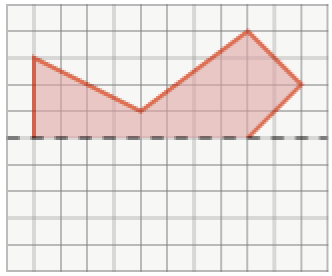


Now, if suppose we are given a part of a figure and its line(s) of symmetry, then can we draw the complete figure?

Yes, we can draw the complete figure by tracing the given part of the figure on the other side of the given line of symmetry.

Let us see this with the help of an example.

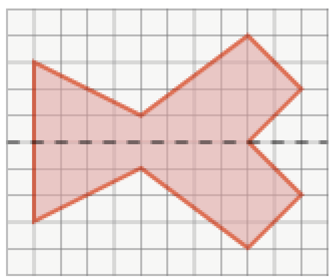
Consider the following figure:



Here, the dotted line is the line of symmetry of the figure.

The complete figure can be obtained by tracing the given figure below its line of symmetry.

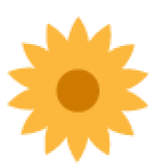
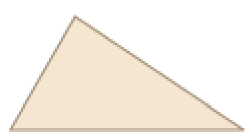
Thus, the complete figure will be represented as:



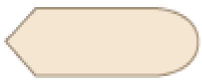
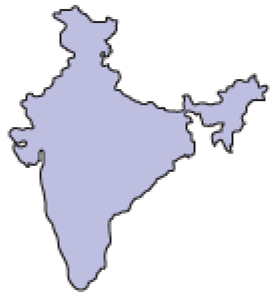
Now, we know how to identify symmetrical figures and their lines of symmetry. Let us discuss some examples based on these concepts.

Example 1:

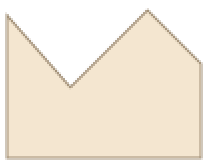
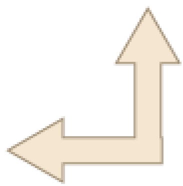
Identify the symmetrical figures out of the following figures. Also draw their lines of symmetry.

(i) **(ii)** **(iii)**

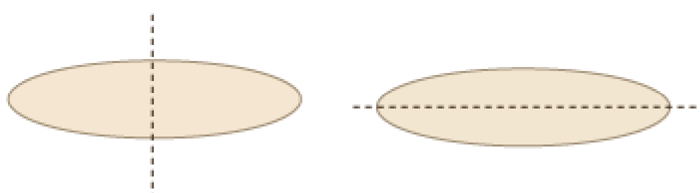
(iv) **(v)** **(vi)**

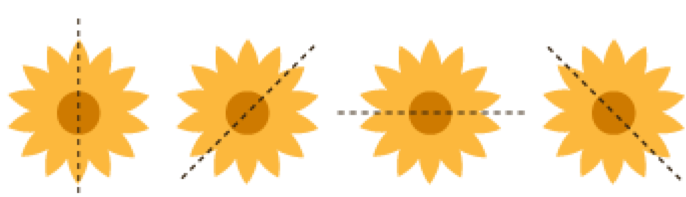
(vii) **(viii)**

Solution:

(i) The given figure is oval (egg-shaped). It has two lines of symmetry. That means the figure is symmetrical.

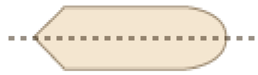


(ii) We can fold the given figure from the centre in any way as it has infinite lines of symmetry. Therefore, the figure is symmetrical. Some of its lines of symmetry are as follows.



(iii) We cannot fold the given triangle to form two identical halves. Therefore, this figure is not symmetrical and does not have any line of symmetry.

(iv) The given figure has a horizontal line of symmetry. Therefore, the figure is symmetrical.

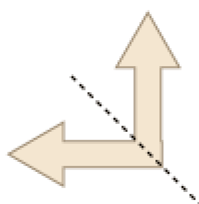


(v) The bottle shown in the given figure is symmetrical, as it has a vertical line of symmetry that divides it into two identical halves.

(vi) The given figure is not symmetrical, as we cannot divide it into two identical halves. Thus, it does not have any line of symmetry.

(vii) We cannot fold the figure in any way in order to divide it into two identical halves. Thus, the figure is not symmetrical and does not have any line of symmetry.

(viii) When we fold the given figure along the dotted line, we obtain two identical halves. Thus, the figure is symmetrical and has one line of symmetry.



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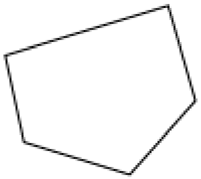
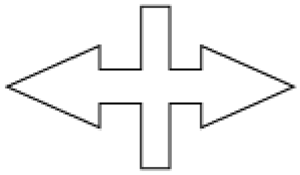
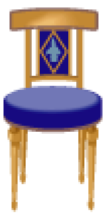
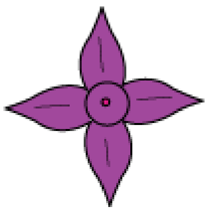
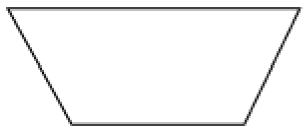
Example 2:

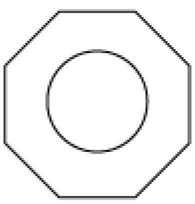
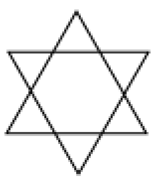
State whether the following figures have single, double, or multiple lines of symmetry. Also, draw their line or lines of symmetry.

  M V

(i) **(ii)** **(iii)** **(iv)**

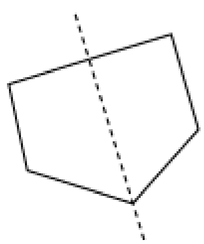
(v) **(vi)** **(vii)**

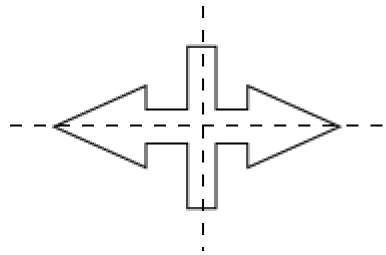
(viii) **(ix)**

Solution:

(i) The given figure has one line of symmetry.

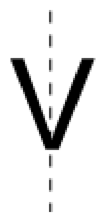


(ii) The given figure has two lines of symmetry.

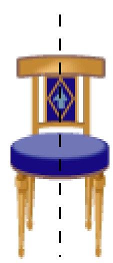


(iii) The given figure has one line of symmetry.

(iv) The given figure has one line of symmetry.

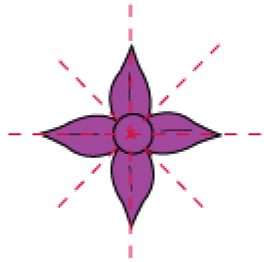


(v) The given figure has one line of symmetry.

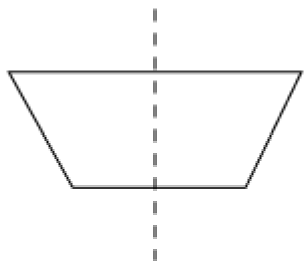


(vi) The given figure has four lines of symmetry, i.e., it has multiple lines of symmetry.

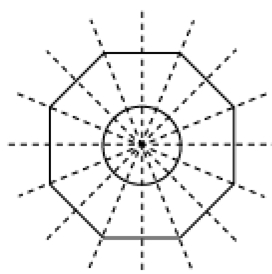
1.



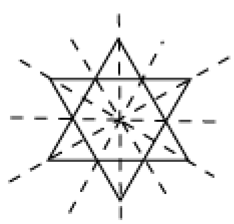
(vii) The given figure has one line of symmetry.



(viii) The given figure has eight lines of symmetry, i.e., it has multiple lines of symmetry.

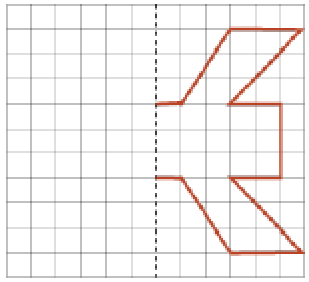


(ix) The given figure has six lines of symmetry, i.e., it has multiple lines of symmetry.

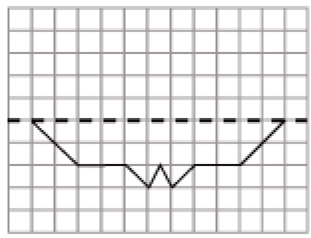


Example 3:
Complete the following figures in which the dotted line shows the line of symmetry.

1.



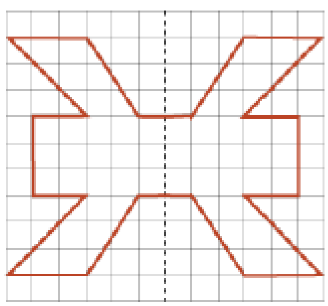
2.



Solution:

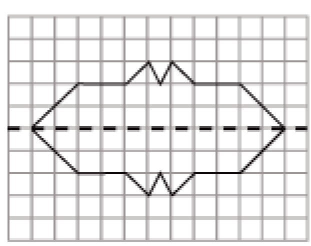
1. The complete figure can be obtained by tracing the given figure to the left of the line of symmetry.

   Thus, the complete figure is represented as:



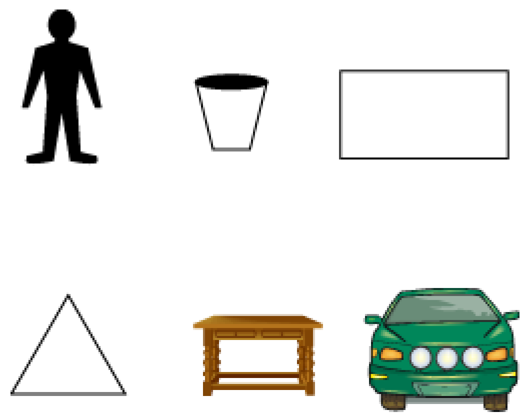
2. The complete figure can be obtained by tracing the given figure above the line of symmetry.

   Thus, the complete figure is represented as:

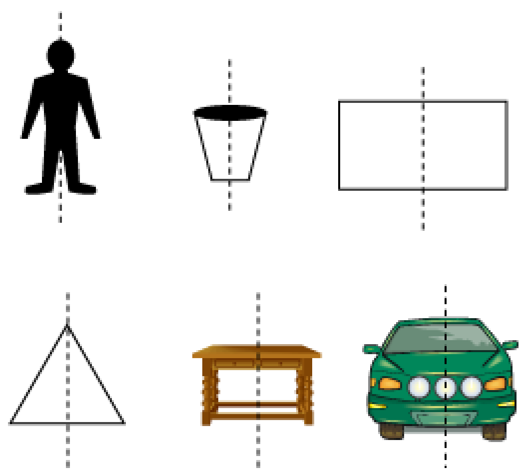


Reflection of Figures

Let us consider the following pictures.



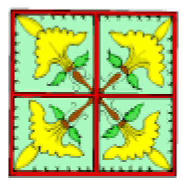
For each picture, let us draw vertical lines exactly at the middle as shown below.



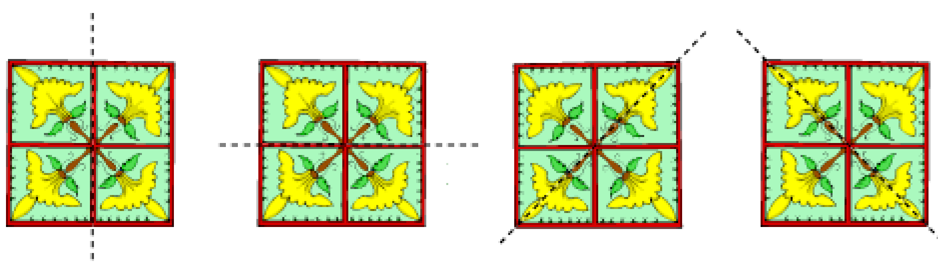
After drawing lines, we can observe that the left half of the pictures is exactly the same as the right half of the pictures. These pictures are known as symmetrical pictures. The line through which the figure is divided is called **line of symmetry.** Here, the dotted lines of these pictures are lines of symmetry. If we consider only one-half of these images and place a mirror instead of the dotted line, then we will get a mirror reflection of the image, which will be the missing half of the original image. Thus, we can also say that the two halves obtained by dividing the figure through the line of symmetry are mirror images of each other. For example, the left portion of each image is the mirror image of the right portion of the image and vice-versa.

We come across different objects in our day to day life where symmetry is shown by mirror reflection. For example, if we look at the following picture, we can see the reflected image of the buildings in the water. The water surface acts as a mirror or as a line of symmetry. We can observe the symmetry of the objects by their reflection, though the image is not very clear.

We can also take the example of Rangoli patterns.

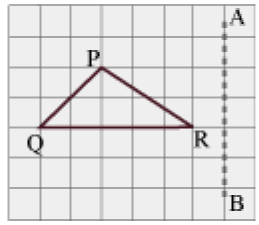


There are many lines of symmetry for these types of patterns as shown below. We can observe reflection pattern in the patterns along their lines of symmetry acting as mirror lines.

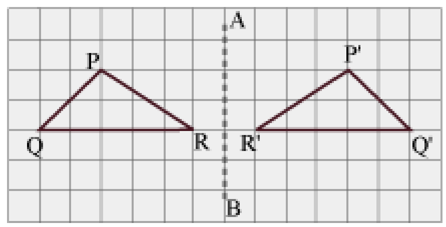


We can see that in each figure, one half of the pattern is the reflection of the other half.

Let us consider the given figure of ΔPQR on a grid paper, where AB is a mirror line.



Let us draw the image of ΔPQR with reference to the mirror line AB.



Here, ΔPQR and ΔP′Q′R′ are symmetrical with reference to the mirror line AB.

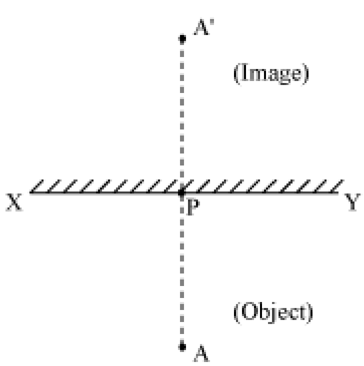
Now, we can say that ΔP′Q′R′ is the mirror image of ΔPQR with reference to the mirror line AB. If we fold the grid paper along the mirror line, then we will observe that the two triangles overlap. It can also be observed that:

- The lengths of the sides of ΔP′Q′R′ are equal to the corresponding sides of ΔPQR.
- The angles of ΔP′Q′R′ are equal to the corresponding angles of ΔPQR.
- Every portion of ΔP′Q′R′ is at the same distance from the mirror line as that of the corresponding portions of ΔPQR.

Now, let us extend the concept of mirror further to study about image of a point.

If we place a point in front of a mirror, then what is the nature of the image formed?

Let XY be a mirror. Let A be a point (object) placed in front of it. We obtain its image A′ as shown below:



Can we notice anything in the above figure?

We can notice that:

1. The distance of the image (A′) behind the mirror is same as the distance of the object (A) from it i.e., PA = PA′
2. The mirror line XY is perpendicular to the line joining the object and the image i.e., XY ⊥ AA′

Here, XY (the mirror line) is called the **axis of reflection** or **mediator**.

What would happen if the point A lies on XY?

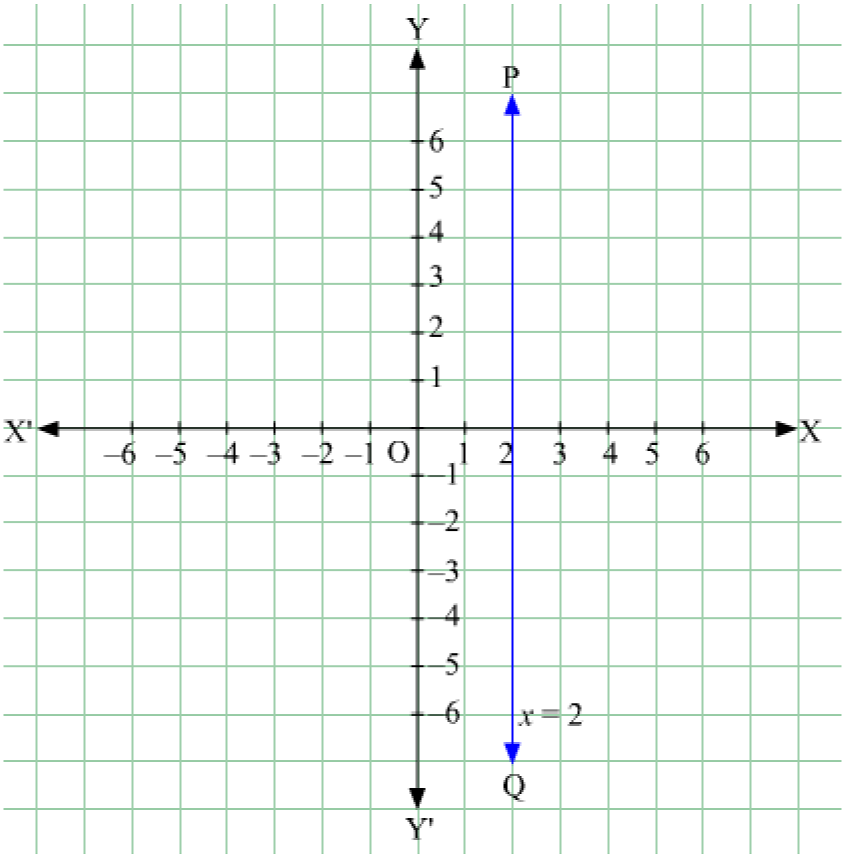
If the point A lies on XY, then its image will be this point itself. In such case, A is called an **invariant point** with respect to mirror line XY.

Now let us go through the following video to study the various methods to find the reflection of a given point about a line, about x-axis, about y-axis, and about origin on a graph paper.

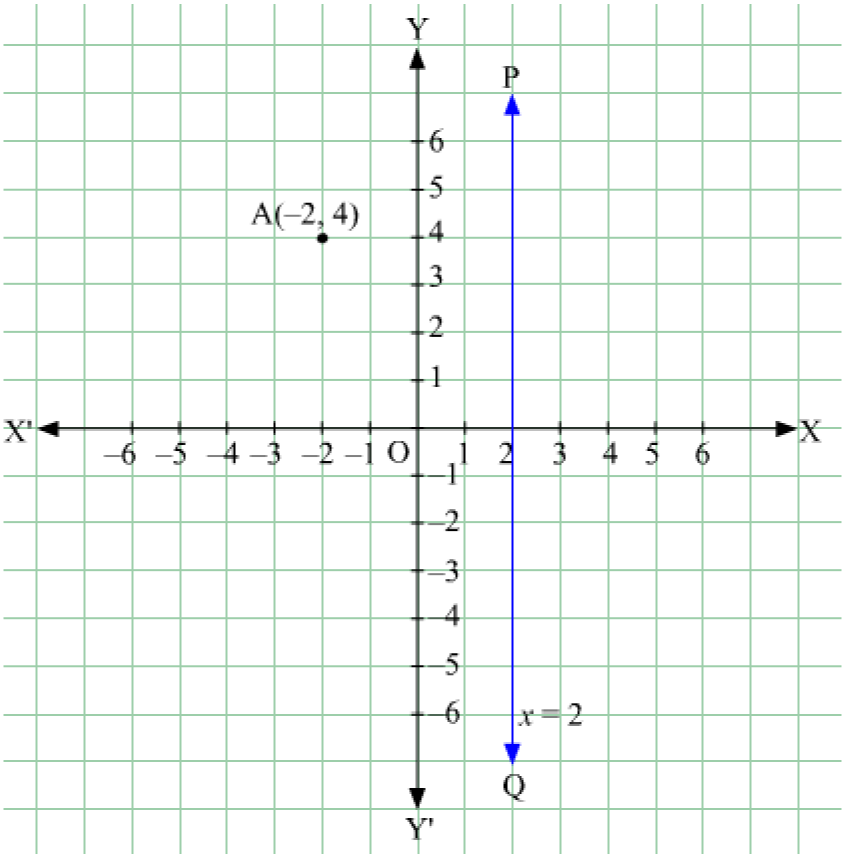
Reflection of a point in the lines $x = a$ and $y = a$.

$x = a$ is a line parallel to the y-axis and at a distance of a units from it.
If we have to find the reflection of point A(−2, 4) from the line $x = 2$ we follow the below given steps:

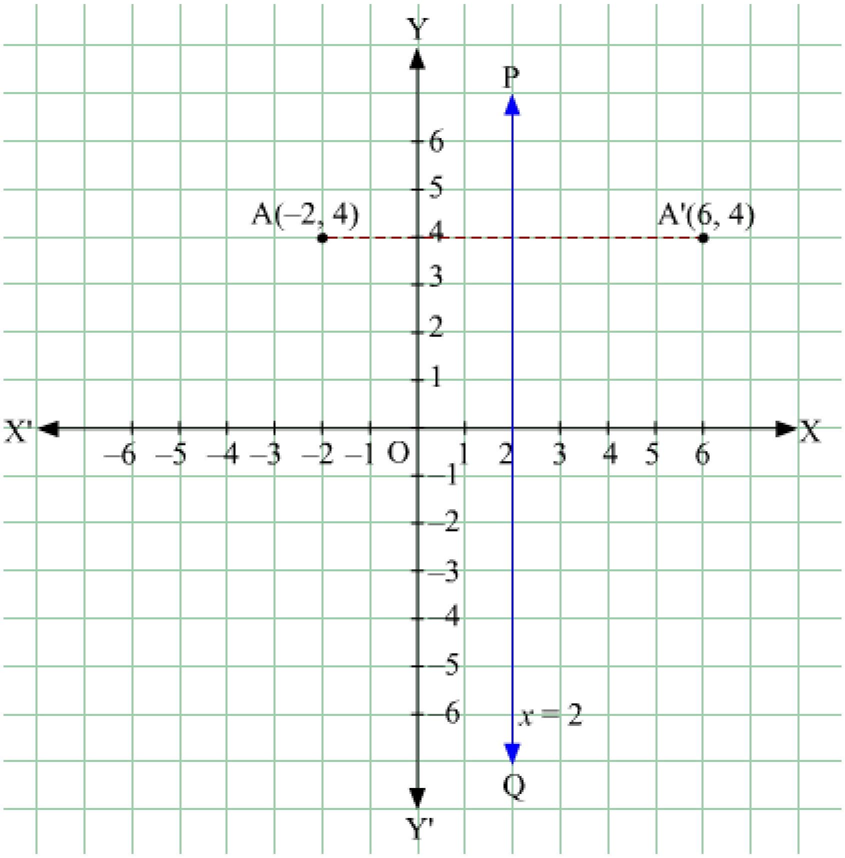
Step 1: Line PQ represents $x = 2$ which is a straight line parallel to the y-axis and at a distance of 2 units from it.



Step 2: Mark a point A(−2, 4) on the same graph.

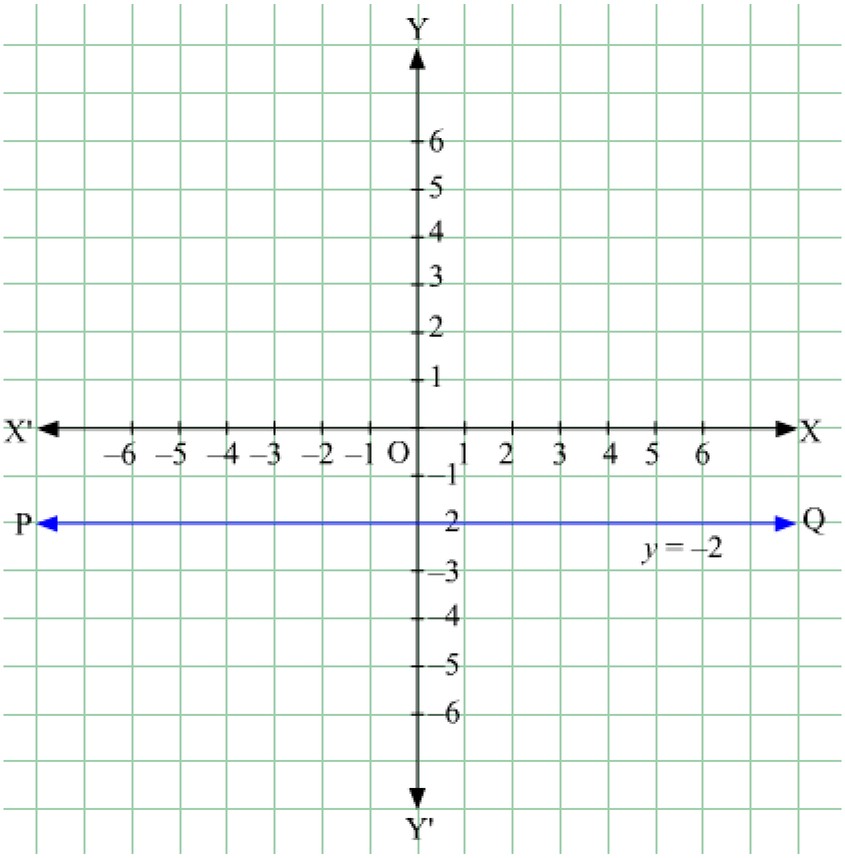


Step 3: From the point A, draw a straight line perpendicular to PQ. Mark a point A' behind the straight line PQ at the same distance as A(−2, 4) is before it. A'(6, 4) is the required reflection of the point A(−2, 4) in the line $x = 2$.

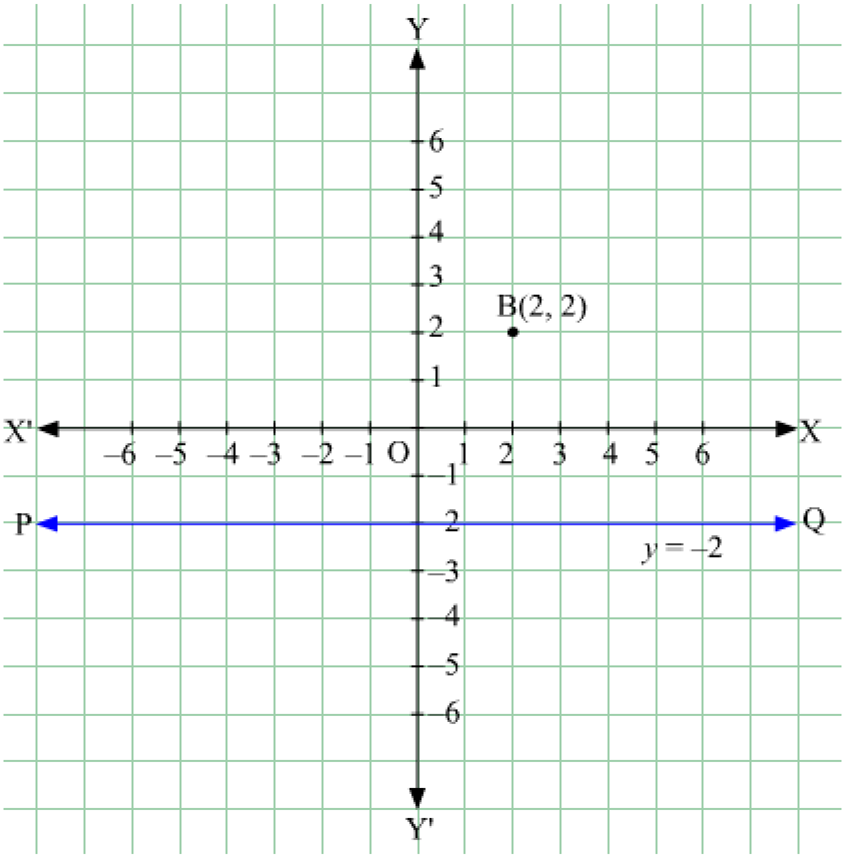


Similarly, we have ***y* = *a*** which is parallel to *x*-axis and is at a distance of *a* units from it.
Suppose we have to find the reflection of point B(2, 2) from the line $y + 2 = 0$ we follow the below given steps:

Step 1: Line PQ represents $y = -2$ which is a straight line parallel to the *x*-axis and at a distance of 2 units from it.

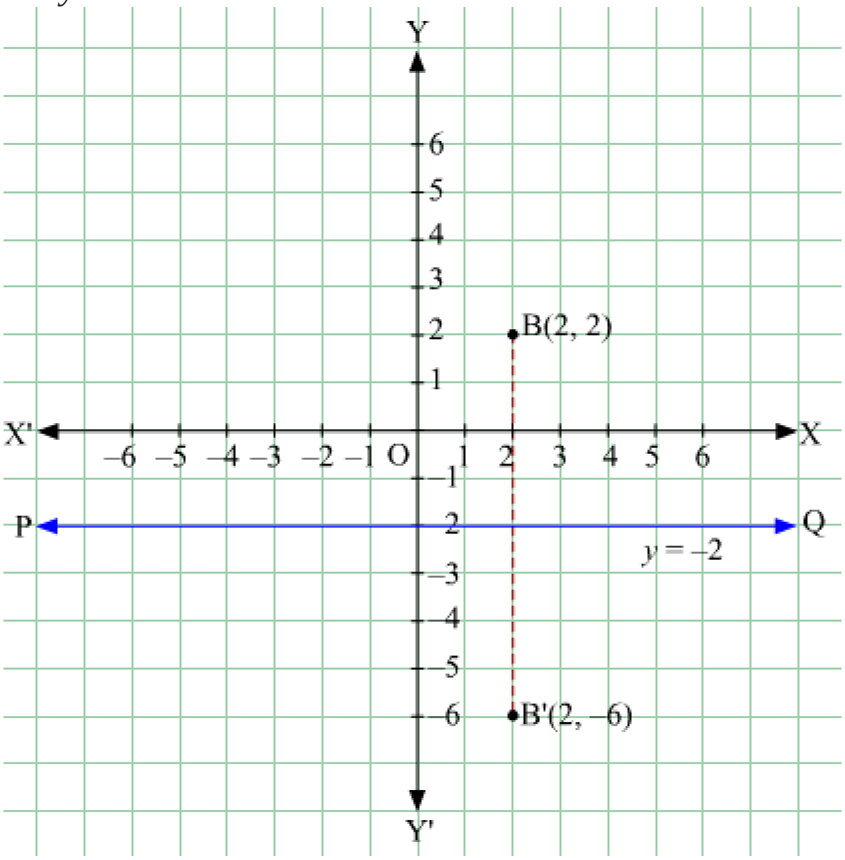


Step 2: Mark a point B(2, 2) on the same graph.



Step 3: From the point B, draw a straight line perpendicular to PQ. Mark a point B' below this straight line PQ at the same distance
as B(2, 2) is above it. B'(2, −6) is the required reflection of the point B(2, 2) in the

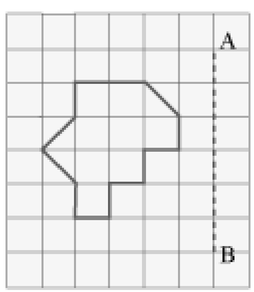
line $y + 2 = 0$.



In order to understand these concepts better, let us look at some examples.

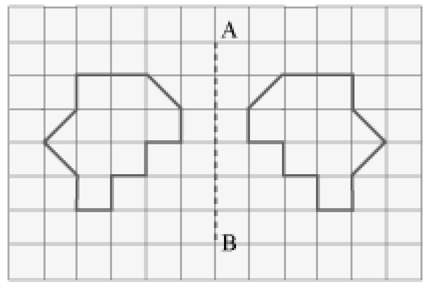
Example 1:

Draw the mirror reflection of the following figure where AB is the mirror line.



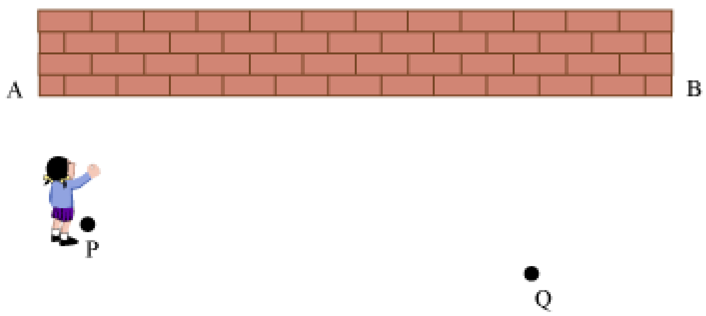
Solution:

The mirror reflection of the given figure with respect to mirror line AB can be drawn as



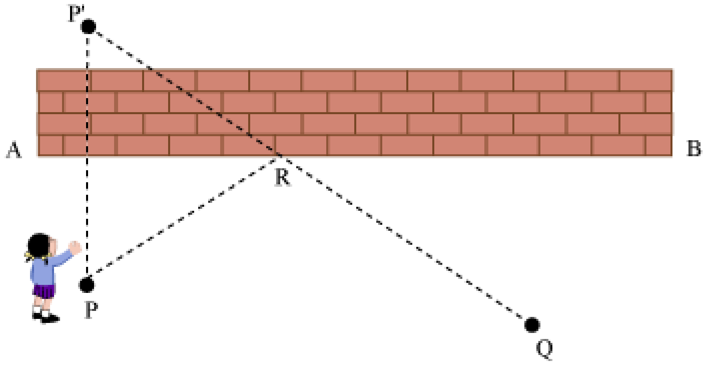
Example 2:

The given figure shows a wall with end points A and B. Sanjana is standing at position P. She has to come to position Q after touching the wall. Find the shortest path for Sanjana to come from P to Q.



Solution:

Let us imagine that wall AB acts as a mirror. Then P′ is the position of the image of point P. The object and its mirror image are at the same distance from the mirror. Therefore, points P and P′ are at the same distance from the wall.



The shortest distance between two points is the straight line joining the two points.

Therefore, the shortest distance between P′ and Q is P′Q. Let us join the points P′ and Q by a straight line which passes through the wall at point R.

Now, P′Q = P′R + RQ

But PR is the mirror reflection of P`R.

Or we can say that PR = P′R

Therefore, P′Q = PR + RQ

The path from P to R and then from R to Q is the shortest path which should be followed by Sanjana.

Example 3:

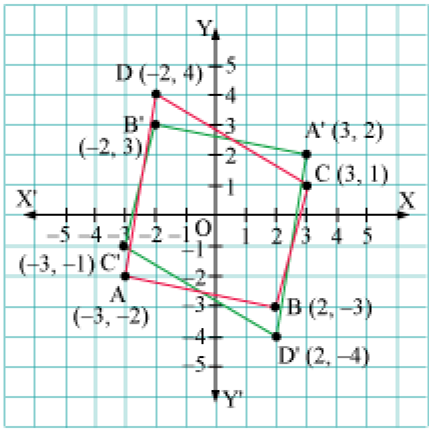
The quadrilateral ABCD whose vertices are A (−3, −2), B (2, −3), C (3, 1), and D (−2, 4) is on a co-ordinate plane. Draw its reflection $A'B'C'D'$ in origin.

Solution:

(1) The reflection of the points A (−3, −2), B (2, −3), C (3, 1), and (D (−2, 4) in the origin are:

$A'(3,2), B'(-2,3), C'(-3,-1)$, and $D'(2,-4)$

By joining $A'B', B'C', C'D'$, and $A'D'$, we obtain the quadrilateral $A'B'C'D'$, which is the reflection of the given quadrilateral ABCD in the origin as shown below.



Example 4:

The line AB joining the points A (−3, −4) and B (2, 1) is on co-ordinate plane.

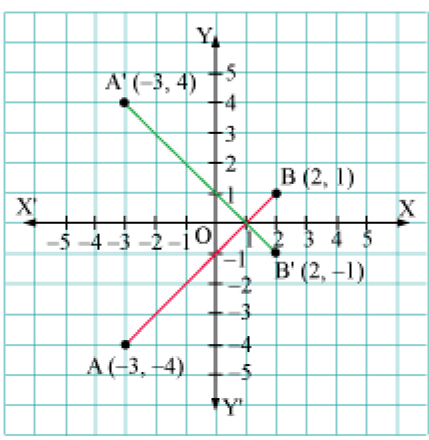
Draw its reflection

(i) $A'B'$ about *x*-axis

(ii) $A''B''$ about *y*-axis

Solution:

(i) The refection of points A (−3, −4) and B (2, 1) about *x*-axis are A' (−3, 4) and B' (2, − 1). By joining $A'B'$, we obtain the reflection of the given line AB as shown below.



(ii) The reflection of points A (−3, −4) and B (2, 1) about *y*-axis are A'' (3, −4) and B'' (− 2, 1).

By joining $A''B''$, we obtain the reflection of the given line AB as shown below.