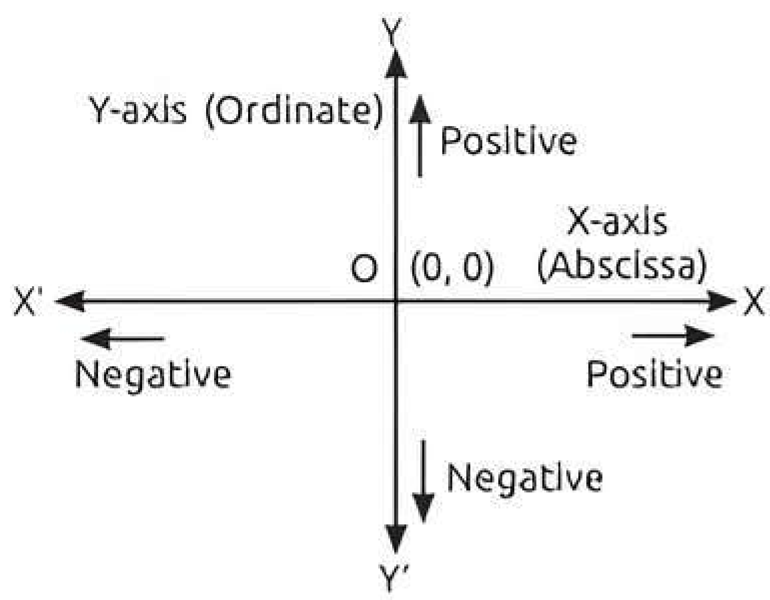
7 Coordinate Geometry

Fastrack Revision

- **Cartesian System:** The system used for describing the position of a point in a plane with reference to two fixed mutually perpendicular lines is termed as the **cartesian system**.

  In a cartesian system,

  (i) Horizontal line XX' is called the X-axis, while vertical line YY' is called Y-axis.



  (ii) The axes XX′ and YY′ divide the plane into four parts called **quadrants**.

  (iii) The perpendicular distance from the Y-axis measured along the X-axis is called ***x*-coordinate** or **abscissa**.

  (iv) The perpendicular distance from the X-axis measured along the Y-axis is called ***y*-coordinate** or **ordinate**.

  (v) (0, 0) is the origin.

- **Distance Formula:** The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the cartesian plane is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

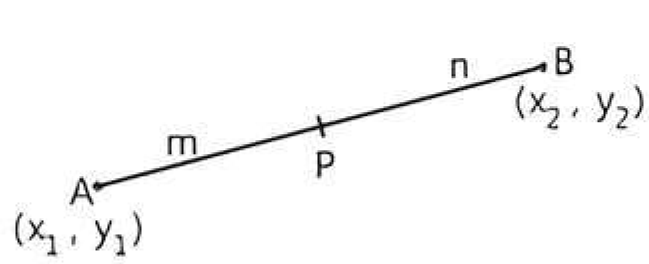
or
$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

  (i) The distance of a point $P(x, y)$ from the origin is $\sqrt{x^2 + y^2}$.

  (ii) Three points are collinear, if sum of two sides is equal to third side.

- **Section Formula for Internal Division:** If $P(x, y)$ divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$, then

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$



- **Mid-point Formula:** If $P(x, y)$ is the mid-point of $A(x_1, y_1)$ and $B(x_2, y_2)$, then

$$x = \frac{x_1 + x_2}{2} \text{ and } y = \frac{y_1 + y_2}{2}$$

Knowledge BOOSTER

1. *In a triangle, sum of lengths of any two sides is greater than the length of third side.*
2. *A triangle is right-angled triangle iff sides of triangle satisfy Pythagoras theorem.*
   Or A triangle is right-angled, if sum of squares of any two sides is equal to square of third largest side.
3. *A triangle is equilateral iff its all sides are equal in length.*
4. *A triangle is isosceles iff its any two sides are equal in length.*
5. *A triangle is right-angled isosceles iff its two sides are equal in length and all its sides satisfy Pythagoras theorem.*
6. *A quadrilateral is a parallelogram iff its opposite sides are equal in length but the diagonals are not equal.*
7. *A quadrilateral is a rectangle iff its opposite sides are equal and diagonals are also equal.*
8. *A quadrilateral is a rhombus iff its all four sides are equal but the diagonals are not equal.*
9. *A quadrilateral is a square iff its all sides are equal and diagonals are also equal.*
10. *In parallelogram, rectangle, square and rhombus, diagonals bisect each other.*
11. *If the vertices of a triangle are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, then centroid of a ΔABC is*

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Practice Exercise

Multiple Choice Questions

Q 1. The distance of the point (–6, 8) from *X*-axis is: *[CBSE 2023]*

a. 6 units b. –6 units c. 8 units d. 10 units

Q 2. The distance of the point (–6, 8) from origin is: *[CBSE 2023]*

a. 6 b. –6 c. 8 d. 10

Q 3. (*x*, *y*) is 5 units from the origin. How many such points lie in the third quadrant? *[CBSE SQP 2023-24]*

a. 0 b. 1
c. 2 d. infinitely many

Q 4. If the distance between A(*k*, 3) and B(2, 3) is 5, then the value of *k* is:

a. 5 b. 6 c. 7 d. 8

Q 5. If the point (*x*, *y*) is equidistant from the points (2, 1) and (1, –2), then:

a. $x+3y=0$ b. $3x+y=0$
c. $x+2y=0$ d. $3x+2y=0$

Q 6. The points (–4, 0), (4, 0) and (0, 3) are the vertices of a/an: *[CBSE 2023]*

a. right triangle b. isosceles triangle
c. equilateral triangle d. scalene triangle

Q 7. The points (2, 4), (2, 6) and $(2+\sqrt{3}, 5)$ are the vertices of:

a. an equilateral triangle
b. an isosceles triangle
c. a right triangle
d. a right angled isosceles triangle

Q 8. The distance between the points $(a\cos\theta+b\sin\theta, 0)$ and $(0, a\sin\theta-b\cos\theta)$ is:

a. a^2+b^2 b. $a+b$
c. a^2-b^2 d. $\sqrt{a^2+b^2}$

Q 9. The area of the triangle formed by the line $\frac{x}{a}+\frac{y}{b}=1$ with the coordinate axes is: *[CBSE 2023]*

a. ab b. $\frac{1}{2}ab$ c. $\frac{1}{4}ab$ d. $2ab$

Q 10. The centre of a circle is (2*a*, *a* – 7). If the circle passes through the point (1, –9) and has diameter $10\sqrt{2}$ units, then the value of *a* is:

a. 9 b. $-\sqrt{3}$
c. $\sqrt{3}$ d. ± 3

Q 11. If $A(3, \sqrt{3}), B(0, 0)$ and $C(3, k)$ are the three vertices of an equilateral triangle ABC, then the value of *k* is: *[CBSE 2021 Term-I]*

a. 2 b. -3
c. $\pm\sqrt{3}$ d. $\sqrt{}$

Q 12. ABCD is a rectangle whose three vertices are (4, 3), (4, 1) and (0, 1). The length of its diagonal is:

a. $2\sqrt{5}$ units b. $\sqrt{5}$ units
c. $\frac{1}{\sqrt{5}}$ units d. $\frac{2}{\sqrt{5}}$ units

Q 13. If the vertices of a parallelogram PQRS taken in order are P(3, 4), Q(–2, 3) and R(–3, –2), then the coordinates of its fourth vertex S are: *[CBSE SQP 2022-23]*

a. (– 2, – 1) b. (–2, –3)
c. (2, – 1) d. (1, 2)

Q 14. If the segment joining the points (*a*, *b*) and (*c*, *d*) subtends a right angle at the origin, then:

a. $ac-bd=0$ b. $ac+bd=0$
c. $ab+cd=0$ d. $ab-cd=0$

Q 15. Points A(–1, *y*) and B(5, 7) lie on a circle with centre O(2, –3*y*). The values of *y* are: *[CBSE 2021 Term-I]*

a. 1, –7 b. –1, 7
c. 2, 7 d. –2, –7

Q 16. If P(–1, 1) is the mid-point of the line segment joining A(–3, *b*) and B(1, *b* + 4), then *b* =

a. 1 b. – 1
c. 2 d. 0

Q 17. In what ratio does the point P(3, 4) divides the line segment joining the points A(1, 2) and B(6, 7)?

a. 1 : 2 b. 2 : 3
c. 3 : 4 d. 1 : 1

Q 18. The coordinates of the circumcentre of the triangle formed by the points O(0, 0), A(*a*, 0) and B(0, *b*) are:

a. (a, b) b. $\left(\frac{a}{2}, \frac{b}{2}\right)$
c. $\left(\frac{b}{2}, \frac{a}{2}\right)$ d. (b, a)

Q 19. If the point P(6, 2) divides the line segment joining A(6, 5) and B(4, *y*) in the ratio 3 : 1, then the value of *y* is: *[CBSE 2020]*

a. 4 b. 3 c. 2 d. 1

Q 20. If A(4, –2), B(7, –2) and C(7, 9) are the vertices of a ΔABC, then ΔABC is: *[CBSE 2021 Term-I]*

a. equilateral triangle
b. isosceles triangle
c. right angled triangle
d. isosceles right angled triangle

Q 21. The ratio in which the X-axis divides the line segment joining the points (–2, 3) and (6, –7) is: *[CBSE 2023]*

a. 1 : 3 b. 3 : 7 c. 7 : 3 d. 1 : 2

Q 22. Point P divides the line segment joining R(–1, 3) and S(9, 8) in ratio k : 1. If P lies on the line $x-y+2=0$, then value of k is: *[CBSE SQP 2021 Term-I]*

a. 2/3 b. 1/2 c. 1/3 d. 1/4

Q 23. The points A, B and C are collinear and AB = BC. If the coordinates of A, B and C are (3, a), (1, 3) and (b, 4) respectively, then the values of a and b are:

a. 2 and –1 b. 1 and –2
c. 1 and 2 d. –1 and –2

Q 24. The base BC of an equilateral ΔABC lies on the Y-axis. The coordinates of C are (0, –3). If the origin is the mid-point of the base BC. What are the coordinates of A and B? *[CBSE 2021 Term-I]*

a. $A(\sqrt{3}, 0), B(0, 3)$ b. $A(\pm 3\sqrt{3}, 0), B(3, 0)$
c. $A(\pm 3\sqrt{3}, 0), B(0, 3)$ d. $A(-\sqrt{3}, 0), B(3, 0)$

Q 25. The equation of the perpendicular bisector of line segment joining points A(4, 5) and B(–2, 3) is: *[CBSE SQP 2021 Term-I]*

a. $2x-y+7=0$ b. $3x+2y-7=0$
c. $3x-y-7=0$ d. $3x+y-7=0$

Assertion & Reason Type Questions

Directions (Q. Nos. 26-30): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reas on (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true and Reason (R) is false
d. Assertion (A) is false and Reason (R) is true

Q 26. Assertion (A): The distance between the points ($\cos\theta$, $\sin\theta$) and ($\sin\theta$, $-\cos\theta$) is 2 units.

Reason (R): The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}.$$

Q 27. Assertion (A): The point P(–4, 6) divides the join of A(–6, 10) and B(3, –8) in the ratio 2 : 7.

Reason (R): If the point C(x, y) divides the join of $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$, then

$$x=\frac{mx_2+nx_1}{m+n} \text{ and } y=\frac{my_2+ny_1}{m+n}.$$

Q 28. Assertion (A): If the coordinates of the mid-points of the sides AB and AC of ΔABC are D(3, 5) and E(–3, –3) respectively, then BC = 20 units.

Reason (R): The line joining the mid-points of two sides of a triangle is parallel to the third side and equal to half of it. *[CBSE SQP 2022-23]*

Q 29. Assertion (A): The coordinates of the points which divide the line segment joining A(2, –8) and B(–3, –7) into three equal parts are $\left(\frac{1}{3}, -\frac{23}{3}\right)$ and $\left(-\frac{4}{3}, -\frac{22}{3}\right)$.

Reason (R): The points which divide AB in the ratio 1 : 3 and 3 : 1 are called points of trisection of AB.

Q 30. Assertion (A): The coordinates of the centroid of a triangle whose vertices are (0, 6), (8, 12) and (8, 0), are $\left(\frac{17}{3}, 5\right)$.

Reason (R): Coordinates of the centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3), are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

Fill in the Blanks Type Questions

Q 31. The angle subtended by joining points A(3, 0) and B(0, –2) to the origin point is ....

Q 32. If the point P($k-1$, 2) is equidistant from the points A(3, k) and B(k, 5), the value of k is/are ....

Q 33. The coordinates of a point on X-axis which is equidistant from the points (–3, 4) and (7, 6), are ....

Q 34. Suppose AB is a line segment and points P and Q are nearer to A and B on a line segment AB such that AP = PQ = QA, then P divides the line segment in the ratio ....

Q 35. The point which lies on the perpendicular bisector of the line segment joining the points A(–2, –5) and B(2, 5) is ....

True/False Type Questions

Q 36. The distance between points P(a sin ϕ, 0) and Q(0, $-a$ cos ϕ) is a.

Q 37. The ratio in which the line segment joining the points (–3, 10) and (6, –8) is divided by (–1, 6) is 2/7.

[NCERT EXERCISE]

Q 38. If the vertices of a triangle are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, then the centroid of a $\triangle ABC$ is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

Q 39. If three points are collinear, then area of triangle is not zero.

Q 40. If point P divides the line joining A and B in the ratio 1 : 1, then point P is the mid-point of AB.

Solutions

1. (c) We know that, the perpendicular distance from X-axis measured along the Y-axis is called y-coordinate.

The distance of the point (–6, 8) from X-axis

= Numerical value of y-coordinate

= 8 units

2. (d) Given, $(-6, 8) \equiv (x_1, y_1)$ and origin $= (0, 0) = (x_2, y_2)$

∴ Distance of the point (–6, 8) from the origin

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0+6)^2 + (0-8)^2}$$

$$= \sqrt{36+64} = \sqrt{100} = 10$$

3. (d) Given; $(x, y) = (x_1, y_1)$ and origin $= (0, 0) = (x_2, y_2)$

Also (x, y) is 5 unit from the origin

$$\therefore \quad \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5$$

$$\Rightarrow \quad (0-x)^2 + (0-y)^2 = 25$$

$$\Rightarrow \quad x^2 + y^2 = 25$$

$$\Rightarrow \quad (-x)^2 + (-y)^2 = 25 \quad ...(1)$$

In third quadrant, the coordinates of the point $= (-x, -y)$ which satisfy eq. (1).

So, there are infinitely many points lie in the third quadrant which satisfy eq. (1).

4. (c) The distance between A and B is

$$AB = \sqrt{(k-2)^2 + (3-3)^2}$$

$$\Rightarrow \quad \sqrt{(k-2)^2} = 5 \quad \text{(given)}$$

Squaring on both sides, we have $(k-2)^2 = 25$

$$k - 2 = \pm 5$$

$$\Rightarrow \quad k = 2 \pm 5$$

$$\Rightarrow \quad k = 7 \text{ or } -3$$

5. (a) Let the points be $P(x, y)$, $A(2, 1)$ and $B(1, -2)$.

Since, P is equidistant from A and B.

$$\therefore \quad AP = BP$$

$$\Rightarrow \quad AP^2 = BP^2$$

$$\Rightarrow \quad (x-2)^2 + (y-1)^2 = (x-1)^2 + (y+2)^2$$

$$\Rightarrow \quad x^2 - 4x + 4 + y^2 - 2y + 1 = x^2 - 2x + 1 + y^2 + 4y + 4$$

$$\Rightarrow \quad -4x - 2y = -2x + 4y$$

$$\Rightarrow \quad 2x + 6y = 0$$

$$\Rightarrow \quad x + 3y = 0$$

6. (b) Let ABC be the triangle whose vertices are A (–4, 0), B (4, 0) and C (0, 3).

> **TiP**
> *In an isosceles triangle, any two sides are equal.*

Here $\quad AB = \sqrt{(4+4)^2 + (0-0)^2} = \sqrt{(8)^2 + 0} = 8$

$$BC = \sqrt{(0-4)^2 + (3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

and $\quad AC = \sqrt{(0+4)^2 + (3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5$

We have, BC = AC = 5

Hence, $\triangle ABC$ is an isosceles triangle.

7. (a) Let ABC be the triangle whose vertices are A(2, 4), B(2, 6) and $C(2+\sqrt{3}, 5)$.

> **TiP**
> *In an equilateral triangle, all sides are equal.*

Here, $\quad AB = \sqrt{(2-2)^2 + (6-4)^2} = \sqrt{0^2 + 2^2} = 2$

$$BC = \sqrt{(2+\sqrt{3}-2)^2 + (5-6)^2}$$

$$= \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{4} = 2$$

and $\quad AC = \sqrt{(2+\sqrt{3}-2)^2 + (5-4)^2}$

$$= \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{4} = 2$$

We have, AB = BC = AC = 2

Hence, $\triangle ABC$ is an equilateral triangle.

8. (d) The distance between points $P(a\cos\theta + b\sin\theta, 0)$ and $Q(0, a\sin\theta - b\cos\theta)$ is given by

$$PQ = \sqrt{(a\cos\theta + b\sin\theta - 0)^2 + (0 - a\sin\theta + b\cos\theta)^2}$$

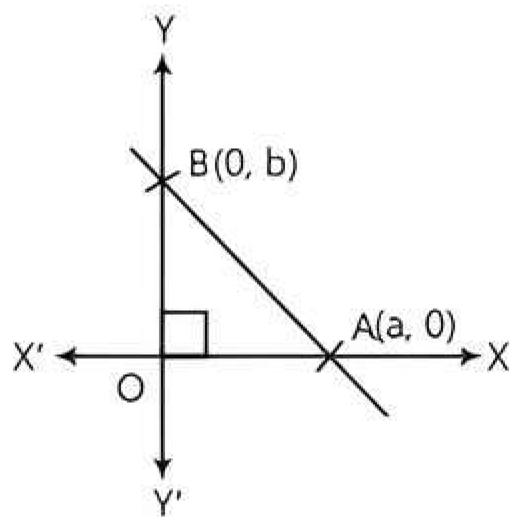
$$= \sqrt{\begin{aligned}&a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta\\ &+ a^2\sin^2\theta + b^2\cos^2\theta - 2ab\cos\theta\sin\theta\end{aligned}}$$

$$= \sqrt{a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)}$$

$$= \sqrt{a^2 + b^2} \qquad (\because \sin^2\theta + \cos^2\theta = 1)$$

9. (b) Given equation of line is $\frac{x}{a} + \frac{y}{b} = 1$. ...(1)

The above eq. (1) is the intercept form of a line, which cut X-axis at point $(a, 0)$ and Y-axis at point $(0, b)$.



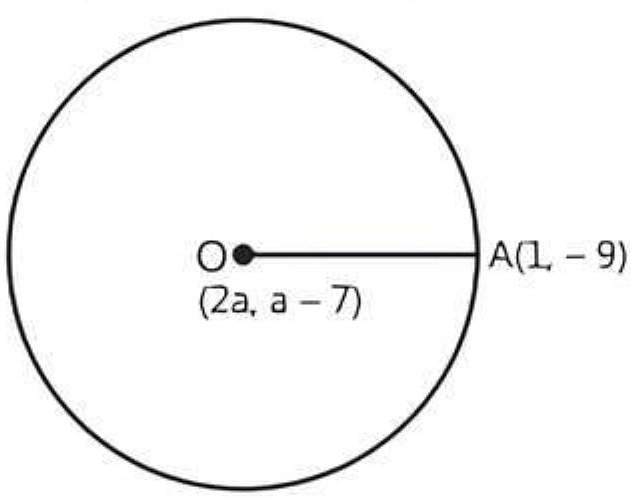
From figure, $OA = a$

and $OB = b$

$\therefore$ The area of the triangle $AOB = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times OA \times OB = \frac{1}{2}ab$$

10. (d) Let $O(2a, a-7)$ be the centre and $A(1, -9)$ be any point through which circle passes.



$$2(OA) = 10\sqrt{2} \qquad (\because \text{Diameter} = 2 \times \text{radius})$$

$$\Rightarrow \quad OA = 5\sqrt{2}$$

$$\Rightarrow \quad OA^2 = (5\sqrt{2})^2 = 50$$

$$\Rightarrow \quad (2a-1)^2 + (a-7+9)^2 = 50$$

$$\Rightarrow \quad 4a^2 + 1 - 4a + a^2 + 4 + 4a = 50$$

$$\Rightarrow \quad 5a^2 = 50 - 5 = 45$$

$$\Rightarrow \quad a^2 = 9$$

$$\Rightarrow \quad a = \pm 3$$

11. (c) Given vertices of an equilateral triangle are $A(3, \sqrt{3}), B(0, 0)$ and $C(3, k)$.

TiP

All sides of an equilateral triangle are equal.

$$\therefore \quad AB = BC$$

$$\Rightarrow \quad \sqrt{(0-3)^2 + (0-\sqrt{3})^2} = \sqrt{(3-0)^2 + (k-0)^2}$$

$$\Rightarrow \quad \sqrt{9+3} = \sqrt{9+k^2}$$

$$\Rightarrow \quad \sqrt{12} = \sqrt{9+k^2}$$

Squaring on both sides, we have

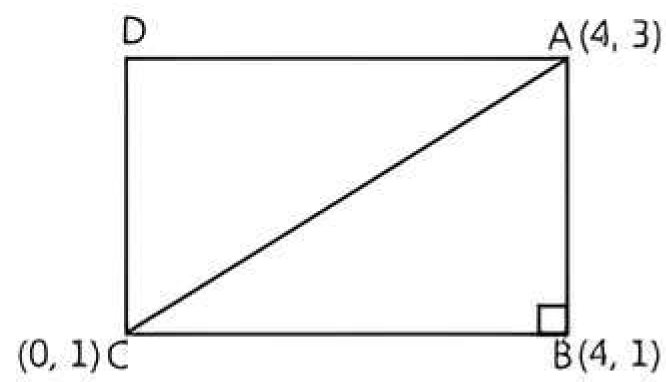
$$12 = 9 + k^2 \Rightarrow k^2 = 3$$

$$\Rightarrow \quad k = \pm\sqrt{3}$$

12. (a) Let the three vertices of rectangle ABCD be $A(4, 3), B(4, 1)$ and $C(0, 1)$.

TiP

Each adjacent sides of a rectangle intersect at right angle.



TR!CK

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

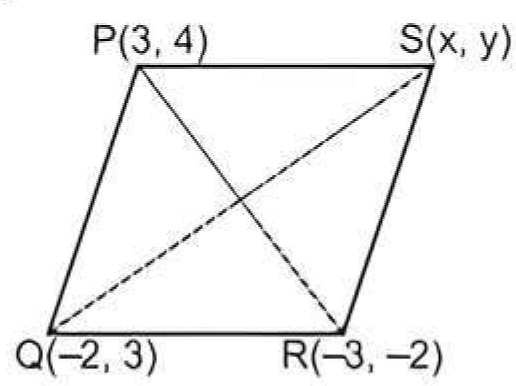
In right ΔABC,

Length of diagonal $AC = \sqrt{(0-4)^2 + (1-3)^2}$

(by Pythagoras theorem)

$$= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5} \text{ units}$$

13. (c) Let coordinates of fourth vertex of a parallelogram be $S(x, y)$.



TiP

In a parallelogram, diagonals intersect at mid-point.

$\therefore$ Mid-point of PR = Mid-point of QS

$$\Rightarrow \quad \left(\frac{3-3}{2}, \frac{4-2}{2}\right) = \left(\frac{-2+x}{2}, \frac{3+y}{2}\right)$$

$$\Rightarrow \quad (0, 1) = \left(\frac{-2+x}{2}, \frac{3+y}{2}\right)$$

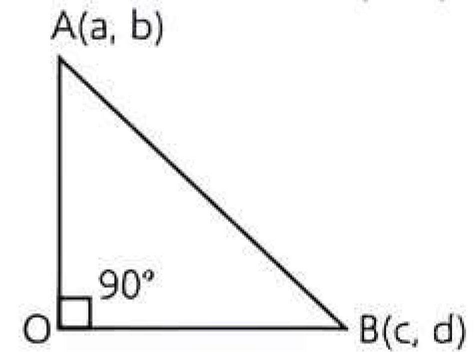
Equating the coordinates both sides,

$$0=\frac{-2+x}{2}, 1=\frac{3+y}{2}$$

$\Rightarrow$ $x=2,\ y=2-3=-1$

Hence, coordinates of fourth vertex is $S(2,-1)$.

14. (b) Let the coordinates be $A(a, b)$, $B(c, d)$, $O(0, 0)$.



Using Pythagoras theorem,

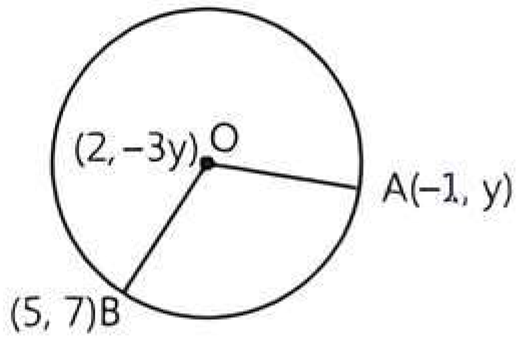
$$AB^2 = AO^2 + BO^2$$

$\Rightarrow$ $(c-a)^2+(d-b)^2=(a-0)^2+(b-0)^2+(c-0)^2+(d-0)^2$

$\Rightarrow$ $c^2+a^2-2ac+d^2+b^2-2bd = a^2+b^2+c^2+d^2$

$\Rightarrow$ $ac+bd=0$

15. (b) Since, OA and OB are the radii of a circle.



$\therefore$ $OA^2 = OB^2$

$\Rightarrow$ $(-1-2)^2+(y+3y)^2=(5-2)^2+(7+3y)^2$

$\Rightarrow$ $9+16y^2=9+49+9y^2+42y$

$\Rightarrow$ $16y^2-9y^2-42y-49=0$

$\Rightarrow$ $7y^2-42y-49=0$

$\Rightarrow$ $y^2-6y-7=0$

TR!CK

$\because$ $7=7\times1$

Here we taken 7 and 1 as a factor of 7.

So, middle term $-6=1-7$.

$\Rightarrow$ $y^2-7y+y-7=0$

$\Rightarrow$ $y(y-7)+1(y-7)=0$

$\Rightarrow$ $(y+1)(y-7)=0$

$\Rightarrow$ $y+1=0$ or $y-7=0$

$\Rightarrow$ $y=-1$ or $y=7$

16. (b) Since, $P(-1, 1)$ is the mid-point of line segment joining $A(-3, b)$ and $B(1, b+4)$.

$\therefore$ $(-1, 1)=\left(\frac{-3+1}{2}, \frac{b+b+4}{2}\right)$

Equating y-coordinate, we get

$$1=\frac{2b+4}{2} \Rightarrow 1=b+2$$

$\Rightarrow$ $b=-1$

17. (b) Let $P(3, 4)$ divides the join of $A(1, 2)$ and $B(6, 7)$ in the ratio $k:1$.

$\therefore$ Coordinates of P are $\frac{6k+1}{k+1}=3$

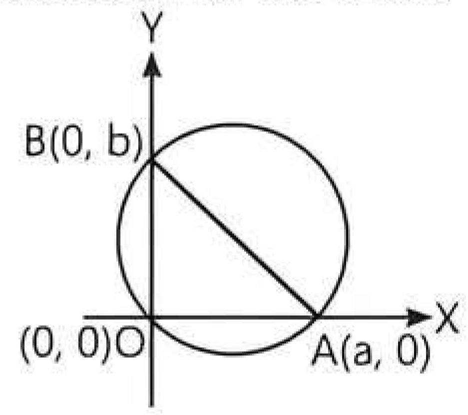
$\Rightarrow$ $6k+1=3k+3 \Rightarrow k=\frac{2}{3}$

and $\frac{7k+2}{k+1}=4$

$\Rightarrow$ $7k+2=4k+4 \Rightarrow k=\frac{2}{3}$

Hence, the required ratio is $\frac{2}{3}:1$ *i.e.*, $2:3$.

18. (b) Since, $\angle AOB = 90^\circ$ and O is a point on the circle. So, AB is a diameter of the circle.



$\therefore$ Circumcentre is the mid-point of diameter AB

$$=\left(\frac{a+0}{2}, \frac{0+b}{2}\right) \text{ i.e., } \left(\frac{a}{2}, \frac{b}{2}\right)$$

19. (d)

TR!CK

If $P(x, y)$ divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m:n$ then

$$x=\left(\frac{mx_2+nx_1}{m+n}, y=\frac{my_2+ny_1}{m+n}\right)$$

$$P(6, 2)=\left(\frac{3\times4+1\times6}{3+1}, \frac{3\times y+1\times5}{3+1}\right)$$

$$=\left(\frac{12+6}{4}, \frac{3y+5}{4}\right)=\left(\frac{9}{2}, \frac{3y+5}{4}\right)$$

On comparing y-coordinate,

$$2=\frac{3y+5}{4} \Rightarrow 3y+5=8 \Rightarrow y=1$$

20. (c) Given vertices of a triangle are $A(4, -2)$, $B(7, -2)$ and $C(7, 9)$.

Now, $AB=\sqrt{(7-4)^2+(-2+2)^2}=\sqrt{3^2+0}=3$

$BC=\sqrt{(7-7)^2+(9+2)^2}=\sqrt{0+11^2}=11$

and $CA=\sqrt{(4-7)^2+(-2-9)^2}=\sqrt{(-3)^2+(-11)^2}$

$=\sqrt{9+121}=\sqrt{130}$

Here we see that $AB \neq BC \neq CA$

Now $(AB)^2+(BC)^2=(3)^2+(11)^2$

$=9+121=130$

$=(CA)^2$

Hence, triangle ABC is a right angled triangle.

21. (b) Let the X-axis divides the join $(-2, 3)$ and $(6, -7)$ in the ratio $k : 1$. Then y-coordinate of X-axis is zero.

$\therefore \quad \frac{-7k+3}{k+1} = 0 \qquad (\because k \neq -1)$

$\Rightarrow \quad -7k + 3 = 0 \quad \Rightarrow k = \frac{3}{7}$ *i.e.*, $3 : 7$

22. (a)

TiP

Coordinates of point P divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ is $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$.

Since, P divides the line segment joining $R(-1, 3)$ and $S(9, 8)$ in ratio $k : 1$.

$\therefore$ Coordinates of P are $\left(\frac{9k-1}{k+1}, \frac{8k+3}{k+1}\right)$

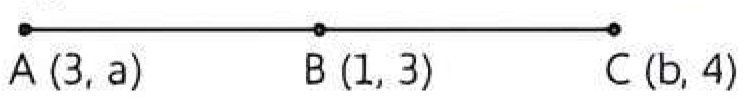
Since, P lies on the line $x - y + 2 = 0$.

Then, $\frac{9k-1}{k+1} - \frac{8k+3}{k+1} + 2 = 0$

$\Rightarrow \quad 9k - 1 - 8k - 3 + 2k + 2 = 0$

$\Rightarrow \quad 3k = 2 \quad \Rightarrow \quad k = \frac{2}{3}$

23. (a) $\because AB = BC$ and A, B and C are collinear.



$\therefore$ B is the mid-point of AC.

TR!CK

The mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

By mid-point formula,

$$(1, 3) = \left(\frac{3+b}{2}, \frac{a+4}{2}\right)$$

On comparing x and y-coordinates, we get

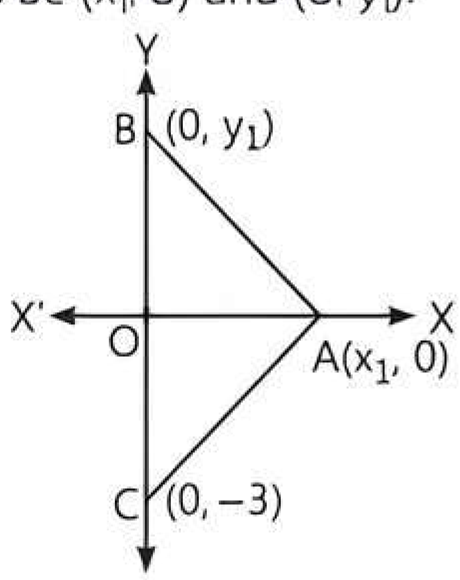
$$1 = \frac{3+b}{2} \Rightarrow 3 + b = 2 \Rightarrow b = -1$$

and $\quad 3 = \frac{a+4}{2} \Rightarrow a + 4 = 6 \Rightarrow a = 2$

Hence, $a = 2$ and $b = -1$.

24. (c) Let coordinates of A and B be $(x_1, 0)$ and $(0, y_1)$.

Since, O is the mid-point of BC.



$\therefore \quad (0, 0) = \left(0, \frac{y_1 - 3}{2}\right)$

$\Rightarrow \quad 0 = \frac{y_1 - 3}{2} \Rightarrow y_1 = 3$

$\therefore$ Coordinate of $B(0, 3)$.

Since, ABC is an equilateral triangle.

$\therefore \quad BC^2 = AC^2$

$\Rightarrow \quad (0)^2 + (-3-3)^2 = (0 - x_1)^2 + (-3-0)^2$

$\Rightarrow \quad 0 + 36 = x_1^2 + 9$

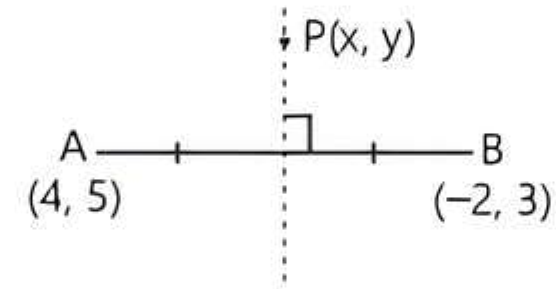
$\Rightarrow \quad x_1^2 = 27 \Rightarrow x_1 = \pm 3\sqrt{3}$

Hence, coordinates are $A(\pm 3\sqrt{3}, 0)$ and $B(0, 3)$.

COMMON ERR(!)R

Sometimes students make a mistake of considering the value of x_1 is only positive, but it may be consider negative value of x_1 also. Because A coordinate will be either on positive X-axis or negative X-axis.

25. (d) Any point on perpendicular bisector will be equidistant from the points $A(4, 5)$ and $B(-2, 3)$.



Let $P(x, y)$ be any point lie on the perpendicular bisector of AB.

$\therefore \quad AP = BP$

$\Rightarrow \quad AP^2 = BP^2$

$\Rightarrow \quad (x-4)^2 + (y-5)^2 = (x+2)^2 + (y-3)^2$ (by distance formula)

$\Rightarrow \quad x^2 + 16 - 8x + y^2 + 25 - 10y = x^2 + 4 + 4x + y^2 + 9 - 6y$

$\Rightarrow \quad 12x + 4y = 28$

$\Rightarrow \quad 3x + y - 7 = 0$

26. (d) **Assertion (A):** Distance between points $A(\cos\theta, \sin\theta)$ and $B(\sin\theta, -\cos\theta)$ is given by

$$AB = \sqrt{(\sin\theta - \cos\theta)^2 + (-\cos\theta - \sin\theta)^2}$$

$$= \sqrt{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta}$$

$$= \sqrt{2(\sin^2\theta + \cos^2\theta)} = \sqrt{2} \text{ units}$$

So, Assertion (A) is false.

Reason (R): It is a true statement.

Hence, Assertion (A) is false but Reason (R) is true.

27. (a) **Assertion (A):** Let $P(-4, 6)$ divides $A(-6, 10)$ and $B(3, -8)$ in the ratio $k : 1$.

Then, $\frac{k \times 3 + 1 \times (-6)}{k+1} = -4$ and $\frac{k \times (-8) + 1 \times 10}{k+1} = 6$

$\Rightarrow \quad 3k - 6 = -4k - 4$ and $-8k + 10 = 6k + 6$

$\Rightarrow \quad 7k = 2 \Rightarrow k = \frac{2}{7}$ and $14k = 4 \Rightarrow k = \frac{2}{7}$

$\therefore$ Required ratio is $2/7$ *i.e.*, $2 : 7$.

So, Assertion (A) is true.

Reason (R) : It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

28. (a) **Assertion (A):**

Now $DE = \sqrt{(-3-3)^2 + (-3-5)^2}$

$= \sqrt{36+64} = \sqrt{100}$

$= 10$ units

Since, D and E are the mid-points of the sides of a triangle.

By using mid-point theorem, the line joining the mid-points of two sides of a triangle is parallel to the third side and equal to half of it.

$\therefore \quad DE = \dfrac{BC}{2}$

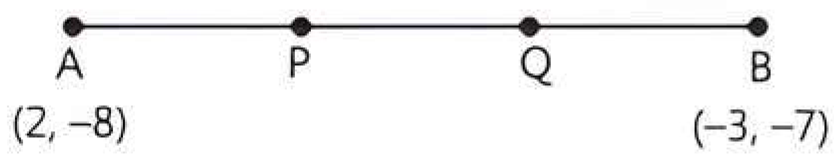
$\Rightarrow \quad BC = 2DE = 2 \times 10 = 20$ units

So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

29. (c) **Assertion (A):** Let P and Q be the points which divide A(2, –8) and B(–3, –7) into three equal parts.



$\therefore \quad AP : PB = 1 : 2$

So, coordinates of P

$= \left(\dfrac{1\times(-3)+2\times2}{1+2}, \dfrac{1\times(-7)+2\times(-8)}{1+2}\right)$

$= \left(\dfrac{-3+4}{3}, \dfrac{-7-16}{3}\right) = \left(\dfrac{1}{3}, -\dfrac{23}{3}\right)$

Also, $AQ : QB = 2 : 1$

$\therefore$ Coordinates of Q

$= \left(\dfrac{2\times(-3)+1\times2}{2+1}, \dfrac{2\times(-7)+1\times(-8)}{2+1}\right)$

$= \left(\dfrac{-6+2}{3}, \dfrac{-14-8}{3}\right) = \left(-\dfrac{4}{3}, -\dfrac{-22}{3}\right)$

So, Assertion (A) is true.

Reason (R): It is false to say that in trisection point, point divide AB in the ratios 3 : 1 and 1 : 3.

Hence, Assertion (A) is true but Reason (R) is false.

30. (d) **Assertion (A):** The, coordinate of the centroid of a triangle with vertices (0, 6), (8, 12) and (8, 0) are

$\left(\dfrac{0+8+8}{3}, \dfrac{6+12+0}{3}\right) = \left(\dfrac{16}{3}, 6\right)$

So, Assertion (A) is false.

Reason (R): It is a true statement.

Hence, Assertion (A) is false but Reason (R) is true.

31. Let O = (0, 0) be the origin.

$\therefore \quad OA = \sqrt{(3-0)^2 + (0-0)^2} = 3$

$OB = \sqrt{(0-0)^2 + (0+2)^2} = 2$

and $\quad AB = \sqrt{(3-0)^2 + (0+2)^2} = \sqrt{9+4} = \sqrt{13}$

$\because \quad AB^2 = OA^2 + OB^2$

$\therefore$ AB subtend right angle at origin.

32. $\because \quad AP = BP$

$\Rightarrow \quad AP^2 = BP^2$ (by distance formula)

$\therefore (k-1-3)^2 + (2-k)^2 = (k-1-k)^2 + (2-5)^2$

$\Rightarrow \quad (k-4)^2 + (2-k)^2 = (-1)^2 + (-3)^2$

$\Rightarrow \quad k^2 + 16 - 8k + 4 + k^2 - 4k = 1 + 9$

$\Rightarrow \quad 2k^2 - 12k + 10 = 0$

$\Rightarrow \quad k^2 - 6k + 5 = 0$

TR!CK

$\because \quad 5 = 5 \times 1$

Here we will take 5 and 1 as a factor of 5.

So, middle term $-6 = -5 - 1$.

$\Rightarrow \quad k^2 - 5k - k + 5 = 0$

$\Rightarrow \quad k(k-5) - 1(k-5) = 0$

$\Rightarrow \quad (k-5)(k-1) = 0$

$\Rightarrow \quad k = 1, 5$

33.

TiP

Ordinate of each point on X-axis is always zero.

Let required point be P(x, 0).

Also, let $\quad A \equiv (-3, 4)$ and $B \equiv (7, 6)$

$\therefore \quad AP = BP$

$\Rightarrow \quad AP^2 = BP^2$

$\Rightarrow \quad (x+3)^2 + (0-4)^2 = (x-7)^2 + (0-6)^2$

(by distance formula)

$\Rightarrow \quad x^2 + 9 + 6x + 16 = x^2 + 49 - 14x + 36$

$\Rightarrow \quad 20x = 60$

$\Rightarrow \quad x = 3$

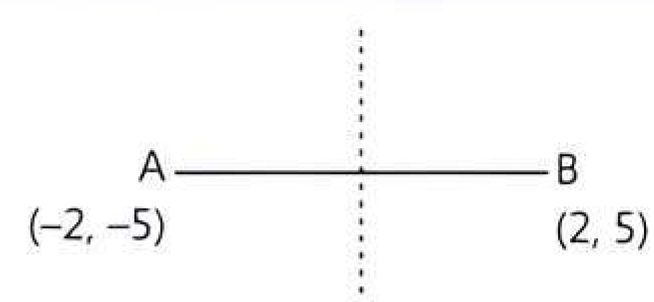
$\therefore$ Point is P(3, 0).

34. 1 : 2

35.

TiP

A point lies on the perpendicular bisector of AB *is equal to the mid-point of* AB.



Perpendicular bisector of AB = Mid-point of AB

$= \left(\dfrac{-2+2}{2}, \dfrac{-5+5}{2}\right) = (0, 0)$

36. The distance between points $P(a \sin \phi, 0)$ and $Q(0, -a \cos \phi)$ is

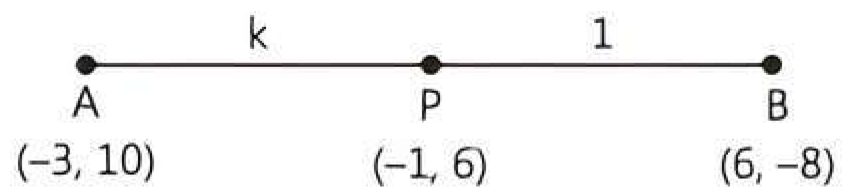
$PQ = \sqrt{(0 - a\sin\phi)^2 + (-a\cos\phi - 0)^2}$

$= \sqrt{a^2\sin^2\phi + a^2\cos^2\phi} = \sqrt{a^2(\sin^2\phi + \cos^2\phi)}$

$= \sqrt{a^2 \times 1} = a \quad [\because \sin^2\theta + \cos^2\theta = 1]$

Hence, given statement is true.

37. Let point P divides the line segment AB in the ratio $k : 1$.



$\therefore$ The coordinates of P $= \left(\frac{6k-3}{k+1}, \frac{-8k+10}{k+1}\right)$

$\Rightarrow \quad (-1, 6) = \left(\frac{6k-3}{k+1}, \frac{-8k+10}{k+1}\right)$

Equating the x and y-coordinates on both sides, we get

$-1 = \frac{6k-3}{k+1}$ and $6 = \frac{-8k+10}{k+1}$

$\Rightarrow \quad -k-1 = 6k-3$ and $6k+6 = -8k+10$

$\Rightarrow \quad 7k = 2$ and $14k = 4$

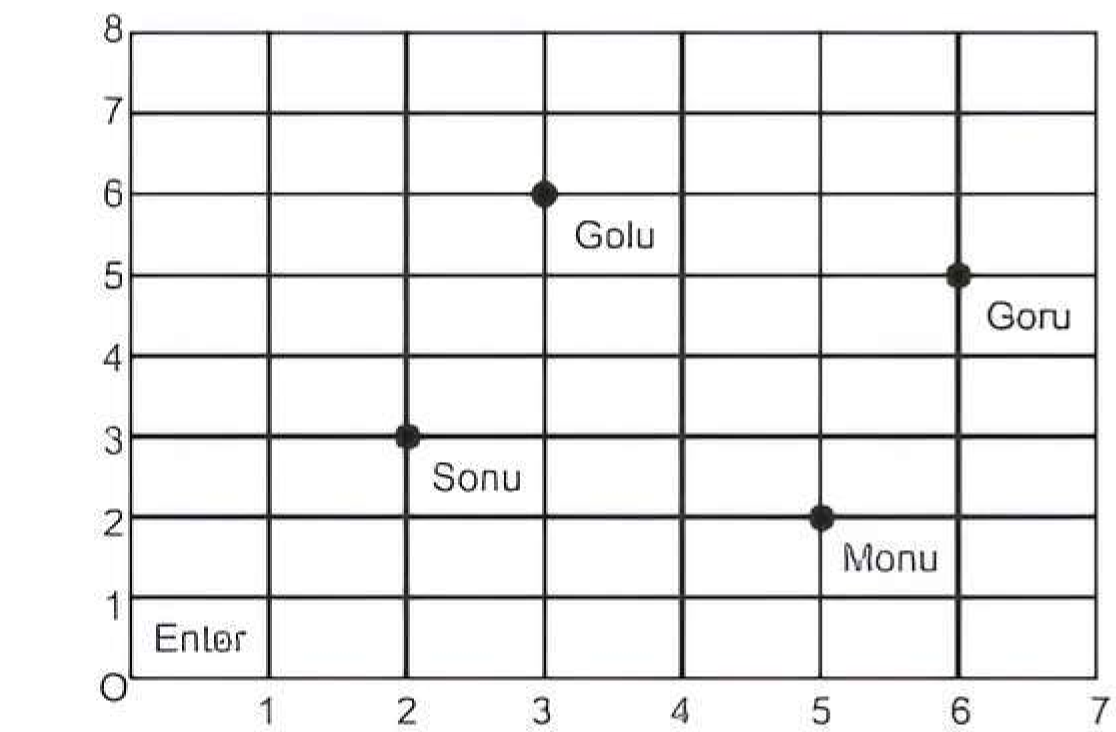
$\Rightarrow \quad k = \frac{2}{7}$ and $k = \frac{2}{7}$

Hence, given statement is true.

38. True **39.** False **40.** True

Case Study Based Questions

Case Study 1



Sonu went to a lab near to his home for COVID 19 test along with his family members. The seats in the waiting area were as per the government norms of distancing during the COVID 19 pandemic (as shown in the above figure). His family members took their seats which are represented as black circular dots.

Based on the above information, solve the following questions:

Q 1. Considering O as the origin, what are the coordinates of seat of Sonu and Goru respectively?

a. (2, 3) and (6, 5)
b. (3, 2) and (5, 6)
c. (3, 6) and (5, 2)
d. (6, 3) and (2, 5)

Q 2. What is the distance between Golu and Monu?

a. $\sqrt{5}$ units b. $2\sqrt{5}$ units
c. $3\sqrt{5}$ units d. $4\sqrt{5}$ units

Q 3. What will be the coordinates of a point exactly between Sonu and Goru where a person can be seated?

a. $\left(\frac{5}{2}, \frac{9}{2}\right)$ b. $\left(\frac{11}{2}, \frac{7}{2}\right)$ c. (4, 4) d. $\left(\frac{9}{2}, 4\right)$

Q 4. Find the area covered by Sonu and its members, if all four seats connected with a rope.

a. $2\sqrt{5}$ sq. units b. $\sqrt{10}$ sq. units
c. $2\sqrt{10}$ sq. units d. 10 sq. units

Q 5. If the doctor divides the rope joining Sonu and Goru in the ratio 1 : 2, then the coordinates of the seat of the doctor is:

a. $\left(\frac{10}{3}, \frac{11}{3}\right)$ b. $\left(\frac{11}{3}, \frac{10}{3}\right)$ c. (4, 4) d. (3, 4)

Solutions

1. The coordinates of seat of Sonu and Goru are (2, 3) and (6, 5) respectively.
So, option (a) is correct.

2. The coordinates of seat of Golu and Monu are (3, 6) and (5, 2) respectively.

TR!CK

The distance between (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

Distance between Golu and Monu

$= \sqrt{(5-3)^2+(2-6)^2} = \sqrt{(2)^2+(-4)^2} = \sqrt{4+16}$

$= \sqrt{20} = 2\sqrt{5}$ units

So, option (b) is correct.

3. The coordinates of a point exactly between Sonu and Goru = Mid-point of Sonu's and Goru's seat.

TR!CK

The mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

$= \left(\frac{2+6}{2}, \frac{3+5}{2}\right) = \left(\frac{8}{2}, \frac{8}{2}\right) = (4, 4)$

So, option (c) is correct.

4. The coordinates of all four members are Sonu (2, 3), Golu (3, 6), Monu (5, 2) and Goru (6, 5).
Distance between Sonu and Monu

$= \sqrt{(5-2)^2+(2-3)^2}$

$= \sqrt{(3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$ units

TiP

In square ABCD, AB = BC = CD = DA and diagonal AC = diagonal BD.

Distance between Monu and Goru

$=\sqrt{(5-6)^2+(2-5)^2}=\sqrt{1+9}=\sqrt{10}$ units

Distance between Goru and Golu

$=\sqrt{(3-6)^2+(6-5)^2}=\sqrt{9+1}=\sqrt{10}$ units

Distance between Golu and Sonu

$=\sqrt{(2-3)^2+(3-6)^2}=\sqrt{1+9}=\sqrt{10}$ units

Distance between Sonu and Goru

$=\sqrt{(2-6)^2+(3-5)^2}=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$ units

Also distance between Golu and Monu is $2\sqrt{5}$ units.

Clearly, the distance between all four members are same and diagonally equal. If all four seats connected with a rope, it forms a square with equal length, *i.e.*, $\sqrt{10}$ units.

TR!CK

Area of square = (side)^2

∴ Area covered by Sonu and its members

$=(\sqrt{10})^2=10$ sq. units

So, option (d) is correct.

5.

TR!CK

The coordinates of the point $P(x, y)$ which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are $\left(\dfrac{m_1x_2+m_2x_1}{m_1+m_2}, \dfrac{m_1y_2+m_2y_1}{m_1+m_2}\right)$.

Coordinates of the seat of the doctor

$=\left(\dfrac{1\times6+2\times2}{1+2}, \dfrac{1\times5+2\times3}{1+2}\right)$

$=\left(\dfrac{6+4}{3}, \dfrac{5+6}{3}\right)=\left(\dfrac{10}{3}, \dfrac{11}{3}\right)$

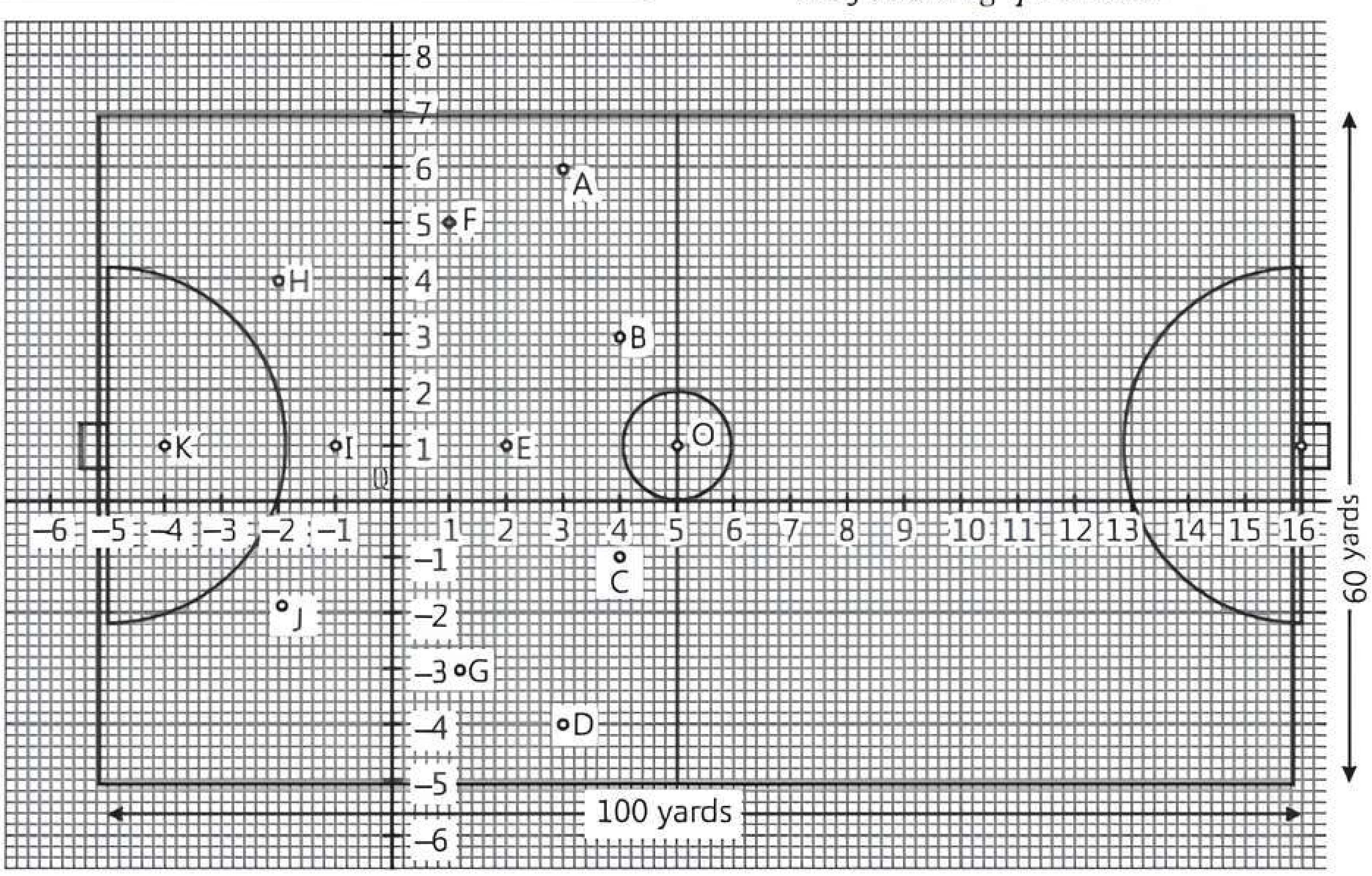
So, option (a) is correct.

Case Study 2

A hockey field is the playing surface for the game of hockey. Historically, the game was played on natural turf (grass) but now-a-days it is predominantly played on an artificial turf. It is rectangular in shape 100 yards by 60 yards. Goals consist of two upright posts placed equidistant from the centre of the backline, joined at the top by a horizontal crossbar. The inner edges of the posts must be 3.66 metres (4 yards) apart and the lower edge of the crossbar must be 2.14 metres (7 feet) above the ground. Each team plays with 11 players on the field during the game including the goalie. Positions you might play include:

- **Forward:** As shown by players A, B, C and D.
- **Midfielders:** As shown by players E, F and G.
- **Fullbacks:** As shown by players H, I and J.
- **Goalie:** As shown by player K.

Based on the picture of a hockey field below, solved the following questions:



Q 1. The coordinates of the centroid of ΔEHJ are:

a. (−2/3, 1) b. (1, −2/3) c. (2/3, 1) d. (−2/3, −1)

Q 2. If a player P needs to be at equal distances from A and G, such that A, P and G are in straight line, then position of P will be given by:

a. (−3/2, 2) b. (2, −3/2)

c. (2, 3/2) d. (−2, −3)

Q 3. The point on X-axis equidistant from I and E is:

a. (1/2, 0) b. (0, −1/2)

c. (−1/2, 0) d. (0, 1/2)

Q 4. What are the coordinates of the position of a player Q such that his distance from K is twice his distance from E and K, Q and E are collinear?

a. (1, 0) b. (0, 1) c. (−2, 1) d. (−1, 0)

Q 5. The point on Y-axis which is equidistant from B and C is:

a. (−1, 0) b. (0, −1) c. (1, 0) d. (0, 1)

Solutions

1. From figure,

Coordinates of $E \equiv (2, 1)$

Coordinates of $H \equiv (-2, 4)$

Coordinates of $J \equiv (-2, -2)$

TiP

Centroid of the triangle with vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

$\therefore$ Centroid of $\Delta EHJ \equiv \left(\frac{2+(-2)+(-2)}{3}, \frac{1+4+(-2)}{3}\right)$

$$= \left(\frac{-2}{3}, 1\right)$$

So, option (a) is correct.

2. From figure,

Coordinates of $A = (3, 6)$

and coordinates of $G \equiv (1, -3)$

Now, if P needs to be at equal distance from A and G, such that A, P and G are collinear, then P will be the mid-point of AG.

TiP

Mid-point of the line segment joining the points (x_1, y_1) and $(x_2, y_2) \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

$\therefore$ Coordinates of P = Mid-point of AG

$$\equiv \left(\frac{3+1}{2}, \frac{-3+6}{2}\right) = \left(2, \frac{3}{2}\right)$$

So, option (c) is correct.

3. From figure,

Coordinates of point $I \equiv (-1, 1)$

and coordinates of point $E \equiv (2, 1)$

Let the point on X-axis equidistant from I and E be $P(x, 0)$.

$\therefore$ $IP = EP$

$\Rightarrow$ $IP^2 = EP^2$

$\Rightarrow$ $(x+1)^2 + (0-1)^2 = (x-2)^2 + (0-1)^2$

(by distance formula)

$\Rightarrow$ $x^2 + 1 + 2x + 1 = x^2 + 4 - 4x + 1$

$\Rightarrow$ $6x = 3$

$\Rightarrow$ $x = \frac{3}{6} = \frac{1}{2}$

$\therefore$ Required point is $P\left(\frac{1}{2}, 0\right)$.

So, option (a) is correct.

4. From figure,

Coordinates of point $K \equiv (-4, 1)$

and coordinates of point $E \equiv (2, 1)$

Let the coordinates of the position of a player Q such that his distance from K is twice his distance from E be $Q(x, y)$.

Then, $KQ : QE = 2 : 1$ ($\because$ K, Q and E are collinear)

By section formula,

$$Q(x, y) = \left(\frac{2\times2+1\times-4}{2+1}, \frac{2\times1+1\times1}{2+1}\right)$$

$$= \left(\frac{4-4}{3}, \frac{2+1}{3}\right) = (0, 1)$$

So, option (b) is correct.

5. From figure,

Coordinates of $B \equiv (4, 3)$

and coordinates of $C = (4, -1)$

Let the point on Y-axis which is equidistant from B and C be $T(0, y)$.

$\therefore$ $BT = CT \Rightarrow BT^2 = CT^2$

$\Rightarrow$ $(0-4)^2 + (y-3)^2 = (0-4)^2 + (y+1)^2$

(by distance formula)

$\Rightarrow$ $16 + y^2 + 9 - 6y = 16 + y^2 + 1 + 2y$

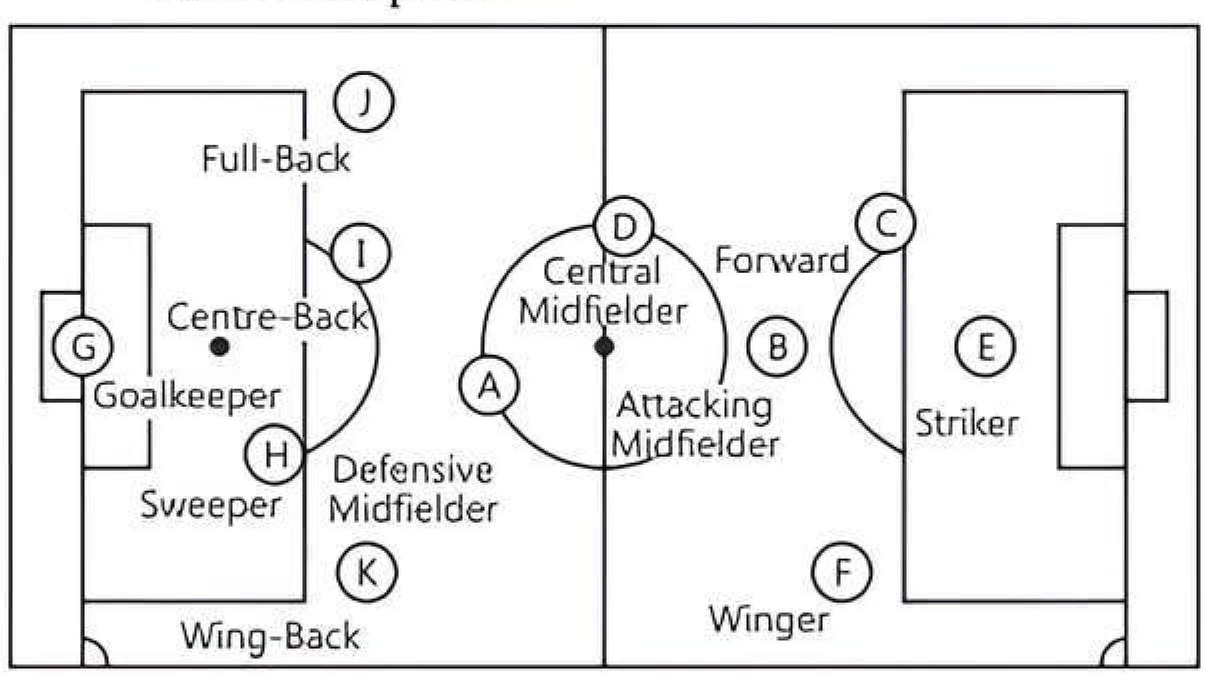
$\Rightarrow$ $8y = 8 \Rightarrow y = \frac{8}{8} = 1$

$\therefore$ Required point is $T(0, 1)$.

So, option (d) is correct.

Case Study 3

Tharunya was thrilled to know that the football tournament is fixed with a monthly timeframe from 20th July to 20th August 2023 and for the first time in the FIFA Women's World Cup's history, two nations host in 10 venues. Her father felt that the game can be better understood if the position of players is represented as points on a coordinate plane.



Based on the above information, solve the following questions: **[CBSE SQP 2023-24]**

Q 1. At an instance, the midfielders and forward formed a parallelogram. Find the position of the central midfielder (D) if the position of other players who formed the parallelogram are : A (1, 2), B (4, 3) and C (6, 6).

Q 2. Check if the Goal keeper G (–3, 5), Sweeper H (3, 1) and Wing-back K (0, 3) fall on a same straight line.

Or

Check if the Full-back J(5, –3) and centre-back I (–4, 6) are equidistant from forward C (0, 1) and if C is the mid-point of IJ.

Q 3. If Defensive midfielder A (1, 4), Attacking midfielder B (2,–3) and Striker E (*a*, *b*) lie on the same straight line and B is equidistant from A and E, find the position of E.

Solutions

1. Given, the coordinates of A, B and C are (1, 2), (4, 3) and (6, 6) respectively.

Let the coordinates of point D be (x, y).

Since A, B, C and D form a parallelogram and in parallelogram, diagonals are bisect each other.

∴ Mid-point of AC = Mid-point of BD

$$\Rightarrow \quad \left[\frac{1+6}{2}, \frac{2+6}{2}\right] = \left[\frac{4+x}{2}, \frac{3+y}{2}\right]$$

$$\Rightarrow \quad \left(\frac{7}{2}, 4\right) = \left(\frac{x+4}{2}, \frac{y+3}{2}\right)$$

Equating x and y-coordinates on both sides, we get

$$\frac{7}{2} = \frac{x+4}{2} \Rightarrow x+4=7 \Rightarrow x=7-4=3$$

and $$4 = \frac{y+3}{2} \Rightarrow y+3=8 \Rightarrow y=8-3=5$$

∴ Position of the central midfielder (D) is (3, 5).

2. Given $G \equiv (-3, 5)$, $H \equiv (3, 1)$ and $K \equiv (0, 3)$

Now, $GH = \sqrt{(3+3)^2+(1-5)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$.

$$HK = \sqrt{(0-3)^2+(3-1)^2} = \sqrt{9+4} = \sqrt{13}$$

and $GK = \sqrt{(0+3)^2+(3-5)^2} = \sqrt{9+4} = \sqrt{13}$

Since, HK + GK = GH

So, G (Goal keeper), H (Sweeper) and K (Wing-back) lie on a same straight line.

Or

Given, $J \equiv (5, -3)$, $I \equiv (-4, 6)$ and $C \equiv (0, 1)$

Now, $CJ = \sqrt{(5-0)^2+(-3-1)^2} = \sqrt{25+16} = \sqrt{41}$

and $CI = \sqrt{(-4-0)^2+(6-1)^2} = \sqrt{16+25} = \sqrt{41}$

Since, $CJ = CI = \sqrt{41}$

So, Full-back (J) and centre-back (I) are equidistant from forward C.

Again, mid-point of $IJ = \left[\frac{-4+5}{2}, \frac{6-3}{2}\right] = \left(\frac{1}{2}, \frac{3}{2}\right)$

But $C \equiv (0, 1)$.

So, C is not the mid-point of IJ.

3. Given A (1, 4), B (2, –3) and E (a, b) lie on the same straight line. Also, B is equidistant from A and E. So, B is the mid-point of AE.

∴ Point B = Mid-point of AE

$$\Rightarrow \quad (2, -3) = \left(\frac{a+1}{2}, \frac{b+4}{2}\right)$$

Equating x and y-coordinates on both sides, we get

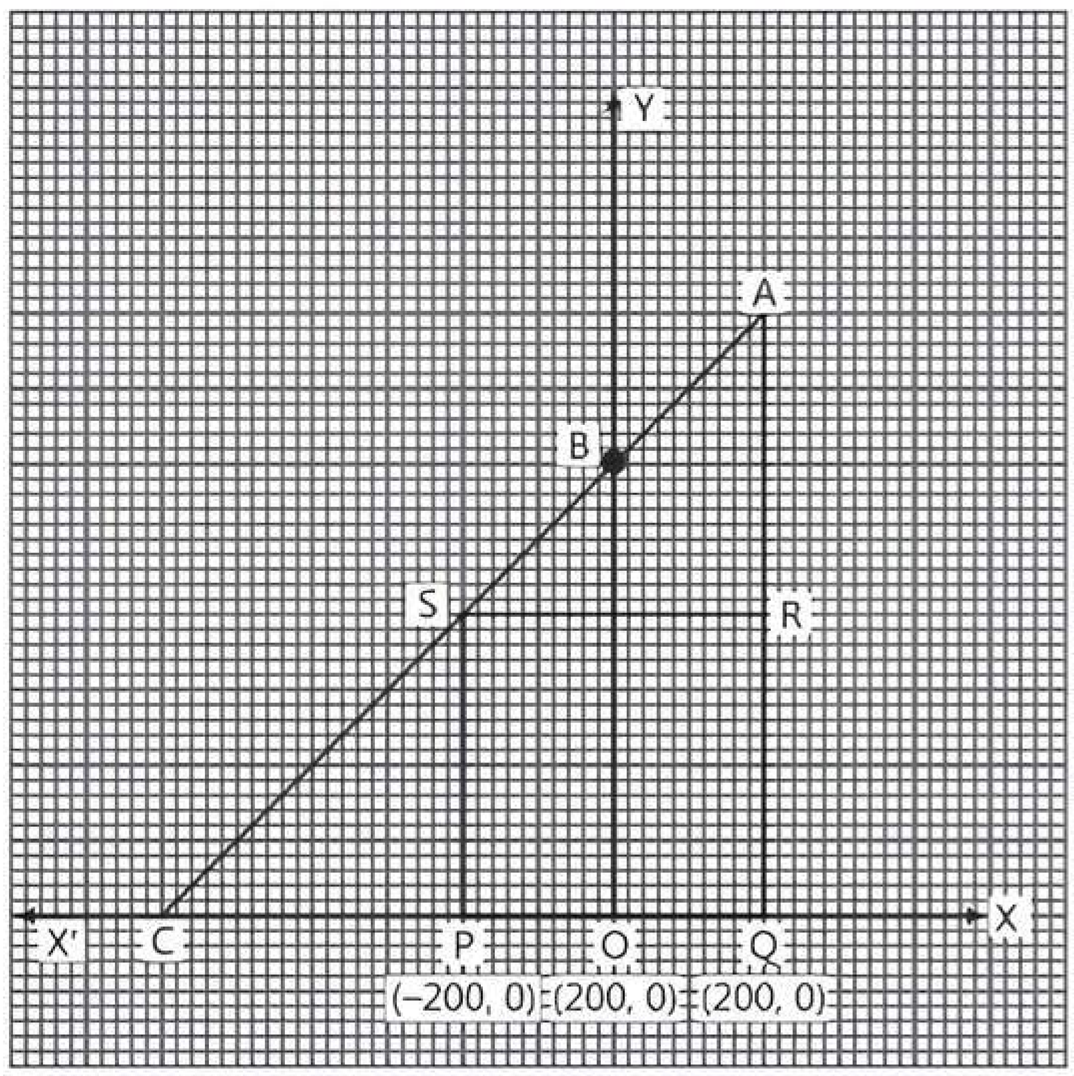
$$2 = \frac{a+1}{2} \Rightarrow a+1=4 \Rightarrow a=3$$

and $$-3 = \frac{b+4}{2} \Rightarrow b+4=-6 \Rightarrow b=-10$$

So, the position of E is at (3, –10).

Case Study 4

Jagdish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O.



Based on the above information, solve the following questions: **[CBSE 2023]**

Q 1. Taking O as origin, coordinates of P are (–200, 0) and of Q are (200, 0). PQRS being a square, what are the coordinates of R and S?

Q 2. What is the area of square PQRS?

Or

What is the length of diagonal PR in square PQRS?

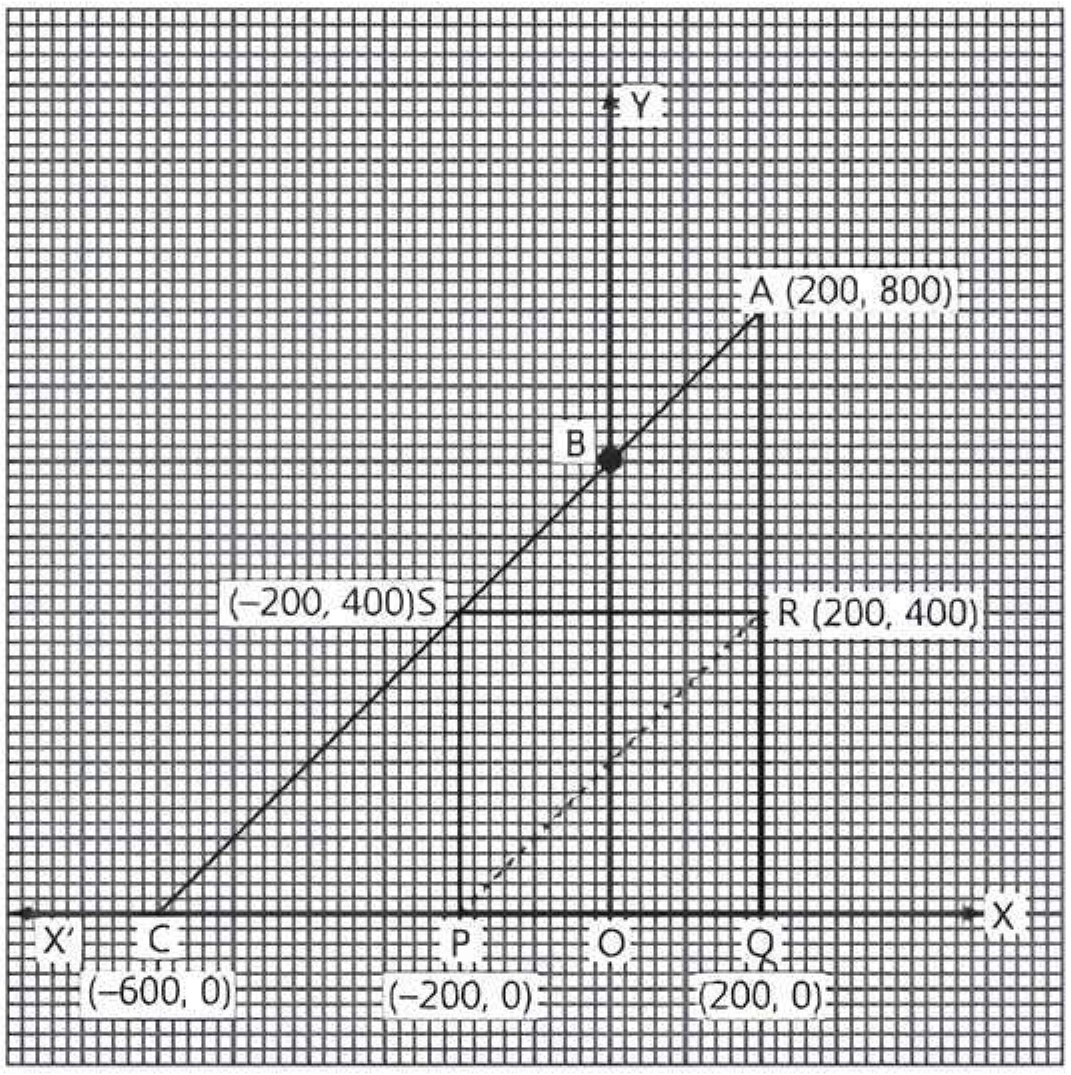
Q 3. If S divides CA in the ratio *k* : 1, what is the value of *k*, where point A is (200, 800)?

Solutions

1. Since PQRS is a square.

∴ Side of the square (PQ) = |OP| + OQ = |– 200 | + 200

= 200 + 200 = 400 units



From figure.

Abscissa of S = Abscissa of P = – 200

Abscissa of R = Abscissa of Q = 200

Ordinate of S = Side of square in positive direction of Y-axis = 400

and ordinate of R = Side of square in positive direction of Y-axis = 400

So, coordinates of R and S are (200, 400) and (–200, 400) respectively.

2. Area of square PQRS = $(PQ)^2$

$$= (200 + 200)^2 + (0 - 0)^2 = (400)^2 + 0$$
$$= 160000 \text{ sq. units}$$

Or

Since, PQ ⊥ RQ (∵ PQRS is a square)

∴ In right ΔPQR,

$$PR^2 = PQ^2 + QR^2$$
$$= (200 + 200)^2 + (0 - 0)^2 + (200 - 200)^2 + (400 - 0)^2$$
$$= (400)^2 + 0 + 0 + (400)^2$$
$$= 2 \times 160000 = 320000$$
$$\Rightarrow \quad PR = 400\sqrt{2} \text{ units}$$

So, length of the diagonal PR is $400\sqrt{2}$ units.

3. From figure, 1 square = 100 units.

Coordinates of C ≡ (–600, 0).

Coordinates of A ≡ (200, 800).

and coordinates of S (–200, 400)

Since, S divides CA in the ratio $k : 1$, then coordinates of point S,

$$(-200, 400) = \left(\frac{200k - 600}{k+1}, \frac{800k + 0}{k+1}\right)$$

(using section formula)

$$\Rightarrow \quad (-200, 400) = \left(\frac{200k - 600}{k+1}, \frac{800k}{k+1}\right)$$

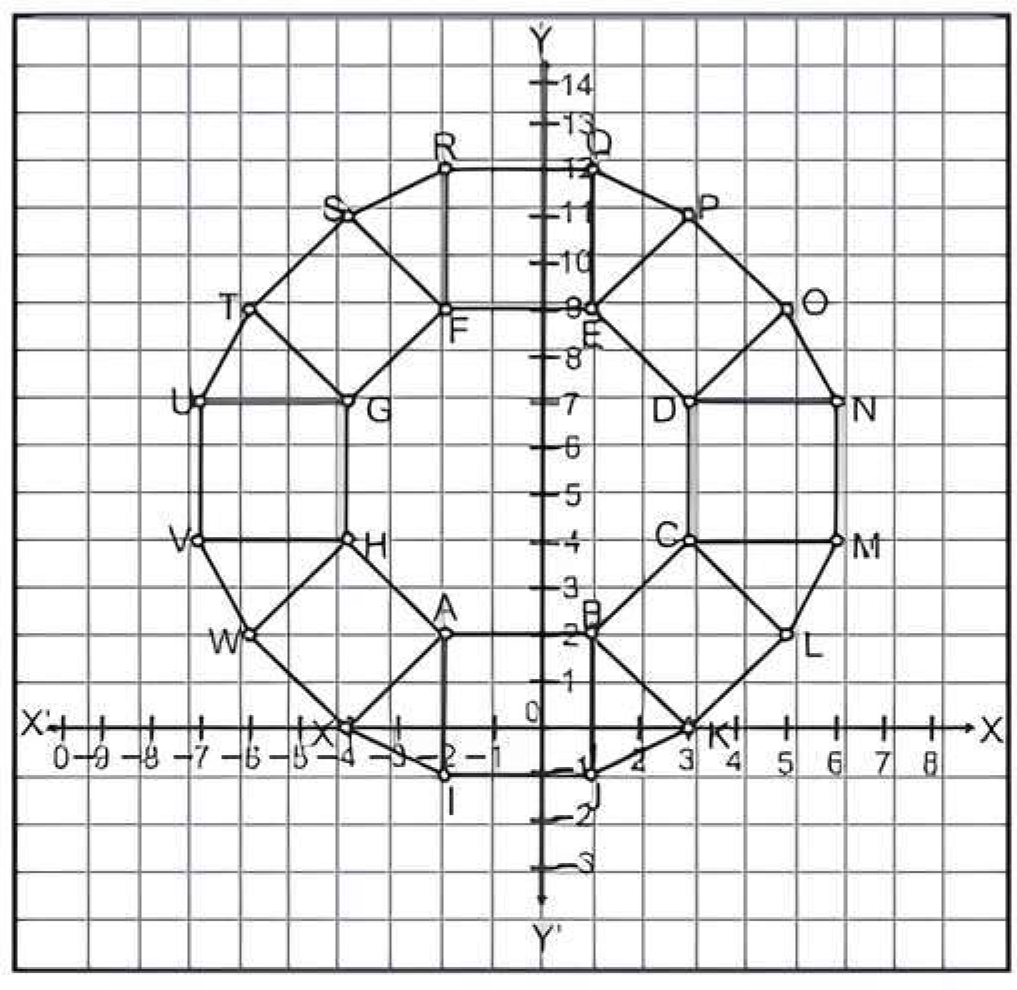
Equating y-coordinate on both sides, we get

$$400 = \frac{800k}{k+1} \Rightarrow 400k + 400 = 800k$$
$$\Rightarrow 400k = 400 \Rightarrow k = 1$$

Case Study 5

A tiling or tessellation of a flat surface is the covering of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps. Historically, tessellations were used in ancient Rome and in Islamic art. You may find tessellation patterns on floors, walls, paintings, etc. Shown below is a tiled floor in the archaeological Museum of Seville, made using squares, triangles and hexagons.

A craftsman thought of making a floor pattern after being inspired by the above design. To ensure accuracy in his work, he made the pattern on the cartesian plane. He used regular octagons, squares and triangles for his floor tessellation pattern.



Based on the above information, solve the following questions: **[CBSE SQP 2022-23]**

Q 1. **What is the length of the line segment joining points B and F?**

Q 2. **The centre 'Z' of the figure will be the point of intersection of the diagonals of quadrilateral WXOP. Then what are the coordinates of Z?**

Q 3. **What are the coordinates of the point on Y-axis equidistant from A and G?**

Or

What is the area of trapezium AFGH?

Solutions

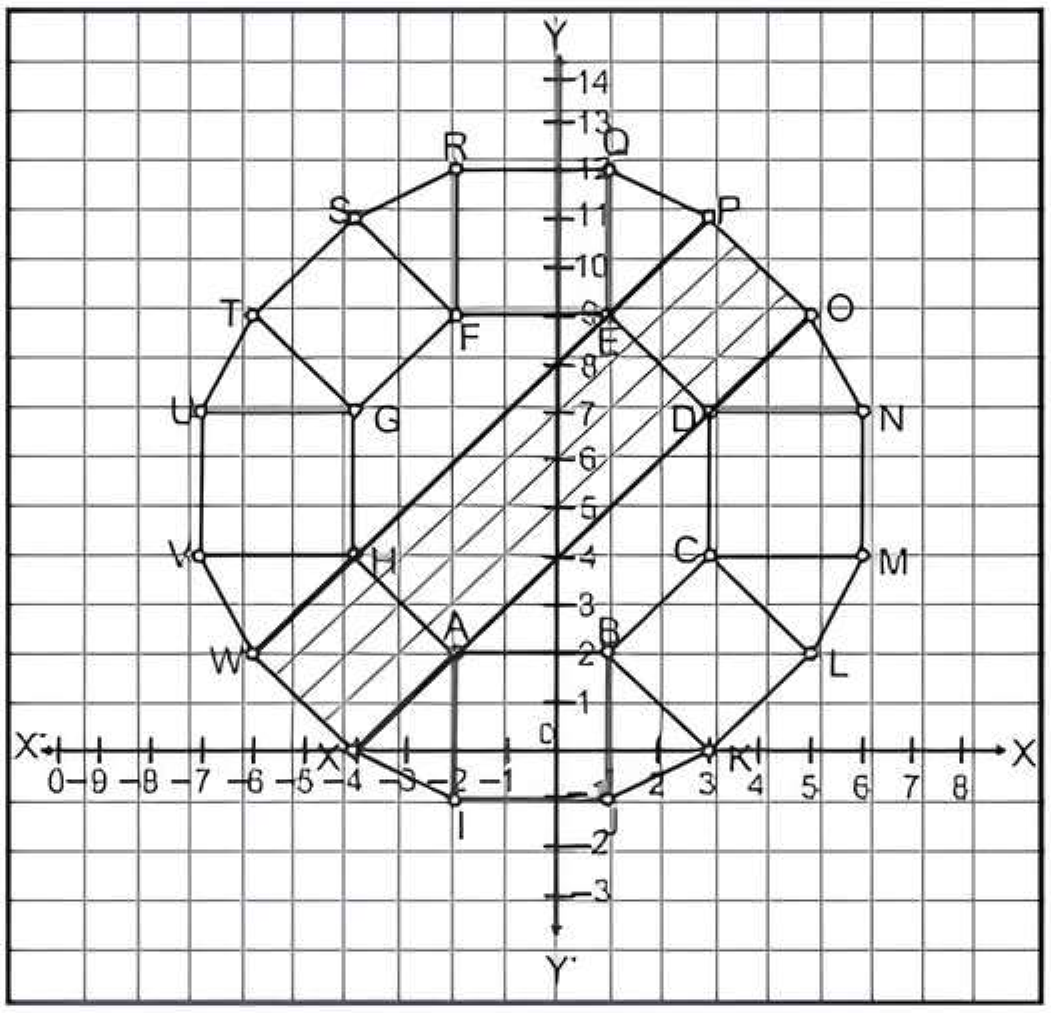
1. From the given figure, the coordinates are B(1, 2), F(– 2, 9).

Now, $BF^2 = (-2-1)^2 + (9-2)^2$ (by distance formula)

$BF^2 = (-3)^2 + (7)^2 = 9 + 49$

$\Rightarrow$ $BF = \sqrt{58}$ units

2.



Here coordinates of WXOP are

W(– 6, 2), X(– 4, 0), O(5, 9), P(3, 11)

Here, $WX = \sqrt{(-4+6)^2 + (0-2)^2}$

$= \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = 2\sqrt{2}$

$XO = \sqrt{(5+4)^2 + (9-0)^2} = \sqrt{(9)^2 + (9)^2} = 9\sqrt{2}$

$OP = \sqrt{(3-5)^2 + (11-9)^2} = \sqrt{(-2)^2 + (2)^2}$

$= \sqrt{4+4} = 2\sqrt{2}$

and $PW = \sqrt{(-6-3)^2 + (2-11)^2}$

$= \sqrt{(-9)^2 + (-9)^2} = 9\sqrt{2}$

Also, $WO = \sqrt{(5+6)^2 + (9-2)^2} = \sqrt{(11)^2 + (7)^2}$

$= \sqrt{121+49} = \sqrt{170}$

and $XP = \sqrt{(3+4)^2 + (11-0)^2} = \sqrt{(7)^2 + (11)^2}$

$= \sqrt{49+121} = \sqrt{170}$

Since, WX = OP and XO = PW

as well as diagonal WO and diagonal XP

So, WXOP is a rectangle.

> **TiP**
>
> *Point of intersection of diagonals of a rectangle is the mid-point of the diagonals.*

Here, the required point Z is mid-point of WO or XP.

$\therefore$ Required point $\equiv \left(\frac{-6+5}{2}, \frac{2+9}{2}\right) = \left(\frac{-1}{2}, \frac{11}{2}\right)$

Hence, coordinates of Z are $\left(-\frac{1}{2}, \frac{11}{2}\right)$.

3. Here, coordinates are A(– 2, 2) and G(– 4, 7).

Let the point on Y-axis be Z(0, y).

Then $AZ^2 = GZ^2$

$\Rightarrow$ $(0+2)^2 + (y-2)^2 = (0+4)^2 + (y-7)^2$

$\Rightarrow$ $(2)^2 + y^2 + 4 - 4y = (4)^2 + y^2 + 49 - 14y$

$\Rightarrow$ $8 - 4y = 65 - 14y$

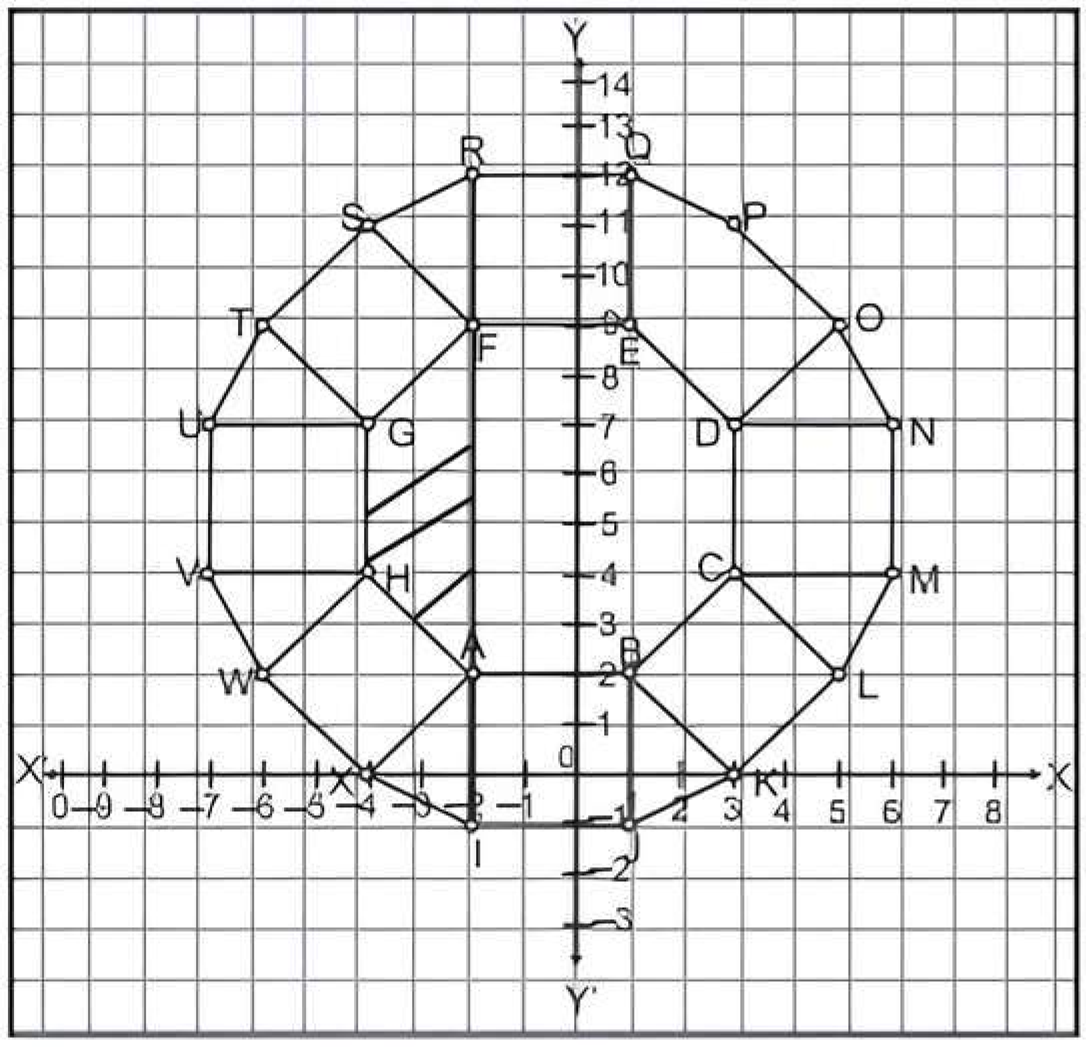
$\Rightarrow$ $10y = 57$

$\Rightarrow$ $y = 5.7$

Hence, the required point is (0, 5.7).

Or

Here, coordinates of trapezium AFGH are A(– 2, 2), F(–2, 9), G(– 4, 7) and H(– 4, 4)



Clearly, GH = 7 – 4 = 3 units

and AF = 9 – 2 = 7 units

So, height of the trapezium AFGH = 2 units

$\therefore$ Area of trapezium $AFGH = \frac{1}{2}(AF + GH) \times \text{Height}$

$= \frac{1}{2}(7+3) \times 2$

$= 10$ sq. units

Very Short Answer Type Questions

Q 1. Find the distance of the point P(–3, 4) from the X-axis.

Q 2. Find the distance of a point P(x, y) from the origin. *[CBSE 2018]*

Q 3. Find the distance between the points (a, b) and (–a, –b). *[CBSE 2019]*

Q 4. Find the value of a, so that the point (3, a) lies on the line represented by 2x – 3y = 5. *[CBSE 2019, 17]*

Q 5. Find the value of x for which the distance between the points A (x, 2) and B (9, 8) is 10 units. *[CBSE 2019]*

Q 6. Show that the points (–2, 3), (8, 3) and (6, 7) are the vertices of a right-angled triangle. *[CBSE 2023]*

Q 7. Find the coordinates of a point A, where AB is diameter of a circle whose centre is (2, –3) and B is the point (1, 4). *[CBSE 2019]*

Q 8. If Q (0, 1) is equidistant from P (5, −3) and R (x, 6), find the values of x. *[CBSE 2023]*

Short Answer Type-I Questions

Q 1. Write the coordinates of a point on X-axis which is equidistant from the points (−3, 4) and (7, 6).

Q 2. Prove that the points (3, 0), (6, 4) and (−1, 3) are the vertices of a right-angled isosceles triangle. *[CBSE 2016]*

Q 3. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from Q (2, −5) and R (−3, 6), find the coordinate of P. *[CBSE 2016]*

Q 4. The point R divides the line segment AB, when A(− 4, 0) and B(0, 6) such that $AR = \frac{3}{4}AB$. Find the coordinates of R. *[CBSE 2019]*

Q 5. Find the ratio in which P(4, m) divides the line segment joining the points A(2, 3) and B(6, −3). Hence find m. *[CBSE 2018]*

Q 6. A line intersects Y-axis and X-axis at points P and Q, respectively. If R (2, 5) is the mid-point of line segment PQ, then find the coordinates of P and Q. *[CBSE 2023]*

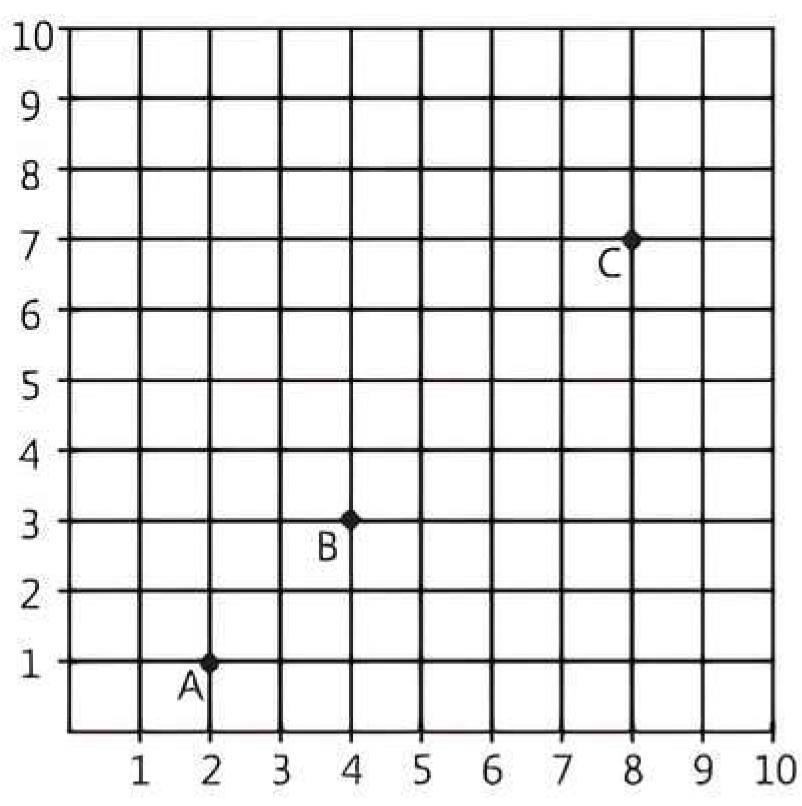
Q 7. The mid-point of the line segment joining A (2a, 4) and B (−2, 3b) is (1, 2a +1). Find the values of a and b. *[CBSE 2019]*

Q 8. Find the ratio in which the line segment joining the points A (6, 3) and B (−2, −5) is divided by X-axis. Also find the coordinates of this point of X-axis. *[CBSE 2023]*

Q 9. If two adjacent vertices of a parallelogram are (3, 2) and (−1, 0) and the diagonals intersect at (2, −5), then find the coordinates of the other two vertices. *[CBSE 2017]*

Q 10. Read the following passage and answer the questions that follows:

The given figure shows the arrangement of desks in a classroom. Adnan, Yuvraj and Deepa are seated at A (2, 1), B (4, 3) and C (8, 7) respectively. Then a new student Sachin joins the class.



(i) Teacher tells Sachin to sit in the middle of Adnan and Deepa. Find the coordinates of the position where he can sit.

(ii) Do you think Adnan, Yuvraj and Deepa are seated in a line? Give reasons for your answer.

Short Answer Type-II Questions

Q 1. The points A (4, 7), B (p, 3) and C (7, 3) are the vertices of a right triangle, right-angled at B. Find the value of p. *[CBSE 2015]*

Q 2. If the point P(x, y) is equidistant from the point A(a + b, b − a) and B(a − b, a + b), prove that bx = ay. *[CBSE 2016]*

Q 3. Show that △ABC with vertices A(−2, 0), B(0, 2) and C(2, 0) is similar to △DEF with vertices D(−4, 0), F(4, 0) and E(0, 4). *[NCERT EXEMPLAR; CBSE 2017]*

Q 4. If P (9a − 2, − b) divides line segment joining A (3a + 1, −3) and B (8a, 5) in the ratio 3 : 1, find the values of a and b. *[NCERT EXEMPLAR; CBSE 2016]*

Q 5. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points P(2, −2) and Q(3, 7)? Also find the value of y. *[CBSE 2017]*

Q 6. If the mid-point of the line segment joining the points A(3, 4) and B(k, 6) is P(x, y) and x + y − 10 = 0, find the value of k. *[NCERT EXEMPLAR; CBSE 2020]*

Q 7. Two vertices of a △ABC are given by A(6, 4) and B(−2, 2) and its centroid is G(3, 4). Find the coordinates of the third vertex C of △ABC. *[V. Imp.]*

Long Answer Type Questions

Q 1. Name the type of quadrilateral formed, if any of the following points (−1, −2), (1, 0), (−1, 2) and (−3, 0). Also give reason for your answer. *[NCERT EXERCISE, V. Imp.]*

Q 2. If A (2, −1), B (3, 4), C (−2, 3) and D (−3, −2) are four points in a plane, show that ABCD is a rhombus but not a square. Find the area of the rhombus.

Q 3. The line segment joining the points A(2, 1) and B(5, −8) is trisected at the points P and Q such that P is nearer to A. Find the coordinates of P and Q. If P also lies on the line given by 2x − y + k = 0, find the value of k. *[CBSE 2019]*

Q 4. Points P, Q and R, in this order, divide a line segment joining A(6, 3) and B(10, 13) in four equal parts. Find the coordinates of P, Q and R. *[CBSE 2017]*

Q 5. If A (−2, 1), B (a, 0), C(4, b) and D (1, 2) are the vertices of a parallelogram ABCD, find the values of a and b. Also, find the length of its sides. *[CBSE 2018]*

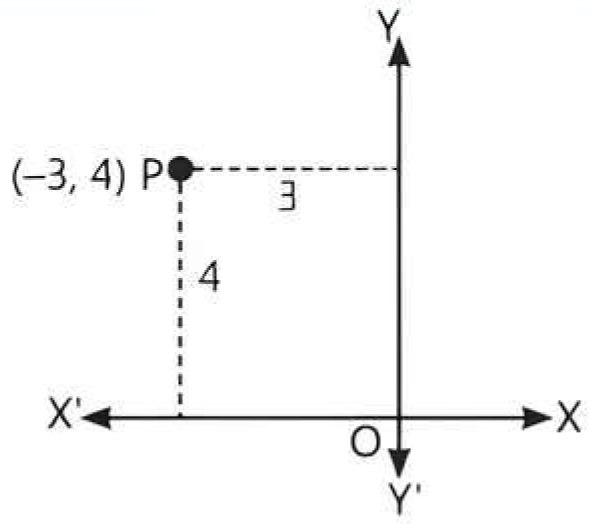
Solutions

Very Short Answer Type Questions

1.

TiP

The perpendicular distance from the X-axis measured along the Y-axis is equivalent to ordinate.



Distance of the point P(–3, 4) from the X-axis = Ordinate of point P (–3, 4) = 4 units

This is also clear from the adjoining figure that the distance of P from X-axis is 4 units.

2.

TR!CK

The distance between two points is

$$d = \sqrt{(\text{difference of abscissa})^2 + (\text{difference of ordinate})^2}$$

Distance between P(x, y) and origin *i.e.*, O(0, 0) is given by, $PO = \sqrt{(0-x)^2+(0-y)^2} = \sqrt{x^2+y^2}$.

3. Distance between (a, b) and (–a, – b) is given by

$$d = \sqrt{(-a-a)^2+(-b-b)^2}$$

$$= \sqrt{(-2a)^2+(-2b)^2}$$

$$= \sqrt{4a^2+4b^2} = 2\sqrt{a^2+b^2} \text{ units}$$

4.

TiP

Every solution of the linear equation is a point lying on the graph of the linear equation.

Since (3, a) lies on $2x - 3y = 5$

$\therefore \quad 2 \times 3 - 3 \times a = 5$

$\Rightarrow \quad 6 - 3a = 5$

$\Rightarrow \quad -3a = 5 - 6 = -1$

Hence, $\quad a = \frac{1}{3}$

5. By using the given condition,

$AB = 10$

$\Rightarrow \quad (AB)^2 = (10)^2$

TiP

The distance between (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}.$$

$\Rightarrow \quad (9-x)^2 + (8-2)^2 = 100$

$\Rightarrow \quad 81 + x^2 - 18x + 36 = 100$

$\Rightarrow \quad x^2 - 18x + 17 = 0$

$\Rightarrow \quad x^2 - (17+1)x + 17 = 0$

$\Rightarrow \quad x(x-17) - 1(x-17) = 0$

$\Rightarrow \quad (x-17)(x-1) = 0 \Rightarrow x = 1, 17$

Hence, the values of x are 1 and 17.

6. Let the points are A (-2, 3), B (8, 3) and C (6, 7). Which are the vertices of ΔABC.

Now $AB = \sqrt{(8+2)^2+(3-3)^2} = \sqrt{(10)^2+(0)^2} = 10$

$BC = \sqrt{(6-8)^2+(7-3)^2}$

$= \sqrt{(-2)^2+(4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

and $CA = \sqrt{(-2-6)^2+(3-7)^2}$

$= \sqrt{(-8)^2+(-4)^2} = \sqrt{64+16}$

$= \sqrt{80} = 4\sqrt{5}$

$\because \quad AB \neq BC \neq CA$

But $BC^2 + CA^2 = (2\sqrt{5})^2 + (4\sqrt{5})^2$

$= 20 + 80 = 100 = (10)^2 = AB^2$

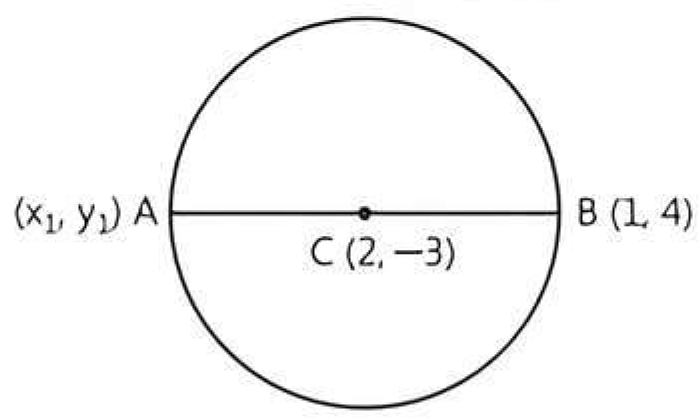
So, by Pythagoras theorem, Δ ABC is a right-angled triangle *i.e.*, the given points are the vertices of a right-angled triangle. **Hence proved.**

7. Let the coordinate of point A be (x_1, y_1).

$\because$ Centre C of a circle is the mid-point of AB.

TiP

Centre of a circle is the mid-point of its diameter.



$\therefore$ Mid-point of $AB = \left(\frac{x_1+1}{2}, \frac{y_1+4}{2}\right)$

$\Rightarrow \quad (2, -3) = \left(\frac{x_1+1}{2}, \frac{y_1+4}{2}\right)$

On equating x and y-coordinates, we get

$$2 = \frac{x_1+1}{2} \text{ and } -3 = \frac{y_1+4}{2}$$

$\Rightarrow \quad x_1 = 4 - 1$ and $y_1 = -6 - 4$

$\Rightarrow \quad x_1 = 3$ and $y_1 = -10$

Hence, coordinate of point A is (3, – 10).

8. Given, Q (0,1) is equidistant from P (5,-3) and R (x, 6).

$\therefore \quad PQ = RQ$

$\Rightarrow \quad PQ^2 = RQ^2$

$\Rightarrow \quad (5-0)^2 + (-3-1)^2 = (0-x)^2 + (6-1)^2$

(by distance formula)

$\Rightarrow \quad 25 + 16 = x^2 + 25$

$\Rightarrow \quad x^2 = 16 \Rightarrow x = \pm 4$

Short Answer Type-I Questions

1. Let the required point on X-axis be $P(x_1, 0)$.

Since, required point is equidistant from the points $A(-3, 4)$ and $B(7, 6)$.

TR!CK

The distance between (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}.$$

$\therefore \quad PA = PB$

$\Rightarrow \quad \sqrt{(-3-x_1)^2+(4-0)^2} = \sqrt{(7-x_1)^2+(6-0)^2}$

$\Rightarrow \quad \sqrt{9+x_1^2+6x_1+16} = \sqrt{49+x_1^2-14x_1+36}$

$\Rightarrow \quad x_1^2+6x_1+25 = x_1^2-14x_1+85$

(squaring on both the sides)

$\Rightarrow \quad 6x_1+14x_1 = 85-25$

$\Rightarrow \quad 20x_1 = 60 \Rightarrow x_1 = \frac{60}{20} = 3$

Hence, the required coordinates are P(3, 0).

2. Let A(3, 0), B(6, 4) and C(-1, 3) be the given points.

Then, $AB = \sqrt{(6-3)^2+(4-0)^2}$

$= \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$ units.

TiP

In a right-angled isosceles triangle ABC,

$AB = AC, \; BC^2 = AB^2 + AC^2$

or $\quad BC = AC, \; AB^2 = BC^2 + AC^2$

or $\quad BC = AB, \; AC^2 = AB^2 + BC^2$

$BC = \sqrt{(-1-6)^2+(3-4)^2}$

TR!CK

The distance between (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}.$$

$= \sqrt{(-7)^2+(-1)^2} = \sqrt{49+1}$

$= \sqrt{50} = 5\sqrt{2}$ units

and $AC = \sqrt{(-1-3)^2+(3-0)^2}$

$= \sqrt{(-4)^2+(3)^2} = \sqrt{16+9} = \sqrt{25} = 5$ units

Here, $\quad AB = AC$

and $\quad BC^2 = AB^2 + CA^2$

(by Pythagoras theorem)

i.e., $\quad (5\sqrt{2})^2 = 5^2 + 5^2 = 25 + 25 = 50$

$\Rightarrow \quad 50 = 50$

Hence, given triangle is a right-angled isosceles.

Hence proved.

3. Let the *y*-coordinate of *P* be *k*, then its *x*-coordinate will be 2*k* by given condition.

So, the coordinates of P, Q and R are $(2k, k)$, $(2, -5)$ and $(-3, 6)$ respectively.

According to the given condition, PQ = PR

$\Rightarrow \quad PQ^2 = PR^2$

TR!CK

The distance between (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}.$$

$\Rightarrow (2-2k)^2+(-5-k)^2 = (-3-2k)^2+(6-k)^2$

$\Rightarrow \quad 4+4k^2-8k+25+k^2+10k$

$= 9+4k^2+12k+36+k^2-12k$

$\Rightarrow \quad 5k^2+2k+29 = 5k^2+45$

$\Rightarrow \quad 2k = 16$

$\Rightarrow \quad k = \frac{16}{2} \Rightarrow k = 8$

Hence, coordinates of P are $(2 \times 8, 8)$ *i.e.*, (16, 8).

4.

TiP

First of all find the ratio of AR : RB, where the point R divides the line segment AB.

Given, $\quad \frac{AR}{AB} = \frac{3}{4}$ or $\frac{AB}{AR} = \frac{4}{3}$,

Subtracting 1 on both sides,

$$\frac{AB}{AR} - 1 = \frac{4}{3} - 1$$

$\Rightarrow \quad \frac{AB - AR}{AR} = \frac{1}{3} \Rightarrow \frac{RB}{AR} = \frac{1}{3}$

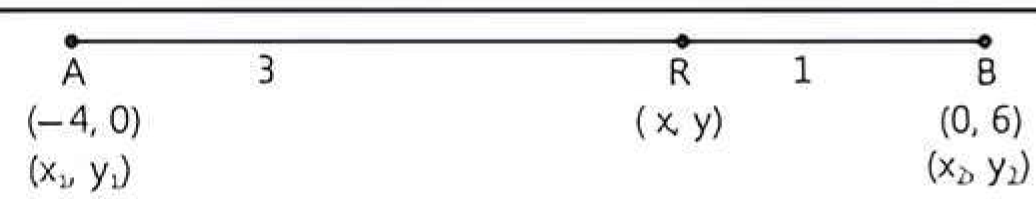
$\Rightarrow \quad \frac{AR}{RB} = \frac{3}{1} = \frac{m_1}{m_2}$ (say)

Here, $\quad x_1 = -4, \; y_1 = 0$

$x_2 = 0, \; y_2 = 6$

TR!CK

The coordinates of a point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$.



By section formula,

$$R(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$= \left(\frac{3(0)+1(-4)}{3+1}, \frac{3(6)+1(0)}{3+1}\right)$$

$(\because m_1 = 3, m_2 = 1)$

$$= \left(\frac{0-4}{4}, \frac{18}{4}\right)$$

$$R(x, y) = \left(-1, \frac{9}{2}\right)$$

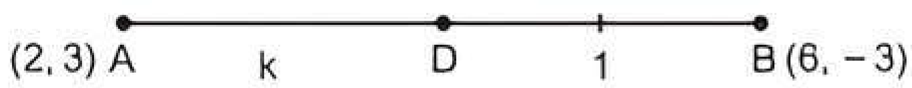
5. Let the ratio in which the line segment joining A(2, 3) and B(6, −3) is divided by point P(4, m) be k : 1.

TR!CK

The coordinates of the point P(x, y) which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$.

Therefore,

$$P(4, m) \equiv P\left(\frac{6k+2}{k+1}, \frac{-3k+3}{k+1}\right)$$



On comparing both sides, we get

$$\frac{6k+2}{k+1} = 4 \Rightarrow 6k+2 = 4k+4 \Rightarrow 2k = 2 \Rightarrow k = 1$$

and $\frac{-3k+3}{k+1} = m \Rightarrow m = \frac{-3(1)+3}{1+1}$ $(\because k = 1)$

$$= \frac{-3+3}{2} = \frac{0}{2} = 0$$

Hence, required ratio is 1 : 1 and $m = 0$

6. We know that the coordinates of the points at X and Y-axes are (x, 0) and (0, y) respectively.

TiPS

- *The ordinate of each point lying on X-axis is zero.*
- *The abscissa of each point lying on Y-axis is zero.*

Let the coordinates of the points P and Q be (0, b) and (a, 0) respectively.

Mid-point of PQ i.e., R ≡ (2, 5) (given)

TiP

The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$\Rightarrow \left(\frac{0+a}{2}, \frac{b+0}{2}\right) \equiv (2, 5)$$

$$\Rightarrow \left(\frac{a}{2}, \frac{b}{2}\right) \equiv (2, 5) \quad \text{(using mid-point formula)}$$

$$\Rightarrow \frac{a}{2} = 2 \text{ and } \frac{b}{2} = -5$$

$$\therefore a = 4 \text{ and } b = -10$$

Hence, the coordinates of P and Q are (0, 10) and (4, 0) respectively.

7. By using given conditions, coordinate of mid-point of A (2a, 4) and B (−2, 3b) = (1, 2a + 1)

$$\Rightarrow \left(\frac{2a-2}{2}, \frac{4+3b}{2}\right) = (1, 2a+1)$$

TR!CK

The mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

On equating the coordinates of x and y both sides, we get

$$\frac{2a-2}{2} = 1 \quad \text{and} \quad \frac{4+3b}{2} = 2a+1$$

$$\Rightarrow 2a-2 = 2 \quad \text{and} \quad 4+3b = 4a+2$$

$$\Rightarrow 2a = 4 \quad \text{and} \quad 2+3b = 4a$$

$$\Rightarrow a = 2 \quad \text{and} \quad 2+3b = 4 \times 2$$

$$\Rightarrow a = 2 \quad \text{and} \quad 3b = 6$$

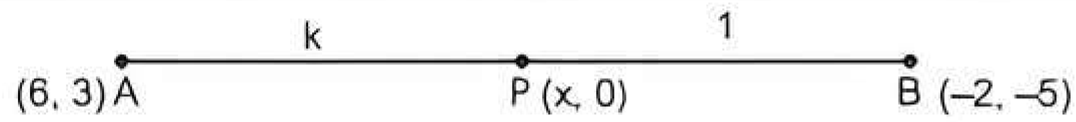
$$\Rightarrow a = 2 \quad \text{and} \quad b = 2$$

Hence, values of a and b are 2 and 2.

8. Let any point on X-axis be P(x, 0). Let point P (x, 0) divides the line joining points A (6, 3) and B (−2, −5) in the ratio k : 1.

TiP

Ordinate of each point on X-axis is always zero.



By using internal division formula,

Coordinate of $P = \left(\frac{-2k+6}{k+1}, \frac{-5k+3}{k+1}\right)$

TR!CK

The coordinates of the point P(x, y) which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$.

$$\Rightarrow (x, 0) = \left(\frac{-2k+6}{k+1}, \frac{-5k+3}{k+1}\right) \quad \text{...(1)}$$

$$\Rightarrow 0 = \frac{-5k+3}{k+1} \Rightarrow -5k+3 = 0 \Rightarrow k = \frac{3}{5}$$

Hence, required ratio is 3 : 5.

∴ From eq. (1),

$$x = \frac{-2k+6}{k+1} \quad \text{...(2)}$$

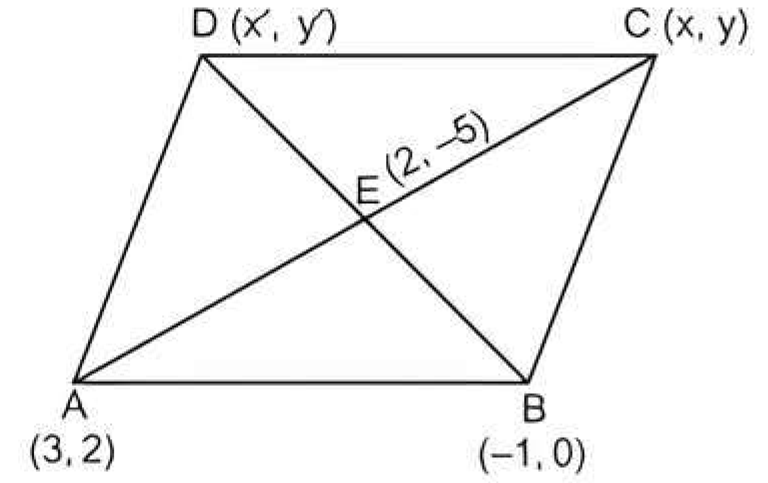
Put $k = \frac{3}{5}$ in eq. (2), we get

$$x = \frac{-2\times\frac{3}{5}+6}{\frac{3}{5}+1} = \frac{-6+30}{3+5} = \frac{24}{8} = 3$$

Hence, coordinate of point P on X-axis is (3, 0).

9. Let ABCD be the parallelogram with two adjacent vertices A(3, 2) and B(−1, 0).

Suppose E(2, −5) be the point of intersection of the diagonals AC and BD.



Let C(x, y) and D(x′, y′) be the coordinates of the other vertices of the parallelogram.

TiP

The diagonals of the parallelogram bisect each other.

Therefore, E is the mid-point of AC and BD.

TR!CK

The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Using the mid-point formula, we have

$$\left(\frac{x+3}{2}, \frac{y+2}{2}\right) \equiv (2, -5)$$

$\Rightarrow$ —— and $\frac{y+2}{2} = -5$

$\Rightarrow$ $x = 1$ and $y = -12$

Also, $\left(\frac{x'-1}{2}, \frac{y'+0}{2}\right) \equiv (2, -5)$

$\Rightarrow$ $\frac{x'-1}{2} = 2$ and $\frac{y'+0}{2} = -5$

$\Rightarrow$ $x' = 5$ and $y' = -10$

So, the coordinates of C and D are (1, –12) and (5, –10).

10.

TiP

The mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

(i) Coordinates of the position where Sachin can sit = Mid-point of the line joining the points represented by A and C.

$$= \left(\frac{2+8}{2}, \frac{1+7}{2}\right)$$

$$= \left(\frac{10}{2}, \frac{8}{2}\right) = (5, 4)$$

(ii) By distance formula,

$$AB = \sqrt{(4-2)^2 + (3-1)^2} = \sqrt{(2)^2 + (2)^2}$$

$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

TR!CK

If three points A, B and C are collinear, then

$AB + BC = AC$

or $AC + BC = AB$

or $AB + AC = BC$

i.e., all three points lie in a line.

$$BC = \sqrt{(8-4)^2 + (7-3)^2} = \sqrt{(4)^2 + (4)^2}$$

$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$AC = \sqrt{(8-2)^2 + (7-1)^2} = \sqrt{(6)^2 + (6)^2}$$

$$= \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

Since, $AB + BC = 2\sqrt{2} + 4\sqrt{2} = 6\sqrt{2} = AC$

Hence, Adnan, Yuvraj and Deepa are seated in a line.

Short Answer Type-II Questions

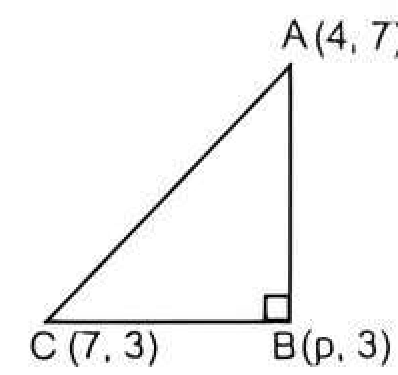
1.

TiP

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$\therefore$ Since, ΔABC is right angled at B.

$AC^2 = AB^2 + BC^2$ (by Pythagoras theorem)



$$\Rightarrow (4-7)^2 + (7-3)^2 = [(4-p)^2 + (7-3)^2] + [(p-7)^2 + (3-3)^2]$$

$$\Rightarrow (-3)^2 + 4^2 = (16 + p^2 - 8p + 16) + (p^2 + 49 - 14p + 0)$$

$$\Rightarrow 9 + 16 = 2p^2 - 22p + 81$$

$$\Rightarrow 2p^2 - 22p + 81 - 25 = 0$$

$$\Rightarrow 2p^2 - 22p + 56 = 0$$

$$\Rightarrow p^2 - 11p + 28 = 0$$

$$\Rightarrow p^2 - 7p - 4p + 28 = 0$$

$$\Rightarrow p(p-7) - 4(p-7) = 0$$

$$\Rightarrow (p-7)(p-4) = 0$$

$$\Rightarrow p - 7 = 0 \text{ or } p - 4 = 0$$

$$\Rightarrow p = 7 \text{ or } p = 4$$

When $p = 7$, then $B(p, 3) = B(7, 3)$ which will coincide with C(7, 3), so this is not possible.

Hence, required value of p is 4.

COMMON ERR(!)R

Some students take both values of p as final answer, but it is wrong. Candidates should cross check the values of p from the given conditions.

2. Given, $PA = PB$

$\Rightarrow \quad (PA)^2 = (PB)^2$

TR!CK

The distance between (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}.$$

$$\Rightarrow \{(a+b)-x\}^2+\{(b-a)-y\}^2 = \{(a-b)-x\}^2+\{(a+b)-y\}^2$$

$$\Rightarrow (a+b)^2+x^2-2(a+b)x+(b-a)^2+y^2-2(b-a)y$$
$$= (a-b)^2+x^2-2(a-b)x+(a+b)^2+y^2-2(a+b)y$$

$$\Rightarrow -2(a+b)x-2(b-a)y = -2(a-b)x-2(a+b)y \quad [\because (a-b)^2=(b-a)^2]$$

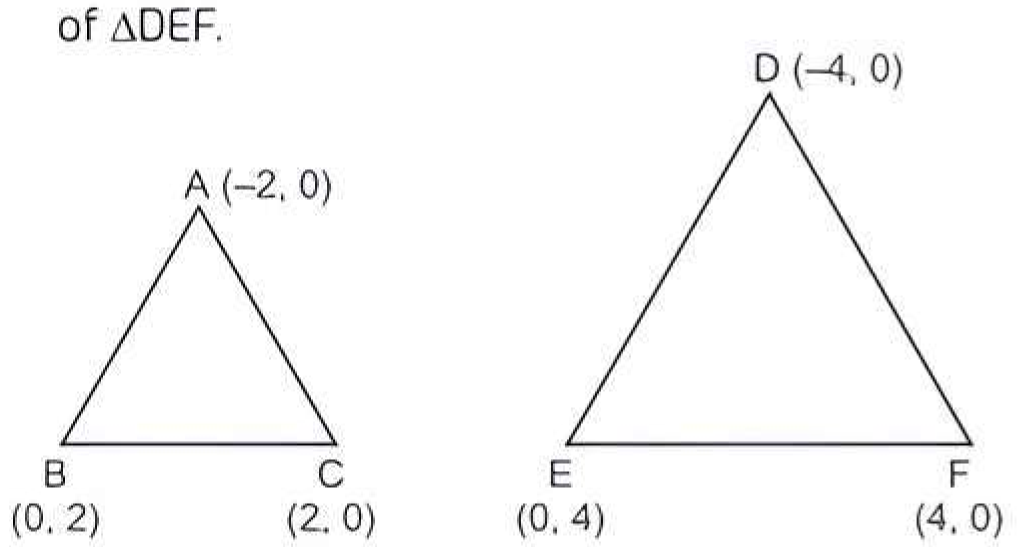
$$\Rightarrow (a+b)x+(b-a)y = (a-b)x+(a+b)y$$

$$\Rightarrow \quad (a+b-a+b)x = (a+b-b+a)y$$

$$\Rightarrow \quad 2bx = 2ay \quad \Rightarrow \quad bx = ay$$

Hence proved.

3. Given, A(–2, 0), B(0, 2) and C(2, 0) are the vertices of ΔABC and D(–4, 0), F(4, 0) and E(0, 4) are the vertices of ΔDEF.



By distance formula,

$$AB = \sqrt{(0+2)^2+(2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units}$$

TR!CK

The distance between (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}.$$

$$BC = \sqrt{(2-0)^2+(0-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units}$$

$$CA = \sqrt{(-2-2)^2+(0-0)^2} = \sqrt{16+0} = 4 \text{ units}$$

$$DE = \sqrt{(0+4)^2+(4-0)^2}$$
$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \text{ units}$$

$$EF = \sqrt{(4-0)^2+(0-4)^2}$$
$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \text{ units}$$

and $FD = \sqrt{(4+4)^2+(0-0)^2}$

$$= \sqrt{64+0} = 8 \text{ units}$$

TR!CK

In two triangles, if sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

$$\therefore \frac{AB}{DE} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}, \frac{BC}{EF} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2} \text{ and } \frac{CA}{FD} = \frac{4}{8} = \frac{1}{2}$$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{1}{2}$$

So, $\Delta ABC \sim \Delta DEF$ **Hence proved.**

4. Given that point P(9a -2, -b) divides AB in the ratio 3 : 1.

$$\therefore \quad AP : PB = m : n = 3 : 1$$

TR!CK

The coordinates of the point P(x, y) which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$.

Using section formula for internal division,

$$x = \frac{mx_2+nx_1}{m+n}$$

$$\Rightarrow \quad 9a-2 = \frac{3\times(8a)+1\times(3a+1)}{3+1}$$

$$\Rightarrow \quad (9a-2)\times 4 = 24a+3a+1$$

$$\Rightarrow \quad 36a-8 = 27a+1$$

$$\Rightarrow \quad 36a-27a = 1+8 \Rightarrow 9a = 9$$

$$\Rightarrow \quad a = \frac{9}{9} = 1$$

and $$y = \frac{my_2+ny_1}{m+n}$$

$$\Rightarrow \quad -b = \frac{3\times 5+1\times(-3)}{3+1}$$

$$\Rightarrow -b = \frac{15-3}{4} = \frac{12}{4} = 3 \quad \Rightarrow \quad b = -3$$

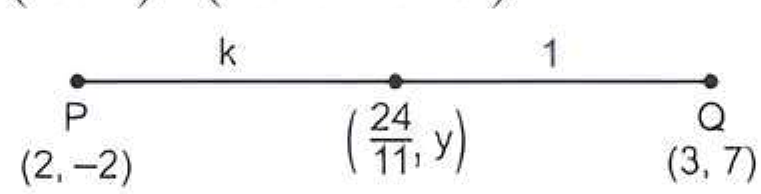
Hence, the values of a and b are 1 and –3 respectively.

5. Let the ratio in which the line segment joining P(2, –2) and Q(3, 7) is divided by point $\left(\frac{24}{11}, y\right)$ be $k : 1$.

TR!CK

The coordinates of the point P(x, y) which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$.

$$\therefore \left(\frac{24}{11}, y\right) \equiv \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$$



On comparing both sides, we get

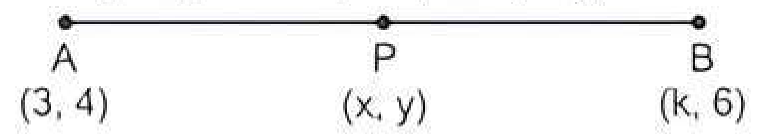
$$\frac{3k+2}{k+1}=\frac{24}{11} \Rightarrow 33k+22=24k+24$$

$$\Rightarrow \quad 9k=2 \quad \Rightarrow \quad k=\frac{2}{9}$$

and $y=\dfrac{7k-2}{k+1}=\dfrac{7\left(\frac{2}{9}\right)-2}{\frac{2}{9}+1}=\dfrac{14-18}{2+9}=\dfrac{-4}{11}$

Hence, the required ratio is 2 : 9 and the value of y is $\frac{-4}{11}$.

6. Given, mid-point of the line segment joining the points $A(3, 4)$ and $B(k, 6)$ is $P(x, y)$.



TiP

The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

∴ Using mid-point formula,

$$x=\frac{3+k}{2} \text{ and } y=\frac{4+6}{2}=5$$

Now, put the above values of x and y in the equation

$$x+y-10=0$$

$$\therefore \quad \frac{3+k}{2}+5-10=0$$

$$\Rightarrow \quad k+3=5\times 2=10$$

$$\Rightarrow \quad k=10-3=7$$

$$\therefore \quad k=7$$

7. Let the coordinates of third vertex C be (x_3, y_3).

Given, $A(6, 4)$, $B(-2, 2)$ and $G(3, 4)$ are the two vertices and a centroid of ΔABC respectively.

TR!CK

If the vertices of a triangle are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3), then centroid of the triangle is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

The coordinates of centroid of a triangle are given as,

$$x=\frac{x_1+x_2+x_3}{3} \Rightarrow 3=\frac{6+(-2)+x_3}{3}$$

$$\Rightarrow 9=4+x_3 \qquad \Rightarrow x_3=9-4=5$$

and $y=\dfrac{y_1+y_2+y_3}{3}$

$$\Rightarrow \quad 4=\frac{4+2+y_3}{3} \Rightarrow 12=6+y_3$$

$$\Rightarrow \quad y_3=12-6=6$$

Hence, the coordinates of the third vertex C of ΔABC are (5, 6).

Long Answer Type Questions

1. Given vertices of a quadrilateral are $A(-1, -2)$, $B(1, 0)$, $C(-1, 2)$ and $D(-3, 0)$.

To identify the type of quadrilateral, we have to find its edges or sides.

Now sides of a quadrilateral are

$$AB=\sqrt{(1+1)^2+(0+2)^2} \text{ (Using distance formula)}$$

TR!CK

Distance between (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}.$$

$$=\sqrt{(2)^2+(2)^2}=\sqrt{4+4}=2\sqrt{2}$$

$$BC=\sqrt{(-1-1)^2+(2-0)^2}$$

$$=\sqrt{(-2)^2+(2)^2}=\sqrt{4+4}=2\sqrt{2}$$

$$CD=\sqrt{(-3+1)^2+(0-2)^2}$$

$$=\sqrt{(-2)^2+(-2)^2}=\sqrt{4+4}=2\sqrt{2}$$

and $DA=\sqrt{(-1+3)^2+(-2-0)^2}=\sqrt{(2)^2+(-2)^2}$

$$=\sqrt{4+4}=2\sqrt{2}$$

Here we see that $AB=BC=CD=DA$

It means given quadrilateral may be either square or rhombus.

Now we further determine the diagonals, so that exact shape of figure will be clear.

Now diagonals of a quadrilateral are

$$AC=\sqrt{(-1+1)^2+(2+2)^2}=\sqrt{0+4^2}=4$$

and $BD=\sqrt{(-3-1)^2+(0-0)^2}=\sqrt{(-4)^2+0}=4$

Here, $AC=BD$

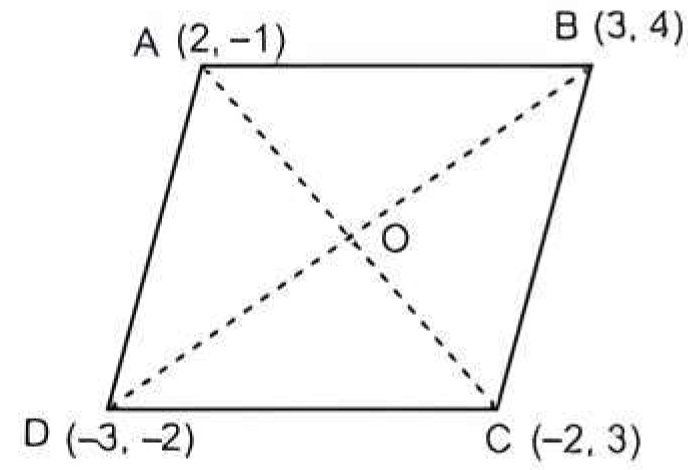
TiP

In a square ABCD, AB = BC = CD = DA and diagonal AC = diagonal BD.

It means, given vertices of a quadrilateral is a square having all sides are equal and diagonals are also equal.

2. $AB=\sqrt{(3-2)^2+(4+1)^2}=\sqrt{1^2+5^2}$

$$=\sqrt{1+25}=\sqrt{26} \text{ units}$$



TR!CK

The distance between (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}.$$

$BC = \sqrt{(-2-3)^2+(3-4)^2}$

$= \sqrt{(-5)^2+(-1)^2} = \sqrt{25+1} = \sqrt{26}$ units

$CD = \sqrt{(-3+2)^2+(-2-3)^2}$

$= \sqrt{(-1)^2+(-5)^2} = \sqrt{1+25} = \sqrt{26}$ units

and $DA = \sqrt{(2+3)^2+(-1+2)^2}$

$= \sqrt{5^2+1^2} = \sqrt{25+1} = \sqrt{26}$ units

Here, $AB = BC = CD = DA = \sqrt{26}$ units

Now, $AC = \sqrt{(-2-2)^2+(3+1)^2} = \sqrt{(-4)^2+4^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ units

and $BD = \sqrt{(-3-3)^2+(-2-4)^2}$

$= \sqrt{(-6)^2+(-6)^2}$

$= \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$ units

So, $AC \neq BD$

In a quadrilateral, if the diagonals are not equal but sides are equal, then quadrilateral is a rhombus, while in case of a square, diagonals are equal.

Hence, given quadrilateral is a rhombus but not a square. **Hence proved.**

TR!CK

Area of rhombus $= \frac{1}{2} \times d_1 \times d_2$

where d_1 and d_2 are diagonals.

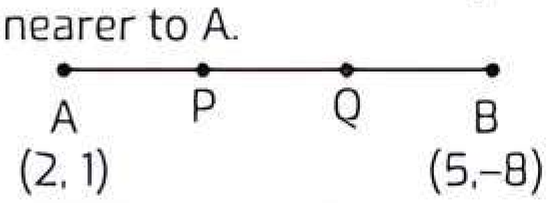
$\therefore$ Area of rhombus ABCD

$= \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24$ sq. units

Hence, the area of rhombus ABCD is 24 sq. units.

3. $\because$ The line AB is trisected at the points P and Q such that P is nearer to A.



$\therefore \frac{AP}{PB} = \frac{1}{2}$ and $\frac{AQ}{QB} = \frac{2}{1}$

$\Rightarrow AP : PB = 1 : 2$ and $AQ : QB = 2 : 1$

TiP

Trisect means to divide three equal parts, i.e., any one point on AB divide the segment into ratio one is to two

Using section formula for internal division, coordinates of P are

$x = \frac{mx_2+nx_1}{m+n} = \frac{1\times5+2\times2}{1+2}$

$= \frac{5+4}{3} = \frac{9}{3} = 3$

TR!CK

The coordinates of the point P(x, y) which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$.

$y = \frac{my_2+ny_1}{m+n} = \frac{1\times(-8)+2\times1}{1+2}$

$= \frac{-8+2}{3} = \frac{-6}{3} = -2$

$\therefore$ Coordinates of P are (3, –2).

Coordinates of Q are

$x = \frac{2\times5+2\times1}{2+1} = \frac{12}{3} = 4$

and $y = \frac{2\times-8+1\times1}{2+1} = \frac{-15}{3} = -5$

$\therefore$ Coordinates of Q are (4, –5)

Since, P lies on $2x - y + k = 0$, so coordinates of P will satisfy the given equation.

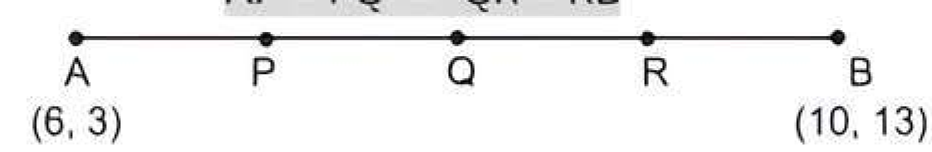
$\therefore \quad 2\times3-(-2)+k = 0$

$\Rightarrow \quad 6+2+k = 0 \Rightarrow k = -8$

Hence, the value of k is –8.

4. Let P, Q and R divide the line segment AB such that

$AP = PQ = QR = RB$



So, $AP : PB = 1 : 3$

$AQ : QB = 2 : 2$

and $AR : RB = 3 : 1$

Let $A(6, 3) = A(x_1, y_1)$

and $B(10, 13) = B(x_2, y_2)$

TR!CK

The coordinates of the point P(x, y) which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$.

Using section formula for internal division,

Coordinates of P are

$x = \frac{mx_2+nx_1}{m+n} = \frac{1\times10+3\times6}{1+3}$

$= \frac{10+18}{4} = \frac{28}{4} = 7$

and $y = \frac{my_2+ny_1}{m+n} = \frac{1\times13+3\times3}{1+3}$

$= \frac{13+9}{4} = \frac{22}{4} = \frac{11}{2}$

So, coordinates of P are $\left(7, \frac{11}{2}\right)$.

Coordinates of Q are $x = \frac{x_1+x_2}{2}$

($\because AQ : QR = 2 : 2$, so Q is the mid-point of AB)

$= \frac{6+10}{2} = \frac{16}{2} = 8$

and $y = \frac{y_1+y_2}{2} = \frac{3+13}{2} = \frac{16}{2} = 8$

So, coordinates of Q are (8, 8).

Coordinates of R are

$$x = \frac{3 \times 10 + 1 \times 6}{3+1} = \frac{30+6}{4} = \frac{36}{4} = 9$$

and $$y = \frac{3 \times 13 + 1 \times 3}{3+1} = \frac{39+3}{4} = \frac{42}{4} = \frac{21}{2}$$

So, coordinates of R are $\left(9, \frac{21}{2}\right)$.

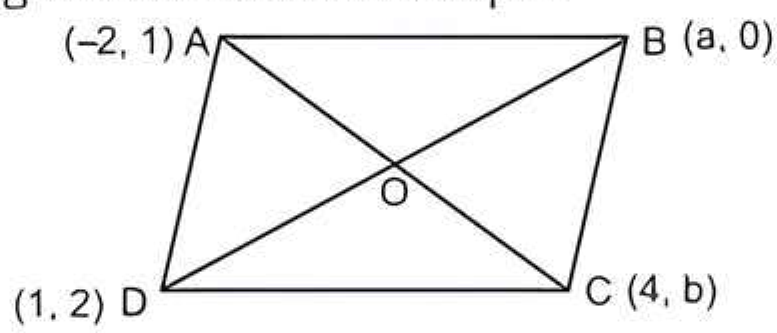
Hence, coordinates of points are P $\left(7, \frac{11}{2}\right)$, Q (8, 8) and R $\left(9, \frac{21}{2}\right)$.

5.

TiP

- *Diagonals of a parallelogram bisect each other at a point.*
- *In a parallelogram, the length of opposite sides are equal.*

In parallelogram, coordinates of mid-point of diagonals AC and BD are equal.



$$\therefore \left(\frac{-2+4}{2}, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, \frac{2+0}{2}\right)$$

(using mid-point formula)

$$\Rightarrow \left(1, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, 1\right)$$

$\Rightarrow 1 = \frac{a+1}{2}$ and $\frac{1+b}{2} = 1$

$\Rightarrow 2 = a + 1$ and $1 + b = 2$

$\Rightarrow a = 2 - 1 = 1$ and $b = 2 - 1 = 1$

So, the coordinates of B and C are (1, 0) and (4, 1) respectively.

TR!CK

The distance between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(\text{difference of abscissa})^2 + (\text{difference of ordinate})^2}$$

Now,

$AB = \sqrt{(1+2)^2 + (0-1)^2} = \sqrt{3^2 + 1^2} = \sqrt{10}$ units

$DC = AB = \sqrt{10}$ units

(Opposite sides of a parallelogram are equal)

and $AD = \sqrt{(1+2)^2 + (2-1)^2} = \sqrt{3^2 + 1^2} = \sqrt{10}$ units

$BC = AD = \sqrt{10}$ units

Hence, $DC = AB = \sqrt{10}$ units and $BC = AD = \sqrt{10}$ units.

Chapter Test

Multiple Choice Questions

Q 1. If the points A (4,5) and B (*x*, 4) are on the circle with centre C(2,2), then the value of *x* is:

a. 5 b. 6 c. 2 d. 3

Q 2. The distance between the points A(0, 6) and B(0, 2) is:

a. 8 b. 6 c. 4 d. 2

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): The value of *y* is 6, for which the distance between the points P(2,–3) and Q(10, *y*) is 10.

Reason (R): The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Q 4. Assertion (A): The distance of the point (2, 11) from the *Y*-axis is 2 units.

Reason (R): The distance of the point (*x*, *y*) from *Y*-axis is *x* unit.

Fill in the Blanks

Q 5. The distance of the point (5, –4) from *X*-axis is ....

Q 6. The image of a point (–4, 5) with respect to *Y*-axis is ....

True/False

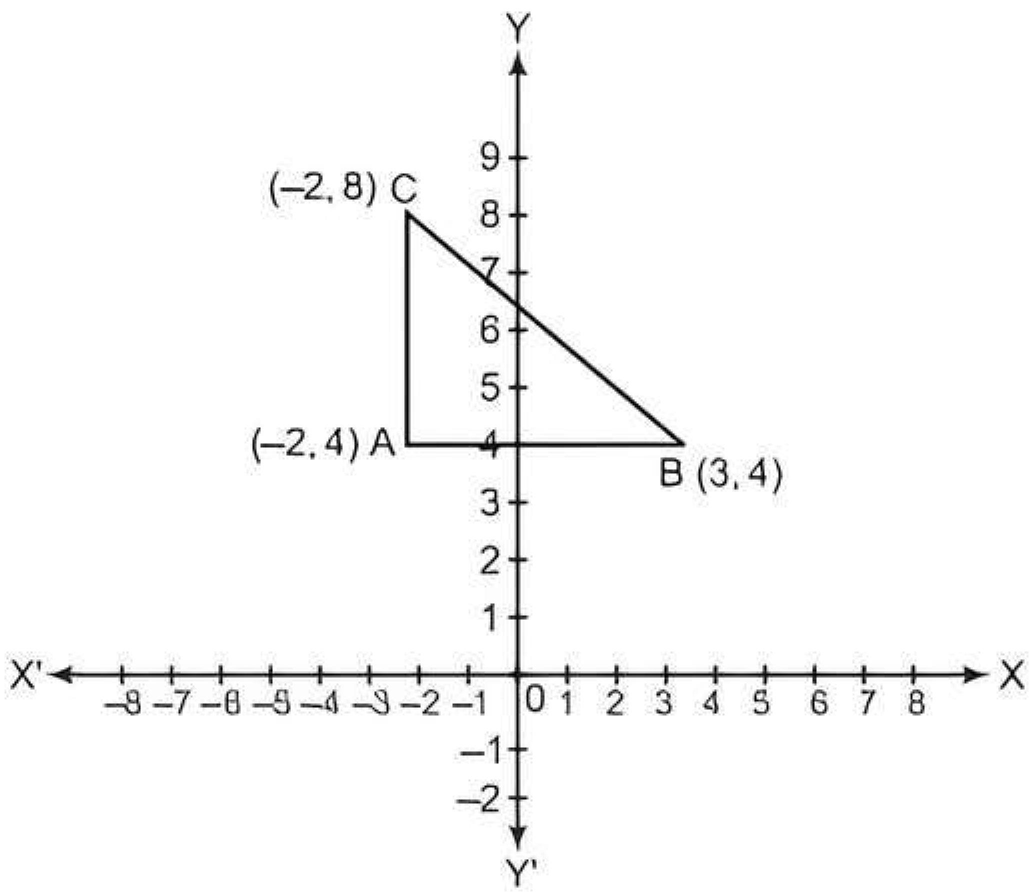
Q 7. If the vertices of a parallelogram taken in order are A(–1, 6), B(2, –5) and C(7, 2), then the fourth vertex is (4, 1).

Q 8. The ratio in which the point divides the line joining the points A(1, 2) and B(–1, 1) internally in the ratio 1 : 2 is $\left(\frac{1}{3}, \frac{5}{3}\right)$.

Case Study Based Question

Q 9. Centonion Public School, have so many branches in different cities of India. One of the branch of the Centonion Public School is in Meerut. In that school hundreds of students study in a classroom.

Out of them one of the girl is standing in the ground having coordinates (3, 4) facing towards west. She moves 5 units in straight line then take right and moves 4 units and stop. Now, she is at her coaching centre. The representation of the above situation on the coordinate axes is shown below:



Based on the above information, solve the following questions:

(i) What is the shortest distance between her school and coaching centre?

(ii) Suppose point D(1, 4) divides the line segment AB in the ratio $k : 1$, then find the value of k.

(iii) If we draw perpendicular lines from points A and B to the X-axis. Find the region covered by these perpendicular lines.

Or

Find the area of $\triangle ABC$.

(iv) Find the image of the mid-point of AB with respect to X-axis.

Very Short Answer Type Questions

Q 10. Find the distance between the points (1, 0) and (2, cot θ).

Q 11. If the coordinates of the mid-point of the line segment joining (−8, 13) and (x, 7) is (4, 10), then find the value of x.

Short Answer Type-I Questions

Q 12. Show that the points P(9, 0), Q(9, 6), R(−9, 6) and S (−9, 0) are the vertices of a parallelogram.

Q 13. If the distance between the points (2, −2) and (−1, x) is 5, then find the possible values of x.

Short Answer Type-II Questions

Q 14. If vertices of a $\triangle ABC$ are A(7, 3), B(5, 3) and (3, −1), then find the length of the median through vertex A.

Q 15. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points P(2, −2) and Q(3, 7)? Also find the value of y.

Long Answer Type Question

Q 16. Points A (−1, y) and B(5, 7) lie on a circle with centre C(2, −3y). Find the value of y. Also, find the radius of the circle.