CBSE Board
Class IX Mathematics

Time: 3 hrs **Total Marks: 80**

General Instructions:

1. **All** questions are **compulsory**.
2. The question paper consists of **30** questions divided into **four sections** A, B, C, and D. **Section A** comprises of **6** questions of 1 mark each, **Section B** comprises of **6** questions of 2 marks each, **Section C** comprises of **10** questions of 3 marks each and **Section D** comprises of **8** questions of 4 marks each.
3. Question numbers **1 to 6** in **Section A** are multiple choice questions where you are to select **one** correct option out of the given four.
4. Use of calculator is **not** permitted.

Section A
(Questions 1 to 6 carry 1 mark each)

1. Simplify : $(5+\sqrt{5})(5-\sqrt{5})$

2. Find the value of p such that (x – 1) is a factor of the polynomial $x^3 + 10x^2 + px$?

3. In ΔABC, $\angle$A = 100° and AB = AC. Find $\angle$B?

4. The cost of notebook is twice the cost of a pen. Write a linear equation in the two variables to represent this statement?

5. Find the range of data: 70, 65, 75, 71, 36, 55, 61, 62, 41, 40, 39, 35.

6. In a parallelogram PQRS, What is the sum of $\angle$R and $\angle$S?

Section B
(Questions 7 to 12 carry 2 marks each)

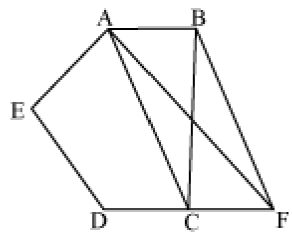
7. Simplify:

$$\left(\frac{81}{16}\right)^{-3/4} \times \left(\frac{25}{9}\right)^{-3/2}$$

8. Factorise: $x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$

9. Where do the following points lie?
   a. (−4, 0)
   b. (−10, 2)
   c. (0, 8)
   d. (10, 4)

10. In the given figure, ABCDE is a pentagon. A line through B and parallel to AC meets DC produced at F. Show that area(ΔACB) = area(ΔACF).



11. A rectangular metallic sheet has dimensions 48 cm × 36 cm. From each corner a square of 8 cm is cut off. An open box is made of the remaining sheet. Find the volume of the box.

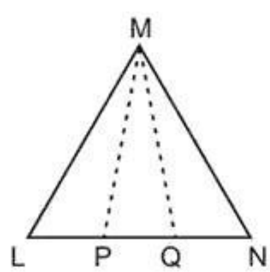
12. Find the value of k, if x = 1, y = 1 is a solution of the equation 9kx + 12ky = 63.

Section C
(Questions 13 to 22 carry 3 marks each)

13. Simplify: $\dfrac{(25)^{\frac{3}{2}} \times (343)^{\frac{3}{5}}}{16^{\frac{5}{4}} \times 8^{\frac{4}{3}} \times 7^{\frac{3}{5}}}$

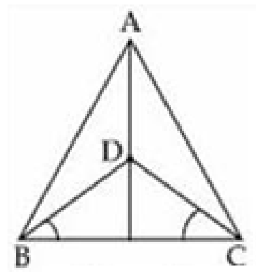
14. (x + 2) is one of the factors of the polynomial $x^3 + 13x^2 + 32x + 20$. Find its remaining factors.

15. In the figure, it is given that LM = MN and LP = QN. Prove that $\Delta LMQ \cong \Delta NMP$



16. The polynomials $p(x) = ax^3 + 3x^2 - 3$ and $q(x) = 2x^3 - 5x + a$ when divided by $(x - 4)$ leave the remainders R_1 and R_2. Find 'a' if $R_1 + R_2 = 0$. Factorise the polynomial.

17. In figure, AB = AC, D is the point in the interior of Δ ABC such that $\angle DBC = \angle DCB$. Prove that AD bisects $\angle BAC$ of Δ ABC.

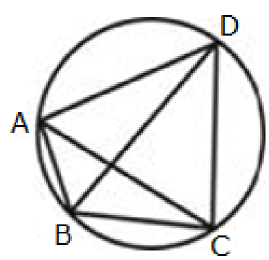


18. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is a

    (a) queen
    (b) non-ace card
    (c) black card

19. Show that the line segments joining the mid points of the opposite sides of a quadrilateral bisect each other.

20. In the given figure, ABCD is a cyclic quadrilateral, in which AC and BD are the diagonals. If $m\angle DBC = 55°$ and $m\angle BAC = 45°$, find $m\angle BCD$.



21. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move over once to level a playground. Find the area of the playground in m^2? $\left[\pi = \frac{22}{7}\right]$

22. The following observations have been arranged in the ascending order.
    29, 32, 48, 50, x, x + 2, 72, 78, 84, 95
    If the median of the data is 63, find the value of x.

Section D
(Questions 23 to 30 carry 4 marks each)

23. Find the value of:

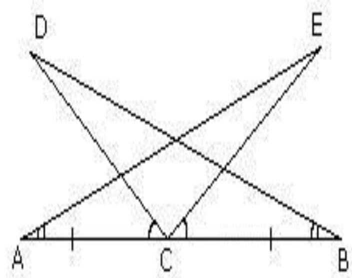
$$\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$

24. How does Euclid's fifth postulate imply the existence of parallel lines? Give a mathematical proof.

25. Simplify:

$$\frac{\left(a^2-b^2\right)^3+\left(b^2-c^2\right)^3+\left(c^2-a^2\right)^3}{\left(a-b\right)^3+\left(b-c\right)^3+\left(c-a\right)^3}$$

26. In the figure, if AC = BC, ∠DCA = ∠ECB and ∠DBC = ∠EAC, then prove that BD = AE.



27. The volume of a cylinder is 6358 cu. cm and its height is 28 cm. Find its radius and the curved surface area.

28. Construct a triangle having a perimeter of 12.5 cm and angles in the ratio of 3 : 4 : 5.

29. If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.

30. Neha and Richa, two students of class IX of a school, together contributed Rs. 100 towards the Prime Minister's Relief Fund, to help earthquake victims. Assume Neha's contribution to be x and that of Richa to be y. Write a linear equation which this data satisfies and draw a graph of the same.

CBSE Board
Class IX Mathematics
Solution

Time: 3 hrs **Total Marks: 80**

Section A

1. $(5+\sqrt{5})(5-\sqrt{5}) = (5)^2 - (\sqrt{5})^2 = 25 - 5 = 20 \quad \left[\because a^2 - b^2 = (a-b)(a+b)\right]$

2. $p(x) = x^3 + 10x^2 + px$
   $(x - 1)$ is the factor of $p(x)$.
   $\therefore\ x - 1 = 0$
   $\therefore\ x = 1$ will satisfy $p(x)$
   $\therefore\ p(1) = 0$
   $\Rightarrow (1)^3 + 10(1)^2 + p(1) = 0$
   $\Rightarrow 1 + 10 + p = 0$
   $\Rightarrow p = -11$

3. Given, AB = AC
   $\therefore\ \angle B = \angle C$ ....(1) $(\because$ angles opp. to equal sides are equal)
   In Δ ABC, we have
   $\angle A + \angle B + \angle C = 180^\circ$
   $\Rightarrow \angle A + \angle B + \angle B = 180^\circ$ ....[From (1)]
   $\Rightarrow 100^\circ + 2\angle B = 180^\circ$ ....$(\because \angle A = 100^\circ)$
   $\Rightarrow 2\angle B = 80^\circ$
   $\Rightarrow \angle B = 40^\circ$

4. Let, the cost of a note book = Rs. x and the cost of a pen = Rs. y
   According to given statement,
   $x = 2y$
   i.e. $x - 2y = 0$

5. Arranging the given data in the Ascending order:
   35, 36, 39, 40, 41, 55, 61, 62, 65, 70, 71, 75
   $\therefore$ Range = Maximum value – Minimum value = 75 – 35 = 40

6. In a Parallelogram PQRS,
   $\angle R$ and $\angle S$ are consecutive interior angles on the same side of the transversal SR.
   Therefore, $m\angle R + m\angle S = 180^\circ$

Section B

7.

$$\left(\frac{81}{16}\right)^{-3/4}\times\left(\frac{25}{9}\right)^{-3/2}$$

$$=\left[\left(\frac{3}{2}\right)^{4}\right]^{-\frac{3}{4}}\times\left[\left(\frac{5}{3}\right)^{2}\right]^{-\frac{3}{2}}$$

$$=\left(\frac{3}{2}\right)^{-3}\times\left(\frac{5}{3}\right)^{-3}$$

$$=\left(\frac{2}{3}\right)^{3}\times\left(\frac{3}{5}\right)^{3}$$

$$=\frac{2^3}{3^3}\times\frac{3^3}{5^3}=\frac{2^3}{5^3}=\frac{8}{125}$$

8.

$$x^2+\frac{1}{x^2}+2-2x-\frac{2}{x}$$

$$=\left(x^2+\frac{1}{x^2}+2\right)-2\left(x+\frac{1}{x}\right)$$

$$=\left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)$$

$$=\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}-2\right)$$

9.

a. Point of the form (a, 0) lie on the x axis.
   So, the point (−4, 0) will lie on the negative side of the x axis.

b. (−, +) are the sign of the coordinate of points in the quadrant II.
   So, the point (−10, 2) lies in the quadrant II.

c. Point of the form (0, a) lie on the y axis.
   So, the point (0, 8) will lie on the positive side of y axis.

d. (+, +) are the sign of the coordinates of points in the quadrant I.
   So, the point (10, 4) lies in the quadrant I.

10. ΔACB and ΔACF lie on the same base AC and are between the same parallel lines AC and BF.

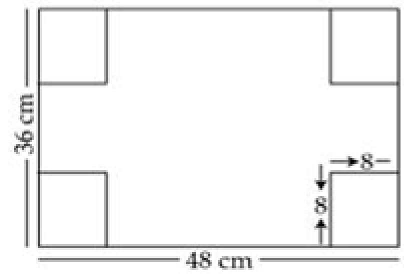
∴ area(ΔACB) = area(ΔACF)

11. Length of the box = l = 48 – 8 – 8 = 32 cm

Breadth of the box = b = 36 – 8 – 8 = 20 cm

Height = h = 8 cm

Volume of the box formed = l × b × h = 32 × 20 × 8 = 5120 cm^3



12. Since x = 1, y = 1 is the solution of 9kx + 12ky = 1, it will satisfy the equation.

∴ 9k(1) + 12k(1) = 63

∴ 9k + 12k = 63

∴ 21k = 63

∴ k = 3

Section C

13.

$$\frac{(25)^{\frac{3}{2}} \times (343)^{\frac{3}{5}}}{16^{\frac{5}{4}} \times 8^{\frac{4}{3}} \times 7^{\frac{3}{5}}}$$

$$= \frac{\left(5^2\right)^{\frac{3}{2}} \times \left(7^3\right)^{\frac{3}{5}}}{\left(2^4\right)^{\frac{5}{4}} \times \left(2^3\right)^{\frac{4}{3}} \times 7^{\frac{3}{5}}}$$

$$= \frac{5^3 \times 7^{\frac{9}{5}}}{2^5 \times 2^4 \times 7^{\frac{3}{5}}}$$

$$= \frac{5^3 \times 7^{\frac{9}{5}}}{2^9 \times 7^{\frac{3}{5}}}$$

$$= \frac{5^3 \times 7^{\frac{6}{5}}}{2^9}$$

14. Let $p(x) = x^3 + 13x^2 + 32x + 20$

$p(-1) = -1 + 13 - 32 + 20 = -33 + 33 = 0$

Therefore $(x + 1)$ is a factor of $p(x)$.

On dividing $p(x)$ by $(x + 1)$, we get

$p(x) \div (x + 1) = x^2 + 12x + 20$

Thus,

$$\begin{aligned} x^3 + 13x^2 + 32x + 20 &= (x + 1)(x^2 + 12x + 20) \\ &= (x + 1)(x^2 + 10x + 2x + 20) \\ &= (x + 1)[x(x + 10) + 2(x + 10)] \\ &= (x + 1)(x + 2)(x + 10) \end{aligned}$$

Hence, $x^3 + 13x^2 + 32x + 20 = (x + 1)(x + 2)(x + 10)$

15. LM = MN (Given)

$\Rightarrow \angle MLN = \angle MNL$ (angles opposite equal sides are equal)

$\Rightarrow \angle MLQ = \angle MNP$

LP = QN (Given)

$\Rightarrow LP + PQ = PQ + QN$ (adding PQ on both sides)

$\Rightarrow LQ = PN$

In ΔLMQ and ΔNMP

LM = MN

$\angle MLQ = \angle MNP$

LQ = PN

$\Delta LMQ \cong \Delta NMP$ (SAS congruence rule)

16. When $p(x) = ax^3 + 3x^2 - 3$ is divided by $(x - 4)$, the remainder is given by

$R_1 = a(4)^3 + 3(4)^2 - 3 = 64a + 45$

When $q(x) = 2x^3 - 5x + a$ is divided by $(x - 4)$, the remainder is given by

$R_2 = 2(4)^3 - 5(4) + a = 108 + a$

Given: $R_1 + R_2 = 0$

$\Rightarrow 65a + 153 = 0$

$\Rightarrow a = \frac{-153}{65}$

By hit and trial we find x = 3 is factor of given polynomial, as

$2(3)^3 - 9 - 39 - 6 = 54 - 54 = 0$

By dividing $2x^3 - x^2 - 13x - 6$ by $x - 3$ we get $2x^2 + 5x + 2$ as quotient.

Now,

$2x^2 + 5x + 2 = 2x^2 + 4x + x + 2 = 2x(x + 2) + 1(x + 2) = (2x + 1)(x + 2)$

So, $2x^3 - x^2 - 13x - 6 = (2x + 1)(x + 2)(x - 3)$

17. In ΔDCB, $\angle DBC = \angle DCB$ (given)

DC = DB [Side opp. To equal $\angle$'s are equal].....(i)

In ΔABD and ΔACD

AB = AC (given)

BD = CD [from (i)]

AD = AD common

$\Delta ABD \cong \Delta ACD$ [SSS Rule]

$\angle BAD = \angle CAD$ (CPCT)

Hence, AD is bisector of $\angle BAC$.

18. Total number of outcomes = n(S) = 52

(a) Let A be the event when the card drawn is a queen.

Total number of queen cards = 4

$\therefore$ n(A) = 4

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(b) Let B be the event when the card drawn is a non-ace card.

Non–ace cards = 52 – 4 = 48

$\therefore$ n(B) = 48

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{48}{52} = \frac{12}{13}$$

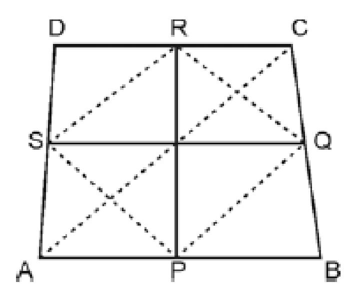
(c) Let C be the event when the card drawn is a black card.

Number of black cards = 26

$\therefore$ n(C) = 26

$$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

19. Let ABCD be a quadrilateral. P, Q, R, and S are the mid points of AB, BC, CD and DA respectively.



Join PQ, QR, RS and SP. Join AC.

In ΔDAC, SR || AC

And $SR = \frac{1}{2}AC$ (Mid-point theorem)

In ΔBAC, PQ || AC

And $PQ = \frac{1}{2}AC$

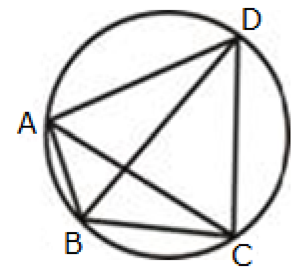
Clearly, PQ || SR and PQ = SR

In quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other and hence it is a parallelogram.

Now, PR and SQ are the diagonals of PQRS and hence PR and SQ bisect each other.

20. Given: ABCD is a cyclic quadrilateral in which AC and BD are diagonals.
$m\angle DBC = 55°$ and $m\angle BAC = 45°$

To find: $m\angle BCD$



Proof: $m\angle CAD = m\angle DBC = 55°$ (Angles in the same segment)

Therefore, $m\angle DAB = m\angle CAD + m\angle BAC$

$= 55° + 45°$

$= 100°$

However, $m\angle DAB + m\angle BCD = 180°$

($\because$ opposite angles of a cyclic quadrilateral)

So, $m\angle BCD = 180° - 100° = 80°$

21. The roller is cylindrical in shape.
Height (h) of cylindrical roller = Length of roller = 120 cm

Radius (r) of the circular end of roller = $\left(\frac{84}{2}\right)\text{cm} = 42\text{ cm}$

C.S.A. of roller = $2\pi rh = 2 \times \frac{22}{7} \times 42 \times 120 = 31680\text{ cm}^2$

Area of field = 500 × C.S.A. of roller = (500 × 31680) cm^2 = 15840000 cm^2

Area of field = 1584 m^2

22. Total observations in the given data set, n = 10 (even)

$$\therefore \text{Median} = \frac{\left(\frac{n}{2}\right)^{th}\text{ observation} + \left(\frac{n}{2}+1\right)^{th}\text{ observation}}{2}$$

$$\therefore 63 = \frac{\left(\frac{10}{2}\right)^{th}\text{ observation} + \left(\frac{10}{2}+1\right)^{th}\text{ observation}}{2}$$

$$\therefore 63 = \frac{5^{th}\text{ observation} + 6^{th}\text{ observation}}{2}$$

$$\therefore 63 = \frac{(x)+(x+2)}{2}$$

$$\therefore 63 = \frac{2x+2}{2}$$

$$\therefore 63 = x+1$$

$$\therefore x = 62$$

Section D

23.

$$\frac{1}{3-\sqrt{8}}=\frac{1}{3-\sqrt{8}}\times\frac{3+\sqrt{8}}{3+\sqrt{8}}=\frac{3+\sqrt{8}}{9-8}=3+\sqrt{8}$$

$$\frac{1}{\sqrt{8}-\sqrt{7}}=\frac{1}{\sqrt{8}-\sqrt{7}}\times\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}=\frac{\sqrt{8}+\sqrt{7}}{8-7}=\sqrt{8}+\sqrt{7}$$

$$\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}=\frac{\sqrt{7}+\sqrt{6}}{7-6}=\sqrt{7}+\sqrt{6}$$

$$\frac{1}{\sqrt{6}-\sqrt{5}}=\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}=\frac{\sqrt{6}+\sqrt{5}}{6-5}=\sqrt{6}-\sqrt{5}$$

$$\frac{1}{\sqrt{5}-2}=\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}=\frac{\sqrt{5}+2}{5-4}=\sqrt{5}+2$$

$$\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$

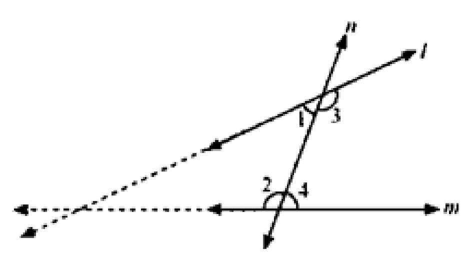
$$=3+\sqrt{8}-\left(\sqrt{8}+\sqrt{7}\right)+\left(\sqrt{7}+\sqrt{6}\right)-\left(\sqrt{6}-\sqrt{5}\right)+\left(\sqrt{5}+2\right)$$

$$=5$$

24. Euclid's 5th postulate states that:

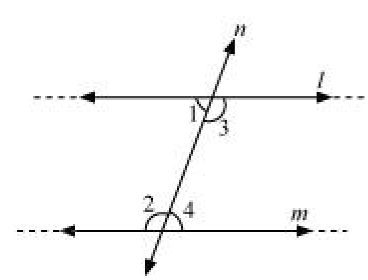
If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

This implies that if n intersects lines l and m and if $\angle 1 + \angle 2 < 180^0$, then $\angle 3 + \angle 4 > 180^0$. In that case, producing line l and further will meet in the side of $\angle 1$ and $\angle 2$ which is less than 180^o.



If $\angle 1 + \angle 2 = 180^0$, then $\angle 3 + \angle 4 = 180^0$

In that case, the lines l and m neither meet at the side of $\angle 1$ and $\angle 2$ nor at the side of $\angle 3$ and $\angle 4$ implying that the lines l and m will never intersect each other. Therefore, the lines are parallel.



25.

Consider $\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

We know that,

If x + y + x = 0 then $x^3 + y^3 + z^3 = 3xyz$

Now, $a^2 - b^2 + b^2 - c^2 - a^2 = 0$

And, a – b +b – c + c – a = 0

$$\therefore \frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$$

$$=\frac{3(a^2-b^2)(b^2-c^2)(c^2-a^2)}{3(a-b)(b-c)(c-a)}$$

$$=\frac{3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{3(a-b)(b-c)(c-a)}$$

$$=(a+b)(b+c)(c+a)$$

26. Given that ∠DCA = ∠ECB

⇒ ∠DCA + ∠ECD = ∠ECB + ∠ECD

⇒ ∠ECA = ∠DCB .....(i)

Now in ΔDBC and ΔEAC

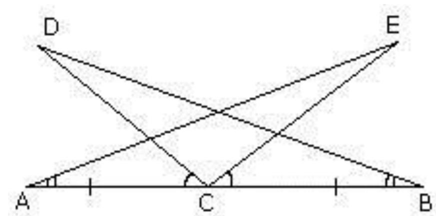
∠DCB = ∠ECA [from (i)]

BC = AC (Given)

∠DBC = ∠EAC (Given)

Δ DBC ≅ Δ EAC (ASA Congruence)

Therefore, BD = AE (CPCT)



27. Let the radius of the cylinder be ‘r’ cm.

$$\therefore \text{Volume of cylinder} = \pi r^2 h = \frac{22}{7}\times r^2 \times 28$$

$$\therefore 6358 = 88r^2$$

$$\therefore r^2 = \frac{6358}{88} = \frac{289}{4}$$

$$\therefore r = \frac{17}{2}\text{ cm} = 8.5\text{ cm}$$

$$\text{Curved surface area of cylinder} = 2\pi rh = 2\times\frac{22}{7}\times\frac{17}{2}\times 28 = 1496\text{ cm}^2.$$

Thus, the radius of the cylinder is 8.5 cm and its curved surface area is 1796 cm^2.

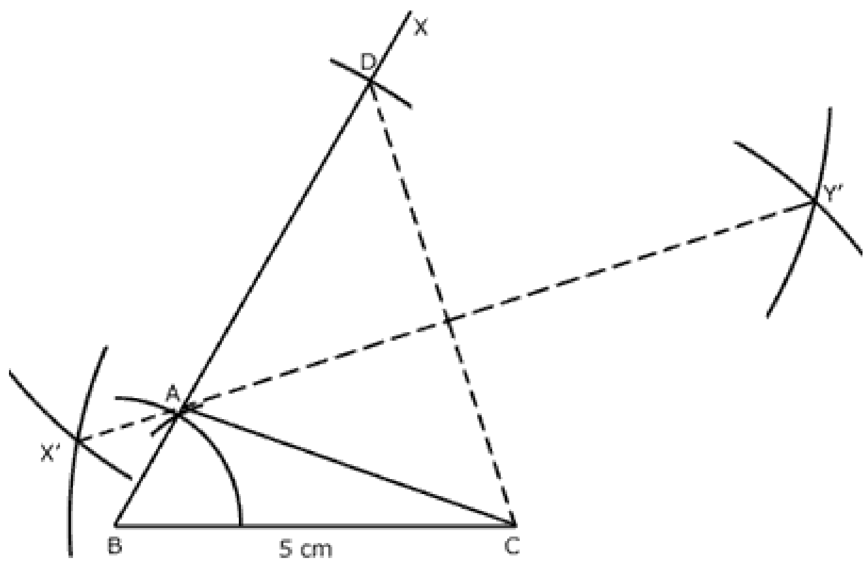
28. Steps of construction:

i. Draw BC = 5 cm

ii. Draw m∠CBX = 60° and cut off BD = 7.7 cm.

iii. Join CD and draw its perpendicular bisector meeting BD at A.

iv. Join AC. ΔABC is the required triangle.



29. Given that AB and CD are two chords of a circle with centre O, intersecting at a point E.

PQ is the diameter through E, such that ∠AEQ = ∠DEQ.

To prove that AB = CD.

Draw perpendiculars OL and OM on chords AB and CD respectively.

Now, m∠LOE = 180° – 90° – m∠LEO ... [Angle sum property of a triangle]

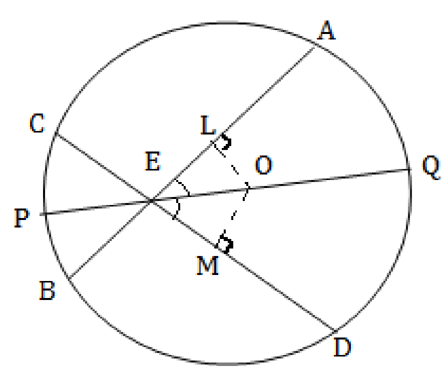
= 90° – m∠LEO

⇒ m∠LOE = 90° – m∠AEQ

⇒ m∠LOE = 90° – m∠DEQ

⇒ m∠LOE = 90° – m∠MEQ

⇒ ∠LOE = ∠MOE



In ΔOLE and ΔOME,

∠LEO = ∠MEO

∠LOE = ∠MOE

EO = EO

ΔOLE ≅ ΔOME

OL = OM

Therefore, chords AB and CD are equidistant from the centre.

Hence, AB = CD

30. According to the given condition,

x + y = 100 ...(i)

Now, put the value x = 0 in equation (i).

$0 + y = 100 \Rightarrow y = 100$

The solution is (0, 100)

Putting the value x = 50 in equation (i), we get

$50 + y = 100 \Rightarrow y = 100 - 50 \Rightarrow y = 50$.

The solution is (50, 50).

Put the value x = 100 in equation (i).

100 + y = 100,

$y = 100 - 100 \Rightarrow y = 0$.

The solution is (100, 0).

x	0	50	100
y	100	50	0

Now, plot the points (0, 100), (50, 50), (100, 0) and draw lines passing through the points.