

Class XI Session 2023-24
Subject - Physics
Sample Question Paper - 9

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions:

1. There are 33 questions in all. All questions are compulsory.
2. This question paper has five sections: Section A, Section B, Section C, Section D and Section E. All the sections are compulsory.
3. Section A contains sixteen questions, twelve MCQ and four Assertion Reasoning based of 1 mark each, Section B contains five questions of two marks each, Section C contains seven questions of three marks each, Section D contains two case study-based questions of four marks each and Section E contains three long answer questions of five marks each.
4. There is no overall choice. However, an internal choice has been provided in section B, C, D and E. You have to attempt only one of the choices in such questions.
5. Use of calculators is not allowed.

Section A

1. Dimensional analysis can be used to [1]
 - a) check the validity of an equation.
 - b) deducing relations among the physical quantities.
 - c) to check integration of the equation.
 - d) check the order of an equation.
2. Two waves of equal amplitude A and equal frequency travel in the same direction in a medium. The amplitude of the resultant wave is [1]
 - a) $2A$
 - b) 0
 - c) Between 0 to $2A$
 - d) A
3. One circular ring and one circular disc both are having the same mass and radius. The ratio of their moments of inertia about the axis passing through their centres and perpendicular to their planes will be [1]
 - a) $1 : 2$
 - b) $2 : 1$
 - c) $1 : 1$
 - d) $4 : 1$
4. At large flow velocities, the flow of fluid becomes: [1]
 - a) viscous
 - b) turbulent
 - c) compressible
 - d) laminar
5. An astronaut is standing on an asteroid when he accidentally drops a wrench. He observes that the gravitational [1]

c) 240 °C

d) 239 °C

13. **Assertion (A):** Fault lines in the earth's crust are like compressed springs. [1]

Reason (R): Compressed springs possess a large amount of kinetic energy.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

14. **Assertion:** First law of thermodynamics allows many processes which actually don't happen. [1]

Reason: First law of thermodynamics must not be violated for any process to happen.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

15. **Assertion (A):** Newton's universal law of gravitation is an inverse square law. [1]

Reason (R): The force is inversely proportional to the square of the distance.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

16. **Assertion (A):** Displacement vector is defined with respect to origin. [1]

Reason (R): Position vector is defined with respect to origin.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

17. The displacement y of a particle in a medium can be expressed as $y = 10^{-6} \sin(100t + 20x + \frac{\pi}{2})$. Where t is in second and x in metre. What is the speed of the wave? [2]

18. The Vander Wall's equation for a gas is $(P + \frac{a}{V^2})(V - b) = RT$ [2]

Determine the dimensions of a and b . Hence write the SI units of a and b .

19. A small spherical ball of radius r falls with velocity v through a liquid having coefficient of viscosity η . Find the viscous drag F on the ball assuming it depends on η , r and v . Take $K = 6\pi$. [2]

20. What is static friction? Is it self adjusting in nature? [2]

21. A body hanging from a spring stretches it by 1 cm at the earth's surface. How much will the same body stretch at a place 1600 km above the earth's surface? Radius of earth 6400 km. [2]

OR

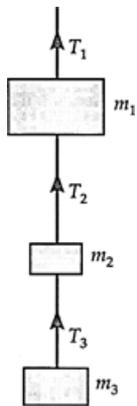
If radius of earth is 6400km, what will be the weight of 1 quintal body if taken to the height of 1600 km above the sea level?

Section C

22. A metallic sphere of radius $1.0 \times 10^{-3}m$ and density $1.0 \times 10^4kg m^{-3}$ enters a tank of water, after a free fall through a distance of h in the earth's gravitational field. If its velocity remains unchanged after entering water, [3]

determine the value of h . Given coefficient of viscosity of water = $1.0 \times 10^{-3} \text{ Nsm}^{-2}$, $g = 10 \text{ ms}^{-2}$ and density of water = $1.0 \times 10^3 \text{ kgm}^{-3}$.

23. The coefficient of volume expansion of glycerin is $49 \times 10^{-5} \text{ K}^{-1}$. What is the fractional change in its density for a 30°C rise in temperature? [3]
24. At $t = 0$, a particle is at rest at origin. Its acceleration is 2 m/s^2 for the first 3 s and -2 m/s^2 for next 3s. Plot the acceleration versus time and velocity versus time graph. [3]
25. A ball is suspended by a cord from the ceiling of a motor car. What will be the effect on the position of the ball if [3]
- the car is moving with uniform velocity
 - the car is moving with accelerated motion and
 - the car is turning towards right?
26. State and explain First Law of Thermodynamics. [3]
27. The masses m_1 , m_2 and m_3 of the three bodies shown in figure are 5, 2 and 3 kg respectively. Calculate the values of the tensions T_1 , T_2 and T_3 when [3]
- the whole system is going upward with an acceleration of 2 ms^{-2} and
 - the whole system is stationary. Given $g = 9.8 \text{ ms}^{-2}$.



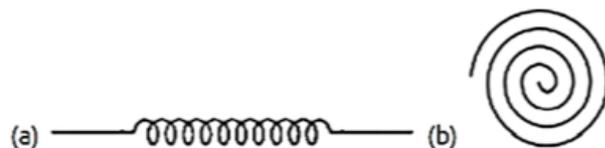
28. A liquid drop of diameter 4mm breaks into 1000 droplets of equal size. Calculate the resultant change in the surface energy. The surface tension of the liquid is 0.07 Nm^{-1} . [3]

OR

A vertical off-shore structure is built to withstand a maximum stress of 10^9 Pa . Is the structure suitable for putting up on top of an oil well in the ocean? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.

Section D

29. **Read the text carefully and answer the questions:** [4]
- There are many types of spring. Important among these are helical and spiral springs as shown in the figure.



Usually, we assume that the springs are massless. Therefore, work done is stored in the spring in the form of the elastic potential energy of the spring. Thus, the potential energy of a spring is the energy associated with the state of compression or expansion of an elastic spring.

- (i) The potential energy of a spring increases in which of the following cases?
- If work is done against conservative force
 - If work is done by non-conservative force

- c) If work is done by conservative force d) If work is done against non-conservative force
- (ii) The potential energy, i.e. $U(x)$ can be assumed zero when
- a) gravitational force is constant b) $x = 0$
- c) infinite distance from the gravitational source d) All of these
- (iii) The ratio of spring constants of two springs is 2 : 3. What is the ratio of their potential energy, if they are stretched by the same force?
- a) 3 : 2 b) 9 : 4
- c) 2 : 3 d) 4 : 9

OR

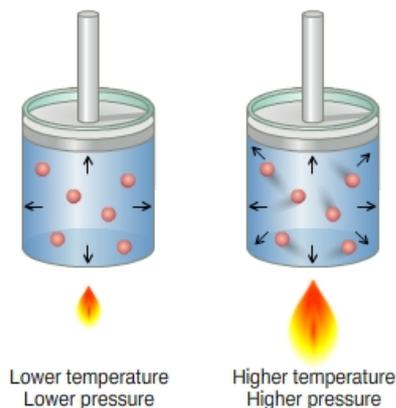
The potential energy of a spring increases by 15 J when stretched by 3 cm. If it is stretched by 4 cm, the increase in potential energy is

- a) 36 J b) 30 J
- c) 27 J d) 33 J
- (iv) The potential energy of a spring when stretched through a distance x is 10 J. What is the amount of work done on the same spring to stretch it through an additional distance x ?
- a) 40 J b) 10 J
- c) 30 J d) 20 J

30. **Read the text carefully and answer the questions:**

[4]

In a gas the particles are always in a state of random motion, all the particles move at different speed constantly colliding and changing their speed and direction, as speed increases it will result in an increase in its kinetic energy.



- (i) If the temperature of the gas increases from 300 K to 600 K then the average kinetic energy becomes:
- a) same b) becomes double
- c) becomes half d) none of these
- (ii) What is the average velocity of the molecules of an ideal gas?
- a) Infinite b) Same
- c) None of these d) Zero

(iii) Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will _____.

- a) decrease
b) none of these
c) increase
d) remains same

(iv) Find the ratio of average kinetic energy per molecule of Oxygen and Hydrogen:

- a) 1:1
b) 4:1
c) 1:2
d) 1:4

OR

The velocities of the three molecules are $3v$, $4v$, and $5v$. calculate their root mean square velocity?

- a) $4.0 v$
b) $4.02 v$
c) $4.08 v$
d) $4.04 v$

Section E

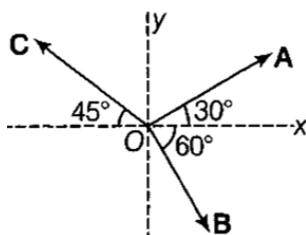
31. Draw a graph to show the variation of P.E., K.E. and total energy of a simple harmonic oscillator with displacement. [5]

OR

Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial ($t = 0$) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).

- a. $x = -2 \sin(3t + \frac{\pi}{3})$
b. $x = \cos(\frac{\pi}{6} - t)$
c. $x = 3 \sin(2\pi t + \frac{\pi}{4})$
d. $x = 2 \cos \pi t$

32. The figure shows three vectors \vec{OA} , \vec{OB} and \vec{OC} which are equal in magnitude (say, F). Determine the direction of $\vec{OA} + \vec{OB} - \vec{OC}$. [5]



OR

- i. Pick out only the vector quantities from the following: Temperature, pressure, impulse, time, power, charge.
ii. Show by drawing a neat diagram that the flight of a bird is an example of composition of vectors.
iii. A man is travelling at 10.8 km h^{-1} in a topless car on a rainy day. He holds his umbrella at an angle 37° to the vertical to protect himself from the rain which is falling vertically downwards. What is the velocity of the rain?
[Given $\cos 37^\circ = \frac{4}{5}$]

33. A metal bar 70 cm long and 4.00 kg in mass supported on two knife-edges placed 10 cm from each end. A 6.00 kg load is suspended at 30 cm from one end. Find the reactions at the knife-edges. (Assume the bar to be of uniform cross-section and homogeneous). [5]

OR

From a uniform disk of radius R , a circular hole of radius $\frac{R}{2}$ is cut out. The centre of the hole is at $\frac{R}{2}$ from the centre of the original disc. Locate the centre of gravity of the resulting flat body.

Solution

Section A

1. (b) deducing relations among the physical quantities.
Explanation: Dimensional analysis is also used to deduce the relation between two or more physical quantities. If we know the degree of dependence of a physical quantity on another, that is the degree to which one quantity changes with the change in another, we can use the principle of consistency of two expressions to find the equation relating these two quantities.

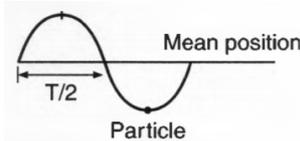
2. (c) Between 0 to 2A
Explanation: Between 0 to 2A

3. (b) 2 : 1
Explanation: $\frac{I_{\text{ring}}}{I_{\text{disc}}} = \frac{MR^2}{\frac{1}{2}MR^2} = 2 : 1$

4. (b) turbulent
Explanation: When any liquid is flowing more than the velocity of its critical velocity then flow becomes turbulent.

5. (b) downward at 2.4 m/s²
Explanation: The acceleration will be downward at 2.4 m/s².

6. (a) $\frac{T}{4}$
Explanation:
It is clear from the figure that the particle will come after a time $\frac{T}{4}$ to its mean position.



7. (c) 10 m/s²
Explanation: $a_{\text{av}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{(-25) - 25}{5} = 10 \text{ m/s}^2$

8. (d) 13 dB
Explanation: $\Delta\beta = \beta_2 - \beta_1$
 $= 10 \log \frac{20I}{I_0} - 10 \log \frac{I}{I_0}$
 $= 10 \times 1.3010 \approx 13 \text{ dB}$

9. (b) 10 g
Explanation: $m = \pi r^2 h \rho$
 $= \pi r^2 \left(\frac{2\sigma \cos \theta}{r \rho g} \right) \rho = \frac{2\pi r \sigma \cos \theta}{g}$
 $\Rightarrow m \propto r$
 $\therefore \frac{m'}{m} = \frac{2r'}{r} = 2$
or $m' = 2m = 2 \times 5g = 10 \text{ g}$

10. (b) 2410 m/sec(perigee),8761 m/sec(apogee)
Explanation: To escape Earth, we need total energy of zero. ($E_{\text{final}} = 0$ because $U \rightarrow 0$ as $R \rightarrow \infty$ and $K \rightarrow 0$ as $v = 0$ at R

→ ∞)

So,

$$K_p + U_p = 0$$

Looking for new velocity at perigee

$$\frac{1}{2}mv_{p,escape}^2 = \frac{GMm}{R_p}$$

$$v_{p,escape} = \sqrt{\frac{2GM}{R_p}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.78 \times 10^6}}$$

$$= 1.084 \times 10^4 \text{ m/sec} = 10840 \text{ m/sec}$$
 Similar calculation at apogee gives

$$v_{a,escape} = \sqrt{\frac{2GM}{R_a}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{10.38 \times 10^6}} = 8.76 \times 10^3 \text{ m/sec} = 8760 \text{ m/sec}$$

If the rocket are fired at perigee,

$$\text{Increase of speed} = v_{p,escape} - v_p = 2.41 \times 10^3 \text{ m/sec} = 2410 \text{ m/sec}$$

If the rocket are fired at apogee,

$$\text{Increase of speed} = v_{a,escape} - v_p = 8.761 \times 10^3 \text{ m/sec} = 8761 \text{ m/sec}$$

11.

(d) 2π s

Explanation: By the conservation of angular momentum,

$$mvR = I\omega_p$$

$$\omega_p = \frac{mvR}{I} = \frac{50 \times 1 \times 2}{200} = 0.5 \text{ rads}$$

$$\omega_m = \frac{v}{R} = \frac{1}{2} = 0.5 \text{ rads}^{-1}$$

Angular velocity of the man relative to the platform,

$$\omega = \omega_m - \omega_p = 0.5 - (-0.5) = 1 \text{ rad s}^{-1}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{1 \text{ rad s}^{-1}} = 2\pi \text{ s}$$

12.

(b) 238°C

Explanation: The base area of the boiler, $A = 0.15 \text{ m}^2$

Thickness of the boiler, $l = 1.0 \text{ cm} = 0.01 \text{ m}$

Boiling rate of water, $R = 6.0 \text{ kg/min}$

Mass, $m = 6 \text{ kg}$

Time, $t = 1 \text{ min} = 60 \text{ s}$

Thermal conductivity of brass, $K = 109 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$

Heat of vaporisation, $L = 2256 \times 10^3 \text{ J kg}^{-1}$

The amount of heat flowing into the water through the brass base of the boiler is

$$Q = \frac{KA(T_1 - T_2)}{l} t \dots\dots(1)$$

where,

T_1 = Temperature of the flame in contact with the boiler

T_2 = Boiling point of water = 100°C

Heat required for boiling the water

$$Q = mL \dots\dots(ii)$$

Equating equations (i) and (ii), we get:

$$mL = \frac{KA(T_1 - T_2)t}{l}$$

$$T_1 - T_2 = \frac{mLl}{KA t}$$

$$= \frac{6 \times 2256 \times 10^3 \times 0.01}{(109 \times 0.15 \times 60)}$$

$$= 137.98^\circ\text{C}$$

Therefore, the temperature of the part of the flame in contact with the boiler is 237.98°C .

13.

(c) A is true but R is false.

Explanation: The earth's crust has many discontinuities and dislocations those are called fault lines. These fault lines in the

earth's crust are like compressed springs. So, the assertion is true.

But the compressed springs possess large amount of potential energy. Hence the reason is false.

14. **(b)** Assertion and reason both are correct statements but reason is not correct explanation for assertion.
Explanation: Assertion and reason both are correct statements but reason is not correct explanation for assertion.
15. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation: According to Newton's universal law of gravitation, everybody in the universe attracts every other body with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
So, this law is an inverse square law since the force is inversely proportional to the square of the distance between the two bodies.
So, the assertion and reason both are true and the reason explains the assertion.
16. **(d)** A is false but R is true.
Explanation: A is false but R is true.

Section B

17. We compare the given wave equation with the standard wave equation,

$$y = A \sin(\omega t + kx + \phi)$$

$$\text{We get } \omega = 100 \text{ rad s}^{-1} \text{ and } k = 20 \text{ rad m}^{-1}$$

$$\text{Speed of the wave, } v = \frac{\omega}{k} = \frac{100}{20} = 5 \text{ ms}^{-1}$$

18. Since dimensionally similar quantities can be added or subtracted, therefore,

$$[P] = \left[\frac{a}{v^2} \right]$$

$$\text{or } [a] = [PV^2] = [ML^{-1} T^{-2}] [L^3]^2 = ML^5 T^{-2}$$

$$\text{Also, } [b] = [V] = L^3$$

The SI unit of a is $\text{kg m}^5 \text{s}^{-2}$ and that of b is m^3 .

19. Let $F = K \eta^a r^b v^c$, then

$$M^1 L^1 T^{-2} = [ML^{-1} T^{-1}]^a [L]^b [LT^{-1}]^c$$

$$= M^a L^{-a+b+c} T^{-a-c}$$

$$\therefore a = 1, -a + b + c = 1, -a - c = -2$$

On solving, $a = b = c = 1$

$$\text{Hence } F = K \eta r v = 6\pi \eta r v \text{ (Stoke's law)}$$

20. Static friction is the force of friction which comes into play between the surfaces of contact of two bodies when there is no relative motion between the bodies inspite of applying an external force on them. Force of static friction is self-adjusting. Its value may vary from zero to limiting friction depending on the value of applied force.

21. In equilibrium $mg = \text{kg}$, $g = \frac{GM}{R^2}$

$$\text{At height } h \text{ } mg' = kx', \quad g' = \frac{GM}{(R+h)^2}$$

$$\frac{g'}{g} = \frac{x'}{x} = \frac{R^2}{(R+h)^2}$$

$$\frac{x'}{x} = \frac{(6400)^2}{(6400+1600)^2} = \frac{6}{25}$$

$$\therefore x = \frac{16}{25} \times 1 \text{ cm} = 0.64 \text{ cm}$$

OR

$$R = 6400 \text{ km} = 6400 \times 10^3 \text{ m}$$

$$h = 1600 \text{ km}$$

$$w = mg = 1 \text{ quintal} = 100 \text{ kg} = 100 \times 9.8 \text{ N} = 980 \text{ N}$$

new weight of body is

$$\text{weight } (w') = mg'$$

gravity at height

$$g' = g \left(\frac{R}{R+h} \right)^2$$

$$w' = 100 \times 9.8 \left(\frac{6400}{1600+6400} \right)^2$$

$$w' = 100 \times 9.8 \text{N} \times 0.64 = 627.2 \text{N}$$

$$w' = 627.2/9.8 = 64 \text{kg}$$

Section C

22. The velocity attained by the sphere after falling freely from height h is

$$v = \sqrt{2gh} \dots (i)$$

After entering water, the velocity of the sphere does not change. So v is also the terminal velocity of the sphere. Hence

$$v = \frac{2}{9} \frac{r^2}{\eta} (\rho - \rho') g$$

$$\text{But } \rho = 10^4 \text{ kgm}^{-3}, \rho' = 10^3 \text{ kgm}^{-3}, r = 10^{-3} \text{m}, g = 10 \text{ ms}^{-2}, \eta = 10^{-3} \text{ Nsm}^{-2}$$

$$\therefore v = \frac{2}{9} \times \frac{(10^{-3})^2 \times (10^4 - 10^3) \times 10}{10^{-3}} = 20 \text{ ms}^{-1}$$

$$\text{From (i), } h = \frac{v^2}{2g} = \frac{20 \times 20}{2 \times 10} = 20 \text{ m}$$

23. Coefficient of volume expansion of glycerin, $\alpha_V = 49 \times 10^{-5} \text{K}^{-1}$

Rise in temperature, $\Delta T = 30^\circ \text{C}$

$$\text{Fractional change in its volume} = \frac{\Delta V}{V}$$

This change is related with the change in temperature as:

$$\frac{\Delta V}{V} = \alpha_V \Delta T$$

$$\Rightarrow \frac{V_{T_2} - V_{T_1}}{V_{T_1}} = \alpha_V \Delta T \quad (V_{T_2} \text{ and } V_{T_1} \text{ are the volumes of glycerin at absolute temperatures } T_2 \text{ and } T_1 \text{ respectively})$$

$$\therefore V_{T_2} - V_{T_1} = V_{T_1} \times \alpha_V \Delta T$$

$$\Rightarrow \frac{m}{\rho_{T_2}} - \frac{m}{\rho_{T_1}} = \frac{m}{\rho_{T_1}} \times \alpha_V \Delta T$$

Where,

m = Mass of glycerine

ρ_{T_1} = Initial density at T_1 when volume was V_{T_1}

ρ_{T_2} = Final density at T_2 when volume becomes V_{T_2}

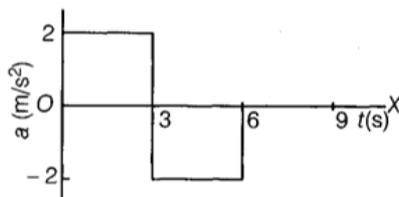
$$\therefore \frac{\rho_{T_1} - \rho_{T_2}}{\rho_{T_2}} = \alpha_V \Delta T$$

Where,

$$\frac{\rho_{T_1} - \rho_{T_2}}{\rho_{T_2}} = \text{Fractional change in density}$$

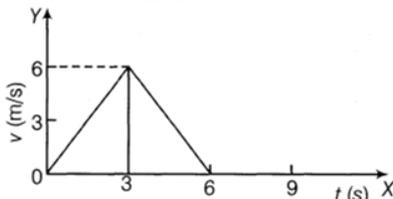
$$\therefore \text{Fractional change in the density of glycerin} = 49 \times 10^{-5} \times 30 = 1.47 \times 10^{-2}$$

24. The acceleration -time graph is



The area enclosed between a - t curve gives change in velocity for the corresponding interval.

At $t = 0$, $v = 0$, hence final velocity at $t = 3$ s will increase to 6 m/s. In next 3 s, the velocity will decrease to zero. Thus, the velocity-time graph is



25. i. The ball will remain suspended vertically.

ii. The ball will move in backward direction.

iii. The ball will move towards left.

26. According to First Law of Thermodynamics, if an amount of heat ΔQ is supplied to a thermodynamic system, a part of it may increase the internal energy of the system by ΔU and the remaining part is used up as the external work done ΔW by the system.

$$\text{Thus, we have } \Delta Q = \Delta U + \Delta W$$

First Law of Thermodynamics follows the conservation law of energy and establishes an exact relation between heat transferred and mechanical work done. It provides a valuable concept of internal energy. It is applicable to every process in nature and to all the three states of matter i.e., solid, liquid and gases. Moreover, change in internal energy of a system may be due to any cause like change in translational or rotational or vibrational kinetic energy or molecular potential energy etc.

27. i. The three bodies together are moving upward with an acceleration of 2 ms^{-2} . The force pulling the system upward is T_1 and the downward force of gravity is $(m_1 + m_2 + m_3)g$.

According to Newton's second law,

$$T_1 - (m_1 + m_2 + m_3)g = (m_1 + m_2 + m_3)a$$

$$\text{or } T_1 = (m_1 + m_2 + m_3)(a + g)$$

$$= (5 + 2 + 3)(2 + 9.8) = 10 \times 11.8$$

$$= 118 \text{ N}$$

Similarly, for the motion of the system $m_2 + m_3$, we can write

$$T_2 = (m_2 + m_3)(a + g)$$

$$= (2 + 3)(2 + 9.8) = 5 \times 11.8$$

$$= 59 \text{ N}$$

For the motion of body of mass m_3 , we have

$$T_3 = m_3(a + g) = 3(2 + 9.8)$$

$$= 35.4 \text{ N}$$

- ii. When the whole system is stationary, $a = 0$. From the above equations, we get

$$T_1 - (m_1 + m_2 + m_3)g = 10 \times 9.8 = 98 \text{ N}$$

$$T_2 = (m_2 + m_3)g = 5 \times 9.8 = 49 \text{ N}$$

$$T_3 = m_3g = 3 \times 9.8 = 29.4 \text{ N}$$

28. Since the diameter of drop = 4mm

$$\text{Radius of drop} = 2\text{mm} = 2 \times 10^{-3}\text{m}$$

$$S = \text{Surface tension} = 0.07 \frac{\text{N}}{\text{m}}$$

Let r be the radius of each of the small droplets volume of big drop = 1000 \times volume of the small droplets

$$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$$

$$\text{or } R = 10r$$

$$\Rightarrow r = \frac{2 \times 10^{-3}}{10} = 2 \times 10^{-4}\text{m}$$

$$\text{original surface area of the drop} = 4\pi R^2$$

$$\text{Total surface area of} = 1000 \times 4\pi r^2 - 4\pi R^2$$

$$= 4\pi [1000 r^2 - R^2]$$

$$\text{Increase in surface} = 4 \times \frac{22}{7} [(1000 \times (2 \times 10^{-4})^2) - [(2 \times 10^{-3})^2]]$$

$$= 4 \times \frac{22}{7} [1000 \times 4 \times 10^{-8} - 4 \times 10^{-6}]$$

$$= 8 \times \frac{22}{7} [10^{-5} - 10^{-6}]$$

$$= \frac{3168}{7} \times 10^{-5}\text{m}^2$$

Increase in surface energy = Surface tension \times Increase in surface area

$$= 0.07 \times \frac{3168}{7} \times 10^{-5}$$

$$= 3168 \times 10^{-8} \text{ J}$$

OR

The maximum allowable stress for the structure is, $P = 10^9 \text{ Pa}$

Depth of the ocean is given by, $d = 3 \text{ km} = 3 \times 10^3 \text{ m}$

Density of water is given by, $\rho = 10^3 \frac{\text{kg}}{\text{m}^3}$

Acceleration due to gravity is, $g = 9.8 \frac{\text{m}}{\text{s}^2}$

The pressure exerted at depth, $d = \rho dg$

$$= 3 \times 10^3 \times 10^3 \times 9.8$$

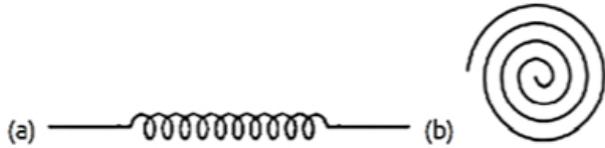
$= 2.94 \times 10^7 \text{ Pa}$

The maximum allowable stress for the structure (10^9 Pa) is greater than the pressure of the sea water ($2.94 \times 10^7 \text{ Pa}$). The pressure exerted by the ocean is less than the pressure that the structure can withstand. Hence, the structure is suitable for putting up on top of an oil well in the ocean.

Section D

29. Read the text carefully and answer the questions:

There are many types of spring. Important among these are helical and spiral springs as shown in the figure.



Usually, we assume that the springs are massless. Therefore, work done is stored in the spring in the form of the elastic potential energy of the spring. Thus, the potential energy of a spring is the energy associated with the state of compression or expansion of an elastic spring.

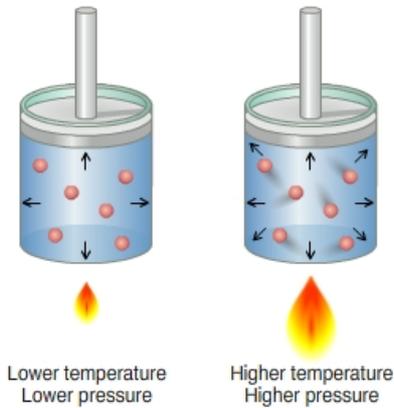
- (i) **(a)** If work is done against conservative force
Explanation: If work is done against conservative force
- (ii) **(d)** All of these
Explanation: All of these
- (iii) **(a)** 3 : 2
Explanation: 3 : 2

OR

- (c)** 27 J
Explanation: 27 J
- (iv) **(c)** 30 J
Explanation: 30 J

30. Read the text carefully and answer the questions:

In a gas the particles are always in a state of random motion, all the particles move at different speed constantly colliding and changing their speed and direction, as speed increases it will result in an increase in its kinetic energy.



- (i) **(b)** becomes double
Explanation: becomes double
- (ii) **(d)** Zero
Explanation: Zero
- (iii) **(d)** remains same
Explanation: remains same
- (iv) **(a)** 1:1
Explanation: 1:1

OR

- (c)** 4.08 v
Explanation: 4.08 v

Section E

31. The potential energy (PE) of a simple harmonic oscillator is

$$PE = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2 \dots (i)$$

When, PE is plotted against displacement x, we will obtain a parabola.

When $x = 0$, $PE = 0$

When $x = \pm A$, $PE = \text{maximum}$

$$= \frac{1}{2}m\omega^2 A^2$$

The kinetic energy (KE) of a simple harmonic oscillator $KE = \frac{1}{2}mv^2$

But velocity of oscillator $v = \omega\sqrt{A^2 - x^2}$

$$\Rightarrow KE = \frac{1}{2}m[\omega\sqrt{A^2 - x^2}]^2$$

$$\text{or } KE = \frac{1}{2}m\omega^2 (A^2 - x^2) \dots (ii)$$

This is also parabola, if we plot KE against displacement x

i.e. $KE = 0$ at $x = \pm A$

and $KE = \frac{1}{2}m\omega^2 A^2$ at $x = 0$

Now, total energy of the simple harmonic oscillator = PE + KE [using Eqs. (i) and (ii)]

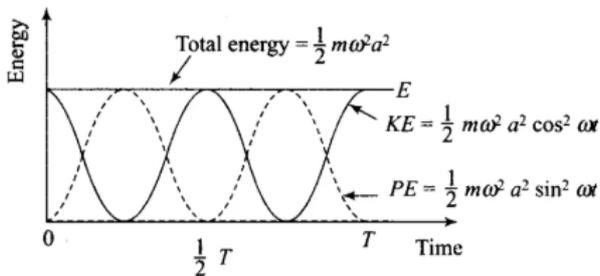
$$= \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m\omega^2 (A^2 - x^2)$$

$$= \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m\omega^2 A^2 - \frac{1}{2}m\omega^2 x^2$$

$$TE = \frac{1}{2}m\omega^2 A^2 = \text{constant}$$

which is a constant and independent of x.

Plotting under the above guidelines KE, PE and TE versus displacement x-graph as follows:



Important point: From the graph, we note that potential energy or kinetic energy completes two vibrations in a time during which S.H.M. completes one vibration.

Thus the frequency of potential energy or kinetic energy is double that of S.H.M.

OR

a. $x = 2 \cos\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right)$

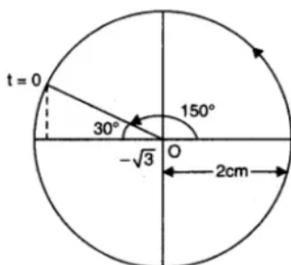
Radius of the reference circle, $r = \text{amplitude of SHM} = 2 \text{ cm}$,

At $t = 0$, $x = -2 \sin \frac{\pi}{3} = \frac{-2\sqrt{3}}{2} = -\sqrt{3} \text{ cm}$

Also $\omega t = 3t$, $\therefore \omega = 3 \text{ rad/s}$

$\cos \phi_0 = -\frac{\sqrt{3}}{2}$, $\phi_0 = 150^\circ$

The reference circle is, thus, as plotted below.



b. $x = \cos\left(t - \frac{\pi}{6}\right)$

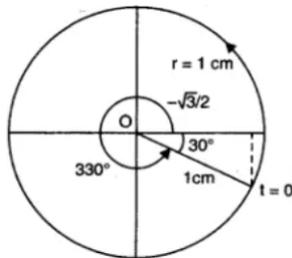
Radius of circle, $r = \text{amplitude of SHM} = 1 \text{ cm}$.

At $t = 0$, $x = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \text{ cm}$

Also $\omega t = 1t \Rightarrow \omega = 1 \text{ rad/s}$

$\cos \phi_0 = \frac{\sqrt{3}}{2}$, $\phi_0 = -\frac{\pi}{6}$

The reference circle is, thus as plotted below.



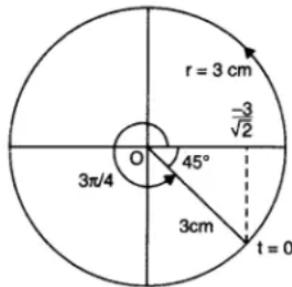
$$c. x = 3 \cos\left(2\pi t + \frac{\pi}{4} + \frac{\pi}{2}\right)$$

Here, radius of reference circle, $r = 3 \text{ cm}$ and at $t = 0$, $x = 3 \sin \frac{\pi}{4} = \frac{\sqrt{3}}{2} \text{ cm}$

$$\omega t = 2\pi t \Rightarrow \omega = 2\pi \text{ rad/s}$$

$$\cos \phi_0 = \frac{\frac{\sqrt{3}}{2}}{3} = -\frac{1}{\sqrt{2}}$$

Therefore, the reference circle is being shown below.



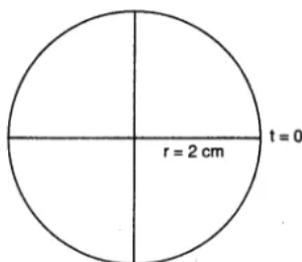
$$d. x = 2 \cos \pi t$$

Radius of reference circle, $r = 2 \text{ cm}$ and at $t = 0$, $x = 2 \text{ cm}$

$$\therefore \omega t = \pi t \text{ or } \omega = \pi \text{ rad/s}$$

$$\cos \phi_0 = 1, \phi_0 = 0$$

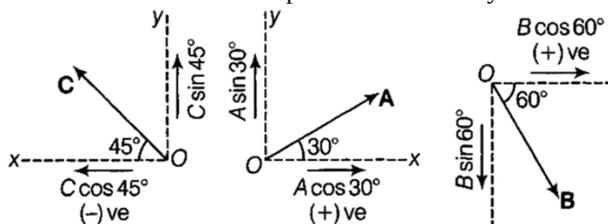
The reference circle is plotted below.



$$32. \text{ Given, } |\vec{OA}| = |\vec{OB}| = |\vec{OC}| = F$$

Angles are 30° , 45° and 60° .

Resolve all the vector components individually



$$R_x = R_{x_1} + R_{x_2} + R_{x_3}$$

Sum of vectors in x-direction (i.e. R_x) and sum of vectors in y-direction (i.e. R_y)

$$R_x = A \cos 30^\circ + B \cos 60^\circ - C \cos 45^\circ$$

$$= \frac{F\sqrt{3}}{2} + \frac{F}{2} - \frac{F}{\sqrt{2}} \quad [\because A = B = C = F]$$

$$= \frac{F}{2\sqrt{2}}(\sqrt{6} + \sqrt{2} - 2)$$

$$R_y = A \sin 30^\circ + C \cos 45^\circ - B \sin 60^\circ$$

$$= \frac{F}{2} + \frac{F}{\sqrt{2}} - \frac{F\sqrt{3}}{2} = \frac{F}{2\sqrt{2}}(\sqrt{2} + 2 - \sqrt{6})$$

Determination of magnitude,

$$\begin{aligned}
 R &= \sqrt{R_x^2 + R_y^2} \\
 &= \sqrt{\left[\frac{F}{2\sqrt{2}}(\sqrt{6} + \sqrt{2} - 2)\right]^2 + \left[\frac{F}{2\sqrt{2}}(\sqrt{2} + 2 - \sqrt{6})\right]^2} \\
 &= \sqrt{F^2(0.435) + F^2(0.116)} \\
 &= \sqrt{F^2(0.550)} = F\sqrt{0.550} \\
 &\Rightarrow R = 0.74 F
 \end{aligned}$$

Determination of direction

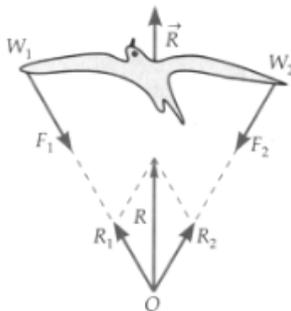
$$\begin{aligned}
 \tan \theta &= \frac{R_y}{R_x} = \frac{\frac{F}{2\sqrt{2}}(\sqrt{2}+2-\sqrt{6})}{\frac{F}{2\sqrt{2}}(\sqrt{6}+\sqrt{2}-2)} \\
 &= \frac{0.97}{1.85} \approx 0.524 \\
 \theta &\approx 27.65
 \end{aligned}$$

This is the angle which R makes with x-axis.

OR

i. Impulse

ii. Flight of a bird. When a bird flies, it pushes the air with forces F_1 and F_2 in the downward direction with its wings W_1 and W_2 . The lines of action of these two forces meet at point O. In accordance with Newton's third law of motion, the air exerts equal and opposite reactions R_1 and R_2 . According to the parallelogram law, the resultant R of the reactions R_1 and R_2 acts on the bird in the upward direction and helps the bird to fly upward.



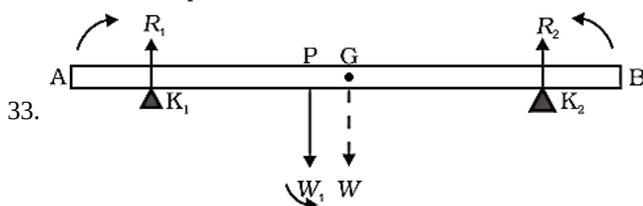
Flight of a bird.

iii. $v_R = 10.8 \text{ km h}^{-1} = 3 \text{ ms}^{-1}$

$$\text{Given: } \cos 37^\circ = \frac{4}{5} \therefore \tan 37^\circ = \frac{3}{4}$$

$$\text{But } \tan 37^\circ = \frac{v_R}{v_M} \text{ or } \frac{3}{4} = \frac{v_R}{3 \text{ ms}^{-1}}$$

$$\text{or } v_R = \frac{9}{4} = 2.25 \text{ ms}^{-1}$$



The figure shows the rod AB, the positions of the knife edges K_1 and K_2 , the centre of gravity of the rod at G, and the suspended load at P.

Note the weight of the rod W acts at its centre of gravity G. The rod is uniform in cross-section and homogeneous; hence G is at the centre of the rod; $AB = 70 \text{ cm}$, $AG = 35 \text{ cm}$, $AP = 30 \text{ cm}$, $PG = 5 \text{ cm}$, $AK_1 = BK_2 = 10 \text{ cm}$ and $K_1G = K_2G = 25 \text{ cm}$. Also,

$W = \text{weight of the rod} = 4.00 \text{ kg}$ and $W_1 = \text{suspended load} = 6.00 \text{ kg}$;

R_1 and R_2 are the normal reactions of the support at the knife edges.

For translational equilibrium of the rod,

$$R_1 + R_2 - W_1 - W = 0 \dots (i)$$

Note W_1 and W act vertically down and R_1 and R_2 act vertically up.

For considering rotational equilibrium, we take moments of the forces. A convenient point to take moments about is G. The moments of R_2 and W_1 are anticlockwise (+ve), whereas the moment of R_1 is clockwise (-ve).

For rotational equilibrium,

$$-R_1(K_1G) + W_1(PG) + R_2(K_2G) = 0 \dots (ii)$$

It is given that $W = 4.00g$ N and $W_1 = 6.00g$

N, where $g =$ acceleration due to gravity. We take $g = 9.8 \text{ m/s}^2$.

With numerical values inserted, from (i)

$$R_1 + R_2 - 4.00g - 6.00g = 0$$

$$\text{or } R_1 + R_2 = 10.00g \text{ N} \dots (iii)$$

$$= 98.00 \text{ N}$$

$$\text{From (ii), } -0.25 R_1 + 0.05 W_1 + 0.25 R_2 = 0$$

$$\text{or } R_1 - R_2 = 1.2g \text{ N} = 11.76 \text{ N} \dots (iv)$$

From (iii) and (iv), $R_1 = 54.88 \text{ N}$,

$$R_2 = 43.12 \text{ N}$$

Thus the reactions of the support are about 55 N at K_1 and 43 N at K_2 .

OR

The centre of mass of an object is the point at which the object can be balanced. Mathematically, it is the point at which the torques from the mass elements of an object sum to zero. The centre of mass is useful because problems can often be simplified by treating a collection of masses as one mass at their common centre of mass. The weight of the object then acts through this point.

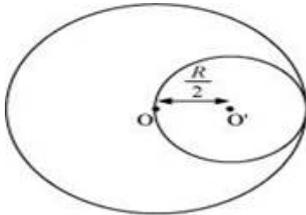
To solve this problem, first we assume that the whole disc was present whose centre of mass lies at the origin from which a small disc was cut out. So CM of remaining portion and cut out disc will lie exactly at the origin i.e Centre of Mass of the original disc at $x = 0$

Mass per unit area of the original disc = σ

Radius of the original disc = R

$$\text{Mass of the original disc, } M = \pi R^2 \sigma$$

The disc with the cut portion is shown in the following figure:



$$\text{Radius of the smaller disc} = \frac{R}{2}$$

$$\text{Mass of the smaller disc, } M' = \pi \left(\frac{R}{2}\right)^2 \sigma = \frac{1}{4} \pi R^2 \sigma = \frac{M}{4}$$

Let O and O' be the respective centers of the original disc and the disc cut off from the original. As per the definition of the centre of mass, the centre of mass of the original disc is supposed to be concentrated at O , while that of the smaller disc is supposed to be concentrated at O' .

It is given that:

$$OO' = \frac{R}{2}$$

After the smaller disc has been cut from the original, the remaining portion is considered to be a system of two masses. The two masses are:

M (concentrated at O), and

$$\left(-M' = \frac{M}{4}\right) \text{ concentrated at } O'$$

(The negative sign indicates that this portion has been removed from the original disc.)

Let x be the distance through which the centre of mass of the remaining portion shifts from point O .

The relation between the centers of masses of two masses is given as:

$$x = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

For the given system

$$x = \frac{M \times 0 - M \times \left(\frac{R}{2}\right)}{M + (-M')} \text{ (here } M' \text{ is } M/4)$$

$$= \frac{\frac{-M}{4} \times \frac{R}{2}}{M - \frac{M}{4}} = \frac{-MR}{8} \times \frac{4}{3M} = \frac{-R}{6}$$

Note that shift in Centre of Mass is very less(only $0.16 R$ or $\frac{R}{6}$) as removed portion has very less mass as compared to the remaining portion.

(The negative sign indicates that the centre of mass gets shifted toward the left of point O and lies at $\frac{R}{6}$ left towards origin.)