CLASS-10+1 CH-7 PERMUTATIONS & COMBINATIONS
EX-7.3 BOOK-NCERT TOPIC- Permutations when all the objects are distinct/not distinct objects

Permutations when all the objects are distinct

$0! = 1$

$= {}^{n}P_r = \frac{n!}{(n-r)!}$

e.g. ZIRA

$n=4 \quad r=4$

${}^{4}P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{24}{1} = 24$

Permutations when all the objects are not distinct objects

1. Google

$= \frac{n!}{P!\,Q!\,r!}$

2. ROSE → 4! → 24
ROES
RSOE
RSEO
RESO
REOS

e.g. Google

$n = 6$
no of O's = 2
G's = 2

$= \frac{6!}{2! \times 2!} = \frac{720}{2 \times 2} = 180$

EXERCISE 7.3

1. How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

Sol

Total no. of digits given = 9 (i.e. 1 to 9)

$n = 9$

FPC

$7 \times 8 \times 9$

we are to form (3) digit no. if no digit is repeated

$r = 3$

$n P_1$

$9P_3 \quad \frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{9 \times 8 \times 7 \times 6!}{6!} = 72 \times 7 = 504$

2. How many 4-digit numbers are there with no digit repeated?

Sol

Total no. of digits = 10 (i.e. 0 to 9)

$9 \times 9 \times 8 \times 7$

$9 \times$

Thousands place can be filled in 9 ways (0 exclude)
Hundreds place " " " 9 (ways) (0 include)
Tens place " " " 8 ways
Unit place " " " 7 ways

By FPC

$= 9 \times 9 \times 8 \times 7$
$= 81 \times 56$
$= 4536$

$81 \times 56 = 486 + 4050 = 4536$

0312

3. How many 3-digit even numbers can be made using the digits 1, 2, 3, 4, 6, 7, if no digit is repeated?

4. Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digit is repeated. How many of these will be even?

5. From a committee of 8 persons, in how many ways can we choose a chairman and a vice chairman assuming one person can not hold more than one position?

Sol Total no. of person $= 8$

$n = 8 \quad r = 2$

$$nP_r = {}^8P_2 = \frac{8!}{(8-2)!} = \frac{8 \times 7 \times 6!}{6!} = 56$$

6. Find n if ${}^{n-1}P_3 : {}^nP_4 = 1 : 9$.

7. Find r if (i) ${}^5P_r = 2\,{}^6P_{r-1}$ (ii) ${}^5P_r = {}^6P_{r-1}$.

Ans 6 $\quad {}^{n-1}P_3 : {}^nP_4 = 1 : 9$ — (1)

$$nP_r = \frac{n!}{(n-r)!}$$

$${}^{n-1}P_3 = \frac{(n-1)!}{(n-1-3)!} = \frac{(n-1)!}{(n-4)!} = \frac{(n-1)(n-2)(n-3)(n-4)!}{(n-4)!}$$

$$= (n-1)(n-2)(n-3)$$

$${}^nP_4 = \frac{n!}{(n-4)!} = \frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!} = n(n-1)(n-2)(n-3)$$

Put in (1)

$$\frac{(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)} = \frac{1}{9}$$

$$\frac{1}{n} = \frac{1}{9}$$

$$\boxed{n = 9}$$

(7) (ii) $\quad {}^5P_r = {}^6P_{r-1}$

$$\frac{5!}{(5-r)!} = \frac{6!}{(6-(r-1))!} = \frac{6!}{(6-r+1)!} = \frac{6!}{(7-r)!}$$

$$\frac{5!}{(5-r)!} = \frac{6 \times 5!}{(7-r)(6-r)(5-r)!}$$

$$1 = \frac{6}{(7-r)(6-r)}$$

$$(7-r)(6-r) = 6$$

$$7(6-r) - r(6-r) = 6$$

$$42 - 7r - 6r + r^2 = 6$$

$$42 - 13r + r^2 - 6 = 0$$

$$r^2 - 13r + 36 = 0$$

$$r^2 - 9r - 4r + 36 = 0$$

$$r(r-9) - 4(r-9) = 0$$

$$(r-9)(r-4) = 0$$

$$r-9 = 0 \quad \text{or} \quad r-4 = 0$$

$$\boxed{r = 9} \quad \text{or} \quad \boxed{r = 4}$$

8. How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?

Sol $n = 8 \quad r = 8$

$= {}^{8}P_{8} = \frac{8!}{(8-8)!} = \frac{8!}{0!} = \frac{8!}{1} = 8! = 40320$

9. How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if.

(i) 4 letters are used at a time, (ii) all letters are used at a time,

(iii) all letters are used but first letter is a vowel?

Sol. MONDAY

(i) $n=6 \quad r=4$

$= {}^6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6\times5\times4\times3\times2!}{2!}$

$= 360$

(ii) $n=6 \quad r=6$

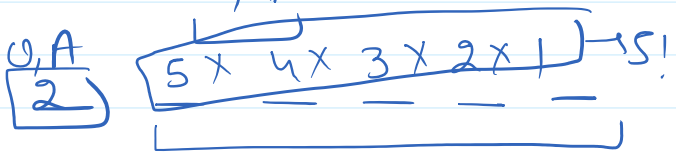
$= {}^6P_6 = 6! = 720$

$n=5 \quad r=5$

${}^5P_5 = 5!$

(iii) Vowel = O, A

$6-1=5$



$= 2 \times 5!$

$= 2 \times 120 = 240$

10. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Sol Total letters = 11

no. of I's = 4
S's = 4
P's = 2

Total arrangements in which four I's are together & not together

$$= \frac{11!}{4!\,4!\,2!}$$

$$= \frac{11\times10\times\cancel{9}^{3}\times\cancel{8}^{2}\times7\times\cancel{6}^{3}\times5\times\cancel{4!}}{\cancel{4!}\times\cancel{4}\times3\times2\times1\times\cancel{2}\times1}$$

$315 \times 11 = 3465$

$$= 11\times10\times3\times7\times3\times5$$

$$= 34650$$

Total no. of arrangements in which four I's comes together

I I I I _ _ _ _ _ _ _ _
1 2 3 4 5 6 7 8

Consider these four I's as one letter

n = 8

$$= 8! \times \frac{4!}{4!} = \frac{\overset{4}{\cancel{8}}\times7\times6\times5\times\cancel{4!}}{\cancel{4!}\times2}$$

$4! \times 2!$

$$= 4\times7\times6\times5$$

$$= 28\times30 = 840$$

Total no. of arrangements in which four I's do not come together

$= 34650 - 840$

$= 33810$

11. In how many ways can the letters of the word PERMUTATIONS be arranged if the

(i) words start with P and end with S, (ii) vowels are all together,

(iii) there are always 4 letters between P and S?