ICSE 2025 EXAMINATION

Sample Question Paper - 2

Mathematics

Time: 2 ½ Hours **Total Marks: 80**

General Instructions:

1. *Answers to this Paper must be written on the paper provided separately.*
2. *You will not be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.*
3. *The time given at the head of this Paper is the time allowed for writing the answers.*
4. *Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*
5. *The intended marks for questions or parts of questions are given in brackets [ ]*

Section A

(Attempt all questions from this section.)

Question 1

Choose the correct answers to the questions from the given options. [15]

i) $\sqrt{8}\left(\sqrt{8}-1\right)$ is always

(a) a rational number
(b) an irrational number
(c) a whole number
(d) a natural number

ii) On a certain sum, the compound interest accrued in one year is Rs. 550. If the rate of interest is 10%, the sum is

(a) Rs. 5000
(b) Rs. 6000
(c) Rs. 4500
(d) Rs. 5500

iii) If $a^2+\frac{1}{a^2}=18$ and $a \neq 0$; then $a-\frac{1}{a}$ is equal to

(a) $2\sqrt{5}$
(b) 4
(c) $3\sqrt{2}$
(d) 16

iv) The value of $\frac{27+8x^3}{9-4x^2}$ is

(a) $\frac{9+4x^2+6x}{3-2x}$

(b) $\frac{9+4x^2+12x}{3-2x}$

(c) $\frac{9+4x^2-6x}{3-2x}$

(d) $\frac{9+4x^2-12x}{3-2x}$

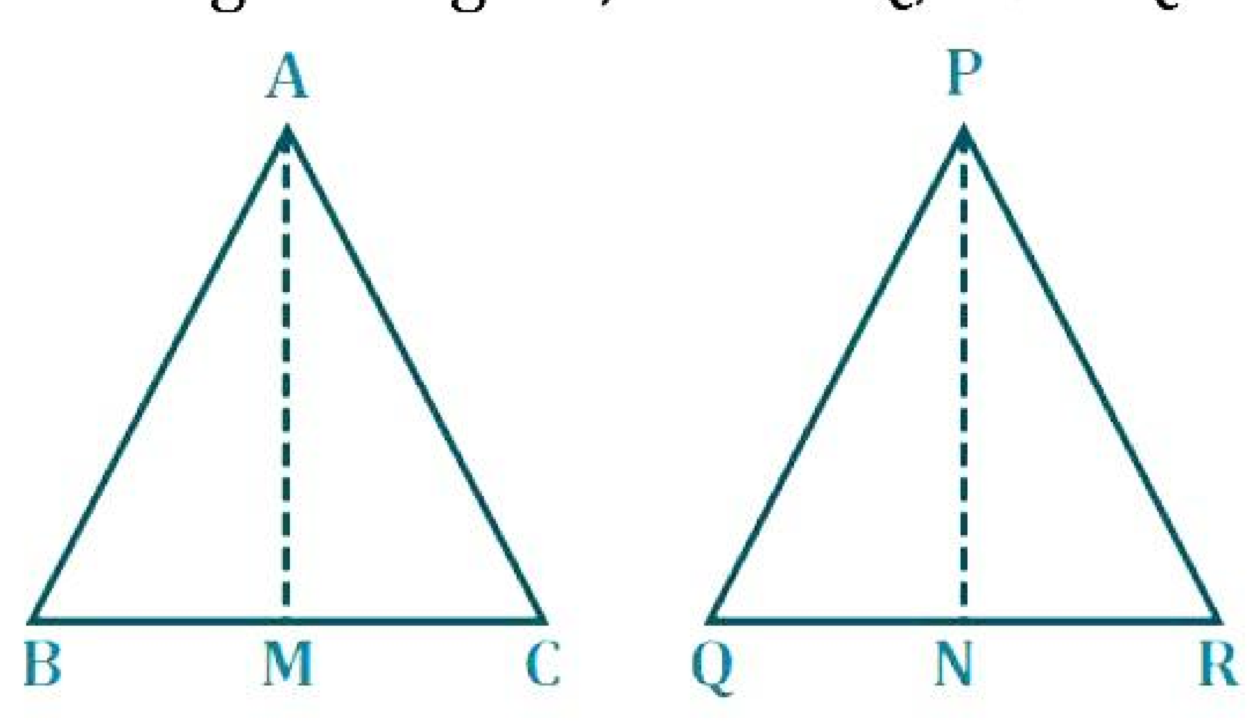
v) Solution of the equations 11x – 7y = 29 and 7x – 11y = 25 is

(a) x = 2, y = 1
(b) x = 2, y = −1
(c) x = −2, y = 1
(d) x = −2, y = −1

vi) The value of $3^3 \times (243)^{-\frac{2}{3}} \times 9^{-\frac{1}{3}}$ is

(a) $3^{-\frac{2}{3}}$
(b) 3
(c) 3^{-1}
(d) 3^7

vii) In the given figure, AB = PQ, BC = QR and median AM = median PN, then



Statement 1: ΔABM and ΔPQN are not congruent.
Statement 2: ΔABC and ΔPQR are congruent.
Which of the following is valid?
(a) Both the statements are true.
(b) Both the statements are false.
(c) Statement 1 is true, and Statement 2 is false.
(d) Statement 1 is false, and Statement 2 is true.

viii) In a rhombus, its diagonals are 30 cm and 40 cm, its perimeter is
(a) 20 cm
(b) 10 cm
(c) 60 cm
(d) 100 cm

ix) In a circle, O is its centre and AB, CD are its two chords. If AB : CD = 3 : 2, then the ratio between ∠AOB and ∠COD is
(a) 1 : 1
(b) 3 : 2
(c) 2 : 5
(d) 3 : 5

x) The median of 80 observations is 50. If each observation is doubled, the resulting median will be
(a) 50
(b) 30
(c) 100
(d) 130

xi) If the mean of a_1 and a_2 is 13 and mean of a_1, a_2 and a_3 is 17. The value of a_3 is
(a) 25
(b) 8
(c) 51/26
(d) 4

xii) The area of a square on the diagonal of a cube is 48 cm^2. Each edge of the cube is
(a) $2\sqrt{6}$ cm
(b) $4\sqrt{3}$ cm
(c) 4 cm
(d) $\sqrt{3}$ cm

xiii) If $\cot A = \sqrt{5}$, the value of $\operatorname{cosec}^2 A - \sec^2 A$ is
(a) $\frac{5}{24}$
(b) $4\frac{4}{5}$
(c) 5
(d) 24

xiv) Lines 4x = 1 and y – 5 = 0 intersect each other at point P. The coordinates of point P are

(a) $\left(\frac{1}{4}, 5\right)$

(b) $\left(-\frac{1}{3}, 5\right)$

(c) $\left(5, \frac{1}{3}\right)$

(d) $\left(5, -\frac{1}{3}\right)$

xv) **Assertion (A):** The distance between the points (–3, 2) and (x, 10) is 10 units. The value of x is 3 or –9.

Reason (R): Distance between any two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(a) A is true, R is false

(b) A is false, R is true

(c) Both A and R are true, and R is the correct reason for A.

(d) Both A and R are true, and R is the incorrect reason for A.

Question 2

i) A sum of money, at compound interest, amounts to Rs. 9100 in 5 years and to Rs. 9828 in 6 years. Find the following without using formula, [4]

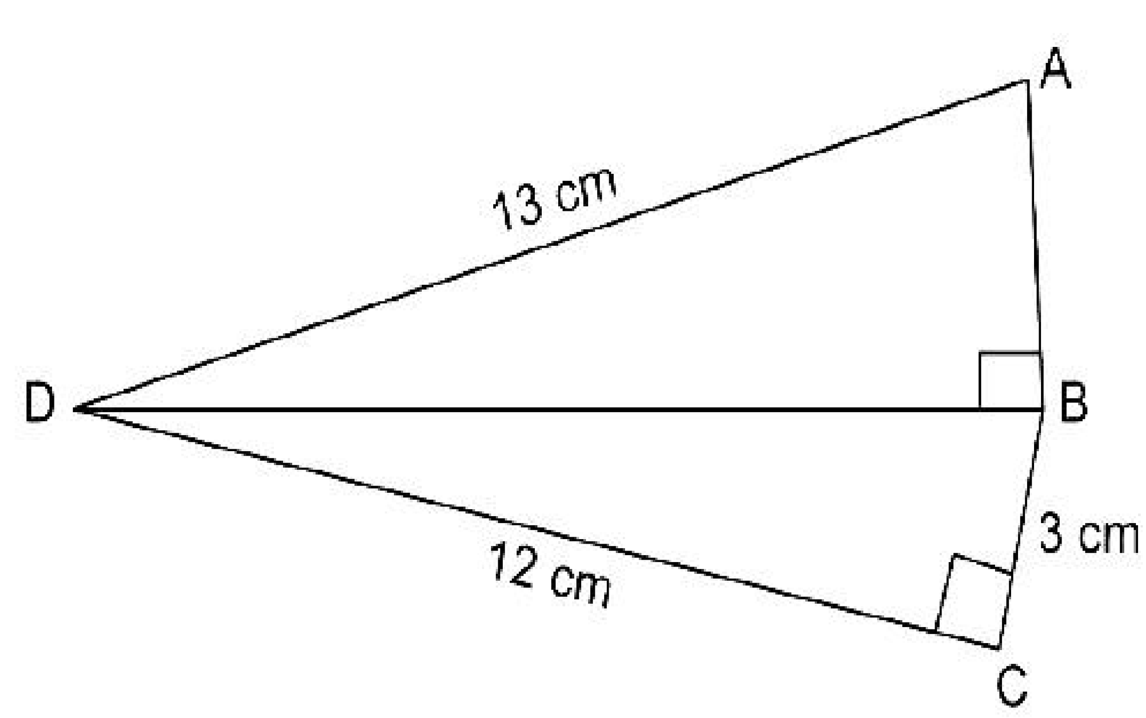
A. rate per cent

B. amount in 7 years

C. amount in 4 years

ii) The sum of the digits of a two-digit number is 12. If the digits are reversed, the new number is 12 less than twice the original number. Find the number. [4]

iii) In the given figure, ABCD is a quadrilateral in which BC = 3 cm, AD = 13 cm, DC = 12 cm and $\angle ABD = \angle BCD = 90^0$. Find the length of AB. [4]



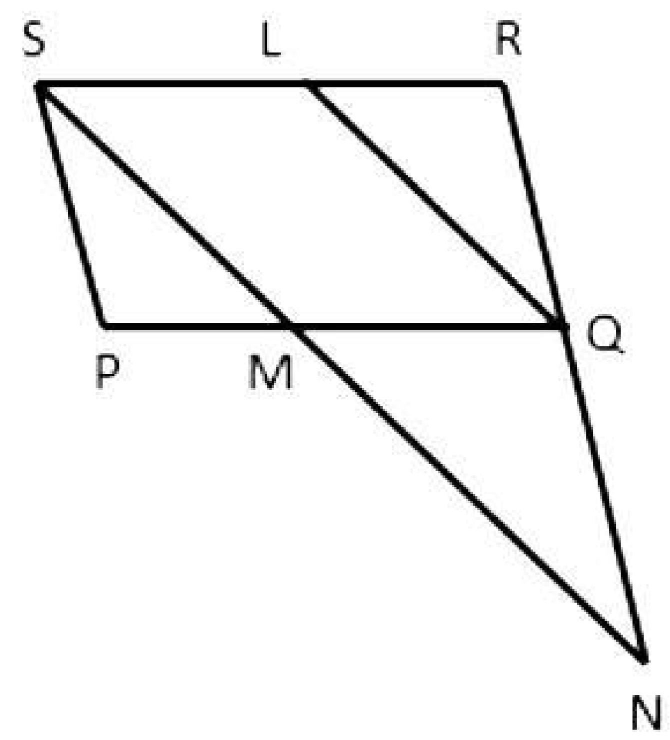
Question 3

i) In a parallelogram PQRS, L is the mid-point of side SR, and SN is drawn parallel to LQ which meets RQ produced at N and cuts side PQ at M. [4]

Prove that:

A. SP = ½ RN

B. SN = 2LQ



ii) In a quadrilateral ABCD, AO and BO are the bisectors of ∠A and ∠B, respectively. Prove that ∠AOB = ½ (∠C + ∠D). [4]

iii) Factorise: [5]

A. $9x^2 - (x^2 - 4)^2$

B. $(x + 1)^6 - (x - 1)^6$

Section B

(Attempt any four questions from this Section.)

Question 4

i) If $x = 2 - \sqrt{3}$, find the value of $\left(x + \frac{1}{x}\right)^3$. [3]

ii) Calculate the amount and the compound interest on Rs. 125000 for $1\frac{1}{2}$ years at the rate of 12% per annum compounded half-yearly. [3]

iii) AB and CD are two chords of a circle with radius r. AB = 2CD and the perpendicular distance of CD from the centre is twice the perpendicular distance of AB from the centre.

Prove that $r = \frac{\sqrt{5}}{2}CD$. [4]

Question 5

i) If $\left(x^2 + \frac{1}{25x^2}\right) = 9\frac{2}{5}$, find the value of $\left(x - \frac{1}{5x}\right)$. [3]

ii) Factorise: $x^2 + \frac{1}{x^2} - 2 - 3x + \frac{3}{x}$ [3]

iii) The mean weight of 8 students is 45.5 kg. Two more students having weights 41.7 kg and 53.3 kg join the group. What is the new mean weight? [4]

Question 6

i) A father is 25 years older than his son. After 5 years, his age will be twice that of his son. Find their present ages. [3]

ii) Simplify: $\frac{5 \times (25)^{n+1} - 25 \times 5^{2n}}{5 \times 5^{(2n+3)} - (25)^{n+1}}$ [3]

iii) Draw a frequency polygon for the following frequency distribution: [4]

Class-interval	1–10	11–20	21–30	31–40	41–50	51–60
Frequency	8	3	6	12	2	7

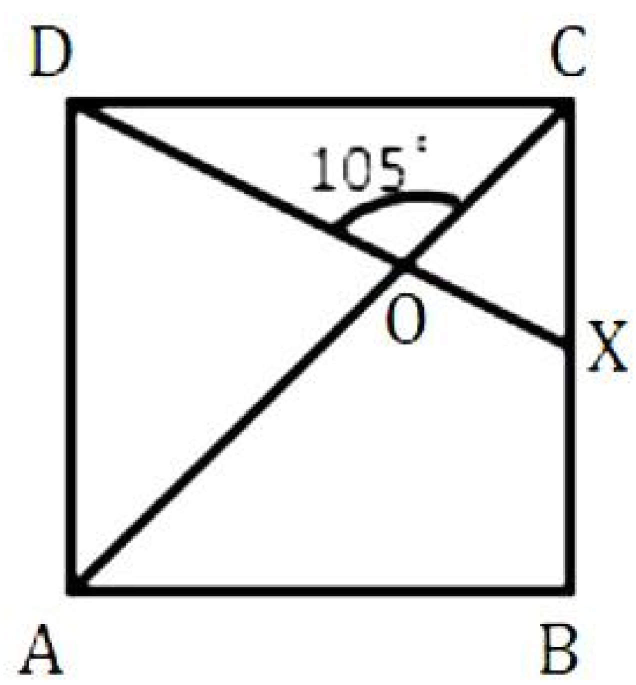
Question 7

i) In triangle ABC, AD is perpendicular to BC. sin B = 0.6, BD = 8 cm and tan C = 1. Find the length of AB, AD, AC and DC. [5]

ii) The perimeter of the isosceles triangle is 42 cm and its base is $1\frac{1}{2}$ times each of the equal sides. Find (a) the length of each side of the triangle, (b) the area of the triangle and (c) the height of the triangle. [5]

Question 8

i) In the given figure, ABCD is a square. A line segment DX cuts the side BC at X and the diagonal AC at O such that $\angle COD = 105°$. Find $\angle OXC$. [3]

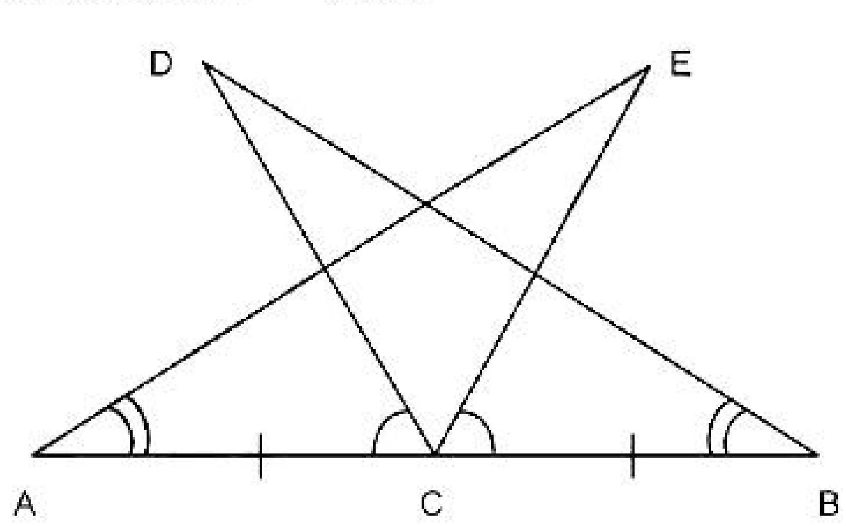


ii) Two circles with centres O and O' intersect at two points A and B. A line PQ is drawn parallel to OO' through A or B intersecting the circles at P and Q. Prove that PQ = 2OO'. [3]

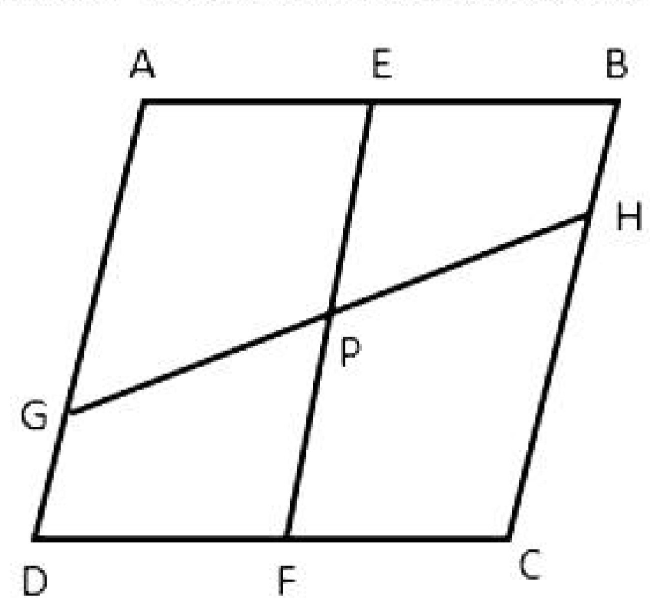
iii) Shanti Sweets Stall placed an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger box of dimensions 25 cm × 20 cm × 5 cm and the smaller box of dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area was required extra. If the cost of the cardboard is Rs. 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind. [4]

Question 9

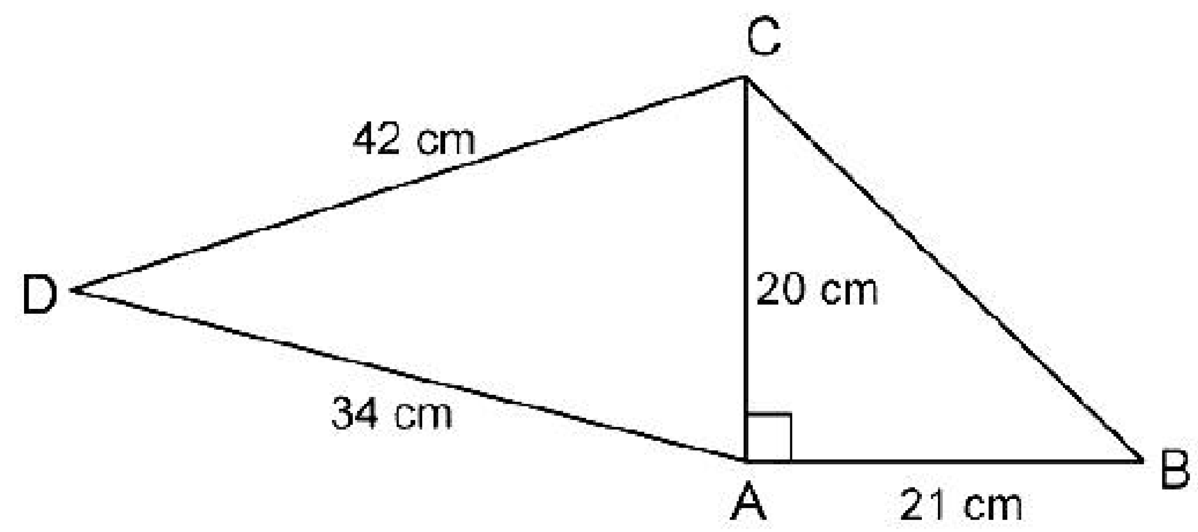
i) In the given figure, C is the midpoint of AB. If $\angle DCA = \angle ECB$ and $\angle DBC = \angle EAC$, prove that DC = EC. [3]



ii) ABCD is a parallelogram. E is the mid-point of AB and F is the mid-point of CD. GH is any line which intersects AD, EF and BC at G, P and H, respectively. Prove that GP = PH. [3]



iii) Find the area of the quadrilateral ABCD in which AB = 21 cm, ∠BAC = 90°, AC = 20 cm, CD = 42 cm and AD = 34 cm. [4]



Question 10

i) How many planks of dimensions (5 m × 25 cm × 10 cm) can be stored in a pit which is 20 m long, 6 m wide and 80 cm deep? [3]

ii) Find the point on the y-axis that is equidistant from points A(–3, 2) and B(5, –2). [3]

iii) Solve the below pair of simultaneous equations graphically. [4]

$2x + 3y = 2$ and $x - 2y = 8$

Solution

Section A

Solution 1

i) Correct option: (b)

Explanation:

$\sqrt{8}\left(\sqrt{8}-1\right)=8-\sqrt{8}$, which is an irrational number.

ii) Correct option: (d)

Explanation:

Let P be the sum.

Here, N = 1, R = 10%

$$\text{C.I.} = \frac{P\times10\times1}{100}$$

$$\Rightarrow 550=\frac{P}{10}$$

$$\Rightarrow P = \text{Rs. } 5500$$

iii) Correct option: (b)

Explanation:

$$\left(a-\frac{1}{a}\right)^2=a^2+\frac{1}{a^2}-2=18-2=16$$

$$\Rightarrow a-\frac{1}{a}=\pm4$$

iv) Correct option: (c)

Explanation:

$$\frac{27+8x^3}{9-4x^2}$$

$$=\frac{3^3+\left(2x\right)^3}{3^2-\left(2x\right)^2}$$

$$=\frac{\left(3+2x\right)\left(9+4x^2-6x\right)}{\left(3-2x\right)\left(3+2x\right)}$$

$$=\frac{9+4x^2-6x}{3-2x}$$

v) Correct option: (b)

Explanation:

$11x - 7y = 29$ ... (I)

$7x - 11y = 25$ ... (II)

Adding (I) & (II), we get

$18x - 18y = 54$

$\Rightarrow x - y = 3$ ... (III)

Subtracting (II) from (I), we get

$4x + 4y = 4$

$\Rightarrow x + y = 1$ ... (IV)

From (III) & (IV), we get

$2x = 4$

$\Rightarrow x = 2$

$\Rightarrow y = -1$

vi) Correct option: (c)

Explanation:

$$3^3 \times (243)^{-\frac{2}{3}} \times 9^{-\frac{1}{3}} = 3^3 \times \left(3^5\right)^{-\frac{2}{3}} \times \left(3^2\right)^{-\frac{1}{3}}$$

$$= 3^3 \times 3^{-\frac{10}{3}} \times 3^{-\frac{2}{3}}$$

$$= 3^{3-\frac{10}{3}-\frac{2}{3}}$$

$$= 3^{3-4}$$

$$= 3^{-1}$$

$$= \frac{1}{3}$$

vii) Correct option: (d)

Explanation:

Given: BC = QR

AM and PN are medians of ΔABC and ΔPQR respectively.

BM = ½ BC = ½ QR = QN

⇒ BM = QN ... (I)

In ΔABM and ΔPQN,

AB = PQ ... (Given)

AM = PN ... (Given)

BM = QN ... From (I)

⇒ ΔABM ≅ ΔPQN ... By SSS congruency criteria

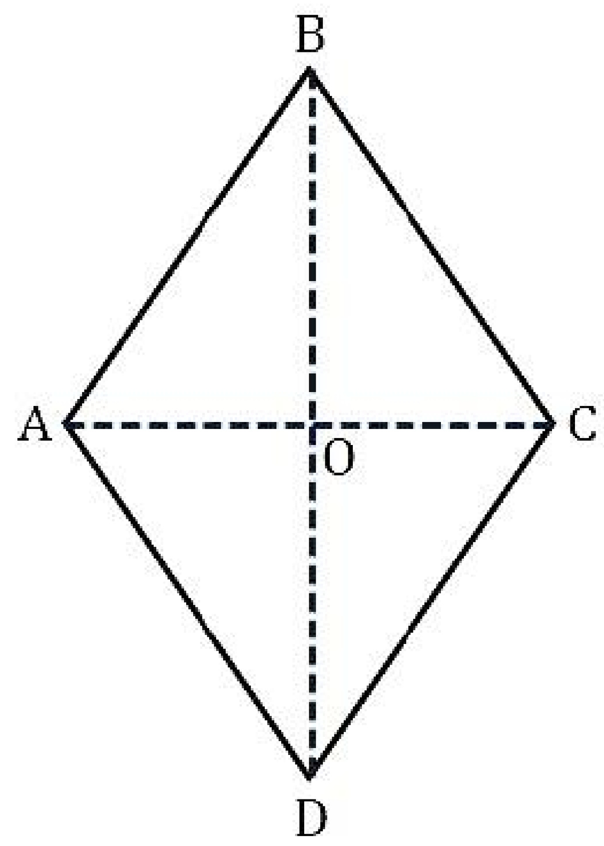
⇒ ∠ABM = ∠PQN

i.e. ∠ABC = ∠PQR

So, ΔABC ≅ ΔPQR ... By SAS congruency criteria

viii) Correct option: (d)

Explanation:



Let ABCD be the rhombus with diagonals AC and BD intersecting at point O.

Let AC = 30 cm and BD = 40 cm.

Diagonals of a rhombus bisect each other at right angles.

⇒ ΔABO is right angle triangle right-angled at O.

⇒ $AB^2 = OA^2 + OB^2$

⇒ $AB^2 = 225 + 400 = 625$ cm^2

⇒ AB = 25 cm

All the sides of a rhombus are equal to each other.

⇒ Perimeter of the rhombus ABCD = 100 cm

ix) Correct option: (b)

Explanation:

Since, chord AB: chord CD = 3: 2

⇒ arc AB : arc CD = 3 : 2

⇒ ∠AOB: ∠COD = 3: 2

x) Correct option: (c)

Explanation:

When each observation is doubled, the observation which was 50 earlier, will now become 100.

The sequencing of the observations will remain as it is as each observation is doubled.

Thus, the new median will be also doubled, that is, 100.

xi) Correct option: (a)

Explanation:

Mean of a_1 and a_2 is 13.

⇒ $a_1 + a_2 = 13 \times 2 = 26$

Mean of a_1, a_2 and a_3 is 17.

⇒ $a_1 + a_2 + a_3 = 17 \times 3 = 51$

⇒ $a_3 = 51 - 26 = 25$

xii) Correct option: (c)

Explanation:

Let 'a' be the edge of a cube.

Length of the diagonal of the cube = $a\sqrt{3}$

Since, area of a square on the diagonal of a cube is 48 cm^2.

$\Rightarrow \left(a\sqrt{3}\right)^2 = 48\ cm^2$

$\Rightarrow 3a^2 = 48\ cm^2$

$\Rightarrow a^2 = 16$ cm

$\Rightarrow a = 4$ cm (Since length can't be negative)

xiii) Correct option: (b)

Explanation:

$$\cot A = \frac{\text{Base}}{\text{Perpendicular}}$$

$$\Rightarrow \sqrt{5} = \frac{B}{P}$$

i.e. if P = x, B = $\sqrt{5}x$

Now, $H^2 = P^2 + B^2$

$\Rightarrow H^2 = x^2 + 5x^2$

$$\Rightarrow H = \sqrt{6}x$$

$$\Rightarrow \operatorname{cosec} A = \frac{H}{P} = \frac{\sqrt{6}x}{x} = \sqrt{6}$$

$$\text{Also, } \sec A = \frac{H}{B} = \frac{\sqrt{6}x}{\sqrt{5}x} = \sqrt{\frac{6}{5}}$$

$$\therefore \operatorname{cosec}^2 A - \sec^2 A = 6 - \frac{6}{5} = \frac{24}{5} = 4\frac{4}{5}$$

xiv) Correct option: (a)

Explanation:

Since, 4x = 1

$$\Rightarrow x = \frac{1}{4}$$

Since, y – 5 = 0

$\Rightarrow y = 5$

So, the point of intersection of the above lines is $\left(\frac{1}{4}, 5\right)$.

Hence, the coordinates of P are $\left(\frac{1}{4}, 5\right)$.

xv) Correct option: (c)

Explanation:

Distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

So, the reason is true.

$\Rightarrow \sqrt{(x+3)^2 + (10-2)^2} = 10$

$\Rightarrow x^2 + 6x + 9 + 64 = 100$

$\Rightarrow x^2 + 6x - 27 = 0$

$\Rightarrow x^2 + 9x - 3x - 27 = 0$

$\Rightarrow (x + 9)(x - 3) = 0$

$\Rightarrow x = -9$ or 3

Thus, the assertion is true.

Hence, both assertion and reason are true, and given reason is the correct reason for assertion.

Solution 2

i)

A. Since the amount in 5 years = Rs. 9100 and the amount in 6 years = Rs. 9828

$\therefore$ 9828 – 9100 = Rs. 728 is the interest in 1 year on Rs. 9100.

$\Rightarrow$ Rate % = $\dfrac{728 \times 100}{9100 \times 1} = 8\%$

B. Amount in 7 years = Amount in 6 years + Interest on it for 1 year

= 9828 + 8% of Rs. 9828 = Rs. 10614.24

C. Let the amount in 4 years = Rs. x

$\Rightarrow$ Amount in 5 years = Amount in 4 years + Interest on it for 1 year

$\Rightarrow 9100 = x + 8\%$ of x

$\Rightarrow 9100 = x + \dfrac{8}{100}x$

$\Rightarrow 9100 = \dfrac{108x}{100}$

$\Rightarrow x = 8425.92 \sim 8426$

Therefore, the amount in 4 years is Rs. 8426.

ii) Let the digit at the units place be x and the digit at the tens place be y.

$\therefore$ The original number = x + 10y

The sum of the digits of a two-digit number is 12.

$\Rightarrow x + y = 12$ ... (i)

If the digits are reversed, then

The new number = 10x + y

If the digits are reversed, the new number is 12 less than twice the original number.

$\Rightarrow 10x + y = 2(x + 10y) - 12$

$\Rightarrow 10x + y = 2x + 20y - 12$

$\Rightarrow 8x - 19y = -12$ ... (ii)

Multiplying equation (i) by 19, we get

$19x + 19y = 228$ ... (iii)

Adding (ii) and (iii), we get

$$\begin{array}{rcl} 8x - 19y &=& -12 \\ 19x + 19y &=& 228 \\ \hline + \quad 27x &=& 216 \end{array}$$

$\Rightarrow \quad x = 8$

Putting $x = 8$ in equation (i), we get

$x + y = 12$

$\Rightarrow y = 12 - 8 = 4$

$\therefore$ The original number $= x + 10y = 8 + 10 \times 4 = 48$

Hence, the required number is 48.

iii) In ΔBCD, $m\angle C = 90^0$

$\therefore$ By Pythagoras' theorem, we get

$BD^2 = BC^2 + CD^2 = 3^2 + 12^2 = 9 + 144 = 153$

$\Rightarrow BD = \sqrt{153}$ cm

In ΔABD, $m\angle B = 90°$

$\therefore$ By Pythagoras' theorem, we get

$AD^2 = AB^2 + BD^2$

$\Rightarrow AB^2 = AD^2 - BD^2 = 13^2 - \left(\sqrt{153}\right)^2 = 169 - 153 = 16$

$\Rightarrow AB = 4$ cm

Solution 3

i)

A. In ΔSRN,

L is the mid-point of SR and LQ || SN

$\Rightarrow$ LQ bisects RN (Mid-point theorem)

$\Rightarrow RQ = QN = ½ RN$

Now, $SP = RQ$ (opposite sides of a parallelogram)

$\Rightarrow SP = ½ RN$

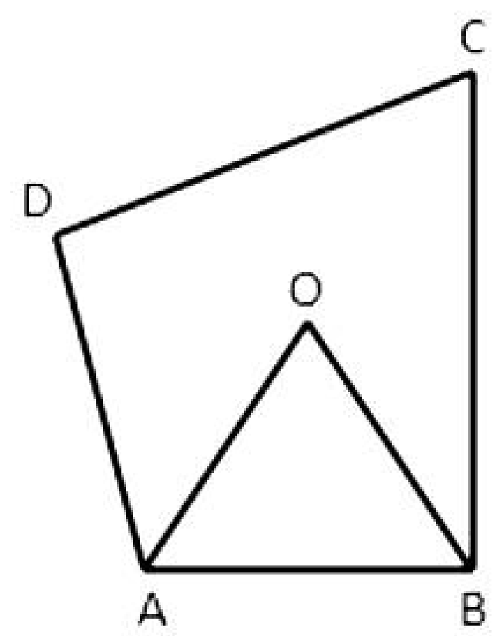
B. In ΔSRN,

L is the mid-point of SR and Q is the mid-point of RN

$\Rightarrow LQ = ½ SN$

$\Rightarrow SN = 2LQ$

ii)



ABCD is a quadrilateral.

OA and OB are the bisectors of $\angle A$ and $\angle B$, respectively.

$\angle OAB = \frac{1}{2} \angle A$ and $\angle OBA = \frac{1}{2} \angle B$ ... (i)

$\angle AOB + \angle OAB + \angle OBA = 180^\circ$

$\angle AOB = 180^\circ - \frac{1}{2} (\angle A + \angle B)$ ... From (i)

Now, sum of the angles of a quadrilateral is 360°.

Hence, $\angle A + \angle B = 360^\circ - (\angle C + \angle D)$

Thus, $\angle AOB = 180^\circ - \frac{1}{2} [360^\circ - (\angle C + \angle D)]$

$\angle AOB = \frac{1}{2} (\angle C + \angle D)$

iii)

A. $9x^2 - (x^2 - 4)^2 = (3x)^2 - (x^2 - 4)^2$

$= (3x + x^2 - 4)(3x - x^2 + 4)$

$= (x^2 + 3x - 4)(4 + 3x - x^2)$

$= (x^2 + 4x - x - 4)(4 + 4x - x - x^2)$

$= [x(x + 4) - 1(x + 4)][4(1 + x) - x(1 + x)]$

$= (x + 4)(x - 1)(1 + x)(4 - x)$

B. $(x + 1)^6 - (x - 1)^6$

Putting $x + 1 = a$ and $x - 1 = b$,

$(x + 1)^6 - (x - 1)^6$

$= (a)^6 - (b)^6$

$= (a^3)^2 - (b^3)^2$

$= (a^3 - b^3)(a^3 + b^3)$

$= (a - b)(a^2 + ab + b^2)(a + b)(a^2 - ab + b^2)$

$= (x + 1 - x + 1)[(x + 1)^2 + (x + 1)(x - 1) + (x - 1)^2](x + 1 + x - 1)[(x + 1)^2 - (x + 1)(x - 1) + (x - 1)^2]$

$= 4x[(x^2 + 2x + 1) + x^2 - 1 + (x^2 - 2x + 1)][(x^2 + 2x + 1) - (x^2 - 1) + (x^2 - 2x + 1)]$

$= 4x(3x^2 + 1)(x^2 + 3)$

Section B

Solution 4

i) Here $x = 2 - \sqrt{3}$

$$\Rightarrow \frac{1}{x} = \frac{1}{2 - \sqrt{3}}$$

$$= \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} \quad \text{(Rationalising the denominator)}$$

$$= \frac{2 + \sqrt{3}}{4 - 3} \qquad \left[\because (a-b)(a+b) = a^2 - b^2\right]$$

$$= 2 + \sqrt{3}$$

Now,

$$\left(x + \frac{1}{x}\right) = \left(2 - \sqrt{3}\right) + \left(2 + \sqrt{3}\right) = 4$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^3 = 4^3$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^3 = 64$$

ii) Given: P = Rs. 125000, r = 12% per annum compounded half-yearly

Time, $n = 1\frac{1}{2}$ years $= \frac{3}{2}$ years

Then,

$$\text{Amount} = P\left(1+\frac{r}{2\times 100}\right)^{n\times 2}$$

$$= 125000\left(1+\frac{12}{2\times 100}\right)^{\frac{3}{2}\times 2}$$

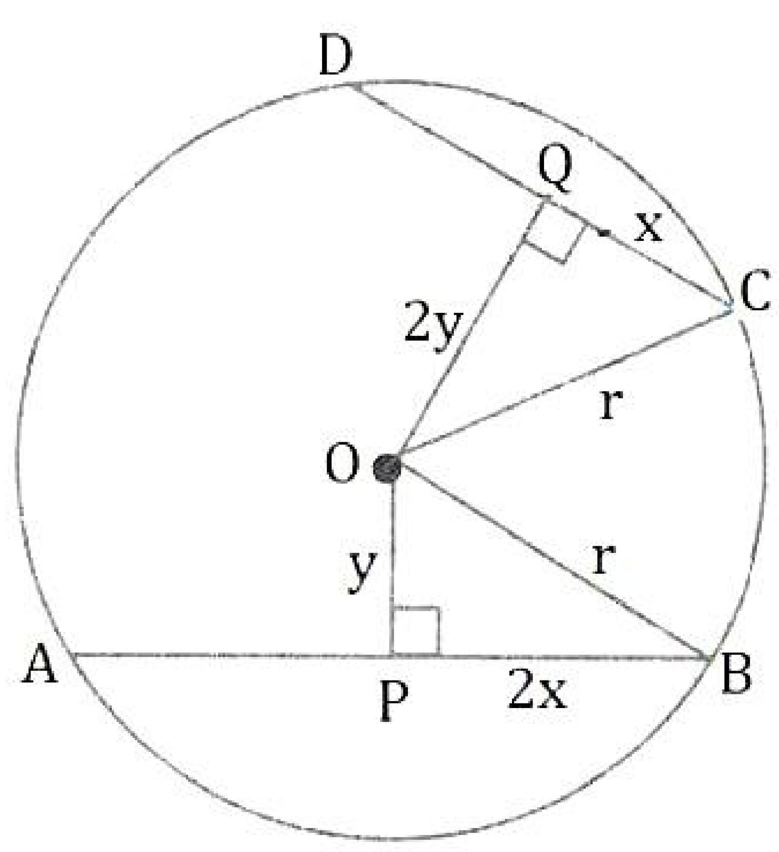
$$= 125000\left(1+\frac{6}{100}\right)^{3}$$

$$= 125000\times\frac{106}{100}\times\frac{106}{100}\times\frac{106}{100}$$

$= \text{Rs.}1,48,877$

Compound interest = A – P = Rs. (148877 – 125000) = Rs. 23877

iii)



To prove: $r = \frac{\sqrt{5}}{2}CD$

Proof :

Given $AB = 2CD$

Let $CD = 2x \Rightarrow AB = 4x$

Let $OP \perp AB$,

$\therefore BP = \frac{1}{2}AB = \frac{1}{2}(4x) = 2x$

Let $OQ \perp CD$,

$\therefore CQ = \frac{1}{2}CD = \frac{1}{2}(2x) = x$

$\because\ OQ = 2OP$

Let $OP = y$

$\therefore OQ = 2y$

In ΔOPB, $\angle P = 90°$

$\therefore OB^2 = OP^2 + BP^2$

$\therefore r^2 = y^2 + (2x)^2$

$\therefore r^2 = y^2 + 4x^2 \quad ....(1)$

In ΔOQC, $\angle Q = 90°$

$\therefore OC^2 = OQ^2 + CQ^2$

$\therefore r^2 = (2y)^2 + x^2$

$\therefore r^2 = 4y^2 + x^2 \quad ....(2)$

From (1) and (2),

$y^2 + 4x^2 = 4y^2 + x^2$

$\therefore 4x^2 - x^2 = 4y^2 - y^2$

$\therefore 3x^2 = 3y^2$

$\therefore x^2 = y^2$

$\therefore x = y \quad [x, y > 0]$

Substituting $x = y$ in (1),

$r^2 = x^2 + 4x^2$

$\therefore r^2 = 5x^2$

$\therefore r = \sqrt{5}x = \frac{\sqrt{5}}{2} \times 2x$

$\therefore r = \frac{\sqrt{5}}{2} \times CD \qquad [\because CD = 2x]$

Solution 5

i) We have

$\left(x^2 + \frac{1}{25x^2}\right) = 9\frac{2}{5}$

$\Rightarrow \left(x^2 + \frac{1}{25x^2}\right) = \frac{47}{5} \quad ....(i)$

We know that $(x - y)^2 = x^2 + y^2 - 2xy$

Therefore

$$\left(x - \frac{1}{5x}\right)^2 = x^2 + \frac{1}{25x^2} - \frac{2}{5}$$

$$= \frac{47}{5} - \frac{2}{5}$$

$$= \frac{45}{5}$$

$$= 9$$

Taking the square root on both sides, we get

$\left(x - \frac{1}{5x}\right) = \pm 3$

ii) $x^2 + \frac{1}{x^2} - 2 - 3x + \frac{3}{x} = \left(x^2 + \frac{1}{x^2} - 2\right) - 3\left(x - \frac{1}{x}\right)$

$$= \left(x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x}\right) - 3\left(x - \frac{1}{x}\right)$$

$$= \left(x - \frac{1}{x}\right)^2 - 3\left(x - \frac{1}{x}\right)$$

$$= \left(x - \frac{1}{x}\right)\left(x - \frac{1}{x} - 3\right)$$

iii) Number of students = 8

Their mean weight= 45.5 kg

Then, sum of weights of 8 students = Mean weight × Number of students

= 45.5 × 8

= 364 kg

Two more students having weights 41.7 kg and 53.3 kg joined the group.

∴ Number of students = 8 + 2 = 10

And, sum of weights of 10 students = 364 + 41.7 + 53.3 = 459

$\Rightarrow$ New mean weight $= \frac{459}{10} = 45.9$

Therefore, the new mean weight is 45.9 kg.

Solution 6

i) Let the father's present age be x years and the son's present age be y years.

The father is 25 years older than his son.

$\Rightarrow x = y + 25$

$\Rightarrow x - y = 25$ ... (i)

After 5 years, his age will be twice that of his son.

$\Rightarrow x + 5 = 2(y + 5)$

$\Rightarrow x + 5 = 2y + 10$

$\Rightarrow x - 2y = 5$ ... (ii)

Subtracting equation (ii) from equation (i), we get

$y = 20$

Putting $y = 20$ in equation (i), we get

$x - 20 = 25$

$\Rightarrow x = 45$

Hence, the present ages of father and son are 45 years and 20 years, respectively.

ii) $$\frac{5\times(25)^{n+1}-25\times5^{2n}}{5\times5^{(2n+3)}-(25)^{n+1}}$$

$$=\frac{5\times\left(5^2\right)^{n+1}-\left(5^2\right)\times5^{2n}}{5\times5^{(2n+3)}-\left(5^2\right)^{n+1}}$$

$$=\frac{5\times5^{2n}\times5^2-5^2\times5^{2n}}{5\times5^{2n}\times5^3-5^{2n}\times5^2}$$

$$=\frac{5^{2n}5^2\left(5-1\right)}{5^{2n}5^2\left(5^2-1\right)}$$

$$=\frac{4}{24}$$

$$=\frac{1}{6}$$

iii) The given frequency distribution table is as shown below:

Class interval	1–10	11–20	21–30	31– 40	41–50	51–60
Frequency	8	3	6	12	2	7

STEPS:

1. The given data is in the inclusive form; convert it in the exclusive form.
2. On making the classes exclusive (continuous), we get the actual class limits as (0.5–10.5), (10.5–20.5), (20.5–30.5), (30.5–40.5), (40.5–50.5) and (50.5–60.5).
3. Find the class mark (mid-value) of each of the given class intervals.
4. The class mark of a class interval = $\frac{\text{lower limit+ upper limit}}{2}$

The frequency distribution table with class marks is given below:

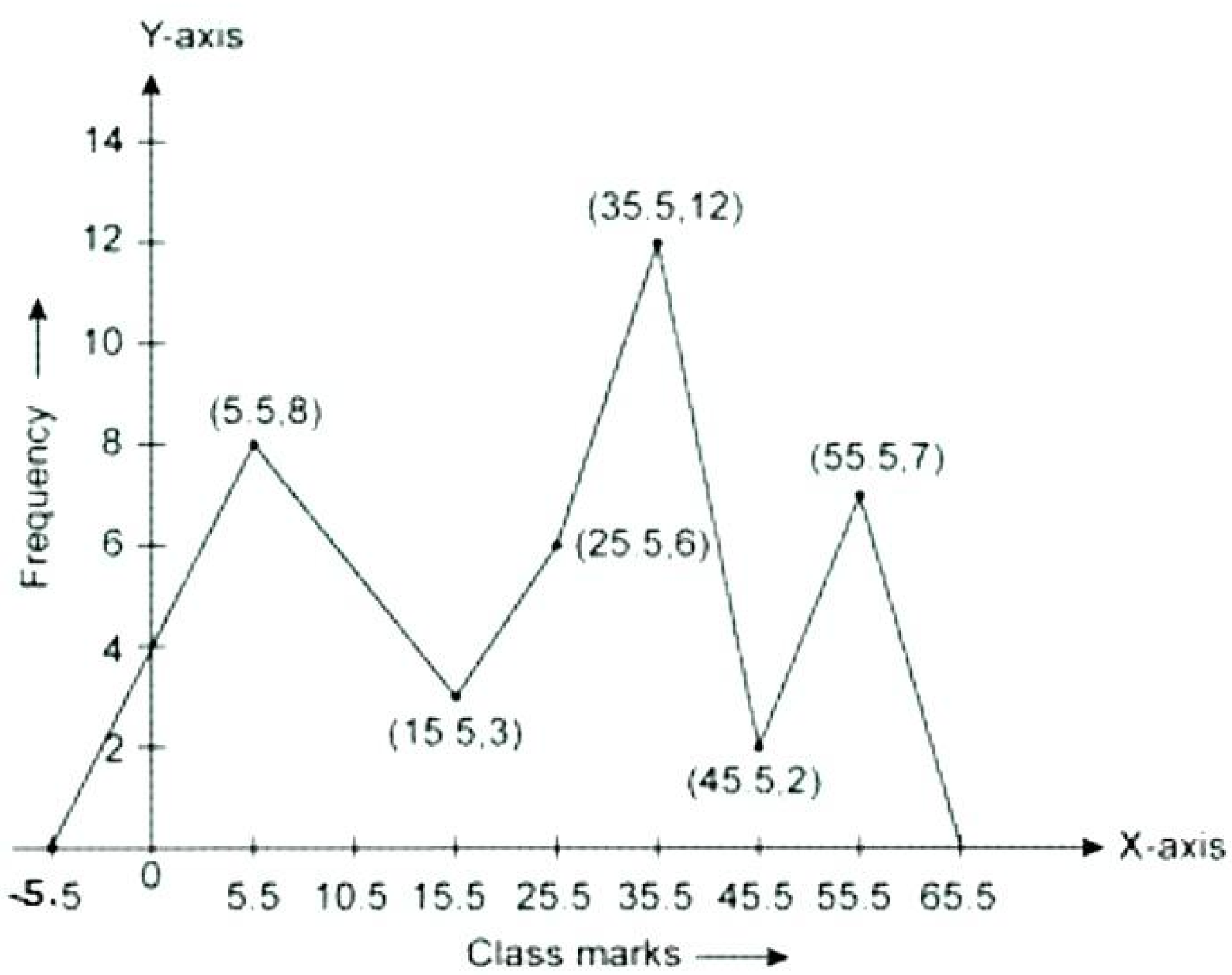
Class Intervals	Class Marks	Frequency
(–10.5) – 0.5	– 5.5	0
0.5 – 10.5	$\frac{0.5+10.5}{2}=\frac{11}{2}=5.5$	8
10.5 – 20.5	15.5	3
20.5 – 30. 5	25.5	6
30.5 – 40. 5	35.5	12
40.5 – 50.5	45.5	2
50.5 – 60.5	55.5	7
60.5 – 70.5	65.5	0

5. In the above table, we have taken imaginary class intervals (–10.5) – 0.5 at the beginning and 60.5 – 70.5 at the end, each with frequency zero.
6. On a graph paper, take class marks along the x-axis and the corresponding frequencies along the y-axis.

7. On this graph paper, plot the points (–5.5, 0), (5.5, 8) … (55.5, 7) and (65.5, 0).
8. Draw line segments joining the consecutive points marked in step 5.

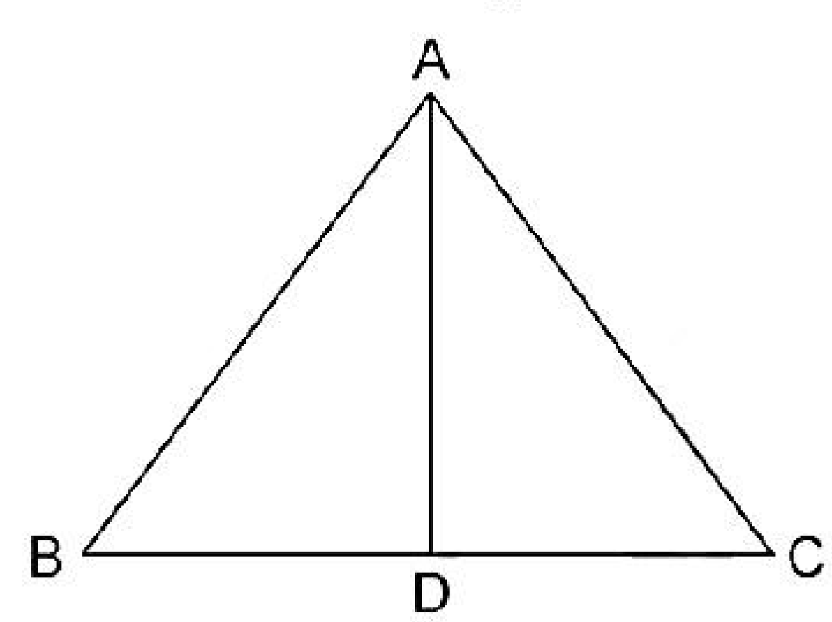
Note: Join the class mark of the class interval just before the first class and the class mark of the class interval just after the last class. This completes the required **frequency polygon**.

∴ The required frequency polygon will be



Solution 7

i) Consider the figure below.



$$\sin B = 0.6 = \frac{6}{10} = \frac{3}{5}$$

i.e. $\frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{3}{5}$

Therefore, if the length of the perpendicular = 3x, then the length of the hypotenuse = 5x

Since, $AD^2 + BD^2 = AB^2$ … By Pythagoras theorem

$\Rightarrow (5x)^2 - (3x)^2 = BD^2$

$\Rightarrow BD^2 = 16x^2$

$\Rightarrow BD = 4x$

Now, BD = 8 cm

$\Rightarrow 4x = 8$ cm

$\Rightarrow x = 2$ cm

Therefore, AB = 5x = 5× 2 = 10 cm

And, AD = 3x = 3 × 2 = 6 cm

Again, $\tan C = \frac{1}{1}$

i.e. $\frac{\text{perpendicular}}{\text{base}} = \frac{AD}{DC} = \frac{1}{1}$

Therefore, if the length of the perpendicular = y, the length of the base = y

Now,

$AD^2 + DC^2 = AC^2 \quad \text{... [Using Pythagoras Theorem]}$

$(y)^2 + (y)^2 = AC^2$

$AC^2 = 2y^2$

$\therefore AC = \sqrt{2}y$

Now, AD = 6 cm

$\Rightarrow y = 6$ cm

Therefore, DC = y = 6 cm

And,

$$
\begin{aligned}
AC &= \sqrt{2}y \\
&= \sqrt{2} \times 6 \\
&= 6\sqrt{2} \text{ cm}
\end{aligned}
$$

ii) In an isosceles triangle, the length of the lateral sides is equal.

Let the length of each lateral side be x cm.

Then, base $= \frac{3}{2} \times x$ cm ....[given]

(a) Perimeter of an isosceles triangle = 42 cm

$\Rightarrow x + x + \frac{3}{2}x = 42$

$\Rightarrow 2x + \frac{3}{2}x = 42$

$\Rightarrow 4x + 3x = 84$

$\Rightarrow 7x = 84$ cm

$\Rightarrow x = \frac{84}{7} = 12$ cm

$\therefore$ Length of lateral side = 12 cm and base $= \frac{3}{2} \times 12 = 18$ cm

Therefore, the length of the sides of an isosceles triangle are 12 cm, 12 cm and 18 cm, respectively.

(b) Let $a = 12$ cm, $b = 12$ cm and $c = 18$ cm

Now, $s = \frac{1}{2}(a+b+c) = \left(\frac{12+12+18}{2}\right) = \left(\frac{42}{2}\right) = 21$ cm

The, area of the triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-12)(21-12)(21-18)}$

$= \sqrt{21\times9\times9\times3}$

$= \sqrt{3\times7\times9\times9\times3}$

$= 27\sqrt{7}\ \text{cm}^2$

(c) Area of a triangle $= \frac{1}{2}\times\text{base}\times\text{height}$

$\Rightarrow 27\sqrt{7} = \frac{1}{2}\times18\times\text{height}$

$\Rightarrow 27\sqrt{7} = 9\times\text{height}$

$\Rightarrow \text{height} = \frac{27\sqrt{7}}{9} = 3\sqrt{7}$ cm

$\therefore$ The height of the triangle is $3\sqrt{7}$ cm.

Solution 8

i) The diagonals bisect the angles of a square.

$\angle OCX = 45^\circ$ ($\angle DCB = 90^\circ$ and CA bisects $\angle DCB$)

Also,

$\angle COD + \angle COX = 180^\circ$ (Linear pair)

$\therefore 105^\circ + \angle COX = 180^\circ$

$\therefore \angle COX = 75^\circ$

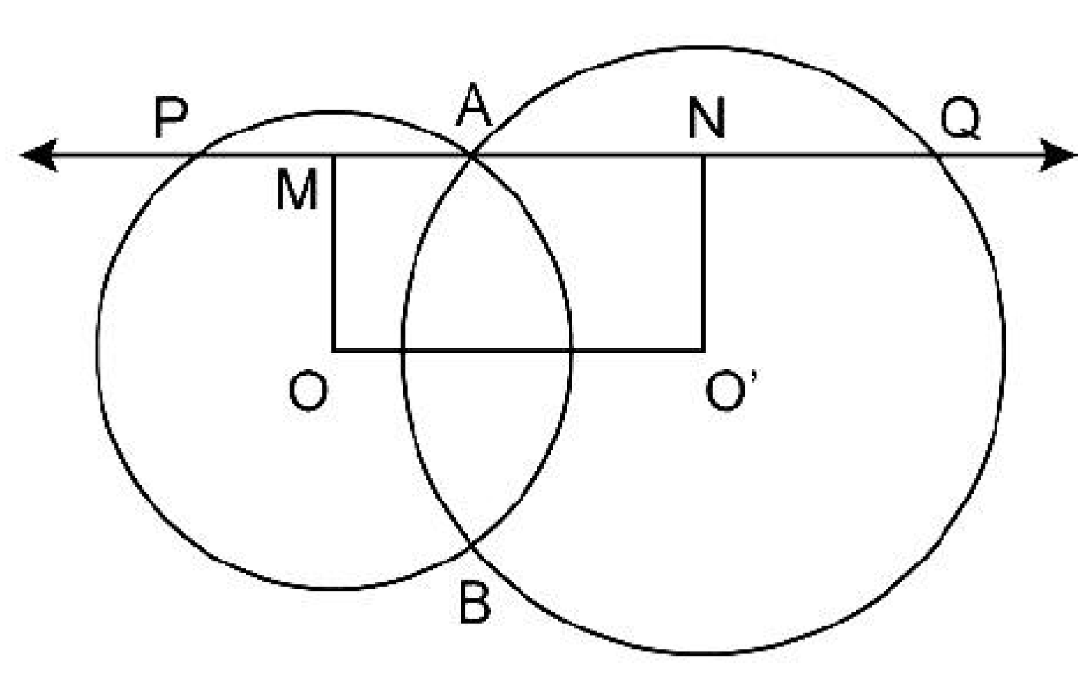
In ΔCOX,

$\angle OCX + \angle COX + \angle OXC = 180^\circ$

$\therefore 45^\circ + 75^\circ + \angle OXC = 180^\circ$

$\therefore \angle OXC = 60^\circ$

ii)



Draw OM $\perp$ PQ and O'N $\perp$ PQ.

$\Rightarrow$ OM $\perp$ AP

$\Rightarrow$ AM = PM (perpendicular from the centre of a circle bisects the chord)

$\Rightarrow$ AP = 2AM ...(i)

And O'N $\perp$ PQ

$\Rightarrow$ O'N $\perp$ AQ

$\Rightarrow$ AN = QN (perpendicular from the centre of a circle bisects the chord)

$\Rightarrow$ AQ = 2AN ...(ii)

Now,

PQ = AP + AQ

$\Rightarrow$ PQ = 2AM + 2AN ... From (i) and (ii)

$\Rightarrow$ PQ = 2(AM + AN)

$\Rightarrow$ PQ = 2MN

$\Rightarrow$ PQ = 2OO' (since MNO'O is a rectangle)

iii) For a bigger box,

Length = 25 cm, Breadth = 20 cm and Height = 5 cm

Total surface area of the bigger box

$= [2(25 \times 20 + 25 \times 5 + 20 \times 5)]\ \text{cm}^2$

$= [2(500 + 125 + 100)]\ \text{cm}^2$

$= 1450\ \text{cm}^2$

Extra area required for overlapping = 5% of 1450 cm^2

$= \left(\frac{1450 \times 5}{100}\right)\text{cm}^2$

$= 72.5\ \text{cm}^2$

Considering all overlaps, the total surface area of 1 bigger box

$= (1450 + 72.5)\ \text{cm}^2$

$= 1522.5\ \text{cm}^2$

Area of the cardboard sheet required for 250 such bigger boxes

$= (1522.5 \times 250)\ \text{cm}^2$

$= 380625\ \text{cm}^2$

For a smaller box,

Length = 15 cm, Breadth = 12 cm and Height = 5 cm

Total surface area of the smaller box

$= [2(15 \times 12 + 15 \times 5 + 12 \times 5]\ \text{cm}^2$

$= [2(180 + 75 + 60)]\ \text{cm}^2$

$= (2 \times 315)\ \text{cm}^2$

$= 630\ \text{cm}^2$

Extra area required for overlapping = 5% of 630 cm^2

$= \left(\frac{630 \times 5}{100}\right)\text{cm}^2$

$= 31.5\ cm^2$

Considering all overlaps, the total surface area of 1 smaller box

$= (630 + 31.5)\ cm^2$

$= 661.5\ cm^2$

Area of the cardboard sheet required for 250 smaller boxes

$= (250 \times 661.5)\ cm^2$

$= 165375\ cm^2$

Total cardboard sheet required $= (380625 + 165375)\ cm^2 = 546000\ cm^2$

Cost of $1000\ cm^2$ cardboard sheet = Rs. 4

Cost of $546000\ cm^2$ cardboard sheet $= \text{Rs}\left(\frac{546000 \times 4}{1000}\right) = \text{Rs. } 2184$

So, the cost of the cardboard sheet required for 250 boxes of each kind will be Rs. 2184.

Solution 9

i) Given : $\angle DCA = \angle ECB$ and $\angle DBC = \angle EAC$

To prove : DC = EC

Proof:

$\angle ACE = \angle ACD + \angle DCE \quad ....(i)$

$\angle BCD = \angle BCE + \angle DCE \quad ....(ii)$

From (i) and (ii),

$\angle ACE = \angle BCD \qquad (\angle ACD = \angle BCE)$

Now, in ΔACE and ΔBCD,

$\angle ACE = \angle BCD$

$AC = BC$

$\angle EAC = \angle DBC$

$\therefore\ \Delta ACE \cong \Delta BCD \qquad$ (By ASA)

$\Rightarrow CE = DC \qquad$ (C.P.C.T.)

ii) AE = ½ AB (E is the mid-point of AB)

DF = ½ DC (F is the mid-point of DC)

$\Rightarrow$ AE = DF and AE || DF (AB = DC and AE || DF, opposite sides of a $||^{gm}$)

$\Rightarrow$ AEFD is a parallelogram.

$\Rightarrow$ AD || EF || BC

Using the intercept theorem, transversal AB makes equal intercepts AE = BE on three parallel lines AD, EF and BC.

Hence, another transversal GH will also make equal intercepts on these parallel lines.

Hence, GP = PH.

iii) $A(\Delta ABC) = \frac{1}{2} \times AB \times AC = \frac{1}{2} \times 21 \times 20 = 210 \text{ cm}^2$

In ΔACD, AC = 20 cm, CD = 42 cm and AD = 34 cm

Let a = 20 cm, b = 42 cm and c = 34 cm

Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{20+42+34}{2}$

$= \frac{96}{2} = 48 \text{ cm}$

$\therefore$ Area of $\Delta ACD = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{48(48-20)(48-42)(48-34)}$

$= \sqrt{48 \times 28 \times 6 \times 14}$

$= \sqrt{16 \times 3 \times 14 \times 2 \times 3 \times 2 \times 14}$

$= 4 \times 3 \times 14 \times 2$

$= 336 \text{ cm}^2$

Thus, the area of quadrilateral ABCD = $A(\Delta ABC) + A(\Delta ACD)$

$= (210 + 336) \text{ cm}^2$

$= 546 \text{ cm}^2$

Solution 10

i) Length of the plank = 5 m = 500 cm

Breadth of the plank = 25 cm

Height of the plank = 10 cm

$\therefore$ Volume of the plank = $(500 \times 25 \times 10) \text{ cm}^3$

Now,

Length of the pit = 20 m = 2000 cm

Breadth of the pit = 6 m = 600 cm

Height of the pit = 80 cm

$\therefore$ Volume of one pit = $(2000 \times 600 \times 80) \text{ cm}^3$

$\therefore$ Number of planks which can be stored = $\frac{\text{Volume of pit}}{\text{Volume of plank}}$

$= \frac{(2000 \times 600 \times 80)}{(500 \times 25 \times 10)}$

$= 768$

ii) Let the required point on the y-axis be C(0, y).

It is given that C(0, y) is equidistant from A(–3, 2) and B(5, –2).

$\Rightarrow AC = CB$

The distance between the given points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow \sqrt{(-3-0)^2 + (2-y)^2} = \sqrt{(5-0)^2 + (-2-y)^2}$

$\Rightarrow \sqrt{(-3)^2 + (2-y)^2} = \sqrt{(5)^2 + (-2-y)^2}$

$\Rightarrow 9 + 4 - 4y + y^2 = 25 + 4 + 4y + y^2$

$\Rightarrow 13 - 4y + y^2 = 29 + 4y + y^2$

$\Rightarrow -16 = 8y$

$\Rightarrow y = -2$

$\therefore$ The required point on the y-axis = (0, –2).

iii)

A. $2x + 3y = 2$

$\Rightarrow x = \frac{2-3y}{2}$

When $y = 2 \Rightarrow x = \frac{2-3\times2}{2} = \frac{2-6}{2} = \frac{-4}{2} = -2$

When $y = 0 \Rightarrow x = \frac{2-3\times(0)}{2} = \frac{2}{2} = 1$

When $y = -2 \Rightarrow x = \frac{2-3\times(-2)}{2} = \frac{2+6}{2} = \frac{8}{2} = 4$

x	–2	1	4
y	2	0	–2

1. Plot the points (–2, 2), (1, 0), (4, –2) on the graph paper, taking 1 cm = 1 unit on both axes.
2. Draw a straight line AB passing through the points plotted.

B. $x - 2y = 8$

$\Rightarrow x = 8 + 2y$

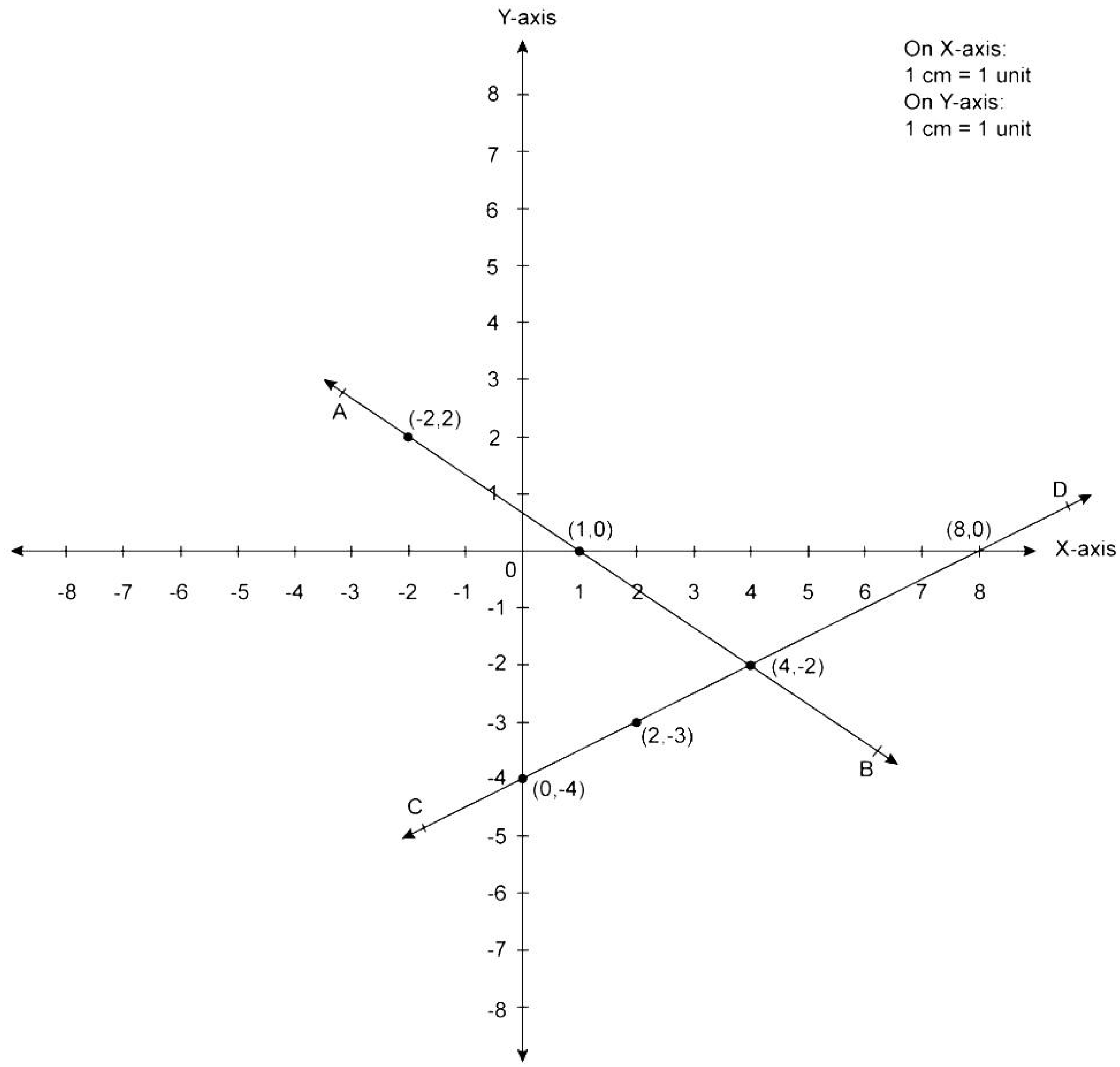
When $y = -3 \Rightarrow x = 8 + 2(-3) = 8 - 6 = 2$

When $y = -4 \Rightarrow x = 8 + 2(-4) = 8 - 8 = 0$

When $y = 0 \Rightarrow x = 8 + 2(0) = 8$

x	2	0	8
y	–3	–4	0

1. Plot the points (2, –3), (0, –4), (8, 0) on the graph paper, taking 1 cm = 1 unit on both axes.
2. Draw a straight line CD passing through the points plotted.



From the graph, lines AB and CD intersect each other at point (4, –2).

⇒ The solution of the given simultaneous equations is (4, –2).