12

Circle

12.01. Introduction

We have seen many objects form childhood those are round in shape such as bangles, coins, wheels of a vehicles, plate, buttons of shirts etc.

Also, the path of the tip of a needle in a wrist is round in shape. The path traced by the tip of the needle is known as circle.

In this chapter, we will discuss about circle and its properties .

12.02. Circle and its parts

Take a compass, and fix a pencil in it, Put its pointed leg on a point on a sheet of paper, rotate the other leg through one revolution. You will see a shape on a paper, it is a circle. You kept in mind that tip of the pencil is produced a point. Actually a circle is group (set) of infinite points which make after rotate other leg (pencil) through revolution. or the collection of all points in a plane, which are at a fixed (constant) distance from a fixed point in a plane, is called a circle.

The fixed point is called a centre of the circle and the fixed distance is called the radius of the circle. In a figure 12.01, O is the centre of circle and Length OP is the radius of circle.

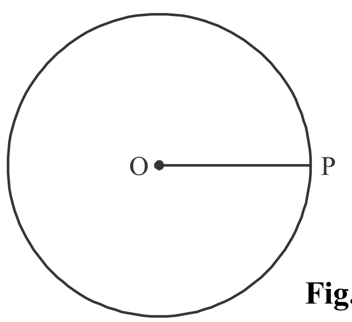


Fig. 12.01

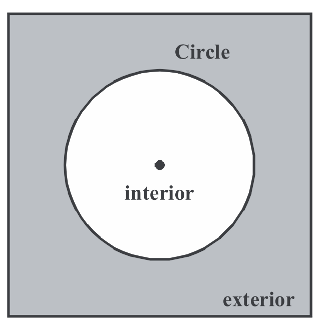


Fig. 12.02

A circle divides the plane on which it lies into three parts. In figure 12.02, They are

(i) inside the circle, which is called the interior of the circle

(ii) the circle

(iii) outside the circle, which is called exterior part of the circle.

the circle and its interior make up the circular region.

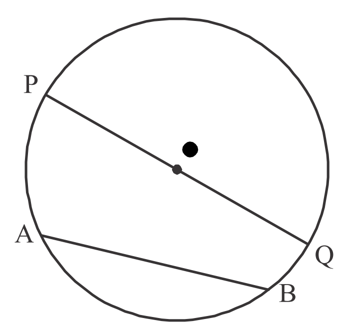


Fig. 12.03

Chord and diameter - If you take two points A and B on a circle, then the line segment AB is called a chord. If the chord, which passes through the centre of the circle, is called diameter of the circle. see the Fig 12.03, chord PQ is a diameter of the circle

Activities –

(i) You make circles with different-different radius in your exercise book and in every circle draw two chords. then measure of all chords. Do you find any other chord of the circle longer than a diameter, exactly No, a diameter is the longest chord of circle, which is equal to two times the radius.

(ii) Draw a circle and see how many diameters can be drawn. Does there exist more than one diamtere? Yes, you can draw infinite numbers of diameters.(see figure 12.04)

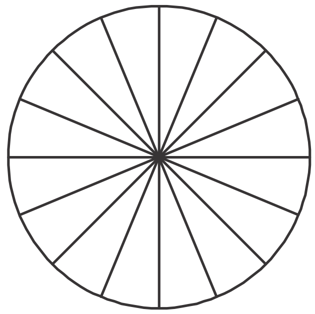


Fig.12.04

Arc : In a figure12.05. There are two points P and Q on the circle. which divides the circle into two parts. You find that one part is longer and the other part is smaller.

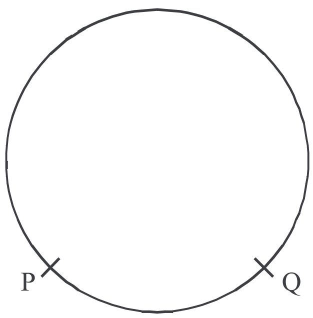


Fig. 12.05

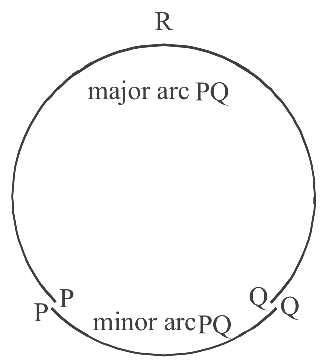


Fig. 12.06

If both the arc viewed sparately according figure 12.06 the shorter one is called minor arc PQ and denoted by PQ. and the longer one is called major arc PQ and denoted by PRQ where R is some point on the arc between P and Q.

When P and Q are ends of a diameter, then both arcs are equal and each is called a semi circle. see Figure 12.07

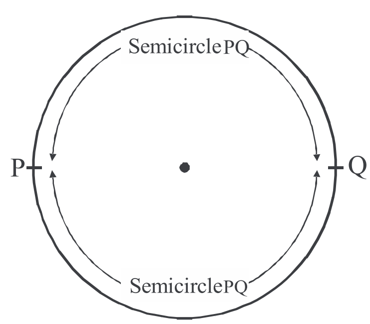


Fig. 12.07

Exercise 12.1

1. **Fill in the blanks :**

(i) Centre of a circle is situated in .... of the circle. (exterior / interior)

(ii) A point whose distance from the center of the circle is grearter then its radius lies in .... of the circle.

(exterior / interior)

(iii) The longest chord of a circle is a .... of the circle.

(iv) An arc is a .... when its ends are the ends of a diameter of circle.

(v) A circle divides the plane, on which it lies in .... parts.

2. **Write true or False : Give reasons for your answers.**

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) In a circle number of equal chords are finite.

(iii) If a circle is divided into three equal arcs, each is a major arc.

(iv) A chord of a circle,which is twice as long as its radius, is a diameter of the circle.

(v) A circle is a plane figure.

(vi) In a plane the set of points are constant distance from the fixed point on the same plane, it is diamter.

(vii) The chord which lies on the centre, it is known as radius.

12.03 Angle Subtended by a Chord at a Point

If P , Q and R lie on the plane which are not in a straight line. Join PR and QR. Then $\angle PRQ$ is called the angle subtended by the line segment PQ at a point R. (see figure 12.08). In Figure 12.09, $\angle AOB$ is the angle subtended by the chord AB at a centre O, $\angle ADB$ and $\angle ACB$ are respectively the angles subtended by AB at points D and C on the major and minor arcs AB.

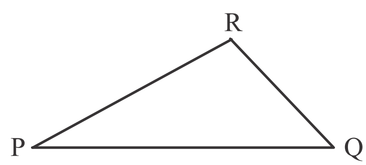


Fig. 12.08

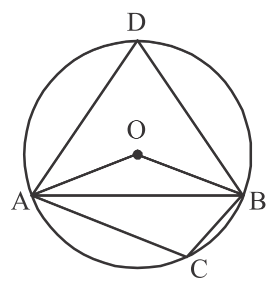
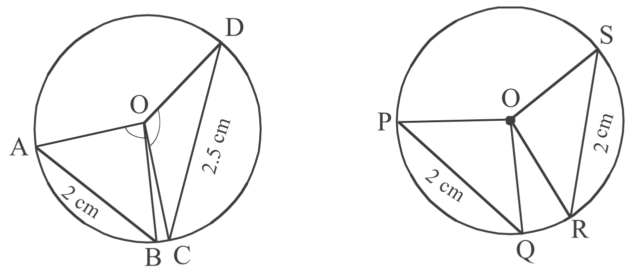


Fig. 12.09

Activity : Let us examine the relationship between the size of the chord and the angle subtended by it at the centre.



(i) Figure 12.10 (ii)

In Figure 12.10 (i) Two chords AB and CD have length 2 cm and 2.5 cm. Here AB < CD then $\angle AOB$ angle subtended by the chord AB on the centre and $\angle COD$ angle subtended by the chord CD on the centre. Have you observed any relation between these angles? Yes, $\angle AOB < \angle COD$. But in Figure 12.10 (ii) What you have seen, You see that $PQ = RS$ then $\angle POQ = \angle ROS$.

or In a circle, the longer chord subtended angle at the centre is greater then the shorter chord subtended angle at the centre.

Let us see to prove these result by the previous results as a theorems.

Theorem 12.1 : Equal chords of a circle subtends equal angles at the centre.

Given : $AB = CD$

To prove : $\angle AOB = \angle COD$

Proof : In ΔAOB and ΔCOD

$OA = OC$ (radii of a circle)

$OB = OD$ (radii of a circle)

$AB = CD$ (given)

$\therefore$ $\Delta AOB \cong \Delta COD$ (SSS rule)

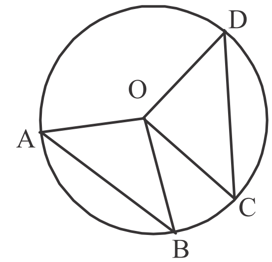


Fig. 12.11

Thus gives $\angle AOB = \angle COD$ (Hence Prove)

Now we can prove its converse.

Theorem 12.2 : If the angles subtended by the chords of a circle at the centre are equal, then chords are equal.

Given : $\angle AOB = \angle COD$

To prove : $AB = CD$

Proof : ΔAOB and ΔDOC

OA = OD (radius of circle)

$\angle AOB = \angle COD$ (given)

OB = OC (radius of a circle)

$\therefore$ $\Delta AOB \cong \Delta DOC$ (SAS rule)

Hence $AB = CD$ **Hence Proved.**

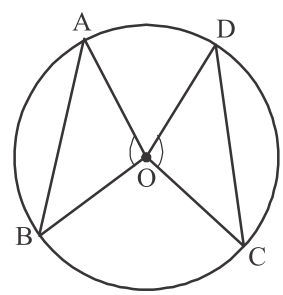


Fig. 12.12

Because the radius of two congruent circles are equal therefore theorem 12.1 and 12.2, we can prove by two congruent circles also.

12.04. Perpendicular from the Centre to a Chord

Theorem 12.3 : The perpendicular from the centre of a circle to the chord bisects the chord.

Given : $OM \perp AB$, AB is a chord

To prove : $AM = BM$

Construct : Join O to A and B

Proof : In ΔOAM and ΔOBM

$OA = OB$ (radius of circles)

$\angle AMO = \angle OMB = 90°$, $OM \perp AB$ (Given)

OM is common

$\Rightarrow$ ΔOAM and ΔOBM are right angle triangles

$\Rightarrow$ $\Delta OAM \cong \Delta OBM$ (RHS rule)

Therefore $AM = BM$ **Hence Proved.**

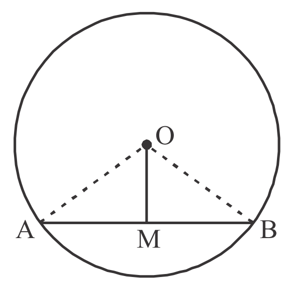


Fig. 12.13

Theorem 12.4 : The line joining the centre of circle to the mid point of chord is perpendicular to the chord.

Given : $AM = BM$

To prove : $OM \perp AB$

Construct : Join O to A and B

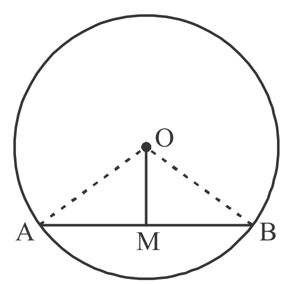


Fig. 12.14

Proof : In ΔOMA and ΔOMB

$OA = OB$ (radius of a circle)

OM is common

AM = BM (given)

i.e. $\Delta OMA \cong \Delta OMB$ (*SSS* rule)

i.e. $\angle OMA = \angle OMB$ (Pair of linear angle)

$\Rightarrow$ $\angle OMA = \angle OMB = 90^{o}$

Hence, $OM \perp AB$ **Hence Proved.**

12.05. Circle passes through three points

You have learnt axiom of class IX that two points are sufficient to draw one and only one straight line. A question arises that do you know that how many points are sufficient to draw one and only one circle?

Take a point P. You have seen that there may be as many circles as you like passing through this point (see fig. 12.15 (i))

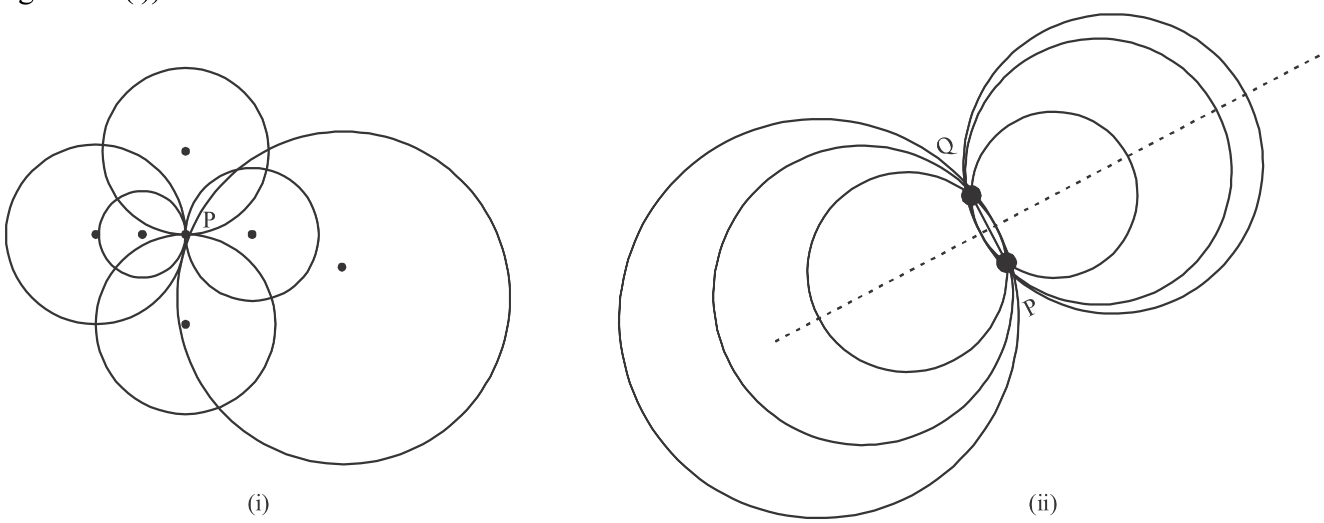


Fig 12.15

Now take two points *P* and *Q*. You again see that there may be an infinite number of circles passing through *P* and *Q* (see fig. 12.15 (ii)). But you can see that centres of every circlers lie on a straight line which is perpendicular bisector of the line *PQ*.

In this order, take three points P, Q and R which is collinear points. Can you draw a circle passing through three collinearr points. No, If the points on a line, then the third point will lie inside or outside the circle which passing through two points (see fig. 12.16)

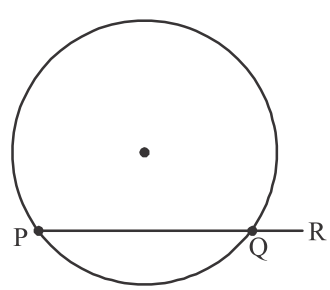


Fig. 12.16

So, let us take three points P, Q and R, which are not on the same line (see fig. 12.17)

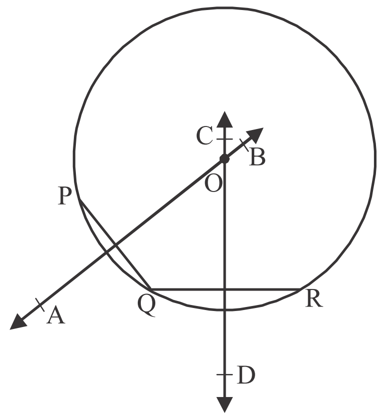


Fig. 12.17

In a fig. 12.15 (ii), You have seen that centres of the circles passes through the two points are lying on the perpendicular bisector.

The centre of the circle which passes through points P, Q and R is lying on the perpendicular bisectors of line segment PQ and QR. So draw perpendicular bisectors of PQ and QR. In fig. 12.17 AB and CD are perpendicular bisectors of PQ and QR respectively.

You have seen that AB and CD are intersect at O. therefore draw the circle form O to P, Q and R (why?) we have studied in class 9 that as every point on the perpendicular bisector of a line segment is equidistant from its end points. i.e., OP = QO ...(i)

and OQ = OR ...(ii)

from (i) and (ii) $\Rightarrow$ OP = OQ = OR

So if draw a circle from O is take OP as radius then it will passes through P, Q and R.

So three points which are not collinear only one circle will pass through them. You know that two lines (perpendicular bisectors) can intersect at only one point which means that the points P, Q and R are at equal distances from the point O. So In other words, there is a unique circle passing through P, Q and R.

This result can be written in the form of following theorem.

Theorem 12.5 There is one and only one circle passing through three given non-collinear points.

12.06. Equal Chords and Their Distances form the Centre

You have studied in class 9, that, Draw infinite line segment from the exterior point to the any line segment, perpendicular will be least . This is the distance from the exterior point to the line segment.

Note : If the point lies on the line, the distance of the line from the point is zero.

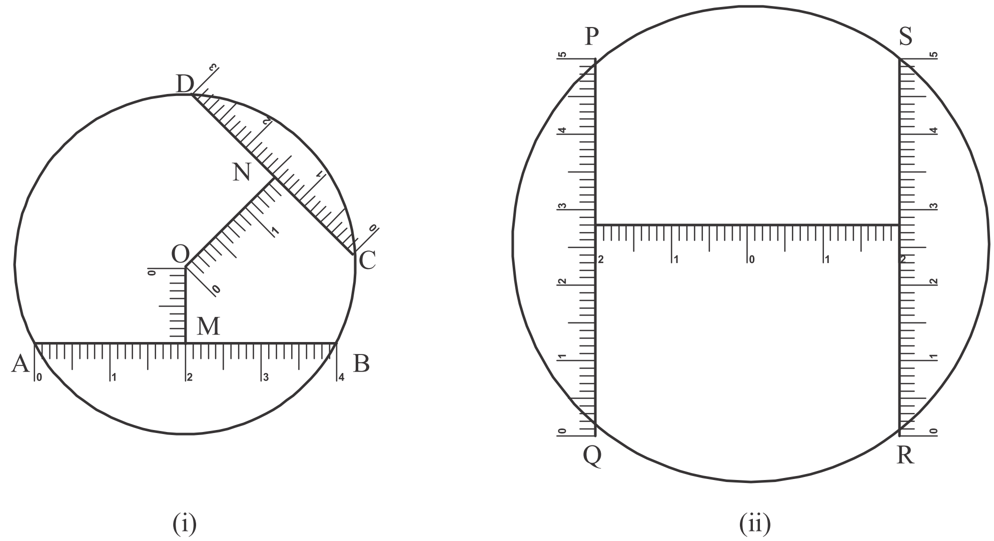


Fig. 12.18

Many chords can be drawn in a circle. You construct a circle and draw more than one chords in the circle. Find the distance of every chord from the centre of circle. What have you observed?

Let us think on an activity-

In fig. 12.18 (i), two chords have 4 cm and 3 cm and distance form the centre is 1 cm and 1.5 cm respectively. so that larger chord is nearer to the centre, then the smaller chord.

In Fig. 12.18 (ii), two chords of equal length 5 cm and having equal distance 2 cm from the centre in othr words equal length of chords are having equal distance form the centre of a circle.

Now again we have to prove for a circle and two congurence circles by the following theorem.

Theorem 12.6 : Equal Chords of a circle are equidistant from the centre.

Given : chord AB = chord CD

To prove : $OM = ON$

Construction : Join OA and OD

Proof : $AM = BM = \frac{1}{2} AB \ldots$ (perpendicular drawn from the centre to the chord is bisecting the chord)

$DN = CN = \frac{1}{2} CD$ (perpendicular drawn from the centre to the chord is bisecting the chrod)

But $AB = CD$ (given)

$AM = DN \ldots$ (i)

In ΔOMA and ΔOND

AM = DN (from (i))

OA = OD (radii of circle)

$\angle OMA = \angle OND = 90^\circ$

$\therefore$ RHS Rule $\Delta OMA \cong \Delta OND$

$\Rightarrow$ OM = ON Hence Proved

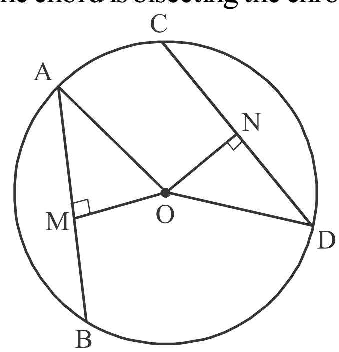


Fig. 12.19

Sub Theorem : In congurent circles equal chords are equidistant from their corresponding centres.

Theorem 12.7 : (Converse of theorem) The chords equidistant from the centre of a circle are equal in length.

Given : Chord AB and CD are having equal distance from the centre O. *i.e.* $OM = ON$

To prove : $AB = CD$

Construction : Join A and D to O

Proof : In ΔOMA and ΔOND

$OM = ON$ (given)

$OA = OD$ (radii of a circle)

$\angle OMA = \angle OND$ ($OM \perp AB$ and $ON \perp CD$)

$\Rightarrow$ RHS rule $\Delta OMA \cong \Delta OND$

$\therefore$ AM = ND

$\Rightarrow$ 2 AM = 2 ND

$\Rightarrow$ AB = CD Hence proved

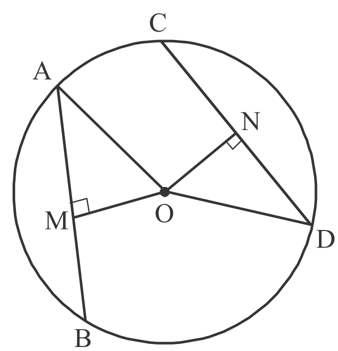


Fig. 12.20

Sub Theorem : Chords of congurent circles are equidistant from the corresponding centres are equal.

Illustrative Example

Example 1. In fig. 12.21, centre of a circle O with radius 5 cm. If $OP \perp AB$, $OQ \perp CD$, AB|| CD, AB = 8 cm and CD = 6 cm. Then find PQ.

Solution : Given : $OP \perp AB$ and $OQ \perp CD$

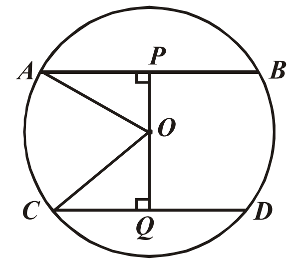


Fig. 12.21

$$\Rightarrow \quad AP = PB = \frac{1}{2}AB = 4\text{ cm}$$

$$CQ = QD = \frac{1}{2}CD = 3\text{ cm}$$

and $OA = OC = 5$ cm (radius)

In ΔOPA by bodhayan theorem

$$OP^2 = OA^2 - AP^2$$

$$OP^2 = 5^2 - 4^2$$

$$= 25 - 16 = 9$$

$\therefore \quad OP = 3$ cm

Similarly In ΔOQC,

$$OQ^2 = OC^2 - CQ^2$$

$$OQ^2 = 5^2 - 3^2 = 25 - 9 = 16$$

$\therefore \quad OQ = 4$ cm

$\Rightarrow$ Therefore $\quad PQ = OP + OQ = 3 + 4 = 7$ cm

Example 2 : In fig.12.22, arc AB = arc CD, To prove $\angle A = \angle B$.

Solution : Given : arc AB = arc CD

To prove : $\angle A = \angle B$

Proof : We know that equal arcs of a circle subtended equal angles at the centre.

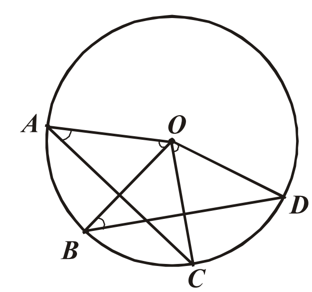


Fig. 12.22

$\therefore \quad \angle AOB = \angle COD$

Adding $\angle BOC$ both sides

$$\angle AOB + \angle BOC = \angle BOC + \angle COD$$

$\Rightarrow \quad \angle AOC = \angle BOD \quad \quad$ . . . (i)

Again ΔAOC and ΔBOD

$OA = OB$ (radii of circle)

$OC = OD$ (radii of circle)

$\angle AOC = \angle BOD$ (from (i))

$\Delta AOC \cong \Delta BOD$ (SAS Rule)

$\therefore$ (Corresponding angles are equal in congruent triangles)

$\Rightarrow \quad \angle A = \angle B$ **Hence Proved.**

Example 3 : In a circle, if two chords AB and AC are equal, prove that, centre of a circle is lie on the bisector of the $\angle BAC$.

Solution : Given : a circle whose centre O, and have equal chord AB and AC

To prove : centre O, lies on the bisector of $\angle BAC$.

Construction : Join O to C and B.

Proof : In ΔAOB and ΔAOC

$BO = OC$ (radii of circle)

$OA = OA$ (common side)

$AB = AC$ (given)

$\Delta AOB \cong \Delta AOC$ (*SSS* rule)

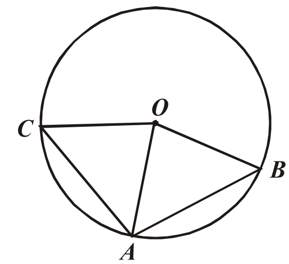


Fig. 12.23

Therefore In congruent triangles corresponding angles are equal.

$\angle OAB = \angle OAC$

centre O, lies on the bisector of $\angle BAC$ **Hence Proved.**

Example 4 : If two circles are intersecting each other at two pionts,then prove that line joiniung their centres will be perpendicular bisector of a common chord.

Solution : Given : Fig. 12.24 is having two circles whose centres are *O* and *P* respectively, which intersect each other on *A* and *B*.

To prove : *OP* is perpendicular bisector of *AB*

Construction : Join *OA, OB* , *PA* and *PB*

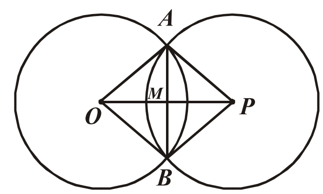


Fig. 12.24

Proof : In ΔOAP and ΔOBP

$AO = BO$ (radii of circle)

$PA = PB$ (radii of circle)

$OP = OP$ (common)

$\Delta OAP \cong \Delta OBP$ (From *SSS* congruence)

Therefore, corresponding angle will be equal of congruence triangles.

$\angle AOP = \angle BOP$

or $\angle AOM = \angle BOM$ .....(i)

Now in ΔAOM and ΔBOM

$OA = OB$ (radii of circle)

$\angle AOM = \angle BOM$ (from equation (i))

$OM = OM$ (common)

$\Delta AOM \cong \Delta BOM$ (from *SAS* congruence)

Therefore, corresponding angle and side will be equal in congruence triangles.

Means $AM = BM$ ....(ii)

$\angle AMO = \angle BMO$ .....(iii)

But $\angle AMO + \angle BMO = 180^\circ$

or $\angle AMO = \angle BMO = 90^\circ$ .....(iv)

From equation (ii) and (iv),

OP is perpendicular bisector of *AB* **Hence proved.**

Example 5 : In a circle of 10 cm radius two chords *AB*=*AC*= 12 cm, then find out the length of chord *BC*.

Solution : In fig. 12.25

ΔABC an insoceles triangle. *AD* is bisector of $\angle BAC$. Therefore *AD* is prependicular bisector of *BC*.

Here, $AC = AB = 12$ cm

$OA = OC = 10$ cm

and $BD = CD$

$\because$ In a ΔADC according to Bodhayan Theorem.

$CD^2 = AC^2 - AD^2$

$CD^2 = 144 - AD^2$ .....(i)

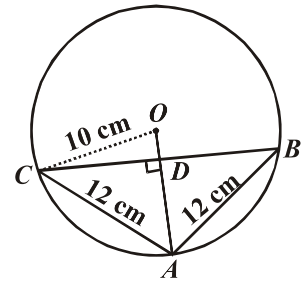


Fig 12.25

Similarly in ΔOCD

$CD^2 = OC^2 - OD^2$

$CD^2 = 100 - (OA - AD)^2 = 100 - (10 - AD)^2$

$CD^2 = 20\,AD - AD^2$ .....(ii)

From the equation (i) and (ii) by soving

$AD = 7.2$ cm

Putting the value of AD in equation (i)

$CD^2 = 144 - (7.2)^2$ or $CD = 9.6$ cm

Hence chord $BC = 2CD = 2 \times 9.6 = 19.2$ cm

Example 6 : Prove that out of two chords, longest chord is nearer to centre of circle.

Solution : Given : In fig. 12.26

A circle whose centre is O and chord $CD >$ Chord AB

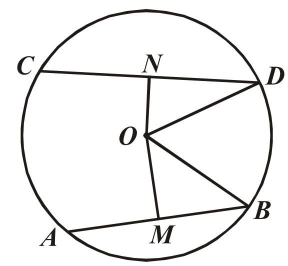


Fig. 12.26

To prove : $ON < OM$

Construction : Join OB and OD

Proof : OM and ON are perpendicular on AB and CD respectively.

So, $MB = \frac{1}{2}AB$ and $ND = \frac{1}{2}CD$ .....(i)

Now in ΔOMB

$MB^2 = OB^2 - OM^2$ .....(ii)

and in ΔOND

$ND^2 = OD^2 - ON^2$ .....(iii)

Given that AB < CD

or $\frac{1}{2}AB < \frac{1}{2}CD$

$MB < ND$ (from equation (ii))

or $MB^2 < ND^2$ .....(iv)

from equation (ii), (iii) and (iv)

$(OB^2 - OM^2) < (OD^2 - ON^2)$

But $OB = OD$ (radius of circle)

So, $-OM^2 < -ON^2$

or, $OM^2 > ON^2$

or $ON < OM$ **Hence Proved.**

Example 7 : In figure 12.27, chord AB = chord CD in a circle, then prove $DQ = BQ$.

Solution : Given : Chord AB = Chord CD

To prove : $DQ = BQ$

Construction : $OL \perp AB$ and $OM \perp CD$ and Join OQ

Proof : $AB = CD$ (given)

or $OL = OM$ .....(i)

Now, in ΔOMQ and ΔOLQ

$OQ = OQ$ (common arm)

$OM = OL$ (from equation (i))

$\angle OMQ = \angle OLQ$ (right angle)

$\angle OMQ \cong \Delta OLQ$ (from RHS)

So the corresponding sides will be equal in congruent triangles

In other words, MQ = LQ .....(ii)

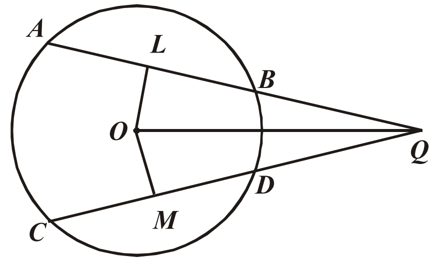


Fig. 12.27

But $MD = \frac{1}{2}CD$ and $LB = \frac{1}{2}AB$

$AB = CD \Rightarrow MD = LB$ .....(iii)

Substracting equation (iii) form (ii)

$MQ - MD = LQ - LB$

Hence $DQ = BQ$ **Hence Proved.**

Exercise 12.2

1. Write True / False and if possible then give reason of your answer.
   - (i) Chords of a circle AB and CD are 3 cm and 4 cm which make angles at centre are of 70° and 50° respectively.
   - (ii) The chords of a circle of length 10 cm and 8 cm. Which apart from the centre are 8 cm and 5 cm respectively.
   - (iii) Two chords of a circle AB and CD which are 4 cm distance from the centre then $AB = CD$.
   - (iv) Two congruence circles of centres O and O' intersects each other at point A and B then $\angle AOB = \angle AO'B$
   - (v) One circle can be drawn by three point which are in a line.
   - (vi) A circle of radius 4 cm can be drawn passing two points A and B, if $AB = 8$ cm.
2. If the radius of a circle is 13 cm and a length of its one chord is 10 cm, then find out the distance of this chord form the centre of circle.
3. Two chords of a circle AB and CD whose lengths are 6 cm and 12 cm respectively are parallel to each other and also situated on one side of centre of circle. If the distance between AB and CD is 3 cm then find out the radius of the circle.
4. In fig. 12.28, AB and CD intersect each other at point E. Then prove that arc DA = arc CD.

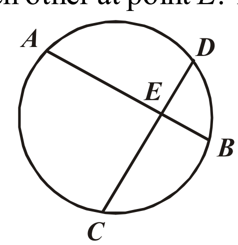


Fig. 12.28

5. In fig. 12.29, AB and CD are two equal chords of a circle, O is the centre of circle. If $OM \perp AB$ and

$ON \perp CD$, then prove that $\angle OMN = \angle ONM$.

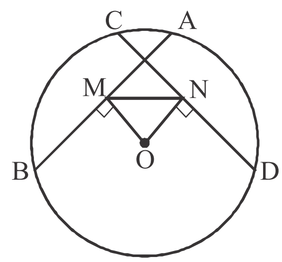


Fig. 12.29

6. In fig. 12.30, O and O' are the centre of circle. $AB || OO'$ then prove that $AB = 2OO'$.

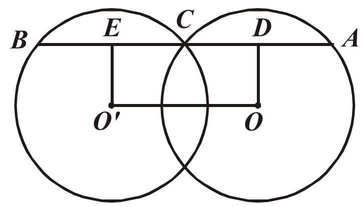


Fig. 12.30

7. AB and CD are two chords of a circle that $AB = 10$ cm and $CD = 24$ cm and $AB || CD$. The distance between AB and CD is 17 cm. Calculate the radius of circle.
8. In a circle of 10 cm radius, the length of two parallel chords are 12 cm and 16 cm. Find out the distance between these if the chords are
   (a) Same side of centre (b) Opposite side of centre.
9. Vertices of a cyclic quadrilateral are situated in such a way that $AB = CD$, then prove that AC = BD.
10. If two equal chords of a circle intertsect each other, then prove their successive parts will be equal to their corresponding parts.
11. Prove that bisector of two parallel chords passes through the centre of circle.

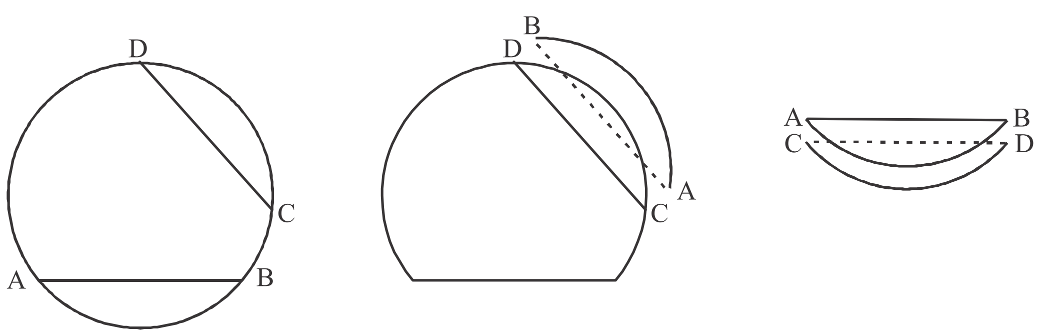
12.07. The Angle Subtended by on arc of a circle

You have learned in previous section that the end points of an arc divide the circle in two parts. If you take two equal chords then we can say about their arc. Is the arc made by a chord equal to arc made by other chord?

Let us solve this puzzle.

Activity :

Draw a circle on a paper. Draw two equal chords AB and CD in it then you will obtain arc AB and CD. (see fig. 12.31 (i))



(i) (ii) (iii)

Fig. 12.31

According to fig. 12.31 (ii), chord AB and chord CD cut in the same direction and try to put AB over CD and try to cover it. What have you noticed ?

. Now cut in same direction of chord CD and according to fig. 12.31 (iii) Try to cover each other. Now you will see that arc AB and CD completely cover each others. In other words, equal chords make congurent arcs.

So, If the two chords of a circle are equal then their arc are congurent and opposite of this if two arcs are congurent then their corresponding chords must be equal.

Here the angle made by arc also may be defined in content to angle made an centre by corresponding chord in other words short arc AB make an angle $\angle AOB$. Let us see fig. 12.32 according this defination and theorem 12.1, we can say that " In any circle congurent arcs (or equal arcs) make equal angle on the centre of circle".

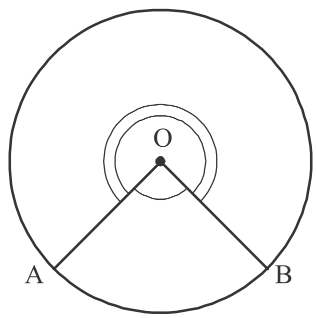
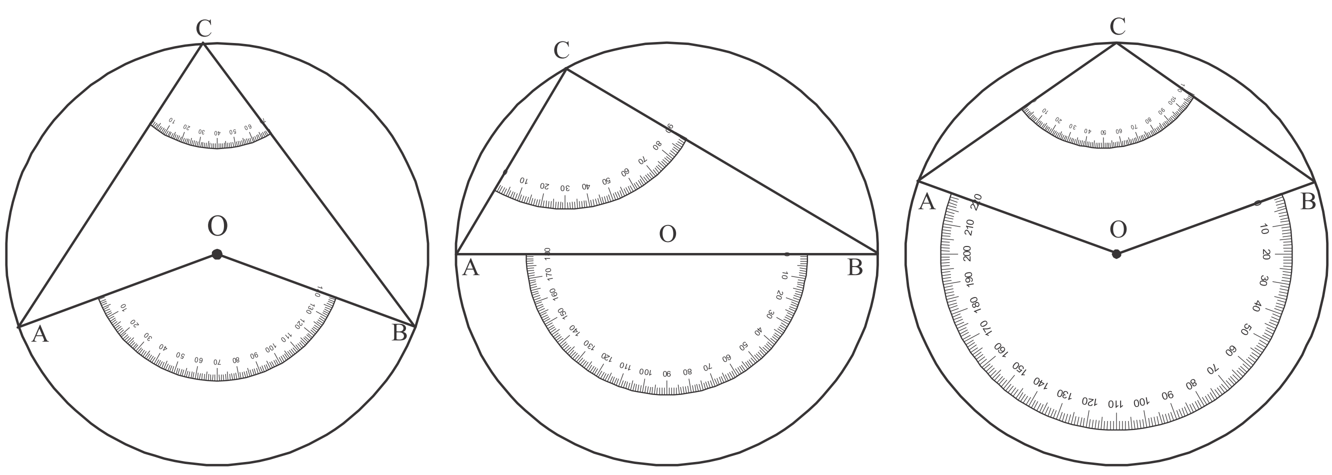


Fig. 12.32

Let us understand the relation between the angle made by an arc and angle of any point in the circle with an activity. In fig 12.33 (i) , (ii) and (iii). you have short arc AB and AB half circle, Big arc AB, made the various angle on the centre of circle. The remaining angles of circle appearing to be measure with the help of photocopy image of protector.



(i) (ii) (iiii)

Fig. 12.33

You have to observe each image carefully, in all angle made by arc AB on the centre $\angle AOB$ and remaining part of circle ACB made angle $\angle ACB$, what relation between them you are able to see

Read the measurement of angles with the help of protector.

In fig. 12.33 (i) $\angle AOB = 140^o$ and $\angle ACB = 70^o$

In (ii) $\angle AOB = 180^{o}$ and $\angle ACB = 90^{o}$

In (iii) obtuse angle $\angle AOB = 220^{o}$ and $\angle ACD = 110^{o}$

It is clear form all the figures the measurement of angle at centre is two times of angle in remaining part of circle is two times of angle in remaining part of circle.

Revise this activity by taking another values of angles.

Let us prove this result with the help of theorem.

Theorem 12.8 : The angle made by an arc on centre is two times of angle made at any point of remaining circle.

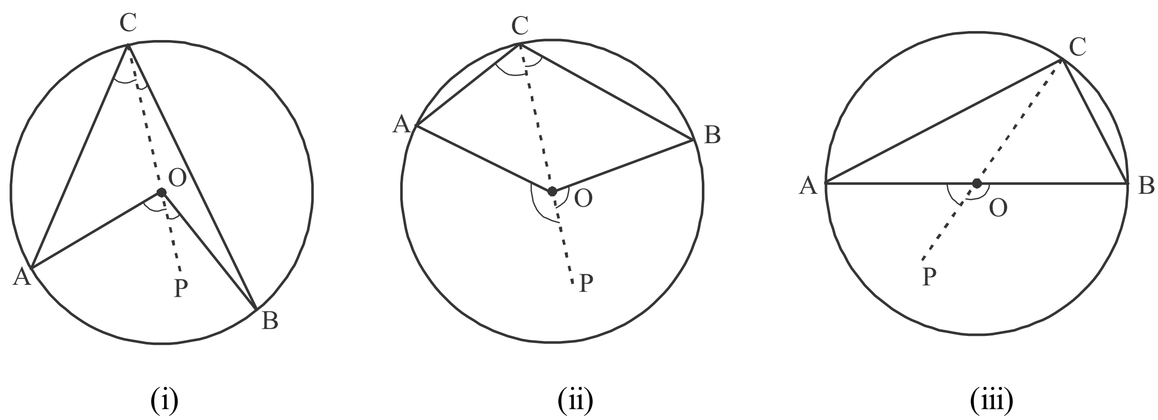


Fig. 12.34

Given : Angle subtended by arc AB to centre O is $\angle AOB$ and angle on remaining part is $\angle ACB$.

To prove : $\angle AOB = 2\angle ACB$

Construction : Produce C to P through O.

Proof : ΔAOC is an isoceles triangle ($\because$ OA = OC are the radius of circle)

So $\angle ACO = \angle OAC$ (opposite angles of equal sides are equal) . . . (i)

In ΔAOC angle $\angle AOP$ is exterior angle

$\angle AOP = \angle ACO + \angle OAC$ $\quad$ ($\angle ACO = \angle OAC$ from eq. (i))

$\angle AOP = \angle ACO + \angle ACO$

$\angle AOP = 2\angle ACO$ . . . (ii)

Similarly $\angle BOP = 2\angle BCO$ . . . (iii)

$\angle AOP + \angle BOP = 2\angle ACO + 2\angle BCO$

$\angle AOP + \angle BOP = 2(\angle ACO + \angle BCO)$

$\angle AOB = 2\angle ACB$

from fig 12.34 (i), (ii) and (iii) **Hence proved**

In fig. 12.34. (iii) $\angle ACB$ is angle made on half circle.

Here $\angle AOB = 180^{o}$; so $\angle ACB = 90^{o}$

Sub theorem : Semi circle subtends right angle.

Let us now discuss on the angles subtended by same arc on remaining parts of circle with an activity.

Activity :

Draw a circle and also draw two chords AC and AD . By drawing the same angle image of protector part is according to fig. 12.35 taking CA and DA as a photocopy image of protector and C and D point.

Now you will see that the angle made in the photocopy of protector. By increasing the other sides . they will meet only at point B.

In fig. $\angle ADB = \angle ACB = 60^\circ$

Hence, angle made by an arc in all remaining part of the circle will be same

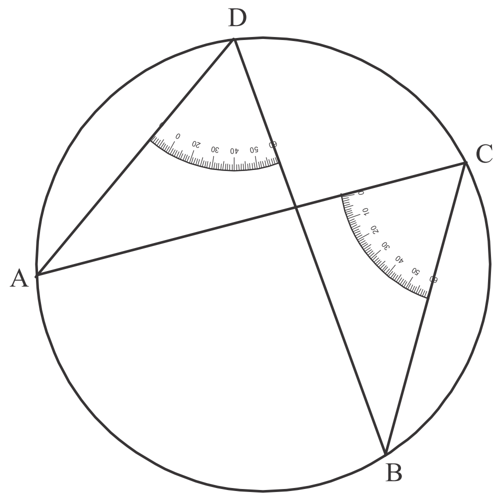


Fig. 12.35

Illustarative Example

Example 1. In Fig. 12.36, AB is diameter and $?\ DAB ?\ 40^\circ$ than find the value of $\angle DCA$.

Solution : AB is diameter of circle , so $\angle ADB = 90^\circ$

Now $\angle DBA = 180^\circ - (90^\circ + 40^\circ)$

$\angle DBA = 50^\circ$

$\therefore$ $\angle DBA$ and $\angle DCA$ are the angle of same sector.

therefore $\angle DCA = \angle DBA = 50^\circ$

$\Rightarrow$ $\angle DCA = 50^\circ$

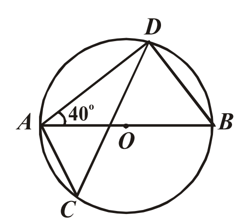


Fig. 12.36

Example 2. In fig. 12.37, arc AB and AC makes angle on O. centre of circle arc 80° and 120° . Find the value of $\angle BAC$ and $\angle BOC$.

Solution : $\angle BOC = 360^\circ - (120^\circ + 80^\circ)$

So, $\angle BOC = 160^\circ$

$\angle BAC = \frac{1}{2}\angle BOC = \frac{1}{2} \times 160^\circ$

therefore $\angle BAC = 80^\circ$

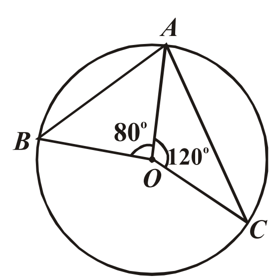


Fig. 12.37

Example 3. In a quadrilateral ABCD, AB=AC=AD, then prove that $\angle BAD = 2(\angle BDC + \angle CBD)$.

Solution : Given : $AB = AC = AD$. In other words, point A is equidistant form B, C and D. So centre of circle is A.

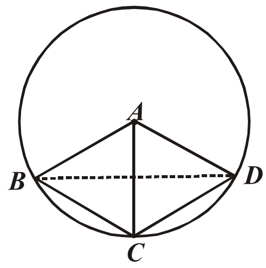


Fig. 12.38

Now arc BC at centre makes $\angle BAC$ and makes $\angle BDC$ on the remaining part of circle.

$\therefore \quad \angle BAC = 2\angle BDC$ . . . (i)

Similarly arc CD makes $\angle DAC$ on centre and $\angle CBD$ on remaining part of circle.

$\therefore \quad \angle CAD = 2\angle CBD$ . . . (ii)

Adding equation (i) and (ii)

$$\angle BAC + \angle CAD = 2(\angle BDC + \angle CBD)$$

$$\angle BAD = 2(\angle BDC + \angle CBD)$$ **Hence proved.**

Example 4. Taking equal side of an triangle isoceles as a diameter, if we draw a circle, prove that it bisects the third unequal side.

Solution : Given : In fig. 12.39 an isoceles ΔABC in which $AB=AC$, taking AC as diameter the circle intersect the unequal side BC at D.

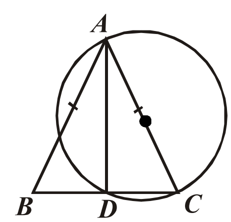


Fig. 12.39

To prove : $BD = DC$

Proof : The circle draw taking AC as diameter

the $\angle ADC$ is an angle of half circle.

So $\quad \angle ADC = 90^\circ$

Now in ΔABD and ΔACD

$AB = AC$ (given)

$AD = AD$ (common arm)

$\angle ADB = \angle ADC$ (right angle)

?*ABD* ? ? *ACD* (from *RHS*)

Hence congruent triangle will have equal corresponding sides

Hence $\quad BD = CD$ **Hence Proved**

Example 5. Prove that the angle of major segment is an acute angle.

Solution : Given : In the fig. there is a circle whose centre is O, then segment is ACB.

To prove : $\angle ACB < 90^\circ$

Construction : Join OA, OB and AB

Proof : Angle subtended by arc AB at centre is $\angle AOB$ and angle on remaining part is $\angle ACB$.

$$\angle ACB = \frac{1}{2}\angle AOB \quad . . . (i)$$

But $\quad \angle AOB < 180^\circ$ (one angle as ΔAOB)

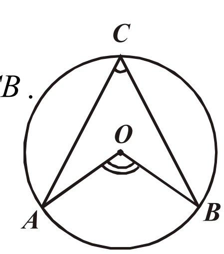


Fig. 12.40

$\therefore \qquad \frac{1}{2} \angle AOB < \frac{1}{2} \times 180^\circ$

Hence $\qquad \frac{1}{2} \angle AOB < 90^\circ \qquad \ldots \text{(ii)}$

from equation (i) and (ii)

$\angle ACB < 90^\circ$ **Hence Proved**

Example 6. AOC is diamtere of a circle and arc $AXB = \frac{1}{2}$ of arc BYC. then find the value of $\angle BOC$.

Solution : arc $AXB = \frac{1}{2}$ arc BYC

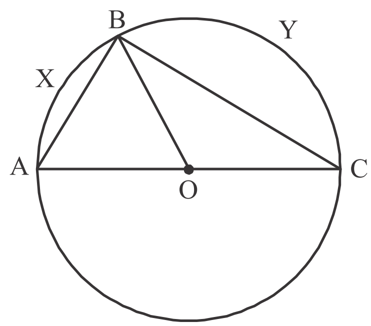


Fig. 12.41

$\therefore \qquad \angle AOB = \frac{1}{2} \angle BOC$

and $\qquad \angle AOB + \angle BOC = 180^\circ$

$\frac{1}{2} \angle BOC + \angle BOC = 180^\circ$

$\angle BOC = \frac{2}{3} \times 180^\circ = 120^\circ$

Example 7. Find the value of x in fig. 12.42.

Solution : $\angle DAC = \angle DBC = 30^\circ$ (Angles of same segment) $\ldots$ (i)

In Δ DBC

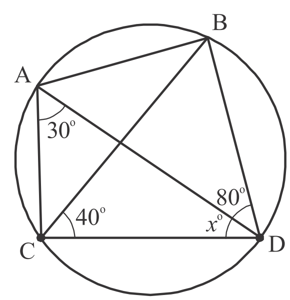


Fig. 12.42

$\angle DBC + \angle DCB + \angle BDC = 180^\circ$

$30^\circ + 40^\circ + \left(x + 80^\circ\right) = 180^\circ$ (from fig. and (i))

$x + 80 = 180 - 70$

$x = 110 - 80$

$x = 30^\circ$

Example 8. In fig. 12.43, $\Delta\, ABC$ is an equilateral triangle, and 'O' is the centre of the circle. Also AO is increased and then it meets the circle on D. Prove that $\Delta\, OBD$ is an equialteral triangle.

Solution : Given : Δ ABC is an equilateral triangle, O is centre of triangle ABC, on producing A it meets with centre at 'D'.

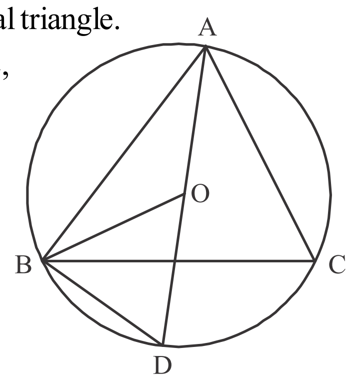


Fig. 12.43

To prove : Δ OBD is an equilateral triangle.

Proof : OB and OD (radius of same circle)

So $\qquad \angle OBD = \angle ODB \qquad \ldots$ (i)

$\because$ Δ ABC is an equilateral triangle

So $\qquad \angle C = 60^\circ \qquad \ldots$ (ii)

$\angle ADB = \angle C$ (from equation (ii) angle made on circle)

So $\angle ADB = 60^\circ$ (from equation (i))

but $\angle ADB$ and $\angle ODB$ (both are same)

but $\angle ODB = 60^\circ$

$\therefore$ $\angle OBD = 60^\circ$ (from equation (i))

But the sum of all the angle of triangle is 180°, so Δ OBD's third angle will also be 60°.

Hence Δ OBD is an equilateral triangle. **Hence proved**

Exercise 12.3

1. State true or false for every statement and also write the justification of your answers.
   (i) The angle subtended by any chord on any two points of circle are equal.
   (ii) In fig. 12.44, AB is diameter of circle and C is any point on the circle then $AC^2 + BC^2 = AB^2$
   (iii) In fig. 12.44, If $\angle ADE = 120^\circ$, then $\angle EAB = 60^\circ$
   (iv) In fig. 12.44, $\angle CAD = \angle CED$

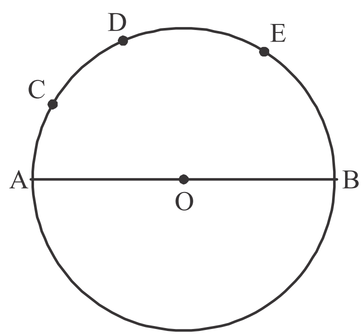


Fig. 12.44

2. In fig. 12.45 ; $\angle ABC = 45^\circ$, then prove that $OA \perp OC$

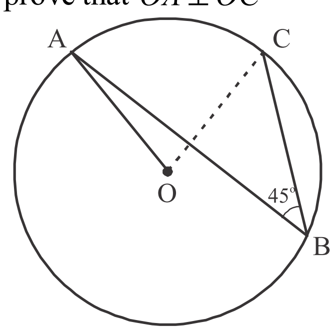


Fig. 12.45

3. O is the centre of a circle ΔABC and D is mid point of base BC then prove that $\angle BOD = \angle A$
4. On common diagonal AB two right triangles ACB and ADB are drawn in such a way that they are situated on opposite side, then prove that $\angle BAC = \angle BDC$
5. Two chords of AB and AC of a circle subtends angle of 90° and 150° at the centre. Find the angle $\angle BAC$ if AB and AC in opposite side of centre.
6. If O is the circumcentre of ΔABC, then prove that $\angle OBC + \angle BAC = 90^\circ$
7. If the length of a chord is equal to its radius, then find out the angle on the major segment in a circle.

8. In fig. 12.46, $\angle ADC = 130^\circ$ and chord BC = chord BE. then find the value of $\angle CBE$.

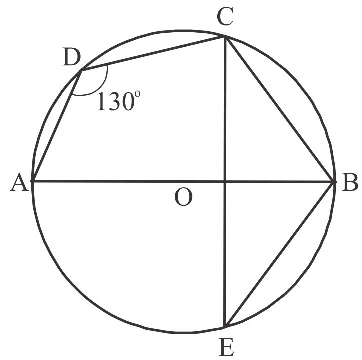


Fig. 12.46

9. In fig. 12.47, angle $\angle ACB = 40^\circ$, then find the value of $\angle OAB$.

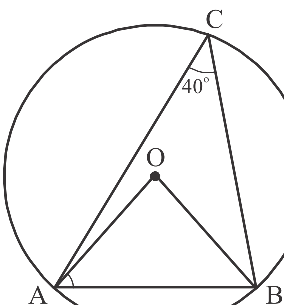


Fig. 12.47

10. In fig. 12.48, AOB is the diameter of a circle and C, D and E are three points situated on semi circle. Find the value of $\angle ACD + \angle BED$

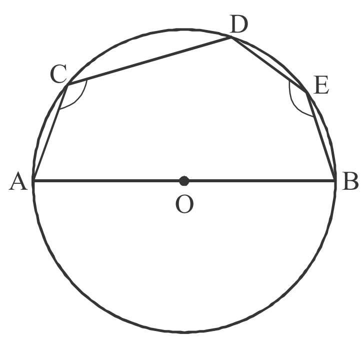


Fig. 12.48

12.8 Cyclic Quadrilateral

Such quadrilateral whose all four vertices are situated on the circle is known as Cyclic Quadrilateral. See fig. 12.49 and 12.50. These Quadrilateral have a special property for that observe carefully on an activity.

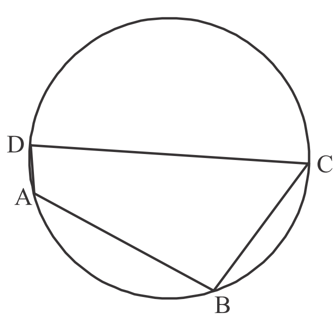


Fig. 12.49

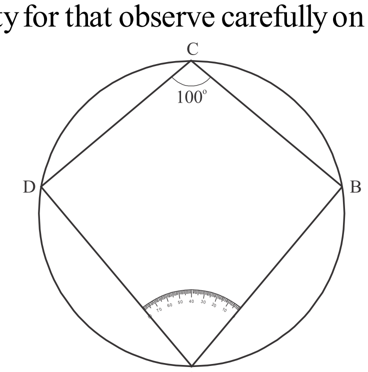


Fig. 12.50

Activity :

1. Draw a circle in your note book.
2. Paste a photocopy of protector at any point of circle, chosen arbitary by you. Here 80^o angle is pasted as shown in fig. 12.50 in $\angle A$.
3. Increase the sides of this angle so that these can touch the circle at any point. Thus you will get $\angle BAD$.
4. Expecting arc DAB, take any point C on the remaining part of circle and complete the quadrilateral.
5. $\angle A$ is opposite to $\angle C$. Find the value of $\angle C$. with the help of protector measure its value. Here you will get the value of $\angle BCD = 100^\circ$ in other words $\angle A + \angle C = 180^\circ$.

Sum of remaining two angles will also be 180^o because sum of all angle of quadrilateral is 360^o.

In other words, the opposite angles of a quadrilateral are supplementary. This can prove by theorem.

Theorem 12.8

Opposite angles of a quadrilateral are supplementary or their total is 180^o

Given : $ABCD$ is a cyclic quadrilateral.

To prove : $\angle A + \angle C = 180^\circ$

$\angle B + \angle D = 180^\circ$

Construction : Join O to B and D

Proof : By arc DAB angle made on centre of circle is x^o and on remaining part angle made is $\angle C$

So, $\angle C = \frac{1}{2} x^o$ . . . (i)

Similarly by arc DCB angle made on centre is y^o and on remaining part angle made is $\angle A$

So, $\angle A = \frac{1}{2} y^\circ$ . . . (ii)

On adding equation (i) and (ii)

$$\angle C + \angle A = \frac{1}{2}\left(x^\circ + y^\circ\right)$$

$$\angle C + \angle A = \frac{1}{2} \times 360^\circ = 180^\circ \qquad \text{. . . (iii)}$$

Since sum of all angles of quadirlateral is 360^o

So, $\angle B + \angle D = 360^\circ - (\angle A + \angle C)$

$\angle B + \angle D = 360° - 180°$ (from (iii))

$\angle B + \angle D = 180°$

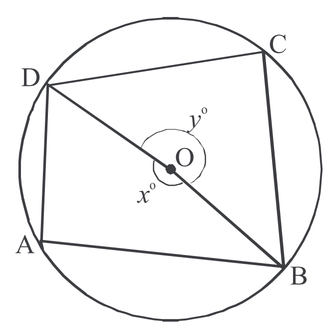


Fig. 12.51

Hence proved

Opposite of this theorem is also true as follows :

Theorem 12.9 : (Converse of theorem 12.8)

Opposite angles of a quadrilateral are supplementary, then it is a cyclic quadrilateral.

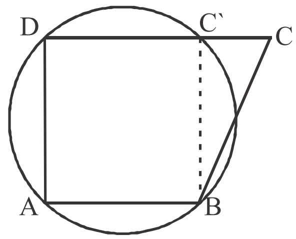


Fig. 12.52

Given : ABCD is a quadirlateral in which

$\angle BAD + \angle BCD = 180^\circ$

and $\angle ABC + \angle ADC = 180^\circ$

To prove : $ABCD$ is a cyclic quadrilateral.

Proof : Let a circle passes through A, B and D but in place of C it passes through C', then joining $C'B$, $ABC'D$ becomes a cyclic quadrilateral.

So $\angle BAD + \angle BC'D = 180^\circ$ (opposite angles of cyclic quadrilateral are supplemantary) . . . (i)

But $\angle BAD + \angle BCD = 180^\circ$ (given) . . . (ii)

from equation (i) and (ii)

$\angle BAD + \angle BC'D = \angle BAD + \angle BCD$

$\Rightarrow \angle BCD = \angle BC'D$ . . . (iii)

But $\angle BC'D$ is an exterior angle of $\Delta BCC'$, $\angle BC'D = \angle BCD + \angle CBC'$ (Exterior angles is sum of interior opposite angles)

$\Rightarrow \angle BC'D > \angle BCD$ . . . (iv)

from equation (iii) and (iv). It is clear $\angle BC'D > \angle BCD$ that is only possible when BC and BC' are coincide or point C and C' are coincide.

So, ABCD is a cyclic quadrilateral

Hence, ABCD is a cyclic quadrilateral **Hence proved**

12.9. Interior Oppostie Angles of Cyclic a Quadrialteral

By increasing a side of a cyclic quadrialteral, the angle made is known as exterior angle of cyclic quadrilateral. (see fig. 12.53) $\angle CBE$ is exterior angle of cyclic quadrilaterel . Similarly you can draw an exterior angle on every vertex of cyclic quadrilateral.

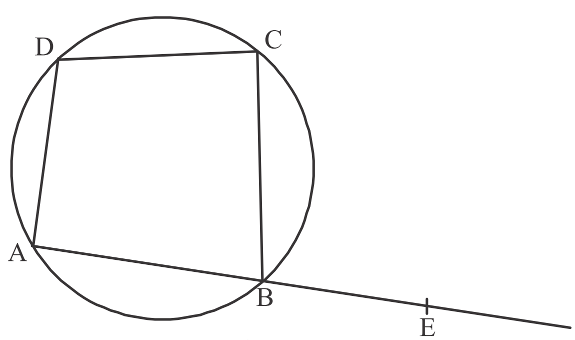


Fig. 12.53

$\angle ABC$ and $\angle ADC$ are known as corresponding and interior opposite angles of exterior angle of $\angle CBE$.

Let us understand what is the relation between exterior and interior opposite angles. Let us try an activity for this.

Activity :

1. Draw a circle.
2. As in fig. 12.53 take any two points *A* and *B* and increase them by joining them up to *E*.

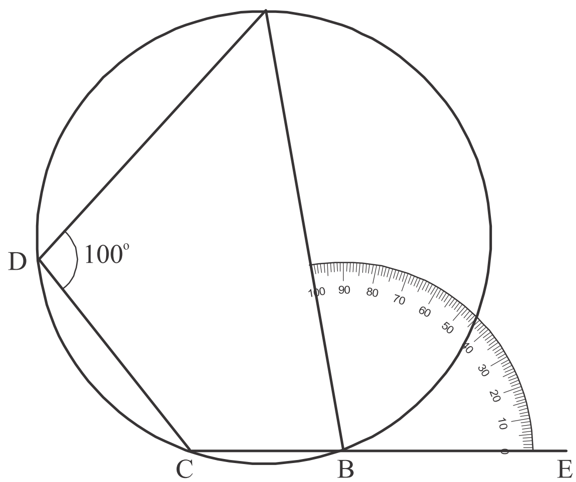


Fig. 12.54

3. From the photocopy of protector cut a portion of a sutaible a measurment (Here angles 100°and paste it in at point B such a way that coincide with base BE)
4. Increase the side *EB* so that it will meet at a point *C* of a circle.
5. Arc of circle ABC expect take a point *D* on the remaining portion of circle complete the cyclic quadrilateral.
6. Measure $\angle ADC$ with the help of protector,you will find that the measurement of angle $\angle ADC = 100^{o}$ i.e. the measurment of exterior angle of cyclic quadrilateral is equal to interior opposite angle of cyclic quadrilateral.

   Let us prove this with the help of theorem.

Theorem 12.10 : By increasing a side of cyclic quadrilateral the make exterior angle is equal to interior opposite angle.

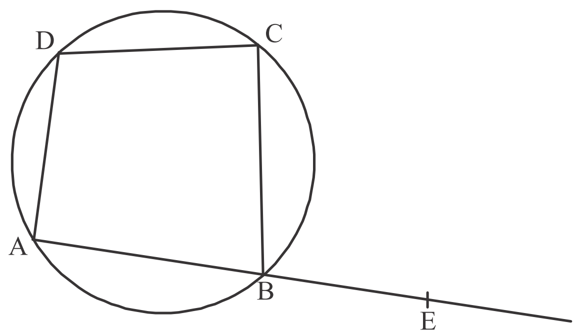


Fig. 12.55

Given : *ABCD* is a cyclic quadrilateral whose *AB* is increased up to *E* point.

To prove : $\angle CBE = \angle ADC$

Proof : *ABCD* is a cyclic quadrilateral

So, $\angle ABC + \angle ADC = 180^{o}$ . . . (i)

$\angle ABC + \angle CBE = 180^{o}$ (from linear pair) . . . (ii)

From equation (i) and (ii)

$$\angle ABC + \angle ADC = \angle ABC + \angle CBE$$

or $\angle ADC = \angle CBE$ **Hence proved**

Can you prove the opposite of this theorem if the exterior angle and interior opposite angles are equal then it is a cyclic quadrilateral.

Theorem 12.11 :

In a quadrilateral, the exterior angle made by increasing a side of quadrilateral, is equal to interior opposite to angle, then it is a cyclic quadrilateral.

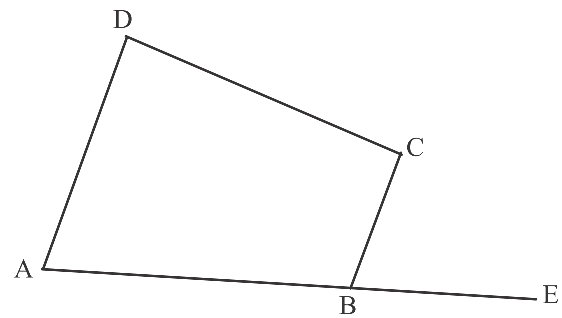


Fig. 12.56

Given : The $\angle CBE$ is exterior angle of quadrialteral $ABCD$,

$$\angle ADC = \angle CBE$$

To prove : ABCD is a cyclic quadrilateral.

Proof : $\angle ADC = \angle CBE$ (given)

Adding $\angle ABC$ on both side

$$\angle ABC + \angle ADC = \angle ABC + \angle CBE$$

$$\angle ABC + \angle CBE = 180^\circ \quad \text{(From linear angle pair)}$$

So, $\angle ABC + \angle ADC = 180^\circ$

Since $\angle ABC$ and $\angle ADC$ are the opposite angles of quadrialteral $ABCD$.

Hence $ABCD$ is a cyclic quadrilateral

Illustrative Examples

Example 1. In fig 12.57, ABCD is a cyclic quadrilateral if $\angle AOC = 136^\circ$ then find the value of $\angle ABC$

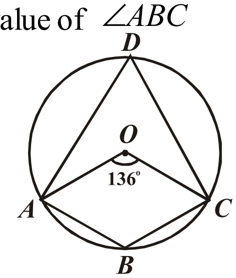


Fig. 12.57

Solution : The arc ABC makes an angle on centre O and remaining part arc

$\angle AOC$ and $\angle ADC$.

So $\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2} \times 136^\circ$

$\angle ADC = 68^\circ$

$\therefore$ $ABCD$ is a quadrilateral there fore sum of oppostie angle will be 180°

$\therefore$ $\angle ADC + \angle ABC = 180^\circ$

$\angle ABC = 180^\circ - 68^\circ$

or $\angle ABC = 112^\circ$

Example 2. In fig. 12.58, $ABCD$ is a cyclic quadrilateral, find the value of x and y.

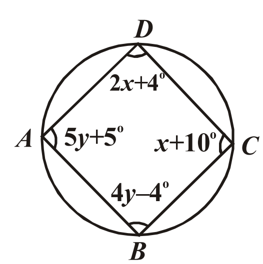


Fig. 12.58

Solution : Opposite angles of a cyclic.

$$(2x + 4^\circ) + (4y - 4^\circ) = 180^\circ$$

or $2x + 4y = 180^\circ$

or $x + 2y = 90^\circ \quad \ldots (1)$

Similarly $(x + 10^\circ) + (5y + 5^\circ) = 180^\circ$

or $x + 5y = 165^\circ \quad \ldots (2)$

By solving equation (i) and (ii)

$$x = 40, \quad y = 25$$

So $\quad x = 40 \quad y = 25$

Example 3. In fig. 12.59, the centre of circle is O and arc BCD subtended an angle 140° at the centre of circle.Find the value of $\angle BAD$ and $\angle DCE$.

Solution : By arc BCD the angle made on centre and remaining part are $\angle BOD$ and $\angle BAD$.

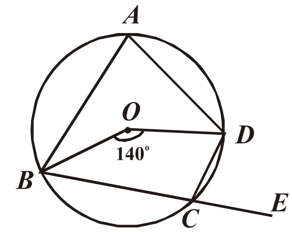


Fig. 12.59

So, $\quad \angle BAD = \frac{1}{2} \times \angle BOD = \frac{1}{2} \times 140^\circ$

So, $\quad \angle BAD = 70^\circ$

But $\angle DCE$, is exterior angle of cyclic quadrilateral

Therefore, exterior angle will be equal to interior opposite angles.

$$\angle DCE = \angle BAD$$

or $\quad \angle DCE = 70^\circ$

Example 4. In fig 12.60, ABCD is a cyclic quadrilateral drawn a line AE parallel to CD and BA is increased upto F. If $\angle ABC = 92^\circ$ and $\angle FAE = 20^\circ$, then find the value of $\angle BCD$.

Solution : $ABCD$ is a cyclic quadirlateral

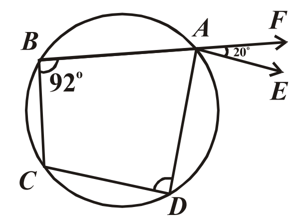


Fig. 12.60

$$\angle ABC + \angle CDA = 180^\circ$$

$\Rightarrow \quad \angle CDA = 180^\circ - 92^\circ$

$\Rightarrow \quad \angle CDA = 88^\circ$

But $\quad CD \,||\, AE$

$$\angle DAE = \angle CDA$$

$$\angle DAE = 88^\circ$$

$$\angle DAF = \angle FAE + \angle DAE = 20^\circ + 88^\circ$$

$$\angle DAF = 108^\circ$$

$$\angle DAB = 180^\circ - 108^\circ = 72^\circ$$

Now, $\quad \angle BCD + \angle DAB = 180^\circ$

$$\angle BCD = 180^\circ - \angle DAB = 180^\circ - 72^\circ$$

$$\angle BCD = 108^\circ$$

Example 5. Prove that, cyclic parallelogram is a rectangle.

Solution : Given : ABCD is a cyclic parallelogram ,

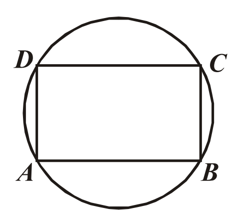


Fig. 12.61

So, $\quad \angle B + \angle D = 180^\circ \quad \ldots$ (i)

$\angle B = \angle D \quad \ldots$ (ii)

From equation (i) and (ii)

$$\angle B = \angle D = 90^\circ$$

Similarly, $\angle A = \angle C = 90^\circ$

Hence, $ABCD$ is a rectangle. **Hence Proved.**

Example 6. If two sides are parallel of a cyclic quadrilateral then prove that remaining sides and diagonal are equal.

Solution : Given : In cyclic quadrilateral ABCD

$AB \parallel DC$

To prove : (i) $AD = BC$

(ii) $AC = BD$

Proof : $AB \parallel DC$ and BC is a transverasl line.

So, $\angle ABC + \angle DCB = 180^\circ$ . . . (i)

But, ABCD is cyclic quadrilateral

So, $\angle ABC + \angle ADC = 180^\circ$ . . . (ii)

From equation (i) and (ii)

$\angle DCB = \angle ADC$ . . . (iii)

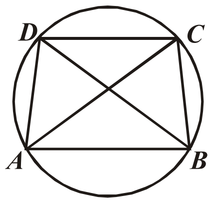


Fig. 12.62

Now, In ΔADC and ΔBCD

$\angle ADC = \angle DCB$ (from equation (iii))

$\angle DAC = \angle DBC$ (angle of same sector)

and $DC = DC$ (common)

$\therefore$ $\Delta ADC \cong \Delta BCD$ (ASA congurence)

So, the corresponding sides are equal of congurence trianlge.

hence $AD = BC$

and $AC = BD$ **Hence proved**

Example 7. In fig. 12.63, $ABCD$ is a quadrilateral, in which $AD = BC$ and $\angle ADC = \angle BCD$, then prove that $ABCD$ is a cyclic quadrilateral.

Solution : Given : In a quadrilateral ABCD, AD = BC and $\angle ADC = \angle BCD$.

To prove : $ABCD$ is a cyclic quadrilateral

Construction : $DN \perp AB$ and $CM \perp AB$ drawn

Proof :

$\angle ADC = \angle BCD$ (given) . . . (i)

$\therefore$ $\angle ADN = \angle ADC - 90^\circ$

$= \angle BCD - 90^\circ$ [from equation (i)]

$\angle ADN = \angle BCM$ . . . (ii)

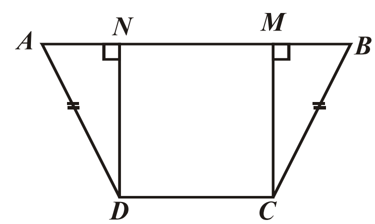


Fig. 12.63

Now, In ΔAND and ΔBMC

$\angle AND = \angle BMC$ (right angle)

$\angle ADN = \angle BCM$ [from equation (ii)]

and $AD = BC$ (given)

$\therefore$ $\Delta AND \cong \Delta BMC$ (AAS congurence)

Therefore the corresponding angle will be equal of congurence triangles.

Hence, $\angle A = \angle B$ . . . (iii)

Similarly $\angle C = \angle D$ . . . (iv)

But $\angle A + \angle B + \angle C + \angle D = 360^\circ$

From equation (iii) and (iv),

$$2\angle B + 2\angle D = 360^\circ$$

$\therefore$ $\angle B + \angle D = 180^\circ$

$\therefore$ $ABCD$ is a cyclic quadrilateral **Hence proved**

Exercise 12.4

1. One angle of cyclic quadrilateral is given, then find the opposite angles.

   (i) 70° (ii) 135° (iii) $112\frac{1}{2}^\circ$ (iv) $\frac{3}{5}$ right angle (v) 165°

2. Find the opposite angle of cyclic qudrilateral in which one angle is

   (i) $\frac{2}{7}$ of other angle (ii) $\frac{11}{4}$ of other angle

3. In fig. 12.64, find the all four angles of a cyclic quadrilateral $ABCD$.

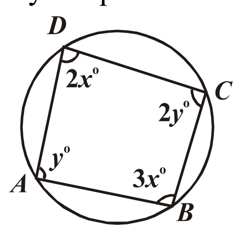


Fig. 12.64

4. In fig. 12.65, few angles are denoted by a, b, c and d. Find the measurement of these angle.

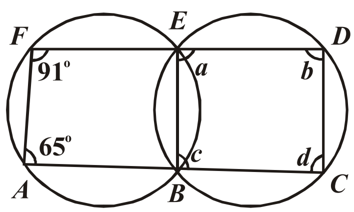


Fig. 12.65

5. If in a cyclic quadrilateral $ABCD$, $AD \parallel BC$, then prove $\angle A = \angle D$.
6. $ABCD$ is a cyclic quadrilateral, on increase AB and DC, meet at point E. Prove that ΔEBC and ΔEDA are similar.
7. Prove that a quadrilateral made from bisector of angles of cyclic quadrilateral is also a cyclic quadrilateral.

Miscellaneous Exercise - 12

1. In a circle of a radius 10 cm the length of chord which is 6 cm apart form the centre is.

   (a) 16 cm (b) 8 cm (c)) 4 cm (d) 5 cm

2. In a circle of radiud 13 cm a 24 cm long chord has been drawn, then the distance of chord from the centre is

   (a) 12 cm (b) 5 cm (c) 6.5 cm (d) 12 cm

3. The measurment of short arc is :

   (a) less the 180° (b) more than 180° (c) 360° (d) 270°

4. Measurment of big arc is :

   (a) less then 180° (b) more than 180° (c) 360° (d) 90°

5. In a circle the chords are at same distance from the centre, than one chord is .... of the other.

(a) double (b) triple (c) half (d) equal

6. Measurment of an arc is 180^0, then the arc is :

(a) big arc (b) short arc (c) circle (d) half circle

7. Number of circles which passes through three collinear points is.

(a) one (b) two (c) zero (d) infinite

8. If in any circle arc AB = arc BA, then arc is :

(a) big arc (b) short arc (c) half circle (d) circle

9. If the diameter of a circle bisect the two chords, then these chord will be :

(a) parallel (b) perpendicular (c) intersecting (d) none of the above

10. If the arc in two congurent circles are equal, then their corresponding chords will be :

(a) parallel (b) equal (c) perpendicular (d) intersecting

11. If AD is a diameter of any circle and AB is chord. If AD = 34 cm, AB = 30 cm, then the distance of AB from centre is :

(a) 17 cm (b) 15 cm (c) 4 cm (d) 8 cm

12. In fig. 12.66, if OA = 5 cm, AB = 8 cm and chord OD is perpendicualr AB, then CD is equal to :

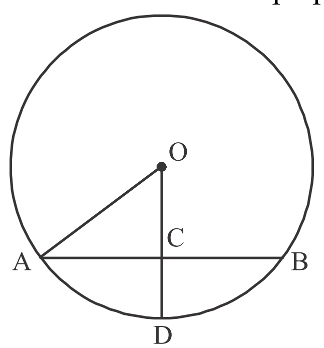


Fig. 12.66

(a) 2 cm (b) 3 cm (c) 4 cm (d) 5 cm

13. If AB = 12 cm, BC = 16 cm and line segment AB is perpendicular on BC, then the radius of circle passing through A, B and C is :

(a) 6 cm (b) 8 cm (c) 10 cm (d) 12 cm

14. In fig. 12.67, if $\angle ABC = 20^o$, then $\angle AOC$ is equal to :

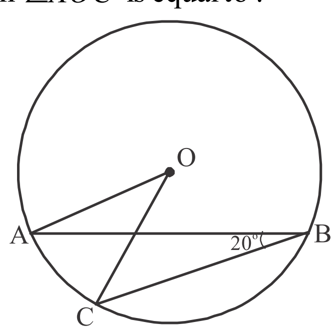


Fig. 12.67

(a) 20^0 (b) 40^0 (c) 60^0 (d) 10^0

15. In fig. 12.68, if AOB is a diameter of circle and AC = BC. then $\angle CAB$ is equal to -

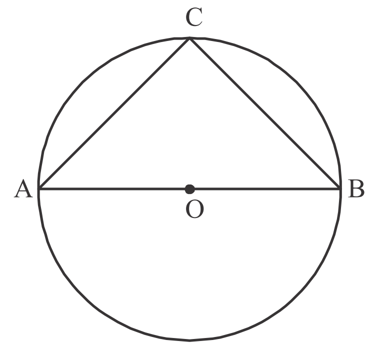


Fig. 12.68

(a) 30° (b) 60° (c) 90° (d) 45°

16. In fig. 12.69, if $\angle OAB = 40^\circ$, then $\angle ACB$ is equal to :

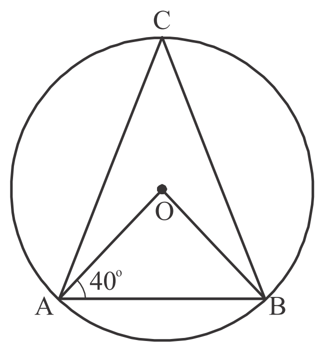


Fig. 12.69

(a) 50° (b) 40° (c) 60° (d) 70°

17. In fig. 12.70, if $\angle DAB = 60^\circ$ and $\angle ABD = 50^\circ$, then $\angle ACB$ is equal to ;

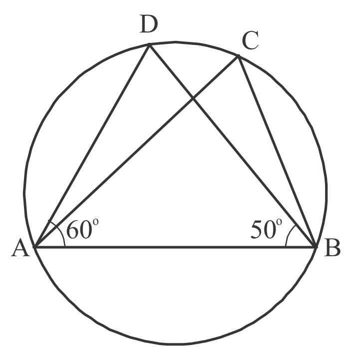


Fig. 12.70

(a) 60° (b) 50° (c) 70° (d) 80°

18. Side AB of a quadrilateral is diameter of its outer circle and $\angle ADC = 140^\circ$ then $\angle BAC$ is equal to :

(a) 80° (b) 50° (c) 40° (d) 30°

19. In fig. 12.75, BC is the diameter and $\angle BAO = 60^\circ$ then $\angle ADC$ is equal to :

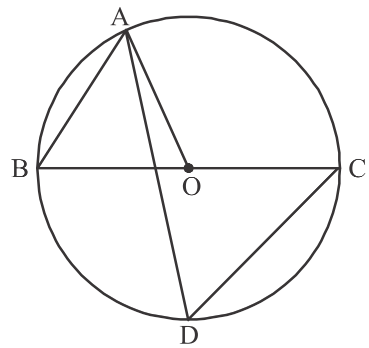


Fig. 12.71

(a) 30° (b) 45° (c) 60° (d) 120°

20. In fig. 12.72. $\angle AOB = 90^\circ$ and $\angle ABC = 30^\circ$ then $\angle CAO$ is equal to :

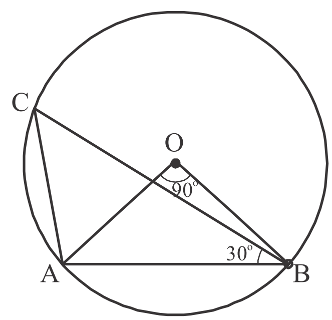


Fig. 12.72

(a) 30° (b) 45° (c) 90° (d) 60°

21. If two equal chords of a circle intersect each other, then prove that two parts of a chord are equal to other are equal to other corresponding both part of chord.
22. If *P, Q* and *R* are the mid points of sides *BC*, *CA* and *AB* of a triangle and *AD* is perpendicular on *BC* form vertex *A*, then prove that *P, Q, R* and *D* are cyclic.
23. *ABCD* is a parallelogram, a circle is drawn passing through *A* and *B* in such a way that it intersect *AD* at *P* and *BC* at *Q*, then prove that *P*, *Q*, *C* and *D* are cyclic
24. Prove that the bisector of any angle of triangle and perpendicular bisector of its opposite side if intersect each other then it intersect on circumcircle.
25. In any circle *AYDZBWCX,* if two chord AB and CD intersect each other at right angle , see fig. 12.73, then prove arc CXA + arc DZB = arc AYD + arc BWC = Semi circle.

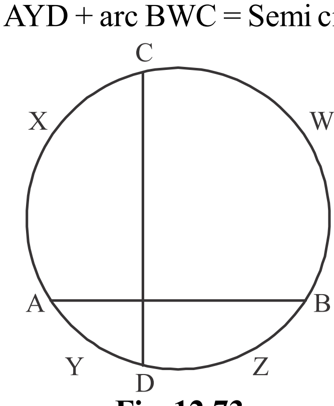


Fig. 12.73

26. If ABC is equilateral triangle in a circle and P is any point on short arc BC, which is not coincidence of B and C. Then prove that PA is bisector of $\angle BPC$.

27. In fig. 12.74, AB and CD are two chords of a circle which intersect each other at point E, then prove that

$\angle AEC = \frac{1}{2}$ (Angle made by arc CXA on centre + Angle made by arc DYB on centre)

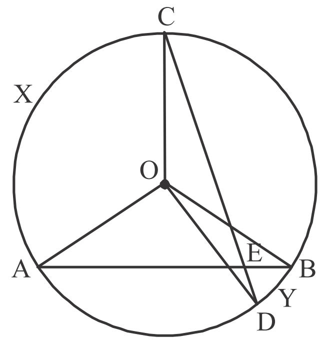


Fig. 12.74

28. If bisectors of oppostite angles in a cyclic quadrilateral *ABCD*, intersect the circular at *P* and *Q* points. then prove that *PQ* is the diameter of circle.

29. The radius of a circle is $\sqrt{2}$ cm . This circle is divided into two parts by 2 cm long chord. Then prove that an angle of 45° is subtends at any point of major segment by this arc.

30. *AB* and *AC* are two chords having radius *r* of a circle in such a way that AB = 2AC. If the distance of *AB* and *AC* from the centre is p and q respectively then prove that $4q^2 = p^2 + 3r^2$

31. In fig. 12.75 , O is the centre of circle $\angle BCO = 30^{\circ}$, then find the value of x and y.

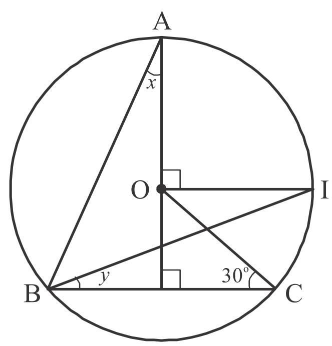


Fig. 12.75

32. In fig. 12.76, O is the centre of circle, Where BD = OD and $CD \perp AB$, then find the value of $\angle CAB$.

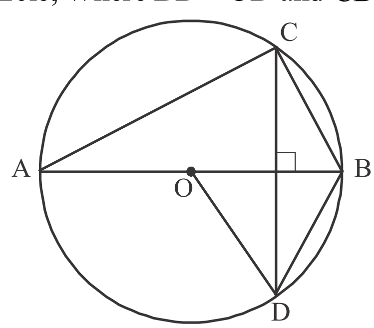


Fig. 12.76

33. Prove that out of all chords which passes through any point of circle, that chord will be smallest which is perpendicular on diameter which passes through that point.

Important Points

1. A circle is a set of all those points lying in the plane which are at a constant distance form a fixed point lying in that plane.
2. Equal chords of a circle (or congurent circles) subetend equal angle at the centre of circle (or corresponding centres).
3. If two chords of any circle (or congurent circle) subtend equal angles at a centre (or on corresponding centres) then chords are equal.
4. The perpendicular drawn from centre of a circle to any chord, it bisects the chord.
5. Any chord passing through centre of circle and bisects any other chord, then it is perpendicular on the chord.
6. Only one circle can be drawn from three non-collinear points .
7. Chords of a circle (or congurent circle) are at equal distance form centre of circle (or corresponding centres of circle).
8. Chords equidistant from centre of a circle, are equal in size.
9. If two arcs of any circle are congurent, then corresponding chord will also be equal and opposite, if two chords of circle are equal then their corresponding arc (long, short) are aslo congurent.
10. Congurent arc of any circle make equal angle at the centre of circle.
11. Angle subtend by any arc at the centre is double of that subtend by same arc in remaining portion of circle at any point.
12. Angles are equal in same segment.
13. The angle in a semi circle subtendended by its base is right angle.
14. If the line segment joining the two points and their inclined line subtend same angle on two other points then all four points will be on the circle subtend equal angle on two other points then all four points will be on the circle.
15. Sum of two opposite angles of cyclic quadrilateral is 180^o.
16. If the sum of two opposite angles of quadrilateral is 180^o, then it will be cyclic quadrilateral.
17. By producing the one side of cyclic quadrilateral the exterior angle so formed is equal to the opposite interior angle.

Answer Sheet

Exercise 12.1

1. (i) Interior (ii) Exterior (iii) Diameter (iv) Semi Circle (v) Three
2. (i) True (ii) False (iii) False (iv) True (v) True (vi) False (vii) False

Exercise 12.2

1. (i) False : Because larger chord subtend larger angle as compared to small chord.

(ii) False : Because larger chord situated at small distance form the circle.

(iii) True : Because both chords are at equal distance form the centre.

(iv) True : Because equal chords of congruent circles subtends equal angle at corresponding centres of circles.

(v) False : Because a circle which passes through two points can not pass through the third point of line.

(vi) True : Because AB is diameter.

2. 12 cm   3. $3\sqrt{5}$ cm   7. 13 cm   8. (i) 2 cm (ii) 14 cm

Exercise 12.3

1. (i) False : If both points are situated on one side (major or minor segment), then become equal otherwise not.

(ii) False : Because $\angle C$ is right angle and $AB^2 = AC^2 + BC^2$

(iii) True : After joining AD, DE, DB and EB , $\angle ADB = 90^\circ$ then $\angle BDE = 120 - 90 = 30^\circ$, here $\angle BDE = \angle EAB = 30^\circ$ as subtended on the same arc segment.

(iv) True : Since chords of congruent circles subtend same angle at the corresponding centres.

(v) False : Since, circle passing through two out of three collinear points can never pass through third collinear point.

(vi) True : $\angle CAD = \angle CED$ as these are angles subtend on the same arc segment after joining AC, CD, AD, DE and CE.

2. 120° 6. 60° 9. 100° 10. 50° 11. 270°

Exercise 12.4

1. (i) 110° (ii) 45° (iii) $67\frac{1}{2}^\circ$ (iv) 126° (v) 15°
2. (i) $45^\circ, 40^\circ$ (ii) $132^\circ, 48^\circ$
3. $\angle A = 60^\circ$, $\angle B = 108^\circ$, $\angle C = 120^\circ$, $\angle D = 72^\circ$
5. $a = 65^\circ, b = 89^\circ, c = 91^\circ, d = 115^\circ$

Miscellaneous Exercise 12

1. (a) 2. (b) 3. (a) 4. (b) 5. (d) 6. (d) 7. (a) 8. (c) 9. (a) 10. (b) 11. (d) 12. (a) 13. (c) 14. (b) 15. (d) 6. (a) 17. (c) 18. (b) 19. (c) 20. (d)