Pair of Linear Equations in Two Variables

Exercise 4.1

Q. 1 A. By comparing the ratios $\dfrac{a_1}{a_2}, \dfrac{b_1}{b_2}, \dfrac{c_1}{c_2},$ **find out whether the lines represented by the following pairs of linear equations intersect at a point, are parallel or are coincident.**

$5x - 4y + 8 = 0$
$7x + 6y - 9 = 0$

Answer : We have,

$5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Here, $a_1 = 5$, $b_2 = -4$, $c_1 = 8$

$a_1 = 7$, $b_2 = 6$, $c_1 = -9$

$\therefore \dfrac{a_1}{a_2} = \dfrac{5}{7}, \dfrac{b_1}{b_2} = -\dfrac{4}{6} = -\dfrac{2}{3}, \dfrac{c_1}{c_2} = \dfrac{8}{9}$

$\therefore$ Two line are intersecting with each at a point.

Q. 1 B. y comparing the ratios $\dfrac{a_1}{a_2}, \dfrac{b_1}{b_2}, \dfrac{c_1}{c_2},$ **find out whether the lines represented by the following pairs of linear equations intersect at a point, are parallel or are coincident.**

$9x + 3y + 12 = 0$
$18x + 6y - 24 = 0$

Answer : We have,

$9x + 3y + 12 = 0$

$18x + 6y - 24 = 0$

Here, $a_1 = 9$, $b_2 = 3$, $c_1 = 12$

$a_1 = 18$, $b_2 = 6$, $c_1 = -24$

$$\therefore \frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{6}, \frac{c_1}{c_2} = \frac{12}{-24} = -\frac{1}{2}$$

∴ Both the lines will coincide.

Q. 1 C. By comparing the ratios $\dfrac{a_1}{a_2}, \dfrac{b_1}{b_2}, \dfrac{c_1}{c_2},$ **find out whether the lines represented by the following pairs of linear equations intersect at a point, are parallel or are coincident.**

6x − 3y + 10 = 0
2x - y + 9 = 0

Answer : We have,

6x − 3y + 10 = 0

2x - y + 9 = 0

Here, $a_1 = 6$, $b_2 = -3$, $c_1 = 10$

$a_1 = 2$, $b_2 = -1$, $c_1 = 9$

$$\therefore \frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1}, \frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1}, \frac{c_1}{c_2} = \frac{10}{9}$$

∴ Two line are parallel to each other.

Q. 2 A. Check whether the following equation are consistent or inconsistent. Solve them graphically.

3x + 2y = 5
2x -3y = 7

Answer : 3x + 2y = 5 2x -3y = 7

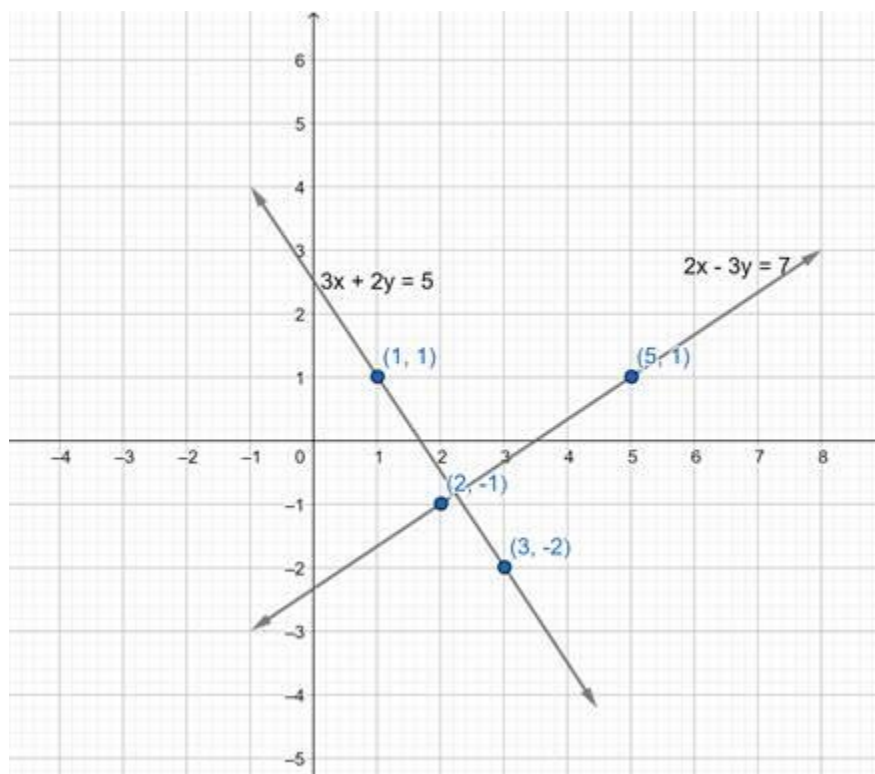| x	1	3
y	1	-2

x	5	2
y	1	-1

Here, $a_1 = 3$, $b_2 = 2$, $c_1 = 5$

$a_1 = 2$, $b_2 = -3$, $c_1 = 7$

$$\therefore \frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2}{-3}, \frac{c_1}{c_2} = \frac{5}{7}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

∴ two equations are consistent.

Q. 2 B. Check whether the following equation are consistent or inconsistent. Solve them graphically.
2x - 3y = 8
4x -6y = 9

Answer : 2x - 3y = 8 4x -6y = 9

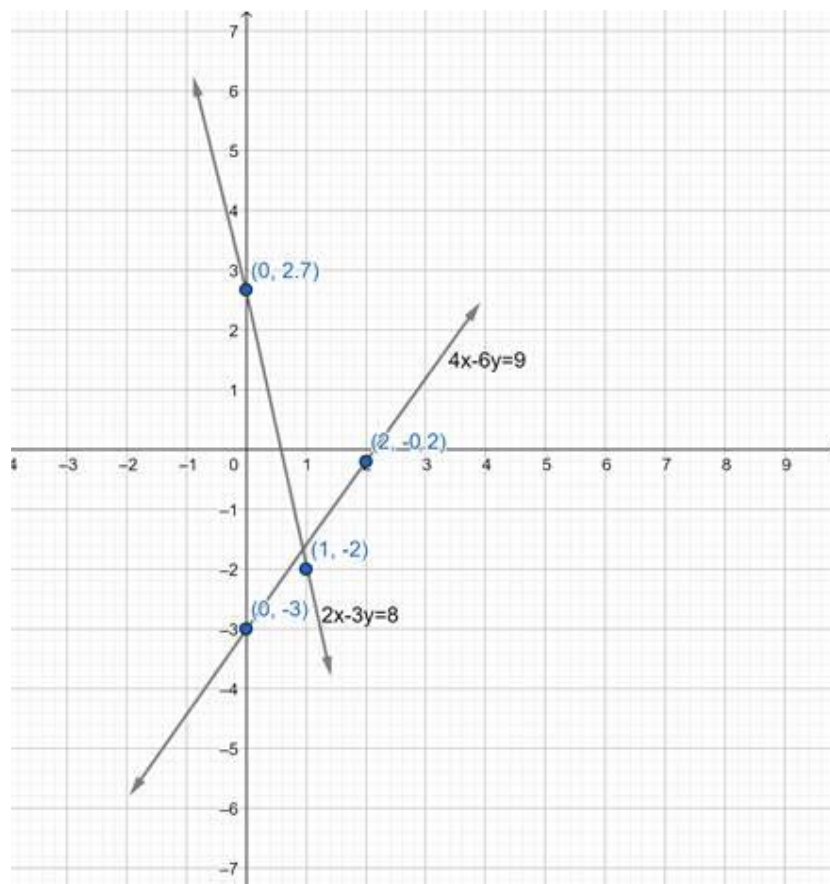| X	1	0
Y	-2	3
x	0	2
y	-3	$-\dfrac{1}{5}$

Here, $a_1 = 2$, $b_2 = -3$, $c_1 = 8$

$a_1 = 4$, $b_2 = -6$, $c_1 = 9$

$$\therefore \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{8}{9}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

∴ both lines are inconsistent

Q. 2 C. Check whether the following equation are consistent or inconsistent. Solve them graphically.

$$\frac{3}{2}x + \frac{5}{3}y = 7$$

9x -10y = 12

Answer :

$\frac{3}{2}x + \frac{5}{3}y = 7$ 9x -10y = 12

x	1	0	2
y	3.3	$\frac{21}{5}$	2.4

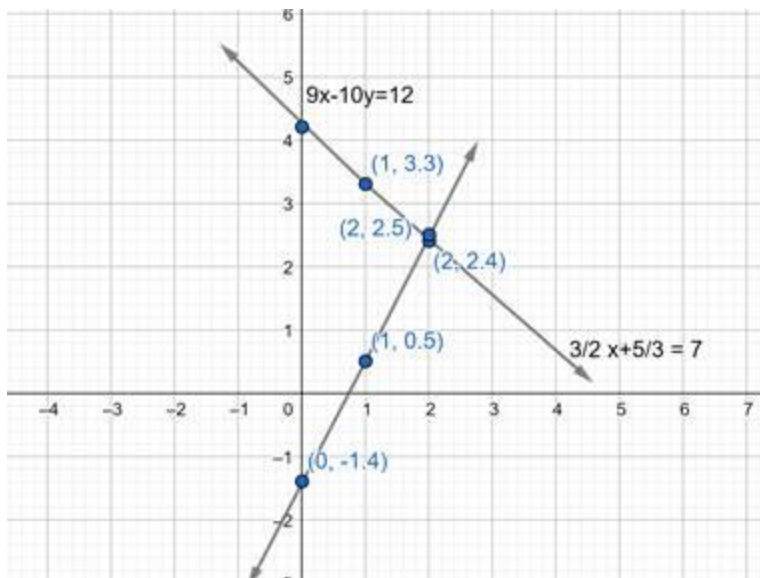| x	1	0	2
y	1/5	$-\frac{14}{10}$	$\frac{2}{5}$

Here, $a_1 = \frac{3}{2}$, $b_2 = \frac{5}{3}$, $c_1 = 7$

$a_1 = 9$, $b_2 = -10$, $c_1 = 12$

$\therefore \frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{1}{6}, \frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = \frac{1}{-6}, \frac{c_1}{c_2} = \frac{7}{12}$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

∴ both lines are consistent

Q. 2 D. Check whether the following equation are consistent or inconsistent. Solve them graphically.

$5x - 3y = 11$
$-10x + 6y = -22$

Answer : $5x - 3y = 11$ $-10x + 6y = -22$

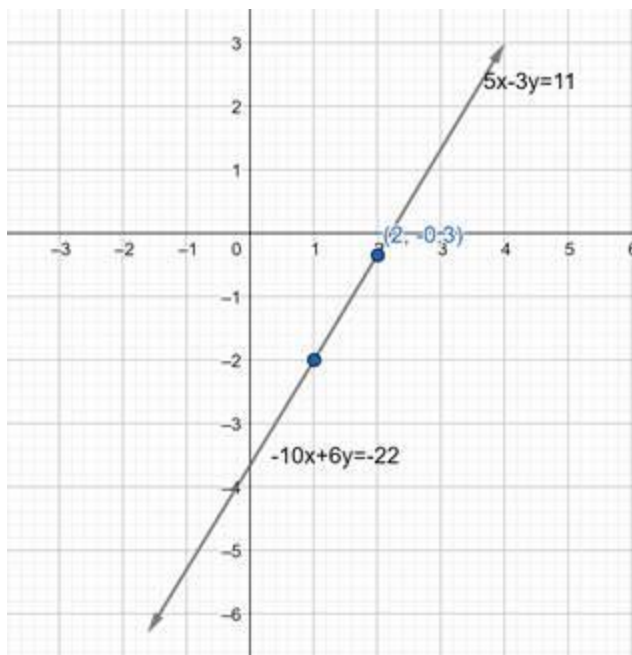| x	1	2
y	-2	$-\dfrac{17}{3}$

x	1	2
y	-2	$-\dfrac{17}{3}$

Here, $a_1 = 5$, $b_2 = -3$, $c_1 = 11$

$a_1 = -10$, $b_2 = 6$, $c_1 = -22$

$$\therefore \frac{a_1}{a_2} = \frac{5}{-10} = \frac{1}{-2}, \frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2}, \frac{c_1}{c_2} = \frac{11}{-22} = \frac{1}{-2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\therefore$ two lines are consistent

Q. 2 E. Check whether the following equation are consistent or inconsistent. Solve them graphically.

$$\frac{4}{3}x + 2y = 8$$

2x + 3y = 12

Answer :

$$\frac{3}{2}x + \frac{5}{3}y = 7 \quad 9x - 10y = 12$$

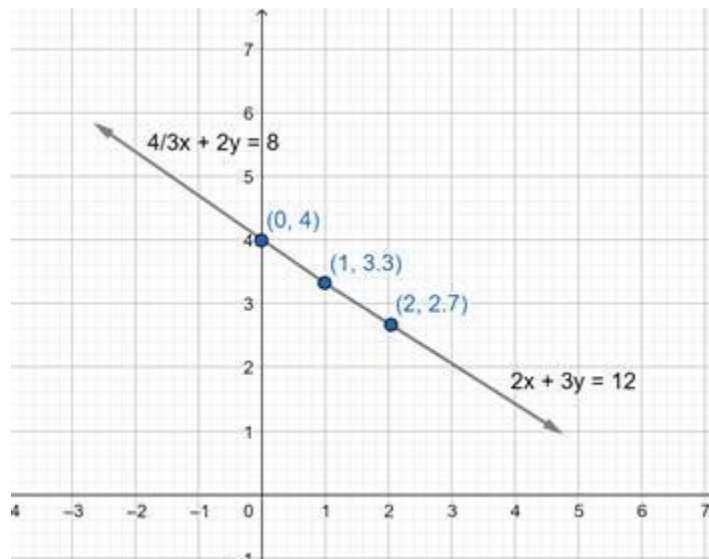| x	0	2
y	4	$\frac{8}{3}$

x	0	2
y	4	$\frac{8}{3}$

Here, $a_1 = \frac{4}{3}$, $b_2 = 2$, $c_1 = 8$

$a_1 = 2$, $b_2 = 3$, $c_1 = 12$

$$\therefore \frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{2}{3}, \frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

∴ two equations are consistent

Q. 2 F. Check whether the following equation are consistent or inconsistent. Solve them graphically.

x + y = 5
2x + 2y = 10

Answer : x + y = 5 2x + 2y = 10

x	0	1	3
y	5	4	2

x	0	1	3
y	5	4	2

Here, $a_1 = 1$, $b_2 = 1$, $c_1 = 5$
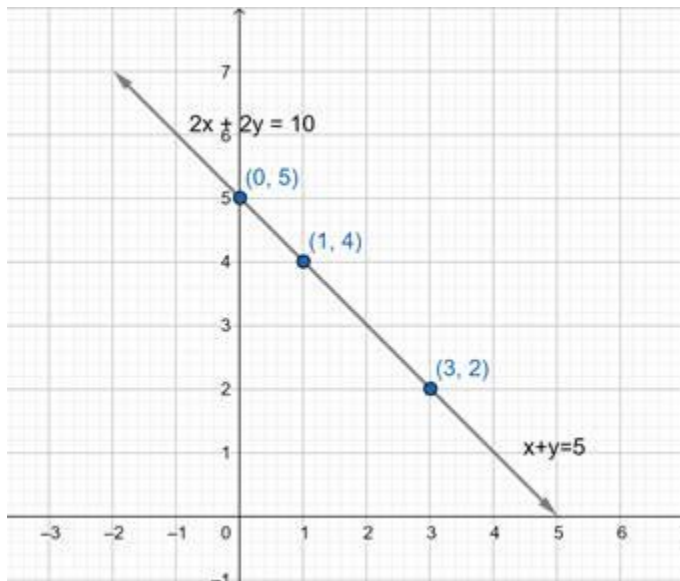
$a_1 = 2$, $b_2 = 2$, $c_1 = 10$

$$\therefore \frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{5}{10} = \frac{1}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\therefore$ two lines are inconsistent

Q. 2 G. Check whether the following equation are consistent or inconsistent. Solve them graphically.

x - y = 8
3x + 3y = 16

Answer : x - y = 8 3x + 3y = 16

X	0	1	6
Y	-8	-7	-2

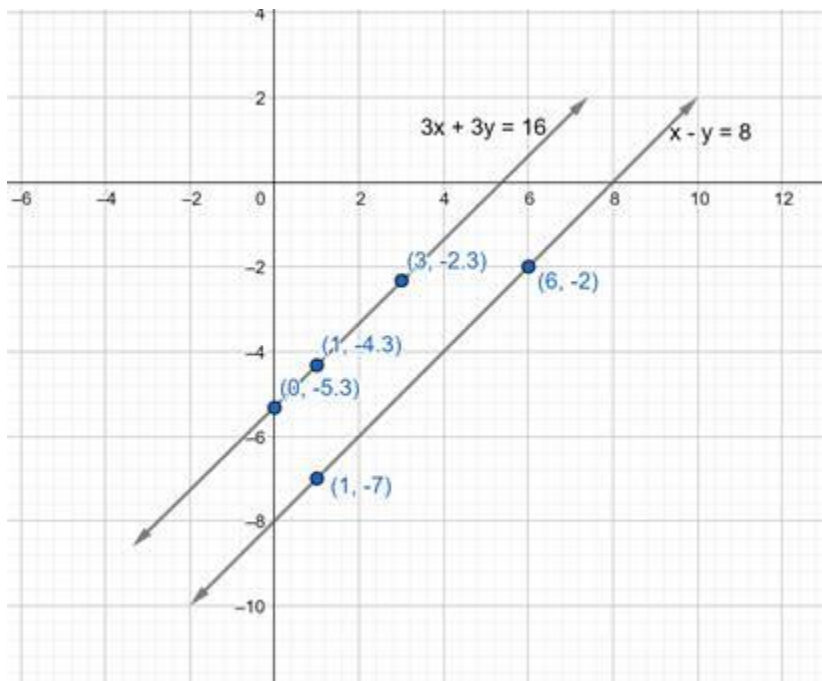| x	0	1	3
y	$-\frac{16}{3}$	$-\frac{13}{3}$	$-\frac{7}{3}$

Here, $a_1 = 1$, $b_2 = -1$, $c_1 = 8$

$a_1 = 3$, $b_2 = 3$, $c_1 = 12$

$$\therefore \frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = -\frac{1}{3}, \frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

∴ two lines are inconsistent

Q. 2 H. Check whether the following equation are consistent or inconsistent. Solve them graphically.

2x + y = 6   4x - 2y = 4

Answer : 2x + y = 6 4x - 2y = 4

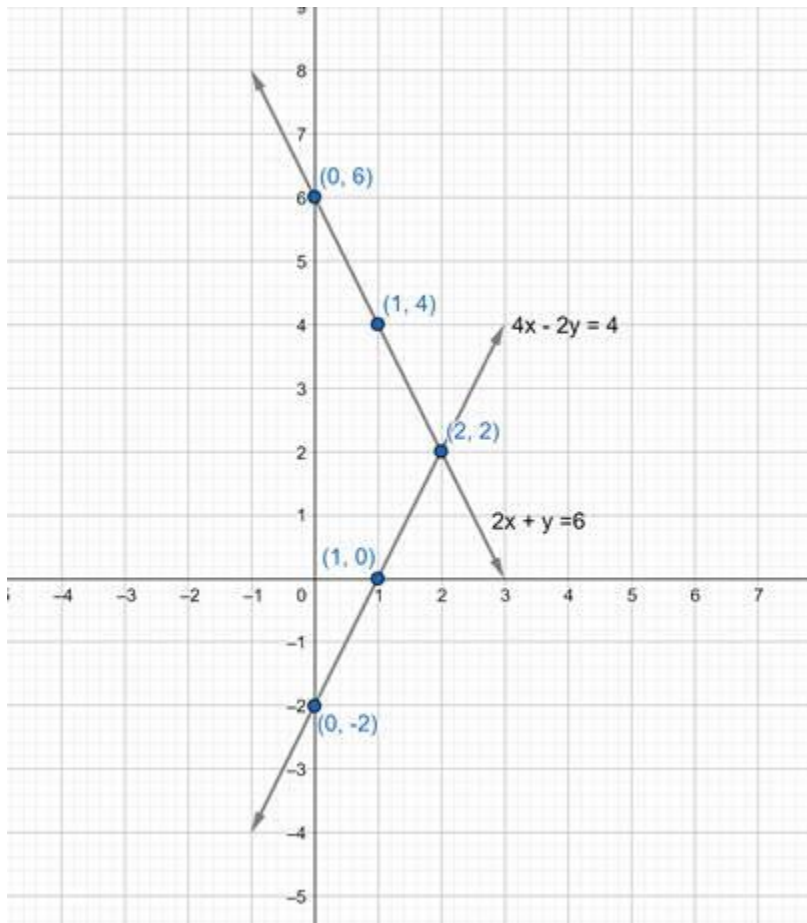| x	0	1	2
y	6	4	2

x	0	1	2
y	-2	0	2

Here, $a_1 = 2$, $b_2 = 1$, $c_1 = 6$

$a_1 = 4$, $b_2 = -2$, $c_1 = 4$

$$\therefore \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = -\frac{1}{2}, \frac{c_1}{c_2} = \frac{6}{4} = \frac{3}{2}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

∴ two lines are consistent

Q. 2 I. Check whether the following equation are consistent or inconsistent. Solve them graphically.

2x - 2y = 2 4x - 4y = 5

Answer :

2x - 2y = 2 4x - 4y = 5

X	0	1	2
Y	-1	0	1

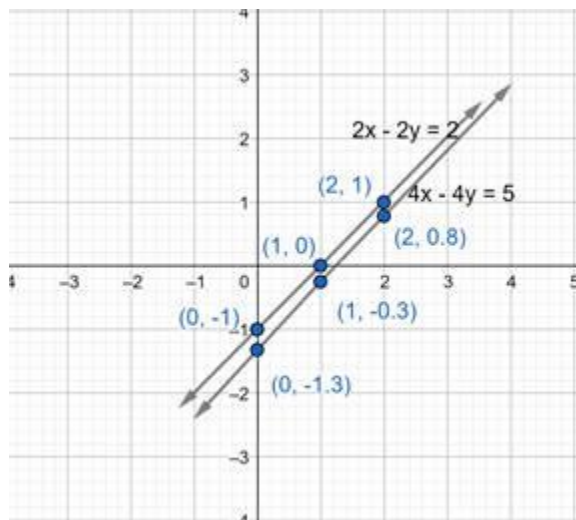| x	0	1	2
y	$-\dfrac{5}{4}$	$-\dfrac{1}{4}$	$\dfrac{3}{4}$

Here, $a_1 = 2$, $b_2 = 2$, $c_1 = 2$

$a_1 = 4$, $b_2 = 4$, $c_1 = 5$

$$\therefore \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{2}{5}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

∴ two lines are inconsistent.

Q. 3. Neha went to a 'sale' to purchase some pants and skirts. When her friend asked her how many of each she had bought, she answered "The number of skirts are two less than twice the number of pants purchased. Also the number of skirts is four less than four times the number of pants purchased."

Help her friend to find how many pants and skirts Neha bought.

Answer : Let the number of pants purchased by Neha be 'x' and no. of skirts purchased be 'y'.

Twice the number of pants = 2x

From the given condition,

$y = 2x - 2$ (I)

i.e. number of skirts are 2 less than 2 twice the number of pants.

$y = 4x - 4$ (II)

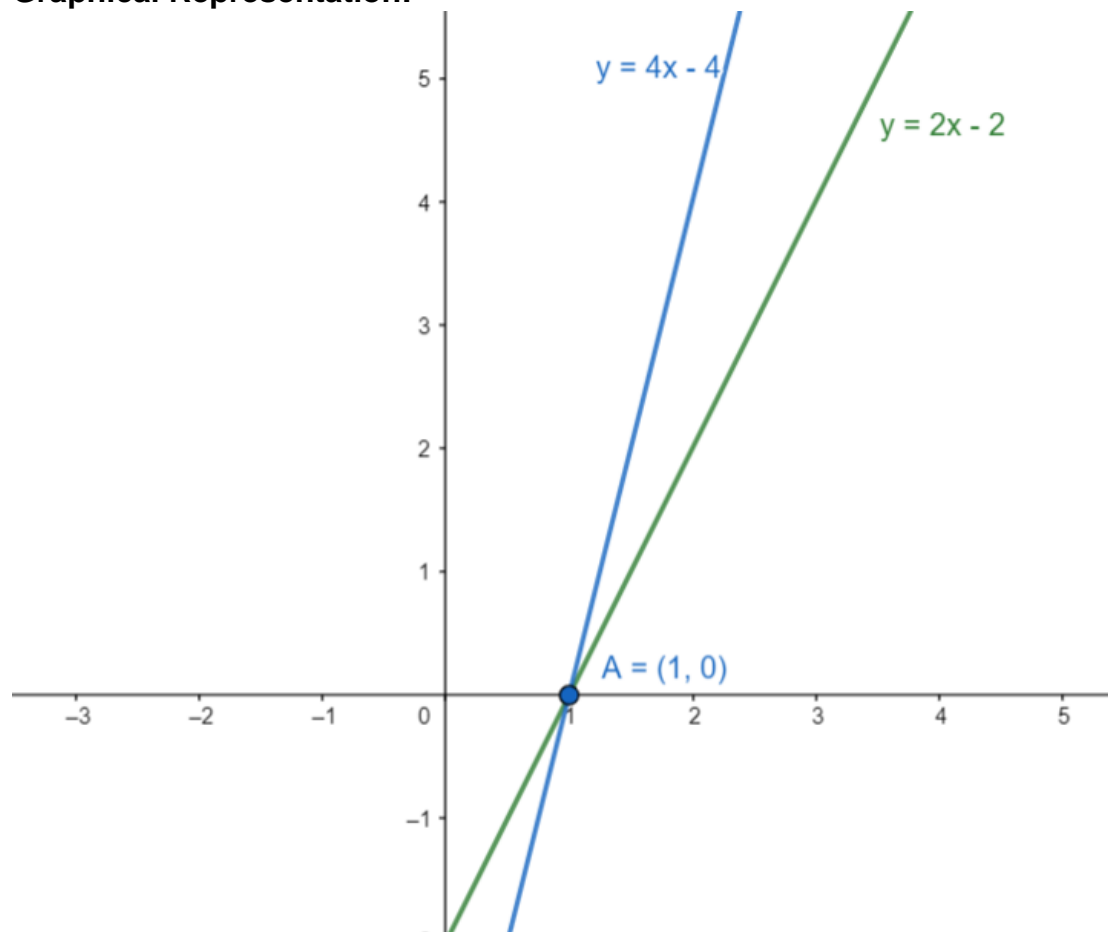i.e. number of skirts are 4 less than four times the number of pants.

Subtracting eq. I from II, we get

$2x = 2$

$\Rightarrow x = 1$

So number of pants purchased = 1 and number of skirts = 0

Graphical Representation:

Q. 4. 10 students of Class-X took part in a mathematics quiz. If the number of girls is 4 more than the number of boys then, find the number of boys and the number of girls who took part in the quiz.

Answer : Let the number of girls and boys in the class be x and y respectively
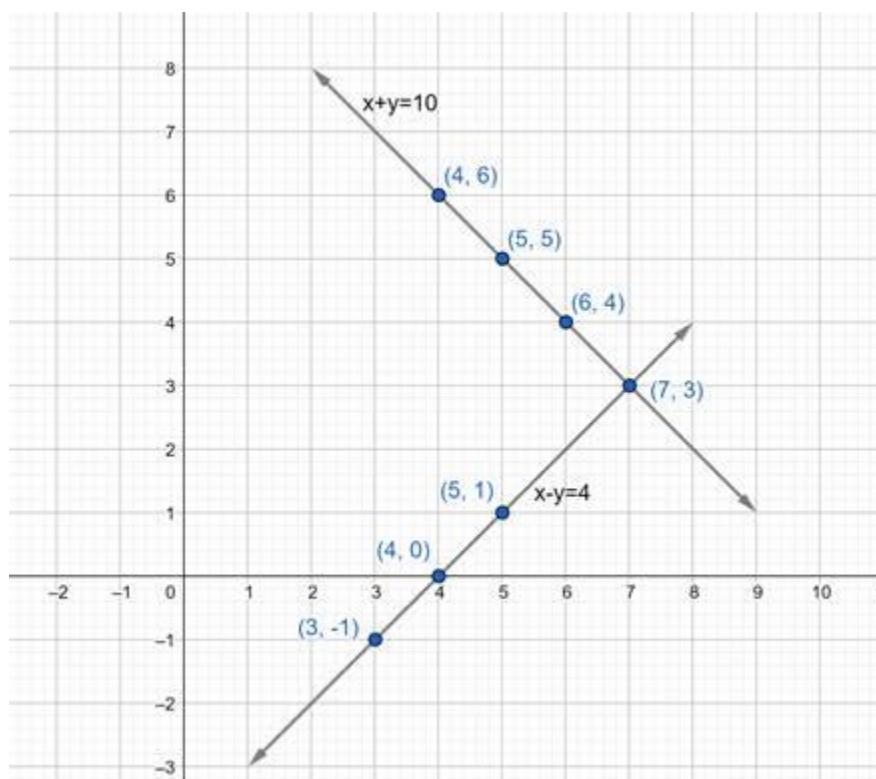
According to given condition, we have:

x + y = 10

x-y = 4

x + y = 10 ⇒ x = 10-y x-y = 4 ⇒ x = 4 + y

x	4	5	6
y	6	5	4

x	5	4	3
y	1	0	-1

From the graph it can be observed that the two lines intersect each other at the point (7, 3)

So, x = 7 and y = 3

Number of Girls = 7;

Number of boys = 3

Q. 5. 5 pencils and 7 pens together cost D50 whereas 7 pencils and 5 pens together cost D46. Find the cost of one pencil and that of one pen.

Answer : Let the cost of one pencil and one pen be ₹x and ₹y respectively.

According to given situation, we have :

5x + 7y = 50

7x + 5y = 46

5x + 7y = 50 7x + 5y = 46

x	3	10	-4
y	5	0	10

x	8	3	-2
y	-2	5	12
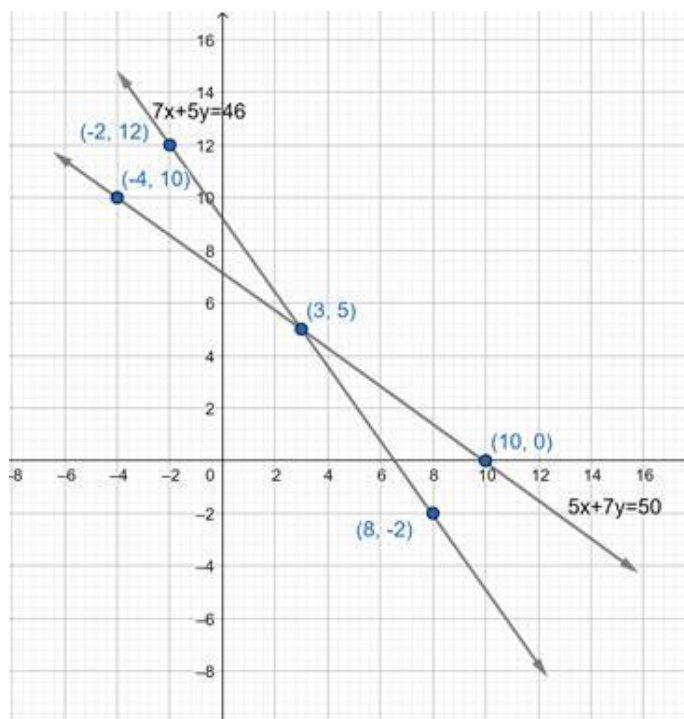
From the graph. It can be observed that the two lines are intersecting each other at (3,5)

x = 3 and y = 5

Cost of pencil = ` 3;

Cost of pen = ` 5

Q. 6. Half the perimeter of a rectangular garden is 36 m. If the length is 4m more than its width, is 36 m. Find the dimensions of the garden.

Answer : Let l be the length and b be the breadth

l = b + 4

l-b = 4

Perimeter = 2(l + b)

$\frac{1}{2}$ × 2(l + b) = 36 (given)

l-b = 4 ……I

l + b = 36 ….II

adding eq. I and II

⇒ 2l = 40

⇒ l = $\frac{40}{2}$ = 20m

Substitute l = 20 in(1)

l-b = 4

20-b = 4

b = 20-4 = 16m

Length = 20 m

Width = 16 m

Q. 7. We have a linear equation 2x + 3y-8 = 0. Write another linear equation in two variables such that the geometrical representation of the pair so formed is intersecting lines.

Now, write two more linear equations so that one forms a pair of parallel lines and the second forms coincident line with the given equation.

Answer : $2x + 3y-8 = 0$

$3x + 2y-7 = 0$(intersecting)

$2x + 3y + 12 = 0$ (parallel lines)

$4x + 6y-16 = 0$ (coincident lines)

Q. 8. The area of a rectangle gets reduced by 80 sq uints if its length is reduced by 5 units and breadth is increased by 2 units. If we increase the length by 10 units and decrease the breadth by 5 units, the area will increase by 50 sq units. Find the length and breadth of the rectangle.

Answer : Let the length be x and breadth be y

According to question,

$xy-80 = (x-5)(x + 2)$

$\Rightarrow xy-80 = xy + 2x-5y-10$

$\Rightarrow -80 + 10 = 2x-5y$

$\Rightarrow 2x-5y = -70$ ….I

$(x + 10)(y-5) = xy + 50$

$\Rightarrow xy-5x + 10y-50 = xy + 50$

$\Rightarrow -5x + 10y = 100$

$\Rightarrow -x + 2y = 20$ ….II

Multiplying Eq. II by 2

$2x-5y = -70$

$\underline{-2x + 4y = 40}$

$-y = -30$

$y = 30$

substituting y = 30 in eq. II

$-x + 2 \times 30 = 20$

-x = 20-60

x = 40

length = 40units and breadth = 30 units

Q. 9. In X class, if three students sit on each bench, one student will be left. If four students sit on each bench, one bench will be left. Find the number of students and the number of benches in that class.

Answer : Let the number no. of students be x

Let the number no. of benches be y

According to given condition, we have

x = 3y + 1 ⇒ x-3y = 1 …..I

x = 4(y-1) ⇒ x = 4y-4 ⇒ x-4y = -4 ….II

Equating eq. I and II

x-3y = 1

-x + 4y = 4

y = 5

Substituting y = 5 in eq. I

x-3×5 = 1

⇒ x-15 + 1 ⇒x = 16

Number of students = 16;

Number of benches = 5

Exercise 4.2

Q. 1. The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save D2000 per month, find their monthly income.

Answer : Let the monthly incomes be represented by x and their expenditure be represented by y.

According to the given income ratios,

Income of the first person is 9x and that of the second persons is 7x.

According to the given expenditure ratios;

Expenditure of the first person is 4y and the second person's is 3y.

Both of them manage to save ₹2000 per month.

i.e. income – expenditure = savings

Then,

9x – 4y = 2000 …I

7x - 3y = 2000 …II

Multiplying I by 3 and II by 4, we get

27x – 12y = 6000 …III

28x – 12y = 8000 …IV

Subtracting III from IV, we get

x = 2000

Putting x = 2000 in I we get,

18000 – 4y = 2000

$\Rightarrow$ 16000 = 4y

$\Rightarrow$ y = 4000

Now,

Income of the first person = 9x = 9× 2000 = 18000

Income of the second person = 7x = 7× 2000 = 14000

Q. 2. The sum of a two digit number and the number obtained reversing the digits is 66. If the digits of the number differ by 2, find the number. how many such numbers are there?

Answer : Let the digit in the unit's place be x and the digit in the tens place be y.

Then, the number = 10y + x

The number obtained by reversing the order of the digits = 10x + y

According to given conditions,

(10y + x) + (10x + y) = 66

$\Rightarrow$ 11(x + y) = 66

$\Rightarrow$ (x + y) = 6

According to second situation, digits differ by 2

So, either x – y = 2 or y – x = 2

Thus , we have the following sets of simuntaneous equations

x + y = 6 …I

x – y = 2 …II

or,

x + y = 6 …III

x – y = 2 …IV

solving equation I and II, we get x = 2 and y = 4

solving equation III and IV , we get x = 4 and y = 2

When x = 4 and y = 2,

Two digit number = (10y + x) = 10(4) + 2 = 42

When x = 2 and y = 4,

Two digit number = (10y + x) = 10(2) + 4 = 24

Hence, the required number is either 24 or 42.

Q. 3. The larger of two supplementary angles exceeds the smaller by 18°. Find the angles.

Answer : Let the larger of the two supplementary angles be 'y' and the smaller angle be 'x'

Therefore, x + y = 180 …I

(∵ sum of supplementary angles is 180°)

Also, y = x + 18 (∵ Given) …..II

Substituting II in I, we get,

x + x + 18 = 180

⇒ 2x + 18 = 180

⇒ 2x = 180 – 18

⇒ 2x = 162

⇒ x = 81°

Substitute x = 81 in II

y = 81 + 18 = 99°

Therefore, measure of larger angle is 99° and the measure of the smaller angle is 81°

Q. 4. The taxi charges in Hyderabad are fixed, along with the charge for the distance covered. For a distance of 10 km., the charge paid is D220. For a journey of 15km. the charge paid is D310.

i. What are the fixed charges and charge per km?
ii. How much does a person have to pay for travelling a distance of 25 km?

Answer : i. Let the fixed charge be x.

Let the charges per km be y.

Total charges to be paid will be the sum of the fixed charges and the charges per km.

For 10kms, the amount paid is ₹220.

x + 10y = 220 …I

For 15kms, the amount paid is ₹ 310

x + 15y = 310 …II

Solving I and II we get

x + 10y = 220

x + 15y = 310

- - -

-5y = -90

y = 18

Substituting the value of y in Eq. I we get

x + 10(18) = 220

x = 220 – 180

x = 40

The Fixed charge are ₹ 40 and the charges per km are ₹18

ii. For 25 km ,

total = 40 + 25(18) = 40 + 450 = 490

hence, for 25 km, the taxi fare is 490

Q. 5. A fraction becomes equal to 4/5 if 1 is added to both numerator and denominator. If however, 5 is subtracted from both numerator and denominator, the fraction becomes equal to 1/2. What is the fraction?

Answer : Let the numerator be x and denominator be y.

Hence, the fraction = x/y

Then, according to first condition, we have

$$\frac{x+1}{y+1} = \frac{4}{5}$$ and

⇒ 5x + 5 = 4y + 4

⇒ 5x – 4y = -1 ….I

According to second condition,

$$\frac{x-5}{y-5} = \frac{1}{2}$$

$\Rightarrow 2x - 10 = y - 5$ ...II

Using the elimination method, multiplying eq. I by 2 and Eq. II by 5

$10x - 8y = -2$

$10x - 5y = 25$

- + - _

$-3y = -27$

$y = 9$

substitute the value of y = 9 in Eq. I

$5x - 4(9) + 1 = 0$

$5x = -1 + 36$

$x = 7$

hence, the given fraction is $\frac{7}{5}$.

Q. 6. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time at different speeds. If the cars travel in the same direction, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

Answer : Let the speed of faster car = x km/h

Speed of slower car = y km/h

Relative speed when both car travels in same direction = x-y

If the cars travel in the same direction, they meet in 5 hours.

$$speed = \frac{distance}{time}$$

Total distance = 100 km

$x-y = \dfrac{100}{5} \Rightarrow x-y = 20$ …1

Relative speed when both car travels in different direction = x + y

If they travel towards each other, they meet in 1 hour.

x + y = 100 …2

Adding Eq. 1 and 2

x-y = 20

x + y = 100

2x = 120

$x = \dfrac{120}{2} = 60$

substituting x = 60 in eq. 2

60 + y = 100

y = 100-60

y = 40 km

Speed of the faster car = 60km/h

Speed of the slower car = 40km/h

Q. 7. Two angles are complementary. The larger angle is 3º less than twice the measure of the smaller angle. Find the measure of each angle.

Answer : Let the larger of the two supplementary angles be 'y' and the smaller angle be 'x'

Therefore, y = 2x – 3 …I

Given two angles are complementary.

x + y = 180°

⇒ x + (2x − 3) = 180

⇒ 3x - 3 = 180

⇒ 3x = 180 + 3

⇒ 3x = 183

⇒ x = 61°

Substitute x = 61 in I

y = 61× 2 − 3 = 122 − 3 = 119°

Therefore, measure of larger angle is 119° and the measure of the smaller angle is 61°

Q. 8. An algebra textbook has a total of 1382 pages. It is broken up into two parts. The second part of the book has 64 pages more than the first part. How many pages are in each part of the book?

Answer : Let the number of pages in the first part be x and in the second part be y.

Total pages number of pages is 1382,

Hence, x + y = 1380 …I

The second part of the book has 64 pages more than the first part

y = x + 64 …II

substituting value of y in equation I

x + x + 64 = 1382

2x = 1382 – 64

2x = 1318

x = 659

substituting the value of x in equation (2),

y = 659 + 64

y = 723

the pages in the first book are 659 and the pages in the second part are 723.

Q. 9. A chemist has two solutions of hydrochloric acid in stock. One is 50% solution and the other is 80% solution. How much of each should be used to obtain 100ml of a 68% solution.

Answer : Let the proportion of the first solution be represented by x and the second solution be represented by y.

First solution contains 50% solution =

$$\frac{50}{100}X = \frac{x}{2}$$

Second solution contains 80% solution =

$$\frac{80}{100}y = \frac{4}{5}y$$

Given that together they can give 100ml of 68% solution.

$$\frac{x}{2} + \frac{4}{5}y = 100 \times \frac{68}{100}$$

$$\frac{x}{2} + \frac{4}{5}y = 68$$

5x + 8y = 680 …I

Total percentage of solution concentration is 100%

x + y = 100 …II

Multiplying Eq. II by 5 and solving with Eq. I

5x + 8y = 680

5x + 5y = 500

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3y = 180

y = 60

substitute in Eq. II

x + 60 = 100

x = 40

Hence, the solutions are 40 and 20ml

Q. 10. Suppose you have D12000 to save. You have to save some amount at 10% and the rest at 15%. How much should be saved at each rate to yield 12% on the total amount saved?

Answer : Let the amount be invested at 10% be x and amount to be invested at 15% be y.

Total amount to be invested is ₹12000

Therefore, x + y = 12000 …I

Simple interest on ₹x at 10% for 1 year = $\frac{x \times 10 \times 1}{100} = \frac{x}{10}$

Simple interest on ₹y at 15% for 1 year = $\frac{y \times 15 \times 1}{100} = \frac{3y}{20}$

Now, total amounts is to be invested at 12%

= $\frac{12000 \times 12 \times 1}{100} = 1440$

$\frac{x}{10} + \frac{3y}{20} = 1440$

2x + 3y = 28800 …II

Multiplying Eq. I by 2 and solving with Eq. II

2x + 3y = 28800

2x + 2y = 12000

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y = 4800

substitute y = 4800 in Eq. I

x + 4800 = 1200

x = 7200

Hence the amount to be invested at 10% is ₹7200 and the amount to be invested at 15% is ₹4800.

Exercise 4.3

Q. 1 A. Solve each of the following pairs of equations by reducing them to a pair of linear equation.

$$\frac{5}{x-1} + \frac{1}{y-2} = 2$$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1$$

Answer :

$$\frac{5}{x-1} + \frac{1}{y-2} = 2$$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1$$

Let $\frac{1}{x-1}$ be p and $\frac{1}{y-2}$ be q,

We get the following pair of linear equations:

5p + q = 2 …III

6p – 3q = 1 …IV

Using the elimination method:

Multiplying eq. III by 3 and adding to IV, we get

15p + 3q = 6

6p – 3q = 1

21p = 7

$$\Rightarrow p = \frac{7}{21} = \frac{1}{3}$$

Substitute the value of p in eq. III

$$5\left(\frac{1}{3}\right) + q = 2$$

$$\Rightarrow q = 2 - \frac{5}{3}$$

$$\Rightarrow q = \frac{1}{3}$$

But, $\frac{1}{x-1} = \frac{1}{3} = p \Rightarrow 3 = x - 1 \Rightarrow x = 4$

$\frac{1}{y-2} = \frac{1}{3} = q \Rightarrow 3 = y - 2 \Rightarrow y = 5$

So, x = 4 and y = 5

Q. 1 B. Solve each of the following pairs of equations by reducing them to a pair of linear equation.

$$\frac{x+y}{xy} = 2$$

$$\frac{x-y}{xy} = 6$$

Answer :

$$\frac{x+y}{xy} = 2$$

$$\frac{x-y}{xy} = 6$$

$$\frac{x+y}{xy} = 2 \Rightarrow \frac{1}{x} + \frac{1}{y} = 2 \dots I$$

$$\frac{x-y}{xy} = 6 \Rightarrow \frac{1}{x} - \frac{1}{y} = 6$$

Substituting $\frac{1}{x} = p$ and $\frac{1}{y} = q$

We get the pair of linear equation,

p + q = 2 …III

p – q = 6 ….IV

using the elimination method,

p + q = 2

p – q = 6

2p = 8

p = 4

substitute the value of p in Eq. III

4 + q = 2

q = -2

but, $\frac{1}{x} = p = 4 \Rightarrow x = \frac{1}{4}$

$\frac{1}{y} = q = -2 \Rightarrow x = -\frac{1}{2}$

Q. 1 C. Solve each of the following pairs of equations by reducing them to a pair of linear equation.

$$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$$

$$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

Answer :

$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$ and $\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

Let $\frac{1}{\sqrt{x}}$ be p and $\frac{1}{\sqrt{y}}$ be q

2p + 3q = 2 …III

4p – 9q = -1 …IV

Multiplying eq. III by 3 and subtracting with eq. IV, we get

6p + 9q = 6

4p – 9q = -1

10p = 5

$$p = \frac{5}{10} = \frac{1}{2}$$

substitute value of p in Eq. III

$$2 \times \frac{1}{2} + 3q = 2$$

$$\Rightarrow 3q = 2 - 1$$

$$\Rightarrow q = \frac{1}{3}$$

But, $\frac{1}{\sqrt{x}} = p = \frac{1}{2} \Rightarrow \sqrt{x} = 2 \Rightarrow x = 2^2 \Rightarrow x = 4$

$\frac{1}{\sqrt{y}} = q = \frac{1}{3} \Rightarrow \sqrt{y} = 3 \Rightarrow y = 3^2 \Rightarrow y = 9$

Q. 1 D. Solve each of the following pairs of equations by reducing them to a pair of linear equation.

$$6x + 3y = 6xy$$
$$2x + 4y = 5xy$$

Answer :

6x + 3y = 6xy

2x + 4y = 5xy

$$\frac{6x + 5y}{xy} = 6 \Rightarrow \frac{6}{x} + \frac{5}{y} = 6 \ ....I$$

$$\frac{2x + 4y}{xy} = 5 \Rightarrow \frac{2}{x} + \frac{4}{y} = 5 \ ....II$$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$

6p + 3q = 6

2p + 4q = 5

Multiplying eq. IV by 3 and subtracting from Eq. III

6p + 3q = 6

6p + 12q = 15
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-9q = -9

q = 1

substitute the value of q in III

6p + 3(1) = 6

$p = \frac{1}{2}$

But, $\frac{1}{x} = p = \frac{1}{2} \Rightarrow x = 2$

$\frac{1}{y} = q = -1 \Rightarrow y = -1$

Q. 1 E. Solve each of the following pairs of equations by reducing them to a pair of linear equation.

$$\frac{5}{x+y} - \frac{2}{x-y} = -1$$

$$\frac{15}{x+y} + \frac{7}{x-y} = 10 \text{ where } x \neq 0, y \neq 0$$

Answer :

$$\frac{5}{x+y} - \frac{2}{x-y} = -1 \text{ and } \frac{15}{x+y} + \frac{7}{x-y} = 10$$

Let $\frac{1}{x+y} = p$ and $\frac{1}{x-y} = q$

5p – 2q = -1 I

15p + 7q = 10 II

Multiply Eq. I by 3 and subtract eq. II

15p – 6q = -3

<u>-15p - 7q = -10</u>

-13q = -13

q = 1

substitute the value of q in Eq. I

5p – 2(1) = -1

$\Rightarrow$ 5p = - 1 + 2

$\Rightarrow$ 5p = 1

$\Rightarrow p = \frac{1}{5}$

But, $\frac{1}{x+y} = p = \frac{1}{5} \Rightarrow 5 = x + y$ ...III

$\frac{1}{x-y} = q = 1 \Rightarrow 1 = x - y \Rightarrow y = x - 1$

Substitute value of y in eq. III

x + x – 1 = 5

$\Rightarrow$ 2x = 6

$\Rightarrow$ x = 3

$\therefore$ y = 3 – 1 = 2

x = 3 and y = 2

Q. 1 F. Solve each of the following pairs of equations by reducing them to a pair of linear equation.

$$\frac{2}{x} + \frac{3}{y} = 13$$

$$\frac{5}{x} - \frac{4}{y} = -2 \text{ where } x \neq 0, y \neq 0$$

Answer :

$\frac{2}{x} + \frac{3}{y} = 13$ and $\frac{5}{x} - \frac{4}{y} = -2$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$

2p + 3q = 13 ...I

5p – 4q = -2 ...II

Multiply Eq. I by 4 and Eq. II by 3 and solve

8p + 12q = 52

15p – 12q = -6

23p = 46

P = 2

Substitute the value of p in Eq. I

2(2) + 3q = 13

$\Rightarrow$ 4 + 3q = 13

$\Rightarrow$ 3q = 9

$\Rightarrow q = 3$

But, $\dfrac{1}{x} = p = 2 \Rightarrow x = \dfrac{1}{2}$

$\dfrac{1}{y} = q = 3 \Rightarrow y = \dfrac{1}{3}$

Q. 1 G

Solve each of the following pairs of equations by reducing them to a pair of linear equation.

$$\dfrac{10}{x+y} + \dfrac{2}{x-y} = 4$$

$$\dfrac{15}{x+y} - \dfrac{5}{x-y} = -2$$

Answer :

$\dfrac{10}{x+y} + \dfrac{2}{x-y} = 4$ and $\dfrac{15}{x+y} - \dfrac{5}{x-y} = -2$

Let $\dfrac{1}{x+y} = p$ and $\dfrac{1}{x-y} = q$

10p + 2q = 4 …I

15p – 5q = -2 …II

Multiply Eq. I by 5 and Eq. II by 2

50p + 10q = 20

30p – 10q = -4

80p = 16

$p = \frac{1}{5}$

substitute value of p in eq. I

$10 \times \frac{1}{5} + 2q = 4$

$\Rightarrow 2 + 2q = 4$

$\Rightarrow 2q = 2$

$\Rightarrow q = 1$

But, $\frac{1}{x+y} = p = \frac{1}{5} \Rightarrow 5 = x + y$ ...III

$\frac{1}{x-y} = q = 1 \Rightarrow 1 = x - y \Rightarrow y = x - 1$

Substitute value of y in eq. III

$x + x - 1 = 5$

$\Rightarrow 2x = 6$

$\Rightarrow x = 3$

$\therefore y = 3 - 1 = 2$

x = 3 and y = 2

Q. 1 H. Solve each of the following pairs of equations by reducing them to a pair of linear equation.

$$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$$

$$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$$

Answer :

$$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4} \text{ and } \frac{1}{[2(3x+y)]} - \frac{1}{[2(3x-y)]} = \frac{1}{8}$$

Let $\frac{1}{3x+y} = m$ and $\frac{1}{3x-y} = n$

$m + n = \frac{3}{4} \Rightarrow 4(m + n) = 3 \Rightarrow 4m + 4n = 3 \ldots (I)$

$\frac{1}{2}m - \frac{1}{2}n = \frac{-1}{8} \Rightarrow 8(m-n) = -1 \times 2 \Rightarrow 8m-8n = -2 \ldots (II)$

Multiply Eq. I by 2

$8m + 8n = 6$

$8m - 8n = -2$

$16m = 4$

$m = \frac{4}{16}$

$m = \frac{1}{4}$

Substituting $m = \frac{1}{4}$ in Eq. II

$8 \times \frac{1}{4} - 8n = -2$

$\Rightarrow 2 - 8n = -2$

$\Rightarrow -8n = -4$

$\Rightarrow n = \frac{1}{2}$

$\therefore m = \dfrac{1}{(3x+y)} \Rightarrow \dfrac{1}{(3x+y)} = \dfrac{1}{4} \Rightarrow 4 = 3x + y \Rightarrow 4 = 3x + y \ \dots \text{III}$

$\therefore n = \dfrac{1}{(3x-y)} \Rightarrow \dfrac{1}{(3x-y)} = \dfrac{1}{2} \Rightarrow 2 = 3x - y \Rightarrow 2 = 3x - y \ \dots \text{IV}$

Equating Eq. III and IV

3x + y = 4

<u>3x-y = 2</u>

6x = 6

x = 1

Substituting x = 1in Eq. III

3×1 + y = 4

$3 + y = 4$

$y = 4 - 3$

$y = 1$

Hence, x = 1 and y = 1

Q. 2 A. Formulate the following problems as a pair of equation and then find their solution.

A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

Answer : Let the speed of the boat in still water be x km/h and speed of the stream be y km/h

Speed of boat upstream = (x – y) km/h

Speed of boat downstream = (x + y) km/h

But, total time of journey is 10hrs.

$$\frac{30}{x-y} + \frac{44}{x+y} = 10$$

$$\frac{40}{x-y} + \frac{55}{x+y} = 13$$

Let $\frac{1}{x-y}$ be a and $\frac{1}{x+y}$ be b

30a + 44b = 10 …I

40a + 55b = 13 …II

Multiply Eq. I by 4 and Eq. II by 3

120a + 176b = 40

120a + 165b = 39

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11b = 1

b = 1/11

substituting value of b in Eq. I

$$30a + 44 \times \frac{1}{11} = 10$$

$\Rightarrow$ 30a + 4 = 10

$\Rightarrow$ 30a = 6

$\Rightarrow a = \frac{6}{30} = \frac{1}{5}$

$\therefore a = \dfrac{1}{x+y} \Rightarrow \dfrac{1}{x+y} = \dfrac{1}{5} \Rightarrow 5 = x + y \Rightarrow 5 = x + y$ III

$\therefore n = \dfrac{1}{x-y} \Rightarrow \dfrac{1}{x-y} = \dfrac{1}{11} \Rightarrow 11 = x - y \Rightarrow 11 = x - y$ IV

x + y = 5

x – y = 11

2x = 16

x = 8

substitute value of x in Eq. III

8 + y = 5

Y = 3

Hence, Speed of boat = 8 km/h;

Speed of stream = 3 km/h

Q. 2 B. Formulate the following problems as a pair of equation and then find their solution.

Rahim travels 600 km to his home partly by train and partly by car. He takes 8 hours if he travels 120 km by train and rest by car. He takes 20 minutes more if he travels 200 km by train and rest by car. Find the speed of the train and the car.

Answer : Let the speed of the train be x km/h and that of the car be y km/h

Also, we know that time $= \dfrac{distance}{speed}$

In situation 1, time spent travelling by train $= \dfrac{120}{x}$ h

And time spent travelling by car $= \dfrac{480}{y}$ h

So, total time taken in 8hour journey is

$$= \frac{120}{x} + \frac{480}{y} = 8$$

$$\Rightarrow \frac{30}{x} + \frac{120}{y} = 2 \ldots\text{I}$$

Again, when he travels 200km by train and rest by car

Time taken by him travel 200km by train $= \frac{200}{x}$ h

And time spent travelling by car $= \frac{400}{y}$ h

Total time taken $= \frac{200}{x} + \frac{400}{y}$

But given time of journey is 8hrs 20 min i.e $80\frac{20}{60} = 8\frac{1}{3}$ h

So, $\frac{200}{x} + \frac{400}{y} = \frac{25}{3} \ldots\text{II}$

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$

30a + 120b = 2 …III

200a + 400b = $\frac{25}{3}$ …IV

Multiply Eq. III by 20 and Eq. IV by 3

600a + 2400b = 40

600a + 1200b = 25

$1200b = 15$

$$b = \frac{15}{1200} = \frac{1}{80}$$

substitute value of b in Eq. III

$$30a + 120 \times \frac{1}{80} = 2$$

$$a = \frac{1}{60}$$

but, $\frac{1}{x} = a = \frac{1}{60} \Rightarrow x = 60$

$\frac{1}{y} = b = \frac{1}{80} \Rightarrow y = 80$

Q. 2 C. Formulate the following problems as a pair of equation and then find their solution.

2 women and 5 men can together finish an embroidery work in 4 days while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone and 1 man alone to finish the work.

Answer : Let the time taken by one woman to finish the work = x days

Work done by one women in one day = 1/x days

Let the time taken by one man to finish the work = y days

Work done by one men in one day = 1/y days

Now, 2 women and 5 men can finish the work in 4 days

So, $\frac{2}{x} + \frac{5}{y} = \frac{1}{4} \Rightarrow \frac{8}{x} + \frac{20}{y} = 1 \dots I$

Also, 3 women and 6 men can finish the work in 3 days

So, $\frac{3}{x} + \frac{6}{y} = \frac{1}{3} \Rightarrow \frac{9}{x} + \frac{18}{y} = 1 \dots II$

Let $\frac{1}{x}$ be a and $\frac{1}{y}$ be b

8a + 20b = 1 …III

9a + 18b = 1 …IV

Multiplying Eq. III by 9 and Eq. IV by 8 and solving

72a + 180b = 9

72a + 144b = 8

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36b = 1

$b = \frac{1}{36}$

substituting value of b in Eq. III

$8a + 20 \times \frac{1}{36} = 1$

288a + 20 = 36

288a = 16

$a = \frac{1}{18}$

$$\frac{1}{x} = a = \frac{1}{18} \Rightarrow x = 18$$

$$\frac{1}{y} = b = \frac{1}{18} \Rightarrow y = 36$$

Number of days by man = 18;

Number of days by woman = 36