# Chapter 8

## Matrices

## <u>Exercise 8.1</u>

**Question 1.**

**Classify the following matrices:**

**i)** $\begin{bmatrix} \mathbf{2} & \mathbf{-1} \\ \mathbf{5} & \mathbf{1} \end{bmatrix}$

**Solution:**

It is square matrix of order 2

ii) [ 2   3   -7]

Solution:

It is row matrix of order $1 \times 3$

iii) $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$

Solution:

It is column matrix of order $3 \times 1$

iv) $\begin{bmatrix} 2 & -4 \\ 0 & 0 \\ 1 & 7 \end{bmatrix}$

Solution:

It is a matrix of order $3 \times 2$

(v) $\begin{bmatrix} 2 & 7 & 8 \\ -1 & \sqrt{2} & 0 \end{bmatrix}$

Solution:

It is a matrix of order $2 \times 3$

**Question 2.**

**i) If a matrix has 4 elements, what are the possible order it can have?**

**Solution:**

It can have $1\times 4, 4 \times 1, or\ 2 \times 2$

ii) If a matrix has 4 elements, what are the possible orders it can have?

Solution:

It can have $1\times 8, 8 \times 1, 2 \times 4, or\ 4 \times 2\ \ order,$

**Question 3 . construct a 2× 2 matrix whose elements $a_{ij}$ are given by**

**i) $a_{ij} = 2i - j$**

**ii) $a_{ij} = i.\ j$**

**Solution:**

i) Given $a_{ij} = 2i - j$

Therefore matrix of order $2 \times 2\ is$

$\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$

ii) Given $a_{ij} = i.\, j$

Therefore matrix of order 2× 2 is

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

Question 4. Find the values of x and y if:

$$\begin{bmatrix} 2x + y \\ 3x - 2y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

**Solution:**

Given

$$\begin{bmatrix} 2x + y \\ 3x - 2y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Now by comparing the corresponding elements,

$2x + y = 5$ ......... i

$3x – 2y = 4$ .........ii

Multiply (i) by 2 and (ii) 1 we get

$4x + 2y = 10$ and $3x – 2y = 4$

By adding we get

$7x = 14$

$X = \frac{14}{7}$

$X = 2$

Substituting the value of x in (i)

$4 + y = 5$

$Y = 5 – 4$

Y = 1

Hence x = 2 and y = 1

**Question 5. Find the value of x if**

$$\begin{bmatrix} \mathbf{3x+y} & \mathbf{-y} \\ \mathbf{2y-x} & \mathbf{3} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{-5} & \mathbf{3} \end{bmatrix}$$

**Solution:**

Given

$$\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$$

Comparing the corresponding terms of given matrix we get

-y = 2

Therefore y = -2

Again we have

3x + y = 1

3x = 1 – y

Substituting the value of y we get

3x = 1 – (-2)

3x = 1+ 2

3x = 3

X = $\frac{3}{3}$

X = 1

Hence x = 1 and y = -2

**Question 6. If**

$$\begin{bmatrix} \boldsymbol{x+3} & \boldsymbol{4} \\ \boldsymbol{y-4} & \boldsymbol{x+y} \end{bmatrix} = \begin{bmatrix} \boldsymbol{5} & \boldsymbol{4} \\ \boldsymbol{3} & \boldsymbol{9} \end{bmatrix}$$

**Find the values of x and y.**

**Solution:**

Given

$$\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$$

Comparing the corresponding terms, we get

X + 3 = 5

X = 5 – 3

X = 2

Again we have

Y – 4 = 3

Y = 3 + 4

Y = 7

Hence x = 2 and y = 7

**Question 7. Find the values of x, y and Z if**

$$\begin{bmatrix} \boldsymbol{x+2} & \boldsymbol{6} \\ \boldsymbol{3} & \boldsymbol{5z} \end{bmatrix} = \begin{bmatrix} \boldsymbol{-5} & \boldsymbol{y^2+y} \\ \boldsymbol{3} & \boldsymbol{-20} \end{bmatrix}$$

**Solution:**

$\begin{bmatrix} x+2 & 6 \\ 3 & 5z \end{bmatrix} = \begin{bmatrix} -5 & y^2+y \\ 3 & -20 \end{bmatrix}$

Comparing the corresponding elements of given matrix, then we get

X + 2 = -5

X = -5 – 2

Also we have 5z = -20

Z = - $\frac{20}{5}$

Z = - 4

Again from given matrix we have

$Y^2 + y - 6 = 0$

The above equation can be written as

$Y^2 + 3y - 2y - 6 = 0$

Y (y + 3) – 2 (y + 3) = 0

Y + 3 = 0 or y – 2 = 0

Y = -3 or y = 2

Hence x = -7, y = -3 2 and Z = -4

**Question 8. Find the values of x, y, a and b if**

$$\begin{bmatrix} \boldsymbol{x-2} & \boldsymbol{y} \\ \boldsymbol{a+2b} & \boldsymbol{3a-b} \end{bmatrix} = \begin{bmatrix} \mathbf{3} & \mathbf{1} \\ \mathbf{5} & \mathbf{1} \end{bmatrix}$$

**Solution:**

Given

$$\begin{bmatrix} x-2 & y \\ a+2b & 3a-b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$$

Comparing the corresponding elements

$X - 2 = 3$ and $y = 1$

$X = 2 + 3$

$X = 5$

Again we have

$A + 2b = 5$ ...... i

$3a - b = 1$ ........ ii

Multiply (i) by 1 and (ii) by 2

$A + 2b = 5$

$6a - 2b = 2$

Now by adding above equations we get

$7a = 7$

$A = \frac{7}{7}$

$A = 1$

Substituting the value of a in (i) we get

$1 + 2b = 5$

$2b = 5 - 1$

$2b = 4$

$B = \frac{4}{2}$

$B = 2$

**Question 9: Find the values of a, b, c and d if**

$$\begin{bmatrix} \boldsymbol{a+b} & \boldsymbol{3} \\ \boldsymbol{5+c} & \boldsymbol{ab} \end{bmatrix} \begin{bmatrix} \boldsymbol{6} & \boldsymbol{d} \\ \boldsymbol{-1} & \boldsymbol{8} \end{bmatrix}$$

**Solution:**

Given

$$\begin{bmatrix} a+b & 3 \\ 5+c & ab \end{bmatrix} \begin{bmatrix} 6 & d \\ -1 & 8 \end{bmatrix}$$

Comparing the corresponding terms, we get

3 = d

D = 3

Also we have

5 + c = -1

C = -1 – 5

C = -6

Also we have,

A + b = 6 and a b = 8

We know that,

$(a – b)^2 = ( a + b)^2 – 4$ ab

$(6)^2 – 32 = 36 – 32 = 4 = (\pm 2)^2$

$a – b = \pm 2$

if a - b = 2

a + b = 6

Adding the above two equations we get

2a = 4

A = $\frac{4}{2}$

A = 2

B = 6 – 4

B = 2

Again we have a – b = -2

And a + b = 6

Adding above equations we get

2a = 4

A = $\frac{4}{2}$

A = 2

Also, b = 6 – 2 = 4

A = 2 and b = 4

## Exercise 8.2

**Question 1. Given that M = $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ *and N* = $\begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$ find, M + 2n**

**Solution:**

Given

M = $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$

N = $\begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$

Now we have to find M + 2n

M + 2N = $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} + 2\begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$

On simplifying we get,

$= \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ -2 & 4 \end{bmatrix}$

$= \begin{bmatrix} 2+4 & 0+0 \\ 1-2 & 2+4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ -1 & 6 \end{bmatrix}$

**Question 2: If A $\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$**

**Find 2A – 3B**

**Solution:**

A = $\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$

B = $\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$

Now we have to find,

2A – 3B = 2 $\begin{bmatrix} 0 & 0 \\ -3 & 1 \end{bmatrix}$ - 3 $\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$

On simplifying we get

= $\begin{bmatrix} 4 & 0 \\ -6 & 2 \end{bmatrix}$ - $\begin{bmatrix} 0 & 3 \\ -6 & 9 \end{bmatrix}$ = $\begin{bmatrix} 4-0 & 0-3 \\ -6+6 & 2-9 \end{bmatrix}$

= $\begin{bmatrix} 4 & -3 \\ 0 & -7 \end{bmatrix}$

**Question 3. sin A** $\begin{bmatrix} \boldsymbol{sinA} & \boldsymbol{-CosA} \\ \boldsymbol{CosA} & \boldsymbol{SinA} \end{bmatrix} + \boldsymbol{cosA} \begin{bmatrix} \boldsymbol{cosA} & \boldsymbol{SinA} \\ \boldsymbol{-SinA} & \boldsymbol{cosA} \end{bmatrix}$

**Solution:**

Given,

Sin A $\begin{bmatrix} sinA & -CosA \\ CosA & SinA \end{bmatrix} + cosA \begin{bmatrix} cosA & SinA \\ -SinA & cosA \end{bmatrix}$

On simplifications, we get

= $\begin{bmatrix} sin^2A & -SinA.CosA \\ SinA.CosA & Sin^2A \end{bmatrix} + cosA \begin{bmatrix} cos^2A & CosA.SinA \\ -CosA.SinA & cos^2A \end{bmatrix}$

= $\begin{bmatrix} sin^2A + cos^2A & -SinA.CosA + CosA.SinA \\ SinA.CosA - CosA.SinA & Sin^2A + cos^2A \end{bmatrix}$

= $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (Since, $Sin^2A + Cos^2A = 1$

**Question 4: A =** $\begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{-2} & \mathbf{3} \end{bmatrix}$ **and B =** $\begin{bmatrix} \mathbf{-2} & \mathbf{-1} \\ \mathbf{1} & \mathbf{2} \end{bmatrix}$**, C =** $\begin{bmatrix} \mathbf{0} & \mathbf{3} \\ \mathbf{2} & \mathbf{-1} \end{bmatrix}$

**Find A + 2B – 3C**

**Solution:**

A = $\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix}$, C = $\begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$

Now we have to find A + 2B – 3C

= $\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ +2 $\begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix}$ - 3 $\begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$

= $\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ + $\begin{bmatrix} -4 & -2 \\ 2 & 4 \end{bmatrix}$ - $\begin{bmatrix} 0 & 9 \\ 6 & -3 \end{bmatrix}$

= On simplifying we get

$\begin{bmatrix} 1-4-0 & 2-2-9 \\ -2+2-6 & 3+4+3 \end{bmatrix}$ = $\begin{bmatrix} -3 & -9 \\ -6 & 10 \end{bmatrix}$

**Question 5: If A=** $\begin{bmatrix} \mathbf{0} & \mathbf{-1} \\ \mathbf{1} & \mathbf{2} \end{bmatrix}$ **and B =** $\begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{-1} & \mathbf{1} \end{bmatrix}$

**Find the matrix X if :**

**i) 3A + X = B**

**ii) x – 3B = 2A**

**Solution:**

Given

A = $\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$

B = $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

Now we have to find

i) 3A + X = B

x = B – 3A

Substituting the values we get

X = $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$

= $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -3 \\ 3 & 6 \end{bmatrix}$

= $\begin{bmatrix} 1-0 & 2+3 \\ -1-3 & 1-6 \end{bmatrix}$

**ii) x – 3B = 2A**

**X = 2A + 3B**

Now substituting the values A and B we get

X = 2 $\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

= $\begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix}$ + $\begin{bmatrix} 3 & 6 \\ -3 & 3 \end{bmatrix}$

= $\begin{bmatrix} 0+3 & -2+6 \\ 2-3 & 4+3 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -1 & 7 \end{bmatrix}$

**Question 6. Solve the matrix equation**

$\begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{5} & \mathbf{0} \end{bmatrix}$**- 3x =** $\begin{bmatrix} \mathbf{-7} & \mathbf{4} \\ \mathbf{2} & \mathbf{6} \end{bmatrix}$

**Solution:**

Given

$\begin{bmatrix} 2 & 1 \\ 5 & 0 \end{bmatrix}$- 3x = $\begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix}$

On simplification we get

$X = \frac{1}{3}\begin{bmatrix} 9 & -3 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$

**Question 7: If $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$+2m=3 $\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$ find the matrix M**

**Solution**

Given,

$\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$+2m=3 $\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$

2M = $\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$ - $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$

= $\begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix}$ - $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$ (After further simplification)

= $\begin{bmatrix} 9-1 & 6-4 \\ 0-(-2) & -9-3 \end{bmatrix}$

= $\begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix}$ (After subtraction of matrices)

M = $\frac{1}{2}\begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & -6 \end{bmatrix}$ (Dividing By 2)

**Question 8: Given A = $\begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$and B = $\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, C = $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$**

**Find the matrix X such that A + 2x = 2B + C**

**Solution:**

A = $\begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$and B = $\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, C = $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

Let x = $\begin{bmatrix} x & y \\ z & t \end{bmatrix}$

A + 2x = 2B + C (Given Condition)

2x = 2B + C – A

$2\begin{bmatrix} x & y \\ z & t \end{bmatrix} = 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$

$= \begin{bmatrix} -6 & 4 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$ (On further calculation)

$= \begin{bmatrix} -6+4-2 & 4+0+6 \\ 8+0-2 & 0+2-0 \end{bmatrix} = \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$

(Addition and subtraction of matrices)

: $2\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$

: $\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix}$ (dividing by 2)

**Question 9: Find x and y if x + y =** $\begin{bmatrix} \mathbf{7} & \mathbf{0} \\ \mathbf{2} & \mathbf{5} \end{bmatrix}$ **and x – y =** $\begin{bmatrix} \mathbf{3} & \mathbf{0} \\ \mathbf{0} & \mathbf{3} \end{bmatrix}$

**Solution:**

Given,

X + y = $\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ ...... (i)

X – y = $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ .......(ii)

Adding (i) and (ii) we get,

2x = $\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 7+3 & 0+0 \\ 2+0 & 5+3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$X = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

Now,

Subtracting (ii) from (i), we get

$2y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$= 2y = \begin{bmatrix} 7-3 & 0-0 \\ 2-0 & 5-3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$

$= y = \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

**Question 10: If $2\begin{bmatrix} \mathbf{3} & \mathbf{4} \\ \mathbf{5} & \boldsymbol{x} \end{bmatrix} + \begin{bmatrix} \mathbf{1} & \boldsymbol{y} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} \mathbf{7} & \mathbf{0} \\ \mathbf{10} & \mathbf{5} \end{bmatrix}$ find the values of x and y**

**Solution:**

Given,

$2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$= \begin{bmatrix} 6+1 & 8+y \\ 10+0 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$ (addition of matrices)

On comparing the corresponding elements, we have

$8 + y = 0$

Then, $y = -8$

And, $2x + 1 = 5$

$2x = 5 - 1 = 4$

$X = \frac{4}{2} = 2$

Therefore x = 2, and y = - 8

**Question 11: If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$ find the values of x and y**

**Solution:**

Given,

$$2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 6+1 & 8+y \\ 10+0 & 2x+1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix} \quad \text{(Addition of matrices)}$$

$$= \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$$

On comparing the corresponding terms, we have

2x + 1 = 5

2x = 5 – 1

$X = \frac{4}{2} = 2$

And,

8 + y = 0

Y = -8

And, z = 7

Therefore, x = 2, y = -8 and Z = 7

**12: If $\begin{bmatrix} 5 & 2 \\ -1 & y+1 \end{bmatrix} - 2\begin{bmatrix} 1 & 2x-1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$ find the values of x and y**

**Solution:**

Given,

$$\begin{bmatrix} 5 & 2 \\ -1 & y+1 \end{bmatrix} - 2\begin{bmatrix} 1 & 2x-1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ -1 & y+1 \end{bmatrix} - \begin{bmatrix} 2 & 4x-2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5-2 & 2-4x+2 \\ -1-6 & y+1+4 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$$ (Subtraction of matrices)

$$\begin{bmatrix} 3 & 4-4x \\ -7 & y+5 \end{bmatrix} \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$$

Now, comparing the corresponding terms, we get

4 – 4x = -8

4 + 8 = 4x

12 = 4x

X = $\frac{12}{4}$

X = 3

And, y + 5 = 2

Y = 2 – 5

Y = -3

Therefore, x = 3 and y = -3

**Question 13: If** $\begin{bmatrix} a & 3 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$

**Find the value of a, b and c.**

**Solution:**

Given

$\begin{bmatrix} a & 3 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$

$= \begin{bmatrix} a+2-1 & 3+b-1 \\ 4+1+2 & 2-2-c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$

$= \begin{bmatrix} a+1 & b+2 \\ 7 & -c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$ (After further calculations)

Next, on comparing the corresponding terms, we have

A + 1 = 5

A = 4

B + 2 = 0

B = -2

-c = 3

C = 3

Therefore, the value of a, b and c are 4, -2 and -3 respectively.

**Question 14: If A =** $\begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix}$ **and B =** $\begin{bmatrix} -2 & 3 \\ 7 & b \end{bmatrix}$, **c =** $\begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$

**And 5A + 2B = C, find the values of a, b and C.**

**Solution:**

Given,

A = $\begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & 3 \\ 7 & b \end{bmatrix}$ , c = $\begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$ and 5A + 2B = C

So, we have

$5\begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix} + 2\begin{bmatrix} -2 & 3 \\ 7 & b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$

$\begin{bmatrix} 10 & 5a \\ -15 & 25 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 14 & 2b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$

$\begin{bmatrix} 10-4 & 5a+6 \\ -15+14 & 25+2a \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$

$\begin{bmatrix} 6 & 5a+6 \\ -1 & 25+2b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$

On comparing the corresponding terms, we get

5a + 6 = 9

5a = 9 – 6

5a = 3 = $\frac{3}{5}$

And,

25 + 2b = -11

2b = -11 – 25

2b = -11 – 25

2b = -36

B = -$\frac{36}{2}$

B = -18

And C = 6

Therefore, the value of a, b and C are $\frac{3}{5}$ , -18 and 6 respectively.

## Exercise 8.3

**1. If A = $\begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$, is the product AB possible? Give a reason. If yes, find AB.**

**Solution:**

Yes, the product is possible because of number of column in A = number of row in B

That is order of matrix is $2 \times 1$

$$AB = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \times 2 + 5 \times 4 \\ 4 \times 2 + (-2) \times 4 \end{bmatrix} = \begin{bmatrix} 6 + 20 \\ 8 - 8 \end{bmatrix} = \begin{bmatrix} 26 \\ 0 \end{bmatrix}$$

**2. If A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$, find AB and BA, is AB = BA?**

**Solution:**

Given

$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$,

$B = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$

Now we have to find $A \times B$

$$\therefore A \times B = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2 - 15 & -2 + 10 \\ 1 - 9 & -1 + 6 \end{bmatrix} = \begin{bmatrix} -13 & 8 \\ -8 & 5 \end{bmatrix}$$

Again have to find $B \times A$

$$B \times A = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 - 1 & 5 - 3 \\ -6 + 2 & -15 + 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -4 & -9 \end{bmatrix}$$

Hence AB is not equal to BA

**3. If A = $\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$ Find AB – 5C**

**Solution:**

Given

A = $\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$,

B = $\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$

C = $\begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

AB = $\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$

On simplification we get

= $\begin{bmatrix} 3 \times 0 + 7 \times 5 & 3 \times 2 + 7 \times 3 \\ 2 \times 0 + 4 \times 5 & 4 \times 2 + 4 \times 3 \end{bmatrix}$

= $\begin{bmatrix} 0 + 35 & 6 + 21 \\ 0 + 20 & 4 + 12 \end{bmatrix} = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix}$

5C = 5 $\begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$

AB – 5C = $\begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix} = \begin{bmatrix} 30 & 52 \\ 40 & -14 \end{bmatrix}$

**4. If A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, find A(BA)**

**Solution:**

Given

A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$BA = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2+2 & 4+1 \\ 1+4 & 2+2 \end{bmatrix}$

$= \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

$A\,(BA) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

$= \begin{bmatrix} 4+10 & 5+8 \\ 8+5 & 10+4 \end{bmatrix}$

$= \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix}$

**5. Give matrices:**

**A =** $\begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{4} & \mathbf{2} \end{bmatrix}$ **and B =** $\begin{bmatrix} \mathbf{3} & \mathbf{4} \\ \mathbf{-1} & \mathbf{-2} \end{bmatrix}$**, C =** $\begin{bmatrix} \mathbf{-3} & \mathbf{1} \\ \mathbf{0} & \mathbf{-2} \end{bmatrix}$

**Find the products of**

**(i) ABC**
**(ii) ACB and state whether they are equal.**

**Solution:**

Given

$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$,

$C = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$

Now consider,

ABC = $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} \times \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$

= $\begin{bmatrix} 6-1 & 8-2 \\ 12-2 & 16-4 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$

= $\begin{bmatrix} 5 & 6 \\ 10 & 12 \end{bmatrix} \times \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$

= $\begin{bmatrix} -15+0 & 5-12 \\ -30+0 & 10-24 \end{bmatrix}$

= $\begin{bmatrix} -15 & -7 \\ -30 & -14 \end{bmatrix}$

ACB = $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \times \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$

= $\begin{bmatrix} -6+0 & 2-2 \\ -12+0 & 4-4 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$

= $\begin{bmatrix} -6 & 0 \\ -12 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$

= $\begin{bmatrix} -18+0 & -24+0 \\ -36+0 & -48+0 \end{bmatrix}$

= $\begin{bmatrix} -18 & -24 \\ -36 & -48 \end{bmatrix}$

$\therefore$ ABC $\neq$ ACB.

**6. Evaluate:** $\begin{bmatrix} \mathbf{4 \sin 30^\circ} & \mathbf{2 \cos 60^\circ} \\ \mathbf{\sin 90^\circ} & \mathbf{2 \cos 0^\circ} \end{bmatrix} \begin{bmatrix} \mathbf{4} & \mathbf{5} \\ \mathbf{5} & \mathbf{4} \end{bmatrix}$

**Solution:**

Given

$\begin{bmatrix} 4 \sin 30^\circ & 2 \cos 60^\circ \\ \sin 90^\circ & 2 \cos 0^\circ \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

$\sin 30° = \frac{1}{2}, \cos 60° = \frac{1}{2}$

$\sin 90° = 1$ and $\cos 0° = 1$

$\therefore \begin{bmatrix} 4 \times \frac{1}{2} & 2 \times \frac{1}{2} \\ 1 & 2 \times 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

$= \begin{bmatrix} 2 \times 4 + 1 \times 5 & 2 \times 5 + 1 \times 4 \\ 1 \times 4 + 1 \times 5 & 1 \times 5 + 2 \times 4 \end{bmatrix}$

$= \begin{bmatrix} 8 + 5 & 10 + 4 \\ 4 + 10 & 5 + 8 \end{bmatrix}$

$= \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$

**7. If A = $\begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix}$ find the matrix AB + BA**

**Solution:**

Given

A = $\begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix}$,

B = $\begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix}$

AB = $\begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix}$

$= \begin{bmatrix} -2 - 12 & 3 - 18 \\ 4 - 16 & -6 - 24 \end{bmatrix}$

$= \begin{bmatrix} -14 & -15 \\ -12 & -30 \end{bmatrix}$

BA = $\begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix} \times \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix}$

= $\begin{bmatrix} -2-6 & 6-12 \\ 4-12 & -12-24 \end{bmatrix}$

= $\begin{bmatrix} -8 & -6 \\ -8 & -36 \end{bmatrix}$

∴ AB + BA

= $\begin{bmatrix} -14 & -15 \\ -12 & -30 \end{bmatrix} + \begin{bmatrix} -8 & -6 \\ -8 & -36 \end{bmatrix}$

= $\begin{bmatrix} -14-8 & -15-6 \\ -12-8 & -30-36 \end{bmatrix}$

= $\begin{bmatrix} -22 & -21 \\ -20 & -66 \end{bmatrix}$

**8. If A = $\begin{bmatrix} \mathbf{1} & \mathbf{-2} \\ \mathbf{2} & \mathbf{-1} \end{bmatrix}$ and B = $\begin{bmatrix} \mathbf{3} & \mathbf{2} \\ \mathbf{-2} & \mathbf{1} \end{bmatrix}$**

**Solution:**

Given,

A = $\begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$

B = $\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$

2B = 2 $\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$

= $\begin{bmatrix} 6 & 4 \\ -4 & 2 \end{bmatrix}$

Now,

$A^2 = A \times A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1-4 & -2+2 \\ 2-2 & -4+1 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$

$\therefore 2B - A^2 = \begin{bmatrix} 6 & 4 \\ -4 & 2 \end{bmatrix} - \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$

$= \begin{bmatrix} 6-(-3) & 4-0 \\ -4-0 & 2-(-3) \end{bmatrix} = \begin{bmatrix} 6+3 & 4 \\ -4 & 2+3 \end{bmatrix}$

$= \begin{bmatrix} 9 & 4 \\ -4 & 5 \end{bmatrix}$

**9. If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$, C = $\begin{bmatrix} 5 & 1 \\ 7 & 4 \end{bmatrix}$, compute**

**(i) A (B + C)**

**(ii) (B + C) A**

**Solution:**

Given,

A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

B = $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

C = $\begin{bmatrix} 5 & 1 \\ 7 & 4 \end{bmatrix}$

(i) A (B + C)

$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \left( \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 1 \\ 7 & 4 \end{bmatrix} \right)$

$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2+5 & 1+1 \\ 4+7 & 2+4 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 7 & 2 \\ 11 & 6 \end{bmatrix}$

$= \begin{bmatrix} 7+22 & 2+12 \\ 21+44 & 6+24 \end{bmatrix} = \begin{bmatrix} 29 & 14 \\ 65 & 30 \end{bmatrix}$

(ii) (B + C) A

$= \begin{bmatrix} 7 & 2 \\ 11 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 7+6 & 14+8 \\ 11+18 & 22+24 \end{bmatrix}$

$= \begin{bmatrix} 13 & 22 \\ 29 & 46 \end{bmatrix}$

**10. If A = $\begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{2} & \mathbf{3} \end{bmatrix}$ and B = $\begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{3} & \mathbf{2} \end{bmatrix}$, C = $\begin{bmatrix} \mathbf{1} & \mathbf{3} \\ \mathbf{3} & \mathbf{1} \end{bmatrix}$, Find the matrix C (B – A).**

**Solution:**

Given,

A = $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ B = $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$, and C = $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$

Now,

B – A = $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

C (B – A) = $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1+3 & -1-3 \\ 3+1 & -3-1 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 4 & -4 \end{bmatrix}$

**11. Let A = $\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{2} & \mathbf{1} \end{bmatrix}$ and B = $\begin{bmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{-1} & \mathbf{0} \end{bmatrix}$ Find $\boldsymbol{A^2 + AB + B^2}$.**

**Solution:**

Given,

A = $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

Now,

$$A^2 = A \times A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 1 + 0 \times 2 & 1 \times 0 + 0 \times 1 \\ 2 \times 1 + 1 \times 2 & 2 \times 0 + 1 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + 0 & 0 + 0 \\ 2 + 2 & 0 + 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 2 + 0 \times 1 & 1 \times 3 + 0 \times 0 \\ 2 \times 2 + 1 \times -1 & 2 \times 3 + 1 \times 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$$

$$B^2 = B \times B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 2 + 3 \times (-1) & 2 \times 3 + 3 \times 0 \\ -1 \times 2 + 0 \times (-1) & -1 \times 3 + 0 \times 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 - 3 & 6 + 0 \\ -2 + 0 & -3 + 0 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

Hence,

$$A^2 + AB + B^2 = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + 2 + 1 & 0 + 3 + 6 \\ 4 + 3 - 2 & 1 + 6 + (-3) \end{bmatrix} = \begin{bmatrix} 4 & 9 \\ 5 & 4 \end{bmatrix}$$

**12. Let A = $\begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{0} & \mathbf{-2} \end{bmatrix}$ and B = $\begin{bmatrix} \mathbf{4} & \mathbf{1} \\ \mathbf{-3} & \mathbf{-2} \end{bmatrix}$ C = $\begin{bmatrix} \mathbf{-3} & \mathbf{2} \\ \mathbf{-1} & \mathbf{4} \end{bmatrix}$ Find $\boldsymbol{A^2 + AC - 5B}$.**

**Solution:**

Given,

A = $\begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$ B = $\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$ and C = $\begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$

Now,

$A^2 + AC - 5B = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix} - 5\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$

$= \begin{bmatrix} 4+0 & 2-2 \\ 0+0 & 0+4 \end{bmatrix} + \begin{bmatrix} -6-1 & 4+4 \\ 0+2 & 0-8 \end{bmatrix} - 5\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$

(Substituting the value from given)

$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix} - \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$

$= \begin{bmatrix} 4-7-20 & 0+8-5 \\ 0+2+15 & 4-8+10 \end{bmatrix} = \begin{bmatrix} -23 & 3 \\ 17 & 6 \end{bmatrix}$

**13. If A = $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$ B = $\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$ find AC + B² – 10C.**

**Solution:**

Given,

A = $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$ B = $\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$

Now,

AC = $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$

$= \begin{bmatrix} 2-3 & 0+12 \\ 5-7 & 0+28 \end{bmatrix}$

$= \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix}$

$B^2 = B \times B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$

$= \begin{bmatrix} 0-4 & 0+28 \\ 0-7 & -4+49 \end{bmatrix}$

$= \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix}$

$10C = 10 \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$

$= \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$

Hence,

$AC + B^2 - 10C = \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix} + \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$

$= \begin{bmatrix} -1-4-10 & 12+28-0 \\ -2-7+10 & 28+45-40 \end{bmatrix}$

$= \begin{bmatrix} -15 & 40 \\ -1 & 33 \end{bmatrix}$

**14. If A = $\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} \end{bmatrix}$ find $A^2$ and $A^3$. Also state that which of these is equal to A.**

**Solution:**

Given,

$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$A^2 = A \times A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Next,

$$A^3 = A^2 \times A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0-1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

From above, it's clearly seen that $A^3 = A$.

**15. If $x = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$ show that $6x - x^2 = 9l$ where $l$ is the unit matrix.**

**Solution:**

Given,

$$x = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$$

Now,

$$x^2 = x \times x = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 16-1 & 4+2 \\ -4-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

Taking L.H.S. we have

$$6x - x^2 = 6\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 6 \\ -6 & 12 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 24-15 & 6-6 \\ -6-6 & 12-3 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$$= 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 9l = \text{R.H.S.}$$

Hence proved.

**16.** $\begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{2} & \mathbf{1} \end{bmatrix}$ **Show that is a solution of the matrix equation $\boldsymbol{x^2 - 2x - 3l = 0}$, where $\boldsymbol{l}$ is the unit matrix of order 2.**

**Solution:**

Given,

$x^2 - 2x - 3l = 0$

$= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow x = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$\therefore x^2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$

Now,

$x^2 - 2x - 3l$

$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 5-2-3 & 4-4+0 \\ 4-4-0 & 5-2-3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore x^2 - 2x - 3l = 0$

Hence proved.

**17. Find the matrix $\boldsymbol{2 \times 2}$ which satisfies the equation.**

$$\begin{bmatrix} \mathbf{3} & \mathbf{7} \\ \mathbf{2} & \mathbf{4} \end{bmatrix}\begin{bmatrix} \mathbf{0} & \mathbf{2} \\ \mathbf{5} & \mathbf{3} \end{bmatrix} + \mathbf{2X} = \begin{bmatrix} \mathbf{1} & \mathbf{-5} \\ \mathbf{-4} & \mathbf{6} \end{bmatrix}$$

**Solution:**

Given,

$$\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0+35 & 6+21 \\ 0+20 & 4+12 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$2X = -\begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} + \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} -34 & -32 \\ -24 & -10 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -34 & -32 \\ -24 & -10 \end{bmatrix} = \begin{bmatrix} -17 & -16 \\ -12 & -5 \end{bmatrix}$$

**18. If A = $\begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \boldsymbol{x} & \boldsymbol{x} \end{bmatrix}$, Find the value of x, so that $\boldsymbol{A^2 - 0}$**

**Solution:**

Given,

$$A = \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix},$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix}\begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix}$$

$$= \begin{bmatrix} 1+x & 1+x \\ x+x^2 & x+x^2 \end{bmatrix} = 0$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

On comparing.

$1 + x = 0$

$\therefore x = -1$

**19.**

**(i)** **Find x and y if** $\begin{bmatrix} -3 & 2 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$

**(ii)** **Find x and y if** $\begin{bmatrix} 2x & x \\ y & 3y \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$

**Solution:**

(i) $\begin{bmatrix} -3 & 2 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -3x & 4 \\ 0 & -10 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -3x & +4 \\ 0 & -10 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$

Comparing the corresponding elements,

$-3x + 4 = -5$

$-3x = -5 - 4 = -9$

$x = \frac{-9}{-3} = 3$

Therefore, $x = 3$ and $y = -10$.

(ii) $\begin{bmatrix} 2x & x \\ y & 3y \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x \times 3 + x \times 2 \\ y \times 3 + 3y \times 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6x + 2x \\ 3y + 6y \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 8x \\ 9y \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$

Comparing, we get

$8x = 16$

$\Rightarrow x = \frac{16}{8} = 2$

And, $9y = 9$

$y = \frac{9}{9} = 1$

**20. Find the values of $x$ and $y$ if $\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$**

**Solution:**

Given,

$\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x+2y & -y \\ 4x & -x+y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x & +y \\ 3x & +y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

On comparing the corresponding elements, we have

$2x + y = 3$ (i)

$3x + y = 2$ (ii)

Subtracting, we get

$-x = 1 \Rightarrow x = -1$

Subtracting the values of $x$ in (i).

$2(-1) + y = 3$

$-2 + y = 3$

$y = 3 + 2 = 5$

Therefore, $x = -1$ and $y = 5$.

## CHAPTER TEST

**1. Find the values of $\boldsymbol{a}$ and $\boldsymbol{b}$ if**

$$\begin{bmatrix} \boldsymbol{a+3} & \boldsymbol{b^2+2} \\ \boldsymbol{0} & \boldsymbol{-6} \end{bmatrix} = \begin{bmatrix} \boldsymbol{2a+1} & \boldsymbol{3b} \\ \boldsymbol{0} & \boldsymbol{b^2-5b} \end{bmatrix}$$

**Solution:**

Given

$$\begin{bmatrix} a+3 & b^2+2 \\ 0 & -6 \end{bmatrix} = \begin{bmatrix} 2a+1 & 3b \\ 0 & b^2-5b \end{bmatrix}$$

Comparing the corresponding elements

$a + 3 = 2a + 1$

$\Rightarrow 2a - a = 3 - 1$

$\Rightarrow a = 2b^2 + 2 = 3b$

$\Rightarrow b^2 - 3b + 2 = 0$

$\Rightarrow b^2 - b - 2b + 2 = 0$

$\Rightarrow b(b-1) - 2(b-1) = 0$

$\Rightarrow (b-1)(b-2) = 0.$

Either $b - 1 = 0$

Then $b = 2$

Hence $a = 2, b = 2$ or 1

**2. Find a, b, c and d if 3** $\begin{bmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{bmatrix} = \begin{bmatrix} \boldsymbol{4} & \boldsymbol{a+b} \\ \boldsymbol{c+d} & \boldsymbol{3} \end{bmatrix} + \begin{bmatrix} \boldsymbol{a} & \boldsymbol{6} \\ \boldsymbol{-1} & \boldsymbol{2d} \end{bmatrix}$

**Solution:**

Given,

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix} + \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix}$$

Now comparing the corresponding elements

$3a = 4 + a$

$a - a = 4$

$2a = 4$

Therefore, $a = 2$

$3b = a + b + 6$

$3b - b = 2 + 6$

$2b = 8$

Therefore, $b = 4$

$3d = 3 + 2d$

$3d - 2d = 3$

Therefore, $d = 3$

$3c = c + d - 1$

$3c - c = 3 - 1$

$2c = 2$

Therefore, $c = 1$

Hence, $a = 2, b = 4, c = 1$ and $d = 3$

**3. Determine the matrices A and B when**

**A + 2B = $\begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix}$ and 2A – B = $\begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix}$**

**Solution:**

Given,

A + 2B = $\begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix}$ … (i)

2A – B = $\begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix}$ … (ii)

Multiplying (i) by 1 and (ii) by 2

A + 2B = $\begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix}$

4A – 2B = 2 $\begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 4 & -2 \end{bmatrix}$

Now, adding we get

5A = $\begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 10 & -5 \end{bmatrix}$

A = $\frac{1}{5}\begin{bmatrix} 5 & 0 \\ 10 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$

From (i) A + 2B = $\begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix}$

= $\begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$ + 2B = $\begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix}$

2B = $\begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 4 & -2 \end{bmatrix}$

∴ B = $\frac{1}{2}\begin{bmatrix} 0 & 2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$

Thus, A = $\begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$

**4.**

**(i)** **Find the matrix B is A = $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ and A² = A + 2B**

**(ii)** **If A = $\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 0 & 1 \\ -2 & 5 \end{bmatrix}$ and C = $\begin{bmatrix} -2 & 0 \\ -1 & 1 \end{bmatrix}$ find A (4B – 3C)**

**Solution:**

(i) Given,

A = $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$

Let B = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^2 = A \times A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 16+2 & 4+3 \\ 8+6 & 2+9 \end{bmatrix}$

$= \begin{bmatrix} 18 & 7 \\ 14 & 11 \end{bmatrix}$

A + 2B = $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} + 2\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$= \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix} = \begin{bmatrix} 4+2a & 1+2b \\ 2+2c & 3+2d \end{bmatrix}$

As, $A^2 = A + 2B$

$\Rightarrow \begin{bmatrix} 18 & 7 \\ 14 & 11 \end{bmatrix} = \begin{bmatrix} 4+2a & 1+2b \\ 2+2c & 3+2d \end{bmatrix}$

Comparing the corresponding elements, we have

$4 + 2a = 18$

$2a = 18 - 4 = 14$

$a = \frac{14}{2}$

$\Rightarrow a = 7$

$1 + 2b = 7$

$2b = 7 - 1 = 6$

$b = \frac{6}{2}$

$\Rightarrow b = 3$

$2 + 2c = 14$

$2c = 14 - 2 = 12$

$2c = 12$

$c = \frac{12}{2}$

$\Rightarrow c = 6$

$3 + 2d = 11$

$2d = 11 - 3$

$d = \frac{8}{2}$

$\Rightarrow d = 4$

Therefore, $a = 7, b = 3, c = 6$ and $d = 4$

$\therefore \text{B} = \begin{bmatrix} 7 & 3 \\ 6 & 4 \end{bmatrix}$

(ii) $\text{A} = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$,

$\text{B} = \begin{bmatrix} 0 & 1 \\ -2 & 5 \end{bmatrix}$,

$\text{C} = \begin{bmatrix} -2 & 0 \\ -1 & 1 \end{bmatrix}$

$4\text{B} - 3\text{C} = 4\begin{bmatrix} 0 & 1 \\ -2 & 5 \end{bmatrix} - 3\begin{bmatrix} -2 & 0 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 4 \\ -8 & 20 \end{bmatrix} - \begin{bmatrix} -6 & 0 \\ -3 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 0-(-6) & 4-0 \\ -8-(-3) & 20-3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+6 & 4-0 \\ -8+3 & 20-3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 4 \\ -5 & 17 \end{bmatrix}$$

Now,

$$A\,(4B - 3C) = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 6 & 4 \\ -5 & 17 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 6 + 2(-5) & 1 \times 4 + 2 \times 17 \\ -3 \times 6 + 4 \times (-5) & -3 \times 4 + 4 \times 17 \end{bmatrix}$$

$$= \begin{bmatrix} 6-10 & 4+34 \\ -18-20 & -12+68 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 38 \\ -38 & 56 \end{bmatrix}$$

**5. If A = $\begin{bmatrix} \mathbf{3} & \mathbf{2} \\ \mathbf{0} & \mathbf{5} \end{bmatrix}$ and B = $\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{2} \end{bmatrix}$ find the each of the following and state it they are equal:**

**(i) (A + B) (A – B)**

**(ii) $A^2 - B^2$**

**Solution:**

Given,

A = $\begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$

(i) (A + B) (A – B)

$$= \left\{\begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\right\} \times \left\{\begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\right\}$$

$= \begin{bmatrix} 3+1 & 2+0 \\ 0+1 & 5+2 \end{bmatrix} \times \begin{bmatrix} 3-1 & 2-0 \\ 0-1 & 5-2 \end{bmatrix}$

$= \begin{bmatrix} 4 & 2 \\ 1 & 7 \end{bmatrix} \times \begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 8-2 & 8+6 \\ 2-7 & 2+21 \end{bmatrix}$

$= \begin{bmatrix} 6 & 14 \\ -5 & 23 \end{bmatrix}$

(ii) $A^2 - B^2$

$= \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9+0 & 6+10 \\ 0+0 & 0+25 \end{bmatrix} - \begin{bmatrix} 1+0 & 0+0 \\ 1+2 & 0+4 \end{bmatrix}$

$= \begin{bmatrix} 9 & 16 \\ 0 & 25 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 9-1 & 16-0 \\ 0-3 & 25-4 \end{bmatrix}$

$= \begin{bmatrix} 8 & 16 \\ -3 & 21 \end{bmatrix}$

Hence, it's clearly seen that $(A + B)(A - B) \neq A^2 - B^2$

**6. If A = $\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ find $A^2 - 5A - 14l$, where $l$ is unit matrix of order 2 × 2.**

**Solution:**

Given,

$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$A^2 = A \times A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix} = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$

5A = $5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix}$

$\therefore A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$

$= \begin{bmatrix} 29 - 15 - 14 & -25 + 25 - 0 \\ -20 + 20 + 0 & 24 - 10 - 14 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

**7. If A = $\begin{bmatrix} \mathbf{3} & \mathbf{3} \\ \boldsymbol{p} & \boldsymbol{q} \end{bmatrix}$ and A² = 0, find p and q.**

**Solution:**

Given

A = $\begin{bmatrix} 3 & 3 \\ p & q \end{bmatrix}$

$A^2 = A \times A = \begin{bmatrix} 3 & 3 \\ p & q \end{bmatrix}\begin{bmatrix} 3 & 3 \\ p & q \end{bmatrix}$

$= \begin{bmatrix} 9 + 3p & 9 + 3q \\ 3p + pq & 3p + q^2 \end{bmatrix}$

But $A^2 = 0$

$\therefore \begin{bmatrix} 9 + 3p & 9 + 3q \\ 3p + pq & 3p + q^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

On comparing the corresponding elements, we have

$9 + 3p = 0$

$3p = -9$

$p = \frac{-9}{3}$

$p = -3$

And,

$9 + 3q = 0$

$3q = -9$

$q = \frac{-9}{3}$

$q = -3$

Therefore, $p = -3$ and $q = -3$

**8. If $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and a, b, c and d.**

**Solution:**

Given,

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -a + 0 & -b + 0 \\ 0 + c & 0 + d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -a & -b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

On comparing the corresponding elements, we have

$-a = 1 \Rightarrow a = -1$

$-b = 0 \Rightarrow b = 0$

$c = 0$ And $d = -1$

Therefore, $a = -1, b = 0, c = 0$ and $d = -1$

**9.** $\begin{bmatrix} \boldsymbol{a-b} & \boldsymbol{b-4} \\ \boldsymbol{b+4} & \boldsymbol{a-2} \end{bmatrix}\begin{bmatrix} \mathbf{2} & \mathbf{0} \\ \mathbf{0} & \mathbf{2} \end{bmatrix} = \begin{bmatrix} \mathbf{-2} & \mathbf{-2} \\ \mathbf{14} & \mathbf{0} \end{bmatrix}$ **Find $\boldsymbol{a}$ and $\boldsymbol{b}$ if**

**Solution:**

Given,

$$\begin{bmatrix} a-b & b-4 \\ b+4 & a-2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 14 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a-2b+0 & 0+2a-2b \\ 2b+8+0 & 0+2a-4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 14 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a-2b & 2a-2b \\ 2b+8 & 2a-4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 14 & 0 \end{bmatrix}$$

On comparing the corresponding terms, we have

$2a - 4 = 0$

$2a = 4$

$a = \frac{4}{2}$

$a = 2$

And, $2a - 2b = -2$

$2(2) - 2b = -2$

$4 - 2b = -2$

$2b = 4 + 2$

$b = \frac{6}{2}$

$b = 3$

Therefore, $a = 2$ and $b = 3$.

**10. If A = $\begin{bmatrix} \sec 60° & \cos 90° \\ -3\tan 45° & \sin 90° \end{bmatrix}$ and B = $\begin{bmatrix} 0 & \cos 45° \\ -2 & 3\sin 90° \end{bmatrix}$**

**Find**

**(i) 2A – 3B**
**(ii) A²**
**(iii) BA**

**Solution:**

$A = \begin{bmatrix} \sec 60° & \cos 90° \\ -3\tan 45° & \sin 90° \end{bmatrix}$ and $B = \begin{bmatrix} 0 & \cos 45° \\ -2 & 3\sin 90° \end{bmatrix}$

$A = \begin{bmatrix} \sec 60° & \cos 90° \\ -3\tan 45° & \sin 90° \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -2 & 3 \end{bmatrix}$

$(\because\ \sec 60° = 2, \cos 90° = 0, \tan 45° = 1, \sin 90° = 1)$

$B = \begin{bmatrix} 0 & \cos 45° \\ -2 & 3\sin 90° \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$ $(\because\ \cot 45° = 1)$

(i) 2A – 3B

$= 2\begin{bmatrix} 2 & 0 \\ -2 & 3 \end{bmatrix} - 3\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -6 & 9 \end{bmatrix}$

$= \begin{bmatrix} 4-0 & 0-3 \\ -6+6 & 2-9 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 0 & -7 \end{bmatrix}$

(ii) $A^2 = A \times A$

$= \begin{bmatrix} 2 & 0 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4+0 & 0+0 \\ -6-3 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -9 & 1 \end{bmatrix}$

(iii) $BA = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 0-3 & 0+1 \\ -4-9 & 0+3 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -13 & 3 \end{bmatrix}$