Class XII Session 2024-25
Subject - Applied Mathematics
Sample Question Paper - 10

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.
3. **Section – A:** It comprises of 20 MCQs of 1 mark each.
4. **Section – B:** It comprises of 5 VSA type questions of 2 marks each.
5. **Section – C:** It comprises of 6 SA type of questions of 3 marks each.
6. **Section – D:** It comprises of 4 LA type of questions of 5 marks each.
7. **Section – E:** It has 3 case studies. Each case study comprises of 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.
8. Internal choice is provided in 2 questions in Section - B, 2 questions in Section – C, 2 questions in Section - D. You have to attempt only one of the alternatives in all such questions.

Section A

1. If $\begin{bmatrix} x+y & x+2 \\ 2x-y & 16 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 1 & 3y+1 \end{bmatrix}$, then the value of x and y are: **[1]**

   a) x = 7, y = 2
   b) x = 5, y = 3
   c) x = 3, y = 5
   d) x = 2, y = 7

2. What does it mean that you calculate a 95% confidence interval? **[1]**

   a) All of these
   b) The process you used will capture the true parameter 95% of the time in long run.
   c) You can be 95% confident that your interval will include the population parameter.
   d) You can be 5% confident that your interval will not include the population parameter.

3. A certain sum of money amounts to ₹ 5832 in 2 years at 8% p.a. compound interest. The sum invested is **[1]**

   a) ₹ 5280
   b) ₹ 5400
   c) ₹ 5200
   d) ₹ 5000

4. The maximum value of Z = 2x + 3y subject to the constraints: x + y $\leq$ 1, 3x + y $\leq$ 4, x, y $\geq$ 0 is: **[1]**

   a) 5
   b) 3

c) 4 d) 2

5. If x is real, the minimum value of x^2 - 8x + 17 is **[1]**

a) 0 b) 1

c) 2 d) -1

6. The probability that a person is not a swimmer is 0.3. The probability that out of 5 persons 4 are swimmers is **[1]**

a) ${}^5C_1$ (0.7) $(0.3)^4$ b) ${}^5C_4$ $(0.7)^4$ (0.3)

c) $(0.7)^4$ (0.3) d) ${}^5C_4$ (0.7) $(0.3)^4$

7. If X is a Poisson variable such that P(X = 1) = 2P(X = 2), then P(X = 0) is **[1]**

a) e^2 b) e

c) 1 d) $\frac{1}{e}$

8. The integrating factor of the differential equation (x log x) $\frac{dy}{dx}$ + y = 2 log x, is given by: **[1]**

a) x b) log (log x)

c) x^2 d) log x

9. A boat goes downstream at u km/hr and upstream at v km/hr. The speed of the boat in still water, in km/hr is **[1]**

a) (u - v) b) u + v

c) $\frac{1}{2}$(u - v) d) $\frac{1}{2}$(u + v)

10. The order of the differential equation $2x^2 \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$, is: **[1]**

a) 2 b) 0

c) 3 d) 1

11. In what ratio must water be mixed with milk to gain $16\frac{2}{3}$% on selling the mixture at cost price? **[1]**

a) 1:6 b) 4:3

c) 6:1 d) 2:3

12. If |x + 3| $\geq$ 10, then **[1]**

a) $x \in (-\infty, -13] \cup [7, \infty)$ b) x $\in$ (-13, 7]

c) $x \in (-\infty, -13) \cup (7, \infty)$ d) x $\in$ (-13, 7)

13. A pipe can fill an empty cistern in 5 hours. A leak develops in the cistern due to which full cistern is emptied in 30 hours. With the leak, the cistern can be filled in **[1]**

a) 12 hours b) 8 hours

c) 6 hours d) 10 hours

14. The optimal value of the objective function is attained at the points **[1]**

a) given by corner points of the feasible region b) given by intersection of inequations with x-axis only

c) given by intersection of inequations with y-axis only d) given by intersection of inequations with the axes only

15. If $\frac{1}{2}\left(\frac{3}{5}x+4\right) \geq \frac{1}{3}(x-6)$, x ∈ R, then **[1]**

a) x ∈ (−∞, 120)

b) x ∈ (−∞, 120]

c) x ∈ [120, ∞)

d) x ∈ (120, ∞)

16. In a school, a random sample of 145 students is taken to check whether a student's average calory intake is 1500 or not. The collected data of average calories intake of sample students is presented in a frequency distribution which is called a **[1]**

a) Sampling distribution

b) Parameter

c) Population sampling

d) Statistics

17. $\int x^3 \log x dx$ is equal to **[1]**

a) $\frac{x^4}{8}\left(\log x - \frac{4}{x^2}\right) + C$

b) $\frac{x^4}{16}(4\log x - 1) + \text{C}$

c) $\frac{x^4}{16}(4\log x + 1) + C$

d) $\frac{x^4 \log x}{4} + C$

18. An orderly set of data arranged in accordance with their time of occurrence is called: **[1]**

a) Time Series

b) Harmonic Series

c) Geometric Series

d) Arithmetic Series

19. Let A, B and C are three matrices of same order. Now, consider the following statements **[1]**

Assertion (A): If A = B, then AC = BC.

Reason (R): If AC = BC, then A = B.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If the demand function of a product is p = 200 - $\frac{x^2}{3}$, then the marginal revenue (MR) of selling 10 units is ₹ 120. **[1]**

Reason (R): MR = $\frac{d}{dx}(\text{R})$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Using three-yearly moving averages, compute the trend values and short term fluctuations for the following data: **[2]**

Year:	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Production (Thousand tonnes):	21	22	23	25	24	22	25	26	27	26

22. Find the amount to which ₹12,000 will accumulate at the effective rate of 3% for 10 years, 4% for 4 years and 5% for 2 years. **[2]**

OR

If money is worth 5% compare the present value of a perpetuity of ₹2,000 payable at the end of each year with that of an ordinary annuity of ₹2,000 per year for 100 years. (Given $(1.05)^{-100}$ = 0.0076)

23. Evaluate the definite integral: **[2]**

$$\int_{1}^{3} \frac{\sqrt[3]{x}}{\sqrt[3]{x}+\sqrt[3]{4-x}} dx$$

24. Mr. Bharti wishes to purchase a flat for ₹ 6000000 with a down payment of ₹ 1000000 and balance in equal monthly payments for 20 years. If bank charges 7.5 % p.a. compounded monthly, calculate the EMI. (Given $(1.00625)^{240} = 4.4608$) **[2]**

OR

Find the effective rate of return which is equivalent to a stated rate of 8% compounded quarterly. [Use $(1.02)^4 = 1.0824$]

25. In what ratio water must be added in milk costing ₹ 60 per litre, so that resulting mixture would be worth ₹ 50 per litre? **[2]**

Section C

26. The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given the number triples in 5 hrs, find how many bacteria will be present after 10 hours. Also, find the time necessary for the number of bacteria to be 10 times the number of the initial present. [Given $\log_e 3 = 1.0986$, $e^{2.1972} = 9$] **[3]**

OR

Solve: $(x^2 + 1)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to the initial condition y(0) = 0.

27. Riya invested ₹ 20,000 in a mutual fund in year 2016. The value of mutual fund increased to ₹ 32,000 in year 2021. Calculate the compound annual growth rate of her investment. [Given, log(1.6) = 0.2041, antilog (0.04082) = 1.098] **[3]**

28. A company has approximated the marginal cost and marginal revenue functions for one of its products by MC = $81 - 16x + x^2$ and MR = $20x - 2x^2$ respectively. Determine the profit-maximizing output and the total profit at the optimal output, assuming fixed cost as zero. **[3]**

29. Four bad eggs are mixed with 10 good ones. If three eggs are drawn one by one with replacement, then find the probability distribution of the number of bad eggs drawn. **[3]**

OR

An urn contains 5 white, 7 red and 8 black balls. If four balls are drawn one by one with replacement, what is the probability that

i. all are white?
ii. only 3 are white?
iii. none is white?
iv. at least three are white?

30. Consider the following data: **[3]**

Year:	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
Production:	137	140	134	137	151	121	124	159	157	169	172	150

Calculate a suitable moving average and show on a graph against the original data.

31. Ten individuals are chosen at random from the population and their heights are found to be in inches 63, 63, 64, 65, 66, 69, 69, 70, 70, 71. Discuss the freedom value of Student's -t and 5% level of significance is 2.62. **[3]**

Section D

32. By using determinants, solve the following system of linear equations: **[5]**

x + y + z = 1

x + 2y + 3z = 4

x + 3y + 5z = 7

OR

The sum of three numbers is 6. If we multiply the third number by 2 and add the first number to the result, we get 7. By adding second and third numbers to three times the first number, we get 12. Using matrices find the numbers.

33. A person can row a boat at 5 km/h in still water. It takes him thrice as long to row upstream as to row downstream. Find the rate at which the stream is flowing. **[5]**

34. If X denotes the number of heads in a single toss of 4 fair coins, then find **[5]**

i. P(X = 3)

ii. P(X < 2)

iii. P(X $\leq$ 2)

iv. P(1 < X $\leq$ 3).

OR

The diameter of shafts produced in a factory confirms to normal distribution. 31% of the shafts have a diameter less than 45 mm and 8% have more than 64 mm. Find the mean and standard deviation of the diameter of shafts.

35. A firm bought a machinery for ₹ 7,40,000 on 1st April, 2020 and ₹ 60,000 is spent on its installation. Its useful life is estimated to be of 5 years. Its scrap value at the end of 5 years is estimated to be ₹ 40,000. Find the amount of annual depreciation and the rate of depreciation. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A real estate company is going to build a new residential complex. The land they have purchased can hold at most 4500 apartments. Also, if they make x apartments, then the monthly maintenance cost for the whole complex would be as follows: Fixed cost = ₹50,00,000. Variable cost = ₹$(160x - 0.04x^2)$

(a) What will be the maintenance cost as a function of x?

(b) If C(x) denote the maintenance cost function, then at what value of x the maximum value of C(x) occur ?

(c) What is the maximum value of C(x)?

OR

What should be the number of apartments, that the complex should have in order to minimize the maintenance cost?

37. **Read the text carefully and answer the questions:** **[4]**

Understanding Perpetuity

An annuity is a stream of cash flows. A perpetuity is a type of annuity that lasts forever, into perpetuity. The stream of cash flows continues for an infinite amount of time. In finance, a person uses the perpetuity calculation in valuation methodologies to find the present value of a company's cash flows when discounted back at a certain rate.

An example of a financial instrument with perpetual cash flows was the British-issued bonds known as consols, which the Bank of England phased out in 2015. By purchasing a consol from the British government, the bondholder was entitled to receive annual interest payments forever.

Perpetuity Present Value Formula

The formula to calculate the present value of perpetuity or security with perpetual cash flows is as follows:

$\text{PV} = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} \cdots = \frac{C}{r}$

where:

PV present value

C = cash flow

r = discount rate

(a) Find the present value of a perpetuity of ₹ 900 payable at the end of each year, if money is worth 5% per annum.

(b) Find the present value of a perpetuity of ₹ 500 payable at the end of each quarter, if money is worth 8% per annum.

(c) Find the present value of a perpetuity of ₹ 300 payable at the beginning of every 6 months, if money is worth 6% per annum.

OR

What amount is received at the end of every 6 months forever, if ₹ 72000 kept in a bank earns 8% per annum compounded half yearly?

38. Aeroplane is an important invention for three reasons. It shortens travel time, is more comfortable and facilitates the transport of heavy cargo. **[4]**

An aeroplane can carry a maximum of 200 passengers.

A profit of ₹400 is made on each executive class ticket and a profit of ₹300 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However at least 4 times as many passenger prefer to travel by economy class than by executive class.

Based on above information answer the following questions.

i. If x tickets of executive class and y tickets of economy class be sold, then write the constraint.
ii. Write the correct pair of constraints.
iii. If profit earned by airlines is represented by Z, then write the constraint.
iv. Airlines are interested to maximise the profit. For this what are the value of x and y i.e.r number of executive class ticket and economy class ticket to be sold?
v. What is the maximum profit earned by airlines?

OR

Read the following text carefully and answer the questions that follow:

A small firm manufacturers gold rings and chains. The total number of rings and chains manufactured per day is almost 24. It takes 1 hour to make ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹ 300 and that on a chain is ₹ 190. Firm is concerned about earning maximum profit on the number of rings (x) and chains (y) that have to be manufactured per day.

i. what is the expression for objective function? (1)
ii. For maximum profit, firm has to make,what should be the number of rings and chains? (1)
iii. What are the Corner points of the feasible region? (2)

OR

What is the maximum profit earned by the firm? (2)

Solution

Section A

1.
   (c) x = 3, y = 5
   Explanation: x = 3, y = 5
2. **(a)** All of these
   Explanation: All of these
3.
   (d) ₹ 5000
   Explanation: Let sum invested be ₹ x, rate = 8%, time = 2 years
   Amount = ₹ 5832
   $\therefore$ 5832 = $x\left(1+\frac{8}{100}\right)^2$
   $\Rightarrow$ 5832 - x $\times$ $\left(\frac{27}{25}\right)^2$
   $\Rightarrow x=\frac{5832\times25\times25}{27\times27}$ = 5000
   $\therefore$ Sum invested = ₹ 5000
4. **(a)** 5
   Explanation: 5
5.
   (b) 1
   Explanation: Let y = x^2 - 8x + 17 = (x^2 - 2 $\times$ 4 $\times$ x + 4^2) + 1 = $(x - 4)^2$ + 1
   We know that if x is a real number then $x^2 \geq 0$
   $\Rightarrow (x - 4)^2 \geq 0$
   Hence minimum value of y is 0 + 1 = 1
6.
   (b) $^5C_4 (0.7)^4 (0.3)$
   Explanation: Here,$\bar{p}$ = 0.3 $\Rightarrow$ p = 0.7 and q = 0.3, n = 5 and r = 4
   $\therefore$Required probability = $^5C_4 (0.7)^4 (0.3)$
7.
   (d) $\frac{1}{e}$
   Explanation: Given P(X = 1) = 2 P(X = 2)
   $\Rightarrow \frac{\lambda^1 e^{-\lambda}}{1!}=2\times\frac{\lambda^2\cdot e^{-\lambda}}{2!}$
   $\Rightarrow \lambda=\lambda^2 \Rightarrow \lambda=0,1 \Rightarrow \lambda=1$
   Now, P(X = 0) $=\frac{\lambda^0\cdot e^{-\lambda}}{0!}=e^{-1}=\frac{1}{e}$
8.
   (d) log x
   Explanation: Given equation can be written as $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{2}{x}$
   Therefore, I.F. = $e^{\int\frac{1}{x\log x}dx}$ = $e^{\log(\log x)}$ = log x
9.
   (d) $\frac{1}{2}$(u + v)
   Explanation: Let the boat and stream speed be x km/hr and y km/hr
   downstream speed = x + y
   u = x + y ...(i)
   upstream speed = x - y

v = x - y ...(ii)

(i) + (ii)

2x = u + v

x = $\frac{u+v}{2}$

$\therefore$ speed of boat in still water = $\frac{u+v}{2}$ km/hr

10. **(a)** 2

Explanation: It is given that equation is $2x^2 \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$

We can see that the highest order derivative present in the given differential equation is $\frac{d^2y}{dx^2}$

Thus, its order is two.

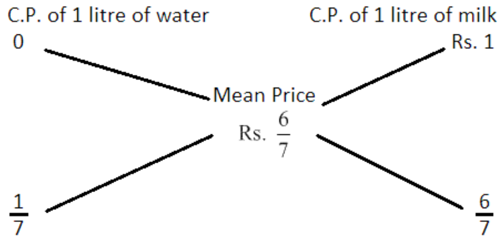
11. **(a)** 1:6

Explanation: Let C.P. of 1 litre milk be ₹ 1

S.P. of 1 litre of mixture = ₹ 1, Gain = $\frac{50}{3}$%

$\therefore$ C.P. of 1 litre of mixture = $100 \times \frac{3}{350} \times 1 = \frac{6}{7}$

By the rule of allegation, we have:



$\therefore$ Ratio of water and milk = $\frac{1}{7}$: $\frac{6}{7}$ = 1:6

12. **(a)** $x \in (-\infty, -13] \cup [7, \infty)$

Explanation: $|x + 3| \geq 10$

$x + 3 \leq -10$ and $x + 3 \geq 10$

$\Rightarrow x \leq -13$ and $x \geq 7$

$\Rightarrow x \in (-\infty, -13][7, \infty)$

13.

(c) 6 hours

Explanation: Tap A can fill the cistern in 5 hour

and leak L can empty the cistern in 30 hours.

So

$\frac{1}{5} - \frac{1}{30} = \frac{6-1}{30} = \frac{5}{3}$ Part can be filled in 1 hours.

$\therefore$ total time to fill the cistern

= $\frac{30}{5}$ = 6 hours

14. **(a)** given by corner points of the feasible region

Explanation: It is known that the optimal value of the objective function is attained at any of the corner points.

Thus, the optimal value of the objective function is attained at the points given by corner points of the feasible region.

15.

(b) $x \in (-\infty, 120]$

Explanation: $x \in (-\infty, 120]$

16. **(a)** Sampling distribution

Explanation: Sampling distribution

17.

(b) $\frac{x^4}{16}(4\log x - 1) + C$

Explanation: $\int \underset{\text{II}}{x^3} \log x dx = \log x \int x^3 dx - \int \left\{\frac{d}{dx}\log x \int x^3 dx\right\} dx$

$= \log x \cdot \frac{x^4}{4} - \int \frac{1}{x} \cdot \frac{x^4}{4} dx = \frac{x^4}{4}\log x - \frac{x^4}{16} + C$

$= \frac{x^4}{16}(4\log x - 1) + C$

18. **(a)** Time Series

Explanation: The organized series of data on the basis of any measure of time is called Time series.

19.

(c) A is true but R is false.

Explanation: For three matrices A, B and C of the same order, if A = B, then AC = BC but the converse is not true.

20.

(d) A is false but R is true.

Explanation: Given P = 200 - $\frac{x^2}{3}$

$\Rightarrow$ R(x) = P·x $\Rightarrow$ R(x) = 200x - $\frac{x^3}{3}$

Now, Marginal Revenue (MR) = $\frac{d}{dx}$R = $\frac{d}{dx}$(200x - $\frac{x^3}{3}$)

$\Rightarrow$ MR = 200-x^2

= $[MR]_{x=10}$ = ₹(200 - 10^2) = ₹100

Assertion is false.

Reason is true.

Section B

21. Computation of trend values

Year	Production (Thousand tonnes)	3-yearly moving totals	3-yearly moving averages Y_c	Short term fluctuations ($Y - Y_c$)
2008	21	-	-	-
2009	22	66	22.00	0
2010	23	70	23.33	-0.33
2011	25	72	24.00	1.00
2012	24	71	23.67	0.33
2013	22	71	23.67	-1.67
2014	25	73	24.33	0.67
2015	26	78	26.00	0.00
2016	27	79	26.33	0.67
2017	26	-	-	-

22. The effective rate is the actual rate compounded annually. Therefore, the required sum S is given by

S = 12000 $(1 + \frac{3}{100})^{10}$ $(1 + \frac{4}{100})^4$ $(1 + \frac{5}{100})^2$

$\Rightarrow$ S = 12000 $(1.03)^{10}$ $(1.04)^4$ $(1.05)^2$

$\Rightarrow$ S = 12000 (1.34391638) (1.16985856) (1.1025) = 20800.10

Hence, the amount is ₹20,800.10

OR

Let P be the present value of a perpetuity of ₹2,000 payable at the end of each year when money is worth 5%. It is given that i = $\frac{5}{100}$ = 0.05 and R = 2,000

$\therefore$ P = $\frac{R}{i}$ $\Rightarrow$ P = ₹$\frac{2,000}{0.05}$ = ₹40,000

Let P_1 be the present value of an ordinary annuity of ₹2,000 per year for 100 years. Then,

$P_1 = R\left\{\frac{1-(1-i)^{-n}}{i}\right\}$

We have, R = 2,000, i = $\frac{5}{100}$ = 0.05 and n = 100

$\therefore$ P_1 = ₹2,000 $\left\{\frac{1-(1.05)^{-100}}{0.05}\right\}$ = ₹2,000$\left(\frac{1-0.0076}{0.05}\right)$ = ₹$\frac{2,000\times0.9924}{0.05}$ = ₹39,696

We observe that the present value of the perpetuity is more than that of ordinary annuity.

23. Let I = $\int_1^3 \frac{\sqrt[3]{x}}{\sqrt[3]{x}+\sqrt[3]{4-x}} dx$ ...(1)

Then by using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

I = $\int_1^3 \frac{\sqrt[3]{1+3-x}}{\sqrt[3]{1+3-x}+\sqrt[3]{4-(1+3-x)}} dx$

$\Rightarrow$ I = $\int_1^3 \frac{\sqrt[3]{4-x}}{\sqrt[3]{4-x}+\sqrt[3]{x}} dx$ ...(2)

On adding (1) and (2), we get

2I = $\int_1^3 \frac{\sqrt[3]{x}+\sqrt[3]{4-x}}{\sqrt[3]{x}+\sqrt[3]{4-x}} dx = \int_1^3 1 dx = [x]_1^3$

$\Rightarrow$ 2I = 3 - 1 $\Rightarrow$ 2I = 2 $\Rightarrow$ I = 1

24. Cost of flat = ₹ 6000000, cash payment ₹ 1000000

So, balance = ₹ 6000000 - ₹ 1000000 = ₹ 5000000

Given P = ₹ 5000000, n = 12 $\times$ 20 = 240 months, i = $\frac{7.5}{1200}$ = 0.00625

$\therefore$ EMI = $\frac{5000000 \times 0.00625 \times (1.00625)^{240}}{(1.00625)^{240}-1}$

= $\frac{5000000 \times 0.00625 \times 4.4608}{3.4608}$ = ₹40279.70

OR

Here, r = 8% p.a., p = 4 quarters

So, effective rate (per rupee) = $\left(1+\frac{8}{400}\right)^4$ - 1 = $(1.02)^4$ - 1

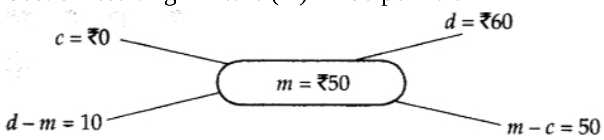
= 1.0824 - 1 = 0.0824

Hence, the effective rate = 0.0824 $\times$ 100% = 8.24%.

25. Cost of water (c) = ₹ 0

Cost of milk (d) = ₹ 60 per litre

Cost of resulting mixture (m) =₹ 50 per litre



So, $\frac{\text{quantity of water}}{\text{quantity of milk}} = \frac{10}{50} = \frac{1}{5}$.

Hence, the required ratio is 1 : 5.

Section C

26. Let A be the amount of bacteria present at time t and A_0 be the initial amount of bacteria. Therefore,we have,

$\frac{dA}{dt} \propto A$

$\frac{dA}{dt} = \lambda A$

$\frac{dA}{A} = \lambda dt$

Integrating both sides,we get,

log A = λt + c ...(i)

when t = 0, A = A_0

log A_0 = 0 + c

c = log A_0

Using equation (i),

log A = λt + log A_0

$\log\left(\frac{A}{A_0}\right) = \lambda t$ ..(ii)

Given, bacteria triples is 5 hours, so A = 3 A_0, when t = 5,

therefore from (ii),we have,

$\log\left(\frac{3A_0}{A_0}\right) = 5\lambda$

log 3 = 5λ

$\lambda = \frac{\log 3}{5}$

Put the value of λ in equation (ii), we have,

$\log\left(\frac{A}{A_0}\right) = \frac{\log 3}{5}t$

Case I: let A_1 be the number of bacteria present in 10 hours, then, we have,

$\log\left(\frac{A_1}{A_0}\right) = \frac{\log 3}{5} \times 10$

$\log\left(\frac{A_1}{A_0}\right)$ = 2 log 3

$\log\left(\frac{A_1}{A_0}\right)$ = 2(1.0986)

$\log\left(\frac{A_1}{A_0}\right)$ = 2.1972

$A_1 = A_0e^{2.1972}$

$A_1 = A_09$

Hence, there will be 9 times the bacteria present is 10 hours.

Case II: Let t_1 be the time necessary for the bacteria to be 10 times, then, we have,

$\log\left(\frac{A}{A_0}\right) = \frac{\log 3}{5} \times t$

$\log\left(\frac{10A_0}{A_0}\right) = \frac{\log 3}{5} \times t_1$

5 log 10 = log 3 t_1

$5\frac{\log 10}{\log 3} = t_1$

Required time is $\frac{5 \log 10}{\log 3}$hours.

OR

The given differential equation can be written as

$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$ ...(i)

This is a linear differential equation of the form $\frac{dy}{dx}$ + Py = Q, where

P = $\frac{2x}{1+x^2}$ and Q = $\frac{4x^2}{1+x^2}$

∴ I.F. = $e^{\int Pdx} = e^{\int \frac{2x}{(1+x^2)dx}} = e^{\log(1+x^2)}$ = 1 + x^2

Multiplying both sides of (i) by I.F. = (1 + x^2), we get

(1 + x^2)$\frac{dy}{dx}$ + 2xy = $4x^2$

Integrating both sides with respect to x, we get

y(1 + x^2) = ∫ $4x^2$ dx + C [Using: y(I.F.) = ∫ Q (I.F.) dx + C]

⇒ y (1 + x^2) = $\frac{4x^3}{3}$ + C ...(ii)

It is given that y = 0, when x = 0. Putting x = 0 and y = 0 in (i), we get

0 = 0 + C ⇒ C = 0

Substituting C = 0 in (ii), we get y = $\frac{4x^3}{3(1+x^2)}$, which is the required solution.

27. Given beginning value of investment = ₹ 20,000

Final value of the investment = ₹ 32,000 No. of years = 5

So, CAGR = $\left(\frac{\text{End Value}}{\text{Beginning Value}}\right)^{\frac{1}{n}} - 1$

=$\left(\frac{32000}{20000}\right)^{\frac{1}{5}} - 1$

= $(1.6)^{\frac{1}{5}} - 1$

x = $(1.6)^{\frac{1}{5}}$

Let,

Taking log both sides, we get

log x = $\frac{1}{5}$log(1.6)

⇒ log x = $\frac{1}{5}$ ×0.2041

⇒ log x = 0.04082

⇒ x = antilog (0.04802)

= 1.098

CAGR = 1.098 - 1 = 0.098

= 9.8%

28. Let P be the profit function. Then,

$\frac{dP}{dx}$ = MR - MC

$\Rightarrow \frac{dP}{dx}$ = (20x - 2x^2) - (81 - 16x + x^2)

$\Rightarrow \frac{dP}{dx}$ = -81 + 36x - 3x^2 ... (i)

$\Rightarrow \frac{dP}{dx}$ = -81 + 36x - 3x^2 and $\frac{d^2P}{dx^2}$ = 36 - 6x

For P to be maximum, we must have

$\frac{dP}{dx}$ = 0

$\Rightarrow$ -81 + 36x - 3x^2 = 0

$\Rightarrow$ -3(x^2 - 12x + 27) = 0

$\Rightarrow$ -3(x - 3) (x - 9) = 0

$\Rightarrow$ x = 3 or, x = 9

We find that

$\left(\frac{d^2P}{dx^2}\right)_{x=9}$ = 36 - 6 $\times$ 9 = -18 < 0

Thus, the output x = 9 gives maximum profit.

From (i), we obtain

$\frac{dP}{dx}$ = -81 + 36x - 3x^2

Integrating both sides, we obtain

P = -81x + 18x^2 - x^3 + k ... (ii)

When x = 0, fixed cost = 0 i.e. there is no profit. So, putting x = 0, P = 0 in (ii), we obtain

P = -81x + 18x^2 - x^3

Putting x = 9, we obtain

P = - 81 $\times$ 9 + 18 $\times$ 9^2 - 9^3 = 0

Hence, there is no profit when 9 items are produced.

29. Since the eggs are drawn one by one with replacement, the events are independent, therefore, it is a problem of binomial distribution

Total number of eggs = 4 + 10 = 14, out of which 4 are bad

If p = probability of drawing a bad egg, then p = $\frac{4}{14} = \frac{2}{7}$, so q = 1 - $\frac{2}{7} = \frac{5}{7}$

Thus, we have a binomial distribution with p = $\frac{2}{7}$, q = $\frac{5}{7}$ and n = 3

If X denotes the number of bad eggs obtained, then X can take values 0, 1, 2, 3

P(0) = $^3C_0$ q^3 = $\left(\frac{5}{7}\right)^3 = \frac{125}{343}$,

P(1) = $^3C_1$ pq^2 = 3 $\cdot \frac{2}{7} \cdot \left(\frac{5}{7}\right)^2 = \frac{150}{343}$,

P(2) = $^3C_2$ p^{2}q = 3 $\cdot \left(\frac{2}{7}\right)^2 \cdot \frac{5}{7} = \frac{60}{343}$ and

P(3) = $^3C_3$ p^3 = $\left(\frac{2}{7}\right)^3 = \frac{8}{343}$

$\therefore$ The required probability distribution is $\begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{125}{343} & \frac{150}{343} & \frac{60}{343} & \frac{8}{343} \end{pmatrix}$.

OR

Let p denote the probability of drawing a white ball from an urn containing 5 white, 7 red and 8 black balls. Then,

$p = \frac{^5C_1}{^{20}C_1} = \frac{5}{20} = \frac{1}{4}$. So, $q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$

Let X be a random variable denoting the number of white balls in 4 draws with replacement. Then, X is a binomial variate with parameters n = 4 p = $\frac{1}{4}$ such that

P(X = r) = Probability that r balls are white $= {}^4C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{4-r}$; r = 0, 1, 2, 3, 4 ...(i)

Now,

i. Probability that all are white = P(X = 4) $= {}^4C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^{4-4} = \left(\frac{1}{4}\right)^4$ [Using (i)]
ii. Probability that only 3 are white = P(X = 3) $= {}^4C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^{4-3} = 3\left(\frac{1}{4}\right)^3$ [Using (i)]
iii. Probability that none is white = P(X = 0) $= {}^4C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^4 = \left(\frac{3}{4}\right)^4$ [Using (i)]

iv. Probability that at least three are white = P(X $\geq$ 3) = P(X = 3) + P(X = 4)

$= {}^4C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^{4-3} + {}^4C_4\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^0$ [Using (i)]

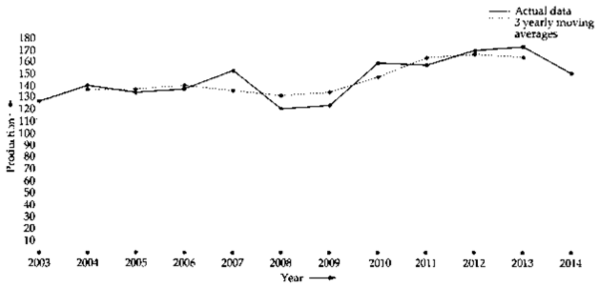
$= 13\left(\frac{1}{4}\right)^4$

30. In order to find which moving average will be appropriate, we will have to estimate the length of the cycle of the above data. We observe that the data has the pattern (137, 140, 134), (137, 151, 121), (124, 159, 157), (169, 172, 150). Thus, we have cycle length of 3. So, we will calculate 3 yearly moving averages as shown in the following table.

Calculation of 3-yearly moving averages

Year	**Production**	**3-yearly moving totals**	**3-yearly moving averages**
2003	137	-	-
2004	140	411	137.00
2005	134	411	137.00
2006	137	422	140.67
2007	151	409	136.33
2008	121	396	132.00
2009	124	404	134.67
2010	159	440	146.67
2011	157	485	161.67
2012	169	498	166.00
2013	172	491	163.67
2014	150	-	-

These moving averages and the original data are plotted on the graph paper to obtain the following graph.



31.

x	$x - \bar{x}$	$(x - \bar{x})^2$
63	-4	16
63	-4	16
64	-3	9
65	-2	4
66	-1	1
69	2	4
69	2	4
70	3	9
70	3	9
71	4	16
$\sum x = 670$		$\sum(x - \bar{x})^2$ = 88

$= \frac{\sum x}{n}$
$= \frac{670}{10}$ = 67
Now, compute the standard deviation using formula as,
$\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$
$= \sqrt{\frac{88}{9}}$
= 3.13 inches
H_0 = The mean of universe, μ = 65 inches, we get
$t = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$
$= \frac{67-65}{\frac{3.13}{\sqrt{10}}}$
$= \frac{2}{\frac{3.13}{3.16}}$
$= \frac{2}{0.9905}$
= 2.02
The number of degree of freedom = n - 1 = 9 Given that the tabulated value for 9 d.f. at level of significance is 2.62.
Since calculated value of t is less than the tabulated value i.e., 2.02 < 2.62, the error has arisen due to fluctuations and we may conclude that the data are consistent with the assumption of mean of height in the universe of 65 inches.

Section D

32. Here
D = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix}$ = 1(10 - 9) - 1(5 - 3) + 1(3 - 2)
= 1 - 2 + 1 = 0
D_1 = $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 3 \\ 7 & 3 & 5 \end{vmatrix}$ = 1(10 - 9) - 1(20 - 21) + 1(12 - 14)
= 1 + 1 - 2 = 0
D_2 = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & 3 \\ 1 & 7 & 5 \end{vmatrix}$ = 1(20 - 21)-1(5 - 3) + 1(7 - 4)
= -1 - 2 + 3 = 0 and
D_3 = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 7 \end{vmatrix}$ = 1(14 - 12) -1(7 - 4) + 1(3 - 2)
= 2 - 3 + 1 = 0
Thus, D = D_1 = D_2 = D_3 = 0, therefore, the given system may or may not be consistent. Let us solve the first two equations for x and y in terms of z. These equations can be written as:
x + y = 1 - z
x + 2y = 4 - 3z
To solve these equations, we use Cramer's rule.
D = $\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$ = 2 - 1 = 1,
D_1 = $\begin{vmatrix} 1-z & 1 \\ 4-3z & 2 \end{vmatrix}$ = 2 - 2z - 4 + 3z = z - 2, and
D_2 = $\begin{vmatrix} 1 & 1-z \\ 1 & 4-3z \end{vmatrix}$ = 4 - 3z - 1 + z = 3 - 2z
By Cramer's rule,
x = $\frac{D_1}{D}$ = $\frac{z-2}{1}$ = z - 2, y = $\frac{D_2}{D}$ = $\frac{3-2z}{1}$ = 3 - 2z.
Let
z = k where k is arbitrary number, then we get
x = k - 2, y = 3 - 2k, z = k, where k is any number.
Note that these values satisfy the third equation i.e. x + 3y + 5z = 7 of the given system. Hence, the system is consistent and it has

infinitely many solutions given by

x = k - 2, y = 3 - 2k, z = k where k is any number.

OR

Let the three numbers be x, y and z. Then, from the given conditions, we obtain

x + y + z = 6 or x + y + z = 6

x + 2z = 7 or, x + 0y + 2z = 7

3x + y + z = 12 or, 3x + y + z = 12

Here $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{vmatrix}$ = 1(0 - 2) - 1(1 - 6) + 1(1 - 0) = -2 + 5 + 1 = 4

$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 7 & 0 & 2 \\ 12 & 1 & 1 \end{vmatrix}$ = 6(0 - 2) - 1(7 - 24) + 1(7 - 0) = -12 + 17 + 7 = 12

$D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 7 & 2 \\ 3 & 12 & 1 \end{vmatrix}$ = 1(7 - 24) - 6(1 - 6) + 1(12 - 21) = -17 + 30 - 9 = 4

and, $D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 0 & 7 \\ 3 & 1 & 12 \end{vmatrix}$ = 1 (0 - 7) - 1(12 - 21) + 6(1 - 0) = -7 + 9 + 6 = 8

$\therefore x = \frac{D_1}{D} = \frac{12}{4}$ = 3, $y = \frac{D_2}{D} = \frac{4}{4}$ = 1 and $z = \frac{D_3}{D} = \frac{8}{4}$ = 2

Thus, the three numbers are 3, 1 and 2.

33. Let the distance covered be d km. and y be speed of stream

speed of boat = 5 km/h

speed of stream = y km/h

speed of boat in upstram(u): x - y km/h

= 5 - y km/h

speed of boat in downstream (v) = x + y km/h

= 5 + y km/h

ATQ.

$\frac{d}{5-y} = 3\left(\frac{d}{5+y}\right)$ $\left[\because T = \frac{D}{S}\right]$

$\frac{1}{5-y} = \frac{3}{5+y}$

5 + y = 3(5 - y)

5 + y = 15 - 3y

y + 3y = 15 - 5

4y = 10

y = $\frac{10}{4}$

y = $\frac{5}{2}$ km/h

y = $2\frac{1}{2}$ km/h

speed of stream is 2.5 km/h

34. A fair coin is tossed 4 times.

$\therefore$ n = 4, probability of getting a head $= p = \frac{1}{2},$ so q$= 1 - \frac{1}{2} = \frac{1}{2}$.

P(r) $= {}^4C_r p^r q^{4-r} = {}^4C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{4-r} = {}^4C_r \left(\frac{1}{2}\right)^4$

i. P(X = 3) = P(3) $= {}^4C_3 \left(\frac{1}{2}\right)^4 = 4 \times \frac{1}{16} = \frac{1}{4}$.

ii. P(X < 2) $= P(0) + P(1) = {}^4C_0 \left(\frac{1}{2}\right)^4 + {}^4C_1 \left(\frac{1}{2}\right)^4$

$= (1+4) \times \left(\frac{1}{2}\right)^4 = \frac{5}{16}$.

iii. P(X $\leq$ 2) = P(0) + P(1) + P(2) $= \left({}^4C_0 + {}^4C_1 + {}^4C_2\right) \left(\frac{1}{2}\right)^4$

= (1 + 4 + 6) $\times \frac{1}{6} = \frac{11}{16}$.

iv. P(1 < X $\leq$ 3) = P(2) + P(3) $= \left({}^4C_2 + {}^4C_3\right) \left(\frac{1}{2}\right)^4$

$= (6+4) \times \frac{1}{16} = \frac{10}{16} = \frac{5}{8}$

OR

Let X denote the diameter of shafts. Let μ be mean and σ be the standard deviation. It is given that X follows a normal distribution.

Let Z be the standard normal variate. Then, Z = $\frac{X-\mu}{\sigma}$

X = 45 $\Rightarrow$ Z = $\frac{45-\mu}{\sigma}$ = Z_1 (Say) and, X = 64 $\Rightarrow$ Z = $\frac{64-\mu}{\sigma}$ = Z_2 (Say).

Now,

P (X < 45) = 0.31

= P(Z < Z_1) = 0.31

= P(Z $\leq$ 0) - P($Z_1 \leq Z \leq 0$) = 0.31 [$\because$ P(Z < Z_1) is less than 0.5 $\therefore$ $Z_1 < 0$]

= 0.5 - P($Z_1 \leq Z \leq 0$) = 0.31

= P($Z_1 \leq Z \leq 0$) = 0.19

= P($0 \leq Z \leq |Z_1|$) = 0.19

= $|Z_1|$ = 0.5 [See Table]

= $-Z_1$ = 0.5 [$\because$ $Z_1 < 0$ $\therefore$ $|Z_1| = -Z_1$]

= Z_1 = -0.5

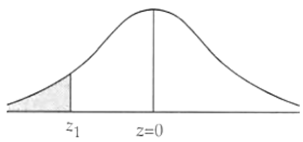
= $\frac{45-\mu}{\sigma}$ = -0.5 [$\because$ $Z_1 = \frac{45-\mu}{\sigma}$]

= μ - 0.5 σ = 4.5 ...(i)

And

P(X > 64) = 0.08

= P(Z > Z_2) = 0.08



= P(Z $\geq$ 0) - P($0 \leq Z \leq Z_2$) = 0.08

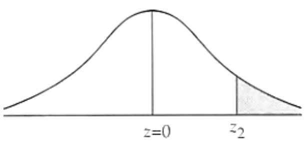
= 0.5 - P($0 \leq Z \leq Z_2$) = 0.08

= P($0 \leq Z \leq Z_2$) = 0.42

= Z_2 = 1.41

= $\frac{64-\mu}{\sigma}$ = 1.41

= μ + 1.41 σ = 64 ...(ii)



Solving (i) and (ii), we get σ = 10 and μ = 49.9

Hence the mean and standard deviation of the diameter of shafts are 49.9 mm and 10 mm. respectively.

35. Cost = 740000 + 60,000 = ₹ 8,00,000.

Determination of the amount of annual Depreciation and Rate of Depreciation = $\frac{\text{Cost of Asset - Estimated Realisable or Scrap}}{\text{No. of year of estimated useful life}}$

= $\frac{(740000 + ₹\ 60,000) - ₹\ 40000}{5.}$

= ₹ 1,52,000

Rate of Depreciation = $\frac{\text{Annual depreciation}}{\text{cos 1 of Asset}} \times 100.$

= $\frac{1,52,000}{800,000} \times 100$ = 19% p.a.

Section E

36. **Read the text carefully and answer the questions:**

A real estate company is going to build a new residential complex. The land they have purchased can hold at most 4500 apartments. Also, if they make x apartments, then the monthly maintenance cost for the whole complex would be as follows:

Fixed cost = ₹50,00,000. Variable cost = ₹$(160x - 0.04x^2)$

(i) Let C(x) be the maintenance cost function, then $C(x) = 5000000 + 160x - 0.04x^2$.

(ii) We have, $C(x) = 5000000 + 160x - 0.04x^2$

Now, C{x) = 160 - 0.08x

For maxima/minima, put C'(x) = 0

$\Rightarrow$ 160 = 0.08x

$\Rightarrow$ x = 2000

(iii) Clearly, from the given condition we can see that we only want critical points that are in the interval [0, 4500]

Now, we have C(0) = 5000000

C(2000) = 5160000 and C(4500) = 4910000

$\therefore$ Maximum value of C(x) would be ₹5160000

OR

The complex must have 4500 apartments to minimise the maintenance cost.

37. **Read the text carefully and answer the questions:**

Understanding Perpetuity

An annuity is a stream of cash flows. A perpetuity is a type of annuity that lasts forever, into perpetuity. The stream of cash flows continues for an infinite amount of time. In finance, a person uses the perpetuity calculation in valuation methodologies to find the present value of a company's cash flows when discounted back at a certain rate.

An example of a financial instrument with perpetual cash flows was the British-issued bonds known as consols, which the Bank of England phased out in 2015. By purchasing a consol from the British government, the bondholder was entitled to receive annual interest payments forever.

Perpetuity Present Value Formula

The formula to calculate the present value of perpetuity or security with perpetual cash flows is as follows:

$\text{PV} = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} \cdots = \frac{C}{r}$

where:

PV present value

C = cash flow

r = discount rate

(i) ₹ 18000

(ii) ₹ 25000

(iii) ₹ 10300

OR

₹ 2880

38. i. Since, Aeroplane can carry a maximum of 200 passengers

$\therefore$ x + y $\leq$ 200

ii. Since, Airline reserves at least 20 seats for executive class

$\Rightarrow$ x $\leq$ 200

Also atleast four times as many passengers prefer to travel by economy class than by executive class.

$\Rightarrow$ y = 4x

$\Rightarrow$ x + 4x $\leq$ 200 [$\because$ x + y $\leq$ 200]

$\Rightarrow$ 5x $\leq$ 200 $\Rightarrow$ x $\leq$ 40

$\Rightarrow$ x $\geq$ 20 and x $\leq$ 40

iii. Profit on executive class = 400x

Profit on executive class = 300y

$\therefore$ Total profit Z = 400x + 300y

iv. We have

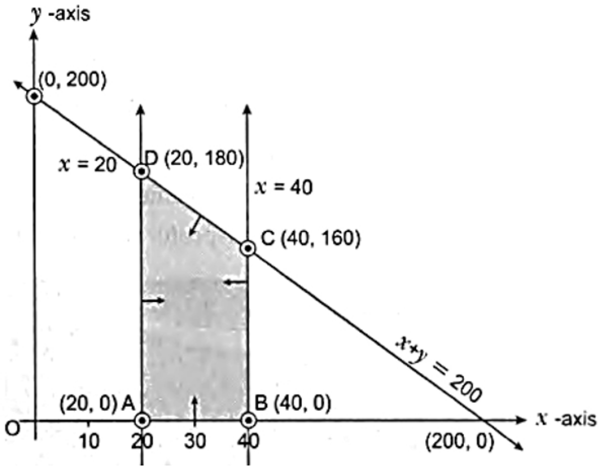
Z = 400x + 300y which is to be maximise under constraints

$x + y \leq 200$

$x \leq 40$

$x \geq 20$, $y \geq 0$

Here, ABCD in bounded feasible region with comer points A(20, 0), B(40, 0), C(40,160), D (20,180).



Now we evaluate Z at each corner points.

Corner point	Z = 400x + 300y
A(20, 0)	8000
B(40, 0)	16000
C(40, 160)	64000 ← Maximum
D(20, 180)	62000

For maximum profit x = 40, y = 160

v. We have

Z = 400x + 300y

$= 400 \times 40 + 300 \times 160$

= 16000 + 48000

= ₹ 64000 [∵ For maximum profit x = 40, y = 160]

OR

i. 300x + 190y

ii. 8, 16

iii. both (0, 24) and (8, 16)

OR

5440