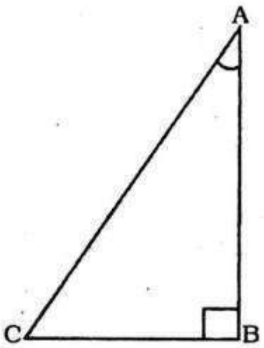
CHAPTER 12

Trigonometric Identities

A Trigonometric Identity is a trigonometric equation which is true for all values of the variable (s).

Let's disuss some trigonometric identities.



In ΔABC, right-angled at B we have:

$AB^2 + BC^2 = AC^2$ ...(i)

Dividing each term of (i) by AC^2, we get

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

$$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = \left(\frac{AC}{AC}\right)^2$$

i.e., $(\cos A)^2 + (\sin A)^2 = 1$

i.e., $\cos^2 A + \sin^2 A = 1$ ....(ii)

This is true for all A such that $0° \leq A \leq 90°$. So, this is a trigonometric identity.

Let us now divide (1) by AB^2. We get

$$\frac{AB^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

or, $$\left(\frac{AB}{AB}\right)^2 + \left(\frac{BC}{AB}\right)^2 = \left(\frac{AC}{AB}\right)^2$$

i.e., $1 + \tan^2 A = \sec^2 A$ ....(iii)

Let us see what we get on dividing (i) by BC^2. We get

$$\frac{AB^2}{BC^2} + \frac{BC^2}{BC^2} = \frac{AC^2}{BC^2}$$

i.e., $$\left(\frac{AB}{BC}\right)^2 + \left(\frac{BC}{BC}\right)^2 = \left(\frac{AC}{BC}\right)^2$$

i.e., $\cot^2 A + 1 = \operatorname{cosec}^2 A$ ....(iv)

Note that cosec A and cot A are not defined for A = 0°. Therefore (iv) is true for all A such that $0° < A \leq 90°$.

Using these identities, we can express each trigonometric ratio in terms of other trigonometric ratios, i.e., if any one of the ratios is known, we can also determine the values of other trigonometric ratios.

Let us see how we can do this using these identities. Suppose we know that

$$\tan A = \frac{1}{\sqrt{3}} \text{ Then, } \cot A = \sqrt{3}$$

Since, $$\sec^2 A = 1 + \tan^2 A = 1 + \frac{1}{3} = \frac{4}{3},\ \sec A = \frac{2}{\sqrt{3}}$$

and $$\cos A = \frac{\sqrt{3}}{2}$$

Again, $$\sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}.$$

Therefore, cosec A = 2.

Example 1 : Express the ratios cos A, tan A and sec A in terms of sin A.

Solution : Since $\cos^2 A + \sin^2 A = 1$, therefore,

$\cos^2 A = 1 - \sin^2 A$, i.e. $\cos A = \pm \sqrt{1 - \sin^2 A}$

This gives $\cos A = \sqrt{1 - \sin^2 A}$

Hence, $$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1 - \sin^2 A}}$$

and $$\sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1 - \sin^2 A}}$$

Example 2 : Prove that

$\sec A (1 - \sin A)(\sec A + \tan A) = 1$

Solution :

LHS = $\sec A (1 - \sin A)(\sec A + \tan A)$

$$= \left(\frac{1}{\cos A}\right)(1 - \sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$$

$$= \frac{(1 - \sin A)(1 + \sin A)}{\cos^2 A} = \frac{1 - \sin^2 A}{\cos^2 A}$$

$$= \frac{\cos^2 A}{\cos^2 A} = 1 = \text{RHS}$$

Example 3 : Prove that

$$\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$$

Solution : LHS $= \dfrac{\cot A - \cos A}{\cot A + \cos A} = \dfrac{\dfrac{\cos A}{\sin A} - \cos A}{\dfrac{\cos A}{\sin A} + \cos A}$

$$= \frac{\cos A\left(\frac{1}{\sin A} - 1\right)}{\cos A\left(\frac{1}{\sin A} + 1\right)} = \frac{\left(\frac{1}{\sin A} - 1\right)}{\left(\frac{1}{\sin A} + 1\right)} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} = \text{RHS}$$

Example 4 : Prove that

$\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{1}{\sec\theta - \tan\theta}$, using the identity $\sec^2\theta = 1 + \tan^2\theta$.

Solution : Since we will apply the identity involving $\sec\theta$ and $\tan\theta$, let us first convert the LHS (of the identity we need to prove) in terms of $\sec\theta$ and $\tan\theta$ by dividing numerator and denominator by $\cos\theta$.

LHS =

$$= \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

$$= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1} = \frac{\{(\tan\theta + \sec\theta) - 1\}(\tan\theta - \sec\theta)}{\{(\tan\theta - \sec\theta) + 1\}(\tan\theta - \sec\theta)}$$

$$= \frac{(\tan^2\theta - \sec^2\theta)(\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

$$= \frac{-1 - \tan\theta + \sec\theta}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

$$= \frac{-1}{\tan\theta - \sec\theta} = \frac{1}{\sec\theta - \tan\theta},$$

which is the RHS of the identity, we are required to prove.

SOLVED OBJECTIVE QUESTIONS

1. Evaluate : $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta}$

(1) $\sec^2\theta$ (2) $2\sec^2\theta$
(3) $2\sec\theta$ (4) $\sec\theta$

2. $\operatorname{cosec}^2\theta + \sec^2\theta = ?$

(1) $\sec^2\theta$ (2) $\operatorname{cosec}^2\theta$
(3) $\operatorname{cosec}^2\theta + \tan^2\theta$ (4) $\operatorname{cosec}^2\theta \cdot \sec^2\theta$

3. $(1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta) = ?$

(1) 1 (2) – 1
(3) 0 (4) 2

4. Evaluate : $\dfrac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta}$

(1) $\dfrac{\sec\theta + 1}{2}$ (2) $\dfrac{\sec\theta - 1}{\sec\theta + 1}$
(3) $\dfrac{\sec\theta + 1}{\sec\theta - 1}$ (4) $\dfrac{2\sec\theta}{\sec\theta + 1}$

5. Evaluate : $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}$

(1) $\sec\theta + \tan\theta$ (2) $\sec\theta - \tan\theta$
(3) $2\sec\theta + \tan\theta$ (4) $2\sec\theta$

6. Evaluate : $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

(1) $7 + \tan^2\theta$
(2) $7 + \tan^2\theta + \cot^2\theta$
(3) $5 + \tan^2\theta + \cot^2\theta$
(4) $5 - \tan^2\theta + \cot^2\theta$

7. Evaluate : $(\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2$

(1) $(1+\sec\theta.\operatorname{cosec}\theta)^2$ (2) $1 + \sec\theta.\operatorname{cosec}\theta$
(3) $1 - \sec\theta$ (4) None of these

8. Evaluate : $\dfrac{1-\cos\theta}{1+\cos\theta}$

(1) $\operatorname{cosec}\theta - \cot\theta$ (2) $(\operatorname{cosec}\theta - \cot\theta)^2$
(3) $\operatorname{cosec}\theta$ (4) $\sin\theta$

9. Evaluate : $\dfrac{\cos\theta}{1-\sin\theta} + \dfrac{\cos\theta}{1+\sin\theta}$

(1) $2\sec\theta$ (2) $2\cos\theta$
(3) $\sec\theta$ (4) $\cos\theta$

10. Evaluate :

$(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

(1) 1 (2) 2
(3) 4 (4) 6

11. Evaluate : $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

(1) $\tan\theta$ (2) $\tan^2\theta$
(3) $\cot\theta$ (4) $\cot^2\theta$

12. $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = ?$

(1) $\operatorname{cosec}\theta$ (2) $2\operatorname{cosec}\theta$
(3) $\tan\theta$ (4) $2\tan\theta$

13. $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = ?$

(1) $\dfrac{1+\sin\theta}{\cos\theta}$ (2) $\dfrac{1-\sin\theta}{\cos\theta}$
(3) $\dfrac{\cos\theta}{1+\sin\theta}$ (4) $\dfrac{1+\sin\theta}{2\cos\theta}$

14. $\tan^2\theta + \cot^2\theta + 2 = ?$

(1) $\operatorname{cosec}^2\theta$ (2) $\sec^2\theta$
(3) $\operatorname{cosec}^2\theta \cdot \sec^2\theta$ (4) $\operatorname{cosec}^2\theta + \sec^2\theta$

15. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then $m^2 - n^2 = ?$

(1) mn (2) $4\sqrt{mn}$
(3) m^2n^2 (4) m^3n^3

16. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then $\cos\theta - \sin\theta = ?$

(1) $2\sin\theta$ (2) $\sqrt{2}\sin\theta$
(3) $\sin\theta$ (4) $\sqrt{3}\sin\theta$

17. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \text{cosec}\theta = q$, then $q(p^2 - 1) = ?$
(1) p (2) $2p$
(3) $3p$ (4) $2p^2$

18. If $\sec\theta + \tan\theta = p$, then $\frac{p^2-1}{p^2+1} = ?$
(1) $\sin\theta$ (2) $\cos\theta$
(3) $\sec\theta$ (4) $\tan\theta$

19. If $x = a\sin\theta$ and $y = b\tan\theta$, then $\frac{a^2}{x^2} - \frac{b^2}{y^2} = ?$
(1) 1 (2) −1
(3) 2 (4) 4

20. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, then $(m^2 + n^2)\cos^2\beta$ =?
(1) n (2) $2n$
(3) n^2 (4) $2n^2$

21. If $\text{cosec}\theta - \sin\theta = l$ and $\sec\theta - \cos\theta = m$, then $l^2m^2(l^2 + m^2 + 3) + 3 = ?$
(1) 1 (2) −1
(3) 2 (4) 4

22. If $x = r\sin A\cos C$, $y = r\sin A\sin C$ and $z = r\cos A$, then $x^2 + y^2 + z^2 = ?$
(1) r (2) r^2
(3) $2r^2$ (4) r^3

23. If $\tan A = n\tan B$ and $\sin A = m\sin B$, then $\frac{m^2-1}{n^2-1} = ?$
(1) $\cos A$ (2) $\cos^2 A$
(3) $2\cos^2 A$ (4) $\cos^3 A$

24. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, then $x^2 + y^2 = ?$
(1) 0 (2) 2
(3) 4 (4) 1

25. $2\sec^2\theta - \sec^4\theta - 2\text{cosec}^2\theta + \text{cosec}^4\theta = ?$
(1) $\cot^4\theta - \tan^4\theta$ (2) $\tan^4\theta - \cot^4\theta$
(3) $\cot^4\theta + \tan^4\theta$ (4) $\cot^6\theta - \cot^4\theta$

26. $(\sin\theta + \text{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = ?$
(1) $\tan^2\theta + \cot^2\theta + 9$ (2) $\tan^2\theta + \cot^2\theta + 7$
(3) $\tan^2\theta + \cot^2\theta + 5$ (4) $\tan^2\theta + \cot^2\theta + 8$

27. $(1 + \cot\theta - \text{cosec}\theta)(1 + \tan\theta + \sec\theta) = ?$
(1) 1 (2) 3
(3) 2 (4) 0

28. $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = ?$
(1) $1 + \sin\theta$ (2) $\frac{1-\sin\theta}{\cos\theta}$
(3) $\frac{\cos\theta}{1+\sin\theta}$ (4) $\frac{\cos\theta}{1-\sin\theta}$

29. If $(1 + \cos\alpha)(1 + \cos\beta)(1 + \cos\gamma) = (1 - \cos\alpha)(1 - \cos\beta)(1 - \cos\gamma)$, then one of the values of each member of this equality is :
(1) $\sin^2\alpha\sin\beta \cdot \sin\gamma$ (2) $\sin\alpha \cdot \sin\beta \cdot \sin\gamma$
(3) $\cos^2\alpha \cdot \cos^2\beta\cos^2\gamma$ (4) $\cos\alpha\cos\beta \cdot \cos\gamma$

30. If $\frac{\sin A}{\sin B} = p$ and $\frac{\cos A}{\cos B} = q$, then $\tan A = ?$
(1) $\pm\frac{p}{q}\sqrt{\frac{q^2-1}{1-p^2}}$ (2) $\pm\frac{q}{p}\sqrt{\frac{q^2-1}{1-p^2}}$
(3) $\pm\frac{q}{p}\sqrt{\frac{1-q^2}{1-p^2}}$ (4) $\pm\frac{q}{p}\sqrt{\frac{q^2+1}{1+p^2}}$

31. If $a\cos^3\theta + 3a\cos\theta.\sin^2\theta = m$ and $a\sin^3\theta + 3a\cos^2\theta.\sin\theta = n$ then $(m+n)^{\frac{2}{3}} + (m-n)^{\frac{2}{3}} = ?$
(1) $2a$ (2) $2a^{\frac{2}{3}}$
(3) $a^{\frac{2}{3}}$ (4) $2a^{\frac{1}{3}}$

32. $2\sec^2\theta - \sec^4\theta - 2\text{cosec}^2\theta + \text{cosec}^4\theta = ?$
(1) $1 + \tan^8\theta$ (2) $1 - \tan^8\theta$
(3) $\frac{1-\tan^8\theta}{\tan^4\theta}$ (4) $\frac{1+\tan^8\theta}{\tan^4\theta}$

33. If $2\tan^2\alpha \cdot \tan^2\beta \cdot \tan^2\gamma + \tan^2\alpha \cdot \tan^2\beta + \tan^2\beta \cdot \tan^2\gamma + \tan^2\gamma \cdot \tan^2\alpha = 1$ then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ = ?
(1) 1 (2) −1
(3) ± 1 (4) 0

34. If $\frac{\cos^4\alpha}{\cos^2\beta} + \frac{\sin^4\alpha}{\sin^2\beta} = 1$ then $\sin^4\alpha + \sin^4\beta = ?$
(1) $2\sin^2\alpha \cdot \sin^2\beta$ (2) $\sin^2\alpha \cdot \sin^2\beta$
(3) $\sin\alpha \cdot \sin\beta$ (4) $2\sin\alpha \cdot \sin\beta$

35. If $10\sin^4\alpha + 15\cos^4\alpha = 6$ then $27\text{cosec}^6\alpha + 8\sec^6\alpha = ?$
(1) 25 (2) 250
(3) 24 (4) 240

36. The least value of $\sec^2\theta + \text{cosec}^2\theta$ is
(1) 3 (2) 8
(3) 4 (4) 7

37. If $\cot\alpha + \tan\alpha = m$ and $\frac{1}{\cos\alpha} - \cos\alpha = n$ then
(1) $m(mn^2)^{\frac{1}{3}} - n(nm^2)^{\frac{1}{3}} = 1$
(2) $m(m^2n)^{\frac{1}{3}} - n(mn^2)^{\frac{1}{3}} = 1$
(3) $n(mn^2)^{\frac{1}{3}} - m(nm^2)^{\frac{1}{3}} = 1$
(4) None of these

38. If $\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$ then $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = ?$

(1) $\frac{1}{y}$ (2) y
(3) $1-y$ (4) $1+y$

39. $1-\frac{\sin^2 y}{1+\cos y}+\frac{1+\cos y}{\sin y}-\frac{\sin y}{1-\cos y}=?$

(1) 0 (2) 1
(3) $\sin y$ (4) $\cos y$

40. If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$, then

(1) $a^2+b^2+2ac=0$ (2) $a^2-b^2+2ac=0$
(3) $a^2+c^2+2ab=0$ (4) $a^2-b^2-2ac=0$

41. $\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}-\frac{1-2\sin^2\theta.\cos^2\theta}{\sin\theta.\cos\theta}=?$

(1) 1 (2) 0
(3) 3 (4) 2

42. If $T_n = \sin^n\theta + \cos^n\theta$ then $\frac{T_3-T_5}{T_1} = ?$

(1) $\sin\theta.\cos\theta$ (2) $\sin^2\theta.\cos^2\theta$
(3) $\sin^2\theta.\cos\theta$ (4) $\sin\theta.\cos^2\theta$

43. $1-\frac{\sin^2\theta}{1+\cot\theta}-\frac{\cos^2\theta}{1+\tan\theta}=?$

(1) $\sin\theta.\cos\theta$ (2) $\sin\theta$
(3) $\cos\theta$ (4) $\sin^2\theta.\cos^2\theta$.

44. If $\cot\theta\,(1+\sin\theta) = 4m$ and $\cot\theta\,(1-\sin\theta) = 4n$, then $(m^2-n^2)^2 = ?$

(1) m^2n^2 (2) m^3n^3
(3) mn (4) m^2n

45. $\sin^6A + \cos^6A + 3\sin^2A.\cos^2A = ?$

(1) 1 (2) 0
(3) 3 (4) 4

46. $3(\sin x-\cos x)^4 + 6(\sin x+\cos x)^2 + 4(\sin^6 x+\cos^6 x) = ?$

(1) 11 (2) 12
(3) 10 (4) 13

47. If (secA + tanA) (secB + tanB) (secC + tan C) = (secA − tan A) (secB − tanB) (secC − tanC), then each one is equal to

(1) 1 (2) −1
(3) ± 1 (4) 0

48. If $3\sin\theta + 5\cos\theta = 5$ then $5\sin\theta - 3\cos\theta = ?$

(1) 3 (2) −3
(3) 4 (4) ± 3

49. $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is possible for

(1) $x = y$ (2) $x < y$
(3) $x > y$ (4) $x^2 = y$

50. $(\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2 = ?$

(1) 1 + secA. cosecA (2) $(1 + \sec A.\operatorname{cosec}A)^2$
(3) secA.cosec A + 2 (4) $(2 + \sec A.\operatorname{cosec}A)^2$

51. $\cot^2A\left(\frac{\sec A-1}{1+\sin A}\right)+\sec^2A\left(\frac{\sin A-1}{1+\sec A}\right)=?$

(1) 0 (2) 1
(3) −1 (4) secA

52. $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}+\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=?$

(1) $2\sin\theta$ (2) $2\cos\theta$
(3) $2\sec\theta$ (4) $2\operatorname{cosec}\theta$

53. $\frac{\cos A.\operatorname{cosec}A-\sin A.\sec A}{\cos A+\sin A}=?$

(1) cosec A − sec A (2) cosec A + sec A
(3) sin A − cos A (4) sin A + cos A

54. $\sec^4A\,(1-\sin^4A) - 2\tan^2A = ?$

(1) 0 (2) 1
(3) −1 (4) 2

55. $\tan^2A.\sec^2B - \sec^2A.\tan^2B = ?$

(1) $\tan^2A - \tan^2B$ (2) $\tan^2A + \tan^2B$
(3) $2\tan^2A$ (4) $2\tan^2B$

56. $\frac{\tan A+\tan B}{\cot A+\cot B}=?$

(1) tan A (2) 2 tan B
(3) tan A + 4 (4) tan A . tan B

57. $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=?$

(1) $\sec\theta+\tan\theta$ (2) $\sec\theta-\tan\theta$
(3) $2\sec\theta$ (4) $2\tan\theta$

58. $\frac{\cos\theta-\sin\theta+1}{\cos\theta+\sin\theta+1}=?$

(1) $\frac{1}{\operatorname{cosec}\theta-\cot\theta}$ (2) $\sec\theta-\tan\theta$
(3) $2\operatorname{cosec}\theta$ (4) $2\cot\theta$

59. $\frac{1}{\sec A+\tan A}+\frac{1}{\sec A-\tan A}=?$

(1) 2 sec A (2) sec A
(3) 2 cos A (4) cos A

60. $(\sec A+\tan A-1)(\sec A-\tan A+1)=?$

(1) tan A (2) 2 tan A
(3) cot A (4) 2 cot A

61. $\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2=?$

(1) $\frac{1-\cos\theta}{1+\cos\theta}$ (2) $\frac{1+\cos\theta}{1-\cos\theta}$
(3) $\frac{\cos\theta}{1-\cos\theta}$ (4) $\frac{\cos\theta}{1+\cos\theta}$

62. $\left(\tan\theta+\frac{1}{\cos\theta}\right)^2+\left(\tan\theta-\frac{1}{\cos\theta}\right)^2=?$

(1) $\frac{1+\sin^2\theta}{1-\sin^2\theta}$ (2) $2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$

(3) $\frac{1-\sin^2\theta}{1+\sin^2\theta}$ (4) $2\left(\frac{1-\sin^2\theta}{1+\sin^2\theta}\right)$

63. $\left(\frac{1}{\sec^2\theta-\cos^2\theta}+\frac{1}{\text{cosec}^2\theta-\sin^2\theta}\right)\sin^2\theta.\cos^2\theta=?$

(1) $\frac{1+\sin^2\theta.\cos^2\theta}{2+\sin^2\theta.\cos^2\theta}$ (2) $\frac{1-\sin^2\theta.\cos\theta}{2+\sin^2\theta.\cos^2\theta}$

(3) $\frac{1-\sin^2\theta.\cos^2\theta}{2+\sin^2\theta.\cos^2\theta}$ (4) 1

64. (1 + cot A – cosec A) (1 + tan A + sec A) = ?
(1) 1 (2) –1
(3) –2 (4) 2

65. $\tan^6\theta+3\tan^2\theta.\sec^2\theta+1=?$
(1) $\tan^6\theta+1$ (2) $\sec^6\theta$
(3) $\sec^4\theta$ (4) $\sec^3\theta$

66. If $x=a\sec\theta+b\tan\theta$ and $y=a\tan\theta+b\sec\theta$ then $x^2-y^2=?$
(1) a^2-b^2 (2) $2a^2$
(3) $2b^2$ (4) a^2+b^2

67. If $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ and $\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta=1$

then, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=?$
(1) 1 (2) –2
(3) 2 (4) –1

68. If $\text{cosec}\,\theta-\sin\theta=a^3$; $\sec\theta-\cos\theta=b^3$ then $a^2b^2(a^2+b^2)=?$
(1) 1 (2) –1
(3) 2 (4) 4

69. If $x=a\cos^3\theta$; $y=b\sin^3\theta$, then $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=?$
(1) 1 (2) 0
(3) 2 (4) 4

70. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=?$
(1) $2\sec\theta$ (2) $\sec\theta$
(3) $2\cos\theta$ (4) $\cos\theta$

71. If $x=a\sec\theta.\cos\phi$, $y=b\sec\theta.\sin\phi$ and $z=c\tan\theta$ then $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=?$
(1) 2 (2) –2
(3) –1 (4) 1

72. If $\cos\theta+\cos^2\theta=1$ then $\sin^{12}\theta+3\sin^{10}\theta+3\sin^8\theta+\sin^6\theta+2\sin^4\theta+2\sin^2\theta-2=?$
(1) 1 (2) 2
(3) 3 (4) 4

73. If $\sec\theta+\tan\theta=p$, then $\frac{p^2-1}{p^2+1}=?$
(1) $\sin\theta$ (2) $\cos\theta$
(3) $\sec\theta$ (4) $\text{cosec}\,\theta$

74. If $\sin\theta+\cos\theta=\sqrt{2}\sin(90°-\theta)$ then $\cot\theta=?$
(1) $\sqrt{2}$ (2) $\sqrt{2}-1$
(3) $\sqrt{2}+2$ (4) $\sqrt{2}+1$

QUESTIONS ASKED IN PREVIOUS SSC EXAMS

75. If $\cos\theta+\sec\theta=\sqrt{3}$, then the value of $\cos^3\theta+\sec^3\theta$ is
(1) 0 (2) 1
(3) –1 (4) $\sqrt{3}$
[SSC Graduate Level Tier-I Exam, 2012]

76. If for all real values of α, $x=\cos^4\alpha+\sin^2\alpha$, then the range in which x lies is
(1) $\frac{3}{4}\le x\le 1$ (2) $\frac{3}{4}\le x\le\frac{13}{15}$
(3) $\frac{13}{16}\le x\le 1$ (4) $\frac{1}{2}\le x\le 2$
[SSC Graduate Level Tier-I Exam, 2012]

77. If $\sec\theta+\tan\theta=2$, then $\sec\theta$ is equal to
(1) $\frac{7}{4}$ (2) $\frac{7}{2}$
(3) $\frac{5}{2}$ (4) $\frac{5}{4}$
[SSC Graduate Level Tier-I Exam, 2012]

78. The least value of $4\,\text{cosec}^2\alpha+9\sin^2\alpha$ is:
(1) 14 (2) 10
(3) 11 (4) 12
[SSC Graduate Level Tier-I Exam, 2012]

79. The simplied value of

$$1-\frac{\sin^2 A}{1+\cos A}+\frac{1+\cos A}{\sin A}-\frac{\sin A}{1-\cos A}$$ is
(1) cosA (2) 0
(3) 1 (4) sinA
[SSC Graduate Level Tier-I Exam, 2012]

80. If $\tan\theta-\cot\theta=a$ and $\cos\theta-\sin\theta=b$, then the value of $(a^2+4)(b^2-1)^2$ is :
(1) 4 (2) 1
(3) 2 (4) 3
[SSC Graduate Level Tier-I Exam, 2012]

81. If $\sin^2 \alpha + \sin^2 \beta = 2$, then the value of $\cos\left(\frac{\alpha+\beta}{2}\right)$ is
(1) 1 (2) –1
(3) 0 (4) 0.5
[SSC Graduate Level Tier-I Exam, 2011]

82. If $\tan \theta . \tan 2\theta = 1$, then the value of $\sin^2 2\theta + \tan^2 2\theta$ is equal to
(1) $\frac{3}{4}$ (2) $\frac{10}{3}$
(3) $3\frac{3}{4}$ (4) 3
[SSC Graduate Level Tier-I Exam, 2011]

83. If $x = \text{cosec}\theta - \sin\theta$ and $y = \sec\theta - \cos\theta$, then the value of $x^2y^2(x^2 + y^2 + 3)$ is
(1) 0 (2) 1
(3) 2 (4) 3
[SSC Graduate Level Tier-I Exam, 2012]

84. If $0 \le \theta \le \frac{\pi}{2}$, $2y \cos\theta = x \sin\theta$ and $2x \sec\theta - y \text{ cosec}\theta = 3$, then the value of $x^2 + 4y^2$ is
(1) 1 (2) 2
(3) 3 (4) 4
[SSC Graduate Level Tier-I Exam, 2012]

85. If $\sin\theta + \sin^2\theta = 1$, then the value of $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta - 1$ is
(1) 0 (2) 1
(3) –1 (4) 2
[SSC Graduate Level Tier-I Exam, 2012]

86. If $\sec x + \cos x = 3$, then $\tan^2 x - \sin^2 x$ is
(1) 5 (2) 13
(3) 9 (4) 4
[SSC Graduate Level Tier-I Exam, 2012]

87. If $\sin p + \text{cosec } p = 2$, then the value of $\sin^7 p + \text{cosec}^7 p$ is
(1) 2^7 (2) 0
(3) 1 (4) 2
[SSC Graduate Level Tier-I Exam, 2012]

88. If $\sin \theta + \sin^2 \theta + \sin^3 \theta = 1$, then $\cos^6 \theta - 4 \cos^4 \theta + 8 \cos^2 \theta$ is equal to
(1) 2 (2) 1
(3) 4 (4) 3
[SSC Graduate Level Tier-I Exam, 2012]

89. Value of $2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4\theta) + 1$ is
(1) 4 (2) 0
(3) 1 (4) 2
[SSC Graduate Level Tier-I Exam, 2012]

90. If $x\sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$ and $x \sin \theta = y \cos \theta$, $\sin \theta \neq \theta$, $\cos \theta \neq 0$, then $x^2 + y^2$ is
(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$
(3) 1 (4) $\sqrt{2}$
[SSC Graduate Level Tier-I Exam, 2012]

91. $\sec^4 \theta - \sec^2 \theta$ is equal to
(1) $\tan^2 \theta - \tan^4 \theta$ (2) $\tan^2 \theta + \tan^4 \theta$
(3) $\cos^4 \theta - \cos^2 \theta$ (4) $\cos^2 \theta - \cos^4 \theta$
[SSC Graduate Level Tier-I Exam, 2012]

92. If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A$ is equal to
(1) 1 (2) $\frac{1}{2}$
(3) 0 (4) –1
[SSC Graduate Level Tier-I Exam, 2012]

93. If $\sec\theta - \text{cosec}\theta = 0$, then the value of $(\sec\theta + \text{cosec}\theta)$ is :
(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{2}{\sqrt{3}}$
(3) 0 (4) $2\sqrt{2}$
[SSC Graduate Level Tier-I Exam, 2012]

94. If $p \sin \theta = \sqrt{3}$ and $p \cos \theta = 1$, then the value of p is :
(1) $\frac{1}{2}$ (2) $\frac{2}{\sqrt{3}}$
(3) $\frac{-1}{\sqrt{3}}$ (4) 2
[SSC Graduate Level Tier-I Exam, 2012]

95. If $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta \neq 0$ and $x \sin\theta - y \cos\theta = 0$, then value of $(x^2 + y^2)$ is :
(1) 1 (2) $\sin\theta - \cos\theta$
(3) $\sin\theta + \cos\theta$
(4) 0
[SSC Graduate Level Tier-I Exam, 2012]

96. If $u_n = \cos^n\alpha + \sin^n\alpha$, then the value of $2u_6 - 3u_4 + 1$ is :
(1) 1 (2) 4
(3) 6 (4) 0
[SSC Graduate Level Tier-I Exam, 2012]

97. If $0 \le \alpha \le \frac{\pi}{2}$ and $2\sin\alpha + 15 \cos^2\alpha = 7$ then the value of $\cot\alpha$ is :
(1) $\frac{1}{2}$ (2) $\frac{5}{4}$
(3) $\frac{3}{4}$ (4) $\frac{1}{4}$
[SSC Graduate Level Tier-I Exam, 2012]

98. If $3\sin^2 \alpha + 7 \cos^2 \alpha = 4$, then the value of $\tan \alpha$ is (where $0 < \alpha < 90°$)
(1) $\sqrt{2}$ (2) $\sqrt{5}$
(3) $\sqrt{3}$ (4) $\sqrt{6}$
[SSC CPO SI & Assistant Intelligence Officer Exam, 2012]

99. The value of
$(\sec\theta - \cos\theta)(\text{cosec}\theta - \sin\theta)(\tan\theta + \cot\theta)$ is

(1) 1 (2) $\frac{3}{2}$
(3) 2 (4) 0

[SSC CPO SI & Assistant Intelligence Officer Exam, 2012]

100. If $\cos^4\theta - \sin^4\theta = \frac{2}{3}$, then the value of $2\cos^2\theta - 1$ is

(1) 0 (2) 1
(3) $\frac{2}{3}$ (4) $\frac{3}{2}$

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

101. If $\tan\theta = 1$, then the value of

$\frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$ is

(1) 2 (2) $2\frac{1}{2}$
(3) 3 (4) $\frac{4}{5}$

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

102. If θ be a positive acute angle satisfying $\cos^2\theta + \cos^4\theta = 1$, then the value of $\tan^2\theta + \tan^4\theta$ is

(1) $\frac{3}{2}$ (2) 1
(3) $\frac{1}{2}$ (4) 0

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

103. The simplified value of $(\sec A - \cos A)^2 + (\text{cosec } A - \sin A)^2 - (\cot A - \tan A)^2$ is

(1) 0 (2) $\frac{1}{2}$
(3) 1 (4) 2

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

104. If θ be an acute angle and $7\sin^2\theta + 3\cos^2\theta = 4$, then the value of $\tan\theta$ is

(1) $\sqrt{3}$ (2) $\frac{1}{\sqrt{3}}$
(3) 1 (4) 0

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

105. The minimum value of $2\sin^2\theta + 3\cos^2\theta$ is

(1) 0 (2) 3
(3) 2 (4) 1

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

106. If $\sin\theta + \text{cosec}\theta = 2$, then the value of $\sin^5\theta + \text{cosec}^5\theta$ when $0° \le \theta \le 90°$, is

(1) 0 (2) 1
(3) 10 (4) 2

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

107. If $2\cos\theta - \sin\theta = \frac{1}{\sqrt{2}}$, $(0° < \theta < 90°)$ the value of $2\sin\theta + \cos\theta$ is

(1) $\frac{1}{\sqrt{2}}$ (2) $\sqrt{2}$
(3) $\frac{3}{\sqrt{2}}$ (4) $\frac{\sqrt{2}}{3}$

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

108. If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$, then the value of $\sin^4\theta - \cos^4\theta$ is

(1) $\frac{1}{5}$ (2) $\frac{2}{5}$
(3) $\frac{3}{5}$ (4) $\frac{4}{5}$

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

109. If $\sec^2\theta + \tan^2\theta = 7$, then the value of θ when $0° \le \theta \le 90°$, is

(1) 60° (2) 30°
(3) 0° (4) 90°

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

110. If $\cos^2\alpha + \cos^2\beta = 2$, then the value of $\tan^3\alpha + \sin^5\beta$ is :

(1) –1 (2) 0
(3) 1 (4) $\frac{1}{\sqrt{3}}$

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

111. If $\sin\theta - \cos\theta = \frac{7}{13}$ and $0 < \theta < 90°$, then the value of $\sin\theta + \cos\theta$ is

(1) $\frac{17}{13}$ (2) $\frac{13}{17}$
(3) $\frac{1}{13}$ (4) $\frac{1}{17}$

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

112. The simplified value of

$(\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y + \tan x \sec y)^2$ is :

(1) −1 (2) 0

(3) $\sec^2 x$ (4) 1

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

113. If $\sin\theta + \text{cosec}\theta = 2$, then value of $\sin^{100}\theta + \text{cosec}^{100}\theta$ is equal to :

(1) 1 (2) 2

(3) 3 (4) 100

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

114. The value of $(\sec\theta - \cos\theta)(\text{cosec}\theta - \sin\theta)(\tan\theta + \cot\theta)$ is

(1) 1 (2) $\frac{3}{2}$

(3) 2 (4) 0

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

115. If θ be acute angle and $\cos\theta = \frac{15}{17}$, then the value of $\cot(90° - \theta)$ is

(1) $\frac{2\sqrt{8}}{15}$ (2) $\frac{8}{15}$

(3) $\frac{\sqrt{2}}{17}$ (4) $\frac{8\sqrt{2}}{17}$

[SSC FCI Assistant Grade-III Exam, 2012]

116. If $\sec^2\theta + \tan^2\theta = \frac{7}{12}$, then

$\sec^4\theta - \tan^4\theta =$

(1) $\frac{7}{12}$ (2) $\frac{1}{2}$

(3) $\frac{5}{12}$ (4) 1

[SSC FCI Assistant Grade-III Exam, 2012]

117. If $\cos x + \cos y = 2$, the value of $\sin x + \sin y$ is

(1) 0 (2) 1

(3) 2 (4) −1

[SSC FCI Assistant Grade-III Exam, 2012]

118. If $\sec\theta + \tan\theta = 2$, then the value of $\sec\theta$ is

(1) $\frac{4}{5}$ (2) 5

(3) $\frac{5}{4}$ (4) $\sqrt{2}$

[SSC Delhi Police S.I. Exam, 19.08.2012]

119. If $\text{cosec}\,\theta - \sin\theta = l$ and $\sec\theta - \cos\theta = m$, then the value of $l^2m^2(l^2 + m^2 + 3)$ is

(1) −1 (2) 0

(3) 1 (4) 2

[SSC Delhi Police S.I. Exam, 19.08.2012]

120. The expression $\frac{\tan 57° + \cot 37°}{\tan 33° + \cot 53°}$ is equal to

(1) tan 33° cot 57°

(2) tan 57° cot 37°

(3) tan 33° cot 53°

(4) tan 53° cot 37°

[SSC Graduate Level Tier-II Exam, 16.09.2012]

121. If $\tan 7\theta \tan 2\theta = 1$, then the value of $\tan 3\theta$ is

(1) $\sqrt{3}$ (2) $-\frac{1}{\sqrt{3}}$

(3) $\frac{1}{\sqrt{3}}$ (4) $-\sqrt{3}$

122. If $\tan\theta = 2$, then the value of $\frac{8\sin\theta + 5\cos\theta}{\sin^3\theta + 2\cos^3\theta + 3\cos\theta}$ is

(1) $\frac{21}{5}$ (2) $\frac{8}{5}$

(3) $\frac{7}{5}$ (4) $\frac{16}{5}$

[SSC FCI Asstt. Grade-III Exam, 11.11.2012 (Ist Sitting)]

123. The value of $(2\cos^2\theta - 1)\left(\frac{1 + \tan\theta}{1 - \tan\theta} + \frac{1 - \tan\theta}{1 + \tan\theta}\right)$ is

(1) 4 (2) 1

(3) 3 (4) 2

[SSC FCI Asstt. Grade-III Exam, 11.11.2012 (IInd Sitting)]

124. If $\cos\theta + \sec\theta = 2$, the value of $\cos^6\theta + \sec^6\theta$ is

(1) 4 (2) 8

(3) 1 (4) 2

[SSC (10+2) Level Data Entry Operator and LDC Exam, 21.10.2012 (Ist Sitting)]

125. The numerical value of

$\frac{5}{\sec^2\theta} + \frac{2}{1 + \cot^2\theta} + 3\sin^2\theta$ is :

(1) 5 (2) 2

(3) 3 (4) 4

[SSC (10+2) Level Data Entry Operator and LDC Exam, 21.10.2012 (IInd Sitting)]

126. The value of $\frac{\cot 30° - \cot 75°}{\tan 15° - \tan 60°}$ is :

(1) 0 (2) 1

(3) $\sqrt{3} - 1$ (4) −1

127. The value of

$\cot\theta.\tan(90^\circ-\theta)-\sec(90^\circ-\theta)\,\text{cosec}\theta+(\sin^2 25^\circ+\sin^2 65^\circ)+\sqrt{3}(\tan 5^\circ\tan 15^\circ.\tan 30^\circ.\tan 75^\circ.\tan 85^\circ)$ is :

(1) 1 (2) –1
(3) 2 (4) 0

[SSC (10+2) Level Data Entry Operator and LDC Exam, 21.10.2012 (IInd Sitting)]

128. If $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}=\frac{5}{4}$, the value of $\frac{\tan^2\theta+1}{\tan^2\theta-1}$ is

(1) $\frac{25}{16}$ (2) $\frac{41}{9}$

(3) $\frac{41}{40}$ (4) $\frac{40}{41}$

[SSC (10+2) Level Data Entry Operator and LDC Exam, 28.10.2012 (Ist Sitting)]

129. If $\tan\left(\frac{\pi}{2}-\frac{\theta}{2}\right)=\sqrt{3}$, the value of $\cos\theta$ is :

(1) 0 (2) $\frac{1}{\sqrt{2}}$

(3) $\frac{1}{2}$ (4) 1

[SSC (10+2) Level Data Entry Operator and LDC Exam, 04.11.2012 (Ist Sitting)]

ANSWERS

1. (2)	**2.** (4)	**3.** (1)	**4.** (3)	**5.** (2)
6. (2)	**7.** (1)	**8.** (2)	**9.** (1)	**10.** (1)
11. (1)	**12.** (2)	**13.** (1)	**14.** (3)	**15.** (2)
16. (2)	**17.** (2)	**18.** (1)	**19.** (1)	**20.** (3)
21. (4)	**22.** (2)	**23.** (2)	**24.** (4)	**25.** (1)
26. (2)	**27.** (3)	**28.** (4)	**29.** (2)	**30.** (1)
31. (2)	**32.** (3)	**33.** (1)	**34.** (1)	**35.** (2)
36. (3)	**37.** (1)	**38.** (2)	**39.** (4)	**40.** (2)
41. (2)	**42.** (2)	**43.** (1)	**44.** (3)	**45.** (1)
46. (4)	**47.** (3)	**48.** (4)	**49.** (1)	**50.** (2)
51. (1)	**52.** (4)	**53.** (1)	**54.** (2)	**55.** (1)
56. (4)	**57.** (1)	**58.** (2)	**59.** (1)	**60.** (2)
61. (1)	**62.** (2)	**63.** (3)	**64.** (4)	**65.** (2)
66. (1)	**67.** (3)	**68.** (1)	**69.** (1)	**70.** (1)
71. (1)	**72.** (1)	**73.** (1)	**74.** (4)	**75.** (1)
76. (1)	**77.** (4)	**78.** (4)	**79.** (1)	**80.** (1)
81. (3)	**82.** (3)	**83.** (2)	**84.** (4)	**85.** (1)
86. (1)	**87.** (4)	**88.** (3)	**89.** (2)	**90.** (3)
91. (2)	**92.** (1)	**93.** (4)	**94.** (4)	**95.** (1)
96. (4)	**97.** (3)	**98.** (3)	**99.** (1)	**100.** (3)
101. (1)	**102.** (2)	**103.** (3)	**104.** (2)	**105.** (2)
106. (4)	**107.** (3)	**108.** (3)	**109.** (1)	**110.** (2)
111. (1)	**112.** (4)	**113.** (2)	**114.** (1)	**115.** (2)
116. (1)	**117.** (1)	**118.** (3)	**119.** (3)	**120.** (2)
121. (3)	**122.** (1)	**123.** (4)	**124.** (4)	**125.** (1)
126. (4)	**127.** (1)	**128.** (3)	**129.** (3)	

EXPLANATIONS

1. (2) $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$

$=\frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$ [On taking LCM]

$=\frac{2}{\cos^2\theta}$ $[\because 1-\sin^2\theta=\cos^2\theta]$

$=2\sec^2\theta$ $\left[\because \sec\theta=\frac{1}{\cos\theta}\right]$

2. (4) $\text{cosec}^2\theta+\sec^2\theta$

$=\frac{1}{\sin^2\theta}+\frac{1}{\cos^2\theta}=\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta\sin^2\theta}$

$=\frac{1}{\sin^2\theta\cos^2\theta}=\frac{1}{\sin^2\theta}\times\frac{1}{\cos^2\theta}=\text{cosec}^2\theta\sec^2\theta$

3. (1) $(1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta)$

$=(1+\tan^2\theta)(1-\sin^2\theta)=\sec^2\theta\cos^2\theta$

$[\because 1+\tan^2\theta=\sec^2\theta \text{ and } 1-\sin^2\theta=\cos^2\theta]$

$=\frac{1}{\cos^2\theta}\cos^2\theta=1$

$\left[\because \sec\theta=\frac{1}{\cos\theta}\therefore \sec^2\theta=\frac{1}{\cos^2\theta}\right]$

4. (3) $\frac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta}=\frac{\frac{\sin\theta}{\cos\theta}+\sin\theta}{\frac{\sin\theta}{\cos\theta}-\sin\theta}$

$=\frac{\sin\theta\left(\frac{1}{\cos\theta}+1\right)}{\sin\theta\left(\frac{1}{\cos\theta}-1\right)}=\frac{\frac{1}{\cos\theta}+1}{\frac{1}{\cos\theta}-1}=\frac{\sec\theta+1}{\sec\theta-1}$

5. (2) $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}}$

[Multiplying and dividing by $(1-\sin\theta)$]

$= \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$

$= \sqrt{\left(\frac{1-\sin\theta}{\cos\theta}\right)^2} = \frac{1-\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \sec\theta - \tan\theta$

6. (2) $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$= (\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta) + (\cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta)$

$= \left(\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\frac{1}{\sin\theta}\right) + \left(\cos^2\theta + \sec^2\theta + 2\cos\theta\frac{1}{\cos\theta}\right)$

$= (\sin^2\theta + \operatorname{cosec}^2\theta + 2) + (\cos^2\theta + \sec^2\theta + 2)$

$= \sin^2\theta + \cos^2\theta + \operatorname{cosec}^2\theta + \sec^2\theta + 4$

$= 1 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 4$

$\left[\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta,\ \sec^2\theta = 1 + \tan^2\theta\right]$

$= 7 + \tan^2\theta + \cot^2\theta$

7. (1) $\left(\sin\theta + \frac{1}{\cos\theta}\right)^2 + \left(\cos\theta + \frac{1}{\sin\theta}\right)^2$

$= \sin^2\theta + \frac{1}{\cos^2\theta} + \frac{2\sin\theta}{\cos\theta} + \cos^2\theta + \frac{1}{\sin^2\theta} + \frac{2\cos\theta}{\sin\theta}$

$= (\sin^2\theta + \cos^2\theta) + \left(\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}\right) + 2\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$= (\sin^2\theta + \cos^2\theta) + \left(\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}\right) + \frac{2(\sin^2\theta + \cos^2\theta)}{\sin\theta\cos\theta}$

$= 1 + \frac{1}{\sin^2\theta\cos^2\theta} + \frac{2}{\sin\theta\cos\theta}$

$= \left(1 + \frac{1}{\sin\theta\cos\theta}\right)^2 = (1 + \sec\theta\operatorname{cosec}\theta)^2$

8. (2) $\frac{1-\cos\theta}{1+\cos\theta} = \frac{1-\cos\theta}{1+\cos\theta}\times\frac{1-\cos\theta}{1-\cos\theta}$

$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{\sin^2\theta}$

$[\because 1 - \cos^2\theta = \sin^2\theta]$

$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = (\operatorname{cosec}\theta - \cot\theta)^2$

9. (1) $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta}$

$= \frac{\cos\theta(1+\sin\theta) + \cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{\cos\theta + \cos\theta\sin\theta + \cos\theta - \cos\theta\sin\theta}{1-\sin^2\theta}$

$= \frac{2\cos\theta}{\cos^2\theta}$ $\quad[\because 1 - \sin^2\theta = \cos^2\theta]$

$= \frac{2}{\cos\theta} = 2\sec\theta$

10. (1) $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta}\right)$

$= \frac{\cos^2\theta}{\sin\theta}\times\frac{\sin^2\theta}{\cos\theta}\times\frac{1}{\sin\theta\cos\theta}$ $\quad[\because \sin^2\theta + \cos^2\theta = 1]$

$= \frac{\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta} = 1$

11. (1) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$= \frac{\sin\theta\{1 - 2(1-\cos^2\theta)\}}{\cos\theta(2\cos^2\theta - 1)} = \frac{\sin\theta(2\cos^2\theta - 1)}{\cos\theta(2\cos^2\theta - 1)} = \tan\theta$

12. (2) $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$

$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{(\sin^2\theta + \cos^2\theta) + 1 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{1 + 1 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$ $\quad[\because \sin^2\theta + \cos^2\theta = 1]$

$$= \frac{2 + 2\cos\theta}{\sin\theta\,(1+\cos\theta)} = \frac{2\,(1+\cos\theta)}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{2}{\sin\theta} = 2\ \text{cosec}\theta$$

13. (1) $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{(\tan\theta + \sec\theta) - 1}{\tan\theta - \sec\theta + 1}$

$$= \frac{(\sec\theta + \tan\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$

$[\because \sec^2\theta - \tan^2\theta = 1]$

$$= \frac{(\sec\theta + \tan\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta)\,[1 - \sec\theta - \tan\theta)]}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta)\,(1 - \sec\theta + \tan\theta)}{(\tan\theta - \sec\theta + 1)}$$

$$= \frac{(\sec\theta + \tan\theta)\,(\tan\theta - \sec\theta + 1)}{(\tan\theta - \sec\theta + 1)}$$

$= \sec\theta + \tan\theta$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1 + \sin\theta}{\cos\theta}$$

ALITER

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{(\sec\theta + \tan\theta) - 1}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\frac{1}{\sec\theta - \tan\theta} - 1}{\tan\theta - \sec\theta + 1} \qquad \left[\because \sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}\right]$$

$$= \frac{1 - \sec\theta + \tan\theta}{\tan\theta - \sec\theta + 1} \times \frac{1}{\sec\theta - \tan\theta}$$

$$= \frac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta$$

$$\left[\because \frac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta\right]$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1 + \sin\theta}{\cos\theta}$$

14. (3) $\tan^2\theta + \cot^2\theta + 2$

$= \tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta \quad [\because \tan\theta\,.\,\cot\theta = 1]$

$= (\tan\theta + \cot\theta)^2$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)^2$$

$$= \left(\frac{1}{\sin\theta\cos\theta}\right)^2 = \frac{1}{\sin^2\theta\cos^2\theta} = \text{cosec}^2\theta\sec^2\theta$$

15. (2) $m^2 - n^2$

$= (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$= 4\tan\theta\sin\theta$

$[\because (a+b)^2 - (a-b)^2 = 4ab]$

and $4\sqrt{mn} = 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$

$$= 4\sqrt{\tan^2\theta - \sin^2\theta} = 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$$

$$= 4\sqrt{\frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}}$$

$$= 4\sqrt{\frac{\sin^2\theta\,(1 - \cos^2\theta)}{\cos^2\theta}}$$

$$= 4\sqrt{\frac{\sin^4\theta}{\cos^2\theta}} = 4\frac{\sin^2\theta}{\cos\theta}$$

$$= 4\sin\theta\frac{\sin\theta}{\cos\theta} = 4\sin\theta\tan\theta$$

$\therefore\ m^2 - n^2 = 4\sqrt{mn}$

16. (2) $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\Rightarrow (\cos\theta + \sin\theta)^2 = 2\cos^2\theta$

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta = 2\cos^2\theta$

$\Rightarrow \cos^2\theta - 2\cos\theta\sin\theta = \sin^2\theta$

$\Rightarrow \cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta = 2\sin^2\theta$

$\Rightarrow (\cos\theta - \sin\theta)^2 = 2\sin^2\theta$

$\Rightarrow \cos\theta - \sin\theta = \sqrt{2}\sin\theta$

ALITER

$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\Rightarrow (\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$

$\Rightarrow \cos^2\theta - \sin^2\theta = 2\sin\theta\cos\theta$

$\Rightarrow (\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = 2\sin\theta\cos\theta$

$$\Rightarrow \cos\theta - \sin\theta = \frac{2\sin\theta\cos\theta}{\cos\theta + \sin\theta}$$

$$\Rightarrow \cos\theta - \sin\theta = \frac{2\sin\theta\cos\theta}{\sqrt{2}\cos\theta}$$

$[\because \cos\theta + \sin\theta = \sqrt{2}\cos\theta]$

$\Rightarrow \cos\theta - \sin\theta = \sqrt{2}\sin\theta$

17. (2) $q(p^2 - 1)$

$= (\sec\theta + \text{cosec}\theta)\,\{(\sin\theta + \cos\theta)^2 - 1\}$

$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)\{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1\}$$

$$= \left(\frac{\sin\theta + \cos\theta}{\cos\theta \sin\theta}\right)(1 + 2\sin\theta\cos\theta - 1)$$

$$= \left(\frac{\sin\theta + \cos\theta}{\cos\theta \sin\theta}\right)(2\sin\theta\cos\theta) = 2(\sin\theta + \cos\theta) = 2p$$

18. (1) $\frac{p^2 - 1}{p^2 + 1} = \frac{(\sec\theta + \tan\theta)^2 - 1}{(\sec\theta + \tan\theta)^2 + 1}$

$$= \frac{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta - 1}{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta + 1}$$

$$= \frac{(\sec^2\theta - 1) + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + 2\sec\theta\tan\theta + (1 + \tan^2\theta)}$$

$$= \frac{\tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + 2\sec\theta\tan\theta + \sec^2\theta}$$

$$= \frac{2\tan^2\theta + 2\tan\theta\sec\theta}{2\sec^2\theta + 2\sec\theta\tan\theta}$$

$$= \frac{2\tan\theta(\tan\theta + \sec\theta)}{2\sec\theta(\sec\theta + \tan\theta)}$$

$$= \frac{\tan\theta}{\sec\theta} = \frac{\sin\theta}{\cos\theta . \sec\theta} = \sin\theta$$

19. (1) $\frac{a^2}{x^2} - \frac{b^2}{y^2} = \frac{a^2}{a^2\sin^2\theta} - \frac{b^2}{b^2\tan^2\theta}$

$[\because x = a\sin\theta, y = b\tan\theta]$

$$= \frac{1}{\sin^2\theta} - \frac{1}{\tan^2\theta} = \text{cosec}^2\theta - \cot^2\theta$$

$[\because 1 + \cot^2\theta = \text{cosec}^2\theta \therefore \text{cosec}^2\theta - \cot^2\theta = 1]$

$= 1$

20. (3) $(m^2 + n^2)\cos^2\beta = \left(\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right)\cos^2\beta$

$\left[\because m = \frac{\cos\alpha}{\cos\beta} \text{ and } n = \frac{\cos\alpha}{\sin\beta}\right]$

$$= \left(\frac{\cos^2\alpha\sin^2\beta + \cos^2\alpha\cos^2\beta}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$$

$$= \cos^2\alpha\left(\frac{\sin^2\beta + \cos^2\beta}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$$

$$= \cos^2\alpha\left(\frac{1}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$$

$$= \frac{\cos^2\alpha}{\sin^2\beta} = \left(\frac{\cos\alpha}{\sin\beta}\right)^2 = n^2$$

21. (4) $l^2m^2(l^2 + m^2 + 3)$

$= (\text{cosec}\theta - \sin\theta)^2(\sec\theta - \cos\theta)^2\{(\text{cosec}\theta - \sin\theta)^2 + (\sec\theta - \cos\theta)^2 + 3\}$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)^2\left(\frac{1}{\cos\theta} - \cos\theta\right)^2$$

$$\left\{\left(\frac{1}{\sin\theta} - \sin\theta\right)^2 + \left(\frac{1}{\cos\theta} - \cos\theta\right)^2 + 3\right\}$$

$$= \left(\frac{1 - \sin^2\theta}{\sin\theta}\right)^2\left(\frac{1 - \cos^2\theta}{\cos\theta}\right)^2$$

$$\left\{\left(\frac{1 - \sin^2\theta}{\sin\theta}\right)^2 + \left(\frac{1 - \cos^2\theta}{\cos\theta}\right)^2 + 3\right\}$$

$$= \left(\frac{\cos^2\theta}{\sin\theta}\right)^2\left(\frac{\sin^2\theta}{\cos\theta}\right)^2\left\{\left(\frac{\cos^2\theta}{\sin\theta}\right)^2 + \left(\frac{\sin^2\theta}{\cos\theta}\right)^2 + 3\right\}$$

$$= \frac{\cos^4\theta}{\sin^2\theta} \times \frac{\sin^4\theta}{\cos^2\theta}\left\{\frac{\cos^4\theta}{\sin^2\theta} + \frac{\sin^4\theta}{\cos^2\theta} + 3\right\}$$

$$= \cos^2\theta \times \sin^2\theta\left\{\frac{\cos^6\theta + \sin^6\theta + 3\cos^2\theta\sin^2\theta}{\cos^2\theta\sin^2\theta}\right\}$$

$= \cos^6\theta + \sin^6\theta + 3\cos^2\theta\sin^2\theta$

$= \{(\cos^2\theta)^3 + (\sin^2\theta)^3\} + 3\cos^2\theta\sin^2\theta$

$= \{(\cos^2\theta + \sin^2\theta)^3 - 3\cos^2\theta\sin^2\theta(\cos^2\theta + \sin^2\theta)\} + 3\sin^2\theta\cos^2\theta$ $[\because a^3 + b^3 = (a + b)^3 - 3ab(a + b)]$

$= \{1 - 3\cos^2\theta\sin^2\theta\} + 3\cos^2\theta\sin^2\theta$

$[\because \cos^2\theta + \sin^2\theta = 1]$

$= 1$

$\therefore$ Required answer = 1 + 3 = 4

22. (2) $x^2 + y^2 + z^2 = r^2\sin^2 A\cos^2 C + r^2\sin^2 A\sin^2 C + r^2\cos^2 A$

$\Rightarrow x^2 + y^2 + z^2 = r^2\sin^2 A(\cos^2 C + \sin^2 C) + r^2\cos^2 A$

$\Rightarrow x^2 + y^2 + z^2 = r^2\sin^2 A + r^2\cos^2 A$

$[\because \cos^2 C + \sin^2 C = 1]$

$\Rightarrow x^2 + y^2 + z^2 = r^2(\sin^2 A + \cos^2 A)$

$\Rightarrow x^2 + y^2 + z^2 = r^2$

23. (2) We have to find $\cos^2 A$ in terms of m and n. this means that the angle B is to be eliminated from the given relations.

Now,

$\tan A = n \tan B \Rightarrow \tan B = \frac{1}{n}\tan A$

$\Rightarrow \cot B = \frac{n}{\tan A}$

and, $\sin A = m \sin B$

$\Rightarrow \sin B = \frac{1}{m}\sin A$

$\Rightarrow \operatorname{cosec} B = \frac{m}{\sin A}$

Substituting the values of cot B in $\operatorname{cosec}^2 B - \cot^2 B = 1$, we get

$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1 \Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2\cos^2 A}{\sin^2 A} = 1$

$\Rightarrow \frac{m^2 - n^2\cos^2 A}{\sin^2 A} = 1$

$\Rightarrow m^2 - n^2\cos^2 A = \sin^2 A$

$\Rightarrow m^2 - n^2\cos^2 A = 1 - \cos^2 A$

$\Rightarrow m^2 - 1 = n^2\cos^2 A - \cos^2 A$

$\Rightarrow m^2 - 1 = (n^2 - 1)\cos^2 A$

$\Rightarrow \frac{m^2 - 1}{n^2 - 1} = \cos^2 A$

24. (4) $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow (x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$

$\Rightarrow x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$ $[\because x\sin\theta = y\cos\theta]$

$\Rightarrow x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$

$\Rightarrow x\sin\theta = \sin\theta\cos\theta$

$\Rightarrow x = \cos\theta$

Now,

$x\sin\theta = y\cos\theta$

$\Rightarrow \cos\theta\sin\theta = y\cos\theta$ $[\because x = \cos\theta]$

$\Rightarrow y = \sin\theta$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

25. (1) $2\sec^2\theta - \sec^4\theta - 2\operatorname{cosec}^2\theta + \operatorname{cosec}^4\theta$

$= 2\sec^2\theta - (\sec^2\theta)^2 - 2\operatorname{cosec}^2\theta + (\operatorname{cosec}^2\theta)^2$

$= 2(1 + \tan^2\theta) - (1 + \tan^2\theta)^2 - 2(1 + \cot^2\theta) + (\cot^2\theta + 1)^2$

$= 2 + 2\tan^2\theta - (1 + \tan^4\theta + 2\tan^2\theta) - 2\cot^2\theta - 2 + \cot^4\theta + 2\cot^2\theta + 1$

$= \cot^4\theta - \tan^4\theta$

26. (2) $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta.\operatorname{cosec}\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta.\sec\theta$

$= (\sin^2\theta + \cos^2\theta) + (\operatorname{cosec}^2\theta + \sec^2\theta) + 4$

$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 4$

$= \tan^2\theta + \cot^2\theta + 7$

27. (3) $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$

$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$

$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\sin\theta + \cos\theta + 1}{\cos\theta}\right)$

$= \frac{(\sin\theta + \cos\theta)^2 - 1}{\sin\theta.\cos\theta}$

$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta - 1}{\sin\theta.\cos\theta}$

$= \frac{2\sin\theta.\cos\theta}{\sin\theta.\cos\theta} = 2$

28. (4) $\frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$

$= \frac{(\sec\theta + \tan\theta)[1 - (\sec\theta - \tan\theta)]}{\tan\theta - \sec\theta + 1}$

$= \frac{(\sec\theta + \tan\theta)(\tan\theta - \sec\theta + 1)}{\tan\theta - \sec\theta + 1} = \sec\theta + \tan\theta$

$= \frac{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\sec\theta - \tan\theta}$

$= \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta} = \frac{1}{\sec\theta - \tan\theta}$

$= \frac{1}{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}} = \frac{\cos\theta}{1 - \sin\theta}$

29. (2) $(1 + \cos\alpha)(1 + \cos\beta)(1 + \cos\gamma) = (1 - \cos\alpha)(1 - \cos\beta)(1 - \cos\gamma)$

Multiplying both sides by $(1 + \cos\alpha)(1 + \cos\beta)(1 + \cos\gamma)$,

$(1 + \cos\alpha)^2(1 + \cos\beta)^2(1 + \cos\gamma)^2$

$= (1 - \cos\alpha)(1 - \cos\beta)(1 - \cos\gamma)(1 + \cos\alpha)(1 + \cos\beta)(1 + \cos\gamma)$

$= (1 - \cos^2\alpha)(1 - \cos^2\beta)(1 - \cos^2\gamma)$

$= \sin^2\alpha.\sin^2\beta.\sin^2\gamma$

$\therefore (1 + \cos\alpha)(1 + \cos\beta)(1 + \cos\gamma) = \pm\sin\alpha.\sin\beta.\sin\gamma$

30. (1) $\frac{\sin A}{\sin B} = p$ and $\frac{\cos A}{\cos B} = q$

$\Rightarrow \frac{\sin A}{\sin B}.\frac{\cos B}{\cos A} = \frac{p}{q}$

$\Rightarrow \frac{\tan A}{\tan B} = \frac{p}{q}$ $\quad \frac{\tan A}{p} = \frac{\tan B}{q} = k$

$\therefore \tan A = kp$ and $\tan B = kq$

Now, sin A = p sin B

$\Rightarrow \dfrac{\tan A}{\sqrt{1+\tan^2 A}} = p\dfrac{\tan B}{\sqrt{1+\tan^2 B}}$

$\Rightarrow \dfrac{pk}{\sqrt{1+p^2k^2}} = p\dfrac{kq}{\sqrt{1+q^2k^2}}$

$\Rightarrow p^2(1+q^2k^2) = p^2q^2(1+p^2k^2)$

$\Rightarrow k^2(q^2 - p^2q^2) = q^2 - 1$

$\Rightarrow k^2 = \dfrac{q^2-1}{q^2(1-p^2)}$

$\Rightarrow k = \pm\dfrac{1}{q}\sqrt{\dfrac{q^2-1}{1-p^2}}$

$\therefore \tan A = \pm\dfrac{p}{q}\sqrt{\dfrac{q^2-1}{1-p^2}}$

31. (2) $m + n = a\cos^3\theta + 3a\cos\theta.\sin^2\theta + a\sin^3\theta + 3a\cos^2\theta.\sin\theta. = a(\cos\theta+\sin\theta)^3$

$m - n = a\cos^3\theta + 3a\cos\theta.\sin^2\theta - a\sin^3\theta - 3a\cos^2\theta.\sin\theta$

$= a(\cos\theta - \sin\theta)^3$

$\therefore \cos\theta + \sin\theta = \dfrac{(m+n)^{\frac{1}{3}}}{a}$ and

$\cos\theta - \sin\theta = \left(\dfrac{m-n}{a}\right)^{\frac{1}{3}}$

$\therefore (\cos\theta+\sin\theta)^2 + (\cos\theta-\sin\theta)^2$

$= \left(\dfrac{m+n}{a}\right)^{\frac{2}{3}} + \left(\dfrac{m-n}{a}\right)^{\frac{2}{3}}$

$\Rightarrow 2(\cos^2\theta + \sin^2\theta) = \dfrac{(m+n)^{\frac{2}{3}}}{(a)^{\frac{2}{3}}} + \dfrac{(m+n)^{\frac{2}{3}}}{(a)^{\frac{2}{3}}}$

$\Rightarrow (m+n)^{\frac{2}{3}} + (m-n)^{\frac{2}{3}} = 2a^{\frac{2}{3}}$

32. (3) $2\sec^2\theta - \sec^4\theta - 2\operatorname{cosec}^2\theta + \operatorname{cosec}^4\theta$

$= 2(1+\tan^2\theta) - (1+\tan^2\theta)^2 - 2(1+\cot^2\theta) + (1+\cot^2\theta)^2$

$= 2(1 + \tan^2\theta - 1 - \cot^2\theta) + (1 + 2\cot^2\theta + \cot^4\theta) - (1 + 2\tan^2\theta + \tan^4\theta)$

$= 2(\tan^2\theta - \cot^2\theta) + (2\cot^2\theta - 2\tan^2\theta) + \cot^4\theta - \tan^4\theta$

$= \dfrac{1}{\tan^4\theta} - \tan^4\theta = \dfrac{1-\tan^8\theta}{\tan^4\theta}$

33. (1) Dividing by $\tan^2\alpha.\tan^2\beta.\tan^2\theta\gamma$,

$2 + \cot^2\gamma + \cot^2\alpha + \cot^2\beta = \cot^2\alpha.\cot^2\beta.\cot^2\gamma$

$\Rightarrow 2 + \operatorname{cosec}^2\gamma - 1 + \operatorname{cosec}^2\alpha - 1 + \operatorname{cosec}^2\beta - 1$

$= (\operatorname{cosec}^2\alpha - 1)(\operatorname{cosec}^2\beta - 1)(\operatorname{cosec}^2\gamma - 1)$

$\Rightarrow \operatorname{cosec}^2\alpha + \operatorname{cosec}^2\beta + \operatorname{cosec}^2\gamma - 1$

$= \operatorname{cosec}^2\alpha.\operatorname{cosec}^2\beta \times \operatorname{cosec}^2\gamma - \operatorname{cosec}^2\alpha.\operatorname{cosec}^2\beta - \operatorname{cosec}^2\beta.\operatorname{cosec}^2\gamma - \operatorname{cosec}^2\gamma.\operatorname{cosec}^2\alpha + \operatorname{cosec}^2\alpha + \operatorname{cosec}^2\beta + \operatorname{cosec}^2\gamma - 1$

$\Rightarrow \operatorname{cosec}^2\alpha.\operatorname{cosec}^2\beta.\operatorname{cosec}^2\gamma = \operatorname{cosec}^2\alpha.\operatorname{cosec}^2\beta + \operatorname{cosec}^2\beta.\operatorname{cosec}^2\gamma + \operatorname{cosec}^2\gamma.\operatorname{cosec}^2\alpha$

$\Rightarrow 1 = \sin^2\gamma + \sin^2\alpha + \sin^2\beta$

34. (1) $\dfrac{\cos^4\alpha}{\cos^2\beta} + \dfrac{\sin^4\alpha}{\sin^2\beta} = 1$

$\Rightarrow \cos^4\alpha.\sin^2\beta + \sin^4\alpha.\cos^2\beta = \cos^2\beta.\sin^2\beta$

$\Rightarrow \cos^4\alpha(1-\cos^2\beta) + \cos^2\beta(1-\cos^2\alpha)^2 = \cos^2\beta(1-\cos^2\beta)$

$\Rightarrow \cos^4\alpha - \cos^4\alpha.\cos^2\beta + \cos^2\beta - 2\cos^2\alpha.\cos^2\beta + \cos^4\alpha.\cos^2\beta = \cos^2\beta - \cos^4\beta$

$\Rightarrow \cos^4\alpha - 2\cos^2\alpha.\cos^2\beta + \cos^4\beta = 0$

$\Rightarrow (\cos^2\alpha - \cos^2\beta)^2 = 0$

$\Rightarrow \cos^2\alpha = \cos^2\beta$

$\Rightarrow 1 - \sin^2\alpha = 1 - \sin^2\beta$

$\Rightarrow \sin^2\alpha = \sin^2\beta$

$\therefore \sin^4\alpha + \sin^4\beta = (\sin^2\alpha - \sin^2\beta)^2 + 2\sin^2\alpha.\sin^2\beta$

$= 2\sin^2\alpha.\sin^2\beta$

35. (2) $10\sin^4\alpha + 15\cos^4\alpha = 6$

$\Rightarrow 10\sin^4\alpha + 15\cos^4\alpha = 6(\sin^2\alpha + \cos^2\alpha)^2$

$\Rightarrow 10\tan^4\alpha + 15 = 6(\tan^2\alpha + 1)^2$ [Dividing by $\cos^4\alpha$]

$\Rightarrow (2\tan^2\alpha - 3)^2 = 0$

$\Rightarrow 2\tan^2\alpha - 3 = 0$

$\Rightarrow \tan^2\alpha = \dfrac{3}{2}$

$\therefore 27\operatorname{cosec}^6\alpha + 8\sec^6\alpha$

$= 27(1+\cot^2\alpha)^3 + 8(1+\tan^2\alpha)^3$

$= 27(1 + \dfrac{2}{3})^3 + 8(1 + \dfrac{3}{2})^3$

$= 27 \times \dfrac{125}{27} + 8 \times \dfrac{125}{8} = 250$

36. (3) $\sec^2\theta + \operatorname{cosec}^2\theta$

$= (1+\tan^2\theta) + (1+\cot^2\theta)$

$= 2 + \tan^2\theta + \cot^2\theta$

$= 2 + (\tan\theta - \cot\theta)^2 + 2\tan\theta.\cot\theta$

$= 4 + (\tan\theta - \cot\theta)^2 \geq 4 \left[\because (\tan\theta - \cot\theta)^2 \geq 0\right]$

37. (1) $\cot\alpha + \tan\alpha = m$

$\Rightarrow 1 + \tan^2\alpha = m\tan\alpha$

$\Rightarrow \sec^2\alpha = m\tan\alpha$ ...(i) and $\frac{1}{\cos\alpha} - \cos\alpha = n$

$\Rightarrow \sec\alpha - \frac{1}{\sec\alpha} = n \Rightarrow \sec^2\alpha - 1 = n\sec\alpha$

$\Rightarrow \tan^2\alpha = n\sec\alpha \Rightarrow \tan^4\alpha = n^2\sec^2\alpha$

$\Rightarrow \tan^4\alpha = n^2m\tan\alpha$ [by (i)]

$\Rightarrow \tan^3\alpha = n^2m$

$\Rightarrow \tan\alpha = (n^2m)^{\frac{1}{3}}$ and $\sec^2\alpha = m(n^2m)^{\frac{1}{3}}$

$\therefore \sec^2\alpha - \tan^2\alpha = 1$

$\Rightarrow m\ (n^2m)^{\frac{1}{3}} - (n^2m)^{\frac{2}{3}} = 1$

$\Rightarrow m\ (mn^2)^{\frac{1}{3}} - n\ (mn^2)^{\frac{1}{3}} = 1$

38. (2) $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} \times \frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}$

$= \frac{(1+\sin\alpha)^2 - \cos^2\alpha}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$

$= 1 + 2\sin\alpha + \sin^2\alpha - 1\ \frac{+\sin^2\alpha}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$

$= \frac{2\sin\alpha(1+\sin\alpha)}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$

$= \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$

39. (4) $1 - \frac{\sin^2 y}{1+\cos y} + \frac{1+\cos y}{\sin y} - \frac{\sin y}{1-\cos y}$

$= \frac{1+\cos y - \sin^2 y}{1+\cos y} + \frac{1-\cos^2 y - \sin^2 y}{\sin y(1-\cos y)}$

$= \frac{\cos y + \cos^2 y}{1+\cos y} + 0 = \cos y$

40. (2) $\sin\theta + \cos\theta = \frac{b}{a}$ and $\sin\theta.\cos\theta = \frac{c}{a}$

$\therefore\ (\sin\theta + \cos\theta)^2 = \frac{b^2}{a^2}$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta = \frac{b^2}{a^2}$

$\Rightarrow 1 + \frac{2c}{a} = \frac{b^2}{a^2}$

$\Rightarrow a^2 + 2ac - b^2 = 0$

41. (2) $\frac{\sin^3\theta}{\cos^3\theta}.\cos^2\theta + \frac{\cos^3\theta}{\sin^3\theta}.$

$\sin^2\theta - \frac{1-2\sin^2\theta.\cos^2\theta}{\sin\theta.\cos\theta}$

$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta} - \frac{1-2\sin^2\theta.\cos^2\theta}{\sin\theta.\cos\theta}$

$= \frac{\sin^4\theta + \cos^4\theta}{\sin\theta.\cos\theta} - \frac{1-2\sin^2\theta.\cos^2\theta}{\sin\theta.\cos\theta}$

$= \frac{(\sin^2\theta+\cos^2\theta)^2 - 2\sin^2\theta.\cos^2\theta}{\sin\theta.\cos\theta}$

$-\frac{1-2\sin^2\theta.\cos^2\theta}{\sin\theta\cos\theta} = 0$

42. (2) $\frac{T_3 - T_5}{T_1} = \frac{\sin^3\theta + \cos^3\theta - (\sin^5\theta + \cos^5\theta)}{\sin\theta + \cos\theta}$

$= \frac{\sin^3\theta - \sin^5\theta + \cos^3\theta - \cos^5\theta}{\sin\theta + \cos\theta}$

$= \frac{\sin^3\theta(1-\sin^2\theta) + \cos^3\theta(1-\cos^2\theta)}{\sin\theta+\cos\theta}$

$= \frac{\sin^3\theta.\cos^2\theta + \cos^3\theta.\sin^2\theta}{\sin\theta + \cos\theta}$

$= \frac{\sin^2\theta.\cos^2\theta(\sin\theta+\cos\theta)}{\sin\theta+\cos\theta}$

$= \sin^2\theta.\cos^2\theta$

43. (1) $1 - \frac{\sin^2\theta}{1+\frac{\cos\theta}{\sin\theta}} - \frac{\cos^2\theta}{1+\frac{\sin\theta}{\cos\theta}}$

$= 1 - \frac{\sin^3\theta}{\sin\theta+\cos\theta} - \frac{\cos^3\theta}{\cos\theta+\sin\theta}$

$= 1 - \frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}$

$= 1 - \frac{(\sin\theta+\cos\theta)(\sin^2\theta+\cos^2\theta-\sin\theta.\cos\theta)}{(\sin\theta+\cos\theta)}$

$= 1 - (1 - \sin\theta\cos\theta) = \sin\theta.\cos\theta.$

44. (3) $\cot\theta(1+\sin\theta) = 4m$ ... (i)

$\cot\theta(1-\sin\theta) = 4n$ ... (ii)

On multiplying,

$\cot^2\theta(1-\sin^2\theta) = 16mn$

$\cot^2\theta.\cos^2\theta = 16mn$

$\cos^4\theta/\sin^2\theta = 16mn$ ... (iii)

Squaring and subtracting the given relation

$\cot^2\theta\ [(1+\sin\theta)^2-(1-\sin\theta)^2] = 16\ (m^2-n^2)$

$\Rightarrow \cot^2\theta.\ 4\sin\theta = 16\ (m^2-n^2)$

$\Rightarrow \cot^2\theta.\ \sin\theta = 4\ (m^2-n^2)$

Squaring both sides again,

$\Rightarrow \cot^4\theta.\ \sin^2\theta = 16\ (m^2-n^2)^2$

$\Rightarrow \dfrac{\cos^4\theta}{\sin^2\theta} = 16\ (m^2-n^2)^2$

$\Rightarrow (m^2-n^2)^2 = mn$ [using (iii)]

45. (1) $(\sin^6A+\cos^6A)+3\sin^2A.\cos^2A$

$= (\sin^2A+\cos^2A)^3 - 3\sin^2A.\cos^2A\ (\sin^2A+\cos^2A) + 3\sin^2A.\cos^2A$

$\left[\because a^3+b^3=(a+b)^3-3ab(a+b)\right]$

$=1-3\sin^2A.\cos^2A+3\sin^2A.\cos^2A = 1$

46. (4) $3(\sin x-\cos x)^4+6(\sin x+\cos x)^2+4(\sin^6x+\cos^6x)$

$=3[(\sin x-\cos x)^2]^2+6(\sin^2x+\cos^2x+2\sin x\cos x)+4(\sin^2x+\cos^2x)$

$[(\sin^2x)^2-\sin^2x.\cos^2x+(\cos^2x)^2]$

$=3[(\sin^2x+\cos^2x-2\sin x.\cos x)^2]+6(1+2\sin x.\cos x)+4[(\sin^2x+\cos^2x)^2-3\sin^2x\cos^2x]$

$=3(1-2\sin x.\cos x)^2+6(1+2\sin x.\cos x)+4(1-3\sin^2x.\cos^2x)$

$=3(1-4\sin x.\cos x+4\sin^2x.\cos^2x)+6+12\sin x.\cos x+4-12\sin^2x.\cos^2x = 13$

47. (3) Let each one be equal to x.

$\therefore x.x = (\sec A+\tan A)(\sec B+\tan B)(\sec C+\tan C)(\sec A-\tan A)$

$(\sec B-\tan B)(\sec C-\tan C)$

$=(\sec^2A-\tan^2A)(\sec^2B-\tan^2B)$

$(\sec^2C-\tan^2C) = 1 \qquad \therefore x=\pm 1$

48. (4) Squaring $3\sin\theta+5\cos\theta=5$,

we get

$9\sin^2\theta+25\cos^2\theta+30\sin\theta\cos\theta=25$

$\Rightarrow 9(1-\cos^2\theta)+25(1-\sin^2\theta)+30\sin\theta.\cos\theta=25$

$\Rightarrow 9=9\cos^2\theta+25\sin^2\theta-30\sin\theta.\cos\theta$

$=(5\sin\theta-3\cos\theta)^2$

$\Rightarrow 5\sin\theta-3\cos\theta=\pm 3$

49. (1) $\sec^2\theta \geq 1$

$\therefore \dfrac{4xy}{(x+y)^2} \geq 1$

$\Rightarrow (x+y)^2 \leq 4xy$

$\Rightarrow (x+y)^2-4xy \leq 0$

$\Rightarrow (x-y)^2 \leq 0 \Rightarrow x=y$

50. (2) $\sin^2A+\sec^2A+2\sin A.\sec A+\cos^2A+\mathrm{cosec}^2A+2\cos A.\mathrm{cosec}A$

$=(\sin^2A+\cos^2A)+(\sec^2A+\mathrm{cosec}^2A)+2\sin A.\sec A+2\cos A.\mathrm{cosec}A$

$= 1+\left(\dfrac{1}{\cos^2A}+\dfrac{1}{\sin^2A}\right)+2\left(\dfrac{\sin A}{\cos A}+\dfrac{\cos A}{\sin A}\right)$

$= 1+\dfrac{\sin^2A+\cos^2A}{\sin^2A.\cos^2A}+2\left(\dfrac{\sin^2A+\cos^2A}{\sin A.\cos A}\right)$

$=1+\mathrm{cosec}^2A.\sec^2A+2\,\mathrm{cosec}A.\sec A$

$=(1+\sec A.\mathrm{cosec}A)^2$

51. (1) $\cot^2A\left(\dfrac{\sec A-1}{1+\sin A}\right)+\sec^2A\left(\dfrac{\sin A-1}{1+\sec A}\right)$

$=\dfrac{\cot^2A(\sec A-1)(\sec A+1)+\sec^2A(\sin A-1)(1+\sin A)}{(1+\sin A)(1+\sec A)}$

$=\dfrac{\cot^2A(\sec^2A-1)+\sec^2A(\sin^2A-1)}{(1+\sin A)(1+\sec A)}$

$=\dfrac{\cot^2A.\tan^2A-\sec^2A(1-\sin^2A)}{(1+\sin A)(1+\sec A)}$

$=\dfrac{1-1}{(1+\sin A)(1+\sec A)}=0$

52. (4) $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}}+\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

$=\sqrt{\dfrac{(1+\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}}+\sqrt{\dfrac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}}$

$=\sqrt{\dfrac{(1+\cos\theta)^2}{1-\cos^2\theta}}+\sqrt{\dfrac{(1-\cos\theta)^2}{1-\cos^2\theta}}$

$=\dfrac{1+\cos\theta}{\sin\theta}+\dfrac{1-\cos\theta}{\sin\theta}=\dfrac{2}{\sin\theta}=2\,\mathrm{cosec}\theta$

53. (1) $\dfrac{\cos A.\dfrac{1}{\sin A}-\sin A.\dfrac{1}{\cos A}}{\cos A+\sin A}=\dfrac{\dfrac{\cos^2A-\sin^2A}{\cos A.\sin A}}{\cos A+\sin A}$

$=\dfrac{(\cos A+\sin A)(\cos A-\sin A)}{(\cos A+\sin A).\cos A.\sin A}$

$=\dfrac{\cos A-\sin A}{\cos A.\sin A}=\dfrac{\cos A}{\cos A.\sin A}-\dfrac{\sin A}{\cos A.\sin A}$

$=\mathrm{cosec}\,A-\sec A$

54. (2) $\sec^4A(1-\sin^4A)-2\tan^2A$

$=\sec^4A-\sec^4A.\sin^4A-2\tan^2A$

$=(1+\tan^2A)^2-\dfrac{\sin^4A}{\cos^4A}-2\tan^2A$

$=1+2\tan^2A+\tan^4A-\tan^4A-2\tan^2A$

$=1$

55. (1) $\tan^2 A \cdot \sec^2 B - \sec^2 A \cdot \tan^2 B$

$= \tan^2 A(1+\tan^2 B) - (1+\tan^2 A)\tan^2 B$

$= \tan^2 A + \tan^2 A \cdot \tan^2 B - \tan^2 B - \tan^2 A \cdot \tan^2 B = \tan^2 A - \tan^2 B$

56. (4) $\dfrac{\tan A + \tan B}{\cot A + \cot B} = \dfrac{\dfrac{\sin A}{\cos A} + \dfrac{\sin B}{\cos B}}{\dfrac{\cos A}{\sin A} + \dfrac{\cos B}{\sin B}}$

$= \dfrac{\dfrac{\sin A.\cos B + \cos A.\sin B}{\cos A.\cos B}}{\dfrac{\cos A.\sin B + \sin A.\cos B}{\sin A.\sin B}}$

$= \dfrac{\sin A.\sin B}{\cos A.\cos B} = \tan A \cdot \tan B$

57. (1) $\dfrac{\sin\theta - (\cos\theta - 1)}{\sin\theta + (\cos\theta - 1)}$

$= \dfrac{\sin\theta - (\cos\theta - 1)}{\sin\theta + (\cos\theta - 1)} \times \dfrac{\sin\theta - (\cos\theta - 1)}{\sin\theta - (\cos\theta - 1)}$

$= \dfrac{\sin^2\theta + \cos^2\theta - 2\cos\theta + 1 - 2\sin\theta(\cos\theta - 1)}{(\sin\theta)^2 - (\cos\theta - 1)^2}$

$= \dfrac{2 - 2\cos\theta - 2\sin\theta.\cos\theta + 2\sin\theta}{\sin^2\theta - \cos^2\theta + 2\cos\theta - 1}$

$= \dfrac{2(1-\cos\theta) + 2\sin\theta(1-\cos\theta)}{1 - \cos^2\theta - \cos^2\theta + 2\cos\theta - 1}$

$= \dfrac{(2+2\sin\theta)(1-\cos\theta)}{2\cos\theta(1-\cos\theta)}$

$= \dfrac{2(1+\sin\theta)}{2\cos\theta} = \dfrac{1+\sin\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$

58. (2) $\dfrac{(\cos\theta + 1) - (\sin\theta)}{(\cos\theta + 1) + (\sin\theta)}$

$= \dfrac{(\cos\theta + 1) - \sin\theta}{(\cos\theta + 1) + \sin\theta} \times \dfrac{(\cos\theta + 1) - \sin\theta}{(\cos\theta + 1) - \sin\theta}$

$= \dfrac{\cos^2\theta + 2\cos\theta + 1 + \sin^2\theta - 2\sin\theta(\cos\theta + 1)}{(\cos\theta + 1)^2 - \sin^2\theta}$

$= \dfrac{2 + 2\cos\theta - 2\sin\theta.(\cos\theta + 1)}{\cos^2\theta + 2\cos\theta + 1 - \sin^2\theta}$

$= \dfrac{2(1+\cos\theta) - 2\sin\theta(\cos\theta + 1)}{2\cos^2\theta + 2\cos\theta}$

$= \dfrac{2(1-\sin\theta)(1+\cos\theta)}{2\cos\theta(1+\cos\theta)} = \dfrac{1-\sin\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta} = \sec\theta - \tan\theta$

59. (1) $\dfrac{1}{\sec A + \tan A} + \dfrac{1}{\sec A - \tan A}$

$= \dfrac{\sec A - \tan A + \sec A + \tan A}{(\sec A + \tan A)(\sec A - \tan A)}$

$[\because \sec^2 A - \tan^2 A = 1]$

$= 2\sec A$

60. (2) $(\sec A + \tan A - 1)\ (\sec A - (\tan A - 1)$

$= \sec^2 A - (\tan A - 1)^2$

$= \sec^2 A - \tan^2 A + 2\tan A - 1$

$= 1 + \tan^2 A - \tan^2 A + 2\tan A - 1$

$= 2\tan A$

61. (1) $\left(\dfrac{1+\sin\theta - \cos\theta}{1+\sin\theta + \cos\theta}\right)^2$

$= \dfrac{1+\sin^2\theta + \cos^2\theta + 2\sin\theta - 2\cos\theta - 2\sin\theta.\cos\theta}{1+\sin^2\theta + \cos^2\theta + 2\sin\theta - 2\cos\theta - 2\sin\theta.\cos\theta}$

$= \dfrac{2+2\sin\theta - 2\cos\theta - 2\sin\theta\cos\theta}{2+2\sin\theta + 2\cos\theta + 2\sin\theta.\cos\theta}$

$= \dfrac{2(1+\sin\theta) - 2\cos\theta(1+\sin\theta)}{2(1+\sin\theta) + 2\cos\theta(1+\sin\theta)}$

$= \dfrac{2(1+\sin\theta)(1-\cos\theta)}{2(1+\cos\theta)(1+\sin\theta)} = \dfrac{1-\cos\theta}{1+\cos\theta}$

62. (2) $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

$\therefore$ Expression $= 2\left(\tan^2\theta + \dfrac{1}{\cos^2\theta}\right)$

$= 2\left(\dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{1}{\cos^2\theta}\right)$

$= 2\left(\dfrac{1+\sin^2\theta}{\cos^2\theta}\right) = 2\left(\dfrac{1+\sin^2\theta}{1-\sin^2\theta}\right)$

63. (3) $\left(\dfrac{1}{\dfrac{1}{\cos^2\theta} - \cos^2\theta} + \dfrac{1}{\dfrac{1}{\sin^2\theta} - \sin^2\theta}\right)$

$$\sin^2\theta \cdot \cos^2\theta = \left(\frac{\cos^2\theta}{1-\cos^4\theta}+\frac{\sin^2\theta}{1-\sin^4\theta}\right)\sin^2\theta.\cos^2\theta$$

$$= \left(\frac{\cos^2\theta}{\sin^2\theta.(1+\cos^2\theta)}+\frac{\sin^2\theta}{\cos^2\theta.(1+\sin^2\theta)}\right)$$

$$\sin^2\theta.\cos^2\theta = \frac{\cos^4\theta}{1+\cos^2\theta}+\frac{\sin^4\theta}{1+\sin^2\theta}$$

$$= \frac{\cos^4\theta(1+\sin^2\theta)+\sin^4\theta(1+\cos^2\theta)}{(1+\sin^2\theta)(1+\cos^2\theta)}$$

$$= \frac{\cos^4\theta+\cos^4\theta.\sin^2\theta+\sin^4\theta+\sin^4\theta.\cos^2\theta}{(1+\sin^2\theta+\cos^2\theta+\sin^2\theta.\cos^2\theta)}$$

$$= \frac{\cos^4\theta+\sin^4\theta+\sin^2\theta\cos^2\theta(\cos^2\theta+\sin^2\theta)}{2+\sin^2\theta.\cos^2\theta}$$

$$= \frac{(\cos^2\theta+\sin^2\theta)^2-2\cos^2\theta.\sin^2\theta+\sin^2\theta.\cos^2\theta}{2+\sin^2\theta.\cos^2\theta}$$

$$= \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta.\cos^2\theta}$$

64. (4) $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A)$

$$= \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$$

$$= \frac{\sin A+\cos A-1}{\sin A}\cdot\frac{\cos A+\sin A+1}{\cos A}$$

$$= \frac{(\sin A+\cos A)^2-1}{\sin A.\cos A}$$

$$= \frac{\sin^2 A+\cos^2 A+2\sin A\cos A-1}{\sin A.\cos A}$$

$$= \frac{2\sin A.\cos A}{\sin A.\cos A}=2$$

65. (2) $\tan^6\theta + 3\tan^2\theta \cdot \sec^2\theta + 1$

$= (\sec^2\theta-1)^3 + 3(\sec^2\theta-1).\sec^2\theta+1$

$= \sec^6\theta - 3\sec^4\theta + 3\sec^2\theta - 1 +$

$3\sec^4\theta - 3\sec^2\theta + 1 = \sec^6\theta$

66. (1) $x^2 - y^2 = (a\sec\theta + b\tan\theta)^2 - (a\tan\theta + b\sec\theta)^2$

$= a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta.\tan\theta - a^2\tan^2\theta - b^2\sec^2\theta - 2ab\tan\theta.\sec\theta$

$= a^2\sec^2\theta - a^2\tan^2\theta - b^2\sec^2\theta + b^2\tan^2\theta$

$= a^2(\sec^2\theta - a^2\tan^2\theta) - b^2(\sec^2\theta - \tan^2\theta)$

$= a^2 - b^2$

67. (3) $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ ... (i)

$\frac{x}{a}\sin\theta + \frac{y}{b}\cos\theta = 1$ ... (ii)

On squaring and adding,

$$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + 2\frac{xy}{ab}\sin\theta.\cos\theta + \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - 2\frac{xy}{ab}\sin\theta.\cos\theta = 2$$

$$\Rightarrow \frac{x^2}{a^2}\cos^2\theta + \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta = 2$$

$$\Rightarrow \frac{x^2}{a^2}(\cos^2\theta + \sin^2\theta) + \frac{y^2}{b^2}(\sin^2\theta + \cos^2\theta) = 2$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$$

68. (1) $\operatorname{cosec}\theta - \sin\theta = a^3$

$$\Rightarrow \frac{1}{\sin\theta} - \sin\theta = a^3$$

$$\Rightarrow \frac{1-\sin^2\theta}{\sin\theta} = a^3$$

$$\Rightarrow \frac{\cos^2\theta}{\sin\theta} = a^3 \Rightarrow a = \frac{(\cos\theta)^{\frac{2}{3}}}{(\sin\theta)^{\frac{1}{3}}}$$

Similarly, $\sec\theta - \cos\theta = b^3$

$$\Rightarrow \frac{\sin^2\theta}{\cos\theta} = b^3 \Rightarrow b = \frac{(\sin\theta)^{\frac{2}{3}}}{(\cos\theta)^{\frac{1}{3}}}$$

$$\therefore a^2b^2(a^2+b^2) = \frac{(\cos\theta)^{\frac{4}{3}}}{(\sin\theta)^{\frac{2}{3}}}\times\frac{(\sin\theta)^{\frac{4}{3}}}{(\cos\theta)^{\frac{2}{3}}}$$

$$\left(\frac{(\cos\theta)^{\frac{4}{3}}}{(\sin\theta)^{\frac{2}{3}}}+\frac{(\sin\theta)^{\frac{4}{3}}}{(\cos\theta)^{\frac{2}{3}}}\right) = (\cos\theta\sin\theta)^{\frac{2}{3}}$$

$$\frac{(\cos^2\theta+\sin^2\theta)^{\frac{4}{3}}}{(\sin\theta)^{\frac{2}{3}}(\cos\theta)^{\frac{2}{3}}} = 1$$

69. (1) $x = a\cos^3\theta \Rightarrow \frac{x}{a} = \cos^3\theta$

$y = b\sin^3\theta \Rightarrow \frac{y}{b} = \sin^3\theta$

$\therefore \left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}}$

$= (\cos^3\theta)^{\frac{2}{3}} + (\sin^3\theta)^{\frac{2}{3}} = \cos^2\theta + \sin^2\theta = 1$

70. (1) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$= \sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}} + \sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$

$= \frac{1+\sin\theta}{\cos\theta} + \frac{1-\sin\theta}{\cos\theta}$

$= \frac{1+\sin\theta+1-\sin\theta}{\cos\theta} = \frac{2}{\cos\theta} = 2\sec\theta$

71. (1) $\frac{x}{a} = \sec\theta.\cos\phi$

$\frac{y}{b} = \sec\theta\,.\,\sin\phi$

$\frac{z}{c} = \tan\theta$

$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$

$= \sec^2\theta.\cos^2\phi + \sec^2\theta.\sin^2\phi - \tan^2\theta$

$= \sec^2\theta(\cos^2\phi + \sin^2\phi) - \tan^2\theta = \sec^2\theta - \tan^2\theta = 1$

72. (1) $\cos\theta + \cos^2\theta = 1$

$\Rightarrow \cos\theta = 1 - \cos^2\theta = \sin^2\theta$

$\therefore$ Expression $= \cos^6\theta + 3\cos^5\theta + 3\cos^4\theta + \sin^6\theta + 2\cos^2\theta + 2\sin^2\theta - 2$

$= \cos^6\theta + \sin^6\theta + 3\cos^5\theta + 3\cos^4\theta + 2 - 2$

$= \cos^6\theta + \sin^6\theta + 3.\cos^4\theta.\cos\theta + 3.\cos^2\theta.\cos^2\theta$

$= \cos^6\theta + \sin^6\theta + 3\cos^4\theta.\sin^2\theta + 3.\cos^2\theta.\sin^4\theta$ $[\because \sin^2\theta = \cos\theta]$

$= (\cos^2\theta + \sin^2\theta)^3 = 1$

73. (1) $\frac{p^2-1}{p^2+1} = \frac{(\sec\theta+\tan\theta)^2-1}{(\sec\theta+\tan\theta)^2+1}$

$= \frac{\sec^2\theta+\tan^2\theta+2\sec\theta.\tan\theta-1}{\sec^2\theta+2\sec\theta.\tan\theta+\tan^2\theta+1}$

$= \frac{\sec^2\theta-1+\tan^2\theta+2\sec\theta.\tan\theta}{\sec^2\theta+2\sec\theta.\tan\theta+\sec^2\theta}$

$= \frac{2\tan^2\theta+2\sec\theta.\tan\theta}{2\sec^2\theta+2\sec\theta.\tan\theta}$

$= \frac{2\tan\theta(\tan\theta+\sec\theta)}{2\sec\theta(\sec\theta+\tan\theta)}$

$= \frac{\tan\theta}{\sec\theta} = \frac{\sin\theta}{\cos\theta}\times\cos\theta = \sin\theta$

74. (4) $\sin\theta + \cos\theta = \sqrt{2}\sin(90^\circ - \theta)$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$\Rightarrow \sin\theta = (\sqrt{2}-1)\cos\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \sqrt{2}-1$

$\Rightarrow \tan\theta = \sqrt{2}-1$

$\cot\theta = \frac{1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)}$

$= \frac{\sqrt{2}+1}{2-1} = \sqrt{2}+1$

75. (1) $\cos\theta + \sec\theta = \sqrt{3}$

On cubing,

$(\cos\theta + \sec\theta)^3 = (\sqrt{3})^3 = 3\sqrt{3}$

$\Rightarrow \cos^3\theta + \sec^3\theta + 3\cos\theta.\sec\theta(\cos\theta + \sec\theta) = 3\sqrt{3}$

$\Rightarrow \cos^3\theta + \sec^3\theta + 3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow \cos^3\theta + \sec^3\theta = 0$

76. (1) $x = \sin^2\alpha + \cos^4\alpha$

$= \sin^2\alpha + (1 - \sin^2\alpha)^2$

$= 1 - \sin^2\alpha + \sin^4\alpha$

$= 1 - \sin^2\alpha(1 - \sin^2\alpha)$

$= 1 - \sin^2\alpha.\cos^2\alpha$

$= 1 - \frac{1}{4}(\sin 2\alpha)^2$

Now,

$0 < \sin^2 2\alpha \le 1$

$\Rightarrow -1 < -\sin^2 2\alpha \le 0$

$\Rightarrow -\frac{1}{4} \le -\frac{1}{4}\sin^2 2\alpha \le 0$

$\Rightarrow 1-\frac{1}{4} \le 1-\frac{1}{4}\sin^2 2\alpha \le 1$

$\Rightarrow \frac{3}{4} \le 1-\frac{1}{4}(\sin^2 2\alpha)^2 \le 1$

$\Rightarrow \frac{3}{4} \le x \le 1$

77. (4) $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{2}$

On adding,

$\sec\theta + \tan\theta + \sec\theta - \tan\theta$

$= 2 + \frac{1}{2}$

$\Rightarrow 2\sec\theta = \frac{5}{2}$

$\Rightarrow \sec\theta = \frac{5}{4}$

78. (4) $4\,\text{cosec}^2\alpha + 9\sin^2\alpha$

$= 4\,\text{cosec}^2\alpha + 4\sin^2\alpha + 5\sin^2\alpha$

$= 4\,[\text{cosec}\,\alpha - \sin\alpha)^2 + 2] + 5\sin^2\alpha$

$= 12$

$[\because\ \text{cosec}\alpha - \sin\alpha \ge 1]$

79. (1) $1 - \frac{\sin^2 A}{1+\cos A} + \frac{1+\cos A}{\sin A} - \frac{\sin A}{1-\cos A}$

$= 1 + \frac{1+\cos A}{\sin A} - \frac{\sin^2 A}{1+\cos A} - \frac{\sin A}{1-\cos A}$

$= 1 + \frac{1+\cos A}{\sin A} - \left(\frac{\sin^2 A(1-\cos A) + \sin A(1+\cos A)}{(1+\cos A)(1-\cos A)}\right)$

$= 1 + \frac{1+\cos A}{\sin A} - \left(\sin A\frac{(\sin A - \sin A.\cos A + (1+\cos A)}{\sin^2 A}\right)$

$= 1 + \frac{1+\cos A}{\sin A} - \frac{\sin A + 1 - \sin A.\cos A + \cos A}{\sin A}$

$= \left(\frac{\sin A + 1 + \cos A - \sin A \; -1 + \sin A.\cos A - \cos A}{\sin A}\right)$

$= \frac{\sin A.\cos A}{\sin A} = \cos A$

80. (1) $\tan\theta - \cot\theta = a$

On squaring

$\tan^2\theta + \cot^2\theta - 2 = a^2$

$\therefore a^2 + 4 = \tan^2\theta + \cot^2\theta + 2$

$= \sec^2\theta + \text{cosec}^2\theta$

$[\because\ 1 + \tan^2\theta = \sec^2\theta;\ 1 + \cot^2\theta = \text{cosec}^2\theta]$

$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta.\cos^2\theta} = \frac{1}{\sin^2\theta.\cos^2\theta}$ .....(i)

Again,

$\cos\theta - \sin\theta = b$

On squaring,

$\cos^2\theta + \sin^2\theta - 2\cos\theta.\sin\theta = b^2$

$\Rightarrow 1 - 2\cos\theta.\sin\theta = b^2$

$\Rightarrow (b^2 - 1) = -2\cos\theta.\sin\theta$ ....(ii)

$\therefore (a^2 + 4)(b^2 - 1)^2$

$= \frac{1}{\sin^2\theta.\cos^2\theta}.4\sin^2\theta.\cos^2\theta$

$= 4$

81. (3) $\sin^2\alpha + \sin^2\beta = 2$

$\because \sin\theta \le 1$

$\therefore \sin\alpha = \sin\beta = 1$

$\therefore \alpha = \beta = 90°$

$\therefore \cos\left(\frac{\alpha+\beta}{2}\right) = \cos 90° = 0$

82. (3) $\tan\theta.\tan 2\theta = 1$

$\Rightarrow \tan\theta = \frac{1}{\tan 2\theta} = \cot 2\theta$

$\Rightarrow \tan\theta = \tan(90° - 2\theta)$

$\Rightarrow \theta = 90° - 2\theta$

$\Rightarrow 3\theta = 90° \Rightarrow \theta = 30°$

$\therefore \sin^2 2\theta + \tan^2 2\theta = \sin^2 60° + \tan^2 60°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\sqrt{3}\right)^2 = \frac{3}{4} + 3 = 3\frac{3}{4}$

83. (2) $x^2y^2(x^2 + y^2 + 3)$

$= (\text{cosec}\theta - \sin\theta)^2(\sec\theta - \cos\theta)^2\{(\text{cosec}\theta - \sin\theta)^2 + (\sec\theta - \cos\theta)^2 + 3\}$

$= \left(\frac{1}{\sin\theta} - \sin\theta\right)^2\left(\frac{1}{\cos\theta} - \cos\theta\right)^2$

$\left\{\left(\frac{1}{\sin\theta} - \sin\theta\right)^2 + \left(\frac{1}{\cos\theta} - \cos\theta\right)^2 + 3\right\}$

$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2\left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2$

$$\left\{\left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2+\left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2+3\right\}$$

$$=\left(\frac{\cos^2\theta}{\sin\theta}\right)^2\left(\frac{\sin^2\theta}{\cos\theta}\right)^2\left\{\left(\frac{\cos^2\theta}{\sin\theta}\right)^2+\left(\frac{\sin^2\theta}{\cos\theta}\right)^2+3\right\}$$

$$=\cos^2\theta\times\sin^2\theta\left(\frac{\cos^6\theta+\sin^6\theta+3\cos^2\theta.\sin^2\theta}{\cos^2\theta.\sin^2\theta}\right)$$

$= \cos^6\theta + \sin^6\theta + 3\cos^2\theta\sin^2\theta$

$$=\left\{(\cos^2\theta)^3+(\sin^2\theta)^3\right\}+3\cos^2\theta.\sin^2\theta$$

$= (\cos^2\theta + \sin^2\theta)^3 - 3\cos^2\theta.\sin^2\theta(\cos^2\theta + \sin^2\theta) + 3\cos^2\theta.\sin^2\theta$

$= 1 - 3\cos^2\theta.\sin^2\theta + 3\cos^2\theta.\sin^2\theta$

$= 1$

84. (4) $2y\cos\theta = x\sin\theta$

$\Rightarrow x\sec\theta = 2y\operatorname{cosec}\theta$

$\therefore 2x\sec\theta - y\operatorname{cosec}\theta = 3$

$\Rightarrow 4y\operatorname{cosec}\theta - y\operatorname{cosec}\theta = 3$

$\Rightarrow 3y\operatorname{cosec}\theta = 3$

$\Rightarrow y\operatorname{cosec}\theta = 1$

$\Rightarrow y = \sin\theta$

$\therefore x\sec\theta = 2y\operatorname{cosec}\theta$

$= 2\sin\theta.\operatorname{cosec}\theta = 2$

$\Rightarrow x = 2\cos\theta$

$\therefore x^2 + 4y^2 = 4\cos^2\theta + 4\sin^2\theta = 4$

85. (1) $\sin\theta + \sin^2\theta = 1$

$\Rightarrow \sin\theta = 1 - \sin^2\theta$

$\Rightarrow \sin\theta = \cos^2\theta$

$\therefore \cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta - 1$

$= (\cos^4\theta + \cos^2\theta)^3 - 1$

$= (\sin^2\theta + \cos^2\theta)^3 - 1 = 1 - 1 = 0$

86. (1) $\sec x + \cos x = 3$

On squaring both sides,

$\sec^2x + \cos^2x + 2 . \sec x . \cos x = 9$

$\Rightarrow \sec^2x + \cos^2x = 9 - 2 = 7$ ...(i)

$[\because \sec x.\cos x = 1]$

Now, $\tan^2x - \sin^2x = \sec^2x - 1 - (1 - \cos^2x)$

$[\because \sec^2x - \tan^2x = 1]$

$= \sec^2x + \cos^2x - 2 = 7 - 2 = 5$

[From equation (i)]

87. (4) $\sin p + \operatorname{cosec} p = 2$

$$\Rightarrow \sin p + \frac{1}{\sin p} = 2$$

$\Rightarrow \sin^2p + 1 = 2\sin p$

$\Rightarrow \sin^2p - 2\sin p + 1 = 0$

$\Rightarrow (\sin p - 1)^2 = 0$

$\Rightarrow \sin p = 1$ and $\operatorname{cosec} p = 1$

$\therefore \sin^7p + \operatorname{cosec}^7p = 1 + 1 = 2$

88. (3) $\sin\theta + \sin^2\theta + \sin^3\theta = 1$

$\Rightarrow \sin\theta + \sin^3\theta = 1 - \sin^2\theta$

$\Rightarrow \sin\theta(1 + \sin^2\theta) = \cos^2\theta$

On squaring both sides,

$\Rightarrow (\sin^2\theta)(1 + \sin^2\theta)^2 = \cos^4\theta$

$\Rightarrow (1 - \cos^2\theta)(1 + (1 - \cos^2\theta))^2 = \cos^4\theta$

$\Rightarrow (1 - \cos^2\theta)(2 - \cos^2\theta))^2 = \cos^4\theta$

$\Rightarrow (1 - \cos^2\theta)(4 - 4\cos^2\theta + \cos^4\theta) = \cos^4\theta$

$\Rightarrow 4 - 4\cos^2\theta + \cos^4\theta - 4\cos^2\theta + 4\cos^4\theta - \cos^6\theta = \cos^4\theta$

$\Rightarrow -\cos^6\theta + 4\cos^4\theta - 8\cos^2\theta + 4 = 0$

$\Rightarrow \cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$

89. (2) $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2[(\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)] - 3(\sin^4\theta + \cos^4\theta) + 1$

$[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

$= 2\sin^4\theta + 2\cos^4\theta - 2\sin^2\theta . \cos^2\theta - 3\sin^4\theta - 3\cos^4\theta + 1$

$= -\sin^4\theta - \cos^4\theta - 2\sin^2\theta\cos^2\theta + 1$

$= -(\sin^4\theta + \cos^4\theta + 2\sin^2\theta . \cos^2\theta) + 1$

$= -(\sin^2\theta + \cos^2\theta)^2 + 1$

$= -1 + 1 = 0$

90. (3) $x\sin^3\theta + y\cos^3\theta = \sin\theta.\cos\theta$

$\Rightarrow (x\sin\theta) . \sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta.\cos\theta$

$\Rightarrow x\sin\theta . \sin^2\theta + x\sin\theta . \cos^2\theta = \sin\theta.\cos\theta$

$\Rightarrow x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta.\cos\theta$

$\Rightarrow x = \cos\theta$

$\therefore x\sin\theta = y\cos\theta$

$\Rightarrow \cos\theta . \sin\theta = y\cos\theta$

$\Rightarrow y = \sin\theta$

$\therefore x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

91. (2) $\sec^4\theta - \sec^2\theta$

$= \sec^2\theta(\sec^2\theta - 1)$

$= (1 + \tan^2\theta)(1 + \tan^2\theta - 1)$

$= \tan^2\theta + \tan^4\theta$

92. (1) $\cos A = 1 - \cos^2 A = \sin^2 A$

$\therefore \sin^2 A + \sin^4 A = \sin^2 A + \cos^2 A = 1$

93. (4) $\sec\theta - \operatorname{cosec}\theta = 0$

$\Rightarrow \sec\theta = \operatorname{cosec}\theta$

$$\Rightarrow \frac{1}{\cos\theta} = \frac{1}{\sin\theta}$$

$\Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \tan\theta = 1 = \tan 45°$

$\Rightarrow \theta = 45°$

$\therefore \sec\theta + \operatorname{cosec}\theta = \sec 45° + \operatorname{cosec} 45°$

$= \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

94. (4) $p \sin\theta = \sqrt{3}$

$p\cos\theta = 1$

On squaring and adding

$p^2\sin^2\theta + p^2\cos^2\theta = 3 + 1$

$\Rightarrow p^2(\sin^2\theta + \cos^2\theta) = 4$

$\Rightarrow p^2 = 4$

$\Rightarrow p = 2$

95. (1) $x\sin^3\theta + y\cos^3\theta = \sin\theta \cdot \cos\theta$

$\Rightarrow x.\sin\theta \cdot \sin^2\theta + y\cos\theta \cdot \cos^2\theta = \sin\theta \cdot \cos\theta$

$\Rightarrow x\sin\theta.\sin^2\theta + x\sin\theta \cdot \cos^2\theta = \sin\theta\cos\theta$

$[x\sin\theta - y\cos\theta = 0;\ x\sin\theta = y\cos\theta]$

$\Rightarrow x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta \cdot \cos\theta$

$\Rightarrow x\sin\theta = \sin\theta.\cos\theta$

$\Rightarrow x = \cos\theta$

$\therefore x\sin\theta = y\cos\theta$

$\Rightarrow \cos\theta.\sin\theta = y\cos\theta$

$\Rightarrow y = \sin\theta$

$\therefore x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

96. (4) $u_n = \cos^n\alpha + \sin^n\alpha$

$\therefore u_6 = \cos^6\alpha + \sin^6\alpha$

$(\cos^2\alpha) + (\sin^2\alpha)^3$

$= (\cos^2\alpha + \sin^2\alpha)^3 - 3\cos^2\alpha.\sin^2\alpha(\sin^2\alpha + \cos^2\alpha)$

$[\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)]$

$= 1 - 3\cos^2\alpha \cdot \sin^2\alpha$

$u_4 = \cos^4\alpha + \sin^4\alpha$

$(\cos^2\alpha)^2 + (\sin^2\alpha)^2$

$= (\cos^2\alpha + \sin^2\alpha)^2 - 2\cos^2\alpha.\sin^2\alpha = 1 - 2\cos^2\alpha \cdot \sin^2\alpha$

$\therefore 2u_6 - 3u_4 + 1$

$= 2(1 - 3\sin^2\alpha\cos^2\alpha) - 3$

$(1 - 2\sin^2\alpha \cdot \cos^2\alpha) + 1$

$= 2 - 3 + 1 = 0$

97. (3) $2\sin\alpha + 15\cos^2\alpha = 7$

$\Rightarrow 2\sin\alpha + 15(1 - \sin^2\alpha) = 7$

$\Rightarrow 2\sin\alpha + 15 - 15\sin^2\alpha = 7$

$\Rightarrow 15\sin^2\alpha - 2\sin\alpha - 8 = 0$

$\Rightarrow 15\sin^2\alpha - 12\sin\alpha + 10\sin\alpha - 8 = 0$

$\Rightarrow 3\sin\alpha(5\sin\alpha - 4) + 2(5\sin\alpha - 4) = 0$

$\Rightarrow (3\sin\alpha + 2)(5\sin\alpha - 4) = 0$

$\Rightarrow \sin\alpha = \frac{4}{5}$

because $\sin\alpha \neq -\frac{2}{3}$

$\cos\alpha = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

$\therefore \cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$

98. (3) $3\sin^2\alpha + 7(1 - \sin^2\alpha) = 4$

$\Rightarrow 3\sin^2\alpha + 7 - 7\sin^2\alpha) = 4$

$\Rightarrow 7 - 4\sin^2\alpha = 4$

$\Rightarrow 4\sin^2\alpha = 3 \qquad \Rightarrow \sin\alpha = \frac{\sqrt{3}}{2}$

$\cos\alpha = \sqrt{1 - \sin^2\alpha} = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

$\therefore \tan\alpha = \frac{\sin a}{\cos a} = \sqrt{3}$

99. (1) $(\sec\theta - \cos\theta)(\mathrm{cosec}\theta - \sin\theta)(\tan\theta + \cot\theta)$

$= \left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$= \left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta}\right)$

$= \frac{\sin^2\theta}{\cos\theta} \cdot \frac{\cos^2\theta}{\sin\theta} \cdot \frac{1}{\sin\theta.\cos\theta} = 1$

100. (3) $\cos^4\theta - \sin^4\theta = \frac{2}{3}$

$\Rightarrow (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) = \frac{2}{3}$

$\Rightarrow \cos^2\theta - \sin^2\theta = \frac{2}{3}$

$\Rightarrow \cos^2\theta - (1 - \cos^2\theta) = \frac{2}{3}$

$\Rightarrow 2\cos^2\theta - 1 = \frac{2}{3}$

101. (1) $\tan\theta = 1 \Rightarrow \theta = 45°$

$\therefore \frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$

$= \frac{8 \times \frac{1}{\sqrt{2}} + \frac{5}{\sqrt{2}}}{\frac{1}{2\sqrt{2}} - \frac{2}{2\sqrt{2}} + \frac{7}{\sqrt{2}}}$

$= \frac{\frac{13}{\sqrt{2}}}{\frac{13}{2\sqrt{2}}} = \frac{13}{\sqrt{2}} \times \frac{2\sqrt{2}}{13} = 2$

102. (2) $\cos^2\theta + \cos^4\theta = 1$

$\Rightarrow \cos^4\theta = 1 - \cos^2\theta = \sin^2\theta$

$\Rightarrow \tan^2\theta = \cos^2\theta$

$\therefore \tan^2\theta + \tan^4\theta = \cos^2\theta + \cos^4\theta$

$= 1$

103. (3) $(\sec A - \cos A)^2 + (\operatorname{cosec} A - \sin A)^2 - (\cot A - \tan A)^2$

$= \sec^2 A + \cos^2 A - 2\sec A \cos A + \operatorname{cosec}^2 A + \sin^2 A - 2\operatorname{cosec} A . \sin A - \cot^2 A - \tan^2 A + 2 \cot A . \tan A$

$= \sec^2 A - \tan^2 A + \cos^2 A + \sin^2 A + \operatorname{cosec}^2 A - \cot^2 A - 2$

$= 3 - 1 = 1$

$\left[\because \sec A . \cos A = 1; \sin A . \operatorname{cosec} A = 1; \tan A . \cot A = 1 \text{ etc}\right]$

104. (2) $7 \sin^2\theta + 3\cos^2\theta = 4$

$\Rightarrow 7\frac{\sin^2\theta}{\cos^2\theta} + 3 = \frac{4}{\cos^2\theta} = 4\sec^2\theta$

$\Rightarrow 7\tan^2\theta + 3 = 4(1 + \tan^2\theta)$

$\Rightarrow 7\tan^2\theta - 4\tan^2\theta = 4 - 3$

$\Rightarrow 3\tan^2\theta = 1$

$\Rightarrow \tan^2\theta = \frac{1}{3}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

105. (2) $2\sin^2\theta + 3\cos^2\theta$

$= 2\sin^2\theta + 2\cos^2\theta + \cos^2\theta$

$= 2(\sin^2\theta + \cos^2\theta) + \cos^2\theta$

$= 2 + \cos^2\theta$

$\because$ Minimum value of $\cos\theta = -1$

$\therefore$ Required minimum value $= 2 + 1 = 3$

106. (4) $\sin\theta + \operatorname{cosec}\theta = 2$

$\Rightarrow \sin\theta + \frac{1}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow (\sin\theta - 1)^2 = 0 \Rightarrow \sin\theta = 1$

$\therefore \sin^5\theta + \operatorname{cosec}^5\theta = 1 + 1 = 2$

107. (3) $2\cos\theta - \sin\theta = \frac{1}{\sqrt{2}}$

$2\sin\theta + \cos\theta = x$ (Let)

On squaring and adding,

$4\cos^2\theta + \sin^2\theta - 4\sin\theta . \cos\theta +$

$4\sin^2\theta + \cos^2\theta + 4\sin\theta . \cos\theta = \frac{1}{2} + x^2$

$\Rightarrow 4(\cos^2\theta + \sin^2\theta) + (\cos^2\theta + \sin^2\theta) = \frac{1}{2} + x^2$

$\Rightarrow \frac{1}{2} + x^2 = 5$

$\Rightarrow x^2 = 5 - \frac{1}{2} = \frac{9}{2} \Rightarrow x = \frac{3}{\sqrt{2}}$

108. (3) $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$

$\Rightarrow \sin\theta + \cos\theta = 3\sin\theta - 3\cos\theta$

$\Rightarrow 4\cos\theta = 2\sin\theta \Rightarrow \tan\theta = 2$

$\therefore \sin^4\theta - \cos^4\theta$

$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$

$= \sin^2\theta - \cos^2\theta$

$= \cos^2\theta(\tan^2\theta - 1)$

$= \frac{\tan^2\theta - 1}{1 + \tan^2\theta} = \frac{4-1}{1+4} = \frac{3}{5}$

109. (1) $\sec^2\theta + \tan^2\theta = 7$

$\Rightarrow 1 + \tan^2\theta + \tan^2\theta = 7$

$\Rightarrow 2\tan^2\theta = 7 - 1 = 6$

$\Rightarrow \tan^2\theta = 3 \Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = 60°$

110. (2) $\cos^2\alpha + \cos^2\beta = 2$

$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta = 2$

$\Rightarrow \sin^2\alpha + \sin^2\beta = 0$

$\Rightarrow \sin\alpha = \sin\beta = 0$

$\Rightarrow \alpha = \beta = 0$

$\therefore \tan^3\alpha + \sin^5\beta = 0$

111. (1) $\sin\theta - \cos\theta = \frac{7}{13}$ ....(i)

$\sin\theta + \cos\theta = x$ ....(ii)

On squaring both equations and adding,

$2(\sin^2\theta + \cos^2\theta) = \frac{49}{169} + x^2$

$\Rightarrow x^2 = 2 - \frac{49}{169} = \frac{338 - 49}{169} = \frac{289}{169} \Rightarrow x = \frac{17}{13}$

112. (4) $(\sec x . \sec y + \tan x . \tan y)^2 - (\sec x . \tan y + \tan x . \sec y)^2$

$= \sec^2 x . \sec^2 y + \tan^2 x . \tan^2 y + 2\sec x . \sec y . \tan x . \tan y - \sec^2 x . \tan^2 y - \tan^2 x . \sec^2 y - 2\sec x . \sec y . \tan x . \tan y$

$= \sec^2 x . \sec^2 y + \tan^2 x . \tan^2 y - \sec^2 x . \tan^2 y - \tan^2 x . \sec^2 y$

$= \sec^2 x . \sec^2 y - \sec^2 x . \tan^2 y - \tan^2 x . \sec^2 y + \tan^2 x . \tan^2 y$

$= \sec^2 x(\sec^2 y - \tan^2 y) - \tan^2 x(\sec^2 y - \tan^2 y)$

$= \sec^2 x - \tan^2 x = 1$

113. (2) $\sin\theta + \operatorname{cosec}\theta = 2$

$\Rightarrow \sin\theta + \frac{1}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow (\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = 1$

$\Rightarrow \operatorname{cosec}\theta = 1$

$\therefore \sin^{100}\theta + \operatorname{cosec}^{100}\theta = 1 + 1 = 2$

114. (1) $(\sec\theta - \cos\theta)(\text{cosec}\theta - \sin\theta)(\tan\theta + \cot\theta)$

$$= \left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta}\right)$$

$$= \frac{\sin^2\theta}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin\theta}\cdot\frac{1}{\sin\theta.\cos\theta} = 1$$

115. (2) $\cos\theta = \frac{15}{17}$

$$\Rightarrow \sec\theta = \frac{1}{\cos\theta} = \frac{17}{15}$$

$$\therefore \cot(90° - \theta) = \tan\theta = \sqrt{\sec^2\theta - 1}$$

$$= \sqrt{\left(\frac{17}{15}\right)^2 - 1} = \sqrt{\frac{289}{225} - 1}$$

$$= \sqrt{\frac{289-225}{225}} = \sqrt{\frac{64}{225}} = \frac{8}{15}$$

116. (1) $\sec^2\theta - \tan^2\theta = 1$

$$\sec^2\theta + \tan^2\theta = \frac{7}{12}$$

$\therefore \sec^4\theta - \tan^4\theta$

$$= (\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta) = 1 \times \frac{7}{12} = \frac{7}{12}$$

117. (1) $\cos x + \cos y = 2$

$\because \cos x \le 1$

$\Rightarrow \cos x = 1; \cos y = 1$

$\Rightarrow x = y = 0°$

118. (3) $\sec\theta + \tan\theta = 2$ ......(i)

$\therefore \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{2}$ ......(ii)

By equations (i) and (ii)

$\therefore \sec\theta + \tan\theta + \sec\theta - \tan\theta$

$$= 2 + \frac{1}{2} = \frac{5}{2}$$

$$\Rightarrow 2\sec\theta = \frac{5}{2} \Rightarrow \sec\theta = \frac{5}{4}$$

119. (3) $(l^2.m^2)(l^2 + m^2 + 3)$

$\Rightarrow (\text{cosec}\,\theta - \sin\theta)^2$

$(\sec\theta - \cos\theta)^2$

$\{(\text{cosec}\,\theta - \sin\theta)^2 + (\sec\theta - \cos\theta)^2 + 3\}$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)^2\left(\frac{1}{\cos\theta} - \cos\theta\right)^2$$

$$\left\{\left(\frac{1}{\sin\theta} - \sin\theta\right)^2 + \left(\frac{1}{\cos\theta} - \cos\right)^2 + 3\right\}$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2\left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2$$

$$\left\{\left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2 + \left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2 + 3\right\}$$

$$= \left(\frac{\cos^2\theta}{\sin\theta}\right)^2\left(\frac{\sin^2\theta}{\cos\theta}\right)^2 \left\{\left(\frac{\cos^2\theta}{\sin\theta}\right)^2 + \left(\frac{\sin^2\theta}{\cos\theta}\right)^2 + 3\right\}$$

$$= \frac{\cos^4\theta}{\sin^2\theta} \times \frac{\sin^4\theta}{\cos^2\theta} \left\{\frac{\cos^4\theta}{\sin^2\theta} + \frac{\sin^4\theta}{\cos^2\theta} + 3\right\}$$

$$= \cos^2\theta \times \sin^2\theta \left\{\frac{\cos^6\theta + \sin^6\theta + 3\cos^2\theta.\sin^2\theta}{\cos^2\theta.\sin^2\theta}\right\}$$

$= \cos^6\theta + \sin^6\theta + 3\cos^2\theta \,.\, \sin^2\theta$

$= \{(\cos^2\theta + \sin^2\theta)^3 - 3\cos^2\theta \,.\, \sin^2\theta\,(\cos^2\theta + \sin^2\theta)\} +$
$3\cos^2\theta \,.\, \sin^2\theta$

$[\because a^3 + b^3 = (a + b)^3 - 3ab\,(a + b)]$

$= 1 - 3\cos^2\theta \,.\, \sin^2\theta + 3\cos^2\theta \,.\, \sin^2\theta = 1$

120. (2) $\frac{\tan 57° + \cot 37°}{\tan 33° + \cot 53°} = \frac{\cot 33° + \tan 53°}{\tan 33° + \cot 53°}$

$[\because \tan(90° - \theta) = \cot\theta,$
$\cot(90° - \theta) = \tan\theta]$

$$= \frac{\frac{1}{\tan 33°} + \tan 53°}{\tan 33° + \frac{1}{\tan 53°}} = \frac{1 + \tan 53°.\tan 33°}{\tan 33°.\tan 53° + 1} \times \frac{\tan 53°}{\tan 33°}$$

$= \tan 53° \,.\, \cot 33° = \cot 37° \,.\, \tan 57°$

121. (3) $\tan 7\theta \,.\, \tan 2\theta = 1$

$$\Rightarrow \tan 7\theta = \frac{1}{\tan 2\theta} = \cot 2\theta$$

$\Rightarrow \tan 7\theta = \tan(90° - 2\theta)$

$\Rightarrow 7\theta = 90° - 2\theta$

$\Rightarrow 9\theta = 90° \Rightarrow \theta = 10°$

$$\therefore \tan 3\theta = \tan 30° = \frac{1}{\sqrt{3}}$$

122. (1) Expression

$$= \frac{8\sin\theta + 5\cos\theta}{\sin^3\theta + 2\cos^2\theta + 3\cos\theta}$$

Dividing numerator and denominator by cos θ,

$$= \frac{8\tan\theta+5}{\tan\theta.\sin^2\theta+2\cos^2\theta+3}$$

$$= \frac{8\tan\theta+5}{2\sin^2\theta+2\cos^2\theta+3}$$

$$= \frac{8\tan\theta+5}{2(\sin^2\theta+\cos^2\theta)+3}$$

$$= \frac{8\times2+5}{5} = \frac{21}{5}$$

123. (4) $(2\cos^2\theta - 1)$

$$\left(\frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= (2\cos^2\theta-1)\left(\frac{(1+\tan\theta)^2+(1-\tan\theta)^2}{1-\tan^2\theta}\right)$$

$$= (2\cos^2\theta-1)\left(2\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)\right)$$

$$= \frac{2\sec^2\theta(2\cos^2\theta-1)}{1-\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{2\sec^2\theta(2\cos^2\theta-1)}{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{2\sec^2\theta.\cos^2\theta(2\cos^2\theta-1)}{2\cos^2\theta-1} = 2$$

124. (4) $\cos\theta + \sec\theta = 2$

$$\Rightarrow \cos\theta + \frac{1}{\cos\theta} = 2$$

$\Rightarrow \cos^2\theta - 2\cos\theta + 1 = 0$

$\Rightarrow (\cos\theta - 1)^2 = 0$

$\Rightarrow \cos\theta = 1$

$\therefore \sec\theta = 1$

$\therefore \cos^6\theta + \sec^6\theta = 1 + 1 = 2$

125. (1) Expression

$$= \frac{5}{\sec^2\theta}+\frac{2}{1+\cot^2\theta}+3\sin^2\theta$$

$$= 5\cos^2\theta+\frac{2}{\text{cosec}^2\theta}+3\sin^2\theta$$

$= 5\cos^2\theta + 2\sin^2\theta + 3\sin^2\theta$

$= 5(\cos^2\theta + \sin^2\theta) = 5$

$$\left[\because 1+\cot^2\theta = \text{cosec}^2\theta\ \frac{1}{\sec\theta} = \cos\theta; \frac{1}{\text{cosec}\theta} = \sin\theta\right]$$

126. (4) $\cot 30° = \cot(90° - 60°)$

$= \tan 60°$

$\cot 75° = \cot(90° - 15°)$

$= \tan 15°$

$$\therefore \frac{\cot 30° - \cot 75°}{\tan 15° - \tan 60°} = \frac{\tan 60° - \tan 15°}{\tan 15° - \tan 60°} = -1$$

127. (1) $\cot\theta.\tan(90°-\theta) - \sec(90°-\theta).\text{cosec}\theta + (\sin^2 25° + \sin^2 65°) + \sqrt{3}(\tan 5°.\tan 15°.\tan 30°.\tan 75°.\tan 85°) = \cot\theta.\cot\theta - \text{cosec}\theta.\text{cosec}\theta + (\sin^2 25° + \cos^2 25°) + \sqrt{3}(\tan 5°.\cot 5°.\tan 15°.\cot 15°.\tan 30°) = (\cot^2\theta - \text{cosec}^2\theta) + (\sin^2 25° + \cos^2 25°) +$

$$\sqrt{3}\times\frac{1}{\sqrt{3}} = -1+1+1 = 1$$

$[\sin(90°-\theta) = \cos\theta; \text{cosec}^2\theta - \cot^2\theta = 1; \tan(90° - \theta) = \cot\theta; \sec(90°-\theta) = \text{cosec}\theta]$

128. (3) $\dfrac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} = \dfrac{5}{4}$

$$\Rightarrow \frac{\cos\theta\left(\frac{\sin\theta}{\cos\theta}+1\right)}{\cos\theta\left(\frac{\sin\theta}{\cos\theta}-1\right)} = \frac{5}{4}$$

$$\Rightarrow \frac{\tan\theta+1}{\tan\theta-1} = \frac{5}{4}$$

$$\Rightarrow \frac{2\tan\theta}{2} = \frac{5+4}{5-4}$$

(By componendo and dividendo)

$\therefore \tan\theta = 9$

$$\therefore \frac{\tan^2\theta+1}{\tan^2\theta-1} = \frac{81+1}{81-1} = \frac{82}{80} = \frac{41}{40}$$

129. (3) $\tan\left(\dfrac{\pi}{2}-\dfrac{\theta}{2}\right) = \sqrt{3}$

$$\Rightarrow \cot\frac{\theta}{2} = \sqrt{3} = \cot 30°$$

$$\Rightarrow \frac{\theta}{2} = 30° \Rightarrow \theta = 60°$$

$$\therefore \cos\theta = \cos 60° = \frac{1}{2}$$