Chapter 4 Matrices and Determinants

Ex 4.7

Answer 1e.

A binomial is a polynomial with two terms. The expression $3x+2$ is an example for a binomial.

A trinomial is a polynomial that consists of three terms. An example for a trinomial is $2a^2+5a+1$.

Answer 1gp.

First, write the left side as a binomial squared.
$(x+3)^2=36$

Take the square root on each side.
$x+3 = \pm 6$

Subtract 3 from each side.
$$x+3-3 = \pm 6-3$$
$$x = -3 \pm 6$$

Now, write as two equations.
$x = -3+6$ or $x = -3-6$

Simplify.
$x = 3$ or $x = -9$

The solutions are 3 and -9.

Answer 1q.

Consider $4x^2=64$
Need to solve the quadratic equation by finding square roots.

On simplification,
$$\Rightarrow x^2=\frac{64}{4}$$
$$\Rightarrow x^2=16$$
$$\Rightarrow x=\pm\sqrt{16}$$
$$\Rightarrow x=\pm 4.$$

Hence the roots of the equation are $\boxed{x=\pm 4}$

Answer 2e.

Consider a quadratic expression of the form x^2+bx.

Completing the square means adding the number $\left(\frac{b}{2}\right)^2$ in order to make the given quadratic expression a perfect square trinomial.

That is,

$$x^2+bx+\left(\frac{b}{2}\right)^2=x^2+2x\left(\frac{b}{2}\right)+\left(\frac{b}{2}\right)^2$$
$$=\left(x+\frac{b}{2}\right)^2$$

Answer 2gp.

Consider the quadratic equation $x^2-10x+25=1$.

Need to solve the quadratic equation by finding square roots.

$x^2-10x+25=1$ Write original equation

$x^2-2x(5)+5^2=1$

$(x-5)^2=1$ Write left side as a binomial square

$x-5=\pm 1$ Apply square root on each side

$x=5\pm 1$ Add 5 on each side

$x=-5+1$ or $x=-5-1$

$x=-4,-6$

Therefore,

The solutions of the equation are $\boxed{x=-4,-6}$.

Answer 2q.

Consider $3(p-1)^2=15$

Need to solve the following quadratic equation by finding square roots.

On simplifying,

$$\Rightarrow (p-1)^2=\frac{15}{3}$$
$$\Rightarrow (p-1)^2=5$$
$$\Rightarrow p-1=\pm\sqrt{5}$$
$$\Rightarrow p=1\pm\sqrt{5}$$

Hence the roots of the equation are $\boxed{p=1\pm\sqrt{5}}$

Answer 3e.

First, write the left side as a binomial squared.

$(x+2)^2 = 9$

Take the square root on each side.

$x + 2 = \pm 3$

Subtract 2 from each side.

$x + 2 - 2 = \pm 3 - 2$

$x = -2 \pm 3$

Now, write as two equations.

$x = -2 + 3$ or $x = -2 - 3$

Simplify.

$x = 1$ or $x = -5$

The solutions are 1 and –5.

Answer 3gp.

First, write the left side as a binomial squared.

$(x-12)^2 = 100$

Take the square root on each side.

$x - 12 = \pm 10$

Add 12 to each side.

$x - 12 + 12 = \pm 10 + 12$

$x = 12 \pm 10$

Now, write as two equations.

$x = 12 + 10$ or $x = 12 - 10$

Simplify.

$x = 22$ or $x = 2$

The solutions are 22 and 2.

Answer 3q.

Consider $16(m+5)^2 = 8$

Need to solve above quadratic equation by finding square roots.

On simplifying,

$$\Rightarrow (m+5)^2 = \frac{8}{16}$$

$$\Rightarrow (m+5)^2 = \frac{1}{2}$$

$$\Rightarrow m+5 = \pm\sqrt{\frac{1}{2}}$$

$$\Rightarrow m+5 = \pm\frac{1}{\sqrt{2}}$$

$$\Rightarrow m = -5 \pm \frac{1}{\sqrt{2}}$$

The roots of the equation are $\boxed{m = -5 \pm \frac{1}{\sqrt{2}}}$

Answer 4e.

Consider the quadratic equation $x^2 + 10x + 25 = 64$

Need to solve the equation by finding square roots.

$x^2 + 10x + 25 = 64$ Write original equation

$x^2 + 2x(5) + 5^2 = 64$

$(x+5)^2 = 64$ Write left side as a binomial square

$x + 5 = \pm\sqrt{64}$ Apply square root on each side

$x + 5 = \pm 8$

$x = -5 \pm 8$ Subtract 5 from each side

$x = -5 + 8$ or $x = -5 - 8$

$x = 3, -13$

Therefore,

The solutions of the equation are $\boxed{x = 3, -13}$.

Answer 4gp.

Consider the quadratic expression $x^2+14x+c$
Need to find the value of c that makes the following expression a perfect square trinomial.

Step 1:
Find half the coefficient of x.

That is, $\frac{14}{2}=7$

Step 2:

Square the result of step1.
That is, $7^2=49$

Step3:

Replace c with the result of step 2.
That gives $x^2+14x+49$
The trinomial $x^2+14x+c$ is a perfect square when $c=49$.
Then

$$\begin{aligned} x^2+14x+49 &= (x+7)(x+7) \\ &= (x+7)^2 \end{aligned}$$

Answer 4q.

To solve the following quadratic equation by finding square roots.

$$-2z^2=424$$
$$z^2=-\frac{424}{2}$$
$$z^2=-212$$
$$z=\pm\sqrt{-212}$$
$$z=\pm\sqrt{4}\cdot\sqrt{-53}$$
$z=\pm2\sqrt{53}i$ are the roots of the given quadratic equation.

Answer 5e.

First, write the left side as a binomial squared.
$(n+8)^2=36$

Take the square root on each side.
$n+8=\pm6$

Subtract 8 from each side.
$$n+8-8=\pm6-8$$
$$n=-8\pm6$$

Now, write as two equations.

$n = -8 + 6$ or $n = -8 - 6$

Simplify.

$n = -2$ or $n = -14$

The solutions are -2 and -14.

Answer 5gp.

STEP 1 First, find half the coefficient of x. In the given expression, the coefficient of x is 22.

$$\frac{22}{2} = 11$$

STEP 2 Square half the coefficient of x.

$$11^2 = 121$$

STEP 3 Replace c with 121 in the given expression.

$$x^2 + 22x + c = x^2 + 22x + 121$$

The expression $x^2 + 22x + c$ is a perfect square trinomial when c is 121.

Factor using the special factoring pattern $a^2 + 2ab + b^2 = (a+b)^2$.

$$x^2 + 22x + 121 = x^2 + 2(x)(11) + 11^2$$
$$= (x + 11)^2$$

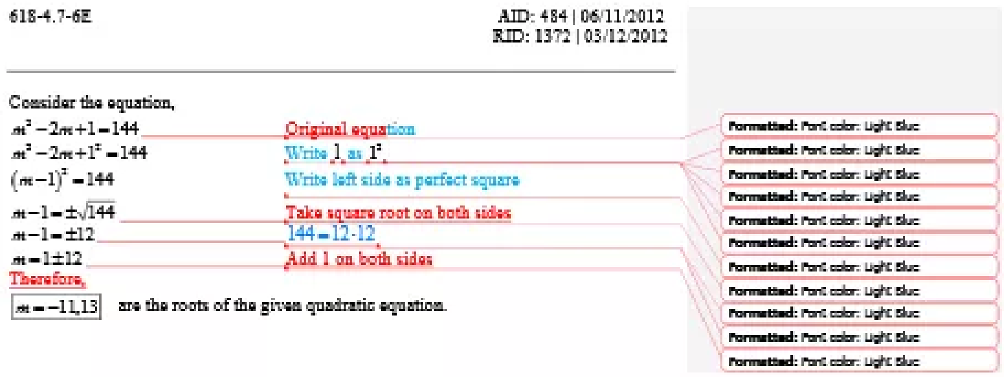
Answer 5q.

To solve the following quadratic equation by finding square roots.

$$s^2 + 12 = 9$$
$$s^2 = -3$$
$$s = \pm\sqrt{-3}$$

$s = \pm\sqrt{3}i$ are the roots of the given quadratic equation.

Answer 6e.



618-4.7-6E

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Consider the equation,

$m^2 - 2m + 1 = 144$ Original equation

$m^2 - 2m + 1^2 = 144$ Write 1 as 1^2.

$(m-1)^2 = 144$ Write left side as perfect square

$m - 1 = \pm\sqrt{144}$ Take square root on both sides

$m - 1 = \pm 12$ $144 = 12 \cdot 12$

$m = 1 \pm 12$ Add 1 on both sides

Therefore,

$m = -11, 13$ are the roots of the given quadratic equation.

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Answer 6gp.

Consider the quadratic expression $x^2 - 9x + c$.
Need to find the value of c that makes the following expression a perfect square trinomial.

Step 1:
Find half the coefficient of x.

That is, $\frac{-9}{2}$

Step 2:

Square the result of step1.

That is, $\left(\frac{-9}{2}\right)^2 = \frac{81}{4}$

Step3:

Replace c with the result of step 2.

That gives $x^2 - 9x + \frac{81}{4}$

The trinomial $x^2 - 9x + c$ is a perfect square when $c = \frac{81}{4}$.

Then

$$x^2 - 9x + \frac{81}{4} = \left(x - \frac{9}{2}\right)\left(x - \frac{9}{2}\right)$$
$$= \left(x - \frac{9}{2}\right)^2$$

Answer 6q.

To solve the following quadratic equation by finding square roots.

$$7x^2 - 4 = -6$$
$$7x^2 = -2$$
$$x^2 = -\frac{2}{7}$$
$$x = \pm\sqrt{-\frac{2}{7}}$$
$$x = \pm\sqrt{\frac{2}{7}}\, i$$ are the roots of the given quadratic equation.

Answer 7e.

First, write the left side as a binomial squared.
$(x-11)^2 = 13$

Take the square root on each side.
$x - 11 = \pm\sqrt{13}$

Now, add 11 to each side.
$$x - 11 + 11 = \pm\sqrt{13} + 11$$
$$x = 11 \pm \sqrt{13}$$

The solutions are $11 + \sqrt{13}$ and $11 - \sqrt{13}$.

Answer 7gp.

First, write the left side in the form $x^2 + bx$. For this, subtract 4 from each side.
$$x^2 + 6x + 4 - 4 = 0 - 4$$
$$x^2 + 6x = -4$$

Square half the coefficient of x.
$$\left(\frac{6}{2}\right)^2 = 3^2 = 9$$

Add 9 to each side of the equation.
$x^2 + 6x + 9 = -4 + 9$

Write the left side as a binomial squared and simplify.
$$(x+3)^2 = -4 + 9$$
$$(x+3)^2 = 5$$

Now, take the square root on each side.
$x + 3 = \pm\sqrt{5}$

Subtract 3 from each side to solve for x.
$$x + 3 - 3 = \pm\sqrt{5} - 3$$
$$x = -3 \pm \sqrt{5}$$

The solutions are $-3 + \sqrt{5}$ and $-3 - \sqrt{5}$.

CHECK

Substitute each solution in the original equation to verify that it is correct.

$-3+\sqrt{5}$	$-3-\sqrt{5}$
$x^2+6x+4=0$	$x^2+6x+4=0$
$\left(-3+\sqrt{5}\right)^2+6\left(-3+\sqrt{5}\right)+4 \stackrel{?}{=} 0$	$\left(-3-\sqrt{5}\right)^2+6\left(-3-\sqrt{5}\right)+4 \stackrel{?}{=} 0$
$9+5-6\sqrt{5}-18+6\sqrt{5}+4 \stackrel{?}{=} 0$	$9+5+6\sqrt{5}-18-6\sqrt{5}+4 \stackrel{?}{=} 0$
$0=0$ ✓	$0=0$ ✓

The solutions check.

Answer 7q.

To write the following expression as a complex number in standard form.

$$(5-3i)+(-2+5i)$$
$$=(5+(-2))+(-3+5)i$$
$$=3+2i$$

Answer 8e.

Consider the equation

$x^2-18x+81=5$	original equation
$x^2-2x(9)+9^2=5$	Write 81 as 9^2
$(x-9)^2=5$	Write left side as perfect square
$x-9=\pm\sqrt{5}$	Take square root on both sides
$x=9\pm\sqrt{5}$	Add 9 on both sides and solve for x

Therefore roots of equation are,

$\boxed{x=9\pm\sqrt{5}}$.

Answer 8gp.

Consider the quadratic equation $x^2-10x+8=0$.

Need to solve the quadratic equation by completing the square.

$x^2-10x+8=0$	Write original equation
$x^2-10x=-8$	Write left side in the form of x^2+bx
$x^2-10x+25=-8+25$	Add $\left(-\frac{10}{2}\right)^2=(-5)^2=25$ to each side
$x^2-10x+25=17$	
$(x-5)^2=17$	Write left side as a binomial square
$x-5=\pm\sqrt{17}$	Apply square root on each side
$x=5\pm\sqrt{17}$	Add 5 on each side

Therefore,

The solutions of the equation $x^2-10x+8=0$ are $\boxed{x=5\pm\sqrt{17}}$.

Answer 8q.

To write the following expression as a complex number in standard form.

$$(-2+9i)-(7+8i)$$
$$=(-2-7)+(9-8)i$$
$$=-9+i$$

Answer 9e.

First, write the left side as a binomial squared.

$(t+4)^2=45$

Take the square root on each side.

$t+4 = \pm\sqrt{45}$

Simplify the radical.

$t+4 = \pm\sqrt{9\cdot 5}$

$t+4 = \pm 3\sqrt{5}$

Now, subtract 4 from each side to solve for t.

$t+4-4 = \pm 3\sqrt{5}-4$

$t = -4 \pm 3\sqrt{5}$

The solutions are $-4+3\sqrt{5}$ and $-4-3\sqrt{5}$.

Answer 9gp.

First, divide each term of the given equation by 2.

$$\frac{2n^2}{2}-\frac{4n}{2}-\frac{14}{2} = \frac{0}{2}$$
$$n^2-2n-7 = 0$$

Write the left side in the form x^2+bx.

$n^2-2n-7+7 = 0+7$

$n^2-2n = 7$

Square half the coefficient of x.

$$\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$$

Add 1 to each side of the equation.

$n^2-2n+1=7+1$

Write the left side as a binomial squared and simplify.

$(n-1)^2 = 7+1$

$(n-1)^2 = 8$

Now, take the square root on each side.

$n - 1 = \pm\sqrt{8}$

Add 1 to each side to solve for n.

$n - 1 + 1 = \pm\sqrt{8} + 1$

$n = 1 \pm \sqrt{8}$

Simplify the radical.

$n = 1 \pm \sqrt{4 \cdot 2}$

$n = 1 \pm 2\sqrt{2}$

The solutions are $1 + 2\sqrt{2}$ and $1 - 2\sqrt{2}$.

Answer 9q.

To write the following expression as a complex number in standard form.

$3i(7-9i)$

$= 21i - 27i^2$

$= 21i - 27(-1)$ (Since $i^2 = -1$)

$= 27 + 21i$

Answer 10e.

Consider the equation

$4u^2 + 4u + 1 = 75$ Original equation

$(2u)^2 + 2(2u)(1) + 1^2 = 75$ Write left side terms as square terms

$(2u+1)^2 = 75$ Write left side as a perfect square

$2u + 1 = \pm\sqrt{75}$ Take square root on both sides

Simplify the expression

$2u + 1 = \pm\sqrt{25} \cdot \sqrt{3}$

$2u + 1 = \pm 5\sqrt{3}$ $25 = 5 \cdot 5$

$2u = -1 \pm 5\sqrt{3}$ Add -1 on both sides

$u = \frac{-1 \pm 5\sqrt{3}}{2}$ Divide by 2

Therefore, the roots of the quadratic equation are $\boxed{u = \frac{-1 \pm 5\sqrt{3}}{2}}$.

Answer 10gp.

Consider the quadratic equation $3x^2+12x-18=0$.
Need to solve the quadratic equation by completing the square.

$3x^2+12x-18=0$	Write original equation
$x^2+4x-6=0$	Divide each side by the coefficient of x^2 3
$x^2+4x=6$	Write left side in the form of x^2+bx
$x^2+4x+4=6+4$	Add $\left(\frac{4}{2}\right)^2=(2)^2=4$ to each side
$x^2+4x+4=10$	
$(x+2)^2=10$	Write left side as a binomial square
$x+2=\pm\sqrt{10}$	Apply square root on each side
$x=-2\pm\sqrt{10}$	Subtract 2 from each side

Therefore,

The solutions of the equation $3x^2+12x-18=0$ are $\boxed{x=-2+\sqrt{10},-2-\sqrt{10}}$.

Answer 10q.

To write the following expression as a complex number in standard form.

$$(8-3i)(-6-10i)$$
$$=-48-80i+18i+30i^2$$
$$=-48+(-80+18)i+30(-1) \qquad (\text{Since } i^2=-1)$$
$$=-48+(-62)i-30$$
$$=-48-30+(-62)i$$
$$=-78-62i$$

Answer 11e.

First, write the left side as a binomial squared.

$(3x-2)^2=-3$

Take the square root on each side.

$3x-2=\pm\sqrt{-3}$

Write the radical in terms of i. We know that $i=\sqrt{-1}$. Thus,

$3x-2=\pm\sqrt{-1\cdot 3}$

$3x-2=\pm i\sqrt{3}$

Now, add 2 to each side.

$3x-2+2=\pm i\sqrt{3}+2$

$3x=2\pm i\sqrt{3}$

Finally, divide each side by 3.

$$\frac{3x}{3} = \frac{2 \pm i\sqrt{3}}{3}$$
$$x = \frac{2 \pm i\sqrt{3}}{3}$$

The solutions are $\frac{2 + i\sqrt{3}}{3}$ and $\frac{2 - i\sqrt{3}}{3}$.

Answer 11gp.

First, apply the distributive property.
$6x^2 + 48x = 12$

Divide each term of the given equation by 6.

$$\frac{6x^2}{6} + \frac{48x}{6} = \frac{12}{6}$$
$$x^2 + 8x = 2$$

Square half the coefficient of x.

$$\left(\frac{8}{2}\right)^2 = 4^2 = 16$$

Add 16 to each side of the equation.
$x^2 + 8x + 16 = 2 + 16$

Write the left side as a binomial squared and simplify.

$$(x+4)^2 = 2 + 16$$
$$(x+4)^2 = 18$$

Now, take the square root on each side.
$x + 4 = \pm\sqrt{18}$

Subtract 4 from each side to solve for x.

$$x + 4 - 4 = \pm\sqrt{18} - 4$$
$$x = -4 \pm \sqrt{18}$$

Simplify the radical.

$$x = -4 \pm \sqrt{9 \cdot 2}$$
$$x = -4 \pm 3\sqrt{2}$$

The solutions are $-4 + 3\sqrt{2}$ and $-4 - 3\sqrt{2}$.

Answer 11q.

To write the following expression as a complex number in standard form.

$$\frac{4i}{-6-11i}$$

$$= \frac{4i(-6+11i)}{(-6-11i)(-6+11i)} \quad \text{Rationalizing numerator and denominator by } (-6+11i)$$

$$= \frac{-24i+44i^2}{(-6)^2-(11)^2 i^2}$$

$$= \frac{-24i+44(-1)}{36-121(-1)} \qquad (\text{Since } i^2=-1)$$

$$= \frac{-44-24i}{157}$$

Answer 12e.

Consider the equation,

$x^2-4x+4=-1$ Original equation

$x^2-2x(2)+2^2=-1$ Write left side terms as square terms

$(x-2)^2=-1$ Write left side as perfect square

$x-2=\pm\sqrt{-1}$ Take square root on both sides

$x-2=\pm i$ $i \cdot i=-1$

$x=2\pm i$ Add 2 on both sides

Therefore, the roots of the equation are $\boxed{x=2\pm i}$.

The correct choice is A.

Answer 12gp.

Consider the quadratic equation $4p(p-2)=100$

Need to solve the quadratic equation by completing the square.

$4p(p-2)=100$ Write the original equation

$p(p-2)=25$ Divide by 4 on each side

$p^2-2p=25$ Write left side in the form of x^2+bx

$p^2-2p+1=25+1$ Add $\left(\frac{-2}{2}\right)^2=(-1)^2=1$ to each side

$p^2-2p+1=26$

$(p-1)^2=26$ Write left side as a binomial square

$p-1=\pm\sqrt{26}$ Apply square root on each side

$p=1\pm\sqrt{26}$ Add 1 on each side

Therefore,

The solutions of the equation $4p(p-2)=100$ are $\boxed{p=1+\sqrt{26},1-\sqrt{26}}$.

Answer 12q.

To write the following expression as a complex number in standard form.

$$\frac{3-2i}{-8+5i}$$

$$=\frac{(3-2i)(-8-5i)}{(-8+5i)(-8-5i)}$$ Rationalizing numerator and denominator by $(-8-5i)$

$$=\frac{-24-15i+16i+10i^2}{(-8)^2-5^2i^2}$$

$$=\frac{-24+(-15+16)i+10(-1)}{64-25(-1)}$$ (Since $i^2=-1$)

In continuation of the above step

$$=\frac{-24+(1)i-10}{64-25(-1)}$$

$$=\frac{-24-10+(1)i}{64-25(-1)}$$

$$=\frac{-34+i}{89}$$

Answer 13e.

STEP 1 First, find half the coefficient of x. In the given expression, the coefficient of x is 6.

$$\frac{6}{2}=3$$

STEP 2 Square half the coefficient of x.

$$3^2=9$$

STEP 3 Replace c with 9 in the given expression.

$$x^2+6x+c=x^2+6x+9$$

The expression x^2+6x+c is a perfect square trinomial when c is 9.

Factor using the special factoring pattern $a^2+2ab+b^2=(a+b)^2$.

$$x^2+6x+9 = x^2+2(x)(3)+3^2$$

$$= (x+3)^2$$

Answer 13gp.

First, prepare to complete the square.

$$y+? = (x^2-8x+?)+17$$

Square half the coefficient of x.

$$\left(\frac{-8}{2}\right)^2=(-4)^2=16$$

Now, complete the square. For this, add 16 to each side of the equation.
$y + 16 = (x^2 - 8x + 16) + 17$

Write $x^2 - 8x + 16$ as a binomial squared.
$y + 16 = (x - 4)^2 + 17$

Solve for y. For this, subtract 16 from each side.

$$y + 16 - 16 = (x - 4)^2 + 17 - 16$$
$$y = (x - 4)^2 + 1$$

A quadratic function in the form $y = a(x - h)^2 + k$ is said to be in vertex form, where (h, k) is the vertex.

On comparing the equation obtained with the vertex form, we get $h = 4$, and $k = 1$. Thus, the vertex of the given function's graph is (4, 1).

Answer 13q.

To write the following quadratic function in vertex form and identify the vertex.

$$y = x^2 - 4x + 9$$
$$x^2 - 4x = y - 9$$

In order to complete the square in $x^2 + bx = c$.

We need to add $\left(\frac{b}{2}\right)^2$ both the sides.

We get $x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$.

In continuation of the above step

$$x^2 - 4x + \left(-\frac{4}{2}\right)^2 = y - 9 + \left(-\frac{4}{2}\right)^2$$
$$x^2 - 4x + 4 = y - 5$$
$$(x - 2)^2 = y - 5$$
$$y = (x - 2)^2 + 5$$

Which is clearly in the vertex form and the vertex is $(2, 5)$.

Answer 14e.

Find the value of c that makes the following expression a perfect square trinomial.
$x^2 + 12x + c$
Compare with $x^2 + bx + c$,
$b = 12$

That is $c = \frac{b^2}{4}$ in order for the quadratic expression to be a perfect square trinomial.

$$c = \frac{12^2}{4}$$
$$= 36$$

Therefore

$x^2 + 12x + c = x^2 + 12x + 36$ Substitute $c = 36$

$= \boxed{(x + 6)^2}$ Write right side expression as perfect square

Answer 14gp.

Consider the quadratic function $y = x^2 + 6x + 3$

Need to write the quadratic function in vertex form and identify the vertex.

$$y = x^2 + 6x + 3 \qquad \text{Write original function}$$

$$y + 9 = (x^2 + 6x + 9) + 3 \qquad \text{Add} \left(\frac{6}{2}\right)^2 = (3)^2 = 9$$

$$y + 9 = (x+3)^2 + 3 \qquad \text{Write } x^2 + 6x + 9 \text{ as a binomial squared}$$

$$y = (x+3)^2 + 3 - 9$$

$$y = (x+3)^2 - 6 \qquad \text{Solve for } y$$

$$y = (x - (-3))^2 + (-6)$$

Therefore,

The vertex form of the function is $y = (x - (-3))^2 + (-6)$ and the vertex is $(-3, -6)$.

Answer 14q.

To write the following quadratic function in vertex form and identify the vertex.

$$y = x^2 + 14x + 45$$

$$x^2 + 14x = y - 45$$

In order to complete the square in $x^2 + bx = c$.

We need to add $\left(\frac{b}{2}\right)^2$ both the sides.

We get $x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$.

In continuation of the above step.

$$x^2 + 14x + \left(\frac{14}{2}\right)^2 = y - 45 + \left(\frac{14}{2}\right)^2$$

$$x^2 + 14x + 49 = y + 4$$

$$(x+7)^2 = y + 4$$

$$y = (x+7)^2 - 4$$

Which is clearly in the vertex form and the vertex is $(-7, -4)$.

Answer 15e.

STEP 1 First, find half the coefficient of x. In the given expression, the coefficient of x is -24.

$$\frac{-24}{2} = -12$$

STEP 2 Square half the coefficient of x.

$(-12)^2 = 144$

STEP 3 Replace c with 144 in the given expression.

$x^2 - 24x + c = x^2 - 24x + 144$

The expression $x^2 - 24x + c$ is a perfect square trinomial when c is 144.

Factor using the special factoring pattern $a^2 - 2ab + b^2 = (a - b)^2$.

$$x^2 - 24x + 144 = x^2 - 2(x)(12) + 12^2$$
$$= (x - 12)^2$$

Answer 15gp.

First, prepare to complete the square.

$$f(x) + ? = (x^2 - 4x + ?) - 4$$

Square half the coefficient of x.

$$\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$

Now, complete the square. For this, add 4 to each side of the equation.

$f(x) + 4 = (x^2 - 4x + 4) - 4$

Write $x^2 - 4x + 4$ as a binomial squared.

$f(x) + 4 = (x - 2)^2 - 4$

Solve for $f(x)$. For this, subtract 4 from each side.

$$f(x) + 4 - 4 = (x - 2)^2 - 4 - 4$$
$$f(x) = (x - 2)^2 - 8$$

A quadratic function in the form $y = a(x - h)^2 + k$ is said to be in vertex form, where (h, k) is the vertex.

On comparing the equation obtained with the vertex form, we get $h = 2$, and $k = -8$. Thus, the vertex of the given function's graph is $(2, -8)$.

Answer 15q.

To write the following quadratic function in vertex form and identify the vertex.

$$f(x) = x^2 - 10x + 17$$
$$x^2 - 10x = f(x) - 17$$

In order to complete the square in $x^2 + bx = c$.

We need to add $\left(\frac{b}{2}\right)^2$ both the sides.

We get $x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$.

In continuation of the above step.

$$x^2-10x+\left(-\frac{10}{2}\right)^2=f(x)-17+\left(-\frac{10}{2}\right)^2$$
$$x^2-10x+25=f(x)+8$$
$$(x-5)^2=f(x)+8$$
$$f(x)=(x-5)^2-8$$

Which is clearly in the vertex form and the vertex is $(5,-8)$.

Answer 16e.

Find the value of c that makes the following expression a perfect square trinomial.

$x^2-30x+c$

Compare with x^2+bx+c,

$b=-30$

Take $c=\frac{b^2}{4}$ in order for the quadratic expression to be a perfect square trinomial.

$$c=\frac{(-30)^2}{4}$$
$$=225$$

Therefore

$x^2-30x+c=x^2-30x+225$ — Substitute $c=225$

$=\boxed{(x-15)^2}$ — Write it as perfect square

Answer 16gp.

Consider the height of the baseball is $y=-16t^2+80t+2$.

Then need to find the maximum height of the baseball.

In order to find the maximum height,

Need to write the given quadratic function in vertex form.

$y=-16t^2+80t+2$

$y=-16(t^2-5t)+2$ — Factor -16 from first two terms

$y=-16\left(t^2-5t+\left(-\frac{5}{2}\right)^2\right)+\left(2+16\left(-\frac{5}{2}\right)^2\right)$ — Add and subtract $16\left(-\frac{5}{2}\right)^2$

$y=-16\left(t-\frac{5}{2}\right)^2+102$ — Simplify

Therefore,

The vertex form of the function is $y=-16\left(t-\frac{5}{2}\right)^2+102$ and the vertex is $\left(\frac{5}{2},102\right)$.

So, the maximum height of the baseball is 102 feet.

Answer 16q.

To write the following quadratic function in vertex form and identify the vertex.

$$g(x)=x^2-2x-7$$

$$x^2-2x=g(x)+7$$

In order to complete the square in $x^2+bx=c$.

We need to add $\left(\frac{b}{2}\right)^2$ both the sides.

We get $x^2+bx+\left(\frac{b}{2}\right)^2=c+\left(\frac{b}{2}\right)^2$.

In continuation of the above step.

$$x^2-2x+\left(-\frac{2}{2}\right)^2=g(x)+7+\left(-\frac{2}{2}\right)^2$$

$$x^2-2x+1=g(x)+8$$

$$(x-1)^2=g(x)+8$$

$$g(x)=(x-1)^2-8$$

Which is clearly in the vertex form and the vertex is $(1,-8)$.

Answer 17e.

STEP 1 First, find half the coefficient of x. In the given expression, the coefficient of x is -2.

$$\frac{-2}{2}=-1$$

STEP 2 Square half the coefficient of x.

$$(-1)^2=1$$

STEP 3 Replace c with 1 in the given expression.

$$x^2-2x+c=x^2-2x+1$$

The expression x^2-2x+c is a perfect square trinomial when c is 1.

Factor using the special factoring pattern $a^2 - 2ab + b^2 = (a-b)^2$.

$$x^2 - 2x + 1 = x^2 - 2(x)(1) + 1^2$$
$$= (x-1)^2$$

Answer 17q.

To write the following quadratic function in vertex form and identify the vertex.

$$y = x^2 + x + 1$$
$$x^2 + x = y - 1$$

In order to complete the square in $x^2 + bx = c$.

We need to add $\left(\frac{b}{2}\right)^2$ both the sides.

We get $x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$.

In continuation of the above step

$$x^2 + x + \left(\frac{1}{2}\right)^2 = y - 1 + \left(\frac{1}{2}\right)^2$$
$$\left(x + \frac{1}{2}\right)^2 = y - \frac{3}{4}$$
$$y = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

Which is clearly in the vertex form and the vertex is $\left(-\frac{1}{2}, \frac{3}{4}\right)$.

Answer 18e.

Find the value of c that makes the following expression a perfect square trinomial.

$x^2 + 50x + c$

Compare with $x^2 + bx + c$,

$b = 50$

Now, $c = \frac{b^2}{4}$ in order for the quadratic expression to be a perfect square trinomial.

$$c = \frac{50^2}{4}$$
$$= 625$$

Therefore

$x^2 + 50x + c = x^2 + 50x + 625$ Substitute $c = 625$

$= \boxed{(x+25)^2}$ Write it as perfect square

Answer 18q.

To write the following quadratic function in vertex form and identify the vertex.

$$y = x^2 + 9x + 19$$
$$x^2 + 9x = y - 19$$

In order to complete the square in $x^2 + bx = c$.

We need to add $\left(\frac{b}{2}\right)^2$ both the sides.

We get $x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$.

In continuation of the above step

$$x^2 + 9x + \left(\frac{9}{2}\right)^2 = y - 19 + \left(\frac{9}{2}\right)^2$$
$$\left(x + \frac{9}{2}\right)^2 = y + \frac{5}{4}$$
$$y = \left(x + \frac{9}{2}\right)^2 - \frac{5}{4}$$

Which is clearly in the vertex form and the vertex is $\left(-\frac{9}{2}, -\frac{5}{4}\right)$.

Answer 19e.

STEP 1 First, find half the coefficient of x. In the given expression, the coefficient of x is 7. Half of 7 is $\frac{7}{2}$.

STEP 2 Square half the coefficient of x.

$$\left(\frac{7}{2}\right)^2 = \frac{49}{4}$$

STEP 3 Replace c with $\frac{49}{4}$ in the given expression.

$$x^2 + 7x + c = x^2 + 7x + \frac{49}{4}$$

The expression $x^2 + 7x + c$ is a perfect square trinomial when c is $\frac{49}{4}$.

Factor using the special factoring pattern $a^2 + 2ab + b^2 = (a + b)^2$.

$$x^2 + 7x + \frac{49}{4} = x^2 + 2(x)\left(\frac{7}{2}\right) + \left(\frac{7}{2}\right)^2$$
$$= \left(x + \frac{7}{2}\right)^2$$

Answer 19q.

Let a student drops a ball from a school roof 45 feet above ground.
We need to find how long is the ball in the air.
Let h be the height of the ball in feet and t be the time in seconds.
Then, $h = -16t^2 + 45$

Put $h = 0$

$$16t^2 = 45$$

$$t^2 = \frac{45}{16}$$

$$t = \pm\sqrt{\frac{45}{16}}$$

$$t = \pm\sqrt{\frac{9}{16}} \cdot \sqrt{5}$$

$$t = \pm\frac{3}{4}\sqrt{5} \qquad \text{(Since } t \not< 0\text{)}$$

$$t = \frac{3}{4}\sqrt{5}$$

Answer 20e.

Find the value of c that makes the following expression a perfect square trinomial.
$x^2 - 13x + c$
Compare with $x^2 + bx + c$,
$b = -13$

Take $c = \frac{b^2}{4}$ in order for the quadratic expression to be a perfect square trinomial.

$$c = \frac{169}{4}$$

Therefore

$$x^2 - 13x + c = x^2 - 13x + \frac{169}{4} \qquad \text{Substitute } c = \frac{169}{4}$$

$$= \boxed{\left(x - \frac{13}{2}\right)^2} \qquad \text{Write right side as a perfect square}$$

Answer 21e.

STEP 1 First, find half the coefficient of x. In the given expression, the coefficient of x is -1. Half of -1 is $-\frac{1}{2}$.

STEP 2 Square half the coefficient of x.

$$\left(-\frac{1}{2}\right)^2 = \frac{1}{4}$$

STEP 3 Replace c with $\frac{1}{4}$ in the given expression.

$$x^2 - x + c = x^2 - x + \frac{1}{4}$$

The expression $x^2 - x + c$ is a perfect square trinomial when c is $\frac{1}{4}$.

Factor using the special factoring pattern $a^2 - 2ab + b^2 = (a - b)^2$.

$$x^2 - x + \frac{1}{4} = x^2 - 2(x)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2$$

$$= \left(x - \frac{1}{2}\right)^2$$

Answer 22e.

Solve the following quadratic equation by completing the square.

$x^2 + 4x = 10$

In order to complete the square in $x^2 + bx = c$.

We need to add $\left(\frac{b}{2}\right)^2$ both the sides.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2.$$

Rewrite equation $x^2 + 4x = 10$ as

$x^2 + 4x + \left(\frac{4}{2}\right)^2 = 10 + \left(\frac{4}{2}\right)^2$ Add $\left(\frac{4}{2}\right)^2$ on both sides

$x^2 + 4x + 4 = 14$ Simplify

$(x+2)^2 = 14$ Write left side as a perfect square

$x + 2 = \pm\sqrt{14}$ Take square root on both sides

$\boxed{x = -2 \pm \sqrt{14}}$ are the roots of the given quadratic equation.

Answer 23e.

The left side of the given equation is already in the form $x^2 + bx$.

First, square half the coefficient of x.

$$\left(\frac{8}{2}\right)^2 = 4^2 = 16$$

Add 16 to each side of the given equation.

$$x^2 + 8x + 16 = -1 + 16$$

Write the left side as a binomial squared and simplify.

$$(x+4)^2 = -1 + 16$$

$$(x+4)^2 = 15$$

Now, take the square root on each side.

$$x + 4 = \pm\sqrt{15}$$

Subtract 4 from each side to solve for x.

$$x + 4 - 4 = \pm\sqrt{15} - 4$$

$$x = -4 \pm \sqrt{15}$$

The solutions are $-4 + \sqrt{15}$ and $-4 - \sqrt{15}$.

CHECK

Substitute each solution in the original equation to verify that it is correct.

$$x = -4 + \sqrt{15}$$

$$x^2 + 8x = -1$$

$$\left(-4+\sqrt{15}\right)^2 + 8\left(-4+\sqrt{15}\right) \stackrel{?}{=} -1$$

$$16 + 15 - 8\sqrt{15} - 32 + 8\sqrt{15} \stackrel{?}{=} -1$$

$$-1 = -1 \checkmark$$

$$x = -4 - \sqrt{15}$$

$$x^2 + 8x = -1$$

$$\left(-4-\sqrt{15}\right)^2 + 8\left(-4-\sqrt{15}\right) \stackrel{?}{=} -1$$

$$16 + 15 + 8\sqrt{15} - 32 - 8\sqrt{15} \stackrel{?}{=} -1$$

$$-1 = -1 \checkmark$$

The solutions check.

Answer 24e.

Solve the following quadratic equation by completing the square.

$$x^2 + 6x - 3 = 0$$

$$x^2 + 6x = 3$$

In order to complete the square in $x^2 + bx = c$.

add $\left(\frac{b}{2}\right)^2$ both the sides.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2.$$

Rewrite equation $x^2+6x=3$ as

$x^2+6x+\left(\frac{6}{2}\right)^2=3+\left(\frac{6}{2}\right)^2$ Add $\left(\frac{6}{2}\right)^2$ on both sides

$x^2+6x+9=12$ Simplify

$(x+3)^2=12$ Write left side as a perfect square

$x+3=\pm\sqrt{12}$ Take square root on both sides

$x+3=\pm\sqrt{4}\cdot\sqrt{3}$

$x+3=\pm2\sqrt{3}$ Simplify

Therefore

$\boxed{x=-3\pm2\sqrt{3}}$ are the roots of the given quadratic equation.

Answer 25e.

First, write the left side in the form x^2+bx. For this, subtract 18 from each side.

$$x^2+12x+18-18 = 0-18$$
$$x^2+12x = -18$$

Square half the coefficient of x.

$$\left(\frac{12}{2}\right)^2 = 6^2 = 36$$

Add 36 to each side of the equation.

$$x^2+12x+36=-18+36$$

Write the left side as a binomial squared and simplify.

$$(x+6)^2 = -18+36$$
$$(x+6)^2 = 18$$

Now, take the square root on each side.

$$x+6=\pm\sqrt{18}$$

Subtract 6 from each side to solve for x.

$$x+6-6 = \pm\sqrt{18}-6$$
$$x = -6\pm\sqrt{18}$$

Simplify the radical.

$$x = -6 \pm \sqrt{9 \cdot 2}$$

$$x = -6 \pm 3\sqrt{2}$$

The solutions are $-6 + 3\sqrt{2}$ and $-6 - 3\sqrt{2}$.

CHECK

Substitute each solution in the original equation to verify that it is correct.

$-6+3\sqrt{2}$

$$x^2 + 12x + 18 = 0$$

$$\left(-6+3\sqrt{2}\right)^2 + 12\left(-6+3\sqrt{2}\right) + 18 \stackrel{?}{=} 0$$

$$36 + 18 - 36\sqrt{2} - 72 + 36\sqrt{2} + 18 \stackrel{?}{=} 0$$

$$0 = 0 \checkmark$$

$-6-3\sqrt{2}$

$$x^2 + 12x + 18 = 0$$

$$\left(-6-3\sqrt{2}\right)^2 + 12\left(-6-3\sqrt{2}\right) + 18 \stackrel{?}{=} 0$$

$$36 + 18 + 36\sqrt{2} - 72 - 36\sqrt{2} + 18 \stackrel{?}{=} 0$$

$$0 = 0 \checkmark$$

The solutions check.

Answer 26e.

Consider,

$x^2 - 18x + 86 = 0$.

$$x^2 - 18x = -86$$

In order to complete the square in $x^2 + bx = c$,

Add $\left(\frac{b}{2}\right)^2$ both the sides.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2.$$

Here, $b = -18$.

$$x^2 - 18x + \left(-\frac{18}{2}\right)^2 = -86 + \left(-\frac{18}{2}\right)^2 \quad \text{Simplify}$$

$$x^2 - 18x + 81 = -86 + 81$$

$$x^2 - 18x + 81 = -5$$

$$(x-9)^2 = -5$$

$$x - 9 = \pm\sqrt{-5}$$

$$x - 9 = \pm\sqrt{5}i$$

$$x = 9 \pm \sqrt{5}i$$

Therefore, the roots of the quadratic equation are $x = \boxed{9+\sqrt{5}i, 9-\sqrt{5}i}$.

Answer 27e.

First, write the left side in the form $x^2 + bx$. For this, subtract 25 from each side.

$$x^2 - 2x + 25 - 25 = 0 - 25$$
$$x^2 - 2x = -25$$

Square half the coefficient of x.

$$\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$$

Add 1 to each side of the equation.

$$x^2 - 2x + 1 = -25 + 1$$

Write the left side as a binomial squared and simplify.

$$(x-1)^2 = -25 + 1$$
$$(x-1)^2 = -24$$

Now, take the square root on each side.

$$x - 1 = \pm\sqrt{-24}$$

Add 1 to each side to solve for x.

$$x - 1 + 1 = \pm\sqrt{-24} + 1$$
$$x = 1 \pm \sqrt{-24}$$

Write in terms of the imaginary unit i.

$$x = 1 \pm \sqrt{-1 \cdot 24}$$
$$x = 1 \pm i\sqrt{24}$$

Simplify the radical.

$$x = 1 \pm i\sqrt{4 \cdot 6}$$
$$x = 1 \pm 2i\sqrt{6}$$

The solutions are $1 + 2i\sqrt{6}$ and $1 - 2i\sqrt{6}$.

Answer 28e.

Consider,

$2k^2 + 16k = -12$.

Divide both sides by 2.

$$k^2 + 8k = -6$$

In order to complete the square in $x^2 + bx = c$.

Add $\left(\frac{b}{2}\right)^2$ both the sides.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2.$$

Here, $b = 8$.

$$k^2 + 8k + \left(\frac{8}{2}\right)^2 = -6 + \left(\frac{8}{2}\right)^2$$ Simplify

$$k^2 + 8k + 16 = -6 + 16$$

$$k^2 + 8k + 16 = 10$$

$$(k + 4)^2 = 10$$

$$k + 4 = \pm\sqrt{10}$$

Therefore, the roots of the equation are $k = \boxed{-4 + \sqrt{10}, -4 - \sqrt{10}}$.

Answer 29e.

First, write the left side in the form $x^2 + bx$. For this, divide each term of the given equation by 3.

$$\frac{3x^2}{3} + \frac{42x}{3} = \frac{-24}{3}$$

$$x^2 + 14x = -8$$

Square half the coefficient of x.

$$\left(\frac{14}{2}\right)^2 = 7^2 = 49$$

Add 49 to each side of the equation.

$$x^2 + 14x + 49 = -8 + 49$$

Write the left side as a binomial squared and simplify.

$$(x + 7)^2 = -8 + 49$$

$$(x + 7)^2 = 41$$

Now, take the square root on each side.

$$x + 7 = \pm\sqrt{41}$$

Subtract 7 from each side to solve for x.

$$x + 7 - 7 = \pm\sqrt{41} - 7$$

$$x = -7 \pm \sqrt{41}$$

The solutions are $-7 + \sqrt{41}$ and $-7 - \sqrt{41}$.

Answer 30e.

Consider,

$4x^2 - 40x - 12 = 0$.

Divide both sides by 4.

$x^2 - 10x - 3 = 0$

$x^2 - 10x = 3$

In order to complete the square in $x^2 + bx = c$.

Add $\left(\frac{b}{2}\right)^2$ both the sides.

$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$.

Here, $b = -10$.

$x^2 - 10x + \left(-\frac{10}{2}\right)^2 = 3 + \left(-\frac{10}{2}\right)^2$ Simplify

$x^2 - 10x + 25 = 3 + 25$

$x^2 - 10x + 25 = 28$

$(x-5)^2 = 28$

$x - 5 = \pm\sqrt{28}$

$x - 5 = \pm\sqrt{4} \cdot \sqrt{7}$

$x - 5 = \pm 2\sqrt{7}$

$x = 5 \pm 2\sqrt{7}$

Therefore, the roots of the equation are $x = \boxed{5+2\sqrt{7}, 5-2\sqrt{7}}$.

Answer 31e.

First, divide each term of the given equation by 3.

$\frac{3s^2}{3} + \frac{6s}{3} + \frac{9}{3} = \frac{0}{3}$

$s^2 + 2s + 3 = 0$

Write the left side in the form $x^2 + bx$.

$s^2 + 2s + 3 - 3 = 0 - 3$

$s^2 + 2s = -3$

Square half the coefficient of x.

$$\left(\frac{2}{2}\right)^2 = 1^2 = 1$$

Add 1 to each side of the equation.

$$s^2 + 2s + 1 = -3 + 1$$

Write the left side as a binomial squared and simplify.

$$(s+1)^2 = -3 + 1$$

$$(s+1)^2 = -2$$

Now, take the square root on each side.

$$s + 1 = \pm\sqrt{-2}$$

Write in terms of i. We know that $i = \sqrt{-1}$. Thus,

$$s + 1 = \pm\sqrt{-1 \cdot 2}$$

$$s + 1 = \pm i\sqrt{2}$$

Subtract 1 from each side to solve for s.

$$s + 1 - 1 = \pm i\sqrt{2} - 1$$

$$s = -1 \pm i\sqrt{2}$$

The solutions are $-1 + i\sqrt{2}$ and $-1 - i\sqrt{2}$.

Answer 32e.

Consider,

$7t^2 + 28t + 56 = 0$.

Divide both sides by 7.

$$t^2 + 4t + 8 = 0$$

$$t^2 + 4t = -8$$

In order to complete the square in $x^2 + bx = c$.

Add $\left(\frac{b}{2}\right)^2$ both the sides.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2.$$

Here, $b = 4$.

$$t^2 + 4t + \left(\frac{4}{2}\right)^2 = -8 + \left(\frac{4}{2}\right)^2 \quad \text{Simplify}$$

$$t^2 + 4t + 4 = -8 + 4$$

$$t^2 + 4t + 4 = -4$$

$$(t+2)^2 = -4$$

$$t + 2 = \pm\sqrt{-4}$$

$$t + 2 = \pm 2i$$

$$t = -2 \pm 2i$$

Therefore, the roots of the equation are $t = \boxed{-2+2i, -2-2i}$.

Answer 33e.

First, divide each term of the given equation by 6.

$$\frac{6r^2}{6} + \frac{6r}{6} + \frac{12}{6} = \frac{0}{6}$$

$$r^2 + r + 2 = 0$$

Write the left side in the form $x^2 + bx$.

$$r^2 + r + 2 - 2 = 0 - 2$$

$$r^2 + r = -2$$

Square half the coefficient of x.

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Add $\frac{1}{4}$ to each side of the equation.

$$r^2 + r + \frac{1}{4} = -2 + \frac{1}{4}$$

Write the left side as a binomial squared and simplify.

$$\left(r + \frac{1}{2}\right)^2 = -2 + \frac{1}{4}$$

$$\left(r + \frac{1}{2}\right)^2 = -\frac{7}{4}$$

Now, take the square root on each side.

$$r + \frac{1}{2} = \pm\sqrt{-\frac{7}{4}}$$

Write in terms of i. We know that $i = \sqrt{-1}$. Thus,

$$r + \frac{1}{2} = \pm\sqrt{-1 \cdot \frac{1}{4} \cdot 7}$$

$$r + \frac{1}{2} = \pm\frac{i\sqrt{7}}{2}$$

Subtract $\frac{1}{2}$ from each side to solve for r.

$$r + \frac{1}{2} - \frac{1}{2} = \pm\frac{i\sqrt{7}}{2} - \frac{1}{2}$$

$$r = -\frac{1}{2} \pm \frac{i\sqrt{7}}{2}$$

The solutions are $-\frac{1}{2} + \frac{i\sqrt{7}}{2}$ and $-\frac{1}{2} - \frac{i\sqrt{7}}{2}$.

Answer 34e.

Consider,

$x^2 + 10x + 8 = -5$.

Subtract both sides by 8.

$$x^2 + 10x + 8 - 8 = -5 - 8$$

$$x^2 + 10x = -13$$

In order to complete the square in $x^2 + bx = c$.

Add $\left(\frac{b}{2}\right)^2$ both the sides.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2.$$

Here, $b = 10$.

$$x^2 + 10x + \left(\frac{10}{2}\right)^2 = -13 + \left(\frac{10}{2}\right)^2$$ Simplify

$$x^2 + 10x + 25 = -13 + 25$$
$$x^2 + 10x + 25 = 12$$
$$(x+5)^2 = 12$$
$$x + 5 = \pm\sqrt{12}$$
$$x + 5 = \pm\sqrt{4}\cdot\sqrt{3}$$
$$x + 5 = \pm 2\sqrt{3}$$
$$x = -5 \pm 2\sqrt{3}$$

Therefore, the roots of the equation are $x = \boxed{-5+2\sqrt{3}, -5+2\sqrt{3}}$.

Answer 35e.

The area of the rectangle is given as 50. From the figure, we note that the length is $x + 10$, and the width is x.

Use the formula for the area of a rectangle to write an equation.
$x(x + 10) = 50$

First, we have to write the left side in the form $x^2 + bx$. For this, apply the distributive property.

$$x(x) + x(10) = 50$$
$$x^2 + 10x = 50$$

Now, square half the coefficient of x.

$$\left(\frac{10}{2}\right)^2 = 5^2$$
$$= 25$$

Add 25 to each side of the equation.
$x^2 + 10x + 25 = 50 + 25$

Write the left side as a binomial squared and simplify.

$$(x+5)^2 = 50 + 25$$
$$(x+5)^2 = 75$$

Take the square root on each side.

$x + 5 = \pm\sqrt{75}$

Subtract 5 from each side to solve for x.

$$x + 5 - 5 = \pm\sqrt{75} - 5$$
$$x = -5 \pm \sqrt{75}$$

Simplify the radical.

$$x = -5 \pm \sqrt{25 \cdot 3}$$
$$x = -5 \pm 5\sqrt{3}$$

Since length cannot be negative, discard the solution $x = -5 - 5\sqrt{3}$. Therefore, the value of x is $-5 + 5\sqrt{3}$.

Answer 36e.

Consider,

The area of a parallelogram of base $x+6$ and height x is 48.

That is, $x(x+6) = 48$

$$x^2 + 6x = 48.$$

In order to complete the square in $x^2 + bx = c$.

Add $\left(\frac{b}{2}\right)^2$ both the sides.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2.$$

Here, $b = 6$.

$$x^2 + 6x + \left(\frac{6}{2}\right)^2 = 48 + \left(\frac{6}{2}\right)^2 \qquad \text{Simplify}$$
$$x^2 + 6x + 9 = 48 + 9$$
$$x^2 + 6x + 9 = 57$$
$$(x+3)^2 = 57$$
$$x + 3 = \pm\sqrt{57}$$
$$x + 3 = \sqrt{57} \qquad (\text{Since } x \text{ can not be negative})$$
$$x = \boxed{-3 + \sqrt{57}}.$$

Answer 37e.

The area of the triangle is given as 40. From the figure, we note that the base is $x+4$, and the height is x.

Use the formula for the area of a triangle to write an equation.

$$\frac{1}{2}x(x+4) = 40$$

Multiply each side by 2.

$$2\cdot\frac{1}{2}x(x+4) = 2\cdot 40$$
$$x(x+4) = 80$$

Write the left side in the form x^2+bx. For this, apply the distributive property.

$$x(x)+x(4) = 80$$
$$x^2+4x = 80$$

Now, square half the coefficient of x.

$$\left(\frac{4}{2}\right)^2 = 2^2 = 4$$

Add 4 to each side of the equation.

$$x^2+4x+4=80+4$$

Write the left side as a binomial squared and simplify.

$$(x+2)^2 = 80+4$$
$$(x+2)^2 = 84$$

Take the square root on each side.

$$x+2=\pm\sqrt{84}$$

Subtract 2 from each side to solve for x.

$$x+2-2 = \pm\sqrt{84}-2$$
$$x = -2\pm\sqrt{84}$$

Simplify the radical.

$$x = -2\pm\sqrt{4\cdot 21}$$
$$x = -2\pm 2\sqrt{21}$$

Since length cannot be negative, discard the solution $x = -2-2\sqrt{21}$. Therefore, the value of x is $-2+2\sqrt{21}$.

Answer 38e.

Consider the area of a trapezoid with parallel sides of length $3x-1$ and $x+9$ and height x is 20.

That is, $\frac{1}{2}(3x-1+x+9)(x)=20$ Simplify

$$\frac{1}{2}(4x+8)x=20$$
$$2(x+2)x=20$$
$$x^2+2x=10$$

In order to complete the square in $x^2+bx=c$.

Add $\left(\frac{b}{2}\right)^2$ both the sides.

$$x^2+bx+\left(\frac{b}{2}\right)^2=c+\left(\frac{b}{2}\right)^2.$$

Here, $b=2$.

$$x^2+2x+\left(\frac{2}{2}\right)^2=10+\left(\frac{2}{2}\right)^2 \quad \text{Simplify}$$
$$x^2+2x+1=10+1$$
$$x^2+2x+1=11$$
$$(x+1)^2=11$$
$$x+1=\pm\sqrt{11} \quad \text{(Since } x \text{ can not be negative)}$$
$$x+1=\sqrt{11}$$
$$x=\boxed{-1+\sqrt{11}}.$$

Answer 39e.

First, we have to factor out -16 from the right side.
$h=-16(t^2-5.6t)$

Square half the coefficient of t.

$$\left(-\frac{5.6}{2}\right)^2=(-2.8)^2=7.84$$

Now, complete the square. For this, subtract $16(7.84)$ from each side of the equation.

$$h-16(7.84)=-16(t^2-5.6t)-16(7.84)$$
$$h-125.44=-16(t^2-5.6t+7.84)$$

Write $t^2 - 5.6t + 7.84$ as a binomial squared.
$h - 125.44 = -16(t - 2.8)^2$

Solve for h. For this, add 125.44 to each side.

$$h - 125.44 + 125.44 = -16(t - 2.8)^2 + 125.44$$
$$h = -16(t - 2.8)^2 + 125.44$$

A quadratic function in the form $y = a(x - h)^2 + k$ is said to be in vertex form, where (h, k) is the vertex.

On comparing the equation obtained with the vertex form, we get $a = -16$, $h = 2.8$, and $k = 125.44$. Thus, the vertex of the given function's graph is (2.8, 125.44).

The y-coordinate of the vertex of a graph represents the maximum or minimum value of the function.

Since $a < 0$, the parabola of the given function opens down and it has the maximum value. Therefore, we can conclude that at 2.8 seconds the water will reach a maximum height of 125.44 feet.

Answer 40e.

Consider,
$y = 0.0085x^2 - 1.5x + 120$.
If it is walked x meters per minute, the rate y of energy is modeled by the above equation.
Write the quadratic function in vertex form and identify the vertex.

$$y = 0.0085x^2 - 1.5x + 120$$
$$y = 0.0085\left(x^2 - \frac{3000}{17}x\right) + 120 \quad \left(\text{Add and Subtract } 0.0085\left(\frac{1500}{17}\right)^2\right)$$
$$y = 0.0085\left(x^2 - \frac{3000}{17}x + \left(\frac{1500}{17}\right)^2\right) + \left(120 - 0.0085\left(\frac{1500}{17}\right)^2\right) \quad \text{Simplify}$$
$$y = 0.0085\left(x - \frac{1500}{17}\right)^2 + \frac{915}{17}$$

Which is clearly in the vertex form and the vertex is $\left(\frac{1500}{17}, \frac{915}{17}\right)$.

Therefore, if it is walked $\frac{1500}{17}$ meters per minute, the rate of energy y used is $\frac{915}{17}$ calories per minute.

Answer 41e.

First, prepare to complete the square.

$y + ? = \left(x^2 - 8x + ?\right) + 19$

Square half the coefficient of x.

$\left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

Now, complete the square. For this, add 16 to each side of the equation.

$y + 16 = (x^2 - 8x + 16) + 19$

Write $x^2 - 8x + 16$ as a binomial squared.

$y + 16 = (x - 4)^2 + 19$

Solve for y. For this, subtract 16 from each side.

$$\begin{aligned} y + 16 - 16 &= (x - 4)^2 + 19 - 16 \\ y &= (x - 4)^2 + 3 \end{aligned}$$

A quadratic function in the form $y = a(x - h)^2 + k$ is said to be in vertex form, where (h, k) is the vertex.

On comparing the equation obtained with the vertex form, we get $h = 4$, and $k = 3$. Thus, the vertex of the given function's graph is (4, 3).

Answer 42e.

Consider,

$y = x^2 - 4x - 1$

Write the quadratic function in vertex form and identify the vertex.

$y = x^2 - 4x - 1$

$x^2 - 4x = y + 1.$

In order to complete the square in $x^2 + bx = c$.

Add $\left(\frac{b}{2}\right)^2$ both the sides.

$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2.$

Here, $b = -4$.

$$x^2 - 4x + \left(-\frac{4}{2}\right)^2 = y + 1 + \left(-\frac{4}{2}\right)^2 \quad \text{Simplify}$$

$$x^2 - 4x + 4 = y + 5$$

$$(x-2)^2 = y + 5$$

$$y = (x-2)^2 - 5$$

Which is clearly in the vertex form and the vertex is $\boxed{(2,-5)}$.

Answer 43e.

First, prepare to complete the square.

$$y + ? = \left(x^2 + 12x + ?\right) + 37$$

Square half the coefficient of x.

$$\left(\frac{12}{2}\right)^2 = (6)^2 = 36$$

Now, complete the square. For this, add 36 to each side of the equation.

$$y + 36 = (x^2 + 12x + 36) + 37$$

Write $x^2 + 12x + 36$ as a binomial squared.

$$y + 36 = (x + 6)^2 + 37$$

Solve for y. For this, subtract 36 from each side.

$$y + 36 - 36 = (x + 6)^2 + 37 - 36$$

$$y = (x + 6)^2 + 1$$

A quadratic function in the form $y = a(x - h)^2 + k$ is said to be in vertex form, where (h, k) is the vertex.

On comparing the equation obtained with the vertex form, we get $h = -6$, and $k = 1$. Thus, the vertex of the given function's graph is $(-6, 1)$.

Answer 44e.

Consider,

$y = x^2 + 20x + 90$

Write the following quadratic function in vertex form and identify the vertex.

$$y = x^2 + 20x + 90$$

$$x^2 + 20x = y - 90$$

In order to complete the square in $x^2 + bx = c$.

Add $\left(\frac{b}{2}\right)^2$ both the sides.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2.$$

Here, $b = 20$.

$$x^2 + 20x + \left(\frac{20}{2}\right)^2 = y - 90 + \left(\frac{20}{2}\right)^2 \quad \text{Simplify}$$

$$x^2 + 20x + 100 = y + 10$$

$$(x+10)^2 = y + 10$$

$$y = (x+10)^2 - 10$$

Which is clearly in the vertex form and the vertex is $\boxed{(-10, -10)}$.

Answer 45e.

First, prepare to complete the square.

$f(x) + ? = \left(x^2 - 3x + ?\right) + 4$

Square half the coefficient of x.

$$\left(\frac{-3}{2}\right)^2 = \frac{9}{4}$$

Now, complete the square. For this, add $\frac{9}{4}$ to each side of the equation.

$$f(x) + \frac{9}{4} = \left(x^2 - 3x + \frac{9}{4}\right) + 4$$

Write $x^2 - 3x + \frac{9}{4}$ as a binomial squared.

$$f(x) + \frac{9}{4} = \left(x - \frac{3}{2}\right)^2 + 4$$

Solve for $f(x)$. For this, subtract $\frac{9}{4}$ from each side.

$$f(x) + \frac{9}{4} - \frac{9}{4} = \left(x - \frac{3}{2}\right)^2 + 4 - \frac{9}{4}$$

$$f(x) = \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$$

A quadratic function in the form $y = a(x - h)^2 + k$ is said to be in vertex form, where (h, k) is the vertex.

On comparing the equation obtained with the vertex form, we get $h = \frac{3}{2}$ and $k = \frac{7}{4}$.

Thus, the vertex of the given function's graph is $\left(\frac{3}{2}, \frac{7}{4}\right)$.

Answer 46e.

We need to write the following quadratic function in vertex form and identify the vertex.

$$g(x) = x^2 + 7x + 2$$

$$\Rightarrow \quad x^2 + 7x = g(x) - 2$$

In order to complete the square in $x^2 + bx = c$.

We need to add $\left(\frac{b}{2}\right)^2$ both the sides.

We get $x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$.

$$\Rightarrow \quad x^2 + 7x + \left(\frac{7}{2}\right)^2 = g(x) - 2 + \left(\frac{7}{2}\right)^2$$

$$\Rightarrow \quad \left(x + \frac{7}{2}\right)^2 = g(x) + \frac{41}{4}$$

$$\Rightarrow \quad g(x) = \left(x + \frac{7}{2}\right)^2 - \frac{41}{4}$$

Which is clearly in the vertex form and the vertex is $\left(-\frac{7}{2}, -\frac{41}{4}\right)$.

Answer 47e.

First, we have to factor out 2 from the first two terms on the right side.
$y = 2(x^2 + 12x) + 25$

Square half the coefficient of x.
$$\left(\frac{12}{2}\right)^2 = 6^2 = 36$$

Now, complete the square. For this, add 2(36) to each side of the equation.
$$y + 2(36) = 2\left(x^2 + 12x\right) + 25 + 2(36)$$
$$y + 72 = 2\left(x^2 + 12x + 36\right) + 25$$

Write $x^2 + 12x + 36$ as a binomial squared.
$y + 72 = 2(x + 6)^2 + 25$

Solve for y. For this, subtract 72 from each side.
$$y + 72 - 72 = 2(x + 6)^2 + 25 - 72$$
$$y = 2(x + 6)^2 - 47$$

A quadratic function in the form $y = a(x - h)^2 + k$ is said to be in vertex form, where (h, k) is the vertex.

On comparing the equation obtained with the vertex form, we get $h = -6$, and $k = -47$. Thus, the vertex of the given function's graph is $(-6, -47)$.

Answer 48e.

We need to write the following quadratic function in vertex form and identify the vertex.
$$y = 5x^2 + 10x + 7$$
$$\Rightarrow \quad y - 7 = 5\left(x^2 + 2x\right)$$
$$\Rightarrow \quad x^2 + 2x = \frac{y-7}{5}$$
In order to complete the square in $x^2 + bx = c$.

We need to add $\left(\frac{b}{2}\right)^2$ both the sides.

We get $x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$.
$$\Rightarrow \quad x^2 + 2x + \left(\frac{2}{2}\right)^2 = \frac{y-7}{5} + \left(\frac{2}{2}\right)^2$$
$$\Rightarrow \quad x^2 + 2x + 1 = \frac{y-7}{5} + 1$$
$$\Rightarrow \quad (x+1)^2 = \frac{y-2}{5}$$
$$\Rightarrow \quad y = 5(x+1)^2 + 2$$
Which is clearly in the vertex form and the vertex is $(-1, 2)$.

Answer 49e.

First, we have to factor out 2 from the first two terms on the right side.

$y = 2(x^2 - 14x) + 99$

Square half the coefficient of x.

$$\left(\frac{-14}{2}\right)^2 = (-7)^2 = 49$$

Now, complete the square. For this, add 2(49) to each side of the equation.

$$y + 2(49) = 2(x^2 - 14x) + 99 + 2(49)$$

$$y + 98 = 2(x^2 - 14x + 49) + 99$$

Write $x^2 - 14x + 49$ as a binomial squared.

$y + 98 = 2(x - 7)^2 + 99$

Solve for y. For this, subtract 98 from each side.

$$y + 98 - 98 = 2(x - 7)^2 + 99 - 98$$

$$y = 2(x - 7)^2 + 1$$

A quadratic function in the form $y = a(x - h)^2 + k$ is said to be in vertex form, where (h, k) is the vertex.

On comparing the equation obtained with the vertex form, we get $h = 7$, and $k = 1$. Thus, the vertex of the given function's graph is (7, 1).

Answer 50e.

In solving the given equation, there is an error in the last step.

It is given, $x = -5 \pm \sqrt{12}$

$\Rightarrow \quad x = -5 \pm 4\sqrt{3}$ Which is wrong.

Because $x = -5 \pm \sqrt{12}$

$\Rightarrow \quad x = -5 \pm \sqrt{4 \cdot 3}$

$\Rightarrow \quad x = -5 \pm \sqrt{4} \cdot \sqrt{3}$

$\Rightarrow \quad x = -5 \pm 2\sqrt{3}$ is the correct solution.

Answer 51e.

In the given case, 4(9) is added to the left side. For balancing an equation, we have to add the same quantity on each side of the equation. Therefore, we have to add 4(9) to the right side also. The error is that 9 is added to the right side and not 4(9).

In order to correct the error, first add 4(9) to each side of the equation given in step 2.

$4(x^2 + 6x + 9) = 11 + 4(9)$

Simplify.

$4(x^2 + 6x + 9) = 47$

Write $x^2 + 6x + 9$ as a binomial squared.

$4(x + 3)^2 = 47$

Divide each side by 4.

$$(x + 3)^2 = \frac{47}{4}$$

Now, take the square root on each side.

$$x + 3 = \pm\sqrt{\frac{47}{4}}$$

$$x + 3 = \pm\frac{\sqrt{47}}{2}$$

Subtract 3 from each side to solve for x.

$$x + 3 - 3 = \pm\frac{\sqrt{47}}{2} - 3$$

$$x = -3 \pm \frac{\sqrt{47}}{2}$$

The solutions are $-3 + \frac{\sqrt{47}}{2}$ and $-3 - \frac{\sqrt{47}}{2}$.

Answer 52e.

We need to solve the following quadratic equation by completing the square.

$$x^2+9x+20=0$$

$$\Rightarrow \quad x^2+9x=-20$$

In order to complete the square in $x^2+bx=c$.

We need to add $\left(\frac{b}{2}\right)^2$ both the sides.

We get $x^2+bx+\left(\frac{b}{2}\right)^2=c+\left(\frac{b}{2}\right)^2$.

$$\Rightarrow \quad x^2+9x+\left(\frac{9}{2}\right)^2=-20+\left(\frac{9}{2}\right)^2$$

$$\Rightarrow \quad \left(x+\frac{9}{2}\right)^2=\frac{1}{4}$$

$$\Rightarrow \quad x+\frac{9}{2}=\pm\sqrt{\frac{1}{4}}$$

$$\Rightarrow \quad x+\frac{9}{2}=\pm\frac{1}{2}$$

$$\Rightarrow \quad x=-\frac{9}{2}\pm\frac{1}{2}$$

$\Rightarrow \quad x=-5,-4$ are the roots of the given quadratic equation.

Answer 53e.

First, write the left side in the form x^2+bx. For this, subtract 14 from each side.

$$x^2+3x+14-14 = 0-14$$

$$x^2+3x = -14$$

Square half the coefficient of x.

$$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

Add $\frac{9}{4}$ to each side of the equation.

$$x^2+3x+\frac{9}{4} = -14+\frac{9}{4}$$

Write the left side as a binomial squared and simplify.

$$\left(x + \frac{3}{2}\right)^2 = -14 + \frac{9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = -\frac{47}{4}$$

Now, take the square root on each side.

$$x + \frac{3}{2} = \pm\sqrt{-\frac{47}{4}}$$

Subtract $\frac{3}{2}$ from each side to solve for x.

$$x + \frac{3}{2} - \frac{3}{2} = \pm\sqrt{-\frac{47}{4}} - \frac{3}{2}$$

$$x = -\frac{3}{2} \pm \sqrt{-\frac{47}{4}}$$

Write in terms of i.

$$x = -\frac{3}{2} \pm \sqrt{-1 \cdot \frac{47}{4}}$$

$$x = -\frac{3}{2} \pm i\sqrt{\frac{47}{4}}$$

Simplify the radical.

$$x = -\frac{3}{2} \pm i\,\frac{\sqrt{47}}{2}$$

The solutions are $-\frac{3}{2} - i\,\frac{\sqrt{47}}{2}$ and $-\frac{3}{2} + i\,\frac{\sqrt{47}}{2}$.

Answer 54e.

We need to solve the following quadratic equation by completing the square.

$$7q^2 + 10q = 2q^2 + 155$$

$\Rightarrow \quad 5q^2 + 10q = 155$

$\Rightarrow \quad q^2 + 2q = 31$

In order to complete the square in $x^2 + bx = c$.

We need to add $\left(\frac{b}{2}\right)^2$ both the sides.

We get $x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$.

$\Rightarrow \quad q^2 + 2q + \left(\frac{2}{2}\right)^2 = 31 + \left(\frac{2}{2}\right)^2$

$\Rightarrow \quad q^2 + 2q + 1 = 32$

$\Rightarrow \quad (q+1)^2 = 32$

$\Rightarrow \quad q + 1 = \pm\sqrt{32}$

$\Rightarrow \quad q + 1 = \pm\sqrt{16} \cdot \sqrt{2}$

$\Rightarrow \quad q + 1 = \pm 4\sqrt{2}$

$\Rightarrow \quad q = -1 \pm 4\sqrt{2}$ are the roots of the given quadratic equation.

Answer 55e.

First, subtract $2x$ from each side to bring the terms containing variable on one side.

$$3x^2 + x - 2x = 2x - 6 - 2x$$

$$3x^2 - x = -6$$

Write the left side in the form $x^2 + bx$. For this, divide each term by 3.

$$\frac{3x^2}{3} - \frac{x}{3} = -\frac{6}{3}$$

$$x^2 - \frac{x}{3} = -2$$

Square half the coefficient of x.

$$\left(-\frac{1}{6}\right)^2 = \frac{1}{36}$$

Add $\frac{1}{36}$ to each side of the equation.

$$x^2 - \frac{x}{3} + \frac{1}{36} = -2 + \frac{1}{36}$$

Write the left side as a binomial squared and simplify.

$$\left(x-\frac{1}{6}\right)^2 = -2+\frac{1}{36}$$

$$\left(x-\frac{1}{6}\right)^2 = -\frac{71}{36}$$

Now, take the square root on each side.

$$x-\frac{1}{6} = \pm\sqrt{-\frac{71}{36}}$$

Add $\frac{1}{6}$ to each side to solve for x.

$$x-\frac{1}{6}+\frac{1}{6} = \pm\sqrt{-\frac{71}{36}}+\frac{1}{6}$$

$$x = \frac{1}{6}\pm\sqrt{-\frac{71}{36}}$$

Write in terms of i.

$$x = \frac{1}{6}\pm\sqrt{-1\cdot\frac{71}{36}}$$

$$x = \frac{1}{6}\pm i\sqrt{\frac{71}{36}}$$

Simplify the radical.

$$x = \frac{1}{6}\pm i\frac{\sqrt{71}}{6}$$

The solutions are $\frac{1}{6}+i\frac{\sqrt{71}}{6}$ and $\frac{1}{6}-i\frac{\sqrt{71}}{6}$.

Answer 56e.

We need to solve the following quadratic equation by completing the square.

$$0.1x^2 - x + 9 = 0.2x$$

$$\Rightarrow \quad 0.1x^2 - 1.2x = -9$$

$$\Rightarrow \quad x^2 - 12x = -90$$

In order to complete the square in $x^2 + bx = c$.

We need to add $\left(\frac{b}{2}\right)^2$ both the sides.

We get $x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$.

$$\Rightarrow \quad x^2 - 12x + \left(-\frac{12}{2}\right)^2 = -90 + \left(-\frac{12}{2}\right)^2$$

$$\Rightarrow \quad x^2 - 12x + 36 = -54$$

$$\Rightarrow \quad (x-6)^2 = -54$$

$$\Rightarrow \quad x - 6 = \pm\sqrt{-54}$$

$$\Rightarrow \quad x - 6 = \pm\sqrt{9} \cdot \sqrt{-6}$$

$$\Rightarrow \quad x - 6 = \pm 3\sqrt{6}i$$

$\Rightarrow \quad x = 6 \pm 3\sqrt{6}i$ are the roots of the given quadratic equation.

Answer 57e.

First, subtract $0.3v$ from each side to bring the terms containing variable on one side.

$$0.4v^2 + 0.7v - 0.3v = 0.3v - 2 - 0.3v$$

$$0.4v^2 + 0.4v = -2$$

Write the left side in the form $x^2 + bx$. For this, divide each term by 0.4.

$$\frac{0.4v^2}{0.4} + \frac{0.4v}{0.4} = \frac{-2}{0.4}$$

$$v^2 + v = -5$$

Square half the coefficient of v.

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Add $\frac{1}{4}$ to each side of the equation.

$$v^2 + v + \frac{1}{4} = -5 + \frac{1}{4}$$

Write the left side as a binomial squared and simplify.

$$\left(v+\frac{1}{2}\right)^2 = -5+\frac{1}{4}$$

$$\left(v+\frac{1}{2}\right)^2 = -\frac{19}{4}$$

Now, take the square root on each side.

$$v+\frac{1}{2} = \pm\sqrt{-\frac{19}{4}}$$

Subtract $\frac{1}{2}$ from each side to solve for v.

$$v+\frac{1}{2}-\frac{1}{2} = \pm\sqrt{-\frac{19}{4}}-\frac{1}{2}$$

$$v = -\frac{1}{2}\pm\sqrt{-\frac{19}{4}}$$

Write in terms of i.

$$v = -\frac{1}{2}\pm\sqrt{-1\cdot\frac{19}{4}}$$

$$v = -\frac{1}{2}\pm i\sqrt{\frac{19}{4}}$$

Simplify the radical.

$$v = -\frac{1}{2}\pm i\frac{\sqrt{19}}{2}$$

The solutions are $-\frac{1}{2}+i\frac{\sqrt{19}}{2}$ and $-\frac{1}{2}-i\frac{\sqrt{19}}{2}$.

Answer 58e.

We need to write a quadratic equation with real number solutions that can be solved by completing the square but not by factorizing.

Consider $x^2+2x-1=0$

The above quadratic equation cannot be solved easily by factorizing.

Because, we cannot easily find two numbers m and n such that n

Such that $m+n=2$ and $mn=-1$

We need to solve the following quadratic equation by completing the square.

$$x^2+2x-1=0$$

$$\Rightarrow \quad x^2+2x=1$$

In order to complete the square in $x^2+bx=c$.

We need to add $\left(\frac{b}{2}\right)^2$ both the sides.

We get $x^2+bx+\left(\frac{b}{2}\right)^2=c+\left(\frac{b}{2}\right)^2$.

$$\Rightarrow \quad x^2+2x+\left(\frac{2}{2}\right)^2=1+\left(\frac{2}{2}\right)^2$$

$$\Rightarrow \quad x^2+2x+1=2$$

$$\Rightarrow \quad (x+1)^2=2$$

$$\Rightarrow \quad x+1=\pm\sqrt{2}$$

$\Rightarrow \quad x=-1\pm\sqrt{2}$ are the roots of the given quadratic equation.

Answer 59e.

a) First, we can graph the function $y = x^2 + 2x$.

For this, substitute some values for x, say -4, and evaluate y.

$$y = (-4)^2 + 2(-4)$$

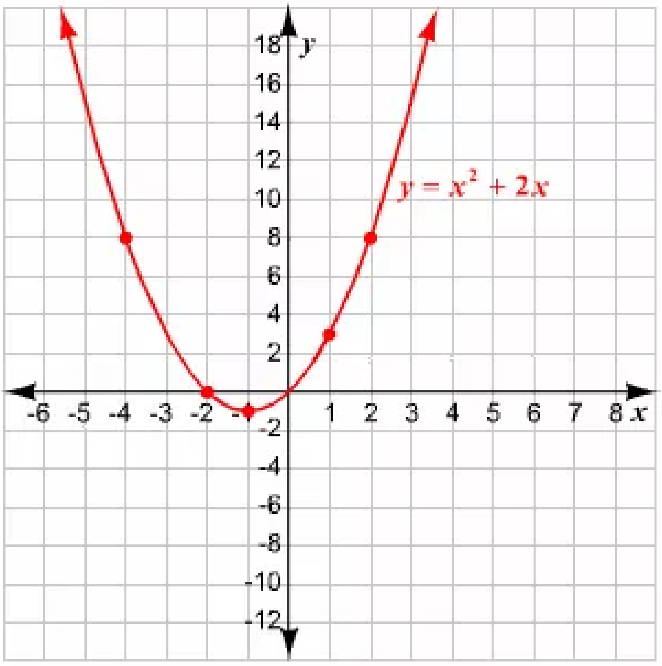
$$= 16 - 8$$

$$= 8$$

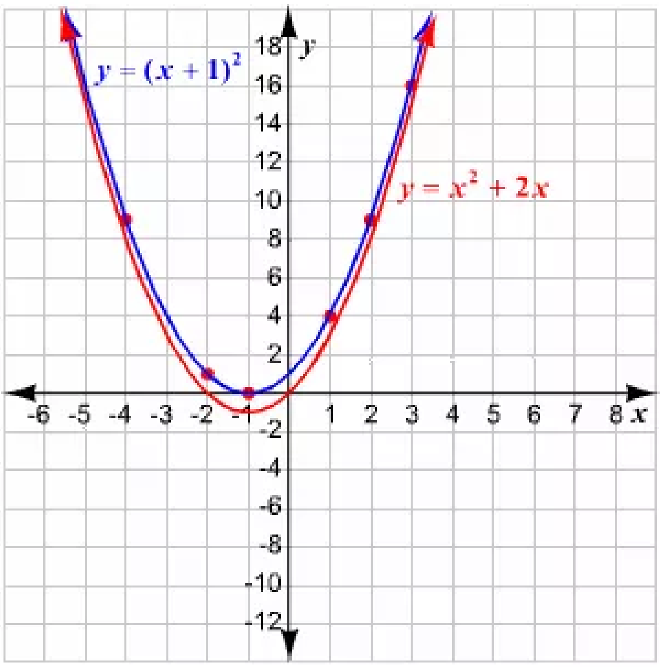
Organize the results in a table.

x	-4	-2	-1	1	2
y	8	0	-1	3	8

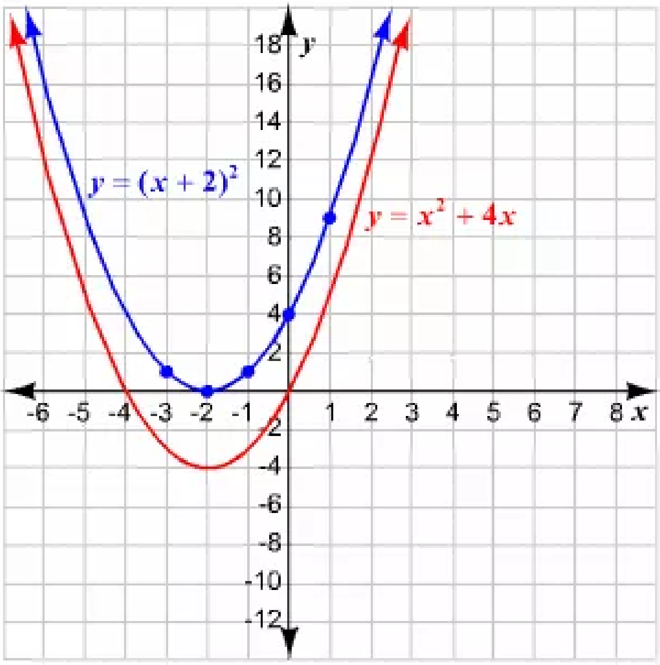
Plot the points on a coordinate plane and connect them with a smooth curve.



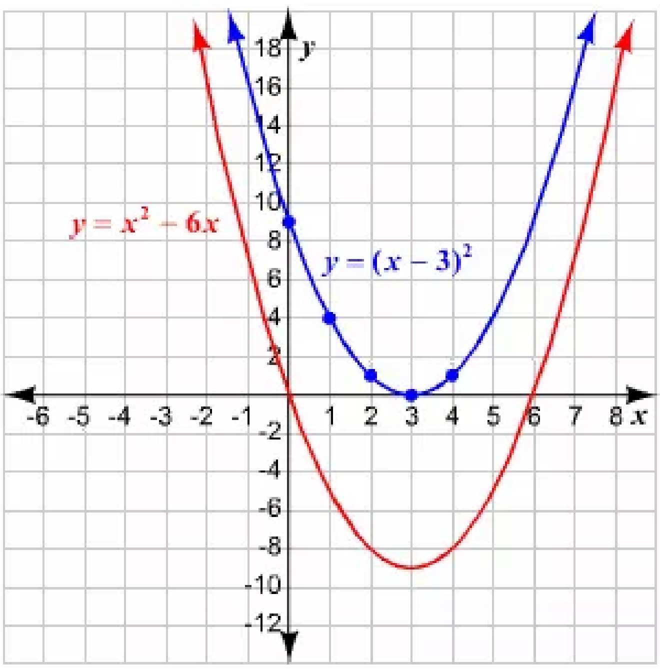
Similarly, graph $y = (x + 1)^2$ on the same coordinate plane.



Graph the second pair of functions on a coordinate plane.



Graph the third pair of functions on a coordinate plane.



b) On comparing each pair of graphs, we find that the graphs of $y = \left(x + \frac{b}{2}\right)^2$ are narrower than the graphs of $y = x^2 + bx$ with their vertices shifted to the x-axis.

When we complete the square of $y = x^2 + bx$, the new equation is $y = \left(x + \frac{b}{2}\right)^2$.

Therefore, both the graphs will be the same.

Answer 60e.

Consider the quadratic equation $x^2+bx+\left(\frac{b}{2}\right)^2=k$

We need to solve the following quadratic equation by finding square roots.

$$x^2+bx+\left(\frac{b}{2}\right)^2=k$$

$$\Rightarrow \quad x^2+2x\left(\frac{b}{2}\right)+\left(\frac{b}{2}\right)^2=k$$

$$\Rightarrow \quad \left(x+\frac{b}{2}\right)^2=k$$

$$\Rightarrow \quad x+\frac{b}{2}=\pm\sqrt{k}$$

$\Rightarrow \quad x=-\frac{b}{2}\pm\sqrt{k}$ are the roots of the given quadratic equation.

The given equation will have only one real solution iff $k=0$, it will have two real solutions iff $k>0$ and imaginary solutions iff $k<0$.

Answer 61e.

First, write the left side in the form x^2+bx. For this, subtract c from each side.

$$x^2+bx+c-c = 0-c$$

$$x^2+bx = -c$$

Square half the coefficient of x.

$$\left(\frac{b}{2}\right)^2=\frac{b^2}{4}$$

Add $\frac{b^2}{4}$ to each side of the equation.

$$x^2+bx+\frac{b^2}{4} = -c+\frac{b^2}{4}$$

Write the left side as a binomial squared and simplify.

$$\left(x+\frac{b}{2}\right)^2 = -c+\frac{b^2}{4}$$

$$\left(x+\frac{b}{2}\right)^2 = \frac{b^2-4c}{4}$$

Now, take the square root on each side.

$$x + \frac{b}{2} = \pm\sqrt{\frac{b^2 - 4c}{4}}$$

Subtract $\frac{b}{2}$ from each side to solve for x.

$$x + \frac{b}{2} - \frac{b}{2} = \pm\sqrt{\frac{b^2 - 4c}{4}} - \frac{b}{2}$$

$$x = -\frac{b}{2} \pm \sqrt{\frac{b^2 - 4c}{4}}$$

Simplify the radical.

$$x = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4c}}{2}$$

The solutions are $-\frac{b}{2} + \frac{\sqrt{b^2 - 4c}}{2}$ and $-\frac{b}{2} - \frac{\sqrt{b^2 - 4c}}{2}$.

Answer 62e.

We need to write the following quadratic function in vertex form and identify the vertex.

$$h = -16t^2 + 32t + 6$$

$$\Rightarrow \quad h - 6 = -16\left(t^2 - 2t\right)$$

$$\Rightarrow \quad h - 6 = -16\left(t^2 - 2t + \left(-\frac{2}{2}\right)^2\right) + 16\left(-\frac{2}{2}\right)^2$$

$$\Rightarrow \quad h - 6 = -16\left(t^2 - 2t + 1\right) + 16$$

$$\Rightarrow \quad h = -16(t - 1)^2 + 22$$

Which is clearly in the vertex form and the vertex is $(1, 22)$.

Therefore the maximum height is 22 feet.

Answer 63e.

In this case, the maximum height represents the maximum value of the function. We know that the maximum value of a quadratic function is the y-coordinate of the vertex.

In order to find the vertex, we have to rewrite the function in the vertex form. For this, first factor out -16 from the first two terms on the right side.

$h = -16(t^2 - 3t) + 4$

Square half the coefficient of t.

$$\left(-\frac{3}{2}\right)^2 = \frac{9}{4}$$

Now, complete the square. For this, add $(-16)\left(\frac{9}{4}\right)$ to each side of the equation.

$$h+(-16)\left(\frac{9}{4}\right) = -16\left(t^2-3t\right)+4+(-16)\left(\frac{9}{4}\right)$$

$$h+(-16)\left(\frac{9}{4}\right) = -16\left(t^2-3t+\frac{9}{4}\right)+4$$

Write $t^2-3t+\frac{9}{4}$ as a binomial squared and simplify.

$$h+(-16)\left(\frac{9}{4}\right) = -16\left(t-\frac{3}{2}\right)^2+4$$

$$h-36 = -16\left(t-\frac{3}{2}\right)^2+4$$

Solve for h. For this, add 36 to each side.

$$h-36+36 = -16\left(t-\frac{3}{2}\right)^2+4+36$$

$$h = -16\left(t-\frac{3}{2}\right)^2+40$$

A quadratic function in the form $y=a(x-h)^2+k$ is said to be in vertex form, where (h, k) is the vertex.

On comparing the equation obtained with the vertex form, we get $a=-16$, $h = \frac{3}{2}$, and $k=40$. Thus, the vertex of the given function's graph is $\left(\frac{3}{2}, 40\right)$.

Therefore, the maximum height of the object is 40 feet.

Answer 64e.

We need to write the following quadratic function in vertex form and identify the vertex.

$$y = (70 - x)(50 + x)$$

$\Rightarrow \quad y = 3500 - 50 + 70x - x^2$

$\Rightarrow \quad y = 3500 + 20x - x^2$

$\Rightarrow \quad x^2 - 20x = 3500 - y$

In order to complete the square in $x^2 + bx = c$.

We need to add $\left(\frac{b}{2}\right)^2$ both the sides.

We get $x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$.

$\Rightarrow \quad x^2 - 20x + \left(-\frac{20}{2}\right)^2 = 3500 - y + \left(-\frac{20}{2}\right)^2$

$\Rightarrow \quad x^2 - 20x + 100 = 3600 - y$

$\Rightarrow \quad y = -(x - 10)^2 + 3600$

Which is clearly in the vertex form and the vertex is $(10, 3600)$.

Therefore the store can maximize the monthly revenue up to $\$3600$ by reducing $\$10$ per skateboard.

Answer 65e.

In this case, the maximum revenue represents the maximum value of the function. We know that the maximum value of a quadratic function is the y-coordinate of the vertex.

In order to find the vertex, we have to rewrite the function in the vertex form. For this, first apply the FOIL method.

$$y = 8000 - 200x + 400x - 10x^2$$

$$y = -10x^2 + 200x + 8000$$

Factor out -10 from the first two terms on the right side.

$$y = -10(x^2 - 20x) + 8000$$

Square half the coefficient of t.

$$\left(-\frac{20}{2}\right)^2 = (-10)^2 = 100$$

Now, complete the square. For this, add $(-10)(100)$ to each side of the equation.

$$y + (-10)(100) = -10\left(x^2 - 20x\right) + 8000 + (-10)(100)$$

$$y + (-10)(100) = -10\left(x^2 - 20x + 100\right) + 8000$$

Write $x^2 - 20x + 8000$ as a binomial squared and simplify.

$$y + (-10)(100) = -10(x-10)^2 + 8000$$
$$y - 1000 = -10(x-10)^2 + 8000$$

Solve for y. For this, add 1000 to each side.

$$y - 1000 + 1000 = -10(x-10)^2 + 8000 + 1000$$
$$y = -10(x-10)^2 + 9000$$

A quadratic function in the form $y = a(x-h)^2 + k$ is said to be in vertex form, where (h, k) is the vertex.

On comparing the equation obtained with the vertex form, we get $a = -10$, $h = 10$, and $k = 9000$. Thus, the vertex of the given function's graph is (10, 9000). Therefore, the maximum revenue is \$9000.

Now, find the maximum cost for a system. For this, evaluate $200 + 10x$ when x is 10.

$$200 + 10(10) = 200 + 100$$
$$= 300$$

Thus, selling a system for \$300 would maximize monthly revenue at \$9000.

Answer 66e.

The path of a ball thrown by a softball player can be modeled by the function

$$y = -0.0110x^2 + 1.23x + 5.50$$

where x is the softball's horizontal position (in feet) and y is the corresponding height (in feet).

(a)

We rewrite the function $y = -0.0110x^2 + 1.23x + 5.50$ into vertex form as given below.

$$y = -0.0110x^2 + 1.23x + 5.50 \quad \text{[The original function]}$$

$$y = -0.0110(x^2 - 111.82x) + 5.50 \quad \text{[Factor } -0.0110 \text{ from first two terms]}$$

$$y + (-0.0110)(?) = -0.0110(x^2 - 111.82x + ?) + 5.50 \quad \text{[Prepare to complete the square]}$$

$$y + (-0.0110)(3125.93) = -0.0110(x^2 - 111.82x + 3125.93) + 5.50 \quad \text{[Add } (-0.0110)(3125.93) \text{ to each side]}$$

$$y - 34.3852 = -0.0110(x - 55.91)^2 + 5.50$$

$$\boxed{y = -0.0110(x - 55.91)^2 + 39.8852} \quad \text{[Solve for } y\text{]}$$

(b)

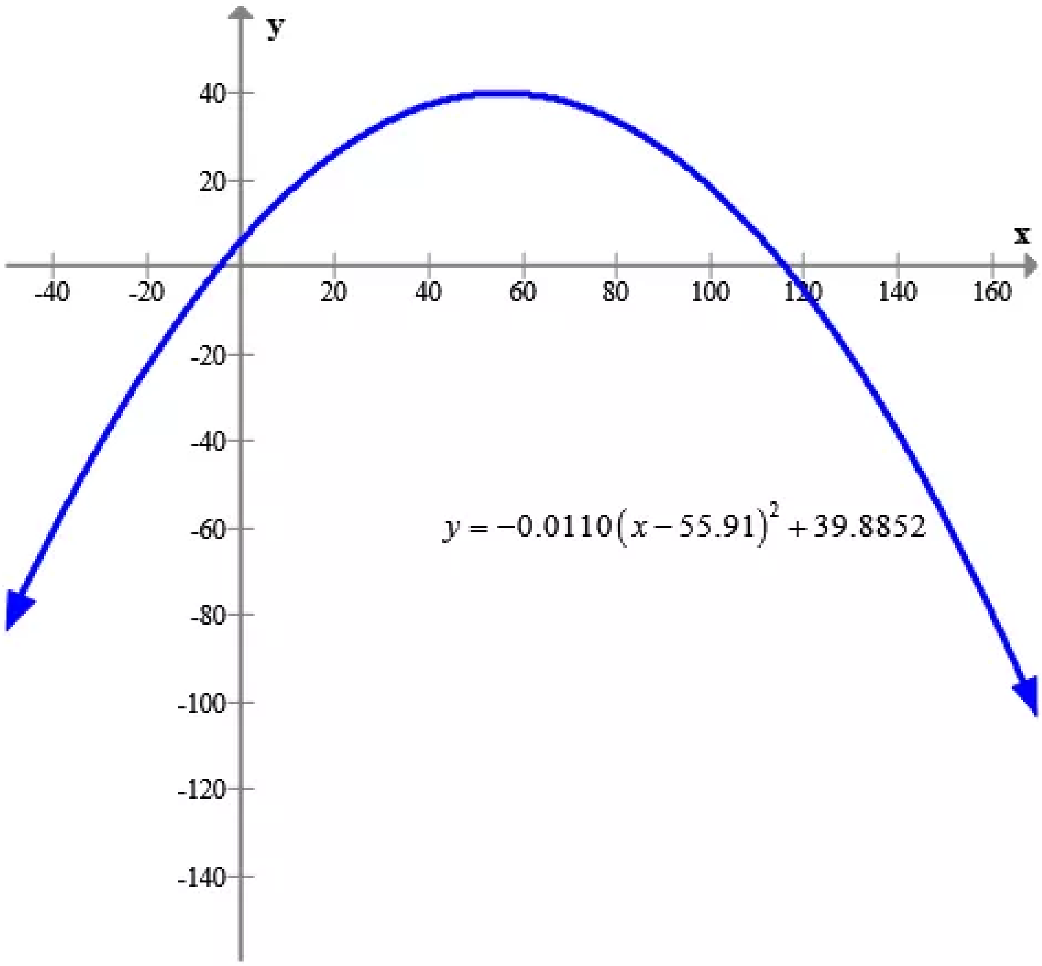
We make a table of values for the function $y=-0.0110x^2+1.23x+5.50$ including values of x from 0 to 120 increments of 10 as given below.

x	0	10	20	30	40	50	60	70
$y=-0.0110x^2+1.23x+5.50$	5.50	16.7	25.7	32.5	37.1	39.5	39.7	37.7

x	80	90	100	110	120
$y=-0.0110x^2+1.23x+5.50$	33.5	27.1	18.5	7.7	-5.3

(c)

We can draw the graph of the function $y=-0.0110x^2+1.23x+5.50$ by plotting the points shown in the table above as given below.



Because the vertex form of the given model is $y = -0.0110(x - 55.91)^2 + 39.8852$, so the vertex is $(55.91, 39.8852)$.

The maximum height of the softball is the y-coordinate of the vertex $(55.91, 39.8852)$ of the parabola with the vertex form of the given model $y = -0.0110(x - 55.91)^2 + 39.8852$.

Hence, the maximum height of the softball is

$$\boxed{39.8852 \text{ feet}}.$$

To find how far the softball travels we have to twice the value of the x-coordinate of the vertex $(55.91, 39.8852)$ of the parabola with the vertex form of the given model $y = -0.0110(x - 55.91)^2 + 39.8852$.

Hence, the softball travels

$$\boxed{55.91 \text{ feet}}.$$

Answer 67e.

a) The eating section is in the shape of a rectangle. The area of a rectangle is the product of its length and width.

From the figure, we find that the length is x, and the width is $120 - 2x$. The area of the section is given as 1500 square feet.

Thus, an equation for the area of the eating section is

$x(120 - 2x) = 1500$.

b) For solving the equation, first apply the distributive property.

$$120x - 2x^2 = 1500$$

$$-2x^2 + 120x = 1500$$

Divide each term by -2.

$$\frac{-2x^2}{-2} + \frac{120x}{-2} = \frac{1500}{-2}$$

$$x^2 - 60x = -750$$

Square half the coefficient of x.

$$\left(-\frac{60}{2}\right)^2 = (-30)^2 = 900$$

Add 900 to each side of the equation.

$x^2 - 60x + 900 = -750 + 900$

$x^2 - 60x + 900 = 150$

Write the left side as a binomial squared and simplify.

$(x - 30)^2 = 150$

Take the square root on each side.

$x - 30 = \pm\sqrt{150}$

Add 30 to each side to solve for x.

$x - 30 + 30 = \pm\sqrt{150} + 30$

$x = 30 \pm \sqrt{150}$

Simplify the radical.

$x = 30 \pm \sqrt{25 \cdot 6}$

$x = 30 \pm 5\sqrt{6}$

Use a calculator to evaluate.

$x \approx 42.25$

or

$x \approx 17.75$

The solutions are 42.25 and 17.75.

Evaluate $120 - 2x$ for $x = 42.25$ and $x = 17.75$.

$120 - 2x = 120 - 2(42.25)$
$= 120 - 84.5$
$= 35.5$

$120 - 2x = 120 - 2(17.75)$
$= 120 - 35.5$
$= 84.5$

Since the width of the eating section cannot be greater than the length of the side of the school, 84.5 ft is not a possible width. Thus, the solution 17.75 must be rejected.

c) In this case, x represents the length and $120 - 2x$ represents the width of the eating section. The possible value for x is 42.25 and the corresponding value for $120 - 2x$ is 35.5. Therefore, the dimensions of the eating section are 42.25 ft by 35.5 ft.

Answer 68e.

Consider that you are given a lump of clay with a volume of 200 cubic centimeters and are asked to make a cylindrical pencil holder. The pencil holder should be 9 c.m high and have an inner radius of 3 cm.

Need to find the thickness x, our pencil holder needs to have if you want to use all of the clay.

Based on the data, We get,

$$\pi(3+x)^2(9)-\pi(3^2)(9-x)=200$$

$$\Rightarrow \quad 9\pi\left\{(3-x)^2-3^2\right\}=200$$

$$\Rightarrow \quad 9\pi x(x+6)=200$$

$$\Rightarrow \quad x^2+6x=\frac{200}{9\pi}$$

In order to complete the square in $x^2+bx=c$.

We need to add $\left(\frac{b}{2}\right)^2$ both the sides.

We get $x^2+bx+\left(\frac{b}{2}\right)^2=c+\left(\frac{b}{2}\right)^2$.

$$\Rightarrow \quad x^2+6x+\left(\frac{6}{2}\right)^2=\frac{200}{9\pi}+\left(\frac{6}{2}\right)^2$$

$$\Rightarrow \quad x^2+6x+9=\frac{81\pi+200}{9\pi}$$

$$\Rightarrow \quad (x+3)^2=\frac{81\pi+200}{9\pi}$$

$$\Rightarrow \quad x+3=\pm\sqrt{\frac{81\pi+200}{9\pi}}$$

$$\Rightarrow \quad x=-3+\sqrt{\frac{81\pi+200}{9\pi}} \qquad (\text{Since } x \not< 0)$$

$$\Rightarrow \quad x\approx 1 \text{ centimeter}$$

Hence the thickness x should be 1 centimeter.

Answer 69e.

Substitute 2 for a, 7 for b, and 5 for c in the expression.
$b^2 - 4ac = 7^2 - 4(2)(5)$

Since there is more than one operation to be performed, let us use the order of operations.

Evaluate the power first.
$7^2 - 4(2)(5) = 49 - 4(2)(5)$

Perform multiplication before subtraction.
$49 - 4(2)(5) = 49 - 40$

Now, subtract.
$49 - 40 = 9$

Therefore, the result is 9.

Answer 70e.

Need to evaluate $b^2 - 4ac$ for the given values of a, b and c
Here $a = 1, b = -6, c = 9$

Then

$$b^2 - 4ac = (-6)^2 - 4(1)(9)$$
$$= 36 - 36$$
$$= 0$$

Hence $\boxed{b^2 - 4ac = 0}$

Answer 71e.

Substitute 4 for a, -1 for b, and 3 for c in the expression.
$b^2 - 4ac = (-1)^2 - 4(4)(3)$

Since there is more than one operation to be performed, let us use the order of operations.

Evaluate the power first.
$(-1)^2 - 4(4)(3) = 1 - 4(4)(3)$

Perform multiplication before subtraction.
$1 - 4(4)(3) = 1 - 48$

Now, subtract.
$1 - 48 = -47$

Therefore, the result is -47.

Answer 72e.

Need to evaluate b^2-4ac for the given values of a,b and c

Here $a=3,b=2,c=-6$

Then

$$b^2-4ac=2^2-4(3)(-6)$$
$$=4+72$$
$$=76$$

Hence $\boxed{b^2-4ac=76}$

Answer 73e.

Substitute -4 for a, 2 for b, and -7 for c in the expression.

$b^2-4ac=2^2-4(-4)(-7)$

Since there is more than one operation to be performed, let us use the order of operations.

Evaluate the power first.

$2^2-4(-4)(-7)=4-4(-4)(-7)$

Perform multiplication before subtraction.

$4-4(-4)(-7)=4-112$

Now, subtract.

$4-112=-108$

Therefore, the result is -108.

Answer 74e.

Need to evaluate b^2-4ac for the given values of a,b and c

Here $a=-5,b=3,c=2$

Then

$$b^2-4ac=3^2-4(-5)(2)$$
$$=9+40$$
$$=49$$

Hence $\boxed{b^2-4ac=49}$.

Add 5 to both the sides to rewrite the equation in slope-intercept form.

$$y-5+5=2x-4+5$$
$$y=2x+1$$

The equation of the line is $y=2x+1$.

Answer 75e.

The slope m of a line is the ratio of the vertical change to the horizontal change.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

In order to find the slope of the line, substitute 9 for y_2, 5 for y_1, 4 for x_2, and 2 for x_1.

$$m = \frac{9-5}{4-2}$$

Evaluate.

$$m = \frac{9-5}{2} = \frac{4}{2} = 2$$

The slope of the line is 2.

Now, use the point-slope form to determine the equation of the line.

The equation $y - y_1 = m(x - x_1)$ is the point-slope form of the line with slope m that contains the point (x_1, y_1).

Substitute 2 for m, 2 for x_1, and 5 for y_1.

$y - 5 = 2(x - 2)$

Use the distributive property to open the parentheses.

$y - 5 = 2x - 4$

Answer 76e.

Need to write an equation of the line that passes through the points $(3,-1)$ and $(6,-3)$

Applying two-point form, $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$ ......(1).

Plugging in the values into (1) ,

$$\frac{y+1}{-2} = \frac{x-3}{3}$$
$$\Rightarrow 3y+3 = -2x+6$$
$$\Rightarrow 2x+3y-3 = 0$$

Hence the equation of the line is $\boxed{2x+3y-3=0}$.

Answer 77e.

The slope m of a line is the ratio of the vertical change to the horizontal change.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

In order to find the slope of the line, substitute 2 for y_2, -4 for y_1, -1 for x_2, and -4 for x_1.

$$m = \frac{2-(-4)}{-1-(-4)}$$

Evaluate.

$$\begin{aligned} m &= \frac{2-(-4)}{3} \\ &= \frac{6}{3} \\ &= 2 \end{aligned}$$

The slope of the line is 2.

Now, use the point-slope form to determine the equation of the line.

The equation $y - y_1 = m(x - x_1)$ is the point-slope form of the line with slope m that contains the point (x_1, y_1).

Substitute 2 for m, -4 for x_1, and -4 for y_1.

$$\begin{aligned} y-(-4) &= 2[x-(-4)] \\ y+4 &= 2(x+4) \end{aligned}$$

Use the distributive property to open the parentheses.

$y + 4 = 2x + 8$

Subtract 4 from both the sides to rewrite the equation in slope-intercept form.

$$\begin{aligned} y+4-4 &= 2x+8-4 \\ y &= 2x+4 \end{aligned}$$

The equation of the line is $y = 2x + 4$.

Answer 78e.

Need to write an equation of the line that passes through the points $(-2,4)$ and $(1,-2)$

Apply two-point form, $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$ ......(1)

Plugging in the values into (1) ,

$$\frac{y-4}{-6}=\frac{x+2}{3}$$
$$\Rightarrow y-4=-2(x+2)$$
$$\Rightarrow y-4=-2x-4$$
$$\Rightarrow 2x+y=0$$

Hence the equation of the line is $\boxed{2x+y=0}$

Answer 79e.

The slope m of a line is the ratio of the vertical change to the horizontal change.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

In order to find the slope of the line, substitute 1 for y_2, -5 for y_1, 1 for x_2, and -1 for x_1.

$$m = \frac{1-(-5)}{1-(-1)}$$

Evaluate.

$$m = \frac{1+5}{1+1}$$
$$= \frac{6}{2}$$
$$= 3$$

The slope of the line is 3.

Now, use the point-slope form to determine the equation of the line.

The equation $y - y_1 = m(x - x_1)$ is the point-slope form of the line with slope m that contains the point (x_1, y_1).

Substitute 3 for m, -1 for x_1, and -5 for y_1.

$$y-(-5) = 3[x-(-1)]$$
$$y+5 = 3(x+1)$$

Use the distributive property to open the parentheses.

$y+5=3x+3$

Subtract 5 from both the sides to rewrite the equation in slope-intercept form.

$$y+5-5 = 3x+3-5$$
$$y = 3x-2$$

The equation of the line is $y = 3x - 2$.

Answer 80e.

Need to write an equation of the line that passes through the points $(6,3)$ and $(8,4)$

Apply two-point form, $\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$ ......(1)

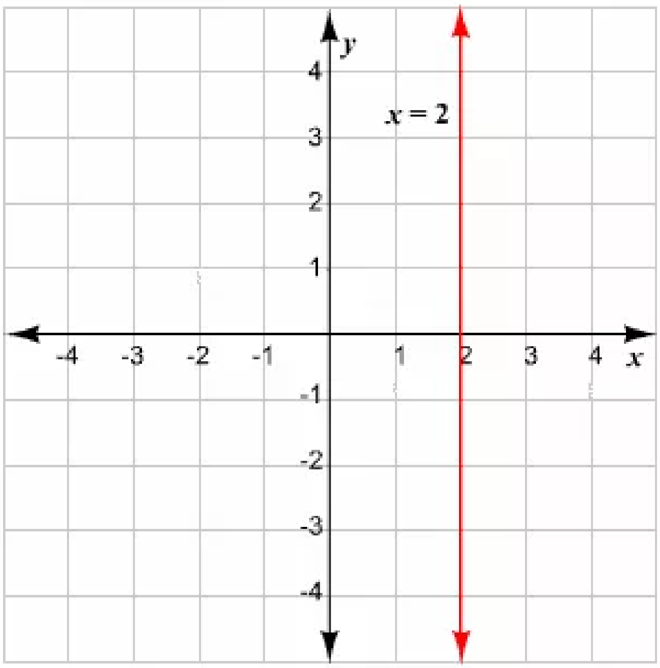
Plugging in the values into (1) ,

$$\frac{y-3}{1}=\frac{x-6}{2}$$
$$\Rightarrow 2y-6=x-6$$
$$\Rightarrow x-2y=0$$

Hence the equation of the line is $\boxed{x-2y=0}$.

Answer 81e.

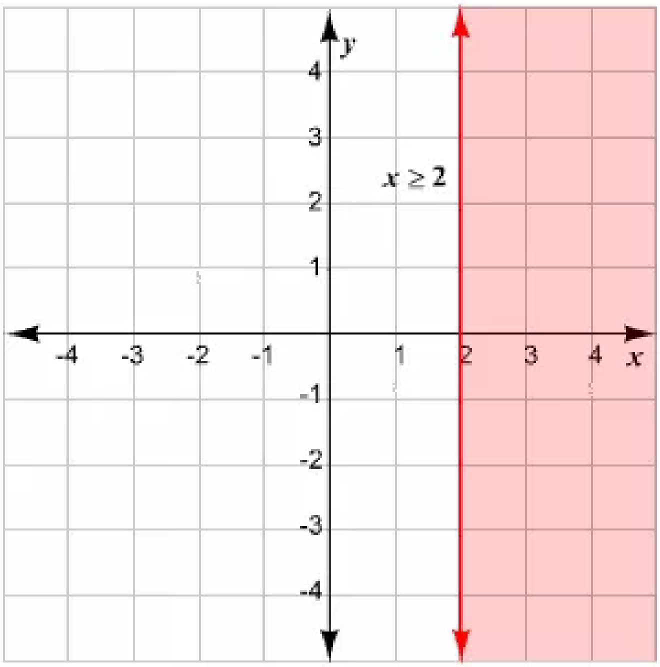
STEP 1 First, we have to graph the equation $x = 2$.
The graph of $x = c$ is the vertical line through $(c, 0)$. Thus, the graph of $x = 2$ is a vertical line through $(2, 0)$. Since $\geq$ is the inequality used, use a solid line.



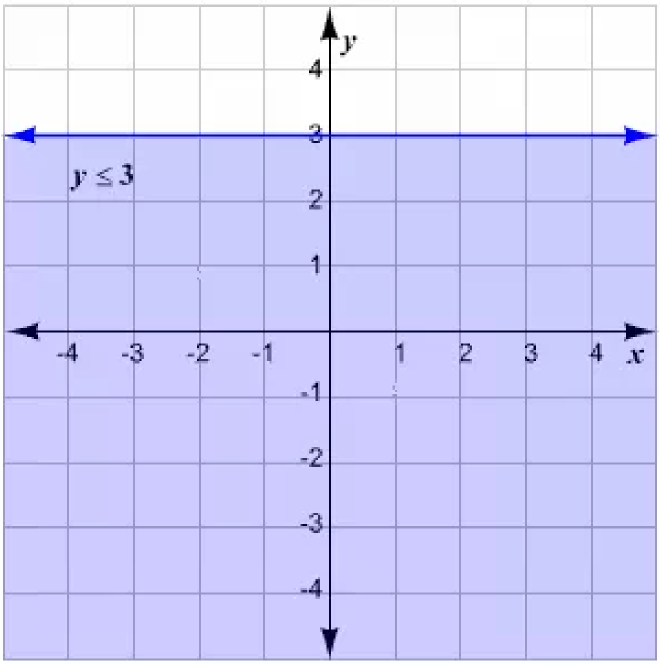
Test a point that is not on the boundary line, say $(1, 1)$.
$1 \geq 2$ False

Therefore, $(1, 1)$ is not a solution.

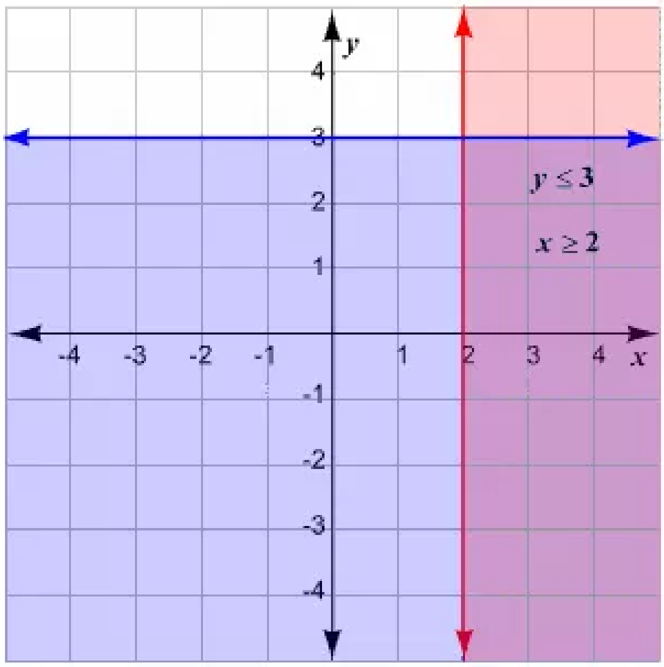
Use **red** to shade the half-plane that does not contain the test point.



Similarly, graph the inequality $y \leq 3$ using **blue**.



STEP 2 Identify the region that is common to both the graphs.



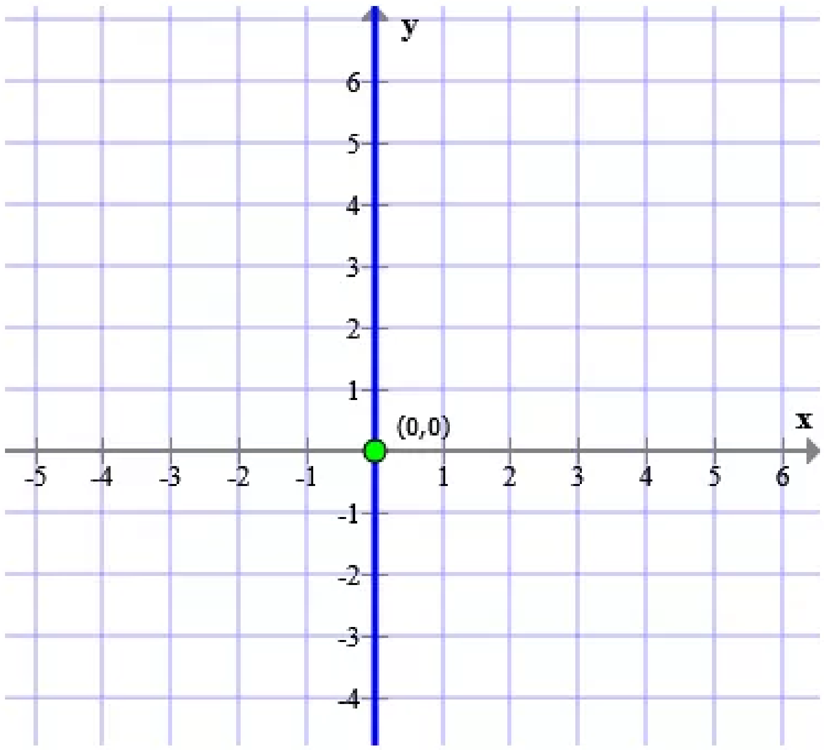
The intersection of the red and blue regions is the graph of the given system. The region shaded in purple is the solution.

Answer 82e.

Now, we are going to graph the inequality

$x \geq 0$

First, we graph the related line $x = 0$ as shown below, which is a vertical line intersecting the x-axis at $(0,0)$.



Because the inequality symbol is greater than or equal, we draw a solid line to indicate that the ordered pairs on the boundary line are solutions.
Now, we choose an ordered pair on one side of the line $x=0$ and test this ordered pair in the inequality $x \geq 0$.

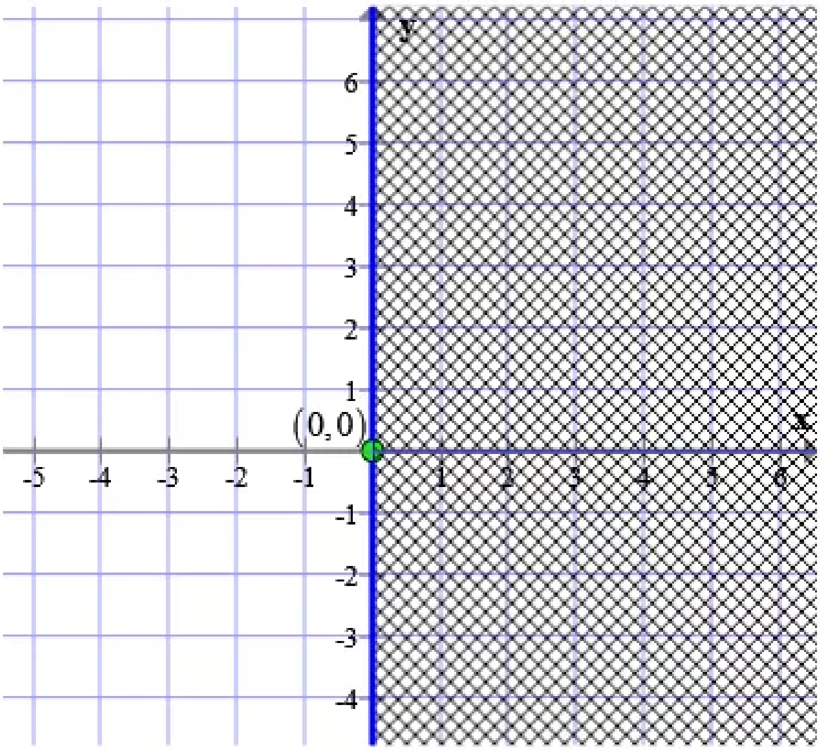
We choose the origin $(-1,0)$.

$$x \geq 0$$

$$-1 \overset{?}{\geq} 0 \qquad [\text{Replace } x \text{ with } -1 \text{ and } y \text{ with } 0]$$

Because the above statement is false, therefore, $(0,0)$ is not a solution for the inequality $x \geq 0$.

So, we shade the side of the line opposite as shown below.



Now, we are going to graph the inequality

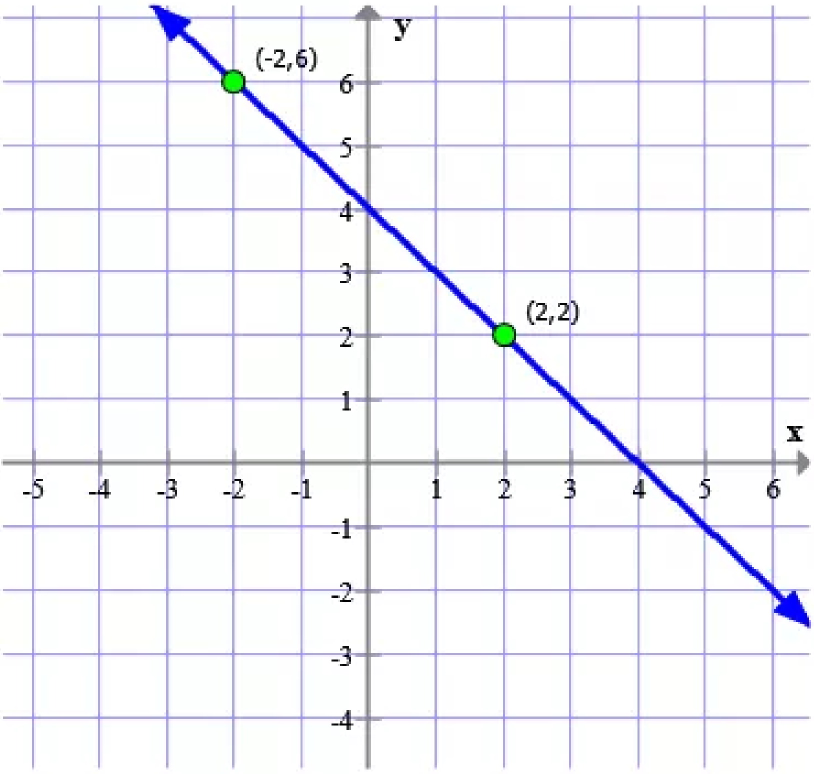
$$x+y<4$$

To graph a linear inequality in two variables,

(1) Graph the related equation (the boundary line). The related equation has an equal sign in place of the inequality symbol. If the inequality symbol is $\geq$ or $\leq$, draw a solid line. If the inequality symbol is > or <, draw a dashed line.

(2) Choose an ordered pair on one side of the boundary line and test this ordered pair in the inequality. If the ordered pair satisfies the inequality, shade the region that contains it. If the ordered pair does not satisfy the inequality, shade the region on the other sides of the boundary line.

First, we graph the related equation $x + y = 4$ as shown below.



Two ordered pairs that satisfy are $(-2,6),(2,2)$ and because the inequality symbol is less than sign, we draw a dashed line to indicate that ordered pairs on the boundary line are solutions as shown above.

Now, we choose an ordered pair on one side of the line and test this ordered pair in the inequality.

We choose the origin $(0,0)$.

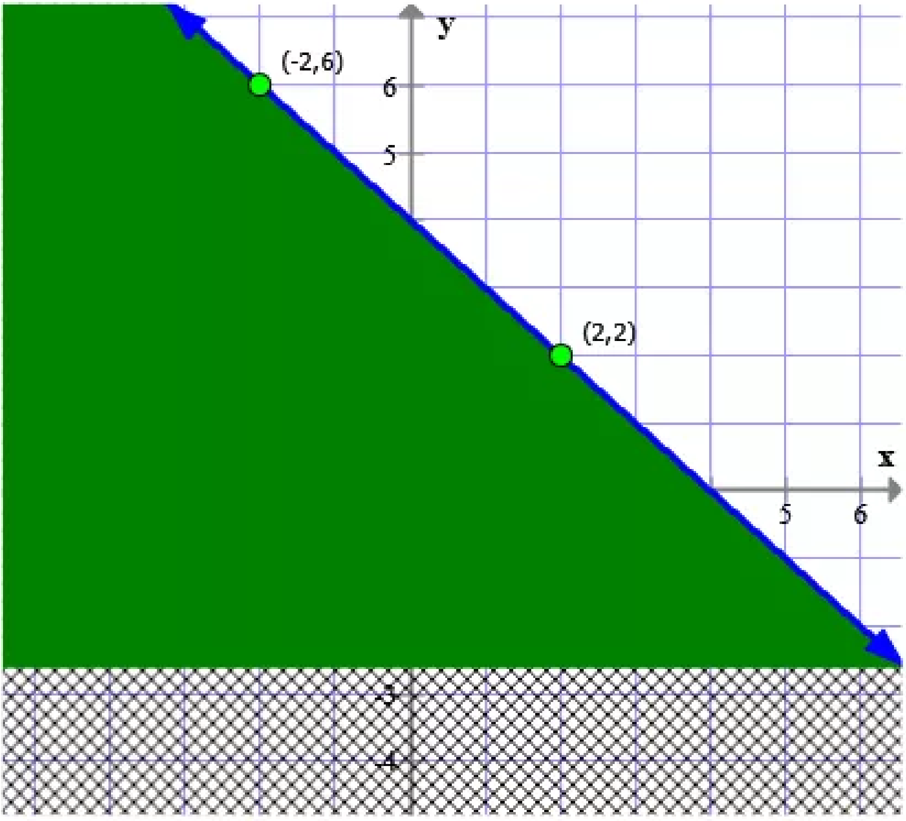
$$x + y < 4$$

$$0 + 0 \overset{?}{<} 4 \qquad [\text{Replace } x \text{ with } 0 \text{ and } y \text{ with } 0]$$

$$0 < 4$$

Because the above statement is true, therefore, $(0,0)$ is a solution for the inequality.

Since $(0,0)$ satisfies the inequality $x+y<4$, we shade the region that contains it as shown below.



We can confirm that the region on the other side of the line should not be shaded by choosing a point in that region, such as $(3,4)$.

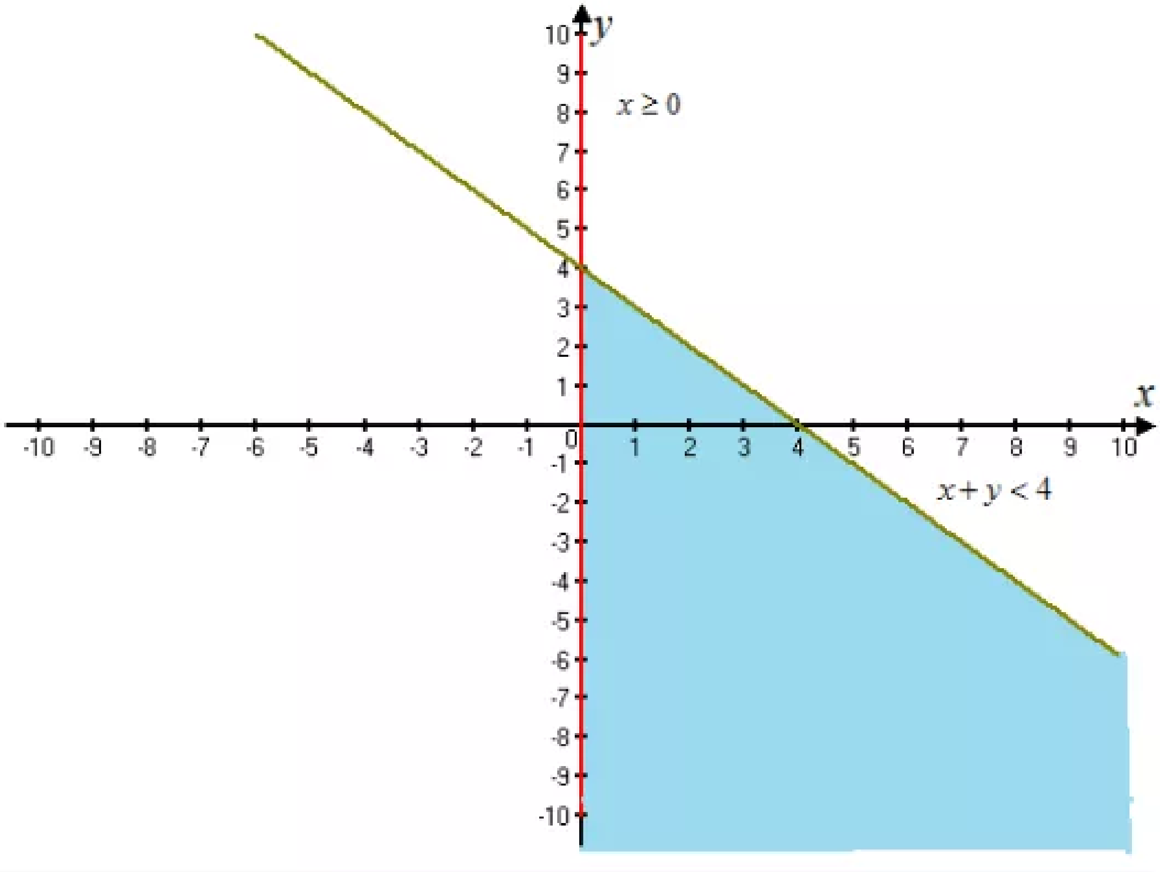
$$x+y<4$$

$$3+4\overset{?}{<}8 \qquad [\text{Replace } x \text{ with 3 and } y \text{ with 4}]$$

$$7<8$$

The above statement is false, which confirms that the region containing the point with coordinates $(3,4)$ should not be shaded.

We draw the graphs of the two inequalities $x \geq 0$ and $x + y < 4$ at the same system of rectangle axes as shown below.



We identify the red region where the two graphs overlap as shown above.
So, this region is the graph of the system.

Answer 83e.

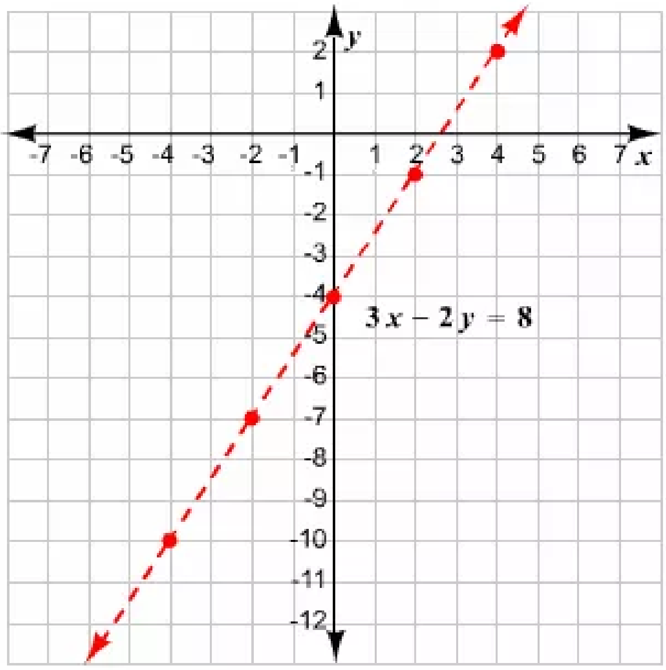
STEP 1 First, we have to graph the equation $3x - 2y = 8$.
For this, substitute some values for x, say, –4 and find the corresponding values for y.

$$3(-4) - 2y = 8$$
$$-12 - 2y = 8$$
$$-2y = 20$$
$$y = -10$$

Organize the results in a table.

x	-4	-2	0	2	4
y	-10	-7	-4	-1	2

Plot these points and join them using a line. Since < is the inequality, use a dashed line.

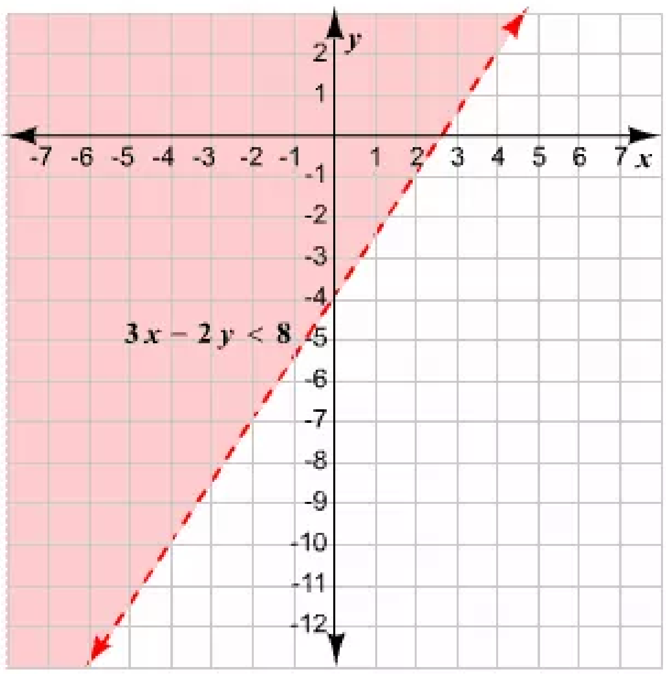


Test a point that is not on the boundary line, say (1, 1).

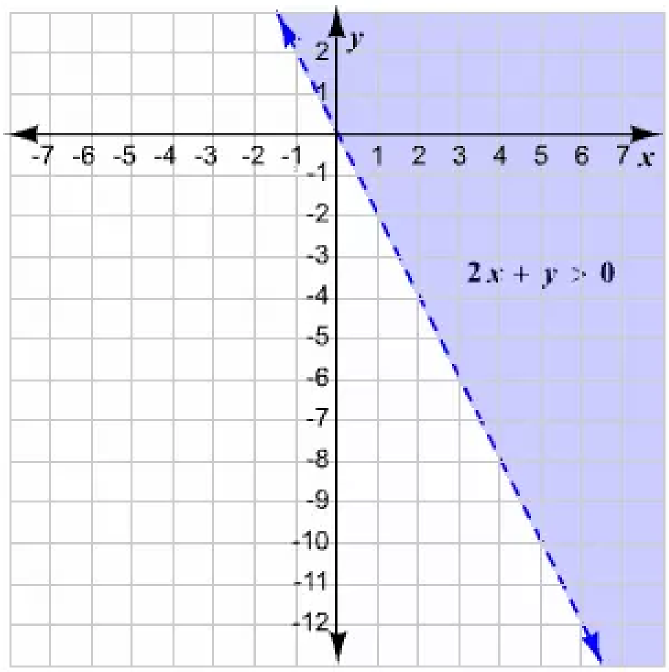
$$3(1) - 2(1) \stackrel{?}{<} 8$$
$$3 - 2 \stackrel{?}{<} 8$$
$$1 < 8 \ \checkmark$$

Therefore, (1, 1) is a solution.

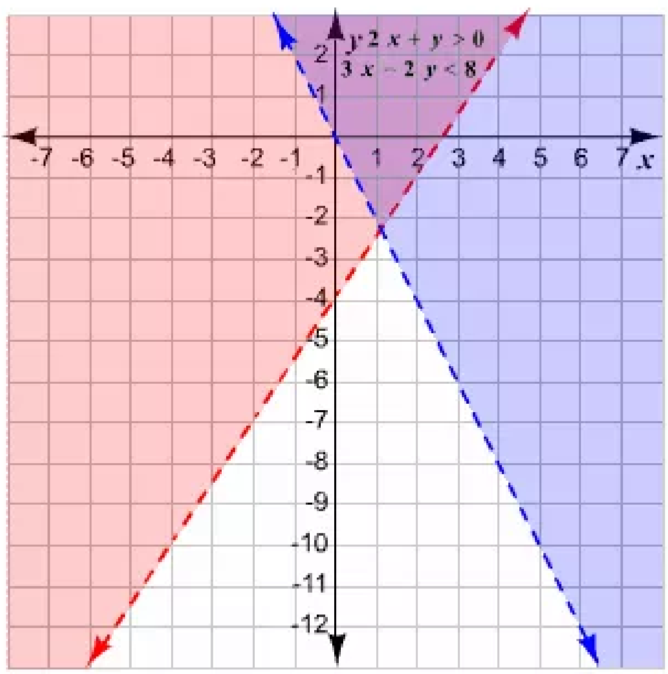
Use **red** to shade the half-plane that contains the test point.



Similarly, graph the inequality $2x + y > 0$ using **blue**.



STEP 2 Identify the region that is common to both the graphs.



The intersection of the red and blue regions is the graph of the given system. The region shaded in purple is the solution.

Answer 84e.

Now, we are going to graph the inequality

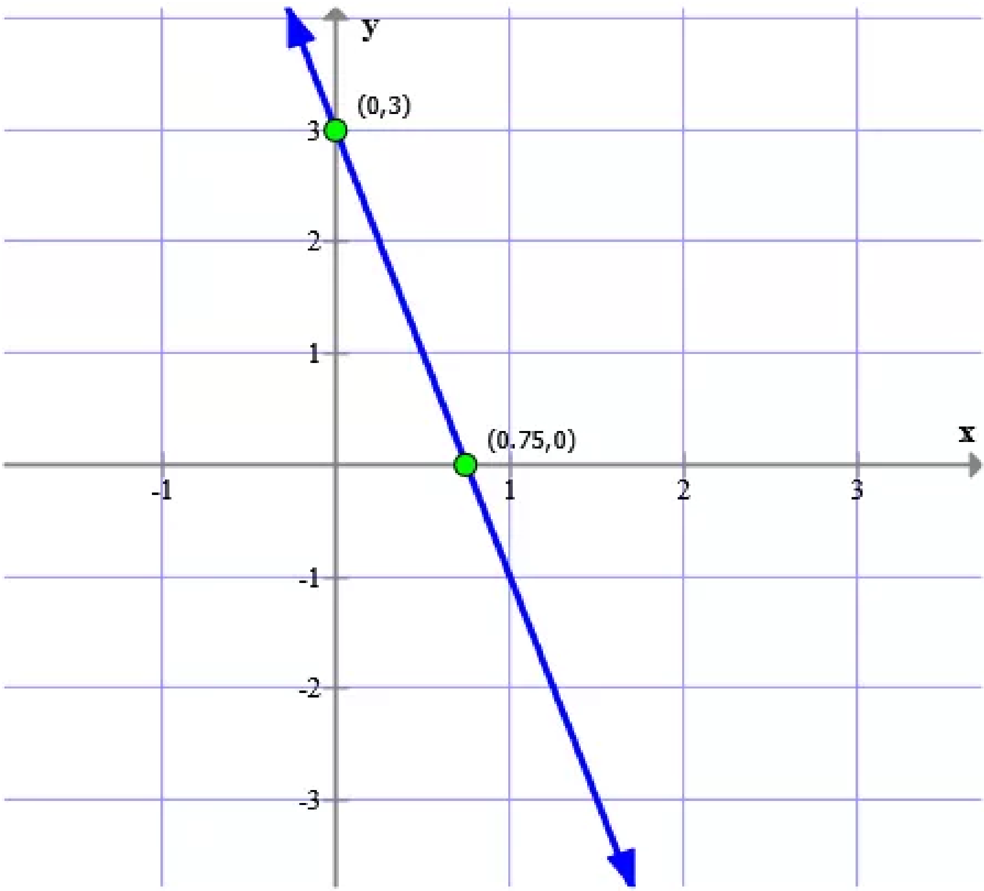
$4x + y \geq 3$

To graph a linear inequality in two variables,

(1) Graph the related equation (the boundary line). The related equation has an equal sign in place of the inequality symbol. If the inequality symbol is $\geq$ or $\leq$, draw a solid line. If the inequality symbol is $>$ or $<$, draw a dashed line.

(2) Choose an ordered pair on one side of the boundary line and test this ordered pair in the inequality. If the ordered pair satisfies the inequality, shade the region that contains it. If the ordered pair does not satisfy the inequality, shade the region on the other sides of the boundary line.

First, we graph the related equation $4x + y = 3$ as shown below.



Two ordered pairs that satisfy are $(0.75,0),(0,3)$ and because the inequality symbol is greater than or equal sign, we draw a solid line to indicate that ordered pairs on the boundary line are solutions as shown above.

Now, we choose an ordered pair on one side of the line and test this ordered pair in the inequality.

We choose the origin $(0,0)$.

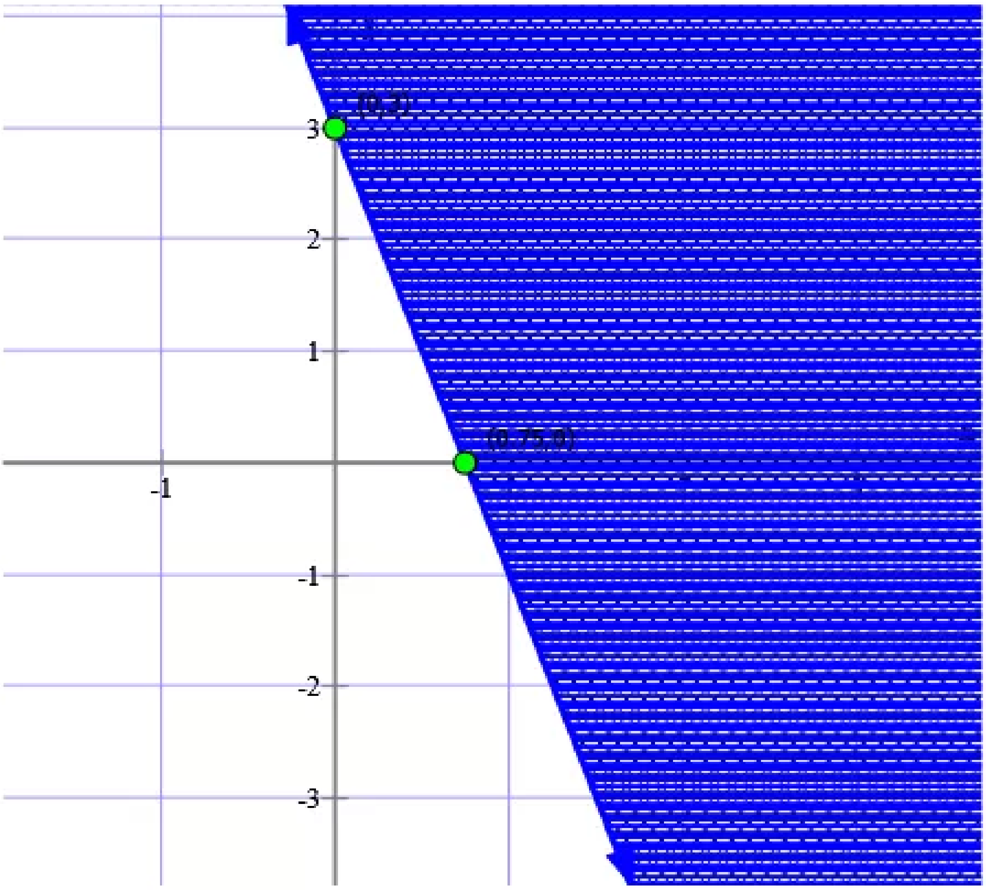
$$4x + y \geq 3$$

$$4(0)+0 \overset{?}{\geq} 4 \qquad \text{[Replace } x \text{ with 0 and } y \text{ with 0]}$$

$$0 \geq 4$$

Because the above statement is false, therefore, $(0,0)$ is not a solution for the inequality $4x + y \geq 3$.

Since $(0,0)$ does not satisfy the inequality $4x+y\geq 3$, we shade the region on the other side of the boundary line as shown below.



We can confirm that the region on the other side of the line should not be shaded by choosing a point in that region, such as $(1,1)$.

$$4x+y\geq 3$$

$$4(1)+1\overset{?}{\geq}3 \qquad \text{[Replace } x \text{ with 1 and } y \text{ with 1]}$$

$$4+1\overset{?}{\geq}3$$

$$5\geq 3$$

The above statement is true, which confirms that we have shaded the correct region of the boundary line.

Now, we are going to graph the inequality

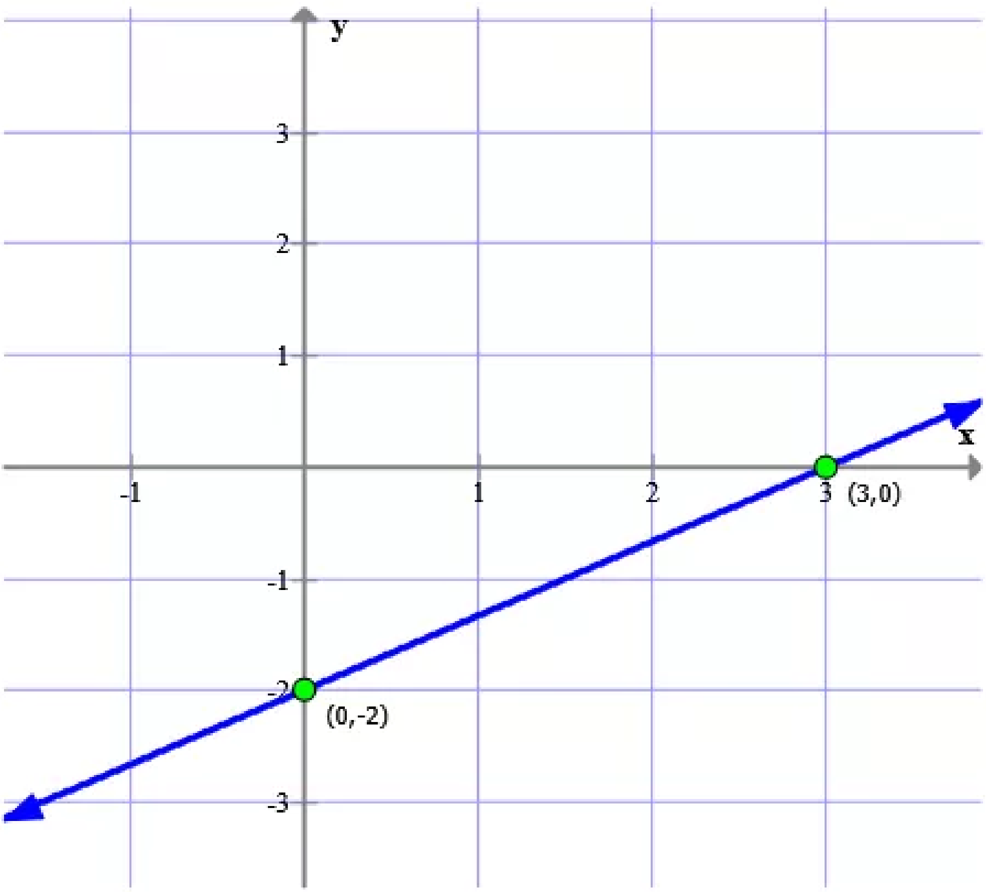
$$2x-3y<6$$

To graph a linear inequality in two variables,

(3) Graph the related equation (the boundary line). The related equation has an equal sign in place of the inequality symbol. If the inequality symbol is $\geq$ or $\leq$, draw a solid line. If the inequality symbol is $>$ or $<$, draw a dashed line.

(4) Choose an ordered pair on one side of the boundary line and test this ordered pair in the inequality. If the ordered pair satisfies the inequality, shade the region that contains it. If the ordered pair does not satisfy the inequality, shade the region on the other sides of the boundary line.

First, we graph the related equation $2x-3y=6$ as shown below.



Two ordered pairs that satisfy are $(0,-2),(3,0)$ and because the inequality symbol is less than sign, we draw a dashed line to indicate that ordered pairs on the boundary line are solutions as shown above.

Now, we choose an ordered pair on one side of the line and test this ordered pair in the inequality.

We choose the origin $(0,0)$.

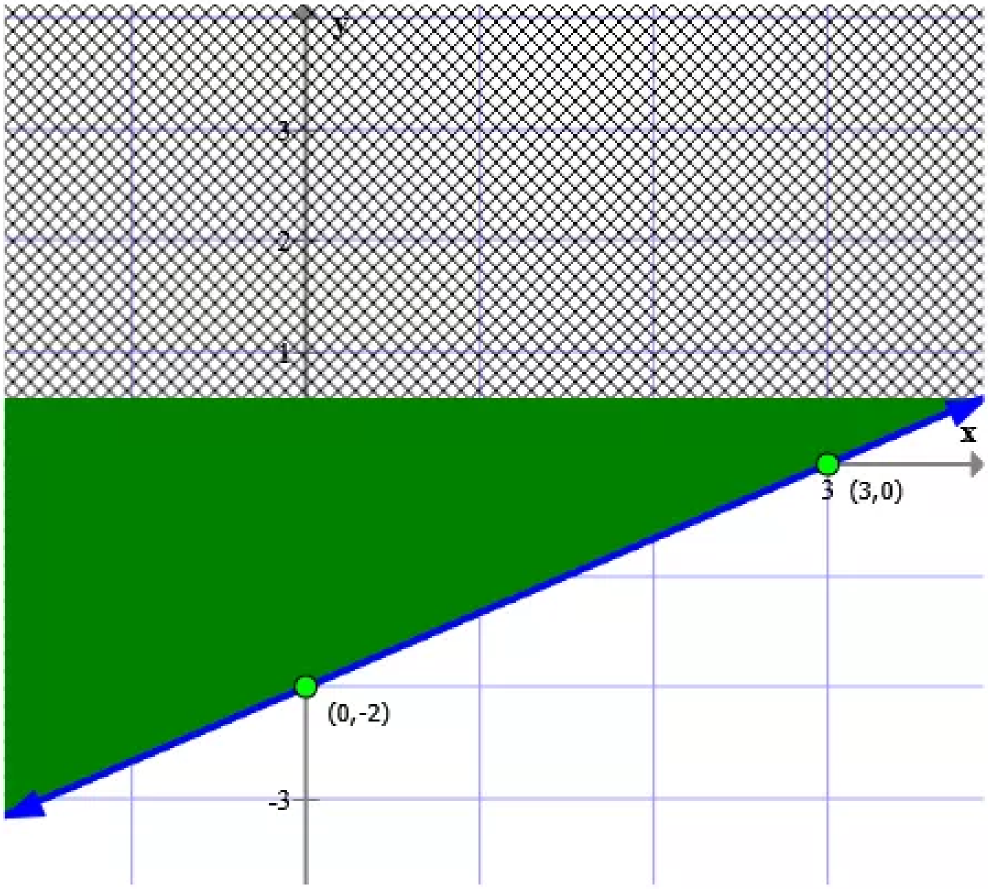
$$2x-3y<6$$

$$2(0)-3(0)\overset{?}{<}6 \qquad \text{[Replace } x \text{ with 0 and } y \text{ with 0]}$$

$$0<6$$

Because the above statement is true, therefore, $(0,0)$ is a solution for the inequality $2x-3y<6$.

Since $(0,0)$ satisfies the inequality $2x-3y<6$, we shade the region that contains it as shown below.



We can confirm that the region on the other side of the line should not be shaded by choosing a point in that region, such as $(2,-2)$.

$$2x-3y<6$$

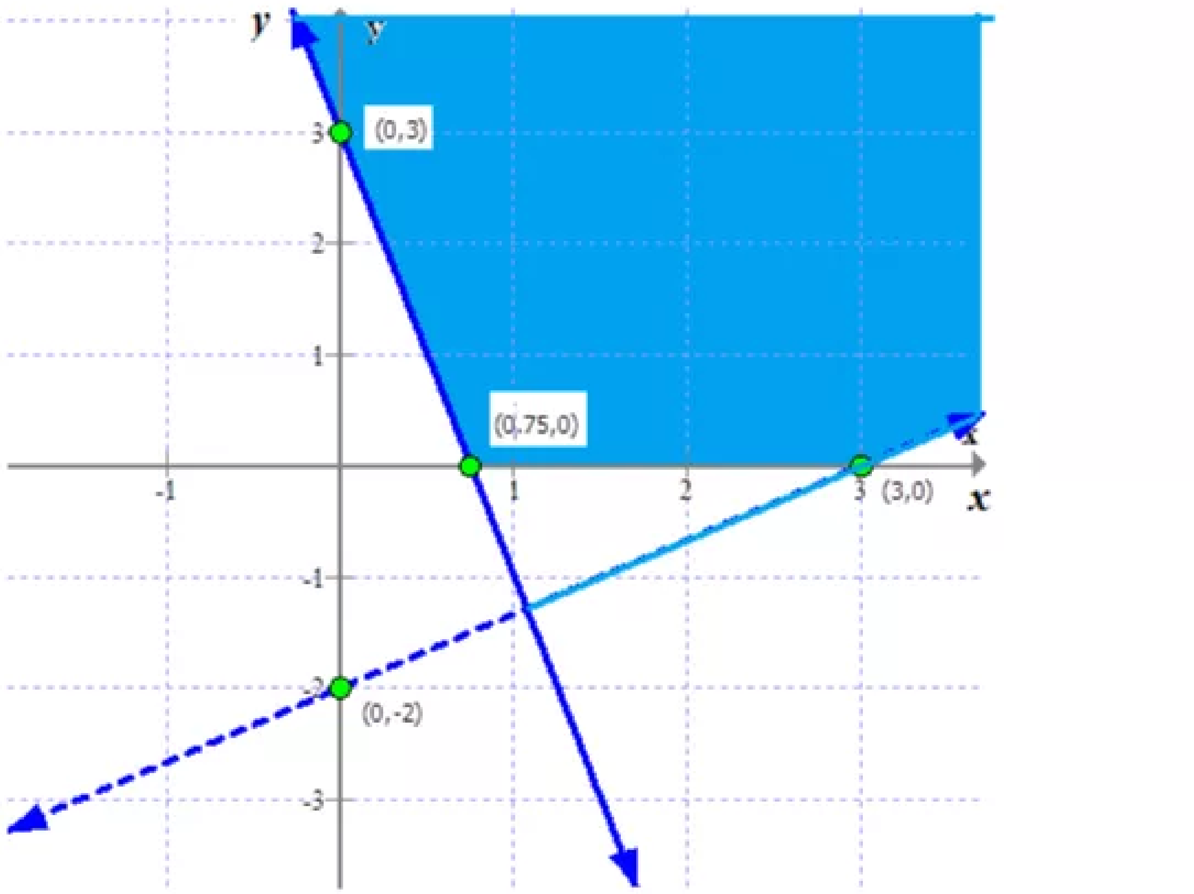
$$2(2)-3(-2)\overset{?}{<}6 \qquad \text{[Replace } x \text{ with 2 and } y \text{ with } -2]$$

$$4+6\overset{?}{<}6$$

$$10<6$$

The above statement is false, which confirms that the region containing the point with coordinates $(2,-2)$ should not be shaded.

We draw the graphs of the two inequalities $x \geq 0$ and $x + y < 4$ at the same system of rectangle axes as shown below.



We identify the red region where the two graphs overlap as shown above.
So, this region is the graph of the system.