Arithmetic Progressions

Selected NCERT Questions

1. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Sol. Let the first term be a and common difference be d.

Now, we have

$a_{11} = 38 \quad \Rightarrow \quad a + (11 - 1)d = 38$

$\Rightarrow \quad a + 10d = 38 \quad ...(i)$

and $\quad a_{16} = 73 \quad \Rightarrow \quad a + (16 - 1)d = 73$

$\Rightarrow \quad a + 15d = 73 \quad ...(ii)$

Now subtracting (ii) from (i), we have

Now, $\quad a + 10d = 38$

$\quad \underline{-a \pm 15d = \pm 73}$

$\quad -5d = -35 \quad$ or $\quad 5d = 35$

$\therefore \quad d = \frac{35}{5} = 7$

Putting the value of d in equation (i), we have

$a + 10 \times 7 = 38 \quad \Rightarrow \quad a + 70 = 38$

$\Rightarrow \quad a = 38 - 70 \quad \Rightarrow \quad a = -32$

We have, $a = -32$ and $d = 7$

Therefore, $\quad a_{31} = a + (31 - 1)d$

$\Rightarrow \quad a_{31} = a + 30d$

$= (-32) + 30 \times 7 = -32 + 210$

$\Rightarrow \quad a_{31} = 178$

2. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Sol. Let a be the first term and d be the common difference.

Since, given AP consists of 50 terms, so $n = 50$

$$a_3 = 12 \quad \Rightarrow \quad a + 2d = 12 \qquad ...(i)$$

Also, $a_{50} = 106 \quad \Rightarrow \quad a + 49d = 106 \qquad ...(ii)$

Subtracting (i) from (ii), we have

$$47d = 94 \quad \Rightarrow \quad d = \frac{94}{47} = 2$$

Putting the value of d in equation (i), we have

$$a + 2 \times 2 = 12 \Rightarrow \quad a = 12 - 4 = 8$$

Here, $a = 8, d = 2$

$\therefore$ 29th term is given by

$$a_{29} = a + (29 - 1)d = 8 + 28 \times 2$$

$$\Rightarrow \quad a_{29} = 8 + 56 \quad \Rightarrow \quad a_{29} = 64$$

3. How many three digit numbers are divisible by 7? **[*Competency Based Question*]**

Sol. Obviously first three digit number and last three digit number divisible by 7, are 105 and 994 respectively.

Now we have an AP

$$105, 112, 119, 126, \ldots\ldots 994$$

Here, $a = 105$ and $d = 7$

Let there be n terms in this AP.

$$\Rightarrow \quad a_n = 994$$

$$\Rightarrow \quad a + (n-1)d = 994 \quad \Rightarrow \quad 105 + (n-1)7 = 994$$

$$\Rightarrow \quad (n-1)7 = 994 - 105 \quad \Rightarrow \quad n - 1 = \frac{889}{7}$$

$$\Rightarrow \quad n - 1 = 127 \quad \Rightarrow \quad n = 127 + 1 = 128$$

Hence 128 three digit numbers are divisible by 7.

4. The sum of the 4^{th} and 8^{th} term of an AP is 24 and the sum of the 6^{th} and 10^{th} term is 44. Find the first three terms of the AP.

Sol. Let a be the first term and d the common difference of given AP.

Now, $a_4 = a + (4-1)d = a + 3d$

$a_8 = a + (8-1)d = a + 7d$

$a_6 = a + (6-1)d = a + 5d$

$a_{10} = a + (10-1)d = a + 9d$

From question,

$$a_4 + a_8 = 24$$

$$\Rightarrow \quad a + 3d + a + 7d = 24 \quad \Rightarrow \quad 2a + 10d = 24 \quad \Rightarrow \quad a + 5d = 12 \quad \ldots (i)$$

Also, $a_6 + a_{10} = 44$

$$a + 5d + a + 9d = 44$$

$$\Rightarrow \quad 2a + 14d = 44 \quad \Rightarrow \quad a + 7d = 22 \qquad \ldots (ii)$$

Subtracting (*i*) from (*ii*), we get

$a + 7d - a - 5d = 22 - 12$

$\Rightarrow \quad 2d = 10 \quad \Rightarrow \quad d = 5$

Putting the value of d in (*i*), we get

$a + 5 \times 5 = 12$

$\Rightarrow \quad a = 12 - 25 \quad \Rightarrow \quad a = -13$

Hence, first term = – 13.

Therefore, required three terms are

$-13, -8, -3$.

5. Subha Rao started work in 1995 at an annual salary of ₹5000 and received increment of ₹200 each year. In which year did her income reach ₹7000?

Sol. According to question, we have an AP.

₹5000, ₹5200, ₹5400, ...

Obviously, 1st term a = ₹5000

Common difference d = ₹200

Let after n years it becomes ₹ 7000.

$\Rightarrow \quad a_n =$ ₹ 7000

$\therefore \quad a_n = a + (n-1)d$

$\Rightarrow \quad 7000 = 5000 + (n-1)\,200 \quad \Rightarrow \quad 7000 - 5000 = (n-1)\,200$

$\Rightarrow \quad n - 1 = \dfrac{2000}{200} \quad \Rightarrow \quad n - 1 = 10 \quad \Rightarrow \quad n = 11$

Hence in 11th year her income will reach ₹7000.

6. How many terms of the AP 9, 17, 25, ... must be taken to give a sum of 636? [*CBSE (AI)* 2017]

Sol.

19. $a = 9$, $d = 8$, $S_n = 636$.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$636 = \frac{n}{2}[18 + (n-1)8]$

$636 = n[9 + (n-1)4]$

$636 = n(9 + 4n - 4)$

$636 = n(5 + 4n)$

$636 = 5n + 4n^2$

$4n^2 + 5n - 636 = 0$

$4n^2 + 53n - 48n - 636 = 0$

~~n & $4n+5$~~

$4n^2 - 48n + 53n - 636 = 0$

$4n(n-12) + 53(n-12) = 0$

$(4n + 53)(n - 12) = 0$

$\therefore n = \frac{-53}{4}$ or 12.

as n is a natural number, $\boxed{n = 12}$

$\therefore$ 12 terms are required to give sum 636.

[*Topper's Answer* 2017]

7. Find the sum of the first 15 multiples of 8. **[*CBSE Delhi 2017 (C)*]**

Sol. The first 15 multiples of 8 are

8, 16, 24, ... 120

Clearly, these numbers are in AP with first term $a = 8$ and common difference, $d = 16 - 8 = 8$

Thus, $S_{15} = \frac{15}{2}[2 \times 8 + (15 - 1) \times 8]$

$= \frac{15}{2}[16 + 14 \times 8] = \frac{15}{2}[16 + 112] = \frac{15}{2} \times 128 = 15 \times 64 = 960$

8. A sum of ₹700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each of the prizes.

Sol. Let the prizes be $a + 60, a + 40, a + 20, a, a - 20, a - 40, a - 60$.

Therefore, the sum of prizes is

$a + 60 + a + 40 + a + 20 + a + a - 20 + a - 40 + a - 60 = 700$

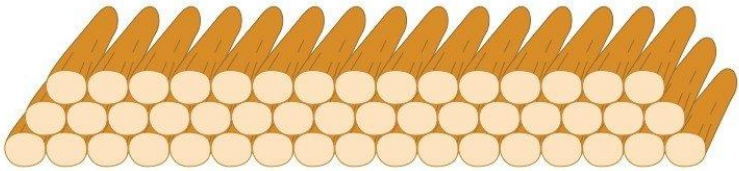
$\Rightarrow \quad 7a = 700 \quad \Rightarrow \quad a = \frac{700}{7} = 100$

Thus, the value of seven prizes are

$100 + 60, 100 + 40, 100 + 20, 100, 100 - 20, 100 - 40, 100 - 60$

i.e., ₹160, ₹140, ₹120, ₹100, ₹80, ₹60, ₹40

9. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see figure). In how many rows are the 200 logs placed and how many logs are in the top row? **[*Competency Based Question*]**



Sol. Since, logs are stacked in each row form a series $20 + 19 + 18 + 17 + \ldots$. Clearly, it is an AP with first term, $a = 20$ and common difference, $d = 19 - 20 = -1$.

$S_n = 200$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow \quad 200 = \frac{n}{2}[2 \times 20 + (n - 1)(-1)]$

$\Rightarrow \quad 400 = n(40 - n + 1)$

$\Rightarrow \quad n^2 - 41n + 400 = 0$

$\Rightarrow \quad n^2 - 25n - 16n + 400 = 0$ (By factorisation)

$\Rightarrow \quad n(n - 25) - 16(n - 25) = 0$

$\Rightarrow \quad (n - 25)(n - 16) = 0$

$\Rightarrow \quad n = 16$ and $n = 25$

Hence, the number of rows is either 25 or 16.

If $n = 16$ then $t_n = a + (n - 1)d$

$= 20 + (16 - 1)(-1)$

$= 20 - 15$

$= 5$

If $n = 25$ then $t_n = a + (n-1)d$

$= 20 + (25 - 1)(-1)$

$= 20 - 24$

$= -4$ (Not possible)

Hence, the number of row is 16 and number of logs in the top row = 5.

10. Which term of the AP 121, 117, 113, ... is its first negative term?

Sol. Given, first term, $a = 121$, common difference, $d = 117 - 121 = -4$

$\because$ nth term of an AP,

$a_n = a + (n-1)d$

$= 121 + (n-1) \times (-4)$

$= 121 - 4n + 4 = 125 - 4n$

For first negative term, $a_n < 0$.

$\Rightarrow \quad 125 - 4n < 0$

$\Rightarrow \quad 125 < 4n$

$\Rightarrow \quad 4n > 125$

$\Rightarrow \quad n > \frac{125}{4}$

$\Rightarrow \quad n > 31\frac{1}{4}$

Least integral value of $n = 32$.

Hence, 32nd term of the given AP is the first negative term.

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. Which of the following two numbers do you think should be the succeeding terms of the pattern given below?

–1/3, –1/12, 1/6, 5/12, __, __

(*a*) 2/3 and 12/11 (*b*) 4/6 and 11/4 (*c*) 2/3 and 11/12 (*d*) 1/2 and 24/22

2. The first term of an AP is p and the common difference is q, then its 10^{th} term is **[*CBSE* 2020(30/1/1)]**

(*a*) $q + 9p$ (*b*) $p - 9q$ (*c*) $p + 9q$ (*d*) $2p + 9q$

3. The common difference of an AP, whose n^{th} term is $a_n = (3n + 7)$, is **[*CBSE* 2020(30/3/1)]**

(*a*) 3 (*b*) 7 (*c*) 10 (*d*) 6

4. Find the nth term of the AP shown below.

___, ___, –36, ___, –44, ... up to n

(*a*) $4n - 24$ (*b*) $-4n - 24$ (*c*) $24 - 4n$ (*d*) $-40 + 4n$

5. Which term of the AP 21, 42, 63, 84, ... is 210? **[*NCERT Exemplar*]**

(*a*) 9th (*b*) 10th (*c*) 11th (*d*) 12th

6. The value of p for which $(2p + 1)$, 10 and $(5p + 5)$ are three consecutive terms of an AP is [*CBSE* 2020(30/3/1)]

(*a*) –1 (*b*) –2 (*c*) 1 (*d*) 2

7. The 4th term from the end of the AP –11, –8, –5, ..., 49 is [*NCERT Exemplar*]

(*a*) 37 (*b*) 40 (*c*) 43 (*d*) 58

8. The next term of the AP $\sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$ is

(*a*) $5\sqrt{2}$ (*b*) $5\sqrt{3}$ (*c*) $3\sqrt{3}$ (*d*) $5\sqrt{3}$

9. The first four terms of an AP, whose first term is –2 and the common difference is –2 are [*NCERT Exemplar*]

(*a*) –2, 0, 2, 4 (*b*) –2, 4, –8, 16 (*c*) –2, –4, –6, –8 (*d*) –2, –4, –8, –16

10. The n^{th} term of the AP $a, 3a, 5a$ is [*CBSE* 2020(30/2/1)]

(*a*) na (*b*) $(2n - 1)a$ (*c*) $(2n + 1)a$ (*d*) $2na$

11. Two APs have same common difference. The first term of one of these is –1 and that of the other is – 8. Then the difference between their 4th term is [*NCERT Exemplar*]

(*a*) –1 (*b*) –8 (*c*) 7 (*d*) –9

12. The 21st term of the AP whose first two terms are –3 and 4, is [*NCERT Exemplar*]

(*a*) 17 (*b*) 137 (*c*) 143 (*d*) –143

13. What would be the last term of an arithmetic progression with 10 terms whose second term is –23 and the third term is –35?

(*a*) –119 (*b*) 119 (*c*) –650 (*d*) 350

14. The first term of an AP is 5 and the last term is 45. If the sum of all the terms is 400, the number of terms is [*CBSE* 2020(30/4/1)]

(*a*) 20 (*b*) 8 (*c*) 10 (*d*) 16

15. Rakesh loves to travel and he travels every year. He has seen 8 different cities in his first year. Thereafter every year he has seen 2 cities. If he had followed this pattern, how many cities did he see by the end of 10 years of travel? [*Competency Based Question*]

(*a*) [5{8 + 9(2)}] cities (*b*) [5{8 + 10(2)}] cities

(*c*) [5{16 + 9(2)}] cities (*d*) [5{16 + 10(2)}] cities

16. The sum of first 16 terms of the AP 10, 6, 2, ... is [*NCERT Exemplar*]

(*a*) –320 (*b*) 320 (*c*) –352 (*d*) –400

17. What is the sum of all the three-digit numbers divisible by 12? [*Competency Based Question*]

(*a*) $\left[\frac{73}{2}(108 + 996)\right]$ (*b*) $\left[\frac{75}{2}(108 + 996)\right]$

(*c*) $\left[\frac{73}{2}(216 + 996)\right]$ (*d*) $\left[\frac{75}{2}(216 + 996)\right]$

18. If the sum of p terms of an AP is q and the sum of q terms is p, then the sum of $p + q$ terms will be

(*a*) 0 (*b*) $p - q$ (*c*) $p + q$ (*d*) $-(p + q)$

19. The first term of an AP is –76 and the sum of first 45 terms is –9360. Which of the following is the last term of this AP?

(*a*) 416 + 76 (*b*) 416 – 76 (*c*) – 416 + 76 (*d*) – 416 – 7

20. Mr. Sharma buys a property every year for 12 years. Every year he buys x acres more than the previous year. If in the 8th year he bought 45 acres of land and in the 5th year he bought 30 acres of land, how many acres did he buy in the last year? [*Competency Based Question*]

(*a*) [5 + 11(5)] acres (*b*) [10 + 11(5)] acres

(*c*) [5 + 11(10)] acres (*d*) [5 + 12(10)] acres

Answers

1. (*c*)	**2.** (*c*)	**3.** (*a*)	**4.** (*b*)	**5.** (*b*)	**6.** (*d*)	**7.** (*b*)
8. (*a*)	**9.** (*c*)	**10.** (*b*)	**11.** (*c*)	**12.** (*b*)	**13.** (*a*)	**14.** (*d*)
15. (*c*)	**16.** (*a*)	**17.** (*b*)	**18.** (*d*)	**19.** (*c*)	**20.** (*b*)	

Very Short Answer Questions

Each of the following questions are of* 1 *mark.

1. For what value of k will $k + 9$, $2k - 1$ and $2k + 7$ are the consecutive terms of an AP?

[*CBSE* 2016(30/2)]

Sol.

2). We have-

Three consecutive terms of AP = $k+9, 2k-1, 2k+7$

Then, $(k+9)(2k+7) = 2(2k-1)$ $\{(a+c = 2b)\}$

$\Rightarrow k+9+2k+7 = 4k-2$

$3k+16 = 4k-2$

$16+2 = 4k-3k$

$\boxed{18 = k}$

[*Topper's Answer* 2016]

2. Write the common difference of the AP $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$ [*CBSE* 2019(30/5/2)]

Sol. Given AP, $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$

$\Rightarrow$ $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, 4\sqrt{3}, \ldots$

$\therefore$ Common difference $= 2\sqrt{3} - \sqrt{3} = \sqrt{3}$

$\Rightarrow$ $d = \sqrt{3}$

3. In an AP, if the common difference(d) = – 4, and the seventh term (a_7) is 4, then find the first term. [*CBSE 2018*(30/1/1)]

Sol.

$d = -4, \qua_7 = 4.$

$t_n = a + (n-1)d.$

$t_7 = a + (7-1)(-4)$

$4 = a + 6 \times (-4)$

$a = 24 + 4$

$a = 28.$

The first term is 28.

[*Topper's Answer* 2018]

4. If 7 times the 7th term of an AP is equal to 11 times its 11th term, then find its 18th term.

[CBSE (F) 2017]

Sol. Given, $7a_7 = 11a_{11}$

$\Rightarrow \quad 7(a + 6d) = 11(a + 10d) \quad$ or $\quad 7a + 42d = 11a + 110d$

$\Rightarrow \quad 4a + 68d = 0 \quad$ or $\quad a + 17d = 0$

Now, $a_{18} = a + 17d = 0$

5. Find the number of terms in the AP:

$$18, 15\frac{1}{2}, 13, \ldots, -47.$$

[CBSE 2019 (30/4/2)]

Sol. Here $-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$ Here, $d = -\frac{5}{2}$ ½

$\Rightarrow \quad n = 27$ ½

[CBSE Marking Scheme 2019(30/4/2)]

6. Find the 9th term from the end (towards the first term) of the AP 5, 9, 13, ..., 185.

[CBSE (Delhi) 2016]

Sol. $l = 185, d = 4$

$l_9 = l - (n-1)\,d = 185 - 8 \times 4 = 153$

7. Find the sum of first 10 multiples of 6. **[CBSE 2019(30/3/1)]**

Sol.

5. First 10 multiples of 6 form AP → 6, 12, 18 . . . . . 60.
(a = 6, l = 60)

Sum of 1st 10 multiples $= \frac{n}{2}[a + l]$

$= \frac{10}{2}[6 + 60]$

$= 330$

[Topper's Answer 2019]

8. Find the sum of the first 100 natural numbers. **[CBSE 2020 (30/5/1)]**

Sol. We have, Sum of first n natural numbers $= \frac{n(n+1)}{2}$

$\therefore$ Sum of first 100 natural numbers $= \frac{100(100+1)}{2} = \frac{100 \times 101}{2} = 50 \times 101 = 5050$

Short Answer Questions–I

Each of the following questions are of 2 marks.

1. Find the middle term of the AP 213, 205, 197, ..., 37. **[Competency Based Question]**

Sol. We have, $a = 213, d = 205 - 213 = -8$ and $l = 37$

Let n be the number of terms of the AP.

$\therefore \quad l = a + (n-1)d$

$\Rightarrow \quad 37 = 213 + (n-1) \times (-8) \quad \Rightarrow \quad (n-1) = \frac{-176}{-8} = 22 \quad \Rightarrow \quad n = 23$

$\therefore$ The middle term will be $\left(\frac{n+1}{2}\right)^{th}$ term

i.e., $\left(\frac{23+1}{2}\right)^{th} = 12^{th}$ term

$\therefore \quad a_{12} = a + 11d = 213 + 11(-8) = 125$

2. For what value of n, are the nth terms of two APs 63, 65, 67,... and 3, 10, 17,... equal?

[*CBSE* (AI) 2017]

Sol. Let nth terms for two given series be a_n and a'_n.

According to question,

$$a_n = a'_n \quad \Rightarrow \quad a + (n-1)d = a' + (n-1)d'$$

$$\Rightarrow \quad 63 + (n-1)2 = 3 + (n-1)7$$

$$\Rightarrow \quad 5n = 65 \quad \Rightarrow \quad n = 13.$$

3. If the ratio of sum of the first m and n terms of an AP is $m^2 : n^2$, show that the ratio of its mth and nth terms is $(2m-1) : (2n-1)$. **[*CBSE (F)* 2016, *Delhi* 2017]**

Sol. $$\frac{S_m}{S_n} = \frac{m^2}{n^2} = \frac{\frac{m}{2}}{\frac{n}{2}} \frac{(2a+(m-1)d)}{(2a+(n-1)d)}$$

$$\Rightarrow \quad \frac{m}{n} = \frac{2a+(m-1)d}{2a+(n-1)d} \quad \Rightarrow \quad 2am + mnd - md = 2an + mnd - nd$$

$$\Rightarrow \quad a(2m-2n) = d(m-n) \quad \Rightarrow \quad 2a = d$$

$$\frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+2(m-1)a}{a+2(n-1)a} = \frac{2m-1}{2n-1}$$ Hence proved.

4. Determine the AP whose third term is 16 and 7^{th} term exceeds the 5th term by 12.

[*CBSE* 2019 (30/4/2)]

Sol. Here $t_3 = 16$ and $t_7 = t_5 + 12$ ½

$\Rightarrow \quad a + 2d = 16 \quad ...(i)$ and $a + 6d = a + 4d + 12 \quad ...(ii)$ ½

From (ii), $d = 6$

From (i), $a = 4$ 1

$\therefore$ AP is 4, 10, 16, ...

[*CBSE Marking Scheme* 2019(30/4/2)]

5. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

[*CBSE (AI)* 2014]

Sol. Natural numbers between 101 and 999 divisible by both 2 and 5 are 110, 120, ... 990.

So, $a = 110, d = 10, a_n = 990$

We know, $a_n = a + (n-1)d$

$$990 = 110 + (n-1)\,10$$

$$(n-1) = \frac{990-110}{10} \quad \Rightarrow \quad n = 88 + 1 = 89$$

6. If the 17^{th} term of an AP exceeds its 10^{th} term by 7, find the common difference.

[*CBSE* 2019 (30/5/1)]

Sol. According to question,

$$a_{17} = a_{10} + 7$$

$$\Rightarrow \quad a + (17-1)\,d = a + (10-1)\,d + 7$$

$$\Rightarrow \quad a + 16d = a + 9d + 7$$

$$\Rightarrow \quad 16d - 9d = 7$$

$$\Rightarrow \quad 7d = 7 \quad \Rightarrow \quad d = 1$$

$\therefore$ Common difference = 1

7. Solve the equation: $1 + 5 + 9 + 13 + ... + x = 1326$. **[*CBSE* 2020 (30/3/1)]**

Sol. Given, $1 + 5 + 9 + 13 + \ldots + x = 1326$

It is an AP with $a = 1, d = 4, a_n = x = l$ and $S_n = 1326$

$$a_n = a + (n-1)d$$

$$x = 1 + (n-1) \times 4 \Rightarrow \frac{(x-1)}{4} = n - 1 \Rightarrow n = \frac{x-1}{4} + 1$$

$$\Rightarrow \quad n = \frac{x+3}{4}$$

Now, $S_n = \frac{n}{2}(a + l)$

$$\Rightarrow \quad 1326 = \frac{x+3}{8}(1 + x) \qquad \Rightarrow \quad \frac{(x+1)(x+3)}{8} = 1326$$

$$\Rightarrow \quad x^2 + 4x + 3 = 10608 \qquad \Rightarrow \quad x^2 + 4x - 10605 = 0$$

$$\therefore \quad x = \frac{-4 \pm \sqrt{16 + 42420}}{2} \qquad \Rightarrow \quad x = \frac{-4 \pm \sqrt{42436}}{2} = \frac{-4 \pm 206}{2}$$

$$x = \frac{-4 + 206}{2} = \frac{202}{2} = 101 \qquad \text{[Ignoring negative value]}$$

$$\therefore \quad x = 101$$

8. The first and the last term of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference. **[*CBSE Delhi* 2014]**

Sol. Let the first term be 'a' and common difference be 'd'.

Given, $a = 5, T_n = 45, S_n = 400$

$$T_n = a + (n-1)d \qquad \Rightarrow \quad 45 = 5 + (n-1)d$$

$$\Rightarrow \quad (n-1)\, d = 40 \qquad \ldots(i)$$

$$S_n = \frac{n}{2}(a + T_n) \qquad \Rightarrow \quad 400 = \frac{n}{2}(5 + 45)$$

$$\Rightarrow \quad n = 2 \times 8 = 16$$

Substituting the value of n in (i)

$$(16 - 1)d = 40 \qquad \Rightarrow \quad d = \frac{40}{15} = \frac{8}{3}$$

9. If S_n, the sum of the first n terms of an AP is given by $S_n = 2n^2 + n$, then find its n^{th} term. **[*CBSE* 2019 (30/5/1)]**

Sol. We have in an AP

$$S_n = 2n^2 + n$$

$$\therefore \quad S_{n-1} = 2(n-1)^2 + (n-1) = 2n^2 - 4n + 2 + n - 1$$

$$\Rightarrow \quad S_{n-1} = 2n^2 - 3n + 1$$

$$\therefore \quad \text{Its } n\text{th term, } t_n = S_n - S_{n-1} = 2n^2 + n - 2n^2 + 3n - 1$$

$$\Rightarrow \quad t_n = 4n - 1$$

Short Answer Questions–II

Each of the following questions are of* 3 *marks.

1. Find a, b and c if it is given that the numbers a, 7, b, 23, c are in AP.

[*NCERT Exemplar, CBSE* 2020 (30/2/1)]

Sol. Given that a, 7, b, 23, c are in AP.

$$\therefore \quad \text{Second term } (a_2) = 7$$

$$\Rightarrow \quad a + (2 - 1)\, d = 7$$

$\Rightarrow \quad a + d = 7 \quad ...(i)$

Also, 4th term $(a_4) = 23$

$\Rightarrow \quad a + 3d = 23 \quad ...(ii)$

From equation (i) and (ii), we have

$$\begin{array}{rrr} a + & d = & 7 \\ a + & 3d = & 23 \\ - & - & - \\ \hline & -2d = -16 & \Rightarrow \quad d = 8 \end{array}$$

Putting $d = 8$ in equation (i), we get

$a + 8 = 7 \quad \Rightarrow \quad a = -1$

Now, $b = 7 + d = 7 + 8 = 15$

$c = 23 + d = 23 + 8 = 31$

2. If m times the m^{th} term of an AP is equal to n times its n^{th} term, show that the $(m + n)^{th}$ term of the AP is zero. **[*CBSE* 2020 (30/3/1)]**

Sol. Given $m[a + (m - 1)\,d] = n[a + (n - 1)\,d]$ **1**

$\Rightarrow \quad a(m - n) + d(m^2 - m - n^2 + n) = 0$

$\Rightarrow \quad (m - n)\,[a + (m + n - 1)d] = 0$ **1**

$\because \quad m \neq n \Rightarrow a + (m + n - 1)\,d = 0$ **½**

$\Rightarrow \quad a_{m+n} = 0$ **½**

[*CBSE Marking Scheme 2020* (30/3/1)]

3. The digits of a positive number of three digits are in AP and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

[*CBSE* 2016 (AI) (30/2)]

Sol.

18) Let three digit of 3 digit no be - $a-d, a, a+d$.

Their sum = 15

$a-d+a+a+d = 15 \Rightarrow 3a = 15 \Rightarrow a = 5$

Required 3 digit no = $100(a-d)+10a+a+d$

$100a-100d+10a+a+d$

$111a-99d$

No. obtained by reversing digit = $100(a+d)+10a+a-d$

$100a+100d+10a+a-d$

$111a+99d$

ATQ –

$111a+99d = 111a-99d-594$

$\Rightarrow \quad 594 = 111a-99d-111a-99d$

$594 = -198d$

$\frac{-594}{198} = d$

$-3 = d.$

The no = $111a-99d$

$111\times5-99\times-3$

$555+297 = 852$

No. $\Rightarrow$ 852

[*Topper's Answer* 2016]

4. If the mth term of an AP is $\frac{1}{n}$ and nth term is $\frac{1}{m}$, then show that its (mn)th term is 1.

[*Competency Based Question*] [*CBSE* 2019(*C*) (30/1/1)]

Sol. Let a and d be the first term and common difference respectively of the given AP. Then

$$a_m = a + (m-1)\,d \quad \Rightarrow \quad a + (m-1)d = \frac{1}{n} \qquad ...(i)$$

and $$a_n = a + (n-1)\,d \quad \Rightarrow \quad a + (n-1)d = \frac{1}{m} \qquad ...(ii)$$

Subtracting (ii) from (i), we have

$$(m-n)\,d = \frac{1}{n} - \frac{1}{m} \quad \Rightarrow \quad (m-n)\,d = \frac{m-n}{mn}$$

$$\therefore \quad d = \frac{1}{mn}$$

Putting $d = \frac{1}{mn}$ in (i), we get

$$a + (m-1)\,\frac{1}{mn} = \frac{1}{n} \quad \Rightarrow \quad a + \frac{m}{mn} - \frac{1}{mn} = \frac{1}{n}$$

$$\Rightarrow \quad a - \frac{1}{mn} = 0 \quad \text{or} \quad a = \frac{1}{mn}$$

$$\therefore \quad mn\text{th term} = a + (mn-1)\,d = \frac{1}{mn} + (mn-1)\,\frac{1}{mn} = \frac{1 + mn - 1}{mn}$$

$$\Rightarrow \quad (mn)\text{th term} = 1$$

5. Which term of the arithmetic progression 3, 15, 27, 39 .... will be 120 more than its 21st term?

[*CBSE* 2019 (30/1/2)]

Sol. We have, $a = 3$ and $d = 12$

$$\therefore \quad a_{21} = a + 20d = 3 + 20 \times 12 = 243$$

Let nth term of the given AP be 120 more than its 21st term. Then,

$$a_n = 120 + a_{21}$$

$$\therefore \quad 3 + (n-1)d = 120 + 243$$

$$\Rightarrow \quad 3 + 12(n-1) = 363 \quad \Rightarrow \quad 12(n-1) = 360$$

$$\Rightarrow \quad n - 1 = 30 \quad \Rightarrow \quad n = 31$$

Hence, 31st term of the given AP is 120 more than its 21st term.

6. Show that the sum of all terms of an AP whose first term is a, the second term is b and the last term is c is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$. **[*Competency Based Question*] [*CBSE* 2020(30/2/1)]**

Sol. In an AP, we have

First term $= a$, Second term $= b$

$$\therefore \quad \text{Common difference, } d = b - a$$

and last term, $a_n = c$

$$\therefore \quad a_n = a + (n-1)\,d$$

$$\Rightarrow \quad c = a + (n-1)\,(b-a)$$

$$\Rightarrow \quad c - a = (n-1)\,(b-a)$$

$$\Rightarrow \quad n - 1 = \frac{c-a}{b-a} \quad \Rightarrow \quad n = \frac{c-a}{b-a} + 1$$

$$\Rightarrow \quad n = \frac{c-a+b-a}{b-a} = \frac{b+c-2a}{b-a}$$

$$\therefore \quad n = \frac{b+c-2a}{b-a}$$

$\therefore$ Sum of all terms of an AP be

$$S_n = \frac{n}{2}(a+l)$$

$$= \left(\frac{b+c-2a}{2(b-a)}\right)(a+c)$$

$$S_n = \frac{(a+c)(b+c-2a)}{2(b-a)}$$

7. The sum of the first 30 terms of an AP is 1920. If the fourth term is 18, find its 11th term. **[*CBSE* 2020(30/4/1)]**

Sol. $\frac{30}{2}[2a + 29d] = 1920$ **1**

$\Rightarrow \quad 2a + 29d = 128 \quad \ldots (i)$

Also, $a_4 = 18 \Rightarrow a + 3d = 18 \quad \ldots (ii)$ **½**

From equation (i) & (ii)

$a = 6, \quad d = 4$ **1**

$\therefore \quad a_{11} = a + 10d = 46$ **½**

[*CBSE Marking Scheme* 2020(30/4/1)]

8. Find the sum of all two digit natural numbers which are divisible by 4. **[*CBSE Delhi* 2017(*C*)]**

Sol. Here $a = 12, d = 4, a_n = 96$

The formula is $a_n = a + (n-1)d$

Therefore $96 = 12 + (n-1) \times 4$

$$\Rightarrow \quad 96 = 8 + 4n \quad \Rightarrow \quad n = \frac{88}{4} \quad \Rightarrow \quad n = 22$$

Apply the formula for sum,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Hence, $S_{22} = 11[24 + 21 \times 4] = 11[24 + 84]$

$$= 11 \times 108 = 1188.$$

9. If the ratio of the sum of first n terms of two AP's is $(7n + 1) : (4n + 27)$, find the ratio of their mth terms. **[*CBSE (AI)* 2016]**

Sol. $$\frac{S_n}{S'_n} = \frac{\frac{n}{2}(2a + (n-1)d}{\frac{n}{2}(2a' + (n-1)d')} = \frac{7n+1}{4n+27} \Rightarrow \frac{a + \frac{n-1}{2}d}{a' + \frac{n-1}{2}d'} = \frac{7n+1}{4n+27} \quad \ldots(i)$$

Since $\frac{t_m}{t'_m} = \frac{a + (m-1)d}{a' + (m-1)d'}$, So replacing $\frac{n-1}{2}$ by $m-1 \Rightarrow n$ by $2m-1$ in (i)

$$= \frac{a + (m-1)d}{a' + (m-1)d'} = \frac{7(2m-1)+1}{4(2m-1)+27} \quad \Rightarrow \quad \frac{t_m}{t'_m} = \frac{14m-6}{8m+23}$$

10. For an AP, it is given that the first term $(a) = 5$, common difference $(d) = 3$, and the n^{th} term $(a_n) = 50$. Find n and sum of first n terms (S_n) of the AP. **[*CBSE* 2020 (30/5/1)]**

Sol. We have, $a = 5, d = 3$ and $a_n = 50$

$\therefore \quad a_n = 50$

$\Rightarrow \quad a + (n-1)d = 50$

$\Rightarrow \quad 5 + (n-1) \times 3 = 50 \quad \Rightarrow \quad (n-1) \times 3 = 45$

$\Rightarrow \quad n - 1 = 15 \quad \Rightarrow \quad n = 16$

$\therefore \quad S_n = \frac{n}{2}(a + l) = \frac{16}{2}(5 + 50) = 8 \times 55 = 440$

11. Find the sum of all two digit natural numbers which when divided by 3 yield 1 as remainder.

[*Competency Based Question*]

Sol. Two digit natural numbers which when divided by 3 yield 1 as remainder are:

10, 13, 16, 19, ..., 97, which forms an AP.

with $a = 10, d = 3, a_n = 97$

$a_n = 97 \Rightarrow a + (n-1)\,d = 97$

or $\quad 10 + (n-1)3 = 97 \quad \Rightarrow \quad (n-1) = \frac{87}{3} = 29 \quad \Rightarrow \quad n = 30$

Now, $\quad S_{30} = \frac{30}{2}[2 \times 10 + 29 \times 3] = 15(20 + 87) = 15 \times 107 = 1605$

Long Answer Questions

Each of the following questions are of 5 marks.

1. The sum of four consecutive numbers in AP is 32 and the ratio of the product of the first and last terms to the product of two middle terms is 7 : 15. Find the numbers.

[*CBSE 2020* (30/1/1), 2018]

Sol. Let four consecutive numbers in AP are $a - 3d,\ a - d,\ a + d,\ a + 3d$.

$\therefore \quad$ Sum $= (a - 3d) + (a - d) + (a + d) + (a + 3d)$

$32 = 4a \quad \Rightarrow \quad a = \frac{32}{4} = 8$

ATQ, $\quad \frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

$\Rightarrow \quad \frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15} \quad \Rightarrow \quad \frac{8^2 - 9d^2}{8^2 - d^2} = \frac{7}{15}$

$\Rightarrow \quad 1 - \frac{8^2 - 9d^2}{8^2 - d^2} = 1 - \frac{7}{15}$ (Subtract from 1 both sides)

$\Rightarrow \quad \frac{8^2 - d^2 - 8^2 + 9d^2}{8^2 - d^2} = \frac{15 - 7}{15}$

$\Rightarrow \quad \frac{8d^2}{64 - d^2} = \frac{8}{15} \quad \Rightarrow \quad \frac{d^2}{64 - d^2} = \frac{1}{15}$

$\Rightarrow \quad 15d^2 = 64 - d^2 \quad \Rightarrow \quad 16d^2 = 64$

$\Rightarrow \quad d^2 = \frac{64}{16} = 4 \quad \therefore \quad d = \pm 2$

When $a = 8, d = 2$ then four numbers are

$8 - 6,\ 8 - 2,\ 8 + 2,\ 8 + 6$

$2,\ 6,\ 10,\ 14$

When $a = 8,\ d = -2$, the four numbers are

$14,\ 10,\ 6,\ 2$

2. Find the sum of the following series :

$5 + (-41) + 9 + (-39) + 13 + (-37) + 17 + ... + (-5) + 81 + (-3)$ **[*CBSE (F)* 2017]**

Sol. The series can be rewritten as,

$(5 + 9 + 13 + ... + 81) + (-41 + (-39) + (-37) + ... + (-5) + (-3))$

For the series $5 + 9 + 13 + ... 81$

$a = 5, d = 4$ and $a_n = 81$

nth term $= a + (n - 1)d = a_n$

$\Rightarrow \quad 5 + (n - 1)4 = 81$

$\Rightarrow \quad 4n = 80 \quad \Rightarrow \quad n = 20$

Sum of 20 terms for this series.

$$S_n = \frac{20}{2}(5 + 81) = 860 \qquad \left[\because S_n = \frac{n}{2}(a + a_n)\right] \qquad ...(i)$$

For the series $(-41) + (-39) + (-37) ... + (-5) + (-3)$

$a = -41, d = 2$ and $a_n = -3$

nth term $= a + (n - 1)d = a_n$

$\Rightarrow \quad -41 + (n - 1)2 = -3$

$\Rightarrow \quad 2n = 40 \quad \Rightarrow \quad n = 20$

Sum of 20 terms for this series

$$S_n = \frac{20}{2}(-41 - 3) = -440 \qquad ...(ii)$$

By adding (i) and (ii), we get

Sum of series $= 860 - 440 = 420$

3. If a^2, b^2, c^2, are in AP, prove that $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in AP. [*Competency Based Question*]

Sol. Since a^2, b^2, c^2, are in AP.

$\therefore \quad b^2 - a^2 = c^2 - b^2 \qquad ...(i)$

Now, $\quad \frac{b}{c+a} - \frac{a}{b+c} = \frac{b^2 + bc - ac - a^2}{(a+c)(b+c)}$

$$= \frac{(b^2 - a^2) + c(b - a)}{(a+c)(b+c)} = \frac{(b-a)(b+a+c)}{(a+c)(b+c)} \qquad ...(ii)$$

Also, $\quad \frac{c}{a+b} - \frac{b}{c+a} = \frac{c^2 + ac - ab - b^2}{(a+b)(c+a)} = \frac{c^2 - b^2 + a(c-b)}{(a+b)(c+a)} = \frac{(c-b)(c+b+a)}{(a+b)(a+c)}$

$$= \frac{(c^2 - b^2)(c+b+a)}{(a+b)(b+c)(c+a)}$$

[Multiplying numerator and denominator by $b + c$]

$$= \frac{(b^2 - a^2)(a+b+c)}{(a+b)(b+c)(c+a)} \quad \text{[Using } (i)\text{]}$$

$$= \frac{(b-a)(a+b+c)}{(b+c)(c+a)} \qquad ...(iii)$$

From (ii) and (iii), we have

$$\frac{b}{c+a} - \frac{a}{b+c} = \frac{c}{a+c} - \frac{b}{c+a}$$

i.e., $\quad \frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in AP.

4. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief? **[*CBSE (Delhi)* 2016]**

Sol. Let total time be n minutes.

Total distance covered by thief = $100\,n$ metres

Total distance covered by policeman = $100 + 110 + 120 + \ldots + (n-1)$ terms

$\therefore \quad 100n = \frac{n-1}{2}[100(2) + (n-2)10]$

$\Rightarrow \quad 200n = (n-1)(180 + 10n)$

$\Rightarrow \quad 10n^2 - 30n - 180 = 0$

$\Rightarrow \quad n^2 - 3n - 18 = 0$

$\Rightarrow \quad (n-6)(n+3) = 0 \quad \Rightarrow \quad n = 6$

Policeman took $(n-1) = (6-1) = 5$ minutes to catch the thief.

5. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses preceding the house numbered X is equal to sum of the numbers of houses following X. Find value of X. **[*Competency Based Question*]**

Sol. The numbers of houses are 1, 2, 3, 4.....49.

The numbers of the houses are in AP, where $a = 1$ and $d = 1$.

Sum of n terms of an AP = $\frac{n}{2}[2a + (n-1)d]$

Let X^{th} number house be the required house.

Sum of number of houses preceding X^{th} house is equal to S_{X-1} *i.e.*,

$$S_{X-1} = \frac{X-1}{2}[2a + (X-1-1)d] \quad \Rightarrow \quad S_{X-1} = \frac{X-1}{2}[2 + (X-2)]$$

$$S_{X-1} = \frac{X-1}{2}(2 + X - 2) \quad \Rightarrow \quad S_{X-1} = \frac{X(X-1)}{2}$$

Sum of numbers of houses following X^{th} house is equal to $S_{49} - S_X$

$$= \frac{49}{2}[2a + (49-1)d] - \frac{X}{2}[2a + (X-1)d]$$

$$= \frac{49}{2}(2 + 48) - \frac{X}{2}(2 + X - 1) = \frac{49}{2}(50) - \frac{X}{2}(X+1)$$

$$= 25(49) - \frac{X}{2}(X+1)$$

Now, we are given that

Sum of number of houses before X is equal to sum of number of houses after X.

i.e., $\quad S_{X-1} = S_{49} - S_X$

$$\Rightarrow \quad \frac{X(X-1)}{2} = 25(49) - X\frac{(X+1)}{2} \quad \Rightarrow \quad \frac{X^2}{2} - \frac{X}{2} = 1225 - \frac{X^2}{2} - \frac{X}{2}$$

$$\Rightarrow \quad X^2 = 1225 \quad \Rightarrow \quad X = \sqrt{1225} \quad \Rightarrow \quad X = \pm 35$$

Since number of houses is positive integer therefore $X = 35$.

6. If the sum of the first p terms of an AP is q and the sum of the first q terms is p; then show that the sum of the first $(p+q)$ terms is $-(p+q)$. **[*CBSE* 2019 (30/5/1)]**

Sol. Let first term be a and common difference of the AP be d.

Given, $S_p = q \quad \Rightarrow \quad \frac{p}{2}(2a + (p-1)d) = q$

$$\Rightarrow \quad 2a + (p-1)d = \frac{2q}{p} \qquad \ldots(i)$$

Also $S_q = p \quad \Rightarrow \quad \frac{q}{2}\{2a + (q-1)d\} = p$

$\Rightarrow \quad 2a + (q-1)d = \frac{2p}{q}$ ...(*ii*)

On subtracting (*ii*) from (*i*), we have

$$(p-q)d = \frac{2q}{p} - \frac{2p}{q} = \frac{2(q^2 - p^2)}{pq}$$

$\Rightarrow \quad d = \frac{-2(p+q)}{pq}$ ...(*iii*)

On putting the value of d in (*i*), we get

$$2a + (p-1) \times \left\{\frac{-2(p+q)}{pq}\right\} = \frac{2q}{p}$$

$\Rightarrow \quad a = \frac{q}{p} + \frac{(p+q)(p-1)}{pq}$ ...(*iv*)

Now, $S_{p+q} = \frac{p+q}{2}(2a + (p+q-1)d)$

$= \frac{p+q}{2}\left(2\frac{q}{p} + \frac{2(p+q)(p-1)}{pq} + (p+q-1) \times \frac{-2(p+q)}{pq}\right)$ (from (*iii*) and (*iv*))

$= (p+q)\left(\frac{q}{p} + \frac{(p+q)(p-1)}{pq} - \frac{(p+q-1)(p+q)}{pq}\right)$

$= (p+q)\left(\frac{q}{p} - \frac{(p+q)}{p}\right) = -(p+q)$

$\therefore \quad S_{p+q} = -(p+q)$

Case Study-based Questions

Each of the following questions are of 4 marks.

1. **Read the following and answer any four questions from (*i*) to (*v*).**

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year. **[*CBSE Question Bank*]**

(i) What is the production during first year?

(a) 2200 (b) 2500 (c) 5000 (d) 8000

(ii) What is the production during 8th year?

(a) 11,000 (b) 20,400 (c) 22,000 (d) 20,000

(iii) Total production in (during) first 3 years is

(a) 21,600 (b) 22,000 (c) 25,000 (d) 30,000

(iv) In which year, the production is ₹29,200?

(a) 9th (b) 12th (c) 14th (d) 15th

(v) The difference of the production during 7th year and 4th year is

(a) 1100 (b) 2200 (c) 4400 (d) 6600

Sol. (i) Let a be the production of TV sets in a factory in first year and d be the fixed number of TV sets increased every year.

Thus we have,

$$a + (6-1)d = 16000 \Rightarrow \quad a + 5d = 16000 \qquad ...(i)$$

$$a + (9-1)d = 22600 \Rightarrow \quad a + 8d = 22600 \qquad ...(ii)$$

Subtracting (i) from (ii), we have

$$3d = 6600 \Rightarrow d = \frac{6600}{3}$$

$$\Rightarrow d = 2200$$

From equation (i), we have

$$a + 5 \times 2200 = 16000 \Rightarrow a = 16000 - 11000$$

$$\Rightarrow a = 5000$$

We have the production during the first year = 5000

$\therefore$ Option (c) is correct.

(ii) Production during 8th year, $a_8 = a + (8-1)d$

$$= a + 7d$$
$$= 5000 + 7 \times 2200$$
$$= 5000 + 15400$$
$$= 20400$$

$\therefore$ Option (b) is correct.

(iii) Total production in (during) first 3 years

$$= \frac{n}{2}(2a + (n-1)d)$$
$$= \frac{3}{2}(2 \times 5000 + 2 \times 2200)$$
$$= 3(5000 + 2200) = 3 \times 7200 = 21600$$

$\therefore$ Option (a) is correct.

(iv) Let in n^{th} year, the production is 29200.

$\Rightarrow \quad a_n = 29200 \Rightarrow \; a + (n-1)d = 29200$

$\Rightarrow \quad 5000 + (n-1) \times 2200 = 29200$

$\Rightarrow \quad (n-1) \times 2200 = 24200$

$\Rightarrow \quad (n-1) = \frac{24200}{2200} = 11$

$\Rightarrow \quad n = 12$

$\therefore$ In 12th year the production is 29200.

$\therefore$ Option (b) is correct.

(v) Difference of the production during 7th year and 4th year

$= (a + 6d) - (a + 3d) = 3d$

$= 3 \times 2200 = 6600$

$\therefore$ Option (d) is correct.

2. Read the following and answer any four questions from (*i*) to (*v*).

Your friend Veer wants to participate in a 200 m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to complete it in 31 seconds. **[*CBSE Question Bank*]**

(*i*) Which of the following terms are in AP for the given situation?

(a) 51,53,55.... (b) 51, 49, 47.... (c) –51, –53, –55.... (d) 51, 55, 59...

(*ii*) What is the minimum number of days he needs to practice till his goal is achieved? **[*Competency Based Question*]**

(a) 10 (b) 12 (c) 11 (d) 9

(*iii*) Which of the following term is not in the AP of the above given situation?

(a) 41 (b) 30 (c) 37 (d) 39

(*iv*) If nth term of an AP is given by $a_n = 2n + 3$ then common difference of the AP is

(a) 2 (b) 3 (c) 5 (d) 1

(*v*) The value of x, for which $2x, x + 10, 3x + 2$ are three consecutive terms of the AP is

(a) 6 (b) –6 (c) 18 (d) –18

Sol. (i) Since Veer currently runs the distance in 51 seconds and with each day of practice it takes him 2 seconds less.

Thus, AP formed is

51, 49, 47, ...

$\therefore$ Option (b) is correct.

(*ii*) Let n be the minimum number of days for AP. 51, 49, 47 .....

$\therefore \quad a_n = 31 \quad \Rightarrow \quad a + (n-1)d = 31$

$\Rightarrow \quad 51 + (n-1) \times (-2) = 31$

$\Rightarrow \quad (n-1) \times (-2) = -20$

$\Rightarrow \quad n - 1 = 10$

$\Rightarrow \quad n = 11$

$\therefore$ Option (*c*) is correct.

(*iii*) Since $a_n = 30 \quad \Rightarrow \quad a + (n-1)d = 30$

$\Rightarrow \quad 51 + (n-1) \times (-2) = 30$

$\Rightarrow \quad (n-1) \times (-2) = -21$

$\Rightarrow \quad n - 1 = \frac{21}{2}$

$\Rightarrow \quad n = \frac{23}{2} \neq$ integer

$\therefore$ 30 is not in AP.

$\therefore$ Option (*b*) is correct.

(*iv*) Given n^{th} term of an AP is

$a_n = 2n + 3$

$\therefore \quad d = a_n - a_{n-1} = (2n + 3) - (2(n-1) + 3)$

$\Rightarrow \quad d = 2$

$\therefore$ Option (*a*) is correct.

(*v*) Since, $2x, x + 10, 3x + 2$ are three consecutive terms are in AP.

$\therefore \quad (x + 10) - 2x = (3x + 2) - (x + 10)$

$\Rightarrow \quad 10 - x = 2x - 8$

$\Rightarrow \quad 18 = 3x \quad \Rightarrow \quad x = 6$

$\therefore$ Option (*a*) is correct.

3. Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of ₹ 1,18,000 by paying every month starting with the first instalment of ₹ 1000. He increases the instalment by ₹ 100 every month.

Based on the above information, answer the following questions:

(*i*) What is the amount paid by him in 30th instalment?

(*ii*) What is the amount paid by him upto 30 instalments?

Sol. (*i*) Since first instalment is of ₹1000 and he increases the instalment by ₹100 every month.

Thus, AP is formed

1000, 1100, 1200, ...

$\therefore a = 1000, d = 100$

Amount paid by him in 30^{th} instalment

$= a + 29d$

$= 1000 + 29 \times 100 = ₹3900$

(*ii*) Amount paid upto 30 instalments

$$= \frac{n}{2}(2a + (n-1)d)$$

$$= \frac{30}{2}(2 \times 1000 + 29 \times 100)$$

$$= \frac{30}{2} \times 4900 = 15 \times 4900 = ₹\,73500$$

PROFICIENCY EXERCISE

■ Objective Type Questions: [1 mark each]

1. Choose and write the correct option in each of the following questions.

(*i*) What are the missing numbers in the pattern given below?

–32, –19.5, –7, ___, 18, 30.5, ___

(*a*) 5.5 and 43 (*b*) 19.5 and 43 (*c*) 5 and 43 (*d*) 5 and 43.5

(*ii*) If the first term of an AP is 2 and the common difference is $\frac{-1}{2}$, what would be the 12th term of the AP?

(*a*) $2+11\left(\frac{1}{2}\right)$ (*b*) $2+11\left(-\frac{1}{2}\right)$ (*c*) $2+12\left(\frac{1}{2}\right)$ (*d*) $2+12\left(-\frac{1}{2}\right)$

(*iii*) Which term of the AP 21, 42, 63, 84 ..... is 210?

(*a*) 9^{th} (*b*) 10^{th} (*c*) 11^{th} (*d*) 12^{th}

(*iv*) Mr. Prasad is collecting donations for a new school building in a village. His wife starts by donating ₹ 1,25,000. His brother also contributes by donating ₹ 54,000. Thereafter every person who contributes in the donation pays ₹ 5,250 more than the previous donor. How much amount was Mr. Prasad able to collect after 23 donations?

(*a*) ₹ 1,64,250 (*b*) ₹ 24,00,750 (*c*) ₹ 25,25,750 (*d*) ₹ 25,70,250

(*v*) Sum of first five multiples of 5 is

(*a*) 45 (*b*) 55 (*c*) 65 (*d*) 75

■ Very Short Answer Questions: [1 mark each]

2. Find the common difference of the AP

$$\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, \ldots (a \neq 0)$$ **[*CBSE* 2019 (30/2/1)]**

3. Find the number of terms in the AP $18, 15\frac{1}{2}, 13, \ldots, -47$. **[*CBSE* 2019 (30/4/2)]**

4. If the n^{th} term of an AP is $pn + q$, find its common difference. **[*CBSE* 2019 (*C*) (30/1/1)]**

5. If in an AP, $a = 15$, $d = -3$ and $a_n = 0$, then find the value of n. [*CBSE* 2019 (30/5/1)]

6. Which term of the AP 4, 7, 10, ... is 64? [*CBSE* 2019 (*C*) (30/1/2)]

7. How many terms of the AP 27, 24, 21 ..... should be taken so that their sum is zero? [*CBSE Delhi* 2016]

8. For what value of k will the consecutive terms $2k + 1$, $3k + 3$ and $5k - 1$ form an AP? [*CBSE (F)* 2016]

9. Write the n^{th} term of the AP $\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, \ldots\ldots$ [*Competency Based Question*]

■ Short Answer Questions–I: [2 marks each]

10. If the 9^{th} term of an AP is zero, then show that its 29^{th} term is double of its 19^{th} term. [*CBSE 2019 (C)*(30/1/1)]

11. Determine k so that $k^2 + 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$ are three consecutive terms of an AP.

12. Write 7th term from the end of the AP 7, 9, 11, 13, ... 213.

13. Find whether –150 is a term of the AP 11, 8, 5, 2, ... [*CBSE Delhi* 2017 (*C*)]

14. Determine the AP whose third term is 16 and 7^{th} term exceeds the 5^{th} term by 12. [*CBSE* 2019(30/4/2)]

15. If the sum of first n terms of an AP is given by $S_n = 3n^2 - 4n$, find the nth term. [*CBSE* 2019 30/1/1)]

16. Find the sum of $7 + 10 + 13 + \ldots + 46$. [*CBSE* 2019 (*C*) (30/1/1)]

17. The 10^{th} term of an AP is (– 4) and its 22^{nd} term is (–16). Find its 38^{th} term. [*CBSE Delhi* 2017(*C*)]

■ Short Answer Questions–II: [3 marks each]

18. In an AP if sum of its first n terms is $3n^2 + 5n$ and its k^{th} term is 164, find the value of k.

19. Determine the AP whose 5th term is 19 and the difference of the 8th term and the 13th term is 20. [*NCERT Exemplar*]

20. How many three digit numbers are divisible by 9? [*Competency Based Question*]

21. Which term of the AP 3, 12, 21, 30,.... will be 90 more than its 50th term? [*CBSE Delhi* 2017 (C)]

22. Find the sum of all odd numbers between 0 and 50. [*CBSE* 2019 (*C*)(30/1/1)]

23. Find the sum of all three digit natural numbers, divisible by 7.

24. Solve the equation: $-4 + (-1) + 2 + \ldots + x = 437$ [*NCERT Exemplar*]

25. Find the sum:

$$\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \ldots \text{ upto } n \text{ terms.}$$ [*Competency Based Question*]

26. Split 207 into three parts such that these are in AP and the product of the two smaller parts is 4623. [*NCERT Exemplar*]

■ Long Answer Questions: [5 marks each]

27. How many terms of the Arithmetic Progression 45, 39, 33, ... must be taken so that their sum is 180? Explain the double answer. [*CBSE* 2019 (30/2/1)]

28. If the ratio of the sum of the first n terms of two APs is $(7n + 1) : (4n + 27)$, then find the ratio of their 9^{th} terms. [*CBSE (AI)* 2017]

29. Find the sum of all the two digit numbers which leave the remainder 2 when divided by 5. [*CBSE* 2019 (30/5/2)]

30. Sums of the first p, q, r terms of an AP are a, b, c respectively. Prove that

$$\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0.$$ **[*Competency Based Question*]**

31. The ratio of the 11th term to the 18th term of an AP is 2 : 3. Find the ratio of the 5th term to the 21st term, and also the ratio of the sum of the first five terms to the sum of the first 21 terms. **[*NCERT Exemplar*]**

32. Ramkali required ₹2500 after 12 weeks to send her daughter to school. She saved ₹100 in the first week and increased her weekly saving by ₹20 every week. Find whether she will be able to send her daughter to school after 12 weeks. **[*CBSE Delhi* 2015]**

Answers

1. (*i*) (*a*) (*ii*) (*b*) (*iii*) (*b*) (*iv*) (*c*) (*v*) (*d*)

2. $-\frac{1}{3}$ **3.** $n = 27$ **4.** $d = p$ **5.** $n = 6$ **6.** $n = 21$ **7.** $n = 19$

8. $k = 6$ **9.** $\frac{1}{m}+n-1$ **11.** $k = 0$ **12.** 201 **13.** No

14. AP is 4, 10, 16 **15.** $6n - 7$ **16.** $S_n = 371$ **17.** -32 **18.** $k = 27$

19. 3, 7, 11,15..... **20.** $n = 100$ **21.** 60^{th} term **22.** 625 **23.** 70336

24. $x = 50$ **25.** $\frac{7n-1}{2}$ **26.** 67, 69, 71 **27.** $n = 6$ or $n = 10$ **28.** 24 : 19

29. $S_n = 981$ **31.** 1 : 3; 5 : 49 **32.** yes

Self-Assessment

Time allowed: 1 hour Max. marks: 40

SECTION A

1. Choose and write the correct option in the following questions. **(3 × 1 = 3)**

(*i*) The 4th term from the end of the AP –11, – 8, – 5, ... 49 is

(*a*) 37 (*b*) 40 (*c*) 43 (*d*) 58

(*ii*) The sum of first 20 odd natural numbers is

(*a*) 281 (*b*) 285 (*c*) 400 (*d*) 421

(*iii*) If the first term of an AP is –5 and the common difference is 2, then the sum of the first 6 terms is **[*NCERT Exemplar*]**

(*a*) 0 (*b*) 5 (*c*) 6 (*d*) 15

2. Solve the following questions. **(2 × 1 = 2)**

(*i*) Which term of the AP – 4, – 1, 2, ... is 101? **[*CBSE* 2019 (30/4/3)]**

(*ii*) Find the sum of first 10 multiples of 6. **[*CBSE* 2019 (30/3/1)]**

SECTION B

■ **Solve the following questions.** **(4 × 2 = 8)**

3. Find the 25^{th} term of the AP $-5, \frac{-5}{2}, 0, \frac{5}{2}$.... **[*CBSE* 2009]**

4. The 4th term of an AP is zero. Prove that the 25th term of the AP is three times its 11th term. **[*CBSE (AI)* 2016]**

5. Find how many integers between 200 and 500 are divisible by 8. **[*CBSE Delhi* 2017]**

6. The sum of the first n terms of an AP is $4n^2 + 2n$. Find the n^{th} term of this AP. **[*CBSE (F)* 2014]**

■ **Solve the following questions.** **(4 × 3 = 12)**

7. If m^{th} term of an AP is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, then find the sum of its first mn terms. **[*Competency Based Question*]**

8. What is the sum of all the three-digit numbers divisible by 12?

9. The sum of first 76 terms of an AP is 21850 and the sum of first 40 terms is 7900. Find the sum of first 100 terms of this AP.

10. If $1 + 4 + 7 + 10 + ... + x = 287$, find the value of x. **[*CBSE (F)* 2017]**

■ **Solve the following questions.** **(3 × 5 = 15)**

11. An arithmetic progression 5, 12, 19, ..., has 50 terms. Find its last term. Hence find the sum of its last 15 terms. **[*CBSE (AI)* 2015]**

12. Ranveer likes to collect stamps. He has a total of 4290 stamps after 10 years of his stamp collection. He counted his total stamps in the 6th year as well and found that he had a total of 1830 stamps. How many stamps did he collect in the last year of his collection? **[*Competency Based Question*]**

13. The sum of the first five term of an AP is 55 and sum of the first ten terms of this AP is 235, find the sum of its first 20 terms.

Answers

1. (*i*) (*b*) (*ii*) (*c*) (*iii*) (*a*) **2.** (*i*) 36^{th} term (*ii*) 330

3. $a_{25} = 55$ **5.** 37 **6.** $8n - 2$ **7.** $\frac{1}{2}(mn+1)$ **8.** 41400 **9.** 34750

10. $x = 40$ **11.** 348;4485 **12.** 708 **13.** 970