

**CBSE Board**  
**Class XII Mathematics**  
**Sample Paper 3**

**Time: 3 hrs**

**Total Marks: 100**

---

**General Instructions:**

1. All the questions are **compulsory**.
  2. The question paper consists of **37** questions divided into **three parts** A, B, and C.
  3. **Part A** comprises of **20** questions of **1 mark** each. **Part B** comprises of **11** questions of **4 marks** each. **Part C** comprises of **6** questions of **6 marks** each.
  4. There is no overall choice. However, an internal choice has been provided in **three questions of 4 marks** each, **four questions of 6 marks** each. You have to attempt only one of the alternatives in all such questions.
  5. Use of calculator is **not** permitted.
- 

**Part A**

**Q1 – Q20 are multiple choice type questions. Select the correct option.**

1. If A and B are square matrices of order 2, then  $\det(A + B) = 0$  is possible only when
  - A.  $\det(A) = 0$  or  $\det(B) = 0$
  - B.  $\det(A) + \det(B) = 0$
  - C.  $\det(A) = 0$  and  $\det(B) = 0$
  - D.  $A + B = 0$
2. If  $a \cdot b = a \cdot c$  and  $a \times b = a \times c$ ,  $a \neq 0$ , then
  - A.  $b = c$
  - B.  $b = 0$
  - C.  $b + c = 0$
  - D.  $b - c = 0$
3. If one ball is drawn at random from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, then the probability that 2 white and 1 black balls will be drawn is
  - A.  $\frac{13}{32}$
  - B.  $\frac{1}{4}$
  - C.  $\frac{1}{32}$

D.  $\frac{3}{16}$

4. The point which does not lie in the half plane  $3x + y \leq 6$  is

- A. (1, 2)
- B. (2, 1)
- C. (1, 3)
- D. (-3, 1)

5. If  $\tan^{-1} \left\{ \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right\} = \alpha$ , then  $x^2 =$

- A.  $\cos 2\alpha$
- B.  $\cos \alpha$
- C.  $\sin 2\alpha$
- D.  $\sin \alpha$

6. The set of points on which the function  $f(x) = |x - 3| \cos x$  is differentiable is

- A.  $\mathbb{R}$
- B.  $\mathbb{R} - \{0\}$
- C.  $\mathbb{Z}$
- D.  $\mathbb{R} - \{3\}$

7. If  $S = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , then  $\text{adj} A$  is

- A.  $\begin{bmatrix} d & c \\ b & a \end{bmatrix}$
- B.  $\begin{bmatrix} -d & c \\ b & -a \end{bmatrix}$
- C.  $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
- D.  $\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$

8. The equation of the normal to the curve  $y = \sin x$  at  $(0, 0)$  is

- A.  $x + y = 0$
- B.  $x - y = 0$
- C.  $y = 0$
- D.  $x = 0$

9. The unit vector perpendicular to the plane passing through the points

$P(\hat{i} - \hat{j} + 2\hat{k})$ ,  $Q(2\hat{i} - \hat{k})$  and  $R(2\hat{j} + \hat{k})$  is

- A.  $8\hat{i} + 4\hat{j} + 4\hat{k}$
- B.  $\frac{1}{\sqrt{6}}(2\hat{i} + \hat{j} + \hat{k})$
- C.  $\frac{1}{2\sqrt{6}}(-5\hat{i} + 2\hat{j} - \hat{k})$
- D.  $\frac{1}{\sqrt{6}}(-5\hat{i} + 2\hat{j} - \hat{k})$

10.  $\int_0^{\pi} \frac{1}{1 + \sin x} dx$  equal

- A. 0
- B.  $\infty$
- C. -2
- D. 2

11.  $\int \frac{1}{1 + e^x} dx$  equals

- A.  $-\log |e^{-x}| + c$
- B.  $\log |1 + e^{-x}| + c$
- C.  $-\log |1 + e^{-x}| + c$
- D.  $\log |e^{-x}| + c$

12. Let  $*$  be a binary operation defined by  $a * b = 3a + 4b - 2$ . Find  $2 * 7$ .

- A. 14
- B. 27
- C. 36
- D. 32

13. If  $A = \{a, b, c\}$  and  $B = \{1, 2, 3\}$  and a function  $f: A \rightarrow B$  is given by  $f = \{(a, 2), (b, 3), (c, 1)\}$ . Then  $f$  is:

- A. One to one
- B. Onto
- C. Bijective
- D. Real

14. Differentiate  $e^{x^2 + \tan x}$  with respect to  $x$ .

- A.  $e^{x^2 + \tan x} (2x + \sec^2 x)$
- B.  $2x + \sec^2 x$

C.  $e^{\tan x} (2x + \sec^2 x)$

D.  $e^{x^2} (2x + \sec^2 x)$

15. Find the angle between the curves  $y^2 = x$  and  $x^2 = y$  at  $(1, 1)$ .

A.  $\tan^{-1} \left( \frac{5}{4} \right)$

B.  $\tan^{-1} \left( \frac{3}{4} \right)$

C.  $\frac{\pi}{2}$

D.  $\frac{3\pi}{2}$

16. Find the equation of a line through  $(-2, 1, 3)$  and parallel to  $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ .

A.  $\frac{x+3}{-2} = \frac{y-4}{1} = \frac{z+8}{3}$

B.  $\frac{x-3}{-2} = \frac{y-5}{1} = \frac{z-6}{3}$

C.  $\frac{x+2}{3} = \frac{y-1}{5} = \frac{z-3}{6}$

D.  $\frac{x+2}{-2} = \frac{y-1}{1} = \frac{z-3}{3}$

17. Area of the region bounded by the circle  $x^2 + y^2 = 4$  in the 1<sup>st</sup> quadrant is equal to

A.  $\frac{\pi}{2}$  units

B.  $\left( 2 + \frac{\pi}{2} \right)$  units

C.  $2\pi$  units

D.  $\pi$  units

18. Find solution of the differential equation  $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$

A.  $\tan^{-1} y = 1 + \frac{x^2}{2} + C$

B.  $\tan^{-1} y = x + \frac{x^3}{3} + C$

C.  $\tan^{-1} x = 1 + \frac{y^2}{2} + C$

D.  $\tan^{-1} x = y + \frac{y^3}{3} + C$

19. Find the principal value of  $\cot^{-1} \left( \sin \left( -\frac{\pi}{2} \right) \right)$

A.  $\frac{3\pi}{4}$

B.  $\frac{\pi}{4}$

C.  $\frac{5\pi}{4}$

D.  $\frac{3\pi}{2}$

20. A line makes angles  $30^\circ$ ,  $60^\circ$  and  $90^\circ$  with the positive directions of the x, y and z axes respectively. Find its direction cosines.

A.  $1, \frac{\sqrt{3}}{2}$  and  $\frac{1}{2}$

B.  $0, \sqrt{3}$  and  $\frac{1}{\sqrt{3}}$

C.  $1, \sqrt{3}$  and  $\frac{1}{\sqrt{3}}$

D.  $0, \frac{1}{2}$  and  $\frac{\sqrt{3}}{2}$

## Part B

21. Differentiate  $\frac{x^3 \sqrt{5+x}}{(7-3x)^5 \sqrt[3]{8+5x}}$  w.r.t x.

**OR**

If  $y = a \cos(\log x) + b \sin(\log x)$ , prove that  $x^2 y'' + xy' + y = 0$ .

22. Find the particular solution of the differential equation  $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$  given that  $y = 1$  when  $x = 0$ .

23. Evaluate:  $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$

**OR**

Evaluate:  $\int_0^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x) dx$

24. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	K	2k	2k	3k	k <sup>2</sup>	2k <sup>2</sup>	7k <sup>2</sup> + k

Determine: (i) k (ii)  $P(X < 3)$  (iii)  $P(X > 5)$  (iv)  $P(1 \leq X < 3)$

25. Solve the following for x:

$$\sin^{-1}(1-x) - 2 \sin^{-1} x = \frac{\pi}{2}$$

**OR**

Solve the Equation:

$$\tan^{-1} \left( \frac{1-x}{1+x} \right) = \frac{1}{2} \tan^{-1} x, (x > 0)$$

26. Find the equation of a plane that is parallel to the x-axis and passes through the line common to two intersecting planes  $\vec{r} \cdot \hat{i} + \hat{j} + \hat{k} - 1 = 0$  and  $\vec{r} \cdot 2\hat{i} + 3\hat{j} - \hat{k} = -4$ .

27. Find the Equation of the plane passing through the point (2, 5, -8), perpendicular to the plane  $2x - 3y + 4z + 1 = 0$ , and  $4x + y - 2z + 6 = 0$ .

28. If  $f: \mathbb{R} \rightarrow A, A = \{x : x \in \mathbb{R}, -1 < x < 1\}, f(x) = \frac{x}{1 + |x|}, x \in \mathbb{R}$ . Show that the function f is a bijective function

29. If x, y and z are different and  $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$  then show that  $xyz = -1$ .

30. Evaluate:  $\int \frac{x^2}{x^4 + x^2 - 2} dx$

31. For what value of k is the following function continuous at  $x = 1$

$$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1}, & x \neq 1 \\ 4k, & x = 1 \end{cases}$$

### Part C

32. Two trainee carpenters A and B earn Rs. 150 and Rs. 200 per day respectively. A can make 6 frames and 4 stools per day while B can make 10 frames and 4 stools per day. How many days shall each work, if it is desired to produce atleast 60 frames and 32 stools at a minimum labour cost? Solve the problem graphically.

OR

A brick manufacturer has two depots, A and B, with stocks of 30,000 and 20,000 bricks respectively. He receives orders from three builders P, Q and R for 15,000, 20,000 and 15,000 bricks respectively. The cost in rupees for transporting 1000 bricks to the builders from the depots are given below:

To \ From	P	Q	R
A	40	20	30
B	20	60	40

How should the manufacturer fulfil the orders so as to keep the cost of transportation minimum?

33. Calculate the area

(i) between the curves  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , and the x-axis between  $x = 0$  to  $x = a$

(ii)

Triangle AOB is in the first quadrant of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , where OA = a and OB = b.

Find the area enclosed between the chord AB and the arc AB of the ellipse

(iii) Find the ratio of the two areas found.

34. Show that the right circular cone of least curved surface and given volume has an altitude equal to  $\sqrt{2}$  times the radius of the base.

**OR**

Find the points at which the function  $f$  given by  $f(x) = (x - 2)^4(x + 1)^3$  is minimum.

35. In answering a question on a MCQ test with 4 choices per question, a student knows the answer, guesses it or copies the answer. Let  $\frac{1}{2}$  be the probability that he knows the answer,  $\frac{1}{4}$  be the probability that he guesses and  $\frac{1}{4}$  be the probability that he copies it.

Assuming that a student, who copies the answer, will be correct with the probability  $\frac{3}{4}$ , what is the probability that student knows the answer, given that he answered it correctly?

Arjun does not know the answer to one of the questions in the test. The evaluation process has negative marking. Which value would Arjun violate if he resorts of unfair means? How would an act like the above hamper his character development in the coming years?

**OR**

Two bags A and B contain 3 red and 4 black balls, and 4 red and 5 black balls respectively. From bag A, one ball is transferred to bag B and then a ball is drawn from bag B. The ball is found to be red in colour. Find the probability that

- The transferred ball is black?
- The transferred ball is red?

36. If  $A = \begin{bmatrix} 0 & -\tan\left(\frac{x}{2}\right) \\ \tan\left(\frac{x}{2}\right) & 0 \end{bmatrix}$  and  $I$  is the identity matrix of order 2, show that  $I + A =$

$$(I - A) \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}.$$

37. Find the equation plane passing through points  $(1, 2, 3)$ ,  $(0, -1, 0)$  and parallel to the line  $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$ .

**OR**

Find the Cartesian equation of the plane passing through the points  $A(0, 0, 0)$  and  $B(3, -1, 2)$  and parallel to the line  $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$



**CBSE Board**  
**Class XII Mathematics**  
**Sample Paper 3 - Solution**

---

**Part A**

**1. Correct option: D**

**Explanation:-**

Determinant A denoted as  $[a_{ij}]$  and determinant B

as  $[b_{ij}]$

$$\Rightarrow A + B = [a_{ij}] + [b_{ij}]$$

$$\Rightarrow A + B = [a_{ij} + b_{ij}]$$

$$\Rightarrow \det(A + B) = \det[a_{ij} + b_{ij}]$$

$$\det(A + B) = 0 \dots \text{(Given)}$$

$$\Rightarrow \det[a_{ij} + b_{ij}] = 0$$

$$\Rightarrow a_{ij} + b_{ij} = 0$$

$$\Rightarrow A + B = 0$$

**2. Correct option: A**

**Explanation:-**

$$\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} \text{ and } \vec{a} \times \vec{b} = \vec{a} \times \vec{c}$$

$$\Rightarrow \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} = 0 \text{ and } \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = 0$$

$$\Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0 \text{ and } \vec{a} \times (\vec{b} - \vec{c}) = 0$$

$$\text{Obviously } \vec{b} - \vec{c} = 0 \Rightarrow \vec{b} = \vec{c}.$$

Also,

$$\Rightarrow |\vec{a}| |\vec{b} - \vec{c}| \cos \theta = 0 \text{ and } |\vec{a}| |\vec{b} - \vec{c}| \sin \theta = 0$$

$$\text{If } \theta = \frac{\pi}{2} \Rightarrow \sin \theta = 1 \Rightarrow \vec{b} - \vec{c} = 0 \Rightarrow \vec{b} = \vec{c}$$

**3. Correct option: A**

**Explanation:-**

Total balls in 1<sup>st</sup> box = 3 white + 1 black = 4

Total balls in 2<sup>nd</sup> box = 2 white + 2 black = 4

Total balls in 3<sup>rd</sup> box = 1 white + 3 black = 4

Probability of 2 white and 1 black balls = P(WWB) + P(WBW) + P(BWW)

$$\begin{aligned}
 &= \frac{3}{4} \times \frac{2}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{2}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{2}{4} \times \frac{1}{4} \\
 &= \frac{18 + 6 + 2}{64} = \frac{13}{32}
 \end{aligned}$$

**4. Correct option: B**

**Explanation:-**

On substituting (2, 1) in  $3x + y \leq 6$ , we get

$7 \leq 6$ , which is not true

Hence, (2, 1) does not lie in the half plane  $3x + y \leq 6$ .

**5. Correct option: C**

**Explanation:-**

$$\tan^{-1} \left\{ \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right\} = \alpha$$

$$\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} = \tan \alpha$$

$$\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \times \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} = \tan \alpha$$

$$\frac{1 - \sqrt{1-x^4}}{x^2} = \tan \alpha$$

$$1 - \sqrt{1-x^4} = x^2 \tan \alpha$$

$$(1 - x^2 \tan \alpha)^2 = 1 - x^4$$

$$x^4 - 2x^2 \tan \alpha + x^4 \tan^2 \alpha = 0$$

$$x^2 (x^2 - 2 \tan \alpha + x^2 \tan^2 \alpha) = 0$$

$$x^2 = \frac{2 \tan \alpha}{1 + \tan^2 \alpha}$$

$$x^2 = \frac{2 \tan \alpha}{\sec^2 \alpha}$$

$$x^2 = 2 \tan \alpha \cos^2 \alpha$$

$$x^2 = 2 \sin \alpha \cos \alpha = \sin 2\alpha$$

**6. Correct option: D**

**Explanation:-**

$$\lim_{x \rightarrow 3^+} \frac{f(x) - f(3)}{x - 3} = \lim_{h \rightarrow 0} \frac{f(3 + h) - f(3)}{h}$$
$$\lim_{x \rightarrow 3^+} \frac{f(x) - f(3)}{x - 3} = \lim_{h \rightarrow 0} \frac{|3 + h - 3| \cos(3 + h) - 0}{h}$$
$$\lim_{x \rightarrow 3^+} \frac{f(x) - f(3)}{x - 3} = \lim_{h \rightarrow 0} \frac{h \cos(3 + h)}{h} = \cos 3$$

$\cos 3$  is not differentiable.

Function is differentiable on  $\mathbb{R} - \{3\}$ .

**7. Correct option: C**

**Explanation:-**

$$S = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$M_{11} = d \Rightarrow A_{11} = d$$

$$M_{12} = c \Rightarrow A_{12} = -c$$

$$M_{21} = b \Rightarrow A_{21} = -b$$

$$M_{22} = a \Rightarrow A_{22} = a$$

$$\Rightarrow \text{Adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

**8. Correct option: A**

**Explanation:-**

$$y = \sin x$$

$$\text{slope of tangent} = \frac{dy}{dx} = \cos x$$

$$\Rightarrow \text{slope of normal} = -\frac{1}{\frac{dy}{dx}} = -\sec x$$

$$\Rightarrow \text{slope of normal at } (0,0) = -\frac{1}{\frac{dy}{dx}} = -1$$

Equation of normal is,

$$y - 0 = -1(x - 0)$$

$$\Rightarrow y = -x$$

$$\Rightarrow x + y = 0$$

**9. Correct option: B**

**Explanation:-**

$$P(\hat{i} - \hat{j} + 2\hat{k}), Q(2\hat{i} - \hat{k}) \text{ and } R(2\hat{j} + \hat{k})$$

$$\vec{PQ} = \hat{i} + \hat{j} - 3\hat{k}$$

$$\vec{PR} = -\hat{i} + 3\hat{j} - \hat{k}$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix}$$

$$\vec{PQ} \times \vec{PR} = 8\hat{i} + 4\hat{j} + 4\hat{k}$$

$$|\vec{PQ} \times \vec{PR}| = 4\sqrt{6}$$

$$\text{Unit vector} = \frac{8\hat{i} + 4\hat{j} + 4\hat{k}}{4\sqrt{6}} = \frac{1}{\sqrt{6}}(2\hat{i} + \hat{j} + \hat{k})$$

**10. Correct option: D**

**Explanation:-**

$$I = \int_0^{\pi} \frac{1}{1 + \sin x} dx$$

$$I = \int_0^{\pi} \frac{1}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x} dx$$

$$I = \int_0^{\pi} \frac{1 - \sin x}{1 - \sin^2 x} dx$$

$$I = \int_0^{\pi} \frac{1 - \sin x}{\cos^2 x} dx$$

$$I = \int_0^{\pi} (\sec^2 x - \tan x \sec x) dx$$

$$I = [\tan x - \sec x]_0^{\pi}$$

$$I = -(-1 - 1)$$

$$I = 2$$

**11. Correct option: C**

**Explanation:-**

$$I = \int \frac{1}{1 + e^x} dx$$

$$I = \int \frac{\frac{1}{e^x}}{\frac{1 + e^x}{e^x}} dx$$

$$I = \int \frac{e^{-x}}{1 + e^{-x}} dx$$

$$\text{Put } 1 + e^{-x} = t \Rightarrow -e^{-x} dx = dt$$

$$e^{-x} dx = -dt$$

$$I = \int \frac{-dt}{t}$$

$$I = -\log |t| + c$$

$$I = -\log |1 + e^{-x}| + c$$

**12. Correct option: D**

**Explanation:-**

$$a * b = 3a + 4b - 2$$

$$2 * 7 = 3 \times 2 + 4 \times 7 - 2 = 32$$

**13. Correct option: C**

**Explanation:-**

$$\text{If } A = \{a, b, c\} \text{ and } B = \{1, 2, 3\}$$

$$\text{The function } f: A \rightarrow B \text{ is given by } f = \{(a, 2), (b, 3), (c, 1)\}$$

Every element of set A is mapped to a unique element of set B, i.e. each element in set B has a unique pre image in B.

So, f is a one-one function

$$\text{Range of } f = \{1, 2, 3\} = B$$

So, f is an onto function

Thus, f is a bijective function

**14. Correct option: A**

**Explanation:-**

$$\frac{d(e^{x^2 + \tan x})}{dx} = e^{x^2 + \tan x} \frac{d}{dx}(x^2 + \tan x) = e^{x^2 + \tan x} (2x + \sec^2 x)$$

**15. Correct option: B**

**Explanation:-**

Given curves are:  $y^2 = x \dots (1)$  and  $x^2 = y \dots (2)$

$$\text{From (1), } \frac{dy}{dx} = \frac{1}{2y}$$

$$\text{Slope of tangent to } y^2 = x \text{ at } (1, 1) = \frac{1}{2 \times 1} = \frac{1}{2}$$

$$\text{From (2), } 2x = \frac{dy}{dx}$$

$$\text{Slope of tangent to } x^2 = y \text{ at } (1, 1) = 2 \times 1 = 2$$

Let angle between both curves be  $\theta$ , then

$$\tan \theta = \left| \frac{2 - \frac{1}{2}}{1 + 2 \times \frac{1}{2}} \right| = \left| \frac{\frac{3}{2}}{2} \right| = \frac{3}{4}$$

$$\Rightarrow \theta = \tan^{-1} \left( \frac{3}{4} \right)$$

**16. Correct option: C**

**Explanation:-**

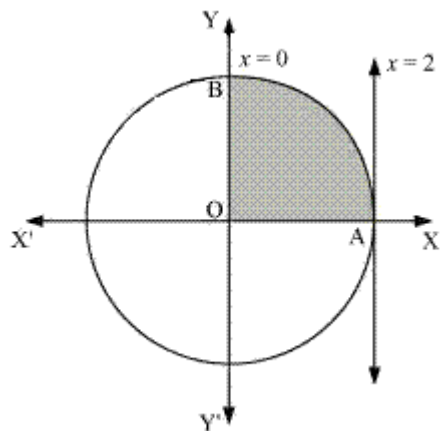
Equation of line through  $(-2, 1, 3)$  and parallel to  $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$  is

$$\frac{x+2}{3} = \frac{y-1}{5} = \frac{z-3}{6}$$

**17. Correct option: D**

**Explanation:-**

The region bounded by the circle  $x^2 + y^2 = 4$  in the 1<sup>st</sup> quadrant is:



Therefore, the required area is

$$\begin{aligned}
 A &= \int_0^2 y \, dx = \int_0^2 \sqrt{4 - x^2} \, dx \\
 &= \left[ \frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 \\
 &= 2 \left( \frac{\pi}{2} \right) \\
 &= \pi \text{ units}
 \end{aligned}$$

**18. Correct option: B**

**Explanation:-**

Given differential equation parallel to  $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$

$$\Rightarrow \frac{dy}{1 + y^2} = (1 + x^2) dx$$

Integrating both the sides, we get

$$\int \frac{dy}{1 + y^2} = \int (1 + x^2) dx$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + C$$

This is the required solution of the given differential equation.

**19. Correct option: A**

**Explanation:-**

$$\cot^{-1} \left( \sin \left( -\frac{\pi}{2} \right) \right) = \cot^{-1} (-1) = \frac{3\pi}{4}$$

**20. Correct option: D**

**Explanation:-**

Let l, m and n be the direction cosines of a line.

$$l = \cos 90^\circ = 0, m = \cos 60^\circ = \frac{1}{2} \text{ and } n = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Hence, the direction cosines of the line are  $0, \frac{1}{2}$  and  $\frac{\sqrt{3}}{2}$ .

## Part B

21. Let  $y = \frac{x^3 \sqrt{5+x}}{(7-3x)^5 \sqrt[3]{8+5x}}$

Taking log on both the sides, we get

$$\log y = 3 \log x + \frac{1}{2} \log(5+x) - 5 \log(7-3x) - \frac{1}{3} \log(8+5x)$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = 3 \cdot \frac{1}{x} + \frac{1}{2} \cdot \frac{1}{5+x} \cdot 1 - 5 \cdot \frac{1}{7-3x} \cdot (-3) - \frac{1}{3} \cdot \frac{1}{8+5x} \cdot 5$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \frac{3}{x} + \frac{1}{2(5+x)} + \frac{15}{7-3x} - \frac{5}{3(8+5x)}$$

$$\Rightarrow \frac{dy}{dx} = y \left[ \frac{3}{x} + \frac{1}{2(5+x)} + \frac{15}{7-3x} - \frac{5}{3(8+5x)} \right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^3 \sqrt{5+x}}{(7-3x)^5 \sqrt[3]{8+5x}} \left[ \frac{3}{x} + \frac{1}{2(5+x)} + \frac{15}{7-3x} - \frac{5}{3(8+5x)} \right]$$

**OR**

$$y = a \cos(\log x) + b \sin(\log x)$$

$$\Rightarrow \frac{dy}{dx} = a \left[ -\sin(\log x) \cdot \frac{1}{x} \right] + b \left[ \cos(\log x) \cdot \frac{1}{x} \right]$$

$$\Rightarrow x \frac{dy}{dx} = xy' = a \left[ -\sin(\log x) \right] + b \left[ \cos(\log x) \right]$$

Differentiating,

$$xy'' + y' = a \left[ -\cos(\log x) \cdot \frac{1}{x} \right] + b \left[ -\sin(\log x) \cdot \frac{1}{x} \right]$$

$$\Rightarrow x^2 y'' + xy' = a \left[ -\cos(\log x) \right] + b \left[ -\sin(\log x) \right] = -y$$

$$\Rightarrow x^2 y'' + xy' + y = 0$$

22. Given differential equation is  $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$

$$\Rightarrow \frac{dy}{dx} = \left( \frac{1}{\frac{x}{y} + \frac{y}{x}} \right) \dots\dots(i)$$

$$\text{Let } v = \frac{y}{x}$$

$$x \frac{dv}{dx} + v = \frac{dy}{dx}$$

From (i), we have

$$x \frac{dv}{dx} + v = \left( \frac{1}{\frac{1}{v} + v} \right)$$



$$\Rightarrow \left( -\frac{1}{v^3} - \frac{1}{v} \right) dv = \frac{1}{x} dx$$

Integrating on both the sides we have

$$\frac{1}{2v^2} - \log v = \log x + C$$

$$\Rightarrow \frac{x^2}{2y^2} = \log \left( \frac{y}{x} \times x \right) + C \dots \dots \dots (ii)$$

Put  $x = 0, y = 1$

$$0 = \log(1) + C$$

$$C = 0$$

From eq (ii) we have

$$\frac{x^2}{2y^2} = \log(y)$$

23. Let  $I = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$

$$= \int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \left[ 1 - (\sin x - \cos x)^2 \right]} dx$$

Put  $\sin x - \cos x = t$

$$\Rightarrow (\cos x + \sin x) dx = dt$$

For  $x = \pi/4, t = 0$  and

For  $x = 0, t = -1$

$$\therefore I = \int_{-1}^0 \frac{1}{9 + 16 \left[ 1 - (t)^2 \right]} dt$$

$$= \int_{-1}^0 \frac{1}{25 - 16(t)^2} dt$$

$$= \int_{-1}^0 \frac{1}{5^2 - (4t)^2} dt$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$$

$$= \frac{1}{10} \left[ \log \left| \frac{5+4t}{5-4t} \right| \right]_{-1}^0$$

$$= \frac{1}{10} \log \left| \frac{1}{\frac{1}{9}} \right|$$

$$= \frac{1}{10} \log |9|$$

OR

$$\int_0^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x) dx$$

$$\int_0^{\frac{\pi}{2}} \left( \log \left( \frac{\sin^2 x}{\sin 2x} \right) \right) dx$$

$$\int_0^{\frac{\pi}{2}} \left( \log \left( \frac{\sin^2 x}{2 \sin x \cos x} \right) \right) dx$$

$$\int_0^{\frac{\pi}{2}} \left( \log \left( \frac{\tan x}{2} \right) \right) dx \quad \dots (i)$$

We get ,

$$I = \int_0^{\frac{\pi}{2}} \log \left( \frac{\tan \left( \frac{\pi}{2} - x \right)}{2} \right) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \left( \frac{\cot x}{2} \right) dx \quad \dots (ii)$$

Adding (i) & (ii)

$$2I = \int_0^{\frac{\pi}{2}} \left[ \log \left( \frac{\tan x}{2} \right) + \log \left( \frac{\cot x}{2} \right) \right] dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log \left[ \left( \frac{\tan x}{2} \right) \left( \frac{\cot x}{2} \right) \right] dx$$

$$\Rightarrow I = \frac{1}{2} \int_0^{\frac{\pi}{2}} \log \left( \frac{1}{4} \right) dx$$

$$\Rightarrow I = \frac{1}{2} \log \left( \frac{1}{4} \right) \times \left( \frac{\pi}{2} \right)$$

$$\Rightarrow I = \frac{1}{2} \log \left( \frac{1}{4} \right)^{\frac{1}{2}} \times \left( \frac{\pi}{2} \right)$$

$$\Rightarrow I = \frac{1}{2} \log \left( \frac{1}{2} \right) \times \left( \frac{\pi}{2} \right)$$

$$\Rightarrow I = \frac{\pi}{2} \log \frac{1}{2}$$

24. (i)  $\sum_{i=0}^7 P(X_i) = 1$

$$\Rightarrow [0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k] = 1$$

$$\Rightarrow 10k^2 + 9k = 1 \Rightarrow 10k^2 + 9k - 1 = 0 \Rightarrow 10k^2 + 10k - k - 1 = 0$$

$$\Rightarrow 10k(k+1) - (k+1) = 0$$

$$\Rightarrow (k+1)(10k-1) = 0$$

$$\Rightarrow k = -1, k = \frac{1}{10}$$

k being a probability cannot be negative

$$\Rightarrow k = \frac{1}{10}$$

$$(ii) P(X < 3) = P(0) + P(1) + P(2) = 0 + k + 2k = 3k = \frac{3}{10}$$

$$(iii) P(X > 5) = P(6) + P(7) = 2k^2 + 7k^2 + k = 2\left(\frac{1}{10}\right)^2 + 7\left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right) = \frac{19}{100}$$

$$(iv) P(1 \leq X < 3) = P(1) + P(2) = k + 2k = 3k = \frac{3}{10}$$

25. Given:  $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

Let  $x = \sin y \Rightarrow y = \sin^{-1}x$

$$\Rightarrow \sin^{-1}(1 - \sin y) - 2\sin^{-1}(\sin y) = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}(1 - \sin y) - 2y = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}(1 - \sin y) = \frac{\pi}{2} + 2y$$

$$\Rightarrow 1 - \sin y = \sin\left(\frac{\pi}{2} + 2y\right)$$

$$\Rightarrow 1 - \sin y = \cos 2y \dots \left( \text{As } \sin\left(\frac{\pi}{2} + x\right) = \cos x \right)$$

Since  $\cos 2y = 1 - 2\sin^2 y$ , we have

$$1 - \sin y = 1 - 2\sin^2 y$$

$$2\sin^2 y - \sin y = 0$$

$$\sin y(2\sin y - 1) = 0$$

$$\sin y = 0 \text{ or } 2\sin y = 1$$

$$\sin y = 0 \text{ or } \sin y = \frac{1}{2}$$

i.e.  $x = 0$  or  $x = \frac{1}{2}$

But  $x = \frac{1}{2}$  does not satisfy the given equation.

Hence,  $x = 0$ .

**OR**

Given:  $\tan^{-1} \left( \frac{1-x}{1+x} \right) = \frac{1}{2} \tan^{-1} x \quad (x > 0)$

$$\Rightarrow 2 \tan^{-1} \left( \frac{1-x}{1+x} \right) = \tan^{-1} x$$

$$\Rightarrow \tan \left[ 2 \tan^{-1} \left( \frac{1-x}{1+x} \right) \right] = \tan \left[ \tan^{-1} x \right]$$

$$\Rightarrow \frac{2 \tan \left[ \tan^{-1} \left( \frac{1-x}{1+x} \right) \right]}{1 - \left( \tan \left[ \tan^{-1} \left( \frac{1-x}{1+x} \right) \right] \right)^2} = x$$

$$\Rightarrow \frac{2 \left( \frac{1-x}{1+x} \right)}{1 - \left( \frac{1-x}{1+x} \right)^2} = x$$

$$\Rightarrow \frac{2(1-x)(1+x)}{(1+x)^2 - (1-x)^2} = x$$

$$\Rightarrow \frac{(1-x^2)}{2x} = x$$

$$\Rightarrow 3x^2 = 1$$

$$\Rightarrow x = \pm \frac{1}{\sqrt{3}}$$

26. The two planes are  $\vec{r} \cdot \hat{i} + \hat{j} + \hat{k} - 1 = 0$  and  $\vec{r} \cdot 2\hat{i} + 3\hat{j} - \hat{k} = -4$

$$\Rightarrow x + y + z - 1 = 0 \text{ and } 2x + 3y - z + 4 = 0$$

The required plane passes through the line common to two intersecting planes

$$\Rightarrow x + y + z - 1 + k(2x + 3y - z + 4) = 0$$

$$\Rightarrow x(1 + 2k) + y(1 + 3k) + z(1 - k) + (-1 + 4k) = 0 \dots 1$$

The required plane is parallel to x-axis whose direction ratios are 1, 0 and 0.

$$\therefore 1(1 + 2k) + 0(1 + 3k) + 0(1 - k) = 0$$

$$\Rightarrow 1 + 2k = 0 \Rightarrow k = -\frac{1}{2}$$

Substituting in (1), we get

$$x \left[ 1 + 2 \left( -\frac{1}{2} \right) \right] + y \left[ 1 + 3 \left( -\frac{1}{2} \right) \right] + z \left[ 1 - \left( -\frac{1}{2} \right) \right] + \left[ -1 + 4 \left( -\frac{1}{2} \right) \right] = 0$$

$$\Rightarrow x(0) + y \left( -\frac{1}{2} \right) + z \left( \frac{3}{2} \right) - 3 = 0$$

$$\Rightarrow -y + 3z - 6 = 0$$

$$\Rightarrow y - 3z + 6 = 0$$

**27.** Equation of the plane passing through the point (2, 5, -8) is:

$$a(x - 2) + b(y - 5) + c(z + 8) = 0$$

If the plane is perpendicular to the plane  $2x - 3y + 4z + 1 = 0$ , then

$$2a - 3b + 4c = 0$$

If the plane is perpendicular to the plane  $4x + y - 2z + 6 = 0$ , then

$$4a + b - 2c = 0$$

On solving equations (2) and (3), we get

$$\frac{a}{6 - 4} = \frac{b}{16 + 4} = \frac{c}{2 + 12}$$

$$\Rightarrow \frac{a}{2} = \frac{b}{20} = \frac{c}{14} \Rightarrow \frac{a}{1} = \frac{b}{10} = \frac{c}{7}$$

On substituting the proportional values of a, b and c in (1), we get

$$(x - 2) + 10(y - 5) + 7(z + 8) = 0$$

$$x + 10y + 7z + 4 = 0$$

**28.** Given:  $f(x) = \frac{x}{1 + |x|}$ ,  $x \in \mathbb{R}$ ;  $-1 < x < 1$

$$\Rightarrow f(x) = \begin{cases} \frac{x}{1 + x}, & x \geq 0 \\ \frac{x}{1 - x}, & x < 0 \end{cases}$$

To show f is one-one i.e.  $f(x_1) = f(x_2) \Rightarrow f(x_1) = f(x_2)$

Case I: Let  $x_1$  be positive and  $x_2$  be negative so  $(x_1) \neq (x_2)$

$$\text{Then } f(x_1) = \frac{x_1}{1 + x_1} \text{ and } f(x_2) = \frac{x_2}{1 - x_2}$$

So  $f(x_1) \neq f(x_2)$  and hence  $(x_1) \neq (x_2) \Rightarrow f(x_1) \neq f(x_2)$

Case II: Let both the numbers  $x_1$  and  $x_2$  be positive.

$$f(x_1) = f(x_2) \Rightarrow \frac{x_1}{1 + x_1} = \frac{x_2}{1 + x_2} \Rightarrow x_1 + x_1 x_2 = x_2 + x_1 x_2 \Rightarrow x_1 = x_2$$

Case III: Let both the numbers  $x_1$  and  $x_2$  be negative.

$$f(x_1) = f(x_2) \Rightarrow \frac{x_1}{1 - x_1} = \frac{x_2}{1 - x_2} \Rightarrow x_1 - x_1 x_2 = x_2 - x_1 x_2 \Rightarrow x_1 = x_2$$

Hence, f is one -one

For onto,

Case I: If x is non-negative then

$$f(x) = y = \frac{x}{1+x} \Rightarrow y + xy = x \Rightarrow x = \frac{y}{1-y}$$

$$\text{Now, } f\left(\frac{y}{1-y}\right) = \frac{\frac{y}{1-y}}{1 + \frac{y}{1-y}} = y$$

Case II: If x is negative then

$$f(x) = y = \frac{x}{1-x} \Rightarrow y - xy = x \Rightarrow x = \frac{y}{1+y}$$

$$\text{Now, } f\left(\frac{y}{1+y}\right) = \frac{\frac{y}{1+y}}{1 - \frac{y}{1+y}} = y$$

Therefore, for each x in A, there exist y in R such that f(y) = x.

Thus, f is an onto function.

Hence, f is a bijective function.

29.

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

$$\Rightarrow (-1)^2 \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow (1 + xyz) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$R_2 \rightarrow R_2 - R_1; R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow (1 + xyz) \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix} = 0$$

$$\Rightarrow (1 + xyz)(y - x)(z - x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y + x \\ 0 & 1 & z + x \end{vmatrix} = 0$$

Applying  $R_3 \rightarrow R_3 - R_2$

$$\Rightarrow (1 + xyz)(y - x)(z - x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & x + y \\ 0 & 0 & z - y \end{vmatrix} = 0$$

$$\Rightarrow (1 + xyz)(x - y)(y - z)(z - x) = 0$$

Given,  $x, y, z$  are different

$$\Rightarrow 1 + xyz = 0 \Rightarrow xyz = -1$$

30. Let  $I = \int \frac{x^2}{x^4 + x^2 - 2} dx$

$$I = \int \frac{x^2}{x^2 - 1} \cdot \frac{x^2}{x^2 + 2} dx$$

$$= \int \frac{x^2}{x - 1} \cdot \frac{x^2}{x + 1} \cdot \frac{1}{x^2 + 2} dx$$

Using partial fraction,

$$\frac{x^2}{x - 1} \cdot \frac{x^2}{x + 1} \cdot \frac{1}{x^2 + 2} = \frac{A}{x - 1} + \frac{B}{x + 1} + \frac{Cx + D}{x^2 + 2}$$

$$\frac{x^2}{x - 1} \cdot \frac{x^2}{x + 1} \cdot \frac{1}{x^2 + 2} = \frac{A(x + 1)(x^2 + 2) + B(x^2 + 2)(x - 1) + (Cx + D)(x - 1)(x + 1)}{(x - 1)(x + 1)(x^2 + 2)}$$

Equating the coefficients from both the numerators we get,

$$A + B + C = 0 \dots\dots(1)$$

$$A - B + D = 1 \dots\dots(2)$$

$$2A + 2B - C = 0 \dots\dots(3)$$

$$2A - 2B - D = 0 \dots\dots(4)$$

Solving the above equations we get,

$$A = \frac{1}{6}, B = -\frac{1}{6}, C = 0, D = \frac{2}{3}$$

Our Integral becomes,

$$\int \frac{x^2}{(x - 1)(x + 1)(x^2 + 2)} dx = \int \frac{1}{6(x - 1)} - \frac{1}{6(x + 1)} + \frac{2}{3(x^2 + 2)} dx$$

$$= \frac{1}{6} \log x - 1 - \frac{1}{6} \log x + 1 + \frac{2}{3} \times \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{x}{\sqrt{2}} \right) + C$$

$$= \frac{1}{6} \left[ \log x - 1 - \log x + 1 + 2\sqrt{2} \tan^{-1} \left( \frac{x}{\sqrt{2}} \right) \right] + C$$

**31.** Given function is  $f(x) = \begin{cases} \frac{x^2 - 1}{x - 1}, & x \neq 1 \\ 4k, & x = 1 \end{cases}$

A function  $f(x)$  is continuous at a point  $x = a$  if  $\lim_{x \rightarrow a} f(x) = f(a)$

Now,

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{(x - 1)(x + 1)}{x - 1}$$

$$= \lim_{x \rightarrow 1} (x + 1) = 2$$

As it is given that  $f(x)$  is continuous at  $x = 1$ , we have

$$\lim_{x \rightarrow 1} f(x) = f(1)$$

$$\Rightarrow 2 = 4k$$

$$\Rightarrow k = \frac{1}{2}$$

## Part C

**32.** Let the two carpenters work for  $x$  days and  $y$  days respectively.

Our problem is to minimize the objective function.

$$C = 150x + 200y$$

Subject to the constraints

$$6x + 10y \geq 60 \Leftrightarrow 3x + 5y \geq 30$$

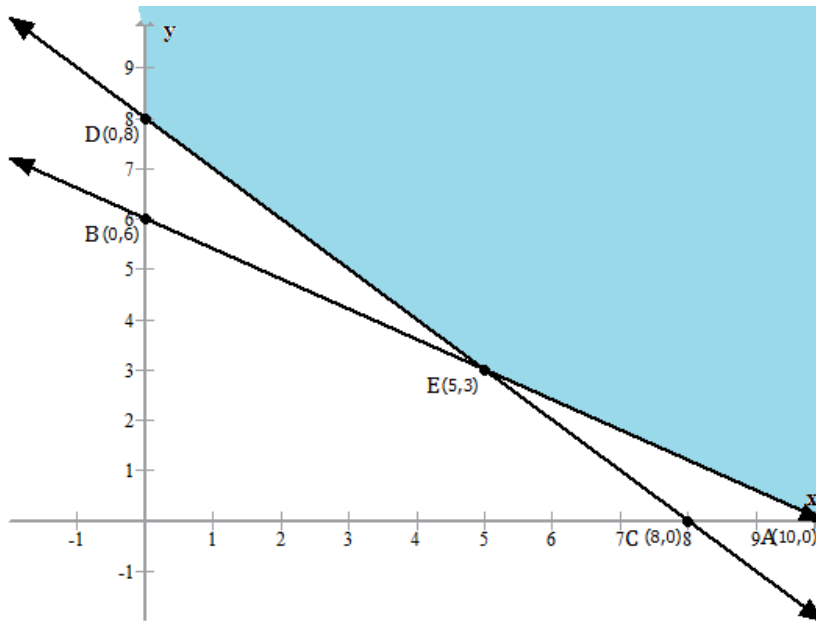
$$4x + 4y \geq 32 \Leftrightarrow x + y \geq 8$$

And

$$x \geq 0, y \geq 0$$

Feasible region is shaded.





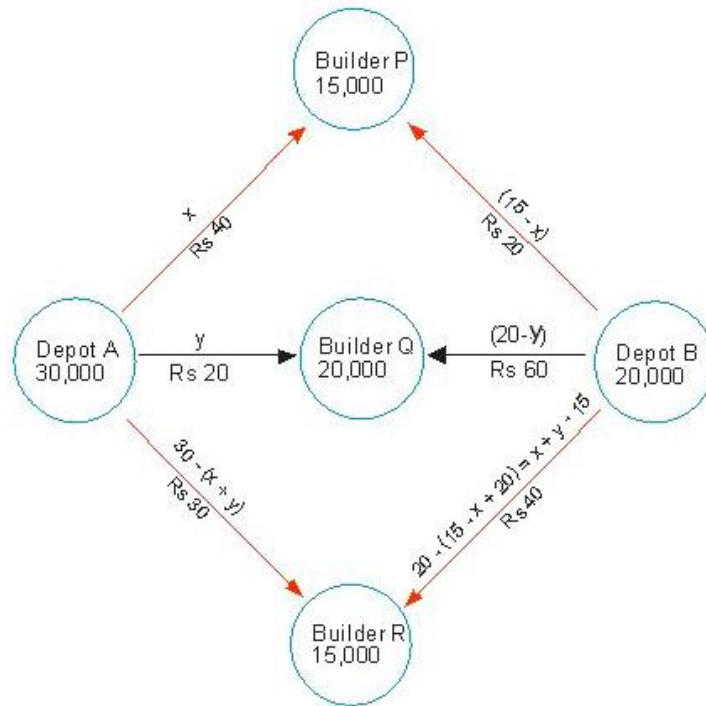
This region is unbounded.

Corner points	Objective function values $C = 150x + 200y$
A(10, 0)	1500
E(5, 3)	1350
D(0, 8)	1600

The labour cost is the least, when carpenter A works for 5 days and carpenter B works for 3 days.

**OR**

Let the depot A transport  $x$  thousand bricks to builder P and  $y$  thousand bricks to builder Q and  $30 - (x + y)$  thousand bricks to builder R. Let the depot B transport  $(15 - x)$  thousand bricks to builder P and  $(20 - y)$  thousand bricks to builder Q and  $(x + y) - 15$  thousand bricks to builder R



Then, the LPP can be stated mathematically as follows:

Minimize  $Z = 30x - 30y + 1800$

Subject to

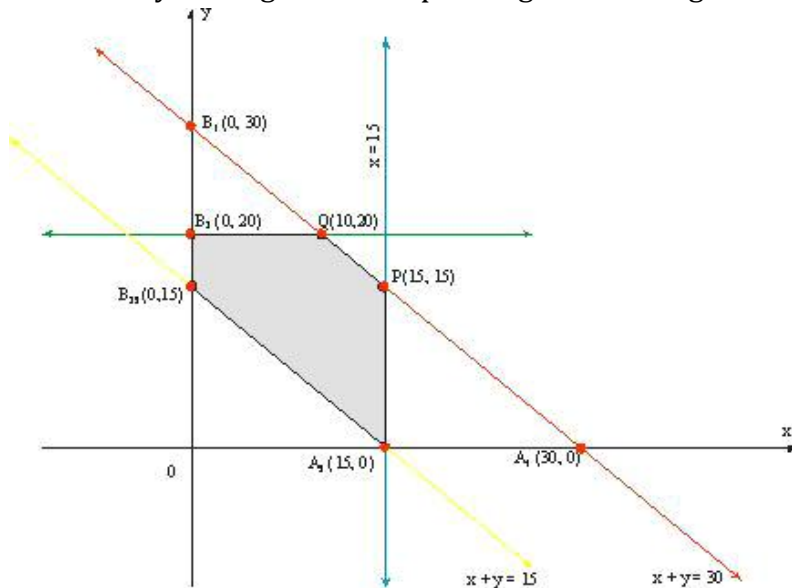
$$x + y \leq 30$$

$$x + y \leq 15$$

$$x \leq 20$$

$$y \leq 15 \text{ and } x \geq 0, y \geq 0$$

To solve this LPP graphically, we first convert inequations into equations and then draw the corresponding lines. The feasible region of the LPP is shaded in the figure given below. The co-ordinates of the corner points of the feasible region  $A_2$   $P$   $Q$   $B_3$   $B_2$  are  $A_2$  (15, 0),  $P$  (15, 15),  $Q$  (10, 20),  $B_3$  (0, 20) and  $B_2$  (0, 15). These points have been obtained by solving the corresponding intersecting lines simultaneously.



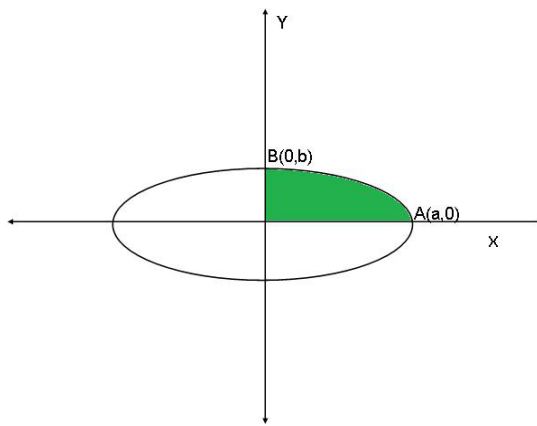
The value of the objective function at the corner points of the feasible region are given in the following table

Point (x, y) $Z = 30x - 30y + 1800$	Value of the objective function
$A_2 (15, 0)$	$Z = 30 \times 15 - 30 \times 0 + 1800 = 2250$
$P (15, 15)$	$Z = 30 \times 15 - 30 \times 15 + 1800 = 1800$
$Q (10, 20)$	$Z = 30 \times 10 - 30 \times 20 + 1800 = 1500$
$B_3 (0, 20)$	$Z = 30 \times 0 - 30 \times 20 + 1800 = 1200$
$B_2 (0, 15)$	$Z = 30 \times 0 - 30 \times 15 + 1800 = 1350$

Clearly, Z is minimum at  $x = 0, y = 20$  and the minimum value Z is 1200.

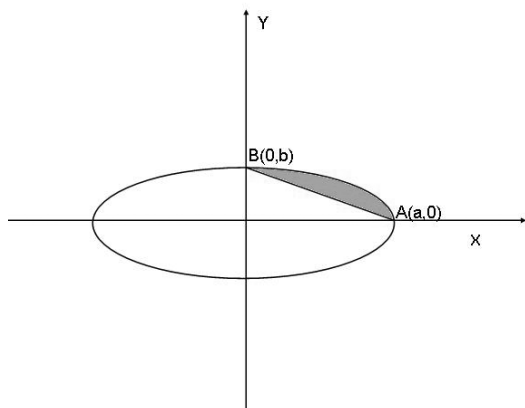
Thus, the manufacturer should supply 0, 20 and 10 thousand bricks to builders P, Q and R from depot A and 15, 0 and 5 thousand bricks to builders P, Q and R from dept B respectively. In this case the minimum transportation cost will be Rs. 1200.

33. (i) Between the curves  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and the x-axis between  $x = 0$  to  $x = a$  is



$$\begin{aligned}
 \int_0^a b \sqrt{1 - \frac{x^2}{a^2}} dx &= \frac{b}{a} \int_0^a \sqrt{a^2 - x^2} dx \\
 &= \frac{b}{a} \left[ \frac{x \sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a \\
 &= \frac{b}{2a} \left[ \left( 0 + a^2 \sin^{-1} 1 \right) - \left( 0 + a^2 \sin^{-1} 0 \right) \right] \\
 &= \frac{b}{2a} \left[ a^2 \times \frac{\pi}{2} \right] \\
 &= \frac{1}{4} \pi ab
 \end{aligned}$$

(ii) Area of triangle AOB is in the first quadrant of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  where OA = a and OB = b



The required area is same as the area enclosed between the chord AB and arc AB of the ellipse

= Area of ellipse in 1<sup>st</sup> quadrant – Area of triangle AOB

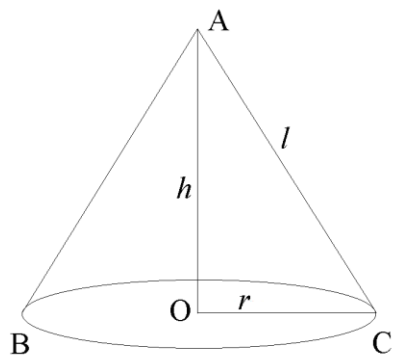
$$= \int_0^a b \sqrt{1 - \frac{x^2}{a^2}} dx - \frac{1}{2} ab$$

$$= \frac{1}{4} \pi ab - \frac{1}{2} ab$$

$$= \frac{(\pi - 2) ab}{4}$$

(iii) Ratio =  $\frac{\frac{1}{4} \pi ab}{\frac{(\pi - 2) ab}{4}} = \frac{\pi}{\pi - 2}$

34.



Here, Volume 'V' of the cone is  $v = \frac{1}{3} \pi r^2 h \Rightarrow r^2 = \frac{3V}{\pi h} \dots (1)$

Surface area  $S = \pi r l = \pi \sqrt{h^2 + r^2} \dots (2)$

Where  $h$  = height of the cone

$r$  = radius of the cone

$l$  = Slant height of the cone

$$S^2 = \pi^2 r^2 (h^2 + r^2) \dots \text{From (2)}$$

Let,  $S_1 = S^2$

Substituting the value of  $r^2$  from equation (1), we have,

$$S_1 = \frac{3\pi V}{h} \left( h^2 + \frac{3V}{\pi h} \right) = 3\pi V h + \frac{9V^2}{h^2}$$

Differentiating  $S_1$  with respect to  $h$ , we get

$$\frac{dS_1}{dh} = 3\pi V - \frac{18V^2}{h^3} \dots \text{(iii)}$$

Take  $\frac{dS_1}{dh} = 0$  for maxima or minima

$$3\pi V - \frac{18V^2}{h^3} = 0$$

$$\Rightarrow h^3 = \frac{6V}{\pi}$$

Differentiating (iii) w.r.t  $h$ , we get

$$\frac{d^2S_1}{dh^2} = \frac{54V^2}{h^4}$$

$$\frac{d^2S_1}{dh^2} > 0 \text{ at } h^3 = \frac{6V}{\pi}$$

Therefore curved surface area is minimum at  $\frac{\pi h^3}{6} = V$

$$\text{Thus, } \frac{\pi h^3}{6} = \frac{1}{3} \pi r^2 h \Rightarrow h^2 = 2r^2 \Rightarrow h = \sqrt{2}r$$

Hence for least curved surface the altitude is  $\sqrt{2}$  times the radius.

**OR**

$$f(x) = (x - 2)^4 (x + 1)^3$$

$$f'(x) = 3(x - 2)^4 (x + 1)^2 + 4(x + 1)^3 (x - 2)^3$$

$$= (x - 2)^3 (x + 1)^2 [3(x - 2) + 4(x + 1)]$$

$$= (x - 2)^3 (x + 1)^2 [3x - 6 + 4x + 4]$$

$$= (x - 2)^3 (x + 1)^2 [7x - 2]$$

$$f'(x) = 0 \Rightarrow (x - 2)^3 (x + 1)^2 [7x - 2] = 0 \Rightarrow x = -1, \frac{2}{7}, 2$$

Let us examine the behavior of  $f'(x)$  slightly to the left and right of each of these three values of  $x$ .

(i)  $x = -1$

When  $x < -1$ ,  $f'(x) > 0$

When  $x > -1$  and  $x < \frac{2}{7}$ ,  $f'(x) > 0$

Therefore,  $x = -1$  is neither a point of local maxima nor minima

(ii)  $x = \frac{2}{7}$

When  $x < \frac{2}{7}$  and  $x > 1$ ,  $f'(x) > 0$

When  $x > \frac{2}{7}$  and  $x < 2$ ,  $f'(x) < 0$

$\Rightarrow x = \frac{2}{7}$  is a point of local maxima

$$f\left(\frac{2}{7}\right) = \left(\frac{2}{7} - 2\right)^4 \left(\frac{2}{7} + 1\right)^3 = \left(-\frac{12}{7}\right)^4 \left(\frac{9}{7}\right)^3 = \frac{2^8 \times 3^{10}}{7^7}$$

Thus, the local maximum value is  $\frac{2^8 \times 3^{10}}{7^7}$ .

(iii)  $x = 2$

When  $x < 2$ ,  $f'(x) < 0$

When  $x > 2$ ,  $f'(x) > 0$

Therefore,  $x = 2$  is a point of local minima

$$f(2) = (2 - 2)^4 (2 + 1)^3 = 0$$

Thus, the local minimum value is 0.

**35.** Let  $E_1$ ,  $E_2$ ,  $E_3$  and  $A$  be the events defined as follows:

Let  $E_1$  be the event that the student knows the answer.

Let  $E_2$  be the event that the student guesses the answer.

Let  $E_3$  be the event that the student copies the answer.

Let  $A$  be the event that the answer is correct.

$$P(E_1) = \frac{1}{2}; P(E_2) = \frac{1}{4}; P(E_3) = \frac{1}{4};$$

Probability that he answers correctly given that he knew the answer is 1.

$$\text{That is, } P(A | E_1) = 1$$

If  $E_2$  has already occurred, then the student guesses.

Since there are four choices out of which only one is correct, therefore, the probability

that he answers correctly given that he has made a guess is  $\frac{1}{4}$ .

$$\text{That is } P(A | E_2) = \frac{1}{4}.$$

$$\text{It is given that, } P(A | E_3) = \frac{3}{4}$$

By Bayes' Theorem, we have,

Required Probability

$$\begin{aligned} &= P(E_1 | A) \\ &= \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)} \\ &= \frac{\frac{1}{2} \times 1}{\frac{1}{2} \times 1 + \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{3}{4}} \\ &= \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{16} + \frac{3}{16}} \\ &= \frac{\frac{1}{2}}{\frac{8}{16} + \frac{1}{16} + \frac{3}{16}} \\ &= \frac{\frac{1}{2}}{\frac{12}{16}} \\ &= \frac{1}{2} \times \frac{16}{12} \\ &= \frac{1}{2} \times \frac{4}{3} \\ &= \frac{2}{3} \end{aligned}$$

Thus, the probability that student knows the answer, given that he answered it correctly is  $\frac{2}{3}$ .

Arjun is dishonest, as he copies from the other student(s).

Copying once may become habit forming as he may continue resort to dishonest means in the coming years.

**OR**

Let  $E_1$  be the event that a red ball is transferred from bag A to bag B

Let  $E_2$  be the event that a black ball is transferred from bag A to bag B

$\therefore E_1$  and  $E_2$  are mutually exclusive and exhaustive.

$P(E_1) = 3/7$  ;  $P(E_2) = 4/7$

Let  $E$  be the event that a red ball is drawn from bag B

$$P(E|E_1) = \frac{4+1}{(4+1)+5} = \frac{5}{10} = \frac{1}{2}$$

$$P(E|E_2) = \frac{3+1}{(5+1)+4} = \frac{4}{10} = \frac{2}{5}$$

$$\therefore \text{ Required probability } = P(E_2|E) = \frac{P(E|E_2)P(E_2)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2)}$$

$$= \frac{\frac{4}{10} \times \frac{4}{7}}{\frac{1}{2} \times \frac{3}{7} + \frac{4}{10} \times \frac{4}{7}}$$

$$= \frac{\frac{16}{70}}{\frac{3}{14} + \frac{16}{70}}$$

$$= \frac{\frac{16}{70}}{\frac{31}{70}}$$

$$= \frac{16}{31}$$

$$\therefore \text{ Required probability } = P(E_1|E) = \frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{3}{7}}{\frac{1}{2} \times \frac{3}{7} + \frac{4}{10} \times \frac{4}{7}}$$

$$= \frac{\frac{3}{14}}{\frac{3}{14} + \frac{16}{70}}$$

$$= \frac{\frac{3}{14}}{\frac{31}{70}}$$

$$= \frac{15}{31}$$



36. We have,

$$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\left(\frac{x}{2}\right) \\ \tan\left(\frac{x}{2}\right) & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\tan\left(\frac{x}{2}\right) \\ \tan\left(\frac{x}{2}\right) & 1 \end{bmatrix}$$

$$I - A = \begin{bmatrix} 1 & \tan\left(\frac{x}{2}\right) \\ -\tan\left(\frac{x}{2}\right) & 1 \end{bmatrix}$$

$$(I - A) \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} = \begin{bmatrix} 1 & \tan\left(\frac{x}{2}\right) \\ -\tan\left(\frac{x}{2}\right) & 1 \end{bmatrix} \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan\left(\frac{x}{2}\right) \\ -\tan\left(\frac{x}{2}\right) & 1 \end{bmatrix} \begin{bmatrix} \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)} & -\frac{2 \tan\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)} \\ \frac{2 \tan\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)} & \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)} \end{bmatrix}$$

for simplicity take  $\tan\left(\frac{x}{2}\right) = t$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix} \begin{bmatrix} \frac{1 - t}{1 + t^2} & -\frac{2t}{1 + t^2} \\ \frac{2t}{1 + t^2} & \frac{1 - t}{1 + t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1 + t^2}{1 + t^2} & -\frac{t(1 + t^2)}{1 + t^2} \\ \frac{t(1 + t^2)}{1 + t^2} & \frac{1 + t^2}{1 + t^2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\left(\frac{x}{2}\right) \\ \tan\left(\frac{x}{2}\right) & 1 \end{bmatrix} = I + A$$

37. Let the equation of required plane be  $\ell x + my + nz + p = 0 \dots (1)$

Plane passes through points  $(1, 2, 3)$  and  $(0, -1, 0)$

$\therefore (1, 2, 3)$  and  $(0, -1, 0)$  satisfies the equation (1)

$$\ell + 2m + 3n + p = 0 \dots (2)$$

$$-m + p = 0$$

$$\Rightarrow p = m \dots (3)$$

$$\text{d.c.'s of line } \frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3} \text{ are } 2, 3, -3$$

d.c.'s normal to plane are  $\ell, m$ , and  $n$  normal to the plane will be  $\perp$  to line

$$\text{i.e., } 2\ell + 3m - 3n = 0 \dots (4)$$

$$\Rightarrow \ell = \frac{3}{2}(n - m)$$

From (2) and (3) we have

$$\ell + 3m + 3n = 0$$

$$\ell = -3(m + n) \dots (5)$$

From (4) and (5)

$$\frac{3}{2}(n - m) = -3(m + n)$$

$$n - m = -2m - 2n$$

$$\Rightarrow 3n = -m \quad \text{or } m = -3n$$

$$\ell = -3(-3n + n) = -3 \times -2n$$

Using  $\ell = 6n, m = -3n$  &  $p = -3n$  in (1) we have required equation as

$$6x - 3y + z - 3 = 0$$

**OR**

Let the equation of plane be  $ax + by + cz + d = 0 \dots (1)$

Since the plane passes through the point A  $(0, 0, 0)$  and B  $(3, -1, 2)$ , we have

$$a \times 0 + b \times 0 + c \times 0 + d = 0$$

$$\Rightarrow d = 0 \dots (2)$$

Similarly for point B  $(3, -1, 2)$ ,  $a \times 3 + b \times (-1) + c \times 2 + d = 0$

$$3a - b + 2c = 0 \dots (\text{Using } d = 0) (3)$$

$$\text{Given equation of the line is } \frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$$

$$\text{We can also write the above equation as } \frac{x-4}{1} = \frac{y-(-3)}{-4} = \frac{x-(-1)}{7}$$

The required plane is parallel to the above line.

$$\text{Therefore, } a \times 1 + b \times (-4) + c \times 7 = 0$$

$$\Rightarrow a - 4b + 7c = 0 \dots (4)$$

Cross multiplying equations (3) and (4), we obtain:

$$\frac{a}{(-1) \times 7 - (-4) \times 2} = \frac{b}{2 \times 1 - 3 \times 7} = \frac{c}{3 \times (-4) - 1 \times (-1)}$$

$$\Rightarrow \frac{a}{-7 + 8} = \frac{b}{2 - 21} = \frac{c}{-12 + 1}$$

$$\Rightarrow \frac{a}{1} = \frac{b}{-19} = \frac{c}{-11} = k$$

$$\Rightarrow a = k, b = -19k, c = -11k$$

Substituting the values of a, b and c in equation (1), we obtain the equation of plane as:

$$kx - 19ky - 11kz + d = 0$$

$$\Rightarrow k(x - 19y - 11z) = 0 \dots \text{From equation (2)}$$

$$\Rightarrow x - 19y - 11z = 0$$

So, the equation of the required plane is  $x - 19y - 11z = 0$