Number System and its Operations

OLYMPIAD EXCELLENCE BOOK **MATHEMATICS**

QUESTIONS

1 If $x = \frac{1}{2-\sqrt{3}}$, what is the value of $x^3 - 2x^2 - 7x + 5$

(a) 2 (b) 3 (c) 5 (d) 9

2. What is the value of $\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$, is being given that $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$

(a) 5.398 (b) 4.258 (c) 5.355 (d) 3.855

3. If the sum of five consecutive integers is S, then the largest of those integers in terms of S is

(a) $\frac{S-10}{5}$ (b) $\frac{S-4}{4}$ (c) $\frac{S+5}{4}$ (d) $\frac{S+10}{5}$

4. If $x = \sqrt[3]{2+\sqrt{3}}$, then $x^3 + \frac{1}{x^3}$, =

(a) 2 (b) 4 (c) 8 (d) 9

5. $(5^{61} + 5^{62} + 5^{63})$ is divisible by

(a) 31 (b) 11 (c) 13 (d) 17

6. The value of: $\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$ is

(a) 1 (b) 2 (c) 3 (d) 8

7. The value of $\frac{1}{\sqrt{6.25}+\sqrt{5.25}} + \frac{1}{\sqrt{4.25}+\sqrt{3.25}} + \frac{1}{\sqrt{5.25}+\sqrt{4.25}} + \frac{1}{\sqrt{3.25}+\sqrt{2.25}}$ is

(a) 1.00 (b) 1.25 (c) 1.50 (d) 2.25

8. $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2} =$

(a) 5 (b) 4 (c) 3 (d) 2

9. The value of $\sqrt{4\sqrt[3]{16\sqrt{4\sqrt[3]{16\sqrt{4\sqrt[3]{16}}}}}}$ ...... is

(a) 2 (b) 2^2 (c) 2^3 (d) 2^5

10. If $m = \sqrt{3+\sqrt{3+\sqrt{3+.....}}}$

$n = \sqrt{3-\sqrt{3-\sqrt{3-.........}}}$

Then among the following the relation between m and n holds is

(a) $m-n+1=0$ (b) $m+n-1=0$ (c) $m+n+1=0$ (d) $m-n-1=0$

11. If $x = \frac{\sqrt{3}}{2}$, then the value of $\sqrt{1+a}+\sqrt{1-a}$ is

(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $2+\sqrt{3}$ (d) $2-\sqrt{3}$

12. **A rational numbers between -3 and 4.**

(a) - 4.5 (b) -3.5 (c) $\frac{13}{2}$ (d) $\frac{1}{2}$

13. **The decimal representation of $\frac{-26}{45}$ is**

(a) $.3\bar{5}$ (b) $-1\overline{55}$ (c) $-.3\overline{55}$ (d) $-0.5\bar{7}$

14. $\mathbf{1.272727 = 1.\overline{27}}$ **can be expressed in the form $\frac{p}{q}$, where p and q are integers an $\mathbf{q} \neq 0$ than it is equal to**

(a) $\frac{106}{99}$ (b) $\frac{127}{99}$ (c) $\frac{14}{11}$ (d) $\frac{27}{99}$

15. **If $\mathbf{A = 2^x, B = 4^y, C = 8^z}$, where $\mathbf{x} = 0.\bar{1}, \mathbf{y = 0.\bar{4}, z = 0.\bar{6}}$, then $\mathbf{A \times B \times C}$ is**

(a) 8 (b) 2 (c) 16 (d) 4

16. **If** $x = 2.\bar{3} - 0.\bar{9}, y = 2.\bar{5} - 0.\bar{5}$, then $x^2 + y^2 - 2xy$ **is**

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{5}$

17. **If** $(3)^{0.\bar{4}+0.\bar{5}} = x, (27)^{0.\overline{21}+0.\overline{12}}$ **then** $x \times y$ **is**

(a) 3^4 (b) 3^3 (c) 3^2 (d) 3^5

18. **If $\mathbf{x = \frac{\sqrt{2}+\sqrt{1}}{\sqrt{2}-\sqrt{1}}}$ and $y = \frac{\sqrt{2}-\sqrt{1}}{\sqrt{2}+\sqrt{1}}$, find the value of $\mathbf{x^2 - y^2}$**

(a) 96 (b) 34 (c) 10 (d) $2\sqrt{2}$

19. **If $\mathbf{a = 3 + 2\sqrt{2}}$ and $\mathbf{b = \frac{1}{a}}$, then $\mathbf{a^2 + b^2 =}$**

(a) 49 (b) 34 (c) 100 (d) 102

20. $\frac{\mathbf{4^{-3} \times a^{-5} \times b^4}}{\mathbf{4^{-5} \times a^{-8} \times b^3}} =$

(a) $\frac{16a^3}{b^7}$ (b) $8\frac{a^2}{b^{-7}}$ (c) $2\frac{a^{-13}}{b^{-7}}$ (d) $\frac{a^8}{b^{-1}}$

21. if $2\sqrt[3]{189} + 3\sqrt[3]{448} - 7\sqrt[3]{56}$ **is simplified, then the resultant answer is**

(a) $8\sqrt[3]{7}$ (b) $6\sqrt[3]{7}$ (c) $4\sqrt[3]{7}$ (d) $9\sqrt[3]{7}$

22. **If** $7\sqrt[4]{162} - 5\sqrt[4]{32} + \sqrt[4]{1250}$ **is simplified, then the resultant value is**

(a) $6\sqrt[3]{2}$ (b) $6\sqrt[4]{2}$ (c) $6\sqrt[5]{2}$ (d) $16\sqrt[4]{2}$

23. **The two irrational numbers lying between $\sqrt{3}$ and $\sqrt{5}$ are**

(a) $15^{\frac{1}{4}}, \frac{3^{\frac{1}{4}}}{1} \times 15^{\frac{1}{8}}$ (b) $6^{\frac{1}{2}}, 2^{\frac{1}{8}} \times 6^{\frac{1}{4}}$ (c) $6^{\frac{1}{8}}, 2^{\frac{1}{6}} \times 6^{\frac{1}{6}}$ (d) $3^{\frac{1}{8}}, 2^{\frac{1}{8}} \times 6^{\frac{1}{8}}$

24. **If** $x = \frac{1}{2+\sqrt{3}}$ **, then the value of** $x^3 - 2x^2 - 7x + 5$ **is**

(a) 1 (b) 2 (c) 3 (d) 4

25. **If** $x = 2 - \sqrt{3}$ **, then the value of** $\mathbf{x^2 + 4x + 4}$ **is.**

(a) $12 + 2\sqrt{3}$ (b) $19 + 8\sqrt{3}$ (c) $12 + 2\sqrt{3}$ (d) $19 - 8\sqrt{3}$

26. **The value of x, when** $2^{x+4}.3^{x+1} = 288$

(a) 1 (b) -1 (c) 0 (d) 2

27. **Which of the following is the value of a in** $\frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} = a + b\sqrt{15}$

(a) 2 (b) -1 (c) -3 (d) 4

28. **The square root of** $0.\overline{4}$ **is**

(a) $0.\overline{6}$ (b) $0.\overline{7}$ (c) $0.\overline{8}$ (d) $0.\overline{9}$

29. **If** $\sqrt{18225} = 135$**, then the value of** $\sqrt{182.25} + \sqrt{1.8225} + \sqrt{0.018225} + \sqrt{0.00018225}$ **is**

(a) 1.49985 (b) 14.9985 (c) 149.985 (d) 1499.85

30. **If** $2^{x-1} + 2^{x+1} = 640$**, the value of x is**

(a) 7 (b) 8 (c) 9 (d) 6

31. **The product of** $\left(0.\overline{09} \times 7.\overline{3}\right)$ **is equal to**

(a) 1 (b) 0 (c) 0 (d) $\frac{1}{2}$

32. $0.142857 - 0.285714$ **is equal to**

(a) 2 (b) 1 (c) 0 (d) $\frac{1}{2}$

33. $\frac{1}{1+2^{x-y}} + \frac{1}{1+2^{y-x}} = ?$

(a) x (b) $x - y$ (c) 1 (d) 0

34. **If** $x + \sqrt{7} = 7 + \sqrt{y}, x + \sqrt{7} = 7 + \sqrt{y}$ **, and x, y are positive integers, then the value of** $\frac{\sqrt{x} + y}{x + \sqrt{y}}$ **is.**

(a) 0 (b) 2 (c) $\frac{1}{2}$ (d) 1

35. **The largest among the numbers** $2^{250}, 3^{150}, 5^{100}$ **and** 4^{200} **is**

(a) 4^{200} (b) 5^{100} (c) 2^{250} (d) 2^{150}

36. $\dfrac{a^{-1}}{a^{-1}+b^{-1}}+\dfrac{a^{-1}}{a^{-1}-b^{-1}}=?$

(a) 0 (b) 1 (c) $\dfrac{2b^2}{b^2-a^2}$ (d) $\dfrac{2b^2}{b^2+a^2}$

37. $\dfrac{x^{-1}}{x^{-1}+y^{-1}}+\dfrac{x^{-1}}{x^{-1}+y^{-1}}=?$

(a) $\dfrac{2y^2}{y^2-x^2}$ (b) $\dfrac{2x^2}{y^2-x^2}$ (c) $\dfrac{2y^2}{y^2+z^2}$ (d) $\dfrac{2x^2}{y^2+x^2}$

38. $\dfrac{\left(a^{x+y}\right)^2\left(a^{y+z}\right)^2\left(a^{z+x+}\right)^2}{\left(a^{4x}.a^{4y}.a^{4z}\right)}=?$

(a) 2a (b) $x+y+z$ (c) 1 (d) 0

39. **The greatest among $\sqrt{11}-\sqrt{9},\sqrt{5}-\sqrt{3},\sqrt{7}-\sqrt{5},\sqrt{13}-\sqrt{11}$ is**

(a) $\sqrt{11}-\sqrt{9}$ (b) $\sqrt{5}-\sqrt{3}$ (c) $\sqrt{7}-\sqrt{5}$ (d) $\sqrt{13}-\sqrt{11}$

40. **The smallest of $\sqrt{6}+\sqrt{3},\ \sqrt{7}+\sqrt{2},\ \sqrt{8}+\sqrt{1},\ \sqrt{5}+\sqrt{4}$ is**

(a) $\sqrt{6}+\sqrt{3}$ (b) $\sqrt{7}+\sqrt{2}$ (c) $\sqrt{8}+\sqrt{1}$ (d) $\sqrt{5}+\sqrt{4}$

ANSWER KEY & HINTS

1. (b): We have,

$$x = \frac{1}{2-\sqrt{3}} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2}$$

$$= \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$$

$$\Rightarrow x-2=\sqrt{3}$$

$$\Rightarrow (x-2)^2 = \left(\sqrt{3}\right)^2$$

$$x^2-4x+4=3 \Rightarrow x^2-4x+1=0$$

$$\therefore x^3-2x^2-7x+5$$

$$= x\left(x^2-4x+1\right)+2(x^2-4x+1)+3$$

$$= x\times 0+2\times 0+3=3$$

2. (a): We have. $\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}$

$$= \sqrt{10}+\sqrt{2^2\times 5}+\sqrt{2^2\times 10}-\sqrt{5}-\sqrt{2^4\times 5}$$

$$= \sqrt{10}+2\sqrt{5}+2\sqrt{10}-\sqrt{5}-2^2-\sqrt{5}$$

$$= \sqrt{10}+2\sqrt{10}+2\sqrt{5}-\sqrt{5}-4\sqrt{5}$$

$$= (1+2)\sqrt{10}+(2-1-4)\sqrt{5}$$

$$= 3\sqrt{10}-3\sqrt{5} = 3\left(\sqrt{10}-\sqrt{5}\right)$$

$$\therefore \frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$$

$$= \frac{15}{3\left(\sqrt{10}-\sqrt{5}\right)} = \frac{5}{\sqrt{10}-\sqrt{5}}$$

$$= \frac{8\left(\sqrt{10}-\sqrt{5}\right)}{\left(\sqrt{10}-\sqrt{5}\right)\left(\sqrt{10}+\sqrt{5}\right)}$$ [Multiplying and dividing by $\sqrt{10}+\sqrt{5}$]

$$= \frac{5\left(\sqrt{10}+\sqrt{5}\right)}{10-5} = \sqrt{10}+\sqrt{5} = 3.162+2.236 = 5.398$$

3. (d): Sum of five consecutive integers = S

$$\therefore \textit{Third integer} = \frac{S}{5}; \quad \therefore \textit{Largest integer} = \frac{S}{5}+2 = \frac{S+10}{5}$$

4. (b): Given $x=\sqrt[3]{2+\sqrt{3}}$

Take cube both side, are get

$$x^3=2+\sqrt{3} \qquad \_\_\_\_\_\_ (1)$$

$$\frac{1}{x^3}=\frac{1}{\left(2+\sqrt{3}\right)}\times\frac{\left(2-\sqrt{3}\right)}{\left(2-\sqrt{3}\right)} \qquad \_\_\_\_\_\_ (2)$$

$$\frac{1}{x^3}=2-\sqrt{3}$$

So, $x^3+\frac{1}{x^3}=2+\sqrt{3}+2-\sqrt{3}=4$

5. (a): $5^{61}+5^{62}+5^{63}=5^{61}(1+5+5^2)=5^{61}\times 31$

which is divisible by 3.

6. (b): $\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$

$$=\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{4+3+2\times 2\times\sqrt{3}}}}$$

$$=\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{(2)^2+(\sqrt{3})^2+2\times 2\times\sqrt{3}}}}$$

$$=\sqrt{-\sqrt{3}\sqrt{3+8\sqrt{\left(2+\sqrt{3}\right)^2}}}$$

$$=\sqrt{-\sqrt{3}\sqrt{3+8\sqrt{\left(2+\sqrt{3}\right)}}}$$

$$=\sqrt{-\sqrt{3}+\sqrt{3+16+8\sqrt{3}}}$$

$$=\sqrt{-\sqrt{3}+\sqrt{\left(\sqrt{3}\right)^2+(4)^2+2\times 4\times\sqrt{3}}}$$

$$=\sqrt{-\sqrt{3}+\sqrt{(4+3)^2}}$$

$$=\sqrt{-\sqrt{3}+4+\sqrt{3}}=\sqrt{4}=2$$

7. (a) $\frac{1}{\sqrt{3.25}+\sqrt{2.25}}=\frac{1}{\left(\sqrt{3.25}+\sqrt{2.25}\right)}\times\frac{\sqrt{3.25}-\sqrt{2.25}}{\sqrt{3.25}-\sqrt{2.25}}=\frac{\sqrt{3.25}-\sqrt{2.25}}{3.25-2.25}=\sqrt{3.25}-\sqrt{2.25}$

Similarly, $\frac{1}{\sqrt{4.25}+\sqrt{3.25}}=\sqrt{4.25}-\sqrt{3.25}$

$$\frac{1}{\sqrt{5.25}+\sqrt{4.25}}=\sqrt{5.25}-\sqrt{4.25}$$

$$\frac{1}{\sqrt{6.25}+\sqrt{5.25}}=\sqrt{6.25}-\sqrt{5.25}$$

$\therefore$ Expression

$$=\sqrt{3.25}-\sqrt{2.25}+\sqrt{4.25}-\sqrt{3.25}+\sqrt{5.25}-\sqrt{4.25}+\sqrt{6.25}-\sqrt{5.25}=\sqrt{6.25}-\sqrt{2.25}=2.5-1.5=1$$

8. (a): Here, $\frac{1}{3-\sqrt{8}}=\frac{\left(3+\sqrt{8}\right)}{\left(3-\sqrt{8}\right)\left(3+\sqrt{8}\right)}$

$$=\frac{3+\sqrt{8}}{9-8}=3+\sqrt{8}$$

$$\frac{1}{\sqrt{8}-\sqrt{7}}=\frac{\sqrt{8}+\sqrt{7}}{\left(\sqrt{8}-\sqrt{7}\right)\left(\sqrt{8}+\sqrt{7}\right)}$$

$=\sqrt{8}+\sqrt{7}$ and…….. so on

Expression $=(3+\sqrt{8})-(\sqrt{8}+\sqrt{7})+(\sqrt{7}+\sqrt{6})-(\sqrt{6}+\sqrt{5})+(\sqrt{5}+\sqrt{2})$

$$=(3+\sqrt{8})-(\sqrt{8}+\sqrt{7})-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2=3+2=5$$

9. **(b)** $x=\sqrt{4\sqrt[3]{16\sqrt{4\sqrt[3]{16.........}}}}$

On squaring both sides, $x^2=4\sqrt[3]{16\sqrt{4\sqrt[3]{16.....}}}$

On cubing, $x^6=64\times16x$

$x=4$

10. **(d)** $m=\sqrt{3+\sqrt{3+\sqrt{3+......}}}$

on squaring both sides,

$m^2=3+m\Rightarrow m^2-m=3$ …..(i)

Again, $n=\sqrt{3-\sqrt{3-\sqrt{3-.....}}}$

On squaring both sides,

$n^2=3-n$

$\Rightarrow n^2+n=3$ …………(ii)

$\therefore m^2-m=n^2+n\Rightarrow\left(m^2-n^2\right)=m+n$

$\Rightarrow(m+n)(m-n)-(m+n)=0$

$\Rightarrow (m+n)(m-n-1)=0$

11. (a) $a=\frac{\sqrt{3}}{2}$ $\quad\therefore \sqrt{1+a}+\sqrt{1-a}$

$=\sqrt{1+\frac{\sqrt{3}}{2}}+\sqrt{1-\frac{\sqrt{3}}{2}}$

$=\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2}}+\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2}}=\frac{\sqrt{4+2\sqrt{3}}}{\sqrt{2}\times\sqrt{2}}-\frac{\sqrt{4-2\sqrt{3}}}{\sqrt{2}\times\sqrt{2}}$

$=\frac{\sqrt{\left(\sqrt{3}+1\right)^2}}{2}+\frac{\sqrt{\left(\sqrt{3}-1\right)^2}}{2}=\frac{\sqrt{3}+1}{2}+\frac{\sqrt{3}-1}{2}=\sqrt{3}$

12. (d) We know that between two rational numbers x and y such that x < y there is a rational numbers $\frac{x+y}{2}$. This is $x<\frac{x+y}{2}<y$ therefore, a rational number between -3 and 4 is $\frac{-3+4}{2}=\frac{1}{2}$ *i.e.*, $-3<\frac{1}{2}<4$

13. (d) By actual division

```
45) 260  (.577
    225
    ---
     350
     315
     ---
      350
      315
      ---
       35
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$\therefore \frac{-16}{45}=-0.5\bar{7}$

14. (c) Let $x=1.\overline{27}$ then $x=1.27272727$ ….(i)

$\Rightarrow 100x=127.272727$ ………….(ii)

On subtraction (i) and (ii) we get

$99x=(127.272727...)-(1.272727)$

$\Rightarrow 99x=126$

$x=\frac{126}{99}=\frac{14}{11}$

Hence, $1.\overline{27}=\frac{14}{11}$

15. (a) $A \times B \times C = 2^x \times 4^y \times 8^z$

$\Rightarrow 2^x \times 2^y \times 2^{3x}$

$\Rightarrow 2^{(x+2y+3z)} \Rightarrow 2^{\left(\frac{1}{9}+\frac{8}{9}+\frac{18}{9}\right)}$

$\Rightarrow 2^3 = 8$

16. (a) $x = 2 + \frac{3}{9} - 1 = \frac{3}{2} \quad y = 2 + \frac{5}{9} - \frac{5}{9} = 2$

$\therefore x^2 + y^2 - 2xy = \frac{9}{4} + 4 - 2.3 \times 2$

$\frac{1}{4}$

17. (c) $x = (3)^{\frac{4}{9}+\frac{5}{9}} = (3)^{\frac{9}{9}} = 3$

$\therefore x.y = 9$

18. (b) $x = \frac{\sqrt{2}+\sqrt{1}}{\sqrt{2}-\sqrt{1}} \times \frac{\sqrt{2}+\sqrt{1}}{\sqrt{2}+\sqrt{1}}$

$\frac{\left(\sqrt{2}+\sqrt{1}\right)^2}{2-1}$

$\Rightarrow x = 2 + 1 + 2\sqrt{2} \Rightarrow x = 3 + 2\sqrt{2}$

And similarly $y = 3 - 2\sqrt{2}$

$x^2 + y^2 = (x+y)^2 - 2xy = (6)^2 - 2(9-8) = 34$

19. (b) $b = \frac{1}{a} = \frac{1}{3+2\sqrt{2}} = 3 - 2\sqrt{2}$

$\therefore a^2 + b^2 = (3+2\sqrt{2})^2 + (3-2\sqrt{2})^2$

$= 2(3^2 + (2\sqrt{2})^2 = 34$

20. (a) $4^{-3+5} \times a^{-5+8} \times b^{-4-3} = \frac{16 \times a^3}{b^7}$

21. (c) $2(27 \times 7)^{1/3} + 3(64 \times 7)^{1/3} - 7(8 \times 7)^{1/3}$

$\Rightarrow 6(7)^{1/3} + 12(7)^{1/3} - 14(7)^{1/3}$

$\Rightarrow (7)^{1/3}\, 6+12-14$

$\Rightarrow 4\times(7)^{1/3}=4\sqrt[3]{7}$

22. (d) $7(81\times 2)^{1/4}-5(16\times 2)^{1/4}+(625\times 2)^{1/4}$

$\Rightarrow 21(2)^{1/4}-10(2)^{1/4}+5(2)^{1/4}$

$\Rightarrow (2)^{1/4}(21-10+5)=16\sqrt[3]{2}$

23. (a) We know that, it a and b are two distinct positive irrational numbers, then $\sqrt{ab}$ is an irrational number lying between a and b.

$\therefore$ Irrational number between $\sqrt{3}$ and $\sqrt{5}$ is $\sqrt{\sqrt{3}\times\sqrt{5}}=\sqrt{\sqrt{15}}=15^{1/4}$

Irrational number between $\sqrt{3}$ and $15^{1/4}$ is $\sqrt{\sqrt{3}\times 15^{1/4}}=3^{1/4}\times 15^{1/8}$

Hence, required irrational numbers are $15^{\frac{1}{4}}$ and $3^{\frac{1}{4}}\times 15^{\frac{1}{8}}$

24. (c) We have,

$$x=\frac{1}{2+\sqrt{3}}=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{2-\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{2-\sqrt{3}}{4-3}=2-\sqrt{3}$$

$\Rightarrow x-2=-\sqrt{3}\Rightarrow (x-2)^2=\left(-\sqrt{3}\right)^2$

$\Rightarrow x^2-4x+4=3\Rightarrow x^2-4x+1=0$

$\therefore x^3-2x^2-7x+5$

$=x\left(x^2-4x+1\right)+2\left(x^2-4x+1\right)+3$

$=x\times 0+2\times 0+3=3.$

25. (d) $\left(2-\sqrt{3}\right)^2+4\left(2-\sqrt{3}\right)+4$

$=4+3-4\sqrt{3}+8-4\sqrt{3}+4=19-8\sqrt{3}$

26. (a) $2^{x+4}3^{x+1}$

$=2^5\times 3^2$

$\therefore x+4=5\ \&\ x+1=2\ \therefore x=1$

27. (d) $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\left(\sqrt{5}-\sqrt{3}\right)^2}{2}$

$= \frac{8-2\sqrt{15}}{2} = 4-\sqrt{25}$ $\quad \therefore a = 4, b = -1$

28. (a) $.\bar{4} = \frac{4}{9} = \frac{2}{3} = .6666...... = .\bar{6}$

29. (b) $\sqrt{182.25} + \sqrt{1.8225} + \sqrt{0.018225} + \sqrt{0.00018225} \Rightarrow 13.5 + 1.35 + 0.135 + 0.0135 \Rightarrow 14.9985$

30. (b) $2^{x-1}\left[1+2^2\right] = 640$

$2^{x-1} = 128 = 2^7 \Rightarrow x-1 = 7, x = 8$

31. (c) $\left(0.\overline{09} \times 7.\bar{3}\right) = \frac{9}{99} \times 7\frac{3}{9} = \frac{1}{11} \times \frac{22}{3} = \frac{2}{3}$

32. (d) $0.\overline{142857} \div 0.\overline{285714}$

$\Rightarrow \frac{142.857}{999999} \div \frac{285714}{99999} \Rightarrow \frac{142857}{285714} = \frac{1}{2}$

33. (c) $\frac{1}{1+2^{x-y}} + \frac{1}{1+2^{y-x}} \Rightarrow \frac{1}{1+\frac{2^x}{2^y}} + \frac{1}{1+\frac{2^y}{2^x}}$

$\Rightarrow \frac{2^y}{2^y+2^x} + \frac{2^x}{2^y+2^y} = \frac{2^x+2^y}{2^x+2^y} = 1$

34. (d) $x + \sqrt{7} = 7 + \sqrt{y}$

$\Rightarrow x = 7 \quad y = 7; \quad \therefore \frac{\sqrt{x}+y}{x+\sqrt{y}} = \frac{\sqrt{7}+7}{7+\sqrt{7}} = 1$

35. (a) (1) $2^{250} = \left(2^5\right)^{50} = 32^{50};$

(2) $3^{150} = \left(3^3\right)^{50} = (27)^{50}$

(3) $3^{100} = \left(5^2\right)^{50} = (25)^{50};$

(4) $4^{200} = \left(4^4\right)^{50} = (256)^{50}$

Hence, 4^{200} is the greatest.

36. (c) We have,

$$\frac{a^{-1}}{a^{-1}+b^{-1}}+\frac{a^{-1}}{a^{-1}-b^{-1}}$$

$$=\frac{1/a}{\frac{1}{a}+\frac{1}{b}}+\frac{1/a}{\frac{1}{a}-\frac{1}{b}}=\frac{1/a}{\frac{b+a}{ab}}+\frac{1/a}{\frac{b-a}{ab}}$$

$$=\frac{1}{a}.\frac{ab}{b+a}+\frac{1}{a}.\frac{ab}{b-a}=\frac{b}{b+a}+\frac{b}{b-a}$$

$$=\frac{b(b-a)+b(b+a)}{(b+a)(b-a)}=\frac{b^2-ab+b^2+ab}{b^2-a^2}$$

$$=\frac{2b^2}{b^2-a^2}$$

37. (a) $\frac{x^{-1}}{x^{-1}+y^{-1}}+\frac{x^{-1}}{x^{-1}-y^{-1}}\Rightarrow\frac{\frac{1}{x}}{\frac{1}{x}+\frac{1}{y}}+\frac{\frac{1}{x}}{\frac{1}{x}-\frac{1}{y}}$

$$\Rightarrow\frac{\frac{1}{x}\times xy}{x+y}+\frac{\frac{1}{x}\times xy}{x-y}\Rightarrow\frac{y}{y+x}-\frac{y}{y-x}\Rightarrow\frac{2y^2}{y^2-x^2}$$

38. (c) $\frac{\left(a^{x+y}\right)^2\left(a^{y+z}\right)^2\left(a^{z+x}\right)^2}{\left(a^{4x}.a^{4y}.a^{4z}\right)}\Rightarrow\frac{a^{2x+2y}.a^{2y+2z}.a^{2z+2x}}{a^{4x}.a^{4y}.a^{4z}}$

$$\Rightarrow\frac{a^{4x+4y+4z}}{a^{4x+4y+4z}}=1$$

39. (b) $\sqrt{11}-\sqrt{9}\times\frac{\sqrt{11}+\sqrt{9}}{\sqrt{11}+\sqrt{9}}=\frac{2}{\sqrt{11}+\sqrt{9}}$;

$$\sqrt{7}-\sqrt{5}\times\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}=\frac{2}{\sqrt{7}+\sqrt{5}}$$

$$\sqrt{5}-\sqrt{3}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{2}{\sqrt{5}+\sqrt{3}};$$

$$\sqrt{13}-\sqrt{11}\times\frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}+\sqrt{11}}=\frac{2}{\sqrt{13}+\sqrt{11}}$$

Hence, $\sqrt{5}-\sqrt{3}$ is the greatest

40. (c) $\left(\sqrt{6}+\sqrt{3}\right)^2=6+3+2\sqrt{18}=9+2\sqrt{18}$

$$\left(\sqrt{7}+\sqrt{2}\right)^2=7+2+2\sqrt{14}=9+2\sqrt{14}$$

$$\sqrt{7}+\sqrt{1}=8+1+2\sqrt{18}=9+2\sqrt{18}$$

$$\sqrt{5}+\sqrt{4}=5+4+2\sqrt{20}=9+2\sqrt{20}$$

Hence $\sqrt{8}+\sqrt{1}$ is smallest