Notes

Polynomials

Polynomials

1. **Polynomials:** If x is a variable, n be a positive integer and $a_0, a_1, a_2 \ldots, a_n$ are real number, then an expression of the form $p(x) = a_0 + a_1x + a_2x^2 + a_nx^n$ is called polynomial, in the variable x. In a polynomial, $p(x) = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_nx^n$, a_0, a_1x, $a_2x^2, \ldots,$ a_nx^2 are known as the terms of the polynomial and $a_0, a_1, a_2 \ldots\ldots$, an are known as their coefficients

2. **Degree of a polynomial:** Let p(x) be a polynomial in x. Then, the highest power of x in p(x) is called the degree of the polynomial p(x). Thus, the degree of the polynomial, $p(x) = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_nx^n$, where an $\neq$ 0 is n.

3. **Constant polynomial:** A polynomial of degree zero is called a constant polynomial e.g. $p(x) = -5$ is a constant polynomial.

4. **Zero polynomial:** The constant polynomial $p(x) = 0$ is called the zero polynomial. The degree of the zero polynomial is not defined since $p(x) = 0 = 0.x = 0.x^2 = 0.x^3 = \ldots$ etc.

5. **Linear polynomial:** A polynomial of degree 1 is called a linear polynomial A linear polynomial is of the form $p(x) = ax + b$, where $a \neq 0$ e.g. $5x + 1$, $-\frac{5}{2}x,\ 2\sqrt{3}x - \sqrt{2}\ etc.$ are linear polynomials.

6. **Quadratic polynomial:** A polynomial of degree 2 *is* called a quadratic polynomial. A quadratic polynomial is of the form $p(x) = ax^2 + bx + c$, where $a \neq 0$.

e.g. $x^2 - 5,\ 5\sqrt{2}x^2 - \frac{1}{\sqrt{3}}x,\ 7x^2 + \sqrt{5}$ etc. are quadratic polynomials.

7. **Cubic polynomial:** A polynomial of degree 3 is called a cubic polynomial A cubic polynomial is of the form $p(x) = ax^2 + bx^2 + cx + d$, where $a \neq 0$.

e.g. $x^3 - 20,\ \sqrt{5}x^3 - \frac{1}{9}x,\ \frac{7}{2}x^3 - \frac{1}{2}x^2 - 4$ etc. are cubic polynomials.

8. **Biquadratic polynomial:** A polynomial of degree 4 is called a biquadratic polynomial. A biquadratic polynomial is of the form $p(x) = ax^4 + bx^3 + cx^2dx + e$, where $a \neq 0$.

e.g. $x^4 - 23, \sqrt{3}x^4 - \frac{1}{9}x,\ \frac{1}{2}x^4 + \frac{3}{4}x - \frac{1}{8}$ etc. are biquadratic polynomials.

9. **Zeros of a polynomial:** A real number k is said to be a zero of the polynomial p(x), if $p(k) = 0$.

10. **Relationship between the Zeros and Coefficients of a Linear Polynomial:** The zero of a linear polynomial

$p(x) = ax + b$ is given by $\alpha = \frac{-b}{a} = \frac{-(\text{constant term})}{(\text{coefficient of x})}$.

A linear polynomial can have at the most one zero.

11. Relationship between the Zeros and Coefficients of a Quadratic Polynomial:

(i) If α and β are the zeros of a quadratic polynomial $p(x) = ax^2 + bx + c,\ a \neq 0$ then

$$\alpha + \beta = \frac{-b}{a} = \frac{-(\text{coefficient of x})}{(\text{coefficien of } x^2)};$$

(ii) A quadratic polynomial whose zeroes are α and β is given by:

$$p(x) = x^2 - (\alpha + \beta) + (\alpha\beta)$$

12. Relationship between the Zeros and Coefficients of a Cubic Polynomial:

(i) If α, β *and* γ are the zeros of $p(x) = ax^3 + bx^2 + cx + d$, then

$$(\alpha + \beta + \gamma) = \frac{-b}{a} = \frac{-(\text{coefficient of } x^2)}{(\text{coefficient of } x^3)};$$

$$(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{c}{a} = \frac{(\text{coefficient of x})}{(\text{coefficient of } x^3)};$$

$$\alpha\beta\gamma = \frac{-d}{a} = \frac{-(constant\ term)}{(\text{coefficient of } x^3)}$$

(ii) A cubic polynomial whose zeros are α, β *and* γ is given by

$$p(x) = \{x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\}$$

Snap Test

1. A real number a is called a zero of the polynomial f(x) if

(a) $f(0) = a$ (b) $f(a) - a$

(c) $f(a) = 0$ (d) $f(a) = f(0)$

(e) None of these

Ans. (c)

Explanation: We know that for the polynomial f(x), if $f(a) = 0$, then a is a zero of the polynomial f(x)

2. If zeros of a quadratic polynomial are $(-3+\sqrt{3})$ and $(-3-\sqrt{3})$, find the polynomial.

(a) $x^2 + 6x + 6$ (b) $x^2 - 6x + 6$

(c) $x^2 + 2x + 4$ (d) $x + 6x - 6$

(e) None of these

Ans. (a)

Explanation: Required polynomial

$$f(x) = [x - (-3 + \sqrt{3})]\,[x - (-3 - \sqrt{3})]$$

$$= [(x + 3) - \sqrt{3}]\ [(x + 3) + \sqrt{3}]$$

$$= (x + 3)^2 - (\sqrt{3})^2 \qquad [\because (a - b)(a + b) = a^2 - b^2]$$

$$= (x^2 + 6x + 9) - 3 = x^2 + 6x + 6.$$

3. **If α and β are the zeros of the polynomial $p(x) = x^2 + 12x + 35$, evaluate $\frac{1}{\alpha}+\frac{1}{\beta}$.**

(a) $\frac{-10}{35}$ (b) $\frac{-12}{35}$

(c) $\frac{-14}{35}$ (d) $\frac{-11}{35}$

(e) None of these

Ans. (b)

Explanation: Given that α *and* β are the zeros of the polynomial $p(x) = x^2 + 12x + 35$, Therefore, $\alpha+\beta = -12$ and $\alpha\beta = 35$.

$$\therefore \frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-12}{35}$$

4. **One of the zeros of the quadratic polynomial $f(x) = 14x^2 - 42k^2x - 9$ is negative of the other. Find the value of k.**

(a) $k = 0$ (b) $k = 1$

(c) $k = 3$ (d) $k = 2$

(e) None of these

Ans. (a)

Explanation: Let α *and* β be the zeros of the polynomial, $f(x) = 14x^2 - 42k^2x - 9$.

Then $\alpha+\beta = \frac{42k^2}{14} = 3k^2$ [Sum of the zeros of f(x)]

Now, let $\beta = (-\alpha)$ [Since one of the zeros of f(x) is negative of the other]

Then, $\alpha+\beta = 0$.

Equating the two values of $(\alpha+\beta)$ we get:

$3k^2 = 0 \Rightarrow k^2 = 0 \Rightarrow k = 0$

5. **The zeros of the cubic polynomial $f(x) = x^3 - 6x^2 - 13x + 42$ are in A.P. Find the its zeros.**

(a) -2, 3 and 7 (b) -1, 2 and 5

(c) -3, 2 and 7 (d) -4, 2 and 5

(e) None of these

Ans. (c)

Explanation: Let $(\alpha - d), \alpha$ and $(\alpha+d)$ be the zeros of the polynomial

$f(x) = x^3 - 6x^2 - 13x + 42$ (Since the zeros are in A.P.)

Then, Sum of the zeros of $f(x) = 6$

i.e. $(\alpha - d) + \alpha + (\alpha + d) = 6 \Rightarrow 3\alpha = 6 \Rightarrow \alpha = 2$

Also, Product of the zeros of $f(x) = -42$

i.e. $(\alpha - d)\,\alpha(\alpha + d) = -42$

$\Rightarrow \alpha(\alpha^2 - d^2) = -42 \Rightarrow 2(2^2 - d^2) = -42$

$\Rightarrow d^2 = 25 \Rightarrow d = \pm 5$

Taking any of the values of d i.e. taking either $d = 5$ or $d = -5$, we get the zeros of f (x) as - 3, 2 and 7.