Chapter 9 Factoring

Answer 1PT.

The example of a prime number is 7.

Since a whole number, greater than 1, whose only factors are 1 and itself, is called a prime number.

Here 7 have only two factors.

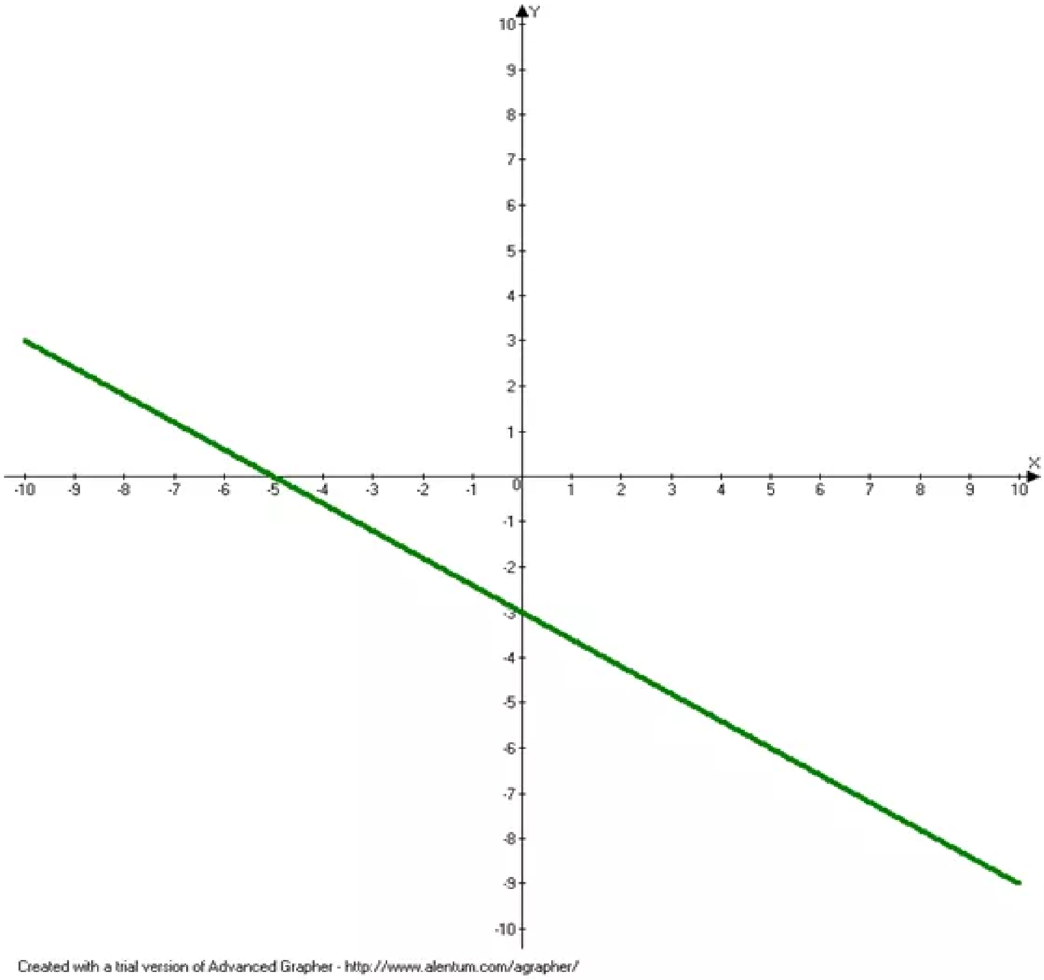
$7 = 1 \cdot 7$

Those are $1, 7$.

Therefore, $\boxed{7}$ is a prime number.

Answer 1STP.

Consider the following graph



By observing the given graph, the line intersect *x* –axis at -5 and intersects *y* –axis at -3

That is *x* – intercept is $= -5$

Y – intercept is $= -3$

The intercepts form of an equation is $\frac{x}{a} + \frac{y}{b} = 1$

Here *a* is *x* – intercept $= -5$

b is *y* – intercept $= -3$

$$\frac{x}{-5}+\frac{y}{-3}=1$$

$$\frac{-x}{5}-\frac{y}{3}+\frac{y}{3}=1+\frac{y}{3} \qquad \left[\text{Add } \frac{y}{3} \text{ on both sides}\right]$$

$$\frac{-x}{5}=1+\frac{y}{3}$$

$$-\frac{x}{5}-1=1+\frac{y}{3}-1 \qquad [\text{Subtract 1 on both sides}]$$

$$\frac{-x}{5}-1=\frac{y}{3}$$

$$3\left(\frac{-x}{5}-1\right)=\frac{y}{3}\cdot 3 \qquad [\text{Multiply with 3 on both sides}]$$

$$3\cdot\frac{-x}{5}-3=y$$

$$y=\frac{-3}{5}x-3$$

Therefore the equation of function graphed is $\boxed{y=\frac{-3}{5}x-3}$

Answer 1VC.

Consider the statement "the number 27 is an example of a prime number".

The objective is to check the given sentence is true or false.

A whole number greater than 1 and whose only factors are 1 and itself is called a prime number.

$$27=3\cdot 9 \quad (27=3\cdot 9)$$

$$=3\cdot 3\cdot 3 \quad (3\cdot 3=9)$$

Therefore 27 has another factor 3.

Therefore 27 is not a prime number, it is composite number.

Therefore the given statement is false.

The number 27 is not a prime number, it is composite number.

Answer 2PT.

Consider the polynomial $x^2 - 36$

Here $36 = 6 \cdot 6$

$= 6^2$

$x^2 - 36 = x^2 - 6^2$

Clearly it is the difference of squares x^2 and 6^2.

The difference of squares property is

$a^2 - b^2 = (a+b)(a-b)$

$x^2 - 36 = x^2 - 62$

$= (x+6)(x-6)$ (Because $a^2 - b^2 = (a+b)(a-b)$)

The factorization of $x^2 - 36$ is $(x+6)(x-6)$.

Answer 2STP.

Consider that the school band, sold tickets to their spring concert everyday at lunch for one week.

Also before they sold any tickets they had gone in their account.

At the end of each day, they recorded the total number of tickets sold and the total amount of money in the bands accounts.

Day	Total number of tickets sold *t*	Total amount in account *a*
Monday	12	\$ 176
Tuesdays	18	\$ 224
Wednesday	24	\$ 272
Thursday	30	\$ 320
Friday	36	\$ 368

The objective is to find the equation that describes the relationship between the total number of tickets sold't' and the amount of money in bands account *a*.

Since In Monday they sold 12 tickets

Amount in account $a = \$176$

Initially, the account had $ 80

Then the cost of 12 tickets $= 176 - 80$

$= 96$

The cost of 12 tickets = 96

The cost of 1 ticket $= \frac{96}{12}$

$= 8$

The relation is

Amount in account $= (\text{cost of 1 ticket}) \cdot \text{number of tickets sold}$

$+ \text{ Initial amount in the account}$

$a = 8 \cdot t + 80$

Check

In Tuesday, number of tickets sold = 18

$a = 8t + 80$

Amount in account, $= 8(18) + 80$

$= 144 + 80$

$= \$224 \text{ True}$

Therefore, the required equation is $\boxed{a = 8t + 80}$

Answer 2VC.

Consider the following statement

" $\underline{2x}$ is the greatest common factor (GCF) of $12x^2$ and $14xy$ ".

The objective is to check the given sentence is true or false.

The GCF of two or more monomials is the product of their common factors when each monomial is in factored form.

$$12x^2 = 2\cdot 6\cdot x^2$$
$$= 2\cdot 2\cdot 3\cdot x\cdot x$$
$$14xy = 2\cdot 7\cdot x\cdot y$$

Circle the common factors

$$= 2\cdot x$$
$$= 2x$$

$2x$ is the greater common factor of $12x^2$ and $14xy$.

<u>The given sentence is True</u>.

Answer 3PT.

Consider the number 63.

The objective is to find the prime factorization of given number.

For this use factor tree method.

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63

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$3\cdot 21$ (Since $3\cdot 21 = 63$)

$3\cdot 7$ $(3\cdot 7 = 21)$

All the factors in branches are primes.

The prime factorization of 63 is $3\cdot 3\cdot 7$.

$$63 = 3\cdot 3\cdot 7$$
$$= 3^2\cdot 7$$

The prime factorization of 63 is $\boxed{3^2\cdot 7}$.

Answer 3STP.

In factoring any polynomial the first step is to find the GCF if it is there.

It will make factoring polynomial much easier because the number of factors of each term will be lower.

It is especially important if the GCF includes a variable.

On forgetting the GCF, if it includes a variable, it could miss a root, add then end up with an incorrect graph for the polynomial.

For example, consider the polynomial

$$\begin{aligned} 4x^2 + 6x &= 2\cdot 2\cdot x\cdot x + 2\cdot 3\cdot x \\ &= 2x[2x+3] \qquad [\text{Factor GCF } 2x] \end{aligned}$$

Thus $4x^2 + 6x = \boxed{2x[2x+3]}$

Answer 3VC.

Consider the following statement

" $\underline{66}$ is an example of a perfect square".

The objective is to check the given sentence is true or false.

$$\begin{aligned} 66 &= 2\cdot 33 \\ &= 2\cdot 3\cdot 11 \end{aligned}$$

A number is perfect square; it is square root of a whole number.

66 is not a perfect square number.

The given sentence is false.

64 is a perfect square number.

Since $\begin{aligned} 64 &= 8\cdot 8 \\ &= 8^2 \end{aligned}$

Therefore, the given statement is False.

$\underline{64}$ is an example of a perfect square.

Answer 4STP.

Today, the refreshment stand at the high school football game sold twice as many bags of popcorn as were sold last Friday

The total sold both the days was 258 bags.

Also the number of bags sold today = n

The number of bags sold last Friday = f

Since, the number of bags sold Today = twice the bags sold last Friday.

$$n = 2 \times f$$
$$n = 2f$$

Also,

Bags sold today plus bags sold last Friday = 258

$$n + f = 258$$

The system represents the given data is

$$\begin{aligned} n + f &= 258 \\ n &= 2f \end{aligned}$$

Therefore, the system of equations represent, given data is

$$\boxed{\begin{aligned} n + f &= 258 \\ n &= 2f \end{aligned}}$$

Answer 4VC.

Consider the following statement

" 61 is a factor of 183 ".

The objective is to check the given sentence is true or false.

For this find the factorization of 183.

$$183 = 3 \cdot 61$$

61 is in the factorization of 183.

Therefore 61 is a factor of 183.

Hence the given statement is true.

Therefore, $\boxed{61}$ is a factor of $\boxed{183}$ is true.

Answer 5PT.

Consider the number 81.

The objective is to find the prime factorization given.

For this use factor tree method.

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81

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$3 \cdot 27 \quad (3 \cdot 27 = 81)$

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$3 \cdot 9 \quad (3 \cdot 9 = 27)$

$3 \cdot 3 \quad (3 \cdot 3 = 9)$

All the factors in last branches are primes.

The prime factorization of 81 is $3 \cdot 3 \cdot 3 \cdot 3$.

$$81 = 3 \cdot 3 \cdot 3 \cdot 3$$
$$= 3^4$$

The prime factorization of 81 is $\boxed{3^4}$.

Answer 5STP.

Consider the polynomial number $5 \cdot 387 \times 10^{-3}$

The objective is to express the given number in standard form.

If the exponent is a positive number, more the decimal point the same number of places to the right as the number of the exponent.

If the exponent is a negative number, move the decimal point the same number of places to the left as the number of the exponent.

In $5 \cdot 387 \times 10^{-3}$, the exponent is -3, it is a negative number, so move the decimal point three places to the left in the first factor $5 \cdot 387$.

In the case $5 \cdot 387$ will become $0 \cdot 005387$.

Note the add zeros in all empty places.

Thus, $\boxed{0 \cdot 005387}$ is the standard form for $5 \cdot 387 \times 10^{-3}$.

Answer 5VC.

Consider the following statement

“the prime factorization for 48 is $\underline{3 \cdot 4^2}$”.

The objective is to check the given sentence is true or false”.

Prime factorization, of 48 is

$$\begin{aligned} 48 &= 2 \cdot 24 \\ &= 2 \cdot 2 \cdot 12 \\ &= 2 \cdot 2 \cdot 2 \cdot 6 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \\ &= 2^4 \cdot 3 \end{aligned}$$

Therefore, the prime factorization of 48 is $2^4 \cdot 3$.

The given sentence is false.

The prime factorization for 48 is $\underline{2^4 \cdot 3}$.

Answer 6PT.

Consider the number -210

The objective is to find the prime factorization of -210.

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$-210 = -1 \cdot 210$

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$= 2 \cdot 105$ (Because $2 \cdot 105 = 210$)

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$= 3 \cdot 35$ (Because $3 \cdot 35 = 105$)

$= 5 \cdot 7$ $(5 \cdot 7 = 35)$

All the factors in last branches are primes.

$$-210 = -1 \cdot 2 \cdot 3 \cdot 5 \cdot 7$$

The factorization of -210 is $\boxed{-1 \cdot 2 \cdot 3 \cdot 5 \cdot 7}$.

Answer 6VC.

Consider the following statement

" $x^2 - 25$ is an example of a perfect square trinomial".

The objective is to check the given sentence is true or false.

Given expression is $x^2 - 25$.

It has two terms.

A perfect square trinomial must contain three terms

Therefore $x^2 - 25$ is not a perfect square trinomial

The given sentence is false.

$$x^2 - 10x + 25 = x^2 - 2 \cdot 5 \cdot x + 5^2$$
$$= (x-5)^2$$

Therefore $x^2 - 10x + 25$ is a perfect square trinomial.

Therefore, the given sentence is false.

$x^2 - 10x + 25$ is an example of a perfect square trinomial.

Answer 7PT.

Consider the set of monomials are $48, 64$.

The objective is to find the GCF of given set of monomials.

Since the GCF of two or more set of monomials is the product of the common factors which are in both the factorization.

For this first factor each monomial completely.

$$48 = 2 \cdot 24 \quad (2 \cdot 24 = 48)$$
$$= 2 \cdot 2 \cdot 12 \quad (2 \cdot 12 = 24)$$
$$= 2 \cdot 2 \cdot 2 \cdot 6 \quad (12 = 2 \cdot 6)$$
$$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \quad (6 = 2 \cdot 3)$$
$$64 = 2 \cdot 32 \quad (64 = 2 \cdot 32)$$
$$= 2 \cdot 2 \cdot 16 \quad (32 = 2 \cdot 16)$$
$$= 2 \cdot 2 \cdot 2 \cdot 8 \quad (2 \cdot 4 = 8)$$
$$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \quad (2 \cdot 2 = 4)$$

Now circle the common factors

$$48 = (2) \cdot (2) \cdot (2) \cdot (2) \cdot 3$$
$$64 = (2) \cdot (2) \cdot (2) \cdot (2) \cdot 2 \cdot 2$$

$GCF =$ Product of factors

$$= 2 \cdot 2 \cdot 2 \cdot 2$$
$$= 4 \cdot 4$$
$$= 16$$

Thus $\boxed{GCF}$ of $\boxed{48, 64}$ is $\boxed{16}$.

Answer 7STP.

The equation is $3x^2 - 48 = 0$

The objective is to find the solution set of given equation

$$3x^2 - 48 = 0$$
$$3 \cdot x^2 - 3 \cdot 16 = 0$$
$$3(x^2 - 16) = 0 \quad \left[\text{Factor GCF } (3x^2, 48) = 3\right]$$
$$3(x^2 - 4 \cdot 4) = 0 \quad [\text{Since } 4 \cdot 4 = 16]$$
$$3(x^2 - 4^2) = 0$$
$$3(x+4)(x-4) = 0 \quad \left[\text{Since } a^2 - b^2 = (a+b)(a-b)\right]$$

The zero product property is if $ab = 0$ then $a = 0 \text{ or } b = 0$ or both

$$x + 4 = 0 \text{ or } x - 4 = 0 \quad [\text{By zero product property adn } 3 \neq 0]$$

Now solve each equation separately

$$x + 4 = 0$$
$$x + 4 - 4 = 0 - 4 \quad [\text{Subtract 4 on both sides}]$$
$$x = -4$$
$$x - 4 = 0$$
$$x - 4 + 4 = 0 + 4 \quad [\text{Add 4 on both sides}]$$
$$x = 4$$

The solution set is $\{-4, 4\}$

Therefore, the solution of given equation is $\boxed{\{-4, 4\}}$

Answer 7VC.

Consider the sentence is "the number 35 is an example of a composite number".

The objective is to check the given sentence is true or false.

A whole number greater than 1 that has more than two factors is a composite number.

$$35 = 1 \cdot 35$$
$$35 = 5 \cdot 7$$

Therefore factors of 35 are $1, 5, 7, 35$

It has 4 factors.

35 is a composite number.

"The number 35 is an example of a composite number".

The given sentence is True.

The number 35 is an example of composite number.

Answer 8PT.

Consider the set of monomials are $28, 75$.

The objective is to find the GCF of given set of monomials.

Since the GCF of two or more monomials is the product of the common factors in each factorization.

$$28 = 2 \cdot 14 \quad (2 \cdot 14 = 28)$$

$$= 2 \cdot 2 \cdot 7 \quad (2 \cdot 7 = 14)$$

$$75 = 3 \cdot 25 \quad (3 \cdot 25 = 75)$$

$$= 3 \cdot 5 \cdot 5 \quad (5 \cdot 5 = 25)$$

Now circle the common factors

$$28 = 2 \cdot 2 \cdot 7$$
$$75 = 3 \cdot 5 \cdot 5$$

Since, they have no common factors.

In this case, the GCF of $28, 75$ is 1.

Therefore GCF of $28, 75$ is $\boxed{1}$.

Answer 8STP.

The equation is $x^2 - 3x + 8 = 6x - 6$

The objective is to find the solution set of given equation.

$$x^2 - 3x + 8 = 6x - 6$$
$$x^2 - 3x + 8 + 6 = 6x - 6 + 6 \quad \text{[Add 6 on both sides]}$$
$$x^2 - 3x + 14 = 6x \quad \text{[Simplify]}$$
$$x^2 - 3x + 14 - 6x = 6x - 6x \quad \text{[Subtract } 6x \text{ on both sides]}$$
$$x^2 - 9x + 14 = 0 \quad \text{[Combine like terms]}$$

To solve the given equation first factor $x^2 - 9x + 14$

Compare $x^2 - 9x + 14$ with $ax^2 + bx + c$

Here $b = -9, c = 14$

$$x^2 - 9x + 14 = (x+m)(x+n)$$
$$= x^2 + (m+n)x + mn$$

Here $m + n = -9, mn = 14$

Now find two numbers m, n such that $m + n = -9$ and $m \cdot n = 14$

Since $m + n$ is negative and mn is positive, then both *m* and *n* must be negative.

Now list all the pairs of negative factors of 14 in those choose a pair whose sum is -9

Factor of 14	Sum of factors
$-1 \cdot -14$	-15
$-2 \cdot -7$	✓ -9

The correct factors are $-2, -7$

$$x^2-9x+14=(x+m)(x+n)$$
$$=(x-2)(x-7) \qquad [m=-2, n=-7]$$
$$x^2-9x+14=0$$
$$(x-2)(x-7)=0$$

The zero product property is if $ab=0$ then $a=0$ or $b=0$ or both

$$x-2=0 \text{ or } x-7=0$$

Now solve each equation completely

$$x-2=0$$
$$x-2+2=0+2 \qquad [\text{Add 2 on both sides}]$$
$$x=2$$
$$x-7=0$$
$$x-7+7=0+7 \qquad [\text{Add 7 on both sides}]$$
$$x=7$$

The solution set is $\{2,7\}$

Check:

To check the proposed solution set substitute each solution in the given equation

Given equation is

$$x^2-3x+8=6x-6$$
$$(2)^2-3(2)+8=6(2)-6 \qquad [\text{Put } x=2]$$
$$4-6+8=12-6$$
$$6=6 \text{ True}$$

$$x^2-3x+8=6x-6$$
$$(7)^2-3(7)+8=6(7)-6 \qquad [\text{Put } x=7]$$
$$49-21+8=42-6 \qquad [\text{Simplify}]$$
$$36=36 \text{ True}$$

The solution set of given equation is $\boxed{\{7,2\}}$

Answer 8VC.

$$x^2-3x-70=(x+m)(x+n)$$

$$=(x+7)(x-10)\quad (m=7, n=-10)$$

$$x^2-3x-70=(x+7)(x-10)$$

Is a product of two polynomials with integral coefficients.

$x^2-3x-70$ is not a prime polynomial.

<u>The given sentence if False</u>.

$2x^2+5x-2$ can not be factored.

Therefore, $\underline{2x^2+5x-2}$ is an example of a prime polynomial.

Answer 9PT.

Consider the set of monomials are $18a^2b^2, 28a^3b^2$.

The objective is to find the GCF of given set of monomials

Since the GCF of two or more monomials is the product of the common factors which are in factorizations of monomials.

For this first find the prime factorizations of given monomials.

$$18a^2b^2=2\cdot 9a^2b^2\quad (2\cdot 9=18)$$

$$=2\cdot 3\cdot 3\cdot a^2\cdot b^2\quad (3\cdot 3=9)$$

$$=2\cdot 3\cdot 3\cdot a\cdot a\cdot b^2\quad (a^2=a\cdot a)$$

$$=2\cdot 3\cdot 3\cdot a\cdot a\cdot b\cdot b\quad (b^2=b\cdot b)$$

$$28a^3b^2=2\cdot 14a^3b^2 \text{ (Since } 28=2\cdot 14\text{)}$$

$$=2\cdot 2\cdot 7a^3b^2\quad (2\cdot 7=14)$$

$$=2\cdot 2\cdot 7\cdot a\cdot a\cdot a\cdot b^2\quad (a^3=a\cdot a\cdot a)$$

$$=2\cdot 2\cdot 7\cdot a\cdot a\cdot a\cdot b\cdot b\quad (b^2=b\cdot b)$$

Now circle the common factors

$$18a^2b^2=(2)\cdot 3\cdot 3\cdot (a)\cdot (a)\cdot (b)\cdot (b)$$

$$28a^3b^2=(2)\cdot 2\cdot 7\cdot (a)\cdot (a)\cdot a\cdot (b)\cdot (b)$$

$GCF =$ Product of factors

$$=2\cdot a\cdot a\cdot b\cdot b$$

$$=2a^2b^2$$

Therefore GCF of given set of monomials is $\boxed{2a^2b^2}$.

Answer 9STP.

Consider that the area of a rectangle is $12x^2 - 21x - 6$

The width is $3x - 6$

The objective is to find the length of rectangle.

For this express $12x^2 - 21x - 6$ as product of factors

$$\begin{aligned} 12x^2 - 21x - 6 &= 3 \cdot 4x^2 - 3 \cdot 7x - 3 \cdot 2 \\ &= 3\left(4x^2 - 7x - 2\right) \qquad [\text{Factor GCF}] \end{aligned}$$

Now compare $4x^2 - 7x - 2$ with $ax^2 + bx + c$

Here $a = 4, b = -7, c = -2$

$$4x^2 - 7x - 2 = 4x^2 + mx + nx - 2$$

Now find two numbers m, n such that $m + n = -7$ and $m \cdot n = 4 \cdot -2 = -8$

Since $m + n$ and $m \cdot n$ are negative then one of *m* or *n* must be negative but not both

List all the factors of -8 in those choose a factor whose sum is -7

Factor of -8	Sum of factors
$-1 \cdot 8$	7
$1 \cdot -8$	✓ -7
$-2 \cdot 4$	2
$2 \cdot -4$	-2

The correct factors are $1, -8$

Thus $3\left(4x^2-7x-2\right)=3\left[4x^2-8x+x-2\right]$

$$=3\left[4x(x-2)+(x-2)\right] \qquad \left[\text{By distributive } (b+c)a=ba+ca\right]$$
$$=(4x+1)3(x-2)$$
$$=(4x+1)(3x-6)$$

Therefore, area of rectangle $=(4x+1)(3x-6)$

Given width of rectangle $=3x-6$

Since area of rectangle $A=\text{length}\cdot\text{width}$

$$(4x+1)(3x-6)=\text{length}\cdot(3x-6)$$
$$\frac{(4x+1)(3x-6)}{(3x-6)}=\text{length}\cdot\frac{(3x-6)}{(3x-6)} \qquad [\text{Divide with } 3x-6].$$
$$4x+1=\text{length} \qquad [\text{Simplify}]$$

Therefore, length of rectangle is $\boxed{4x+1}$

Answer 9VC.

Consider sentence is "expressions with four or more unlike terms can sometimes be factored by grouping ".

The objective is to check the given sentences is true or false

The given sentence is true.

The distributive property can also be used to factor some polynomials having a fair or more terms.

This method is called factoring by grouping because pairs of term are graped together and factored.

The given sentence is true.

Answer 10PT.

Consider the polynomial $25y^2-49w^2$

The objective is to factor the given polynomial.

$25y^2-49w^2=5\cdot 5y^2-49w^2$ (Since $5\cdot 5=25$)

$=5^2y^2-7\cdot 7w^2 \quad (7\cdot 7=49)$

$=(5y)^2-7^2w^2 \quad \left(a^m\cdot a^m=(ab)^m\right)$

$=(5y)^2-(7w)^2$

The difference of squares property is

$a^2-b^2=(a+b)(a-b)$

$=(5y+7w)(5y-7w)$

[Since $a^2-b^2=(a+b)(a-b)$]

Thus,

$25y^2-49w^2=(5y+7w)(5y-7w)$

Therefore the factorization of given polynomial is $\boxed{(5y+7w)(5y-7w)}$.

Answer 10STP.

Consider that the *y*-intercept of a line is -1

The objective is to find the equation of a line that has *y* intercept -1 and is perpendicular to the graph of $2-2y=-5x$.

Let $y=mx+c$ be the required line.

Where c is *y*-intercept.

m is scope of line,

Since $y=mx+c$ is perpendicular to $2-2y=-5x$

Product of these scope is -1.

For this first find the scope of $2-2y=-5x$

$$2-2y=-5x$$

$$2-2y-2=-5x-2 \qquad (\text{Subtract } 2 \text{ on both sides})$$

$$-2y=-5x-2$$

$$\frac{-2y}{-2}=\frac{-5x}{-2}-\frac{-2}{-2} \qquad (\text{Divide with } -2 \text{ on both sides})$$

$$y=\frac{5}{2}x+1$$

Scope $=\frac{5}{2}$

Thus,

$$m\cdot\frac{5}{2}=-1$$

$$m=\frac{-1}{\frac{5}{2}}=\frac{-2}{5}$$

Thus,

$$y=mx+c$$

$$=\frac{-2}{5}x+(-1) \qquad \left(\text{Scope}=\frac{-2}{5}, \text{y-intercept } c=-1\right)$$

$$y=\frac{-2}{5}x-1$$

Therefore, equation of required line is $\boxed{y=\frac{-2}{5}x-1}$.

Answer 10VC.

Consider the sentence is " $\underline{(b-7)(b+7)}$ is the factorization of a difference of squares".

The objective is to check the given sentence is true or false

Since the difference of squares is

$$a^2-b^2=(a+b)(a-b)$$
$$(b-7)(b+7)=(b+7)(b-7)$$
$$=b^2-7^2$$

Therefore $(b-7)(b+7)$ is the factorization of a difference of squares b^2-7^2.

The given sentence is <u>true</u>.

Answer 11E.

Consider the integer 28.

The objective is to find the prime factorization of 28

Use factor tree to find prime factorization.

28

$2\cdot 14$ (Because $2\cdot 14=28$)

$2\cdot 7$ $(2\cdot 7=14)$

All the factors in the last branch of the factor tree are primes.

Thus, the prime factorization of 28 is

$$2\cdot 2\cdot 7=2^2\cdot 7$$

Therefore, the prime factorization of 28 is $\boxed{2^2\cdot 7}$.

Answer 11PT.

Consider the polynomial $t^2 - 16t + 64$.

The objective is to find the factors of given polynomial.

$t^2 - 16t + 64 = t^2 - 2\cdot 8\cdot t + 8\cdot 8$ (Since $16 = 2\cdot 8, 8\cdot 8 = 64$)

$= t^2 - 2\cdot 8\cdot t + 8^2 \quad \left(8^2 = 8\cdot 8\right)$

The perfect squares property is

$(a-b)^2 = a^2 - 2ab + b^2$

$= (t-8)^2 \quad (a = t, b = 8)$

Thus,

$t^2 - 16t + 64 = (t-8)^2$

Therefore the factorization of given polynomial is $\boxed{(t-8)^2}$.

Answer 11STP.

The equation is $6|x-2| = 18$

The objective is to find the values for *x* to make the given equation true.

$$6|x-2| = 18$$
$$\frac{6}{6}|x-2| = \frac{18}{6} \qquad [\text{Divide with 6 on both sides}]$$
$$|x-2| = 3$$

Since $|x| = a$ implies $x = a$ and $x = -a$

$|x-2| = 3$ implies

$x - 2 = 3$ and $x - 2 = -3$

Now solve each equation completely

$$x - 2 = 3$$
$$x - 2 + 2 = 3 + 2 \qquad [\text{Add 2 on both sides}]$$
$$x = 5$$
$$x - 2 = -3$$
$$x - 2 + 2 = -3 + 2 \qquad [\text{Add 2 on both sides}]$$
$$x = -1$$

The values of *x* are 5, -1

Therefore, the values of *x* are $\boxed{-1, 5}$

Answer 12PT.

Consider the polynomial $x^2 + 14x + 24$.

The objective is to factor the given polynomial.

Compare $x^2 + 14x + 24$ with $x^2 + bx + c$

Here $b = 14$,

$c = 24$

$$x^2 + 14x + 24 = (x+m)(x+n)$$
$$= x^2 + (m+n)x + mn$$

Here $m + n = 14$,

$mn = 24$.

Since $m+n, mn$ are positive, then find two values of m, n such that

$m + n = 14$,

$mn = 24$, m, n are positive.

Now list all the factors of

$mn = 24$, in those choose a pair whose sum is 14.

Factors of 24	Sum of factors
$1, 24$	25
$2, 12$	14
$3, 8$	11
$4, 6$	10

The correct factors are $2, 12$.

Thus $x^2 + 14x + 24 = (x+m)(x+n)$

$= (x+2)(x+12) \quad (m = 2, n = 12)$

Check:- To check the proposed solution, multiply the factors using $FOIL$ method.

$$(x+2)(x+12)=\overset{F}{x\cdot x}+\overset{O}{12\cdot x}+\overset{I}{2\cdot x}+\overset{L}{2\cdot 12}$$

($FOIL$ Method)

$$=x^2+12x+2x+24$$

(Simplify)

$$=x^2+14x+24$$ True

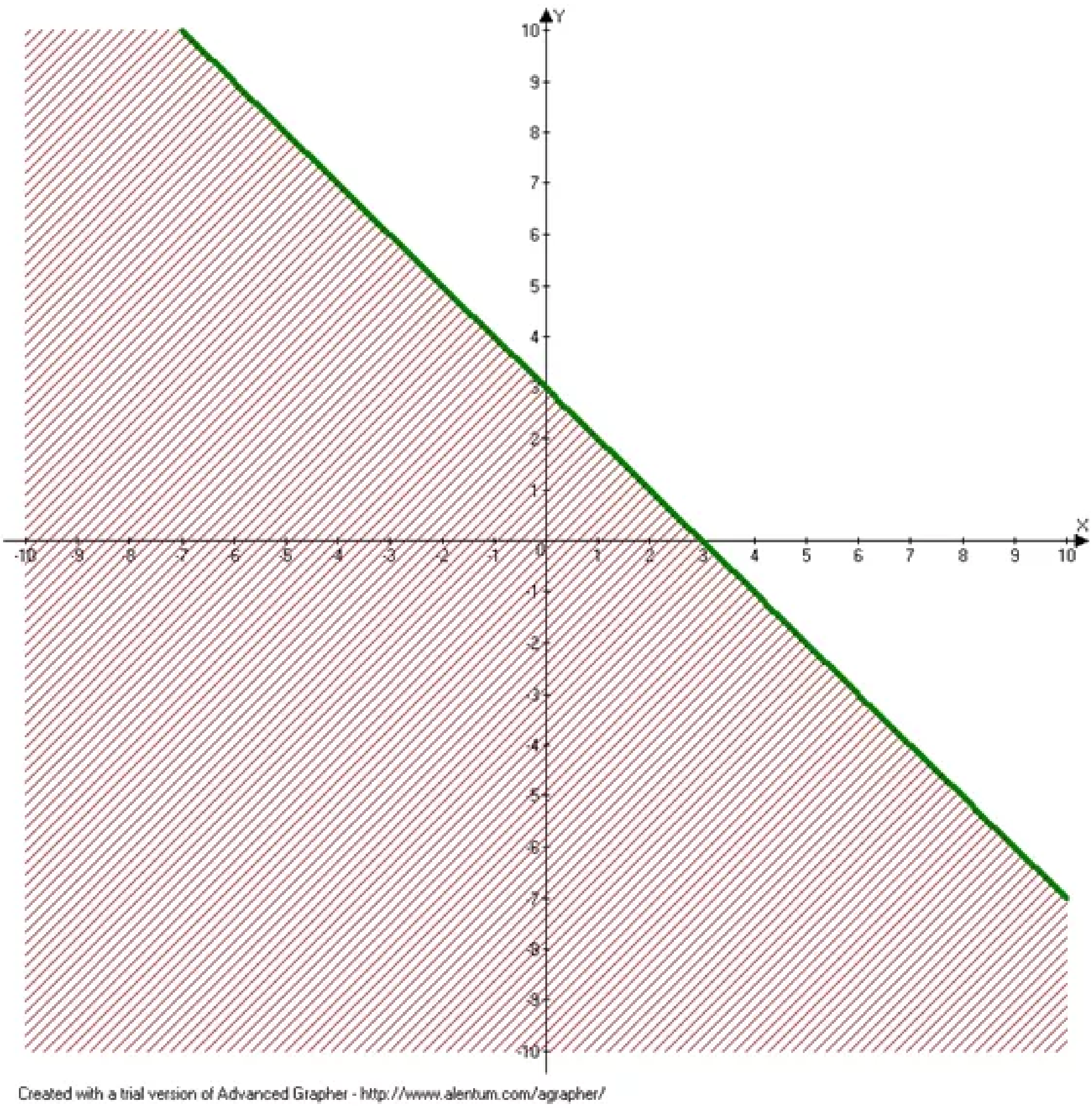
Therefore, the factorization of given polynomial is $\boxed{(x+2)(x+12)}$.

Answer 12STP.

The inequality is $x+y\le 3$

The objective is to draw the graph of given inequality.

The graph of $x+y\le 3$ is



Answer 13E.

Consider the integer 150

The objective is to find the prime factorization of 150.

For this use factor Tree.

\

/

150

\

/

$2 \cdot 75 \quad (2 \cdot 75 = 150)$

\

/

$5 \cdot 15 \quad (5 \cdot 15 = 75)$

$3 \cdot 5 \quad (3 \cdot 5 = 15)$

All the factors in the last branch of the factor tree are primes.

Thus, the prime factorization of 150 is $2 \cdot 5 \cdot 3 \cdot 5$

$= 2 \cdot 3 \cdot 5^2$

Therefore, the prime factorization of 150 is $\boxed{2 \cdot 3 \cdot 5^2}$.

Answer 13PT.

Consider the polynomial $28m^2 + 18m$.

The objective is to find the factorization of given polynomial.

$28m^2 + 18m = 2 \cdot 14m \cdot m + 2 \cdot 9 \cdot m$ (Since $2 \cdot 14 = 28, m \cdot m = m^2$)

$= 2 \cdot m \cdot 14m + 2m \cdot 9$

$= 2m(14m + 9)$ (Factor $GCF\left(28m^2, 18m\right) = 2m$)

Thus the factorization of $28m^2 + 18m$ is $\boxed{2m(14m+9)}$.

Answer 13STP.

A movie theater charges \$ $7\cdot50$ for each adult ticket and \$4 for each child ticket.

Total number of ticket sold =145

Amount collected for 145 ticket =\$790.

Let *x* number of adult ticket were sold.

Then number of child tickets = total number of tickets − number of adult tickets.

= $245-x$.

Amount collected for 145 tickets = \$790.

Number of adult tickets $\cdot\$7\cdot5$ plus number of child tickets $\cdot\$4=790.$

$$
\begin{aligned}
x(7\cdot5)+(145-x)4&=790\\
7\cdot5x+145\cdot4-x\cdot4&=790\\
7\cdot5x-4x+580&=790 && \text{(Simplify)}\\
3\cdot5x+580&=790 && \text{(Combine like terms)}
\end{aligned}
$$

$$
\begin{aligned}
3\cdot5x+580-580&=790-580 && \text{(Subtract 580 on both side)}\\
3\cdot5x&=210\\
\frac{3\cdot5x}{3\cdot5}&=\frac{210}{3\cdot5} && \text{(Divide with} 3\cdot5 \text{ on both side)}\\
x&=60
\end{aligned}
$$

Therefore, the number of adult tickets sold is $\boxed{60.}$.

Answer 14E.

Consider the integer 301.

The objective is to find the prime factorization of each integer.

For this use factor tree.

\

/

301

$7\cdot43$ (Because $7\cdot43=301$)

Therefore all factors in the last branch of the factor tree are primes.

Thus, the prime factorization of 301 is $7\cdot43$.

Therefore, the prime factorization of $\boxed{301}$ is $\boxed{4\cdot43}$.

Answer 14PT.

Consider the polynomial $a^2 - 11ab + 18b^2$

The objective is to factor given polynomial.

Compare $a^2 - 11ab + 18b^2$ with $a^2 + cx + d$.

Here $c = -11b,$

$d = 18b^2$

$$a^2 - 11ba + 18b^2 = (a+n)(a+n)$$
$$= a^2 + (m+n)a + mn$$

Here $m + n = -11b$,

$mn = 18b^2$

Now find two functions $m + n$ such that

$m + n = -11b$,

$mn = 18b^2$, since $m + n$ is negative, mn is positive.

Thus both m, n are negative.

List all the negative factors of $18b^2$ in those choose a pair whose sum is $-11b$.

Factors of $18b^2$	Sum of factors
$-1, -18b^2$	$-1 - 18b^2$
$-2, -9b^2$	$-2, -9b^2$
$-3, -6b^2$	$-3 - 6b^2$
$-b, -18b$	$-19b$
$-2b, -9b$	$-11b$
$-3b, -6b$	$-9b$

The correct factors are $-2b, -9b$.

$$a^2 - 11ab + 18b^2 = (a+m)(a+n)$$

$$= (a + -2b)(a + (-9b))$$

$$(m = -2b, n = -9b)$$

$$= (a-2b)(a-9b)$$

Check: To Check the factors use $FOIL$ method.

$$(a-2b)(a-9b) = a \cdot \overset{F}{a} + a \cdot (\overset{O}{-9}b) + a \cdot (\overset{I}{-2}b) + (\overset{L}{-2}b) \cdot (-9b)$$

($FOIL$ Method)

$$= a^2 - 9ab - 2ab + 18b^2$$

(Simplify)

$= a^2 - 11ab + 18b^2$ True

Therefore, the factorization of given polynomial is $\boxed{(a-2b)(a-9b)}$.

Answer 14STP.

Consider the system of equations $3x + y = 8$

$4x - 2y = 14$

The objective is to solve the given system.

Now we solve this by substitution method.

$3x + y = 8$ .... (1)

$4x - 2y = 14$ .... (2)

$$
\begin{aligned}
3x + y &= 8 \\
3x + y - 3x &= 8 - 3x \quad [\text{Subtract } 3x \text{ on both sides}] \\
y &= 8 - 3x
\end{aligned}
$$

Substitute value of y in equation (2)

$$
\begin{aligned}
4x - 2y &= 14 \\
4x - 2(8 - 3x) &= 14 \quad [\text{Substitute } y = 8 - 3x] \\
4x - 16 + 6x &= 14 \\
10x - 16 &= 14 \\
10x - 16 &= 14 \quad [\text{Combine like terms}] \\
10x - 16 + 16 &= 14 + 16 \quad [\text{Add 16 on both sides}] \\
10x &= 30 \\
\frac{10x}{10} &= \frac{30}{10}
\end{aligned}
$$

$$
\begin{aligned}
x &= 3 \\
y &= 8 - 3x \\
&= 8 - 3(3) \quad [\text{put } x = 3] \\
&= 8 - 9 \\
&= -1
\end{aligned}
$$

The solution of given system is $\boxed{x = 3, y = -1}$

Answer 15PT.

Consider the polynomial $12x^2+23x-24$.

The objective is to factor the given polynomial.

Compare $12x^2+23x-24$ with ax^2+bx+c

Here $a=12$,

$b=23,$

$c=-24$

$12x^2+23x-24=12x^2+mx+nx-24$

That is find two numbers m,n such that

$m+n=23$ and

$mn=12\cdot-24$
$=-288$ is negative.

Thus either m or n must be negative but not both

For this list all the factors of

$mn=-288$ in those chose a pair factor whose sum is

$m+n=23$

Factors of -288	Sum of factors
-1.288	287
$1.-288$	-287
-2.144	142
$2.-144$	-142
-3.96	93
$3.-96$	-93
-4.72	68

$4.-72$	-68
-6.48	42
$6.-48$	-42
-8.36	28
$8.-36$	-28
-9.32	23
$9.-32$	-23
-12.24	12
$12.-24$	-12
-16.18	2
$16.-18$	-2

The correct factors are $-9,32$.

$12x^2+23x-24=12x^2+mx+nx-24$

$=12x^2-9x+32x-24 \quad (m=-9, n=32)$

$=3\cdot 4\cdot x\cdot x-3\cdot 3\cdot x+8\cdot 4x-8\cdot 3$

$=3x(4x-3)+8(4x-3)$ (Factor GCF)

$=(3x+8)(4x-3)$ (By distributive $(b+c)a=ba+ca$)

Check: Check the factors by multiplying using $FOIL$ method.

$(3x+8)(4x-3)=\overset{F}{3}x\cdot 4x+(\overset{O}{-3})\cdot 3x+\overset{I}{8}\cdot 4x+\overset{L}{8}\cdot(-3)$

($FOIL$ methed)

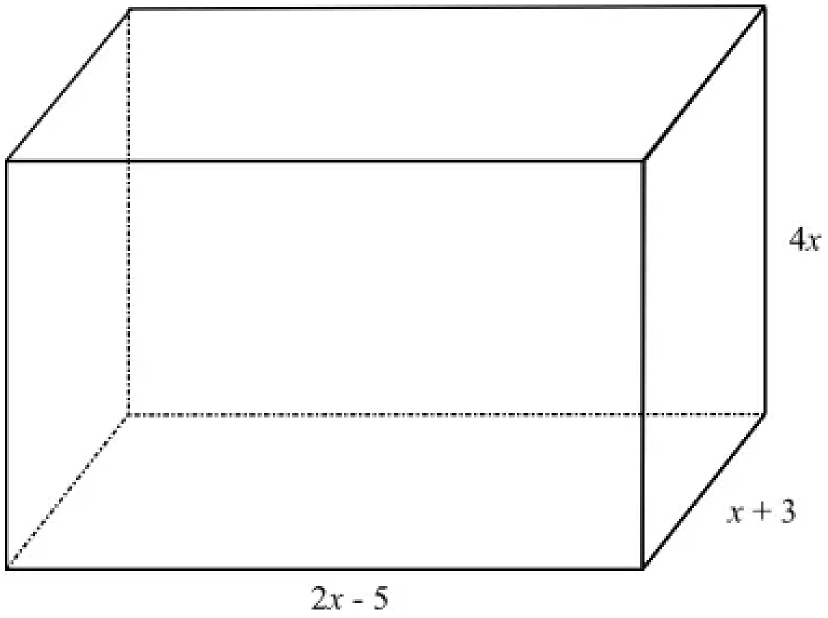
$=12x^2-9x+32x-24$ (Simplify)

$=12x^2+23x-24$ True

Therefore the factorization of given polynomial is $\boxed{(3x+8)(4x-3)}$.

Answer 15STP.

The rectangular prism is



From the figure length of prism $l = 2x - 5$

Width of prism $b = x + 3$

Height of prism $h = 4x$

Since the volume of rectangular prism $= lbh$

Where *l* is length of prism.

b is width of prism

h is height of prism.

Volume of rectangular prism $= lbh$

$$
\begin{aligned}
&= (2x-5)(x+3)4x && [\text{Put } l = 2x-5, b = x+3, h = 4x] \\
&= (2x-5)(x \cdot 4x + 3 \cdot 4x) && [\text{By distributive } (b+c)a = ba + ca] \\
&= (2x-5)(4x^2 + 12x) \\
&= 2x \cdot 4x^2 + 2x \cdot 12x + (-5) \cdot 4x^2 + (-5)12x && [\text{By FOIL method}] \\
&= 8x^3 + 34x^2 - 20x^2 - 60x && [\text{Simplify}] \\
&= 8x^3 + 4x^2 - 60x
\end{aligned}
$$

Therefore, volume of prism is $\boxed{8x^3 + 4x^2 - 60x}$

Answer 16E.

Consider the integer -378.

The objective is to find the prime factorization of -378.

For this use factor tree.

$-378 = -1 \cdot 378$

$2 \cdot 189$ (Because $2 \cdot 189 = 378$)

$3 \cdot 63 \quad (3 \cdot 63 = 189)$

$3 \cdot 21 \quad (3 \cdot 21 = 63)$

$3 \cdot 7 \quad (3 \cdot 7 = 21)$

All the factors in the last branch of the factor tree are primes.

Thus the prime factorization of -378 is

$-1 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 7 = -1 \cdot 2 \cdot 3^3 \cdot 7$

Therefore, the prime factorization of $\boxed{-378}$ is $\boxed{-1 \cdot 2 \cdot 3^3 \cdot 7}$.

Answer 16PT.

Consider the polynomial $2h^2-3h-18$

The objective is to factor of given polynomial.

Compare $2h^2-3h-18$ with ax^2+bx+c

Here $a=2, b=-3, c=-18$

$2h^2-3h-18=2h^2+mh+nh-18$

Now find two numbers m,n such that $m+n=-3, m\cdot n=(-18)\cdot 2=-36$

Since $m+n$ is negative then either *m* or *n* must be negative. For this list all the factors of -36 in those on factor is negative.

In the list of pair of factors of -36 choose a pair whose sum is -3

Factors of -36	Sum of factors
$-1\cdot 36$	35
$1\cdot -36$	-35
$-2\cdot 18$	16
$2\cdot -18$	-16
$-3\cdot 12$	9
$3\cdot -12$	-9
$-4\cdot 9$	5
$4\cdot -9$	-5
$-6\cdot 6$	0

There is no pair of factors, such that $m+n=-3$

Thus the given polynomial cannot be factored using integers.

Therefore, the given polynomial is $\boxed{\text{Prime}}$

Answer 17E.

Consider the set of monomials $35, 30$.

The objective is to find the GCF of given set of monomials.

Since the GCF of two or more monomials is the product of their common factors when each monomial is in factored form.

For this first find the prime factorization of 35 and 30.

$35 = 5 \cdot 7$ (Since $5 \cdot 7 = 35$)

$30 = 2 \cdot 15 \quad (2 \cdot 15 = 30)$

$= 2 \cdot 3 \cdot 5 \quad (3 \cdot 5 = 15)$

Circle the common factors.

$$35 = (5) \cdot 7$$
$$30 = 2 \cdot 3 \cdot (5)$$

Therefore $GCF =$ Product of common prime factors.

$= 5$

Therefore, $\boxed{GCF}$ of $\boxed{35}$ and $\boxed{30}$ is $\boxed{5}$.

Answer 17PT.

Consider the polynomial $6x^3 + 15x^2 - 9x$

The objective is to factor the given polynomial.

$$\begin{aligned} 6x^3 + 15x^2 - 9x &= 3 \cdot 2 \cdot x \cdot x^2 + 3 \cdot 5 \cdot x \cdot x - 3 \cdot 3 \cdot x \\ &= 3x\left[2x^2 + 5x - 3\right] \qquad (\text{Factor GCF } 3x) \end{aligned}$$

Now factor $2x^2 + 5x - 3$

Compare $2x^2 + 5x - 3$ with $ax^2 + bx + c$

$a = 2, b = 5, \quad c = -3$

$2x^2 + 5x - 3 \quad 2x^2 + mx + nx - 3$

Now find two number *m*, *n* such that $m + n = 5$ and $mn = -3.2 = -6.$ since $m + n$ is positive and mn is negative then either *m* or *n* must be negative but not both.

Now list all the factor of $mn = -6$ in those one factor is negative,

In those choose a pair of factor whose sum is 5.

Factor of -6	Sum of factors
[illegible]	✓

-5	5
$1\cdot -6$	-5
$-2\cdot 3$	1
$2\cdot -3$	-1

The correct factors are -1, 6.

$$3x\left[2x^2+5x-3\right]=3x\left[2x^2+mx+nx-3\right]$$
$$=3x\left[2x^2-x+6x-3\right] \qquad (m=-1, n=6.)$$
$$=3x\left[x\cdot 2x+-1\cdot x+3\cdot 2x+-1\cdot 3\right]$$
$$=3x\left[x(2x-1)+3(2x-1)\right] \qquad (\text{Factor the GCF})$$
$$=3x\left[(x+3)(2x-1)\right] \qquad \text{By distributive } (b+c)a=ba+ca$$

Thus the factorization of give polynomial is $\boxed{3x\left[(x+3)(2x-1)\right]}$

Answer 17STP.

The expression is $(x+t)x+(x+t)y$

The objective is to express the given expression as product of two factors

The distributive property is $a(b+c)=ab+ac$ and $(b+c)a=ba+ca$

$$(x+t)x+(x+t)y=(x+t)[x+y] \qquad [\text{By distributive}]$$

Therefore, $\boxed{(x+t)x+(x+t)y=(x+t)(x+y)}$

Answer 18E.

Consider the set of integers are $12, 18, 40$.

The objective is to find the GCF of given set of integers.

Since the GCF of two or more integers is the product of the prime factors common to the integers.

For this first find prime factorization of given integers

$12 = 2 \cdot 6$ (Since $2 \cdot 6 = 12$)

$= 2 \cdot 2 \cdot 3 \quad (2 \cdot 3 = 6)$

$18 = 2 \cdot 9 \quad (2 \cdot 9 = 18)$

$40 = 2 \cdot 20 \quad (2 \cdot 20 = 40)$

$= 2 \cdot 2 \cdot 10 \quad (2 \cdot 10 = 20)$

$= 2 \cdot 2 \cdot 2 \cdot 5 \quad (2 \cdot 5 = 10)$

Now circle the common factors

$12 = (2) \cdot 2 \cdot 3$

$18 = (2) \cdot 3 \cdot 3$

$40 = (2) \cdot 2 \cdot 2 \cdot 5$

$GCF =$ Product of common prime factors.

$= 2$

Therefore $\boxed{GCF}$ of given set of integers is $\boxed{2}$.

Answer 18PT.

Consider the polynomial $64p^2 - 63p + 16$

The objective is to factor the given polynomial.

Compare $64p^2 - 63p + 16$ with $ap^2 + 6p + c$

Here $a = 64, b = -63, c = 16$

$$\begin{aligned} 64p^2 - 63p + 16 &= 64p^2 + mp + np + 16 \\ &= 64p^2 + (m+n)p + 16 \end{aligned}$$

Here $m + n = -63$ and $mn = 64 \cdot 16 = 1024$

Since $m + n$ is negative and mn is positive, then both m, n are negative.

List all the factors of 1024 in those choose a pair whose sum is -63

Factors of 1024	Sum of factors
$-1 \cdot -1024$	-1025
$-2 \cdot -512$	-514
$-4 \cdot -256$	-260
$-8 \cdot -128$	-134
$-16 \cdot -64$	-80
$-32 \cdot -32$	-64

There exists no pair of factors whose sum is -63.

Thus $64p^2 - 63p + 16$ is not factored.

Therefore, the given polynomial is $\boxed{\text{Prime}}$

Answer 18STP.

Consider that the product of two consecutive odd integers is 195.

The objective is to find the integers.

Since the general form of on odd integer is $2x+1$

The consecutive odd integers are $2x+1, 2x+3$

Given that the product of consecutive odd integers is 195.

$$(2x+1)(2x+3)=195$$
$$2x\cdot 2x+2x\cdot 3+1\cdot 2x+1\cdot 3=195 \quad \text{[By FOIL method]}$$
$$4x^2+6x+2x+3=195$$
$$4x^2 8x+3=195 \quad \text{[Combine like terms]}$$
$$4x^2+8x+3-195=195-195 \quad \text{[Subtract 195 on both sides]}$$
$$4x^2+8x-192=0$$

Now factor $4x^2+8x-192$

Compare $4x^2+8x-192$ with ax^2+bx+c

$$a=4, b=8, c=-195$$

$$4x^2+8x-192=4x^2+mx+nx-192$$
$$=4x^2+(m+n)x-192$$

Now find two numbers *m*, *n* such that $m+n=8$ and $mn=4\cdot -192=-768$

Since $m+n$ is positive and *mn* is negative then one of *m* or *n* must be negative but not both.

The suitable factors are $32, -24$

$$4x^2+8x-192=4x^2+32x-24x-192$$
$$=4x(x+8)-24(x+8) \quad \text{[Factor the GCF]}$$
$$=(4x-24)(x+8) \quad \text{[By distributive } (b+c)a=ba+ca\text{]}$$

$$4x^2+8x-192=0$$
$$(4x-24)(x+8)=0$$
$$4(x-6)(x+8)=0$$

The zero product property is if $ab=0$ then $a=0 \text{ or } b=0$ or both

$$x-6=0 \text{ or } x+8=0$$

$$x-6=0$$

$$x-6+6=0+6 \qquad \text{[Add 6 on both sides]}$$

$$x=6$$

$$x+8=0$$

$$x+8-8=0-8 \qquad \text{[Subtract 8 on both sides]}$$

$$x=-8$$

Therefore, $x=6 \text{ or } x=-8$

For $x=6$, the odd integers are

$$=2x+1, \quad \text{or} \quad =2x+3$$

$$=2(6)+1, \text{or} \quad =2(6)+3 \qquad [x=6]$$

$$=13, \quad \text{or} \quad =15$$

For $x=-8$, the odd integers are

$$=2x+1, \quad \text{or} \quad =2x+3$$

$$=2(-8)+1, \text{or} \quad =2(-8)+3 \qquad [x=-8]$$

$$=-15, \quad \text{or} \quad =-13$$

Therefore, the odd integers are $\boxed{13,15 \text{ or } -15,-13}$

Answer 19E.

Consider the set of monomials $12ab, 4a^2b^2$.

The objective is to find the GCF of given set of monomials.

Since the GCF of two or more monomials is the product of their common factors when each monomial is in factored form.

For this first find the factorization of each monomial

$12ab = 2 \cdot 6ab$ (Because $2 \cdot 6 = 12$)

$= 2 \cdot 2 \cdot 3ab \quad (2 \cdot 3 = 6)$

$= 2 \cdot 2 \cdot 3 \cdot a - b$

$4a^2b^2 = 2 \cdot 2a^2b^2$ (Since $2 \cdot 2 = 4$)

$= 2 \cdot 2 \cdot a \cdot a \cdot b^2$ (Since $a^2 = a \cdot a$)

$= 2 \cdot 2 \cdot a \cdot a \cdot b \cdot b \quad \left(b^2 = b \cdot b\right)$

Now circle the common factors

$12ab = (2) \cdot (2) \cdot 3 \cdot (a) \cdot (b)$

$4a^2b^2 = (2) \cdot (2) \cdot a \cdot (a) \cdot (b) \cdot b$

$GCF =$ Product of common factors

$= 2 \cdot 2 \cdot a \cdot b$

$= 4ab$ (Simplify)

Therefore, $\boxed{GCF}$ of given set of monomials is $\boxed{4ab}$.

Answer 19PT.

Consider the polynomial $2d^2+d-1$

The objective is to factor of given polynomial.

Compare $2d^2+d-1$ with ax^2+bx+c

Here $a=2, b=1, c=-1$

$$2d^2+d-1=2d^2+md+nd-1$$

Now find two numbers m, n such that $m+n=1, m\cdot n=2\cdot -1=-2$

Since $m+n$ is positive and mn is negative then either *m* or *n* negative but not both.

List all the pair of factors of $m\cdot n=-2$, in those choose a pair whose sum is 1.

Factors of -2	✓ Sum of factors
$-1\cdot 2$	1
$1\cdot -2$	-1

The correct factors are $-1, 2$

$$\begin{aligned}2d^2+d-1&=2d^2+md+nd-1\\&=2d^2+2d-d-1 \quad [m=2, n=-1]\\&=2\cdot d\cdot d+2\cdot d+(-1)\cdot d+(-1)\\&=2d(d+1)+-1(d+1)[\text{Factor the GCF}]\end{aligned}$$

$$=(2d-1)(d+1) \quad [\text{By Distributive } (b+c)a=ba+ca]$$

<u>Check:</u> To check the factor, multiply the factors using FOIL method

$$\begin{aligned}(2d-1)(d+1)&=2d\cdot d+2d\cdot 1+(-1)d+(-1)\cdot 1 \quad [\text{FOIL Method}]\\&=2d^2+2d-d-1 \quad [\text{Simplify}]\\&=2d^2+d-1 \quad \text{True}\end{aligned}$$

Therefore, the factorization of given polynomial is $\boxed{(2d-1)(d+1)}$

Answer 19STP.

Consider the equation $2x^2+5x-12=0$

The objective is to find the solution set of given equation by factoring.

For this first factor $2x^2+5x-12$

Now compare $2x^2+5x-12$ with ax^2+bx+c

Here $d=2,b=5,c=-12$

$$\begin{aligned}2x^2+5x-12&=2x^2+mx+nx-12\\&=2x^2+(m+n)x-12\end{aligned}$$

Now find two numbers m,n such that $m+n=5$ and $m\cdot n=2\cdot -12=-24$

Since $m+n$ is positive and mn is negative then one of *m* or *n* must be negative but not both

List all the factors of -24 in those choose a pair whose sum is 35

Factor of -24	Sum of factors
$-1\cdot 24$	23
$1\cdot -24$	-23
$-2\cdot 12$	10
$2\cdot -12$	✓ -10
$-3\cdot 8$	5
$3\cdot -8$	-5
$-4\cdot 6$	2
$4\cdot -6$	-2

The correct factors are $-3,8$

$$
\begin{aligned}
2x^2+5x-12 &= 2x^2+mx+nx-12 \\
&= 2x^2-3x+8x-12 && [m=-3, n=3] \\
&= x(2x-3)+4(2x-3) && [\text{Factor the GCF}] \\
&= (x+4)(2x-3) && [\text{By distributive } (b+c)a = ba+ca]
\end{aligned}
$$

$$
\begin{aligned}
2x^2+5x-12 &= 0 \\
(x+4)(2x-3) &= 0
\end{aligned}
$$

The zero product property is if $ab=0$ then $a=0$ or $b=0$ or both

$x+4=0$ or $2x-3=0$

Now solve

$$
\begin{aligned}
x+4 &= 0 \\
x+4-4 &= 0-4 && [\text{Subtract 4 on both sides}] \\
x &= -4
\end{aligned}
$$

$$
\begin{aligned}
2x-3 &= 0 \\
2x-3+3 &= 0+3 && [\text{Add 3 on both sides}] \\
x &= 3 \\
\frac{2x}{2} &= \frac{3}{2} && [\text{Divide with 2 on both sides}]
\end{aligned}
$$

$$x = \frac{3}{2}$$

The solution set is $\left\{-4, \frac{3}{2}\right\}$

Check:

To check the proposed solution set substitute each solution in the given equation and verify.

$$2x^2+5x-12=0$$

$$2(-4)^2+5(-4)-12=0 \qquad [\text{Put } x=-4]$$

$$32-20-12=0 \qquad [\text{Simplify}]$$

$$32-32=0$$

$$0=0 \ \text{ True}$$

$$2x^2+5x-12=0$$

$$2\left(\frac{3}{2}\right)^2+5\left(\frac{3}{2}\right)-12=0 \qquad \left[\text{Put } x=\frac{3}{2}\right]$$

$$2\cdot\frac{9}{4}+\frac{15}{2}-12=0 \qquad [\text{Simplify}]$$

$$\frac{9}{2}+\frac{15}{2}-12=0$$

$$\frac{9+15-24}{2}=0$$

$$0=0 \text{ True}$$

Therefore, the solution set of given equation is $\boxed{\left\{-4,\frac{3}{2}\right\}}$

Answer 20E.

Consider the set of monomials $16mrt, 30m^2r$.

The objective is to find the GCF of given set of monomials.

Since the GCF of two or more monomials is the product of their common factors when each monomial is in factored form.

For this first find the factorization of each monomial.

$$16mrt = 2 \cdot 8mrt \quad (2 \cdot 8 = 16)$$

$$= 2 \cdot 2 \cdot 4 \cdot mrt \quad (8 = 24)$$

$$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot mrt \quad (4 = 2 \cdot 2)$$

$$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot m \cdot r \cdot t$$

$30m^2r = 2 \cdot 15m^2r$ (Because $30 = 2 \cdot 15$)

$$= 2 \cdot 3 \cdot 5m^2r \quad (3 \cdot 5 = 15)$$

$$= 2 \cdot 3 \cdot 5 \cdot m \cdot m \cdot r \quad (m^2 = m \cdot m)$$

Now circle the common factors

$$16mrt = (2) \cdot 2 \cdot 2 \cdot 2 \cdot (m) \cdot (r) \cdot t$$

$$30m^2r = (2) \cdot 3 \cdot 5 \cdot (m) \cdot m \cdot (r)$$

$GCF =$ Product of common factors

$$= 2 \cdot m \cdot r$$

$$= 2mr$$

Thus, GCF of given set of monomials is $\boxed{2mr}$.

Answer 20PT.

Consider the polynomial $36a^2b^3 - 45ab^4$.

The objective is to factor the given polynomial.

For this first find the GCF of $36a^2b^2, 45ab^4$.

Since the GCF is the product of common factors which are in both factorizations.

$$36a^2b^3 = 2 \cdot 18 \cdot a^2b^3 \quad (2 \cdot 18 = 36)$$

$$= 2 \cdot 2 \cdot 9 \cdot a^2b^3 \quad (2 \cdot 9 = 18)$$

$$= 2 \cdot 2 \cdot 3 \cdot 3 \cdot a^2b^3 \quad (3 \cdot 3 = 9)$$

$$= 2 \cdot 2 \cdot 3 \cdot 3 \cdot a \cdot ab^3 \quad (a \cdot a = a^2)$$

$$= 2 \cdot 2 \cdot 3 \cdot 3 \cdot a \cdot a \cdot b \cdot b \cdot b$$

$$(b^3 = b \cdot b \cdot b)$$

$$45ab^4 = 5 \cdot 9ab^4 \quad (5 \cdot 9 = 45)$$

$$= 5 \cdot 3 \cdot 3ab^4 \quad (3 \cdot 3 = 9)$$

$$= 5 \cdot 3 \cdot 3 \cdot a \cdot b \cdot b \cdot b \cdot b \quad (b^4 = b \cdot b \cdot b \cdot b)$$

Circle the common factors

$$36a^2b^3 = 2 \cdot 2 \cdot (3) \cdot (3) \cdot (a) \cdot a \cdot (b) \cdot (b) \cdot (b)$$

$$45ab^4 = (3) \cdot (3) \cdot 5 \cdot (a) \cdot b \cdot (b) \cdot (b) \cdot (b)$$

$$GCF = 3 \cdot 3 \cdot a \cdot b \cdot b \cdot b$$

$$= 9ab^3$$

$$36a^2b^3 - 45ab^4 - 9 \cdot 4 \cdot a \cdot a \cdot b^3 \cdot b - 9 \cdot 5 \cdot a \cdot b^3 \cdot b$$

$$= 9ab^3 \cdot a - 9ab^3 \cdot 5b$$

$$= 9ab^3(4a - 5b) \quad \text{(Factor } GCF\text{)}$$

Thus the factorization of given polynomial is $\boxed{9ab^3(4a-5b)}$.

Answer 20STP.

Consider the polynomial number $2x^2+7x+3$

The objective is to factor the given polynomial.

Compare $2x^2+7x+3$ with ax^2+bx+c

There $a=2, b=7, c=3.$

$$
\begin{aligned}
2x^2+7x+3 &= 2x^2+mx+nx+3 \\
&= 2x^2+(m+n)x+3
\end{aligned}
$$

Now find two numbers m,n such that $m+n=7, mn=3,2=6$. Since $m+n$ and mn are positive, then both m,n are positive.

List all the factors of $mn=6$ in those choose a pair whose sum is 7.

Factor of 6	✓ Sum of factor
$1\cdot 6$	7
$2\cdot 3$	5

The correct factors are 1, 6

$$
\begin{aligned}
2x^2+7x+3 &= 2x^2+mx+nx+3 \\
&= 2x^2+x+6x+3 && (m=1, n=6) \\
&= x(2x+1)+3(2x+1) && (\text{factor GCF}) \\
&= (x+3)(2x+1) && (\text{By disttributive}(b+c)a=ba+ca)
\end{aligned}
$$

Check,

To check the factorizations multiply the factors using FOIL Method.

$$
\begin{aligned}
& \quad\;\; \text{F} \qquad \text{O} \qquad \text{I} \qquad \text{L} \\
(x+3)(2x+1) &= x\cdot 2x+x\cdot 1+3\cdot 2x+3\cdot 1 && (\text{FOIL Method}) \\
&= 2x^2+x+6x+3 && (\text{Combine like terms}) \\
&= 2x^2+7x+3 \qquad \text{True}
\end{aligned}
$$

Therefore, the factorization of given polynomial is $\boxed{(x+3)(2x+1)}$

Answer 21E.

Consider the set of monomials $20n^2, 25np^5$.

The objective is to find the GCF of given set of monomials.

Since the GCF of two or more monomials is the product of their common factors when each monomial is in factored form.

For this first find the factorization of each monomial.

$20n^2 = 2 \cdot 10n^2$ (Because $2 \cdot 10 = 20$)

$= 2 \cdot 2 \cdot 5n^2 \quad (2 \cdot 5 = 10)$

$= 2 \cdot 2 \cdot 5 \cdot n \cdot n \quad (n^2 = n \cdot n)$

$25np^5 = 5 \cdot 5np^5 \quad (25 = 5 \cdot 5)$

$= 5 \cdot 5 \cdot n \cdot p \cdot p \cdot p \cdot p \cdot p$

$(p^5 = p \cdot p \cdot p \cdot p \cdot p)$

Now circle the common factors

$20n^2 = 2 \cdot 2 \cdot (5) \cdot (n) \cdot n$

$25np^5 = 5 \cdot (5) \cdot (n) \cdot p \cdot p \cdot p \cdot p \cdot p$

$GCF =$ Product of common factors

$= 5 \cdot n$

$= 5n$

Thus, GCF of given set of monomials is $\boxed{5n}$.

Answer 21PT.

Consider the polynomial

$$\begin{aligned}36m^2+60mn+25n^2 &= 6\cdot 6m^2+2\cdot 5\cdot 6\cdot m\cdot n+5\cdot 5n^2\\ &= 6^2m^2+2\cdot 5\cdot 6\cdot m\cdot n+5^2n^2\\ &= (6m)^2+2\cdot 5\cdot 6\cdot m\cdot n+(5n)^2 \qquad \left[a^m\cdot b^m=(ab)^m\right]\\ &= (6m)^2+2\cdot(6m)(5n)+(5n)^2\end{aligned}$$

Clearly it is a perfect square trinomial.

Since the first term is perfect square $(6m)^2$

Last term is perfect square

middle term is twice the product of first and last terms $2\cdot 6m\cdot 5n$

Since $(a+b)^2=a^2+2ab+b^2$

$$\begin{aligned}36m^2+60mn+25n^2 &= (6m)^2+2\cdot(6m)\cdot(5n)+(5n)^2\\ &= (6m+5n)^2\end{aligned}$$

Therefore, the factorization of given polynomial is $\boxed{(6m+5n)^2}$

Answer 21STP.

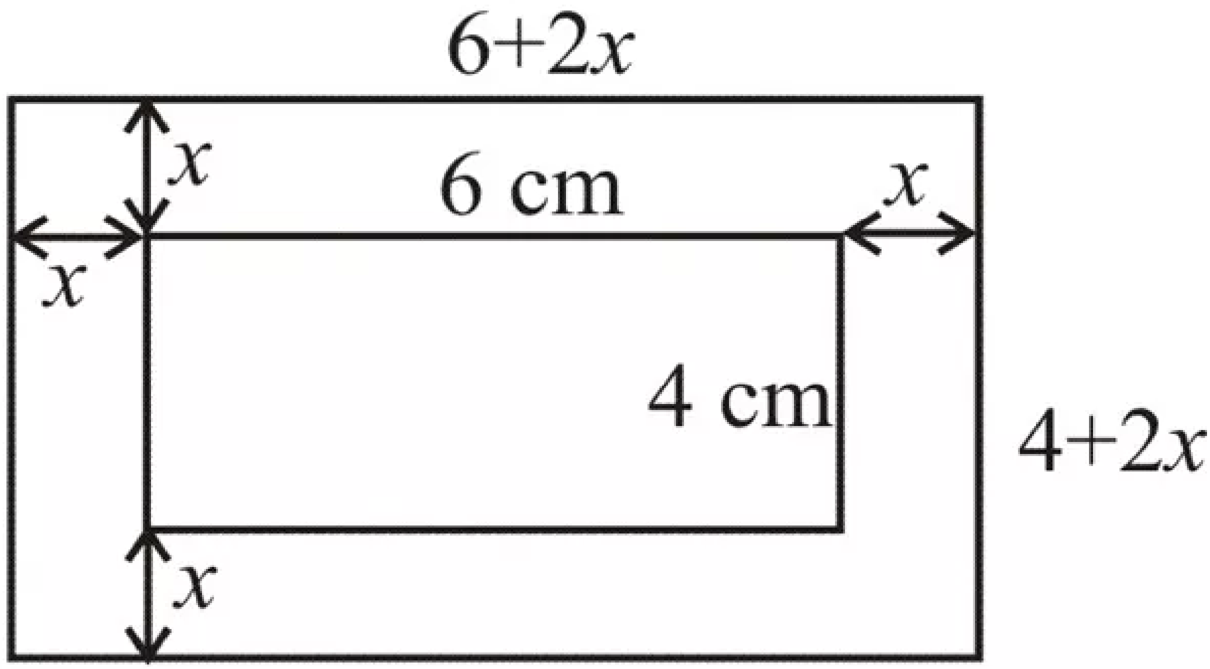
Consider that the length and width of an advertisement in the local newspaper has to be increase by the same amount in order to double its area.

The original advertisement has a length of 6 centimeters and a width of 4 centimeters.

(a)

Length of original advertisement $= 6 \text{ cm}$

Width of original advertisement $= 4 \text{ cm}$



Let x cm be the amount to be increased to double the are.

Length of enlarged advertisement $= (6+2x) \text{ cm}$

Width of enlarged advertisement $= (4+2x) \text{ cm}$

Area of original advertisement $= 2 \times \text{length} \times \text{width}$

$= 6 \times 4$
$= 24$

Area of enlarged advertisement $= 2 \times \text{Area of original advertisment}$

$= 2 \times 24$
$= 48$

That is area of enlarged advertisement $(6+2x)(4+2x) = 48$

$$(6+2x)(4+2x) = 48$$
$$6 \cdot 4 + 6 \cdot 2x + 2x \cdot 4 + 2x \cdot 2x = 48 \quad [\text{By FOIL method}]$$
$$24 + 12x + 8x + 4x^2 = 48$$
$$4x^2 + 20x + 24 = 48$$
$$4x^2 + 20x + 24 - 48 = 48 - 48 \quad [\text{Subtract 48 on both sides}]$$
$$4x^2 + 20x - 24 = 0$$

Therefore, the equation represents to area of enlarged advertisement is $\boxed{4x^2 + 20x - 24 = 0}$

(b)

Area of enlarged advertisement is

$$4x^2+20x-24=0$$

$$4x^2+24x-4x-24=0$$

$$4x(x+6)-4(x+6)=0 \qquad [\text{Factor GCF}]$$

$$(x+6)(4x-4)=0 \qquad [\text{By distributive } (b+c)a=ba+ca]$$

$$4(x-1)(x+6)=0$$

The zero product property is if $ab=0$ then $a=0$ or $b=0$ or both

$$x-1=0 \text{ or } x+6=0$$

$$x-1=0$$

$$x-1+1=0+1 \qquad [\text{Add 1 on both sides}]$$

$$x=1$$

$$x+6=0$$

$$x+6-6=0-6 \qquad [\text{Subtract 6 on both sides}]$$

$$x=-6$$

$$x=1 \text{ or } x=-1$$

Since length always positive, so consider $x=1$

The new dimensions of advertisement is length $=6+2x$

$$=6+2(1) \qquad [x=1]$$

$$=8$$

Width $=4+2x$

$$=4+2(1)$$

$$=6$$

The new dimensions of advertisement is $\boxed{\begin{array}{l}\text{length}=8\text{ cm}\\ \text{width}=6\text{ cm}\end{array}}$

(c)

New area of advertisement

= Area of enlarged Advertisement – Area of original advertisement

$$=48-24$$

$$=24$$

Therefore, new area of advertisement is $\boxed{24 \text{ square centimeters}}$

Answer 22E.

Consider the set of monomials $60x^2y^2, 35xz^3$.

The objective is to find the GCF of GCF of given set of monomials.

Since the GCF of two or more monomials is the product of their common factors when each monomial is in factored form.

For this first find the factorization of each monomial.

$$60x^2y^2 = 2\cdot 30x^2y^2 \quad (60 = 2\cdot 30)$$

$$= 2\cdot 2\cdot 15x^2y^2 \quad (30 = 2\cdot 15)$$

$$= 2\cdot 2\cdot 3\cdot 5x^2y^2 \quad (15 = 3\cdot 5)$$

$$= 2\cdot 2\cdot 3\cdot 5\cdot a\cdot x\cdot y^2 \quad (x\cdot x = x^2)$$

$$= 2\cdot 2\cdot 3\cdot 5\cdot x\cdot x\cdot y\cdot y$$

$$(y^2 = y\cdot y)$$

$$35xz^3 = 5\cdot 7xz^3 \quad (35 = 5\cdot 7)$$

$$= 5\cdot 7\cdot x\cdot z\cdot z\cdot z \quad (z^3 = z\cdot z\cdot z)$$

Now circle the common factors.

$$60x^2y^2 = 2\cdot 2\cdot 3\cdot (5)\cdot (x)\cdot x\cdot y\cdot y$$

$$35xz^3 = (5)\cdot 7\cdot (x)\cdot z\cdot z\cdot z$$

$GCF =$ Product of common factors

$= 5\cdot x$

$= 5x$

Thus, the GCF of given set of monomials is $\boxed{5x}$.

Answer 22PT.

Consider the polynomial a^2-4.

The objective is to factor the given polynomial.

The difference of square property is

$$a^2-b^2=(a+b)(a-b)$$

$a^2-4=a^2-2\cdot 2$ (Since $2\cdot 2=4$)

$=a^2-2^2$ $\left(x\cdot x=x^2\right)$

$=(a+2)(a-2)$ (Here $a=a,b=2$)

Thus, $a^2-4=(a+2)(a-2)$

Therefore, the factorization of given polynomial is $\boxed{(a+2)(a-2)}$.

Answer 22STP.

Consider that the area of rectangular plot of land is $\left(6c^2+7c-3\right)$sq. miles.

(a)

The objective is to find the algebraic expression for length and width

For this factor area $=6c^2+7c-3$

$=6c^2+9c-2c-3$

$=3c(2c+3)-1(2c+3)$ [Factor the GCF]

$=(3c-1)(2c+3)$ [By distributive $(b+c)a=ba+ca$]

The algebraic expression for length and width is $\boxed{(3c-1)(2c+1)}$

(b)

Given area of rectangle = 21 square miles

Thus, $6c^2 + 7c - 3 = 21$

$$6c^2 + 7c - 3 - 21 = 21 - 21 \quad \text{[Subtract 21 on both sides]}$$

$$6c^2 + 7c - 24 = 0$$

$$6c^2 + 16c - 9c - 24 = 0$$

$$2c(3c+8) - 3(3c+8) = 0 \quad \text{[Factor GCF]}$$

$$(2c-3)(3c+8) = 0 \quad \left[\text{By distributive } (b+c)a = ba + ca\right]$$

The zero product property is of $ab = 0$ then $a = 0 \text{ or } b = 0$ or both

$$2c - 3 = 0 \text{ or } 3c + 8 = 0$$

$$2c - 3 = 0$$

$$2c - 3 + 3 = 0 + 3 \quad \text{[Add 3 on both sides]}$$

$$2c = 3$$

$$\frac{2c}{2} = \frac{3}{2} \quad \text{[Divide with 2 on both sides]}$$

$$c = \frac{3}{2}$$

$$\frac{2c}{2} = \frac{3}{2} \quad \text{[Divide with 2 on both sides]}$$

$$c = \frac{3}{2}$$

$$c = \frac{-8}{3}$$

Since *c* is length it is always positive.

So consider the positive values for *c*.

$$c = \frac{3}{2}$$

Therefore, the value of *c* is $\boxed{\frac{3}{2}}$

(c)

Here the objective is to find the length and widths from (a)

Area $=(3c-1)(2c+3)$

length $=2c+3$

width $=3c-1$

from (b)

$$c=\frac{3}{2}$$

Length $=2c+3$

$$=2\left(\frac{3}{2}\right)+3 \qquad \left[\text{Put } c=\frac{3}{2}\right]$$

$$=\frac{6}{2}+3$$

$$=3+3$$

$$=6$$

Width $=3c-1$

$$=3\left(\frac{3}{2}\right)-1 \qquad \left[\text{Put } c=\frac{3}{2}\right]$$

$$=\frac{9}{2}-1$$

$$=\frac{9-2}{2}$$

$$=\frac{7}{2}$$

Therefore, $\boxed{\begin{array}{l}\text{Length}=6\text{ miles}\\ \text{Width}=\frac{7}{2}\text{ miles}\end{array}}$

Answer 23E.

Consider the polynomial $13x+26y$.

The objective is to factor the given polynomial,

To factor $13x+26y$, first find GCF of $13x, 26y$ and then write $13x, 26y$ as product of GCF and use distributive to factor GCF.

$$13x = 13\cdot x$$

$$26y = 2\cdot 13y \quad (26 = 2\cdot 13)$$

$$= 2\cdot 13\cdot y$$

Now circle the common factors in $13x, 26y$

$$13x = (13)\cdot x$$
$$26y = 2\cdot(13)\cdot y$$

$GCF =$ Product of common factors

$$= 13$$

Therefore,

$$\begin{aligned} 13x+26y &= 13\cdot x + 2\cdot 13\cdot y \\ &= 13\cdot x + 13\cdot 2y \end{aligned}$$

$= 13(x+2y)$ (By distributive $a(b+c) = ab+ac$)

Therefore, the factorization of $13x+26y$ is $\boxed{13(x+2y)}$.

Answer 23PT.

Consider the polynomial $4my-20m+3py-15p$

The objective is the factor given polynomial

For this first group the terms having common factors and then factor the GCF. After use distributive property to factor completely

$$\begin{aligned} 4my-20m+3py-15p &= (4my-20m)+(3py-15) \\ &= (4\cdot m\cdot y - 5\cdot 4\cdot m)+(3\cdot p\cdot y - 3\cdot 5\cdot p) \\ &= 4m(y-5)+3p(y-5) && [\text{Factor GCF}] \\ &= (4m+3)(y-5) && [\text{By Distributive } (b+c)a = ba+ca] \end{aligned}$$

Therefore, the factorization of given polynomial is $\boxed{(4m+3)(y-5)}$

Answer 23STP.

Consider that, Madison is building a fenced, rectangular dog pen.

The width of the pen will be 3 yards less than the length.

Total area enclosed is 28 square yards.

(a)

Let L represents length of the pen.

Width of pen = 3 yards less than the length

$$= \text{length} - 3$$
$$= L - 3$$

Area of rectangular pen $= \text{length} \cdot \text{width}$

$$= L(L-3)$$

Given area of pen = 28

$$L(L-3) = 28$$

Therefore, equation showing area of the pen in terms of lengths $\boxed{L(L-3) = 28}$

(b)

From (a)

$$
\begin{aligned}
L(L-3) &= 28 \\
L \cdot L - 3 \cdot L &= 28 && [\text{By distributive property } a(b+c) = ab + ac] \\
L^2 - 3L &= 28 \\
L^2 - 3L - 28 &= 28 - 28 && [\text{Subtract 28 on both sides}] \\
L(L-3) &= 28 \\
L \cdot L - 3 \cdot L &= 28 && [\text{By distributive property } a(b+c) = ab + ac] \\
L^2 - 3L &= 28 \\
L^2 - 3L - 28 &= 28 - 28 && [\text{Subtract 28 on both sides}]
\end{aligned}
$$

The zero product property is if $ab = 0$ then $a = 0 \text{ or } b = 0$ or both

$$
\begin{aligned}
L - 7 = 0 &\text{ or } L + 4 = 0 \\
L - 7 &= 0 \\
L + 7 - 7 &= 0 + 7 && [\text{Add 7 on both sides}] \\
L &= 7 \\
L + 4 &= 0 \\
L + 4 - 4 &= 0 - 4 && [\text{Subtract 4 on both sides}] \\
L &= -4
\end{aligned}
$$

The solution set is $\{7, -4\}$

Since length always positive, consider $L = 7$

Therefore, length of pen $\boxed{7 \text{ yards}}$

(c)

The objective is to find the number of yards of fencing that will heed to enclose the pen completely

For this find perimeter of pen

Length of pen $L = 7$

Width of pen $= L - 3$

$= 7 - 3$

$= 4$

Perimeter of rectangular pen $= 2(l + b)$

$= 2(7 + 4) \quad [l = 7, b = 4]$

$= 2(11)$

$= 22$

Therefore, to enclose the pen completely it requires $\boxed{22 \text{ yards}}$ of fencing.

Answer 24E.

Consider the polynomial $24a^2b^2 - 18ab$

The objective is to factor the given polynomial.

To factor $24a^2b^2 - 18ab$, first find GCF of $24a^2b^2, 18ab$ and write $24a^2b^2, 18ab$ as product of GCF.

Then use distributive property to factor.

$$24a^2b^2 = 2\cdot 12a^2b^2 \quad (24 = 2\cdot 12)$$

$$= 2\cdot 2\cdot 6a^2b^2 \quad (2\cdot 6 = 12)$$

$$= 2\cdot 2\cdot 2\cdot 3\cdot a^2b^2 \quad (6 = 2\cdot 3)$$

$$= 2\cdot 2\cdot 2\cdot 3\cdot a\cdot ab^2 \quad (a^2 = a\cdot a)$$

$$= 2\cdot 2\cdot 2\cdot 3\cdot a\cdot a\cdot b\cdot b \quad (b^2 = b\cdot b)$$

$$18ab = 2\cdot 9ab \quad (18 = 2\cdot 9)$$

$$= 2\cdot 3\cdot 3\cdot ab \quad (9 = 3\cdot 3)$$

Now circle the common factors in $24a^2b^2, 18ab$.

$$24a^2b^2 = (2)\cdot 2\cdot 2\cdot (3)\cdot (a)\cdot a\cdot (a)\cdot b$$
$$18ab = (2)\cdot 3\cdot (3)\cdot (a)\cdot (b)$$

$GCF =$ Product of common factors

$$= 2\cdot 3\cdot a\cdot b$$
$$= 6ab$$

$$24a^2b^2 - 18ab = 2\cdot 2\cdot 2\cdot 3\cdot a\cdot a\cdot b\cdot b - 2\cdot 3\cdot 3\cdot a\cdot b$$

$= 6ab\cdot 4ab - 6ab\cdot 3$ (Product of GCF)

$$= 6ab\cdot 4ab + (-1)\cdot 6ab\cdot 3$$
$$= 6ab\cdot 4ab + 6ab\cdot (-1)\cdot 3$$

$= 6ab\left[4ab + (-1)3\right]$ (By distributive $a(b+c) = ab + ac$)

$$= 6ab(4ab - 3)$$

Therefore thus, the factorization of given polynomial is $\boxed{6ab(4ab-3)}$.

Answer 24PT.

Consider the polynomial $5a^2b+5a^2-10a$

The objective is to factor the given polynomial.

$$15a^2b+5a^2-10a=\left(15a^2b+5a^2\right)-10a$$

$$=\left(3\cdot5\cdot a^2b+5a^2\right)-10a \qquad (3\cdot5=15)$$

$$=5a^2(3b+1)-10a \qquad \left(\text{Factor GCF }5a^2\right)$$

$$=5a^2(3b+1)-2\cdot5a \qquad (2\cdot5=10)$$

$$=5\cdot a\cdot a(3b+1)-2\cdot5a \qquad \left(a^2=a\cdot a\right)$$

$$=5a\left[a(3b+1)-2\right] \qquad (\text{Factor GCF }5a)$$

Thus the factorization of given polynomial is $\boxed{5a\left[a(3b+1)-2\right]}$

Answer 25E.

Consider the polynomial $26ab+18ac+32a^2$.

The objective is to factor the given polynomial.

To factor, $26ab+18ac+32a^2$, find GCF of $26ab, 18ac, 32a^2$ and write each term as product of GCF.

$$26ab=2\cdot13\cdot a\cdot b \quad (26=2\cdot13)$$

$$18ac=2\cdot9\cdot a\cdot c \quad (18=2\cdot9)$$

$$=2\cdot3\cdot3\cdot a\cdot c \quad (9=3\cdot3)$$

$$32a^2=2\cdot16\cdot a^2 \quad (2\cdot16=32)$$

$$=2\cdot2\cdot8\cdot a\cdot a \quad (16=2\cdot8)$$

$$=2\cdot2\cdot2\cdot2\cdot2\cdot a\cdot a$$

$$(8=2\cdot2\cdot2)$$

Now circle the common factors

$$26ab=(2)\cdot13\cdot(a)\cdot b$$

$$18ac=(2)\cdot3\cdot3\cdot(a)\cdot c$$

$$32a^2=(2)\cdot2\cdot2\cdot2\cdot2\cdot(a)\cdot a$$

$GCF=$ Product of common factors

$= 2 \cdot a$

$26ab + 18ac + 32a^2 = 2 \cdot 13 \cdot ab + 2 \cdot 3 \cdot 3 \cdot a \cdot c + 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot a \cdot a$

$= 2a \cdot 13b + 2a(9c) + 2a \cdot 16a$

$= 2a[13b + 9c + 16a]$

(Factor GCF)

Thus the factorization of given polynomial is $\boxed{2a(13b+9c+16a)}$.

Answer 25PT.

The equation is $6y^2 - 5y - 6$

The objective is to find given polynomial

Compare $6y^2 - 5y - 6$ with $ax^2 + bx + c$

Here $a = 6, b = -5, c = -6$

$6y^2 - 5y - 6 = 6y^2 + mx + nx - 6$

Now find two numbers m, n such that $m + n = 6 = -5$ and $m \cdot n = a \cdot c = 6 \cdot -6 = -36$

Since $m + n, mn$ are negative then either *m* or *n* must be negative but not both

For this list all the factors of -36 in those choose a pair whose sum is -5

Factor of -36	Sum of factors
$-1 \cdot 36$	35
$1 \cdot -36$	-35
$-2 \cdot 18$	16
$2 \cdot -18$	-16
$-3 \cdot 13$	10
$3 \cdot -13$	-10
$-4 \cdot 9$	✓ 5
$4 \cdot -9$	-5
$6 \cdot -6$	0

The correct factors are $4, -9$

$$6y^2-5y-6=6y^2+my+ny-6$$
$$=6y^2-9y+4y-6 \qquad [m=4, n=-9]$$
$$=2\cdot3\cdot y\cdot y-3\cdot3y+2\cdot2y-2\cdot3$$
$$=3y(2y-3)+2(2y-3) \qquad [\text{Factors GCF}]$$
$$=(3y+2)(2y-3) \qquad \begin{bmatrix}\text{By distributive}\\(b+c)a=ba+ca\end{bmatrix}$$

Check:

To check the factorization, product the factors using FOIL method

$$(3y+2)(2y-3)=3y\cdot2y+3y\cdot(-3)+2(2y)+2\cdot(-3) [\text{FOIL method}]$$
$$=6y^2-9y+4y-6 \qquad [\text{Simplify}]$$
$$=6y^2-5y-6 \text{ True}$$

Therefore, the factorization of given polynomial is $\boxed{(3y+2)(2y-3)}$

Answer 26E.

Consider the polynomial $a^2-4ac+ab-4bc$

The objective is to factor the given polynomial.

$$a^2-4ac+ab-4bc=(a^2-4ac)+(ab-4bc)$$

(Group terms with common factors)

$$=(a\cdot a-4\cdot a\cdot c)+(a\cdot b-4\cdot b\cdot c)$$

$$=a(a-4c)+b(a-4c)$$

(Factor the GCF)

$$=(a+b)(a-4c)$$

(By distributive $(b+c)a=ba+ca$)

Therefore the factorization of given polynomial is $\boxed{(a+b)(a-4c)}$.

Answer 26PT.

Consider the polynomial $4s^2 - 100t^2$

The objective is to factor the given polynomial

$$\begin{aligned} 4s^2 - 100t^2 &= 2\cdot 2s^2 - 100t^2 && [2\cdot 2 = 4] \\ &= 2^2 s^2 - 10\cdot 10\cdot t^2 && [10\cdot 10 = 100] \\ &= (2s)^2 - 10t^2 t^2 && \left[10^2 = 10\cdot 10\right] \\ &= (2s)^2 - (10t)^2 && \left[a^m b^m = (ab)^m\right] \end{aligned}$$

The difference of squares property is $a^2 - b^2 = (a+b)(a-b)$

$$\begin{aligned} &= (2s+10t)(2s-10t) && [\text{Difference of squares property}] \\ &= (2s + 2\cdot 5t)(2\cdot s - 2\cdot 5t) \\ &= 2(s+5t)2(s-5t) && [\text{Factor of GCF}] \\ &= 4(s+5t)(s-5t) \end{aligned}$$

Therefore, the factorization of given polynomial is $\boxed{4(s+5t)(s-5t)}$

Answer 27E.

Consider the polynomial $4rs + 12ps + 2mr + 6mp$

The objective is to factor the given polynomial.

Since the polynomial has 4 terms, to factor it first group the terms having common factors.

And then use distributive property

$$\begin{aligned} &4rs + 12ps + 2mr + 6mp \\ &\qquad = (4rs + 12ps) + (2mr + 6mp) \\ &= (4\cdot r\cdot s + 4\cdot 3\cdot p\cdot s) + (2\cdot m\cdot r + 2\cdot 3\cdot m\cdot p) \end{aligned}$$

$= 4s(r+3p) + 2m(r-3p)$ (Factor the GCF)

$= (4s+2m)(r+3p)$ (By distributive $(b+c)a = ba + ca$)

$= 2(2s+m)(r+3p)$ (Take 2 as common factor)

Therefore the factorization of given polynomial is $\boxed{2(2s+m)(r+3p)}$.

Answer 27PT.

Consider the polynomial $x^3-4x^2-9x+36$

The objective is to factor the given polynomial

For this first group the terms having common factors

And then factor of GCF in each group. After that

Use distributive property $(b+c)a=ba+ca$

$$\begin{aligned} x^3-4x^2-9x+36 &= (x^3-4x^2)+(-9x+36) && [\text{Group the terms}] \\ &= (x\cdot x^2-4\cdot x^2)+[-9\cdot x+(-9)(-4)] \\ &= x^2(x-4)+(-9)[x-4] && [\text{Factor the GCF}] \\ &= (x^2-9)(x-4) && [\text{By distributive property}] \end{aligned}$$

Therefore, the factorization of given polynomial is $\boxed{(x^2-9)(x-4)}$

Answer 28E.

Consider the polynomial $24am-9an+40bm-15bn$.

The objective is to factor the given polynomial.

Since the polynomial has 4 terms, group the terms have common factors and then use distributive property to factor the given polynomial.

$$24am-9an+40bm-15bn$$

$$=(24am-9an)+(40bm-15bn)$$

$$=(3\cdot 8\cdot am-3\cdot 3\cdot a\cdot n)+(5\cdot 8\cdot b\cdot m-5\cdot 3\cdot b\cdot n)$$

$=3a(8m-3n)+5b(8m-3n)$ (Factor the GCF)

$=(3a+5b)(8m-3n)$ (By distributive $(b+c)a=(ba+ca)$)

The factorization of given polynomial is $\boxed{(3a+5b)(8m-3n)}$.

Answer 29E.

Consider the equation

$x(2x-5)=0$

The objective is to find the solution set of given equation.

$x(2x-5)=0$

The zero product property is of

$ab=0$ then

$a=0$ or

$b=0$ or both

$x(2x-5)=0$

$x=0$

Or, $2x-5=0$ (By zero product property)

Now solve each equation separately.

$x=0$,

$2x-5=0$

$2x-5+5=0+5$ (Add 5 on both sides)

$2x=5$

$\frac{2x}{2}=\frac{5}{2}$ (Divide with 2 on both sides)

$x=\frac{5}{2}$

The solution set is $\left\{0,\frac{5}{2}\right\}$.

Check: To check the proposed solution set substitute each solution in the given equation.

Given equation is

$x(2x-5)=0$

$0(2(0)-5)=0$ (Put $x=0$)

$0(-5)=0$

$0=0$ True

For, $x=\frac{5}{2}$,

$x(2x-5)=0$

$\frac{5}{2}\left(2\left(\frac{5}{2}\right)-5\right)=0$ (Put $x=\frac{5}{2}$)

$\frac{5}{2}(5-5)=0$ (Simplify)

$\frac{5}{2}(0)=0$

$0=0$ True

The solution set of given equation is $\boxed{\left\{0,\frac{5}{2}\right\}}$.

Answer 30E.

Consider the equation

$$(3n+8)(2n-6)=0$$

The objective is to find the solution set of given equation.

$$(3n+8)(2n-6)=0$$

The zero product property is if

$ab=0$ then

$a=0$ or

$b=0$ or both

$3n+8=0$ or

$2n-6=0$ (By zero product property)

Now solve each equation completely.

$$3n+8=0$$

$3n+8-8=0-8$ (Subtract 8 on both sides)

$$3n=-8$$

$\frac{3n}{3}=\frac{-8}{3}$ (Divide with 3 on both sides)

$$n=\frac{-8}{3}$$

$$2n-6=0$$

$2n-6+6=0+6$ (Add 6 on both sides)

$$2n=6$$

$\frac{2n}{2}=\frac{6}{2}$ (Divide with 2 on both sides)

$$n=3$$

The solution set is $\left\{\frac{-8}{3},3\right\}$.

Check: To check the proposed solution set substitute each solution in the given equation.

Given equation is

$$(3n+8)(2n-6)=0$$

$$\left(3\cdot\left(\frac{-8}{3}\right)+8\right)\left(2\left(\frac{-8}{3}\right)-6\right)=0$$ (Put $n=\frac{-8}{3}$)

$$(-8+8)\left(\frac{-16}{3}-6\right)=0$$ (Simplify)

$$0\left(\frac{-16}{3}-6\right)=0$$

$0=0$ True

For $n=3$,

$$(3n+8)(2n-6)=0$$

$$(3(3)+8)(2(3)-6)=0$$ (Put $n=3$)

$$(9+8)(6-6)=0$$ (Simplify)

$$17(0)=0$$

$0=0$ True

The solution set of given equation is $\boxed{\left\{\frac{-8}{3},3\right\}}$.

Answer 30PT.

Consider the equation $(4x-3)(3x+2)=0$

The objective is to find the solution set of given equation.

The zero product property is of $ab=0$ then $a=0 \text{ or } b=0$ or both.

$$(4x-3)(3x+2)=0$$
$$4x-3=0 \text{ or } 3x+2=0 \qquad [\text{By zero product property}]$$

Now solve each equation separately.

$$4x-3=0$$
$$4x-3+3=0+3 \qquad [\text{Add 3 on both sides}]$$
$$4x=3$$
$$\frac{4x}{4}=\frac{3}{4} \qquad [\text{Divide with 4 on both sides}]$$
$$x=\frac{3}{4}$$
$$3x+2=0$$
$$3x+2-2=0-2 \qquad [\text{Subtract 2 on both sides}]$$
$$3x=-2$$
$$\frac{3x}{3}=\frac{-2}{3} \qquad [\text{Divide with 3 on both sides}]$$
$$x=\frac{-2}{3}$$

The solution set is $\left\{-\frac{2}{3},\frac{3}{4}\right\}$

<u>Check:</u> To check the proposed solution set. Substitute each solution in the given equation.

Given equation is $(4x-3)(3x+2)=0$

$$\left[4\left(\frac{-2}{3}\right)-3\right]\left[3\left(\frac{-2}{3}\right)+2\right]=0 \qquad \left[\text{Put } x=\frac{-2}{3}\right]$$
$$\left(\frac{-8}{3}-3\right)(-2+2)=0 \qquad [\text{Simplify}]$$
$$\left(\frac{-8}{3}-3\right)=0$$
$$0=0 \quad \text{True}$$

$$(4x-3)(3x+2)=0$$
$$\left[\not{4}\left(\frac{3}{\not{4}}\right)-3\right]\left[3\left(\frac{3}{4}\right)+2\right]=0 \qquad \left[\text{Put } x=\frac{3}{4}\right]$$
$$(3-3)\left(\frac{9}{4}+2\right)=0 \qquad [\text{Simplify}]$$
$$0\left(\frac{9}{4}+2\right)=0$$
$$0=0 \quad \text{True}$$

Therefore, the solution set of given equation is $\boxed{\left\{\frac{-2}{3},\frac{3}{4}\right\}}$

Answer 31E.

Consider the equation

$4x^2 = -7x$

The objective is to find the solution set of given equation.

$4x^2 = -7x$

$4x^2 + 7x = -7x + 7x$ (Add $7x$ on both sides)

$4x^2 + 7x = 0$

$4 \cdot x \cdot x + 7 \cdot x = 0 \quad \left(x^2 = x \cdot x\right)$

$x(4x+7) = 0$ (Factor $GCF\left(4x^2, 7x\right) = x$)

The zero product property is of

$ab = 0$ then

$a = 0$ or

$b = 0$ or both.

$x = 0$

Or, $4x + 7 = 0$ (By zero product property)

Now solve each equation completely.

$4x + 7 = 0$

$4x + 7 - 7 = 0 - 7$ (Subtract 7 on both sides)

$4x = -7$

$\frac{4x}{4} = \frac{-7}{4}$ (Divide with 4 on each side)

$x = \frac{-7}{4}$

The solution set is $\left\{0, \frac{-7}{4}\right\}$.

Check:- To check the proposed solution set substitute each solution in the given equation.

Given equation is

$4x^2 = -7x$

$40^2 = -7(0)$ (Put $x = 0$)

$0 = 0$ True

For $x = -\frac{7}{4}$,

$4x^2 = -7x$

$4\left(\frac{-7}{4}\right)^2 = -7\left(\frac{-7}{4}\right)$ (Put $x = \frac{-7}{4}$)

$4\frac{49}{4 \cdot 4} = \frac{49}{4}$ (Simplify)

$\frac{49}{4} = \frac{49}{4}$ True

The solution set of given equation is $\boxed{\left\{0, \frac{-7}{4}\right\}}$.

Answer 31PT.

Consider the equation $18s^2 + 72s = 0$

The objective is to find the solution set of given equation.

$$18s^2 + 72s = 0$$
$$18 \cdot s \cdot s + 18 \cdot 4s = 0 \quad [\text{Since } 18 \cdot 4 = 72]$$
$$18s \cdot s + 18s \cdot 4 = 0$$
$$18s[s + 4] = 0 \quad \left[\text{Take GCF } (18s^2, 72s) = 18s \text{ as factor}\right]$$

The zero product property is of $ab = 0$ then $a = 0 \text{ or } b = 0$ or both.

$$18s(s + 4) = 0$$
$$18s = 0 \text{ or } s + 4 = 0 \quad [\text{By zero product property}]$$

Now solve each equation separately.

$$18s = 0$$
$$\frac{18s}{18} = \frac{0}{18} \quad [\text{Divide with 18 on both sides}]$$
$$s = 0$$

$$s + 4 = 0$$
$$s + 4 - 4 = 0 - 4 \quad [\text{Subtract 4 on both sides}]$$
$$s = -4$$

The solution set is $\{0, -4\}$

Check: To check the proposed solution set. Substitute each solution in the given equation.

Given equation is $18s^2+72s=0$

$$18(0)^2+72(0)=0 \qquad [\text{Put } s=0]$$

$$0+0=0$$

$$0=0 \text{ True}$$

$$18s^2+72s=0$$

$$18(-4)^2+72(-4)=0 \qquad [\text{Put } s=-4]$$

$$18\cdot16-72\cdot4=0 \qquad [\text{Simplify}]$$

$$288-288=0$$

$$0=0 \text{ True}$$

Therefore, the solution set of given equation is $\boxed{\{0,-4\}}$

Answer 32E.

Consider the trinomial $y^2+7y+12$

The objective is to factor the given trinomial.

Compare $y^2+7y+12$ with x^2+bx+c

Here $b=7$,

$c=12$

Now $y^2+7y+12=(y+m)(y+n)$

$=y^2+(m+n)y+mn$

Now find two numbers m,n such that

$m+n=7$ and

$mn=12$

Since $m+n$ and mn are positive then m,n must be positive

To find m,n list all the factors of

$mn=12$, in those choose a pair whose sum is 7.

Factors of 12	Sum of factors
$1,12$	13
$2,6$	8
$3,4$	7

The correct factors are $3,4$.

$y^2+7y+12=(y+m)(y+n)$

$=(y+3)(y+4) \quad (m=3, n=4)$

Check: To check the factors, multiply the factors using $FOIL$ Method.

$$(y+3)(y+4)=\overset{F}{y\cdot y}+\overset{O}{y\cdot 4}+\overset{I}{3\cdot y}+\overset{L}{3\cdot 4}$$

($FOIL$ method)

$=y^2+4y+3y+12$ (Simplify)

$=y^2+7y+12$ True

(Combine like terms)

Therefore, the factors of given trinomial is $\boxed{(y+3)(y+4)}$.

Answer 32PT.

Consider the equation

$$4x^2 = 36$$

The objective is to find the solution set of given equation

$$4x^2 = 36$$

$$\frac{4x^2}{4} = \frac{36}{4}$$ (Divide with 4 on both sides)

$$x^2 = 9$$

The square root property is if

$n > 0,$ then

$$x^2 = n$$

$$x = \pm\sqrt{n}$$

$$x^2 = 9$$

$$\Rightarrow \quad x = \pm\sqrt{9}$$ (By square root property)

$$x = \pm 3$$

$$\sqrt{9} = 3$$

The solution set is $\{-3, 3\}$.

Check:- To check the proposed solution set substitute each equation in the given equation.

Given equation is

$$4x^2 = 36$$

$$4(-3)^2 = 36$$ (Put $x = -3$)

$$4 \cdot 9 = 36$$

$36 = 36$ True

$$4x^2 = 36$$

$$4 \cdot 3^2 = 36$$ (Put $x = 3$)

$$4 \cdot 9 = 36$$ (Simplify)

$36 = 36$ True

The solution set of given equation is $\boxed{\{-3, 3\}}$.

Answer 33E.

Consider the trinomial $x^2 - 9x - 36$.

The objective is to find the factorization of given trinomial.

Compare $x^2 - 9x - 36$ with $x^2 + bx + c$.

$b = -9,$

$c = -36$

$$x^2 - 9x - 36 = (x+m)(x+n)$$
$$= x^2 (m+n)x + mn$$

Here $m + n = -9,$

$mn = -36$

$m+n, mn$ are negative, then either m or n negative but not both.

For this list all the pair of factors of

$mn = -36$ in those one factor is negative choose a pair whose sum is -9.

Factors of -36	Sum of factors
$1.-36$	-35
-1.36	35
-2.18	16
$2.-18$	-16
-3.12	9
$3.-12$	-9
-4.9	5
$4.-9$	-5

$6, -6$	0

The correct factors are $3, -12$.

$$x^2 - 9x - 36 = (x+m)(x+n)$$

$$= (x+3)(x-12) \quad (m = 3, n = -12)$$

Check: To check the factors multiply the factors using $FOIL$ method.

$$(x+3)(x-12) = x \cdot \overset{F}{x} + (\overset{O}{-12}) \cdot x + \overset{I}{3} \cdot x + 3 \cdot (\overset{L}{-12})$$

($FOIL$ Method)

$$= x^2 - 12x + 3x - 36$$

(Simplify)

$$= x^2 - 9x - 36 \text{ True}$$

Therefore the factorization of given trinomial is $\boxed{(x+3)(x-12)}$.

Answer 33PT.

Consider the equation $t^2 + 25 = 10t$

The objective is to find the solution set of given equation.

$$t^2 + 25 = 10t$$
$$t^2 + 25 - 10t = 10t - 10t \quad [\text{Subtract } 10t \text{ on both sides}]$$
$$t^2 - 10t + 25 = 0$$
$$t^2 - 2 \cdot 5 \cdot t + 5 \cdot 5 = 0 \quad [\text{Since } 2 \cdot 5 = 10]$$
$$t^2 - 2 \cdot 5 \cdot t + 5^2 = 0$$

The perfect squares property is $(a-b)^2 = a^2 - 2ab + b^2$

$$t^2 - 2 \cdot 5 \cdot t + 5^2 = 0$$
$$(t-5)^2 = 0 \quad \text{By perfect square property}$$

The zero product property is of $ab = 0$ then $a = 0 \text{ or } b = 0$ or both.

$$(t-5)^2 = 0$$
$$(t-5)(t-5) = 0$$
$$t - 5 = 0 \text{ or } t - 5 = 0$$

Now solve each equation separately.

$$t - 5 = 0$$
$$t - 5 + 5 = 0 + 5 \quad [\text{Add 5 on both sides}]$$
$$t = 5$$

The solution set is $\{5\}$

Check: To check the proposed solution set substitute each solution in the given equation.

Given equation is

$$t^2 + 25 = 10t$$
$$(5)^2 + 25 = 10(5) \quad [\text{Put } t = 5]$$
$$25 + 25 = 50$$
$$50 = 50 \quad \text{True}$$

Therefore, the solution set of given equation is $\boxed{\{5\}}$

Answer 34E.

Consider the trinomial $b^2 + 5b - 6$.

The objective is to factor the given trinomial.

Compare $b^2 + 5b - 6$ with $x^2 + ax + c$.

Here $a = 5$,and

$c = -6$

$$b^2 + 5b - 6 = (b+m)(b+n)$$
$$= b^2 + (m+n)b + mn$$

Here $m + n = 5,$

$mn = -6$

Since $m+n$ is positive and mn is negative, then one of m or n must be negative but not both.

Now list all the factors of

$mn = -6$, in those choose a pair whose sum is 5.

Factor of -6	Sum of factors
-1.6	5
$1.-6$	-5
-2.3	1
$2.-3$	-1

The correct factors are $-1, 6$.

Thus, $b^2 + 5b - 6 = (b+m)(b+n)$

$= (b-1)(b+6) \quad (m = -1, n = 6)$

Check: To check the factors multiply the factors using $FOIL$ method

$$(b-1)(b+6) = \overset{F}{b \cdot b} + \overset{O}{6 \cdot b} + \overset{I}{(-1) \cdot b} + \overset{L}{(-1) \cdot 6}$$

($FOIL$ method)

$= b^2 + 6b - b - 6$ (Simplify)

$= b^2 + 5b - 6$ True

Therefore, the factorization of given trinomial is $\boxed{(b-1)(b+6)}$.

Answer 34PT.

Given equation is $a^2 - 9a - 52 = 0$

The objective is to find the solution set of given equation.

For this first factor $a^2 - 9a - 52$

Compare $a^2 - 9a - 52$ with $a^2 + ba + c$

Here $a = -9, c = -52$

$$a^2 - 9a - 52 = (a+m)(a+n)$$
$$= a^2 + (m+n)a + mn$$

Here $m + n = -9$, $mn = -52$

Now find two numbers m, n such that $m + n = -9, mn = -52$ both are negative then one of *m*, *n* must be negative but not both.

For this list all the pair of factors of -52 in those choose a pair whose sum is -9

Factor of -52	Sum of factors
$-1 \cdot 52$	51
$1 \cdot -52$	-51
$-2 \cdot 26$	24
$2 \cdot -26$	-24
$-4 \cdot 13$	9
$4 \cdot -13$	✓ -9

The correct factors are $4, -13$

$$a^2 - 9a - 52 = (a+m)(a+n)$$
$$= (a+4)(a-13) \qquad [m = 4, n = -13]$$
$$a^2 - 9a - 52 = 0$$
$$(a+4)(a-13) = 0 \qquad [\text{By factorization}]$$

The zero product property is if $ab = 0$ then $a = 0$ or $b = 0$ or both

$$a+4 = 0 \text{ or } a = -13 = 0$$
$$a+4 = 0$$
$$a+4-4 = 0-4 \qquad [\text{Subtract 4 on both sides}]$$
$$a = -4$$
$$a-13 = 0$$
$$a-13+13 = 0+13 \qquad [\text{Add 13 on both sides}]$$
$$a = 13$$

The solution is $\{-4, 13\}$

Check:

To check the proposed solution set substitute each solution in the given equation and verify.

Given equation is

$$a^2 - 9a - 52 = 0$$
$$(-4)^2 - 9(-4) - 52 = 0 \qquad [\text{Put } a = -4]$$
$$16 + 36 - 52 = 0 \qquad [\text{Simplify}]$$
$$52 - 52 = 0$$
$$0 = 0 \text{ True}$$
$$a^2 - 9a - 52 = 0$$
$$(13)^2 - 9(13) - 52 = 0 \qquad [\text{Put } a = 13]$$
$$169 - 117 - 52 = 0 \qquad [\text{Simplify}]$$
$$169 - 169 = 0$$
$$0 = 0 \text{ True}$$

The solution set of given equation is $\boxed{\{-4, 13\}}$

Answer 35E.

Consider the trinomial $18-9r+r^2$.

The objective is to factor the given trinomial completely

$$18-9r+r^2 = r^2-9r+18$$

Compare it with x^2+bx+c

$$b=-9,$$
$$c=18$$

$$r^2-9r+18 = (r+m)(r+n)$$
$$= r^2+(m+n)r+mn$$

Here $m+n=-9$ and

$$mn=18$$

That is we have to find m,n such that

$$m+n=-9,$$

$$mn=18$$

Since $m+n$ is negative and mn is positive then $m+n$ both m,n are negative.

For this list all the negative factors of 18 is those choose a pair whose sum is -9.

Factors of 18	Sum of factors
$-18,-1$	-19
$-2,-9$	-11
$-3,-6$	-9

The correct factors are $-3,-6$

$$r^2-9r+18=(r+m)(r+n)$$

$$=(r-3)(r-6) \quad (m=-3, n=-6)$$

Check: To check the factors, multiply the factors using $FOIL$ method.

$$(r-3)(r-6)=r\cdot \overset{F}{r}+r\cdot(\overset{O}{-6})+(\overset{I}{-3})r+(\overset{L}{-3})\cdot(-6)$$

($FOIL$ method)

$$=r^2-6r-3r+18 \text{ (Because simplify)}$$

$$=r^2-9r+18 \text{ True}$$

Therefore, the factorization of given trinomial is $\boxed{(r-3)(r-6)}$.

Answer 35PT.

The equation is $x^3-5x^2-66x=0$

The objective is to find the solution set of given equation.

For this first factor x^3-5x^2-66x

$$x^3-5x^2-66x = x\cdot x^2-5x.x-66\cdot x$$
$$= x\left[x^2-5x-66\right]$$

Now compare $x^2-5x-66$ with ax^2+bx+c

Here $b=-5, c=-66$

$$x^2-5x-66=(x+m)(x+n)$$
$$=x^2+(m+n)x+mn$$

Now find two numbers m,n such that $m+n=-5$ and $m\cdot n=-66$ negative

Since $m+n, mn$ are negative then one of *m* or *n* must be negative but not both.

List all the factors of -66 in those choose a pair whose sum is -5

Factor of -66	Sum of factors
$-1\cdot 66$	65
$1\cdot -66$	-65
$-2\cdot 33$	31

$2\cdot -33$	-31
$-3\cdot 22$	19
$3\cdot -22$	-19
$-6\cdot 11$	5
$6\cdot -11$	✓ -5

The correct factors are $6, -11$

Thus $x^2-5x-66=(x+m)(x+n)$

$=(x+6)(x-11) \qquad [m=6, n=-11]$

Therefore $x^3-5x^2-66x=0$

$x(x^2-5x-66)=0$

$x(x+6)(x-11)=0$

The zero product property is if $ab=0$ then $a=0$ or $b=0$ or both

$x=0$ or $x+6=0$ or $x-11=0$

Now solve each equation completely

$$x=0$$

$$x+6=0$$

$$x+6-6=0-6 \qquad [\text{Subtract 6 on both sides}]$$

$$x=-6$$

$$x-11=0$$

$$x-11+11=0+11 \qquad [\text{Add 11 on both sides}]$$

$$x=11$$

The solution set is $\{0,-6,11\}$

Check:

To check the proposed solution set substitute each solution in the given equation and verify.

Given equation is

$$x^3 - 5x^2 - 66x = 0$$

$$(0)^3 - 5(0)^2 - 66(0) = 0 \qquad [\text{Put } x = 0]$$

$$0 = 0 \text{ True}$$

$$x^3 - 5x^2 - 66x = 0$$

$$(-6)^3 - 5(-6)^2 - 66(-6) = 0 \qquad [\text{Put } x = -6]$$

$$-216 - 180 + 396 = 0 \qquad [\text{Simplify}]$$

$$-396 + 396 = 0$$

$$0 = 0 \text{ True}$$

$$x^3 - 5x^2 - 66x = 0$$

$$(11)^3 - 5(11)^2 - 66(11) = 0 \qquad [\text{Put } x = 11]$$

$$1331 - 605 - 726 = 0 \qquad [\text{Simplify}]$$

$$1331 - 1331 = 0$$

$$0 = 0 \text{ True}$$

The solution set of given equation is $\boxed{\{0, -6, 11\}}$

Answer 36E.

Consider the equation $a^2 + 6ax - 40x^2$

The objective is to solve the given equation.

Compare $a^2 + 6ax - 40x^2$ with $a^2 + ba + c$

Here $b = 6x, c = -40x^2$

$$a^2 + 6ax - 40x^2 = (a+m)(a+n)$$
$$= a^2 + (m+n)a + mn$$

Here $m + n = 6x, mn = -40x^2$

Now find m, n such that $m + n = 6x$ is positive and $mn = -40x^2$ is negative and one of *m* or *n* is negative

List all the factors of $mn = -40x^2$ in those choose a pair whose sum is $6x$

Factor of $-40x^2$	Sum of factors
$-1 \cdot 40x^2$	$-1+40x^2$
$1 \cdot -40x^2$	$1-40x^2$
$-2 \cdot 20x^2$	$-2+20x^2$
$2 \cdot -20x^2$	$2-20x^2$
$4 \cdot -10x^2$	$4-10x^2$
$-4 \cdot 10x^2$	$-4+10x^2$
$-5 \cdot 8x^2$	$-5+8x^2$
$5 \cdot -8x^2$	$5-8x^2$
$-x \cdot 40x$	$39x$
$x \cdot -40x$	$-39x$
$-2x \cdot 20x$	$18x$
$2 \cdot -20x$	$-18x$
$4x \cdot -10x$	$-6x$
$-4x \cdot 10x$	$6x$

$-5x \cdot 8x$	$3x$
$5x \cdot -8x$	$-3x$

The correct factors are $-4x, 10x$

$$\begin{aligned} a^2+6ax-40x^2 &= (a+m)(a+n) \\ &= (a-4x)(a+10x) \quad [m=-4x, n=10x] \end{aligned}$$

Check:

To check the factorizations multiply the factors using FIOIL method.

$$\begin{aligned} (a-4x)(a+10x) &= a \cdot a + a \cdot 10x + (-4x) \cdot a + (-4x) \cdot 10x \qquad [\text{FOIL Method}] \\ &= a^2 + 10ax - 4ax - 40x^2 \\ &= a^2 + 6ax - 40x^2 \quad \text{True} \end{aligned}$$

Therefore, the solution set of given equation is $\boxed{\{(a-4x)(a+10x)\}}$

Answer 36PT.

Consider the equation $2x^2 = 9x + 5$

The objective is to find the solution set of given equation.

$$2x^2 = 9x + 5$$

$$2x^2 - 9x = 9x + 5 - 9x \qquad [\text{Subtract } 9x \text{ on both sides}]$$

$$2x^2 - 9x = 5$$

$$2x^2 - 9x - 5 = 5 - 5 \qquad [\text{Subtract 5 on both sides}]$$

$$2x^2 - 9x - 5 = 0$$

For this first factor $2x^2 - 9x - 5$

Now compare $2x^2 - 9x - 5$ with $ax^2 + bx + c$

Here $a = 2, b = -9, c = -5$

$$2x^2 - 9x - 5 = 2x^2 + mx + nx - 5$$
$$= 2x^2 + (m+n)x - 5$$

Now find two numbers m, n such that $m + n = -9$ and $m \cdot n = 2 \cdot -5 = -10$

Since $m + n, mn$ are negative then one of *m* or *n* must be negative but not both.

List all the factors of $mn = -10$ in those choose a pair whose sum is -9

Factor of -10	Sum of factors
$-1 \cdot 10$	✓ 9
$1 \cdot -10$	-9
$-2 \cdot 5$	3
$2 \cdot -5$	-3

The correct factors are $1, -10$

Thus $2x^2-9x-5=2x^2+mx+nx-5$

$=2x^2+x-10x-5$ $\quad [m=1, n=-10]$

$=x(2x+1)+(-5)(2x+1)$ $\quad$ [Factor the GCF]

$=(x-5)(2x+1)$ $\quad$ [By distributive $(b+c)a=ba+ca$]

$$2x^2-9x-5=0$$
$$(x-5)(2x+1)=0$$

The zero product property is if $ab=0$ then $a=0$ or $b=0$ or both

$x-5=0$ or $2x+1=0$

Now solve each equation completely

$$x-5=0$$
$$x-5+5=0+5 \quad \text{[Add 5 on both sides]}$$
$$x=5$$
$$2x+1=0$$
$$2x+1-1=0-1 \quad \text{[Subtract 1 on both sides]}$$
$$2x=-1$$
$$\frac{2x}{2}=\frac{-1}{2} \quad \text{[Divide with 2 on both sides]}$$
$$x=\frac{-1}{2}$$

The solution set is $\left\{5, \frac{-1}{2}\right\}$

Check:

To check the proposed solution set substitute each solution in the given equation and verify.

Given equation is

$$2x^2=9x+5$$
$$2\left(\frac{-1}{2}\right)^2=9\left(\frac{-1}{2}\right)+5 \quad \left[\text{Put } x=\frac{-1}{2}\right]$$
$$2\cdot\frac{1}{4}=\frac{-9}{2}+5 \quad \text{[Simplify]}$$
$$\frac{1}{2}=\frac{-9+10}{2} \quad \text{[Simplify]}$$
$$\frac{1}{2}=\frac{1}{2} \quad \text{True}$$

$$2x^2 = 9x + 5$$
$$2(5)^2 = 9(5) + 5 \qquad [\text{Put } x = 5]$$
$$50 = 45 + 5 \qquad [\text{Simplify}]$$
$$50 = 50 \quad \text{True}$$

The solution set of given equation is $\boxed{\left\{\frac{-1}{2}, 5\right\}}$

Answer 37E.

Consider the equation $m^2 - 4mn - 32n^2$

The objective is to solve the given equation.

Compare $m^2 - 4mn - 32n^2$ with $m^2 + bm + c$

Here $b = -4n, c = -32n^2$

$$m^2 - 4mn - 32n^2 = (m + x)(m + y)$$
$$= m^2 + (x + y)m + xy$$

Here $x + y = -4n$ and $xy = -32n^2$

Now find x, y such that $x + y = -4n$ is positive and $xy = -32n^2$.

Since $x + y, xy$ are negative then one of x or y must be negative.

List all the factors of $xy = -32n^2$ in those choose a pair whose sum is $-4n$

Factor of $-32n^2$	Sum of factors
$-1\cdot 32n^2$	$-1+32n^2$
$1\cdot -32n^2$	$1-32n^2$
$-2\cdot 16n^2$	$-2+16n^2$
$2\cdot -16n^2$	$2-16n^2$
$-4\cdot 8n^2$	$-4+8n^2$
$4\cdot -8n^2$	$4-8n^2$
$-n\cdot 32n$	$31n$
$-2n\cdot 16n$	$14n$
$2n\cdot -16n$	$-14n$
$-4n\cdot 8n$	✓ $4n$
$4n\cdot -8n$	$-4n$

The correct factors are $4n, -8n$

Thus $m^2-4mn-32n^2=(m+x)(m+y)$

$=(m+4n)(m-8n)$

Check:

To check the factorization multiply the factors using FIOIL method.

$$(m+4n)(m-8n) = m\cdot m + m\cdot(-8n) + 4n\cdot m + 4n(-8n) \qquad [\text{FOIL Method}]$$
$$= m^2 - 8mn + 4mn - 32n^2 \qquad [\text{Simplify}]$$
$$= m^2 - 4mn - 32m^2 \quad \text{True}$$

Therefore, the solution set of given equation is $\boxed{\{(m+4n)(m-8n)\}}$

Answer 37PT.

Consider the equation $3b^2+6=11b$

The objective is to solve find the solution set of given equation

$$3b^2+6=11b$$
$$3b^2+6-11b=11b-11b \qquad [\text{Subtract } 11b \text{ on both sides}]$$
$$3b^2+6-11b=0 \qquad [\text{Combine like terms}]$$

First factor $3b^2-11b+6$

Compare $3b^2-11b+6$ with ab^2+cb+d

Here $a=3, c=-11, d=6$

$$3b^2+6=11b=3b^2+mb+nb+6$$
$$=3b^2+(m+n)b+6$$

Now find two numbers m,n such that $m+n=-11$ and $mn=3\cdot 6=18$

Since $m+n$ is negative and mn is positive then both *m* and *n* are negative.

List all the negative pairs of $mn=18$ is those choose a pair whose sum is -11

Factor of 18	Sum of factors
$-1\cdot-18$	-19
$-2\cdot-9$	✓ -11
$-3\cdot-6$	-9

The correct factors are $-2,-9$

$$3b^2+6=11b=3b^2+mb+nb+6$$
$$=3b^2-2b-9b+6 \quad [m=-2,n=-9]$$
$$=b(3b-2)-3(3b-2) \quad [\text{Factor the GCF}]$$
$$=(b-3)(3b-2) \quad [\text{By distributive } (b+c)a=ba+ca]$$

$$3b^2-11b+6=0$$
$$(b-3)(3b-2)=0$$

By zero product property if $ab=0$ then $a=0 \text{ or } b=0$ or both

$$b-3=0 \text{ or } 3b-2=0 \quad [\text{By zero product property}]$$

Now solve each equation completely

$$b-3=0$$
$$b-3+3=0+3 \quad [\text{Add 3 on both sides}]$$
$$b=3$$
$$3b-2=0$$
$$3b-2+2=0+2 \quad [\text{Add 2 on both sides}]$$
$$3b=2$$
$$\frac{3b}{3}=\frac{2}{3} \quad [\text{Divide with 3 on both sides}]$$
$$b=\frac{2}{3}$$

The solution set is $\left\{3,\frac{2}{3}\right\}$

Check:

To check the proposed solution set, substitute each solution in the given equation and verify

Given equation is $3b^2+6=11b$

$$3(3)^2+6=11(3) \qquad [\text{put } b=3]$$
$$27+6=33 \qquad [\text{Simplify}]$$
$$33=33 \text{ True}$$

$$3b^2+6=11b$$
$$3\left(\frac{3}{2}\right)^2+6=11\left(\frac{3}{2}\right) \qquad \left[\text{put } b=\frac{3}{2}\right]$$
$$3\cdot\frac{4}{3\cdot 3}+6=\frac{22}{3} \qquad [\text{Simplify}]$$
$$\frac{4}{3}+6=\frac{22}{3}$$
$$\frac{4+18}{3}=\frac{22}{3}$$
$$\frac{22}{3}=\frac{22}{3} \text{ True}$$

Therefore, the solution set of given equation is $\boxed{\left\{\frac{2}{3},3\right\}}$

Answer 38E.

Consider the equation $y^2 + 13y + 40 = 0$

The objective is to solve the given equation.

For this first factor the polynomial $y^2 + 13y + 40$

Compare $y^2 + 13y + 40$ with $y^2 + by + c$

Here $b = 13, c = 40$

$$\begin{aligned} y^2 + 13y + 40 &= (y+m)(y+n) \\ &= y^2 + (m+n)y + mn \end{aligned}$$

Now find two numbers m, n such that $m + n = 13, mn = 40$

Since $m + n, mn$ are positive then both m, n are positive

List all the pair of factors of $mn = 40$ in those choose a pair whose sum is 13

Factor of 40	Sum of factors
$1 \cdot 40$	41
$2 \cdot 20$	22
$4 \cdot 10$	14
$5 \cdot 8$	✓ 13

The correct factors are 5, 8

$$y^2+13y+40=(y+m)(y+n)$$
$$=(y+5)(y+8) \qquad [m=5, n=8]$$

Thus $y^2+13y+40=0$

$$(y+5)(y+8)=0$$

The zero product property is if $ab=0$ then $a=0 \text{ or } b=0$ or both

$$y+5=0 \text{ or } y+8=0$$

Now solve each equation completely

$$y+5=0$$
$$y+5-5=0-5 \qquad [\text{Subtract 5 on both sides}]$$
$$y=-5$$
$$y+8=0$$
$$y+8-8=0-8 \qquad [\text{Subtract 8 on both sides}]$$
$$y=-8$$

The solution set is $\{-5,-8\}$

Check:

To check the proposed solution substitute each solution in the given equation

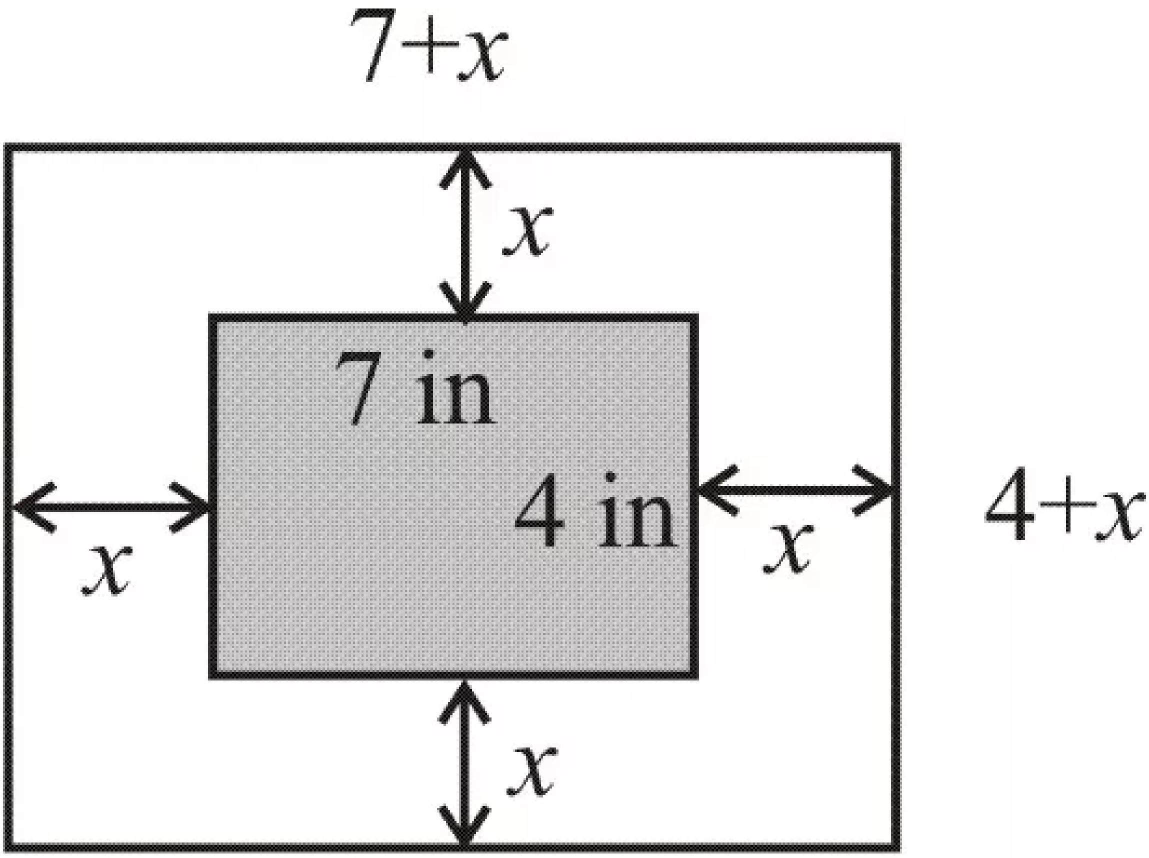
Given equation is $y^2+13y+40=0$

$$(-5)^2+13(-5)+40=0 \qquad [\text{Put } y=-5]$$
$$25-65+40=0 \qquad [\text{Simplify}]$$
$$65-65=0$$
$$0=0 \text{ True}$$
$$(-8)^2+13(-8)+40=0 \qquad [\text{Put } y=-8]$$
$$64-104+40=0 \qquad [\text{Simplify}]$$
$$104-104=0$$
$$0=0 \text{ True}$$

Therefore, the solution set of given equation is $\boxed{\{-5,-8\}}$

Answer 38PT.

Consider that a rectangle is 4 inches wide by 7 inches long.



Also length and width are increased by same amount, the area is increased by 26 square inches.

The objective is to find the dimensions of new rectangle

Length of old rectangle $l = 7$ in

Width of old rectangle $b = 4$ in

Area of old rectangle $A = lb$

$= 7 \cdot 4 \qquad [\text{Put } l = 7, b = 4]$

$= 28$ Square inches

The new rectangle is formed by adding same amount to length and width

Let x be the added amount

Length of new rectangle = length of old rectangle + added amount

$= 7 + x$

Width of new rectangle = width of old rectangle plus added amount

$= 4 + x$

Given area of new rectangle = Area of old rectangle plus 26 square inches.

That is $(7+x)(4+x) = 28 + 26$

$$(7+x)(4+x) = 54$$

$$7\cdot 4 + 7\cdot x + x\cdot 4 + x\cdot x = 54 \quad \text{[By FOIL method]}$$

$$x^2 + 11x + 28 = 54 \quad \text{[Combine like terms]}$$

$$x^2 + 11x + 28 - 54 = 54 - 54 \quad \text{[Subtract 54 on both sides]}$$

$$x^2 + 11x - 26 = 0$$

$$x^2 + 11x - 26 = 0$$

$$x^2 + 2x + 13x - 26 = 0 \quad \text{[Factor]}$$

$$x(x-2) + 13(x-2) = 0 \quad \text{[Take factor the GCF]}$$

$$(x+13)(x-2) = 0 \quad \text{[By distributive } (b+c)a = ba + ca\text{]}$$

The zero product property is if $ab = 0$ then $a = 0$ or $b = 0$ or both

$$x + 13 = 0 \text{ or } x - 2 = 0$$

$$x + 13 = 0$$

$$x + 13 - 13 = 0 - 13 \quad \text{[Subtract 13 on both sides]}$$

$$x = -13$$

$$x - 2 = 0$$

$$x - 2 + 2 = 0 + 2 \quad \text{[Add 2 on both sides]}$$

$$x = 2$$

Therefore, $x = -13$ or $x = 2$

Since length always positive consider $x = 2$

Length of new rectangle $= 7 + x$

$= 7 + 2 \qquad [\text{put } x = 2]$

$= 9$

Width of new rectangle $= 4 + x$

$= 4 + 2 \qquad [\text{put } x = 2]$

$= 6$

Therefore, the dimensions of new rectangle are length $= \boxed{9 \text{ inches}}$, width $= \boxed{6 \text{ inches}}$

Answer 39E.

Consider the equation $x^2-5x-66=0$

The objective is to solve the given equation.

For this first factor the polynomial $x^2-5x-66$

Compare $x^2-5x-66$ with x^2+bx+c

Here $b=-5, c=-6$

$$x^2-5x-66=(x+m)(x+n)$$
$$=x^2+(m+n)x+mn$$

$m+n=-5, mn=-66$

Now find two numbers m,n such that $m+n=-5, mn=-66$

Since $m+n, mn$ are negative then either m or n negative but not both

List all the factors of -66 in those choose a pair whose sum is -5

Factor of -66	Sum of factors
$-1\cdot 66$	65
$1\cdot -66$	-65
$-2\cdot 33$	31
$2\cdot -33$	-31
$-3\cdot 22$	19
$3\cdot -22$	-19
$6\cdot -11$	✓ -5
$-6\cdot 11$	5

The correct factors are 6, -11

$$x^2 - 5x - 66 = (x+m)(x+n)$$
$$= (x+6)(x-11) \qquad [m=6, n=-11]$$

$$x^2 - 5x - 66 = 0$$
$$(x+6)(x-11) = 0$$

The zero product property is if $ab = 0$ then $a = 0$ or $b = 0$ or both

$$x+5 = 0 \text{ or } x-11 = 0$$

Now solve each equation completely

$$x+6 = 0$$
$$x+6-6 = 0-6 \qquad [\text{Subtract 6 on both sides}]$$
$$x = -6$$
$$x-11 = 0$$
$$x-11+11 = 0+11 \qquad [\text{Add 11 on both sides}]$$
$$x = 11$$

The solution set is $\{-6, 11\}$

The correct factors are 6, -11

$$x^2 - 5x - 66 = (x+m)(x+n)$$
$$= (x+6)(x-11) \qquad [m=6, n=-11]$$

$$x^2 - 5x - 66 = 0$$
$$(x+6)(x-11) = 0$$

The zero product property is if $ab = 0$ then $a = 0$ or $b = 0$ or both

$$x+5 = 0 \text{ or } x-11 = 0$$

Now solve each equation completely

$$x+6 = 0$$
$$x+6-6 = 0-6 \qquad [\text{Subtract 6 on both sides}]$$
$$x = -6$$
$$x-11 = 0$$
$$x-11+11 = 0+11 \qquad [\text{Add 11 on both sides}]$$
$$x = 11$$

The solution set is $\{-6, 11\}$

Check:

To check the proposed solution substitute each solution in the given equation

Given equation is $x^2-5x-66=0$

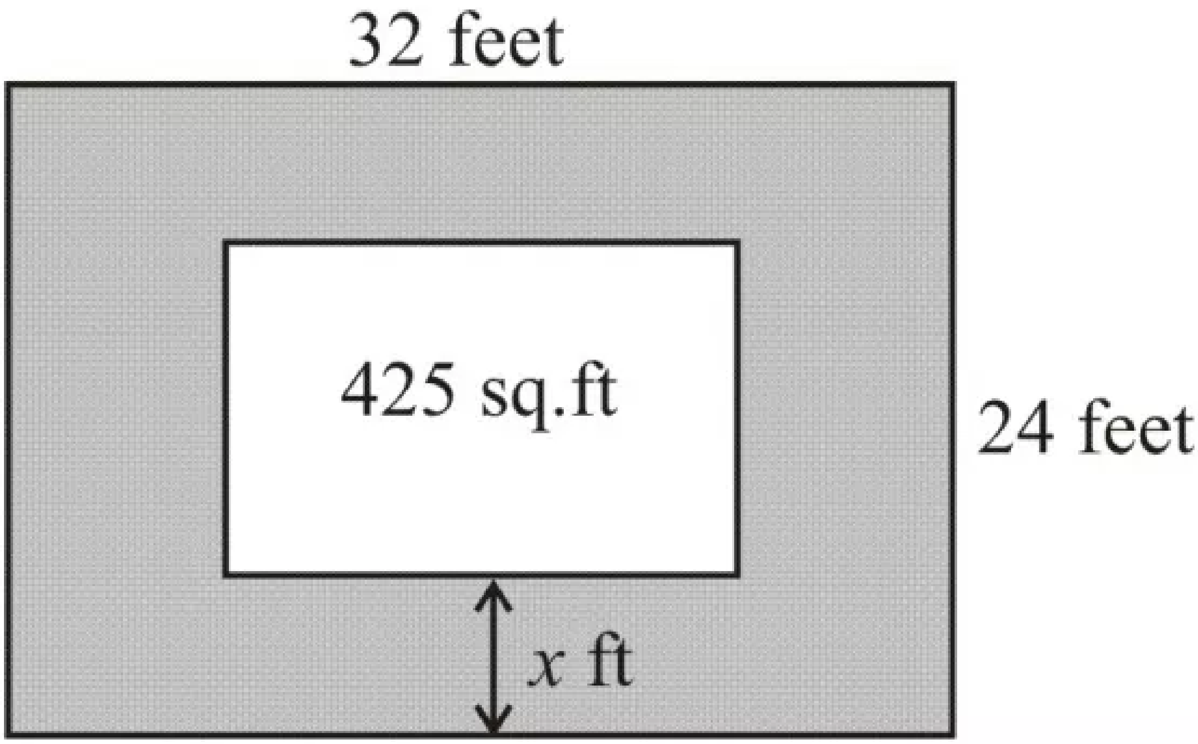
$$(-6)^2-5(-6)-66=0 \quad [\text{Put } x=-6]$$
$$36+30-66=0 \quad [\text{Simplify}]$$
$$66-66=0$$
$$0=0 \ \text{True}$$

$$x^2-5x-66=0$$
$$(11)^2-5(11)-66=0 \quad [\text{Put } x=11]$$
$$121-55-66=0 \quad [\text{Simplify}]$$
$$121-66=0$$
$$0=0 \ \text{True}$$

Therefore, the solution set of given equation is $\boxed{\{-6,11\}}$

Answer 39PT.

Consider that a rectangular lawn is 24 feet wide by 32 feet long.



Also a side walk will be built along the inside edges of all four sides.

Area of remaining lawn = 425 sq. ft

The objective is to find the wide of the walk.

Let of wide of walk = x ft

Then length of remaining lawn = length of lawn – [2×wide of walk]

$$=32-2x$$

Width of remaining lawn = width of lawn – 2×wide of walk

$$=24-2x$$

Area of Remaining lawn = 425 sq. ft

Length × width = 425 sq. ft

$$\begin{aligned}(32-2x)(24-2x)&=425\\ 32\cdot 24+32(-2x)+24(-2x)+(-2x)(-2x)&=425 &&[\text{FOIL method}]\\ 768-64x-48x+4x^2&=425 &&[\text{Simplify}]\\ 4x^2-112x+768&=725 &&[\text{Combine like terms}]\end{aligned}$$

$$4x^2-112x+768-425=425-425 \quad [\text{Subtract 425 on both sides}]$$

$$4x^2-112x+343=0$$

$$4x^2-98x-14x+343=0$$

$$2x(2x-49)-7(2x-49)=0 \quad [\text{Factor the GCF}]$$

$$(2x-7)(2x-49)=0 \quad \begin{bmatrix}\text{By distributive}\\ (b+c)a=ba+ca\end{bmatrix}$$

The zero product property is if $ab=0$ then $a=0$ or $b=0$ or both

$$2x-7=0 \text{ or } 2x-49=0$$

$$2x-7=0$$

$$2x-7+7=0+7 \qquad [\text{Add 7 on both sides}]$$

$$2x=7$$

$$\frac{2x}{2}=\frac{7}{2} \qquad [\text{Divide with 2 on both sides}]$$

$$x=\frac{7}{2}$$

And

$$2x-49=0$$

$$2x-49+49=0+49 \qquad [\text{Add 49 on both sides}]$$

$$2x=49$$

$$\frac{2x}{2}=\frac{49}{2} \qquad [\text{Divide with 2 on both sides}]$$

$$x=\frac{49}{2}$$

$$x=\frac{7}{2} \text{ or } x=\frac{49}{2}$$

If $x=\frac{49}{2}$, then width of side walk is $x=24.5$ is more than width of lawn 24

$x=\frac{49}{2}$ is not possible

Thus $x=\frac{7}{2}$

Therefore, wide of walk is $\boxed{\frac{7}{2}\text{ feet}}$

Answer 40E.

Consider the equation $m^2-m-12=0$

The objective is to find the solution set of given equation

For this first factor m^2-m-12

Compare m^2-m-12 with m^2+bm+c

Here $b=-1, c=-12$

$$\begin{aligned} m^2+bm+c &= (m+x)(m+y) \\ &= m^2+(x+y)m+xy \end{aligned}$$

Here $x+y=-1, xy=-12$

Now find two numbers x, y such that $x+y=-1$ negative and $xy=-12$ negative. Thus either *x* or *y* must be negative.

List all the factors of $xy=-12$ in those chose a pair whose sum is -1

Factors of -12	Sum of factors
$-1\cdot 12$	11
$-1\cdot 12$	-11
$-2\cdot 6$	4
$2\cdot -6$	-4
$-3\cdot 4$	✓ 1
$3\cdot -4$	-1

The correct factor are $3,-4$

$$m^2-m-12=(m+x)(m+y)$$
$$=(m+3)(m-4) \qquad [x=3, y=-4]$$

Thus $m^2-m-12=0$

$$(m+3)(m-4)=0$$

By zero product property of $ab=0$ then $a=0$ or $b=0$ or both

$m+3=0$ or $m-4=0$

Now solve each equation completely

$$m+3=0$$

$$m+3-3=0-3 \quad \text{[Subtract 3 on both sides]}$$

$$m=-3$$

$$m-4=0$$

$$m-4+4=0+4 \quad \text{[Add 4 on both sides]}$$

$$m=4$$

The solution set is $\{-3,4\}$

Check:

To check the proposed solution set substitute each solution in the given equation

Given equation is $m^2-m-12=0$

$$(-3)^2-(-3)-12=0 \quad [\text{put } m=-3]$$

$$9+3-12=0 \quad \text{[Simplify]}$$

$$12-12=0$$

$$0=0 \ \text{ True}$$

$$m^2-m-12=0$$

$$4^2-4-12=0 \quad [\text{put } m=4]$$

$$16-4-12=0 \quad \text{[Simplify]}$$

$$16-16=0$$

$$0=0 \ \text{ True}$$

Therefore, the solution set of given equation is $\boxed{\{-3,4\}}$

Answer 41E.

Consider the polynomial $2a^2-9a+3$

The objective is to factor the given polynomial.

Compare $2a^2-9a+3$ with dx^2+bx+c

Here $d=2, b=-9, c=3$

$$\begin{aligned}2a^2-9a+3 &= 2a^2+ma+na+3\\ &= 2a^2+(m+n)a+3\end{aligned}$$

Now find two numbers *m*, *n* such that $m+n=-9, mn=2\cdot 3=6$

Since $m+n$ negative and mn is positive then both m,n are negative.

Now list all the negative factors of $mn=6$ in those chose a pair whose sum is -9

Factors of 6	Sum of factors
$-1\cdot -6$	-7
$-2\cdot -3$	-5

There exists no pair of factors whose sum is -9

Thus $2a^2-9a+3$ is not factored.

Therefore, $2a^2-9a+3$ is prime

Answer 42E.

Consider the polynomial $2m^2+13m-24$

The objective is to factor the given polynomial.

Compare $2m^2+13m-24$ with am^2+bm+c

Here $a=2, b=13, c=-24$

$$\begin{aligned}2m^2+13m-24 &= 2m^2+pm+qm-24\\ &= 2m^2+(p+q)m-24\end{aligned}$$

Now find two numbers *p*, *q* such that $p+q=13, pq=2\cdot -24=-48$

Since $p+q$ is positive and $pq=-48$ is negative then both *p* or *q* must be negative but not both.

List all the factors of -48 in those choose a pair whose sum is 13

Factors of -48	Sum of factors
$-1\cdot 48$	47
$1\cdot -48$	-47
$-2\cdot 24$	22
$2\cdot -24$	✓ -22
$-3\cdot 16$	13
$3\cdot -16$	-13
$-4\cdot 12$	8
$4\cdot -12$	-8
$-6\cdot 8$	2
$6\cdot -8$	-2

The correct factors are $-3,16$

$$\begin{aligned}2m^2+13m-24 &= 2m^2+pm+qm-24 \\ &= 2m^2-3m+16m-24 && [p=-3, q=16] \\ &= m(2m-3)+8(2m-3) && [\text{Factor GCF}] \\ &= (m+8)(2m-3)\end{aligned}$$

Check:

To check the proposed factorization multiply the factors using FOIL method.

$$\begin{aligned}(m+8)(2m-3) &= m\cdot 2m + m(-3) + 8(2m) + 8(-3) \quad &[\text{FOIL method}]\\ &= 2m^2 - 3m + 16m - 24 \quad &[\text{Simplify}]\\ &= 2m^2 + 13m - 24 \ \ \text{True}\end{aligned}$$

Therefore, the factorization of given trinomial is $\boxed{(m+8)(2m-3)}$

Answer 43E.

Consider the polynomial $25r^2+20r+4$

The objective is to factor the given polynomial.

Compare $25r^2+20r+4$ with ar^2+br+c

Here $a=25, b=20, c=4$

$$\begin{aligned}25r^2+20r+4 &= 25r^2+mr+nr+4\\ &= 25r^2+(m+n)r+4\end{aligned}$$

Now find two numbers *m*, *n* such that $m+n=20$ and $mn=ac=25\cdot 4=100$

Since $m+n$ and mn are positive then both m and n are positive

Now list all the positive factors of 100 in those choose a pair whose sum is 20

Factors of 100	Sum of factors
$1\cdot 100$	101
$2\cdot 50$	52
$4\cdot 25$	29
$5\cdot 20$	✓ 25
$10\cdot 10$	20

The correct factors are $10, 10$

$$\begin{aligned}25r^2+20r+4 &= 25r^2+mr+nr+4\\ &= 25r^2+10r+10r+4 && [m=10, n=10]\\ &= 5r(5r+2)+2(5r+2) && [\text{Factor GCF}]\\ &= (5r+2)(5r+2)\end{aligned}$$

Thus the factorization of $25r^2+20r+4$ is $(5r+2)^2$

Check:

To check the proposed factorization multiply the factors using FOIL method.

$$(5r+2)(5r+2) = 5r\cdot 5r + 2\cdot 5r + 2\cdot 5r + 2\cdot 2 \qquad [\text{FOIL method}]$$
$$= 25r^2 + 10r + 10r + 4 \qquad [\text{Simplify}]$$
$$= 25r^2 + 20r + 4 \quad \text{True}$$

Therefore, the factorization of given trinomial is $\boxed{(5r+2)(5r+2)}$

Answer 44E.

Consider the polynomial $6z^2 + 7z + 3$

The objective is to factor the given polynomial.

Compare $6z^2 + 7z + 3$ with $az^2 + bz + c$

Here $a = 6, b = 7, c = 3$

Now

$$6z^2 + 7z + 3 = 6z^2 + mz + nz + 3$$
$$= 6z^2 + (mn)z + 3$$

Now find two numbers *m*, *n* such that $m + n = 7, mn = 6\cdot 3 = 18$

Since $m + n$ and mn are positive then both m, n are positive.

Now list all the factors of 18 in those choose a pair whose sum is 7

Factors of 18	Sum of factors
$1\cdot 18$	19
$2\cdot 9$	11
$3\cdot 6$	9

There exists no pair of factors whose sum is 17

Thus $6z^2 + 7z + 3$ is not factored.

Therefore, $6z^2 + 7z + 3$ is $\boxed{\text{prime}}$

Answer 45E.

Consider the polynomial $12b^2 + 17b + 6$

The objective is to factor the given polynomial.

Compare $12b^2 + 17b + 6$ with $ab^2 + db + c$

Here $a = 12, d = 17, c = 6$

Now

$$\begin{aligned} 12b^2 + 17b + 6 &= 12b^2 + mb + nb + 6 \\ &= 12b^2 + (m+n)b + 6 \end{aligned}$$

Now find two numbers *m*, *n* such that $m + n = 17$ positive and $mn = 12 \cdot 6 = 72$ positive

Since $m + n, mn$ are positive then roots m, n are must be positive

List all the factors of 72 in those choose a pair whose sum is 17

List of factors of 72	List of factors
$1 \cdot 72$	73
$2 \cdot 36$	38
$3 \cdot 24$	27
$4 \cdot 18$	22
$6 \cdot 12$	18
$8 \cdot 9$	17

The correct factors are 8,9

$$\begin{aligned} 12b^2 + 17b + 6 &= 12b^2 + mb + nb + 6 \\ &= 12b^2 + 8b + 9b + 6 && [m = 8, b = 9] \\ &= 4b(3b+2) + 3(3b+2) && [\text{Factor GCF}] \\ &= (4b+3)(3b+2) && [\text{By distributive } (b+c)a = ba + ca] \end{aligned}$$

Check:

To check the proposed factorization, multiply the factors using FOIL method.

$$\begin{aligned} (4b+3)(3b+2) &= 4b \cdot 3b + 4b \cdot 2 + 3 \cdot 3b + 3 \cdot 2 && [\text{By FOIL method}] \\ &= 12b^2 + 8b + 9b + 6 && [\text{Simplify}] \\ &= 12b^2 + 17b + 6 \quad \text{True} \end{aligned}$$

Therefore, the factorization of given trinomial is $\boxed{(4b+3)(3b+2)}$

Answer 46E.

Consider the polynomial $3n^2-6n-45$

The objective is to factor the given polynomial.

$$\begin{aligned}3n^2-6n-45&=3\cdot n^2-3\cdot 2\cdot n-3\cdot 15\\&=3\left(n^2-2n-15\right)\qquad [\text{Factor GCF } 3]\end{aligned}$$

Now factor $n^2-2n-15$

Compare $n^2-2n-15$ with ab^2+db+c

Here $a=-2, c=-15$

Now

$$\begin{aligned}n^2-3n-15&=(n+p)(n+q)\\&=n^2+(p+q)n+pq\end{aligned}$$

Now find two numbers p, q such that $p+q=2, pq=-15$

Since $p+q, pq$ are negative then one of p or q must be negative but not both.

List all the factors of -15 in those choose a pair whose sum is -2

List of factors of -15	List of factors
$-1\cdot 15$	14
$1\cdot -15$	-14
$-3\cdot 5$	✓ 2
$3\cdot -5$	-2

The correct factors are 3, -5

$$\begin{aligned}n^2-2n-15&=(n+p)(n+q)\\&=(n+3)(n-5)\qquad [p=3, q=-5]\\3n^2-6n-45&=3\left[n^2-2n-15\right]\\&=3(n+3)(n-5)\end{aligned}$$

Check:

To check the proposed factorization, multiply the factors using FOIL method.

$$\begin{aligned}3(n+3)(n-5)&=3\left[n\cdot n+(-5)\cdot n+3\cdot n+3\cdot(-5)\right]\qquad [\text{By FOIL method}]\\&=3\left[n^2-5n+3n-15\right]\qquad [\text{Simplify}]\\&=3\left[n^2-2n-15\right]\\&=3n^2-6n-45\ \ \text{True}\end{aligned}$$

Therefore, the factorization of given trinomial is $\boxed{3(n+3)(n-5)}$

Answer 47E.

The equation is $2r^2-3r-20=0$

The objective is to find the solution set of given equation.

For this first factor $2r^2-3r-20$

Now compare $2r^2-3r-20$ with ax^2+bx+c

Here $a=2,b=-3,c=-20$

$$\begin{aligned}2r^2-3r-20&=2r^2+mr+nr-20\\&=2r^2+(m+n)r-20\end{aligned}$$

Now find two numbers m,n such that $m+n=-3$ and $m\cdot n=3\cdot-20=-40$

Since $m+n,mn$ are negative then either *m* or *n* are negative but not both

Now list all the factors of -40 in those choose a pair whose sum is -3

Factor of -40	Sum of factors
$-1\cdot 40$	39
$1\cdot -40$	-39
$-2\cdot 20$	18
$2\cdot -20$	-18
$-4\cdot 10$	6
$4\cdot -10$	-6
$-5\cdot 8$	✓ 3
$5\cdot -8$	-3

The correct factors are $5,-8$

$$2r^2-3r-20=2r^2+mr+nr-20$$

$$=2r^2-8r+5r-20 \quad [m=-8, n=5]$$

$$=2r(r-4)+5(r-4) \quad [\text{Factors GCF}]$$

$$=(2r+5)(r-4) \quad \left[\begin{array}{l}\text{By distributive}\\ (b+c)a=ba+ca\end{array}\right]$$

$$2r^2-3r-20=0$$

$$(2r+5)(r-4)=0$$

The zero product property is if $ab=0$ then $a=0 \text{ or } b=0$ or both

$$2r+5=0 \text{ or } r-4=0$$

Now solve each equation completely

$$2r+5=0$$

$$2r+5-5=0-5 \quad [\text{Subtract 5 on both sides}]$$

$$2r=-5$$

$$\frac{2r}{2}=\frac{-5}{2} \quad [\text{Divide with 2 on both sides}]$$

$$r=\frac{-5}{2}$$

$$r-4=0$$

$$r-4+4=0+4 \quad [\text{Add 4 on both sides}]$$

$$r=4$$

The solution set of given equation is $\left\{\frac{-5}{2},4\right\}$

Check:

To check the proposed solution set substitute each solution in the given equation

Given equation is

$$2r^2 - 3r - 20 = 0$$

$$2\left(\frac{-5}{2}\right)^2 - 3\left(\frac{-5}{2}\right) - 20 = 0 \qquad \left[\text{Put } r = \frac{-5}{2}\right]$$

$$2 \cdot \frac{25}{4} + \frac{15}{2} - 20 = 0$$

$$\frac{25}{2} + \frac{15}{2} - 20 = 0 \qquad [\text{Simplify}]$$

$$\frac{25 + 15 - 40}{2} = 0$$

$$\frac{40 - 40}{2} = 0$$

$$0 = 0 \;\; \text{True}$$

$$2r^2 - 3r - 20 = 0$$

$$2(4)^2 - 3(4) - 20 = 0 \qquad [\text{Put } r = 4]$$

$$32 - 12 - 20 = 0 \qquad [\text{Simplify}]$$

$$32 - 32 = 0$$

$$0 = 0 \text{ True}$$

The solution set of given equation is $\boxed{\left\{\frac{-5}{2}, 4\right\}}$

Answer 48E.

The equation is $3a^2 - 13a + 14 = 0$

The objective is to find the solution set of given equation.

For this first factor $3a^2 - 13a + 14$

Now compare $3a^2 - 13a + 14$ with $dx^2 + bx + c$

Here $d = 3, b = -13, c = 14$

$$3a^2 - 13a + 14 = 3a^2 + ma + na + 14$$
$$= 3a^2 + (m+n)a + 14$$

Now find two numbers m, n such that $m + n = -13$ and $m \cdot n = 3 \cdot 14 = 42$

Since $m + n$ is negative and $m \cdot n$ is positive then both *m* and *n* are negative

Now list all the factors of 42 in those choose a pair whose sum is -13

Factor of 42	Sum of factors
$-1 \cdot -42$	-43
$-2 \cdot -21$	-23
$-3 \cdot -14$	✓ -17
$-7 \cdot -6$	-13

The correct factors are $-6, -7$

$$3a^2-13a+14=3a^2+ma+na+14$$
$$=3a^2-6a-7a+14 \qquad [m=-6, n=-7]$$
$$=3a(a-2)-7(a-2)$$
$$=(3a-7)(a-2) \qquad [\text{By distributive } (b+c)a=ba+ca]$$

$$3a^2-13a+14=0$$
$$(3a-7)(a-2)=0$$

The zero product property is if $ab=0$ then $a=0$ or $b=0$ or both

$$3a-7=0 \text{ or } a-2=0$$

Now solve each equation completely

$$3a-7=0$$
$$3a-7+7=0+7 \qquad [\text{Add 7 on both sides}]$$
$$3a=7$$
$$\frac{3a}{3}=\frac{7}{3} \qquad [\text{Divide with 3 on both sides}]$$
$$a=\frac{7}{3}$$

$$a-2=0$$
$$a-2+2=0+2 \qquad [\text{Add 2 on both sides}]$$
$$a=2$$

The solution set of given equation is $\left\{\frac{7}{3},2\right\}$

Check:

To check the proposed solution set substitute each solution in the given equation

Given equation is $3a^2-13a+14=0$

$$3\left(\frac{7}{3}\right)^2-13\left(\frac{7}{3}\right)+14=0 \qquad \left[\text{Put } a=\frac{7}{3}\right]$$
$$3\cdot\frac{49}{9}-\frac{91}{3}+14=0 \qquad [\text{Simplify}]$$
$$\frac{49}{3}-\frac{91}{3}+14=0$$
$$\frac{49-91+42}{3}=0 \qquad [\text{Simplify}]$$
$$\frac{91-91}{3}=0$$
$$0=0 \text{ True}$$

$$3a^2 - 13a + 14 = 0$$
$$3(2)^2 - 13(2) + 14 = 0 \; [\text{Put } a = 2]$$
$$12 - 26 + 14 = 0$$
$$26 - 26 = 0$$
$$0 = 0 \text{ True}$$

The solution set of given equation is $\boxed{\left\{\frac{7}{3}, 12\right\}}$

Answer 49E.

Consider the polynomial $40x^2 + 2x = 24$

The objective is to find the solution set of given equation

$$40x^2 + 2x = 24$$
$$40x^2 + 2x - 24 = 24 - 24 \qquad [\text{Subtract 24 on both sides}]$$
$$40x^2 + 2x - 24 = 0$$
$$2 \cdot 20x^2 + 2 \cdot x - 2 \cdot 12 = 0$$
$$2\left[20x^2 + x - 12\right] = 0 \qquad [\text{Factor the GCF 2}]$$

Now factor the polynomial $40x^2 + 2x - 24$

Compare $40x^2 + 2x - 24$ with $ax^2 + bx + c$

Here $a = 20, b = 1, c = -12$

$$\begin{aligned} 20x^2 + x - 12 &= 20x^2 + mx + nx - 12 \\ &= 20x^2 + (m+n)x - 12 \end{aligned}$$

Now find two numbers m, n such that $m + n = 1$ and $mn = 20 \cdot (-12) = -240$

Since $m + n$ is positive and mn is negative then either *m* or *n* must be negative but not both

List all the factors of $mn = -240$ in those chose a pair whose sum is 1

Factors of -240	Sum of factors
$-1 \cdot 240$	239
$1 \cdot -240$	-239
$-2 \cdot 120$	118
$2 \cdot -120$	-118
$-3 \cdot 80$	77
$3 \cdot -80$	-70
$-4 \cdot 60$	56
$4 \cdot -60$	-56
$-5 \cdot 48$	43
$5 \cdot -48$	-43
$-6 \cdot 40$	34
$6 \cdot -40$	-34
$-8 \cdot 30$	22
$8 \cdot -30$	-22

$-10 \cdot 24$	14
$10 \cdot -24$	-14
$-12 \cdot 20$	8
$12 \cdot -20$	-8
$-15 \cdot 16$	✓ 1
$15 \cdot -16$	-1

The correct factor are $-15,16$

$$\begin{aligned} 20x^2+x-12 &= 20x^2+mx+nx-12 \\ &= 20x^2-15x+16x-12 && [m=-15, n=16] \\ &= 5x(4x-3)+4(4x-3) && [\text{Factor GCF}] \\ &= (5x+4)(4x-3) && [\text{By distributive } (b+c)a = ba+ca] \end{aligned}$$

$$40x^2+2x=24$$

$$2(20x^2+x-12)=0$$

$$2[5x+4](4x-3)=0$$

By zero product property of $ab=0$ then $a=0 \text{ or } b=0$ or both

$$5x+4=0 \text{ or } 4x-3=0$$

$$5x+4=0$$

$$5x+4-4=0-4 \qquad [\text{Subtract 4 on both sides}]$$

$$5x=-4$$

$$\frac{5x}{5}=\frac{-4}{5} \qquad [\text{Divide with 5 on both sides}]$$

$$x=\frac{-4}{5}$$

$$4x-3=0$$

$$4x-3+3=0+3 \qquad [\text{Add 3 on both sides}]$$

$$4x = 3$$

$$\frac{4x}{4} = \frac{3}{4} \qquad \text{[Divide with 4 on both sides]}$$

$$x = \frac{3}{4}$$

The solution set is $\left\{\frac{-4}{5}, \frac{3}{4}\right\}$

Check:

To check the proposed solution substitute each solution in the given equation

Given equation is $40x^2 + 2x = 24$

$$40\left(\frac{-4}{5}\right)^2 + 2\left(\frac{-4}{5}\right) = 24 \qquad \left[\text{put } x = \frac{-4}{5}\right]$$

$$40 \cdot \frac{16}{5 \cdot 5} - \frac{8}{5} = 24$$

$$8 \cdot \frac{16}{5} - \frac{8}{5} = 24$$

$$\frac{128 - 8}{5} = 24$$

$$\frac{120}{5} = 24$$

$$24 = 24 \text{ True}$$

The equation is $40x^2 + 2x = 24$

$$40\left(\frac{3}{4}\right)^2 + 2\left(\frac{3}{4}\right) = 24 \qquad \left[\text{put } x = \frac{3}{4}\right]$$

$$40 \cdot \frac{9}{4 \cdot 4} + \frac{6}{4} = 24$$

$$10 \cdot \frac{9}{4} + \frac{6}{4} = 24$$

$$\frac{90 + 6}{4} = 24$$

$$\frac{96}{4} = 24$$

$$24 = 24 \text{ True}$$

Therefore, the solution set of given equation is $\boxed{\left\{\frac{-4}{5}, \frac{3}{4}\right\}}$

Answer 50E.

Consider the polynomial $2y^3 - 128y$

The objective is to factor the given polynomial

$$2y^3 - 128y = 2 \cdot y \cdot y \cdot y - 2 \cdot 64 \cdot y$$

$$= 2y(y \cdot y - 64)$$ (Factor $GCF(2y^3, 128y) = 2y$)

$$= 2y(y^2 - 64)$$

The difference of squares property is

$$a^2 - b^2 = (a+b)(a-b)$$

$$2y^3 - 128y = 2y(y^2 - 64)$$

$$= 2y(y^2 - 8^2) \quad (8^2 = 64)$$

$$= 2y(y+8)(y-8)$$

Therefore, the factorization of given polynomial is $\boxed{2y(y+8)(y-8)}$.

Answer 51E.

Consider the polynomial $9b^2 - 20$

The objective is to factor the given polynomial.

$$9b^2 = 3 \cdot 3 \cdot b \cdot b \quad (9 = 3 \cdot 3, b^2 = b \cdot b)$$

$$20 = 2 \cdot 10 \quad (20 = 2 \cdot 10)$$

$$= 2 \cdot 2 \cdot 5$$

Since $9b^2, 20$ have no common factors.

Therefore, $9b^2 - 20$ has no factorization.

Therefore, $9b^2 - 20$ is a prime.

Answer 52E.

Consider the polynomial $\frac{1}{4}n^2-\frac{9}{16}r^2$

The objective is to factor the given polynomial.

$\frac{1}{4}n^2-\frac{9}{16}r^2=\frac{1}{4}\cdot n^2-\frac{1}{4}\cdot\frac{9}{4}r^2$ (Because $\frac{9}{16}=\frac{9}{4}+\frac{1}{4}$)

$=\frac{1}{4}\left[n^2-\frac{9}{4}r^2\right]$ (Factor $GCF=\frac{1}{4}$)

$=\frac{1}{4}\left[n^2-\frac{3\cdot3}{2\cdot2}r^2\right]$ (Because $9=3\cdot3, 4=2\cdot2$)

$=\frac{1}{4}\left[n^2-\frac{3^2}{2}r^2\right]$

$=\frac{1}{4}\left[n^2-\left(\frac{3}{2}\right)^2\cdot r^2\right]$ (Because $\frac{a^m}{b^m}=\left(\frac{a}{b}\right)^m$)

$=\frac{1}{4}\left[n^2-\left(\frac{3}{2}r\right)^2\right]$ (Because $a^m\cdot b^n=(ab)^m$)

$=\frac{1}{4}\left[n+\frac{3}{2}r\right]\left(n-\frac{3}{2}r\right)$ [Since $a^2-b^2=(a+b)(a-b)$]

Therefore the factorization of given polynomial is $\boxed{\frac{1}{4}\left(n+\frac{3}{2}r\right)\left(n-\frac{3}{2}r\right)}$.

Answer 53E.

Consider the equation

$b^2 - 16 = 0$

The objective is to find the solution set of given equation.

$b^2 - 16 = 0$

$b^2 - 4 \cdot 4 = 0$ (Because $4^2 = 16$)

$b^2 - 4^2 = 0$

The difference of squares property is

$a^2 - b^2 = (a+b)(a-b)$

$b^2 - 4^2 = 0$

$(b+4)(b-4) = 0$ ($a^2 - b^2 = (a+b)(a-b)$)

The zero product property is of

$ab = 0$ then

$a = 0$

Or, $b = 0$ or both

$b + 4 = 0$

Or, $b - 4 = 0$

Now solve each equation completely.

$b + 4 = 0$

$b + 4 - 4 = 0 - 4$ (Subtract 4 on each side)

$b = -4$

$b - 4 + 4 = 0 + 4$ (Add 4 on each side)

$b = 4$

The solution set is $\{-4, 4\}$.

Check: To check the proposed solution set, substitute each solution in the given equation.

Given equation is

$b^2 - 16 = 0$

$4^2 - 16 = 0$ (Put $b = 4$)

$16 - 16 = 0$

$0 = 0$ True

For $b = -4$,

$b^2 - 16 = 0$

$(-4)^2 - 16 = 0$ (Put $b = -4$)

$16 - 16 = 0$

$0 = 0$ True

The solution set of given equation is $\boxed{\{-4, 4\}}$.

Answer 54E.

Consider the equation

$25 - 9y^2 = 0$

The objective is to find the solution set of given equation.

$25 - 9y^2 = 0$

$5 \cdot 5 - 9y^2 = 0 \quad (5 \cdot 5 = 25)$

$5^2 - 3 \cdot 3y^2 = 0 \quad (3 \cdot 3 = 9)$

$5^2 - 3^3 y^2 = 0$

$5^2 - (3y)^2 = 0 \quad \left(a^m b^m = (ab)^m\right)$

The difference of squares property is

$a^2 - b^2 = (a+b)(a-b)$

$5^2 - (3y)^2 = 0$

$\Rightarrow \quad (5+3y)(5-3y)$

$= 0$

The zero product property is if

$ab = 0$ then

$a = 0$ or

$b = 0$ or both.

$5 + 3y = 0$

Or $5-3y=0$

Now solve each equation completely.

$5+3y=0$

$5+3y-5=0-5$ (Subtract 5 on both sides)

$3y=-5$

$\frac{3y}{3}=\frac{-5}{3}$ (Divide with 3 on both sides)

$y=\frac{-5}{3}$

$5-3y=0$

$3y+5-3y=0+3y$ (Add $3y$ on both sides)

$5=3y$

$\frac{5}{3}=\frac{3y}{3}$ (Divide with 3 on both sides)

$\frac{5}{3}=y$

The solution set is $\left\{\frac{-5}{3},\frac{5}{3}\right\}$

Check: For $y=\frac{-5}{3}$,

$25-9y^2=0$

$25-9\left(\frac{-5}{3}\right)^2=0$ (Put $y=\frac{-5}{3}$)

$25-9\cdot\frac{25}{9}=0$ (Simplify)

$25-25=0$

$0=0$ True

For $y=\frac{5}{3}$,

$25-9y^2=0$

$25-9\left(\frac{5}{3}\right)^2=0$ (Put $y=\frac{5}{3}$)

$25-9\cdot\frac{25}{9}=0$

$25-25=0$

$\Rightarrow \quad 0=0$ True

The solution set of given equation is $\boxed{\left\{\frac{-5}{3},\frac{5}{3}\right\}}$.

Answer 55E.

Consider the equation

$16a^2 - 81 = 0$

The objective is to find the solution set of given equation.

$16a^2 - 81 = 0$

$4 \cdot 4a^2 - 81 = 0$ (Since $4 \cdot 4 = 16$)

$4^2 a^2 - 9 \cdot 9 = 0$ $(9 \cdot 9 = 81)$

$(4a)^2 - 9 \cdot 9 = 0$ (Because $a^m \cdot b^m = (ab)^m$)

$(4a)^2 - 9^2 = 0$ $(9 \cdot 9 = 9^2)$

The difference of squares property is

$a^2 - b^2 = (a+b)(a-b)$

$(4a)^2 - 9^2 = 0$

$(4a+9)(4a-9) = 0$ (By difference of squares)

The zero product property is if

$ab = 0$

Then $a = 0$

Or, $b = 0$ or both.

$4a + 9 = 0$

Or, $4a - 9 = 0$ (By zero product property)

Now solve each equation separately.

$4a + 9 = 0$

$4a + 9 - 9 = 0 - 9$ (Subtract 9 on both sides)

$4a = -9$

$\frac{4a}{4} = \frac{-9}{4}$ (Divide with 4 on both sides)

$a = \frac{-9}{4}$

$4a - 9 = 0$

$4a - 9 + 9 = 0 + 9$ (Add 9 on both sides)

$4a = 9$

$\frac{4a}{4} = \frac{9}{4}$ (Divide with 4 on both sides)

$a = \frac{9}{4}$

The solution set is $\left\{\frac{-9}{4}, \frac{9}{4}\right\}$.

Check: To check the proposed solution set substitute each solution in the given equation.

Given equation is

$16a^2 - 81 = 0$

$16\left(\frac{-9}{4}\right)^2 - 81 = 0$ (Put $a = \frac{-9}{4}$)

$16 \cdot \frac{81}{16} - 81 = 0$ (Simplify)

$81 - 81 = 0$

$0 = 0$ True

For $a = \frac{9}{4}$,

$16a^2 - 81 = 0$

$16\left(\frac{9}{4}\right)^2 - 81 = 0$ (Put $a = \frac{9}{4}$)

$16 \cdot \frac{81}{16} - 81 = 0$ (Simplify)

$81 - 81 = 0$

$0 = 0$ True

Thus, the solution set is $\boxed{\left\{\frac{-9}{4}, \frac{9}{4}\right\}}$.

Answer 56E.

Consider the polynomial $a^2+18a+81$.

The objective is to factor the given polynomial.

For this use perfect square property

$a^2+18a+81=a^2+2\cdot9\cdot a+81$ (Because $18=2\cdot9$)

$=a^2+2\cdot9\cdot a+9\cdot9$ (Since $1=9\cdot9$)

$=a^2+2\cdot9\cdot a+9^2$ ($9\cdot9=9^2$)

The perfect squares property is

$(a+b)^2=a^2+2ab+b^2$

$=(a+9)^2$ (Here $a=a,b=9$)

Thus,

$a^2+18a+81=(a+9)^2$

The factorization of given polynomial is $\boxed{(a+9)^2}$.

Answer 57E.

Consider the polynomial $9k^2-12k+4$

The objective is to factor the given polynomial

$9k^2-12k+4=3\cdot3k^2-12k+4$ (Since $9=3\cdot3$)

$=3^2k^2-12k+4$ $(3\cdot3=3^2)$

$=(3k)^2-2\cdot6k+4$ $(12=2\cdot6)$

$=(3k)^2-2\cdot2\cdot3k+4$ $(6=2\cdot3)$

$=(3k)^2-2\cdot3k\cdot2+2^2$ $(4=2^2)$

The perfect squares property is

$(a-b)^2=a^2-2ab+b^2$

$=(3k-2)^2$ (Because here $a=3k,b=2$)

Thus

$9k^2-12k+4=(3k-2)^2$

Therefore, the factorization of given polynomial is $\boxed{(3k-2)^2}$.

Answer 58E.

Consider the equation $4-28r+49r^2$

The objective is to factor the given polynomial.

$4-28r+49r^2 = 2\cdot 2-28r+49r^2$ (Because $2\cdot 2=4$)

$=2^2-2\cdot 14r+7\cdot 7r^2$ $(2\cdot 14=28)$

$=2^2-2\cdot 2\cdot 7r+7^2r^2$ (Since $14=2\cdot 7, 7\cdot 7=7^2$)

$=2^2-2\cdot 7\cdot(2r)+(7r)^2$

$\left(a^m\cdot a^m=(ab)^m\right)$

The prefect square property is

$\left((a-b)^2=a^2-2ab+b^2\right)$

$=(2-7r)^2$ (Here $a=2, b=7r$)

Thus

$4-28r+49r^2=(2-7r)^2$

Therefore, the factorization of given polynomial is $\boxed{(2-7r)^2}$.

Answer 59E.

Consider the polynomial $32n^2-80n+50$

The objective is to factor the given polynomial.

$32n^2-80n+50 = 2\cdot 16n^2-2\cdot 40n+2\cdot 25$

$=2\left(16n^2-40n+25\right)$ (Factor $GCF\cdot 2$)

$=2\left(4\cdot 4n^2-40n+25\right)$ $(4\cdot 4=16)$

$=2\left(4^2n^2-2\cdot 20n+25\right)$ $\left(4\cdot 4=4^2, 40=2\cdot 20\right)$

$=2\left[(4n)^2-2\cdot 4\cdot 5n+5\cdot 5\right]$ $(20=4\cdot 5, 25=5\cdot 5)$

$=2\left[(4n)^2-2\cdot(4\cdot n)^5+5^2\right]$ $\left(5^2=5\cdot 5\right)$

The perfect square property is

$(a-b)^2=a^2-2ab+b^2$

$=2[4n-5]^2$ (Here $a=4n, b=5$)

Thus

$32n^2-80n+50=2(4n-5)^2$

Therefore, the factorization of given polynomial is $\boxed{2(4n-5)^2}$.

Answer 60E.

Consider the equation

$$6b^2 - 24b^2 + 24b = 0$$

The objective is to find the solution set of given equation.

$$6b^2 - 24b^2 + 24b = 0$$

$$6 \cdot b \cdot b^2 - 6 \cdot 4 \cdot b \cdot b + 6 \cdot 4 \cdot b = 0 \quad \left(b^3 = b \cdot b^2, 24 = 6 \cdot 4\right)$$

$6b\left[b^2 - 4b + 4\right] = 0$ (Factor the $GCF\ 6b$)

$$6b\left[b^2 - 2 \cdot 2b + 2 \cdot 2\right] = 0 \quad (2 \cdot 2 = 4)$$

$$6b\left(b^2 - 2 \cdot 2 \cdot b + 2^2\right) = 0 \quad \left(2 \cdot 2 = 2^2\right)$$

The perfect square property is

$$(a-b)^2 = a^2 - 2ab + b^2$$

$6b(b-2)^2 = 0$ (Here $a = b, b = 2$)

The zero product property is if

$ab = 0$ then

$a = 0$

Or, $b = 0$ or both.

$$6b = 0$$

Or, $(b-2)^2 = 0$

Now solve each equation separately.

$$6b = 0$$

$\frac{6b}{6} = \frac{0}{6}$ (Divide with 6 on both sides)

$$b = 0$$

Now $(b-2)^2 = 0$

$$b - 2 = 0$$

$b - 2 + 2 = 0 + 2$ (Add 2 on both sides)

$$b = 2$$

The solution set is $\{0, 2\}$.

Check: To check the proposed solution set substitute each solution in the given equation.

Given equation is

$6b^3-24b^2+24b=0$

$60^3-24(0)^2+24(0)=0$ (Put $b=0$)

$0-0+0=0$

$0=0$ True

For $b=2$,

$6b^3-24b^2+24b=0$

$6(2^3)-24(2^2)+24(2)=0$ (Put $b=2$)

$6\cdot 8-24(4)+48=0$ (Simplify)

$48-96+48=0$

$96-96=0$ (Simplify)

$0=0$ True

The solution set of given equation is $\boxed{\{0,2\}}$.

Answer 61E.

Consider the equation

$49m^2-126m+81=0$

The objective is to find the solution set of given equation.

$49m^2-126m+81=0$

$7\cdot 7m^2-126m+81=0$ $(7\cdot 7=49)$

$7^2m^2-2\cdot 63m+81=0$ $(2\cdot 63=126)$

$(7m)^2-2\cdot 7\cdot 9m+81=0$ $(7\cdot 9=63)$

$(7m)^2-2\cdot 7m\cdot 9+9\cdot 9=0$ (Since $9\cdot 9=81$)

$(7m)^2-2\cdot 7m\cdot 9+9^2=0$

The perfect square property is

$(a-b)^2=a^2-2ab+b^2$

$(7m-9)^2=0$

$7m-9+9=0+9$ (Add 9 on both sides)

$7m=9$

$\frac{7m}{7}=\frac{9}{7}$ (Divide with 7 on both sides)

$m=\frac{9}{7}$

The solution set is $\left\{\frac{9}{7}\right\}$.

Check: To check the proposed solution substitutes it in the given equation.

Given equation is

$$49m^2-126m+81=0$$

$$49\left(\frac{9}{7}\right)^2-126\left(\frac{9}{7}\right)+81=0$$ (Put $m=\frac{9}{7}$)

$$49\cdot\frac{9^2}{72}-126\frac{9}{7}+81=0$$

$$49\cdot\frac{81}{49}-7\cdot18\frac{9}{7}+81=0$$

$81-162+81=0$ (Simplify)

$$162-162=0$$

$0=0$ True

The solution set is $\boxed{\left\{\frac{9}{7}\right\}}$.

Answer 62E.

Given equation is

$$(c-9)^2=144$$

The objective is to find the solution set of given equation.

$$(c-9)^2=144$$

The square root property is

If $n>0$,

$x^2=n$ then

$$x=\pm\sqrt{n}$$

$$(c-9)^2=144$$

$c-9=\pm\sqrt{144}$ (By square root property)

$c-9=\pm\sqrt{(12)^2}$ (Since $144=12^2$)

$c-9=\pm\sqrt{12}$ (Since $\sqrt{a^2}=a$)

$c-9+9=9\pm12$ (Add 9 on both sides)

$$c=9\pm12$$

$$c=9+12$$

Or, $c=9-12$

$$c=21$$

Or, $c=-3$

The solution set is $\{-3,21\}$.

Check: To check the proposed solution set substitute each solution in the given equation.

Given equation is

$(c-9)^2 = 144$

$(-3-9)^2 = 144$ (Put $c = -9$)

$(-12)^2 = 144$

$144 = 144$ True

For $c = 21$,

$(c-9)^2 = 144$

$(21-9)^2 = 144$ (Put $c = 21$)

$(12)^2 = 144$

$144 = 144$ True

Therefore, the solution set of given equation is $\boxed{\{-3, 21\}}$.

Answer 63E.

Consider the equation

$$144b^2 = 36$$

The objective is to find the solution set of given equation.

$$144b^2 = 36$$

$12 \cdot 12b^2 = 36$ (Since $12 \cdot 12 = 144$)

$$12^2 b^2 = 36$$

$(12b)^2 = 36$ (Since $a^m \cdot b^m = (ab)^m$)

The square root property is, if

$n > 0$,

$x = n$ then

$$x = \pm\sqrt{n}$$

$$(12b)^2 = 36$$

$$12b = \pm\sqrt{36}$$

$12b = \pm\sqrt{6^2}$ (Because $6^2 = 36$)

$$12b = \pm 6 \quad \left(\sqrt{a^2} = a\right)$$

$\frac{12b}{12} = \frac{\pm 6}{12}$ (Divide with 12 on both sides)

$$b = \pm\frac{6}{12}$$

$$b = +\frac{6}{12}$$

Or, $b = -\frac{6}{12}$

$$b = \frac{1}{2}$$

Or, $b = \frac{-1}{2}$

The solution set is $\left\{\frac{-1}{2}, \frac{1}{2}\right\}$.

Check: To check the proposed solution set substitute each solution in the given equation.

Given equation is

$144b^2 = 36$

$144\left(-\frac{1}{2}\right)^2 = 36$ (Put $b = -\frac{1}{2}$)

$144 \cdot \frac{1}{4} = 36$

$36 \cdot 4 \cdot \frac{1}{4} = 36$ (Simplify)

$36 = 36$ True

$144b^2 = 36$

\ $144\left(\frac{1}{2}\right)^2 = 36$ (Put $b = \frac{1}{2}$)

$36 \cdot 4 \cdot \frac{1}{4} = 36$ (Simplify)

$36 = 36$ True

The solution set of given equation is $\boxed{\left\{-\frac{1}{2}, \frac{1}{2}\right\}}$.