Chapter 6

Polygons

6.1 Leela made some shapes on paper with the help of pencil and scale.

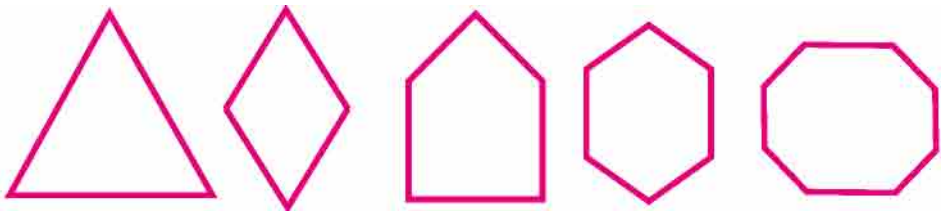


Figure 6.1

Leela asked to Kapil , can you identify these shapes?

Kapil - In previous classes, we learned that on the basis of number of sides we can identify the different shapes. Just like in figure 6.1,(I) a shape made of three sides is triangle,(ii) a shape made of four sides is quadrilateral, (iii) it is pentagon.

Sushila - Identify (iv) and (v) shape.

Kapil - It is hexagon and octagon as it contains 6 and 8 sides.

These closed shapes are made of many sides. A closed shape made up of three or more than three sides is called a polygon.

6.2 Diagonal of Polygon

Diagonal of polygon is made of by joining each vertices to the other vertices other than the adjacent vertices. See the figure 6.2, in figure(i) vertices A is joined with C and D other than E and B and get diagonal AC and AD respectively. Similarly, other two diagonal can be drawn by the various vertices.

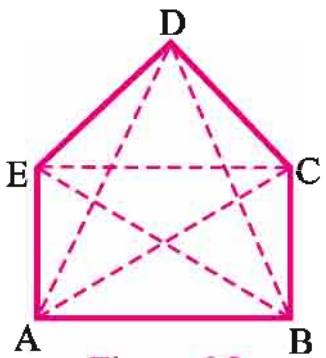
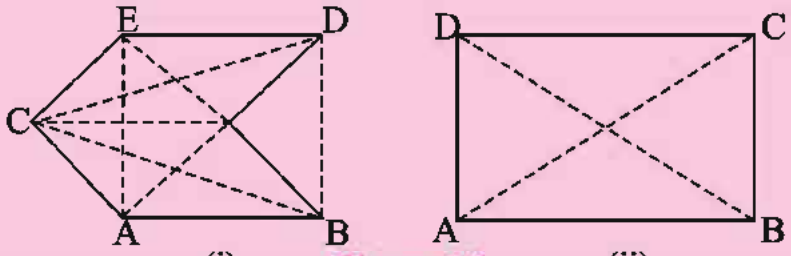


Figure 6.2

Do and Learn

See the diagonal in figure 6.3 and name them.

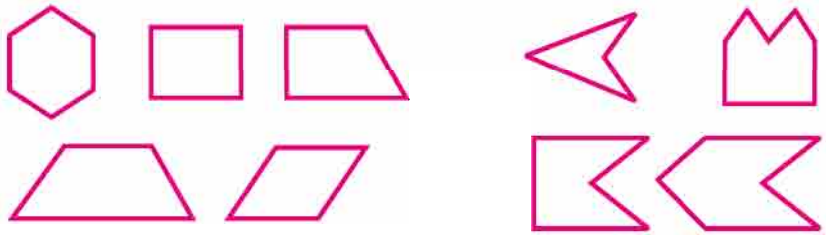


(i) Figure 6.3 (ii)

6.3 Concave and Convex Polygon

Given below some polygon shapes in two groups;

Group A Group B



(i) Figure 6.4 (ii)

Give the name A, B, C, D, E to vertices and draw the diagonal from each vertices of the polygons.

Are all the diagonals of polygons of group A interior?

Are all the diagonals of polygons of group B interior?

Does any polygon of the group has the diagonal exterior of the polygon?

After drawing the diagonals of polygons of an each group, you will find that all the diagonals of group A's polygons are internal and in group B's polygons some of the diagonals are external.

Convex polygon does not have diagonal outside the polygon i.e, they are inside the polygons while in concave polygon, diagonals are inside and outside.

Note: In convex polygon, each angle is less than 180^0 and in concave polygon, at least one angle is more than 180^0.

6.4 Regular and Irregular Polygon

See the following polygon figures:

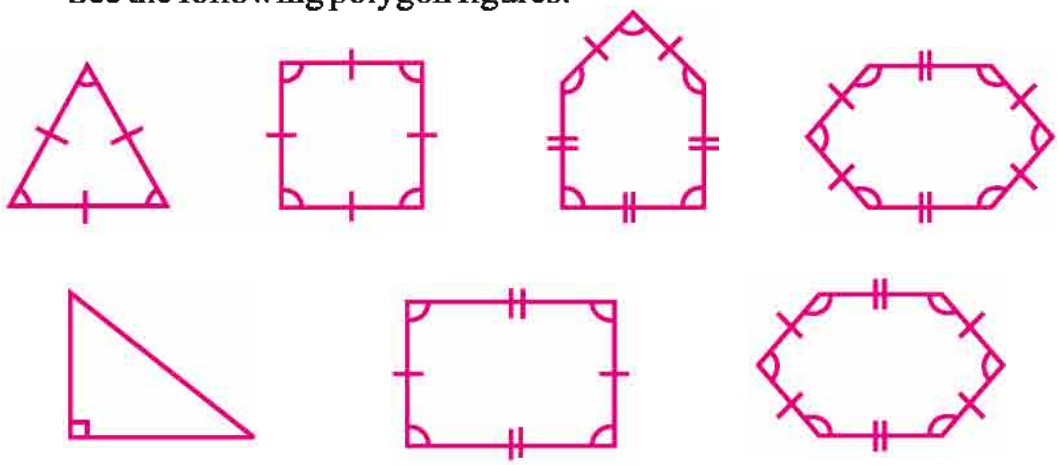


Figure 6.5

In these figures, ∧ ∧ symbol is used for equal side and ⌊⌋ symbol is used for equal angles.

A equiangular polygon has all angles equal while a regular polygon has all sides equal. In the above figure, which polygons are equiangular regular polygons? Are all regular polygons? Are all regular polygons equiangular also?

You will find that all the angles of every equiangular polygons are equal but it is not necessary that equiangular polygon is regular polygon since rectangular has equal angles but not equal sides.

Sum of the interior angles of polygon:

All the students draw a polygon in their note book and make a point inside this polygon.

1. Join the insider point to all vertices of the polygon. How many triangles are made? Discuss on it. What you conclude?
2. For example, a pentagon is drawn which has 5 angles.
3. There are 5 triangle in this figure.

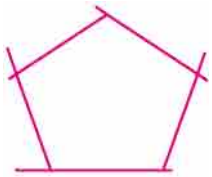


Figure 6.6

4. n-side polygon has – angles.
5. To join n-vertices with its center, – triangles are obtain.


Figure 6.7

The addition of total angles of all the triangles are equal to the number of triangles × 180^0 (since, sum of all the angles of a triangle is 180^0)

Thus, sum of all the angles of triangles made up of pentagon $= 5 \times 180^0$

Similarly, for hexagon $= 6 \times 180^0$

Thus, sum of all the angles of triangles made up of n-sides polygon $= n \times 180^0$

We know that the sum of all angles made on center $= 360^0$

1. So the sum of interior angles of pentagon is

$= 5 \times 180^0 - 360^0$
$= 5 \times 180^0 - 2 \times 180^0$
$= 180^0 (5 - 2)$

2. Sum of interior angles of hexagon is

$= 6 \times 180^0 - 360^0$
$= 6 \times 180^0 - 2 \times 180^0$
$= 180^0 (6 - 2)$

Similarly, the sum of interior angles of n-sides polygon is $= n \times 180^0 - 360^0$
$= n \times 180^0 - 2 \times 180^0$
$= 180^0 (n - 2)$
$= 180^0$ (no. of sides of polygon -2)
$= 180^0 (n-2)$

we can get the sum of all interior angles of the polygon by subtracting 2 from the number of sides and multiplying it with 180^0,

For example, if n = 7, then the sum of interior angles $= (7-2)180^0$
$= 5 \times 180^0 = 900^0$

if n = 4, then the sum of interior angles $= (4-2)180^0 = 2 \times 180^0 = 360^0$

Thus, if we have to find out an interior angle of any regular polygon then divided the total sides n by the number of sum of all interior angles.

So ,each interior angle of n-sided regular polygon will be $= \frac{(n-2)\ 180^0}{n}$

6.5 Sum of the measure of the exterior angle of a polygon

Exterior angle of a Polygon:

By increasing the sides in one direction (clockwise or anti-clockwise) outer of a polygon, the resultant angle (which is complimentary of interior angles) of that is exterior angles of a polygon.

In figure 6.8, angle$\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$ and $\angle 6$ are exterior angles of a polygon.
In figure, sum of exterior and interior angle on each vertices is a linear angle-pair.
So, sum of angles made on vertices =

number of sides $\times 180^0$

$= n \times 180^0$.....(i)

Interior angles of a polygon $= (n-2) \times 180^0$.....(ii)

Sum of exterior angles $= (i) - (ii)$

$= n \times 180^0 - (n-2) \times 180^0$

$= 180^0 \times \{n - (n-2)\}$

$= 180^0 \times \{(n - n + 2)\}$

$= 180^0 \times 2$

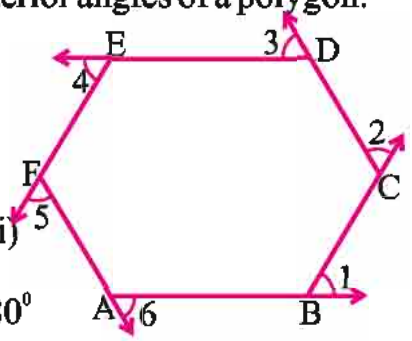


Figure 6.8

Thus, the sum of all exterior angles of a convex polygon is $= 360^0$

If there are n- sides in any regular polygon, then the value of each exterior angle will be $= \frac{360^0}{n}$

Ex. 1: Find the value of each exterior angle of a regular pentagon.

Sol: Value of all exterior angle of a polygon $= 360^0$

Number of sides of a polygon $= 5$

Value of an exterior angle $= \frac{360^0}{5}$

$= 72^0$

Ex. 2: If value of every exterior angle of a regular polygon is 60^0 then find the number of sides.

Sol: value of exterior angles = number of sides $\times$ value of an exterior angle

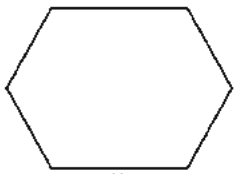
$360^0 = n \times 60^0$

$n = \frac{360}{60}$

$n = 6$ sides

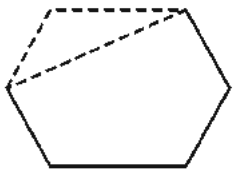
Activity:

Draw any polygon on piece of paper (hexagon is shown in figure).

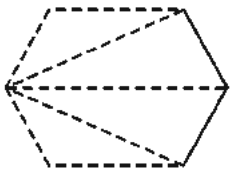

(i)

Adjoining the adjacent vertices, cut by the scissors and separate the triangle.

Similarly, make more triangles.


(ii)

How many triangles you have made?
Think and discuss. Is there any relationship between number of sides of polygon and number of triangles?



(iii)

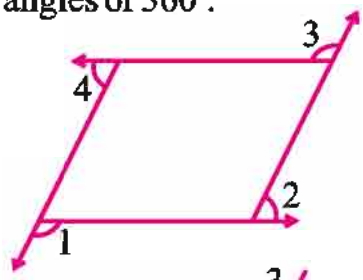
Figure 6.9

Is number of triangle less by two then number of sides?
Hence, the number of triangle made up of six-sides polygon is = 4
Sum of interior angles of a triangle is = 180^0
Sum of interior angles will be = 4×180^0

Activity:

Sum of exterior angles of a polygon is 4 right angles or 360^0.

1. Draw a polygon on a paper and make its exterior angles.



2. Now separate the exterior angles with the help of a scissors.

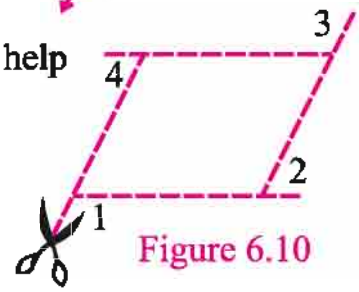
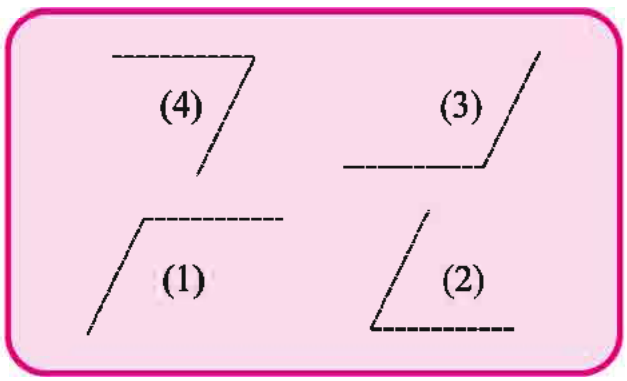
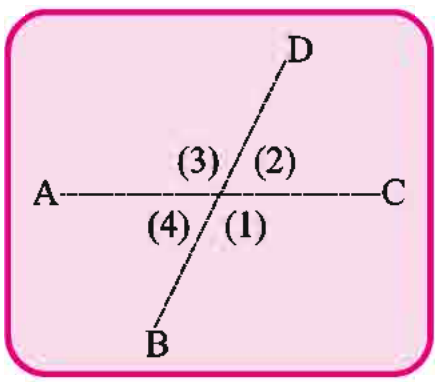


Figure 6.10

3. Now join the cutting parts.





In figure, by moving from part(1) and to reach the same, one round is complete. So $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^0$

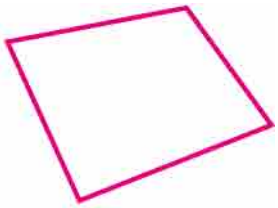
Exercise 6.1

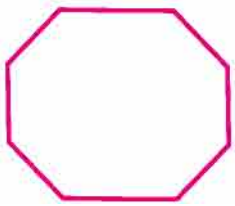
1. Draw the diagonal with the help of a pencil in the following figure and tell-

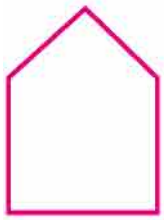
(i) 

(ii) 

(iii) 

(iv) 

(v) 

(vi) 

(i) In which shapes, diagonal will be inside?

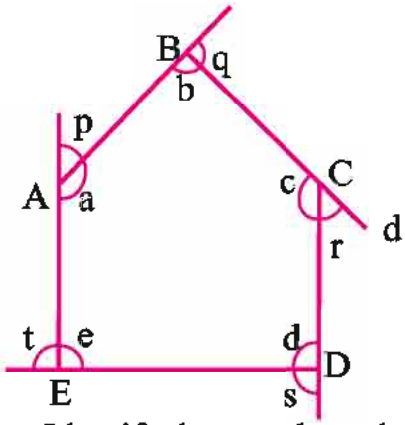
(ii) In which shapes, diagonal will be outside?

(iii) Identify the type of polygon (concave or convex)?

2. In the given polygon ABCDE

(i) Write the name of interior angles.

(ii) Write the name of exterior angles.



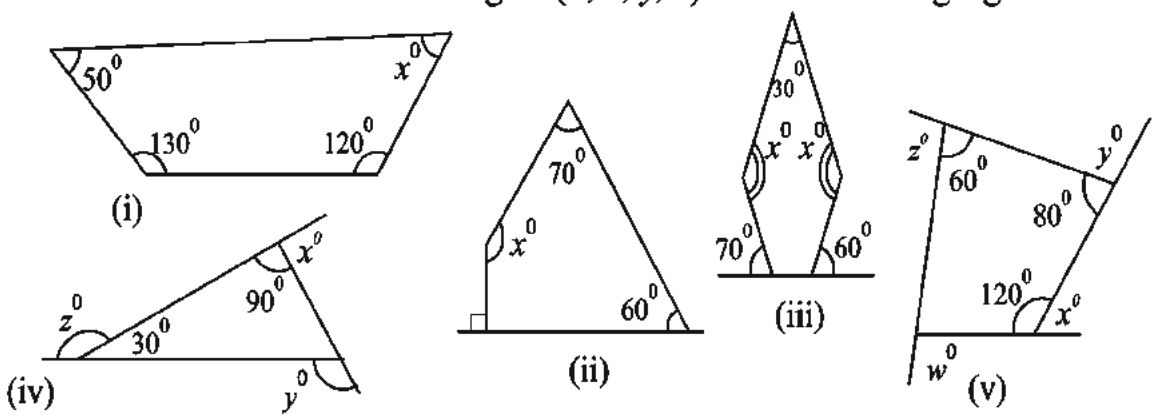
3. Define the regular polygon. Identify that regular polygon in which there are:

(i) 5 sides

(ii) 6 sides

(iii) 8 sides

4. Find the values of unknown angles (w, x, y, z) in the following figure:



5. Find the number of sides of a regular polygon whose measure of an each exterior angle is 45^0.
6. Find the number of sides of a regular polygon if its each interior angle is 165^0.
7. Find the number of sides of that regular polygon whose each exterior angle is 24^0.
8. Find the value of every interior angle of that regular polygon which has 10 sides.
9. If interior angles of any polygon is 115^0 then will it be regular polygon?
10. One interior angle of a hexagon is 165^0 and the measure of remaining interior angle is x^0 then find out the measure of all the angles.
11. By increasing the sides of a triangle in a single direction, obtained exterior angles are 110^0, 115^0 and x^0, then find the value of x.
12. Find the sum of all interior angles of a regular heptagon.

6.6 Characteristic of Quadrilateral

You know that the closed shape made up of four line segments is called quadrilateral. There are four angles, four vertices and two diagonals in any quadrilateral.

In quadrilateral ABCD,

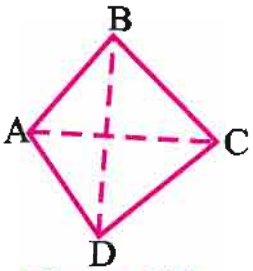


Figure 6.11

Vertices are A, B, C and D.
Sides are AB, BC, CD and DA.
Angles are $\angle ABC$, $\angle BCD$, $\angle CDA$, and $\angle DAB$.
Diagonals are AC and BD.

6.7 Different Types of Quadrilaterals

We have divided the triangle on the basis of their sides like equilateral triangle, isosceles triangles and scalene triangle and on the basis of their angles like acute angle triangle, right angle triangle and obtuse angle triangle.

Likewise some quadrilaterals name specific due to its sides and angles. Some such types of quadrilaterals are known as square, rectangle, parallelogram, trapezium and kite.

6.7.1 Trapezium

A quadrilateral having exactly one pair of parallel sides , is called a trapezium.

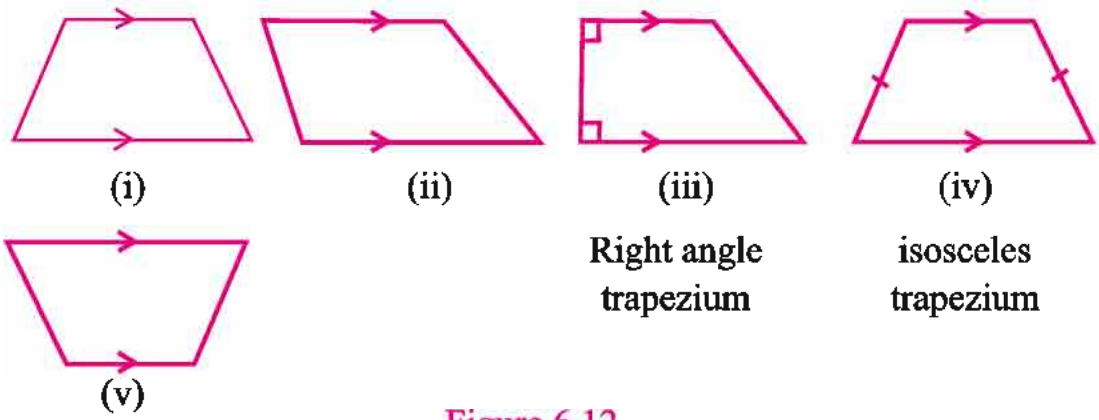


Figure 6.12

(arrow shows the parallel sides)

Above figure shows the trapezium but (iii) and (iv) are special kind of trapezium.

- A trapezium is said to be a right angle trapezium if its two angles are right angles as shown in figure(iii).
- A trapezium is said to be an isosceles trapezium, if its non-parallel sides are equal as shown in figure(iv).

Activity

Take set squares from your friends geometry box and arrange them in the following manner.

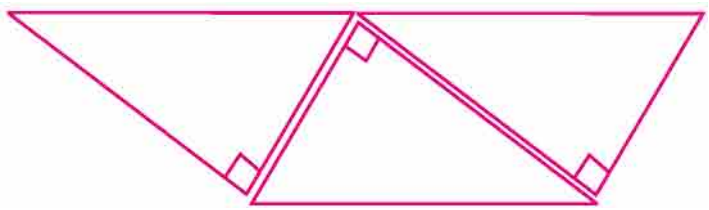


Figure 6.13

The obtained figure is trapezium.

Now by the use of set square, get the different shapes of trapezium.

6.7.2. Kite

A quadrilateral is a kite, if it has two pairs of adjacent sides equal.

As shown in figure, AB, BC and CD, DA are two equal pair of adjacent sides respectively.

so AB = BC and AD = CD

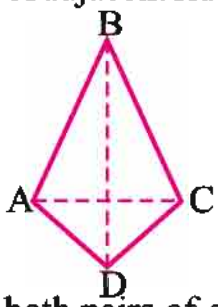


Figure 6.14

6.7.3 Parallelogram

A quadrilateral is a parallelogram if its both pairs of opposite sides are equal and parallel. Following are the characteristics of parallelogram:

1. Each pair of opposite sides is equal.
2. Each pair of opposite angles is equal.
3. Diagonal bisects each other.

Activity

You with your friends take two similar set of set squares into the geometry box and arrange them in the following way:

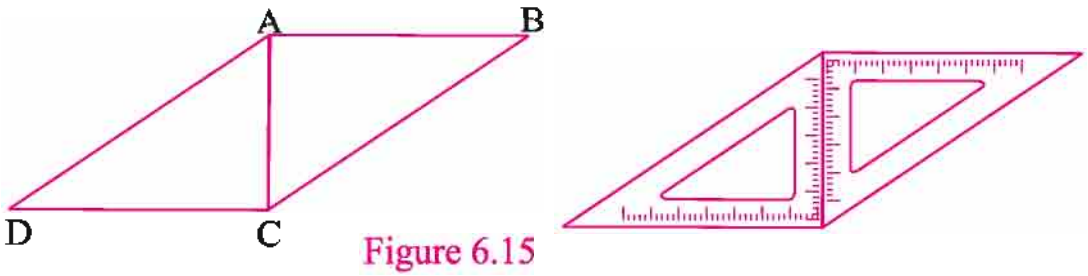


Figure 6.15

In figure, AB ||DC, AD||BC and AB = DC, AD = BC

Thus,

1. In a parallelogram both pairs of opposite sides are equal and parallel.
2. Each pair of opposite angles is equal.

Take a set of set- square and arrange them in the following way and find that the pairs of opposite angles are equal or not.

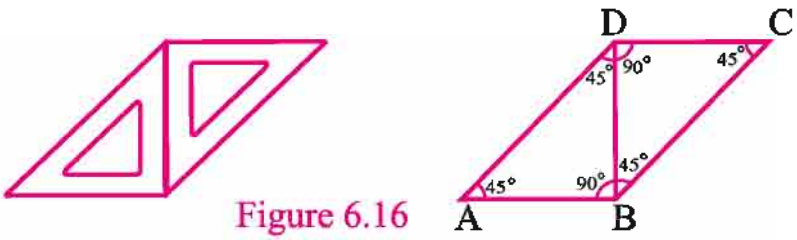


Figure 6.16

By the above figure it is clear that pairs of opposite angles are equal. The set-square of 30^0-60^0-90^0, the opposite pairs are 30^0-30^0 and 150^0-150^0.

3. Diagonal bisects each other. Draw parallelogram ABCD and measure AO, OB, OC and OD. On which conclusion you reach? Is OA = OC and OB = OD?

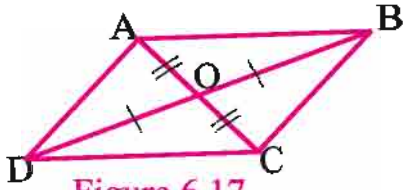


Figure 6.17

It is clear that diagonal AC and BD bisect each other at point O. So OA= OC and OB = OD.

6.7.4 Special Cases of Parallelogram

(i) Rectangle : A parallelogram in which each angle is a right angle is called a rectangle.

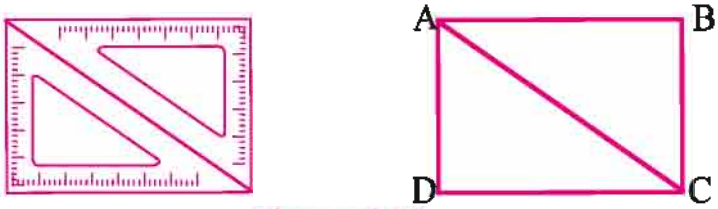


Figure 6.18

Arrange the two set square of 30^0-60^0-90^0 according to the figure and check that the opposite angles are right angle. $\angle A = \angle C = \angle B = \angle D$ are right angles and measure of AC and BD i.e., AC= BD.

Qualities of Rectangle

1. Opposite sides are equal.
2. Each angle is right angle.
3. Diagonals are same and bisect each other.

(ii) **Square:** A Rectangle in which all sides are equal is called square. Take two set of set-square of angle. Set them according to the figure and check that all

the angles are right angle. Diagonals are perpendiculars to each other and $\angle A = \angle C = \angle B = \angle D$

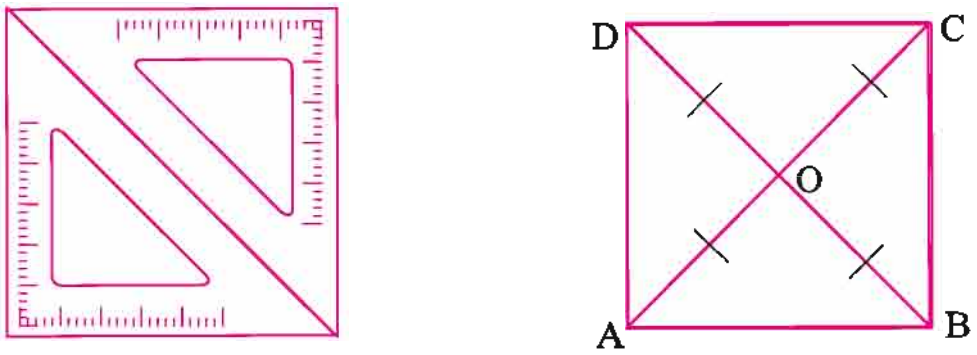


Figure 6.19

Qualities of Square:

1. Each side is same.
2. Diagonal bisects each other.
3. Diagonals are perpendicular.

6.7.5 Rhombus: A parallelogram having all sides equal and diagonal bisects perpendicular to each other. Take four set -square of 30^0-60^0-90^0 and arrange them according to the following figure. You will see that the diagonal bisects each other on right angle and all sides are equal.

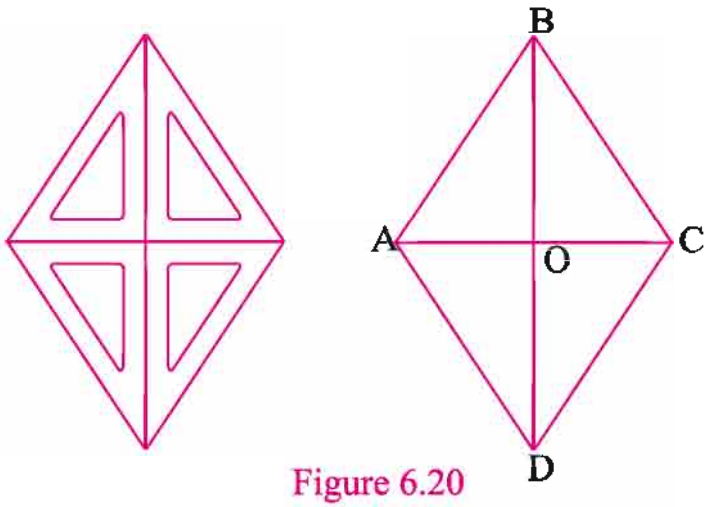


Figure 6.20

In figure, AC and BD bisect each other on right angle. OA=OC and OB = OD.

Do and Learn

On the basis of characteristics of quadrilateral tick the ✓or X on the given place.

	Paralle logram	Rectangle	Rhombus	Square	Trapezium whose non-parallel sides are equal	Trapezium	Kite
Opposite sides are parallel	✓	✓	✓	✓	X	X	X
Opposite sides are equal.							
Opposite angles are equal.							
Diagonals make congr-uent triangle							
Diagonals bisect each other.							
Diagonals are perpendicular on each other.							
Diagonals are equal.							
All angles are right angle.							
All sides are equal.							

Exercise 6.2

1. Fill in the blanks choosing the right option-

(i) Adjacent angles of a parallelogram are.....(equal/ supplement).

(ii) Diagonals of a rectangle are.....(equal/ perpendicular bisect).

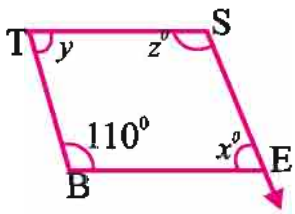
(iii) In any trapezium AB||CD, if A= 100^0 then the value of D will be..... (100^0/80^0)

(iv) If in any quadrilateral, diagonal bisect each other on right angle then, it is

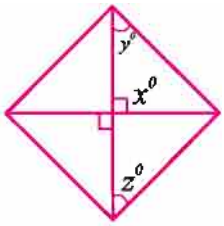
called.....(parallelogram/ rhombus).

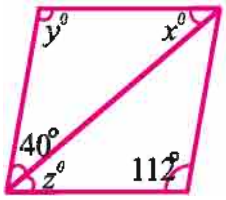
(v) All squares are .....(congruent/ similar).

2. In figure, BEST is a parallelogram.
Find the value of x,y,z,



3. In following parallelograms, find the unknown value of x,y,z.

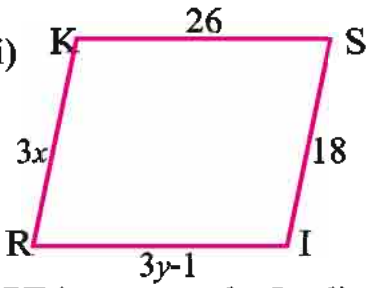




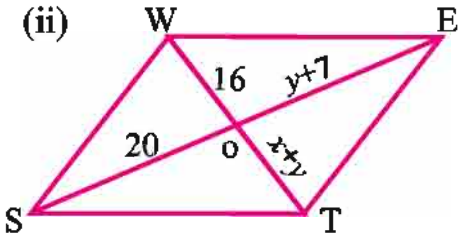
4. In any parallelogram, the ratio of two adjacent angles is 1: 5. Find the value of all angles of parallelogram.

5. Following figures RISK and STEW are parallelogram. Find the value of x and y(length in cm).

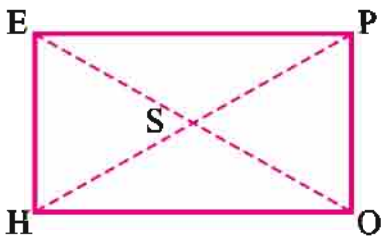
(i)



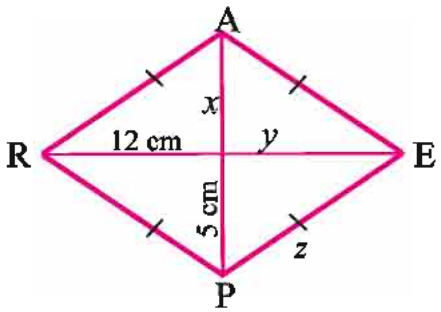
(ii)



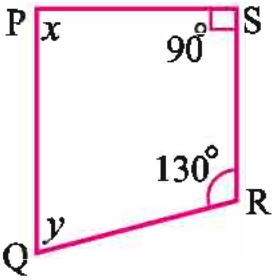
6. HOPE is a rectangle. Its diagonals intersect each other at point S. Find the value of x. If SH = $2x$+4 and SE = $3x$+1.



7. PEAR is a rhombus. Find the value of x,y and z, also write the causes.



8. In trapezium PQRS, PQ || SR find the value of $\angle x$ and $\angle y$.



We Learnt

1. A simple closed curve made up of three or more than three line segments is called polygon.
2. In any triangle, number of diagonal is less than two the number of sides. The sum of interior angles of a polygon is $(n-2)180^0$.
3. Those polygons whose all diagonals are the interior, are called convex polygon.
4. Those polygons in which at least one diagonal is exterior, are called concave polygon.
5. A polygon having all sides equal measure, is called a regular polygon.
6. Sum of all exterior angles of a polygon is 360^0.
7. A closed curve made up of four simple line segment is called quadrilateral. There are four angles, four vertices, four sides and two diagonals in any quadrilateral.
8. A quadrilateral having exactly one pair of parallel sides, is called a trapezium.
9. A quadrilateral is a kite, if it has two pairs of equal adjacent sides.