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In earlier classes, during the study of coordinate geometry, we studied about coordinate axes, coordinate plane, plotting of points in a plane, distance between two points, section formula etc. All these are the basic concept of coordinate geometry. In this chapter, we shall continue the study of coordinate geometry to understand the properties of straight lines.

STRAIGHT LINES

CHAPTER CHECKLIST

- A Recall of Coordinate Geometry
- Slope of a Straight Line
- Various Forms of Equation of Line
- Distance of a Point from a Line

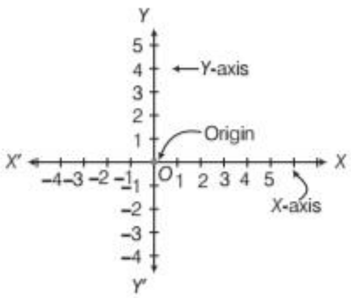
|TOPIC 1|

A Recall of Coordinate Geometry

COORDINATE AXES AND COORDINATE PLANE

Coordinate Axes

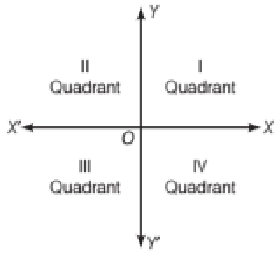
Let $X'OX$ and $Y'OY$ be two perpendicular lines intersect at O. Then, point O is called **origin** and the lines $X'OX$ and $Y'OY$ are called **coordinate axes**.



The horizontal line $X'OX$ is called X-axis and the vertical line $Y'OY$ is called Y-axis.

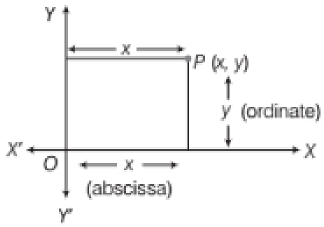
Coordinate Plane

The intersection of X-axis and Y-axis divide the plane into four parts. These four parts are called **quadrant**, $\left(\frac{1}{4}\text{th part}\right)$ numbered I, II, III and IV anti-clockwise from OX. Thus, the plane consists of the axes and four quadrants, is known as XY-plane or **cartesian plane** or **coordinate plane** and the axes are known as **coordinate axes**. These coordinate axes are also called *rectangular axes* as they are perpendicular to each other.



Coordinates of a Point in Cartesian Plane

Let P be any point in a plane. If the distance of point P from Y-axis is x and the distance of point P from X-axis is y. Then, the coordinates of a point P are (x, y) where, x is called x-coordinate or abscissa and y is called the y-coordinate or ordinate.



The coordinates of a point on the X-axis are of the form $(x, 0)$ and of a point on the Y-axis are of the form $(0, y)$.

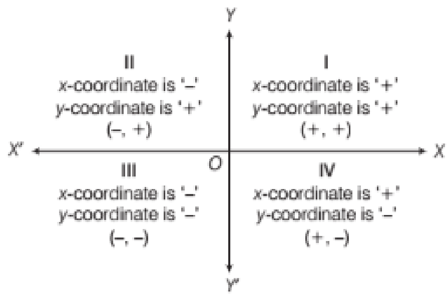
Note

(i) Abscissa of any point on Y-axis is zero.
(ii) Ordinate of any point on X-axis is zero.
(iii) Coordinates of the origin are (0, 0).

Sign Convention of Coordinates

The signs of coordinates of points in all the four quadrants are expressed in the table given below

Quadrant	Sign of x-coordinate	Sign of y-coordinate	Point
I	+	+	(+, +)
II	−	+	(−, +)
III	−	−	(−, −)
IV	+	−	(+, −)



EXAMPLE |1| In which quadrant, the following points lie?

(i) (6, − 3) (ii) (−4, 1)
(iii) (−1, − 1) (iv) (5, 4)

Sol. (i) Let $A = (6, -3)$

Since, x-coordinate of A is positive and its y-coordinate is negative, therefore A lies in the fourth quadrant.

(ii) Let $B = (-4, 1)$

Since, x-coordinate of B is negative and its y-coordinate is positive, therefore B lies in the second quadrant.

(iii) Let $C = (-1, -1)$

Since, x-coordinate of C is negative and its y-coordinate is also negative, therefore C lies in the third quadrant.

(iv) Let $D = (5, 4)$

Since, x-coordinate of D is positive and its y-coordinate is also positive, therefore D lies in the first quadrant.

DISTANCE FORMULA

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

EXAMPLE |2| Find the distance between the points $P(a\cos\alpha, a\sin\alpha)$ and $Q(a\cos\beta, a\sin\beta)$.

Sol. The distance between P and Q is

$$PQ = \sqrt{(a\cos\beta - a\cos\alpha)^2 + (a\sin\beta - a\sin\alpha)^2}$$

[by distance formula]

$$= \sqrt{a^2(\cos^2\beta + \cos^2\alpha - 2\cos\alpha\cos\beta) + a^2(\sin^2\beta + \sin^2\alpha - 2\sin\beta\sin\alpha)}$$

$$= a\sqrt{(\cos^2\beta + \sin^2\beta) + (\cos^2\alpha + \sin^2\alpha) - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)}$$

$$= a\sqrt{1 + 1 - 2\cos(\alpha - \beta)} = a\sqrt{2[1 - \cos(\alpha - \beta)]}$$

[$\because \cos^2\theta + \sin^2\theta = 1$ and $\cos A\cos B + \sin A\sin B = \cos(A - B)$]

$$= a\sqrt{2 \times 2\sin^2\left(\frac{\alpha - \beta}{2}\right)} \quad [\because \cos\theta = 1 - 2\sin^2\theta/2]$$

$$= 2a\sin\left(\frac{\alpha - \beta}{2}\right) \text{ units}$$

EXAMPLE |3| Find the point on X-axis which is equidistant from the points (3, 2) and (−5, − 2).

Sol. Let the point on X-axis be $P(x, 0)$, which is equidistant from (say) A (3, 2) and (say) $B(-5, -2)$.

Since, P is equidistant from A and B. So,

$\therefore \quad PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow (3-x)^2 + (2-0)^2 = (-5-x)^2 + (-2-0)^2$ [by distance formula]

$\Rightarrow 9 + x^2 - 6x + 4 = 25 + x^2 + 10x + 4$

$\Rightarrow 16x + 16 = 0 \Rightarrow x = -1$

Thus, point on X-axis is $(-1, 0)$.

APPLICATION OF DISTANCE FORMULA

Distance formula is also used to identify that which geometrical figure is represented by the given points. Conditions correspond- ing to three and four points are given below:

1. Three points will represent
   - (i) an **equilateral triangle** iff all sides are equal.
   - (ii) an **isosceles triangle** iff two sides are equal.
2. Four points will represent
   - (i) a **parallelogram** iff opposite sides are equal.
   - (ii) a **rectangle** iff opposite sides are equal and diagonals are also equal.
   - (iii) a **rhombus** iff all the four sides are equal.
   - (iv) a **square** iff all the four sides are equal and diagonals are also equal.

EXAMPLE |4| Show that four points (0, − 1), (6, 7), (−2, 3) and (8, 3) are the vertices of a rectangle.

Sol. Let, A (0, −1), B (6, 7), C (−2, 3) and D (8, 3) be the given points. Then,

$AD = \sqrt{(8-0)^2 + (3+1)^2}$ [by distance formula]

$= \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$ units

$CB = \sqrt{(6+2)^2 + (7-3)^2} = \sqrt{64+16} = \sqrt{80}$

$= 4\sqrt{5}$ units

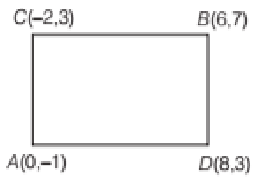
$AC = \sqrt{(-2-0)^2 + (3+1)^2}$

$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ units

and $BD = \sqrt{(8-6)^2 + (3-7)^2}$

$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ units

Thus, $AD = CB$ and $AC = BD$



So, ADBC is a parallelogram.

Now, $AB = \sqrt{(6-0)^2 + (7+1)^2}$

$= \sqrt{36+64} = \sqrt{100} = 10$ units

and $CD = \sqrt{(8+2)^2 + (3-3)^2} = \sqrt{100} = 10$ untis

Clearly, $AB = CD$

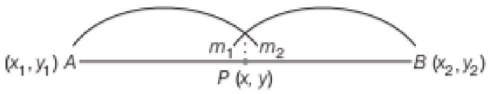
Hence, ADBC is a rectangle.

SECTION FORMULAE

1. Internal Division

If point $P(x, y)$ divides the line segment AB, obtained by joining $A(x_1, y_1)$ and $B(x_2, y_2)$, internally in the ratio $m_1 : m_2$, then coordinates of P are

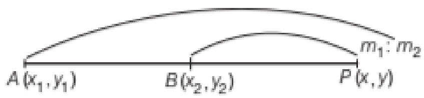
$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2},\ y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$



2. External Division

If point $P(x, y)$ divides the line segment AB, obtained by joining $A(x_1, y_1)$ and $B(x_2, y_2)$, externally in the ratio $m_1 : m_2$, then coordinates of P are

$$x = \frac{m_1x_2 - m_2x_1}{m_1 - m_2};\quad y = \frac{m_1y_2 - m_2y_1}{m_1 - m_2}$$



Note

The coordinates of mid-points of line segment obtained by joining $A(x_1, y_1)$ and $B(x_2, y_2)$ are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

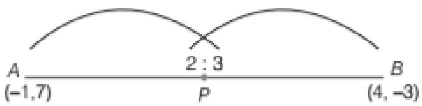
EXAMPLE |5| Find the coordinates of the point which divides the join of (− 1, 7) and (4, − 3) in the ratio 2 : 3

(i) internally. (ii) externally.

Sol. Let $A(x_1, y_1) \equiv A(-1, 7)$ and $B(x_2, y_2) \equiv B(4, -3)$.

Again, let $P(x, y)$ be the required point.

(i) Here, P divides AB internally in the ratio 2 : 3.



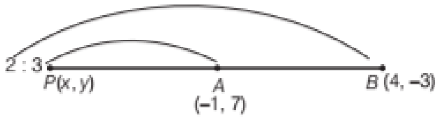
$$\therefore\ P(x, y) \equiv P\left(\frac{2\times 4+3\times(-1)}{2+3}, \frac{2\times(-3)+3\times 7}{2+3}\right)$$

[by internal division formula and $m_1 = 2$, $m_2 = 3$]

$$\Rightarrow P(x, y) \equiv P\left(\frac{8-3}{5}, \frac{-6+21}{5}\right) \Rightarrow P(x, y) \equiv P(1, 3)$$

Thus, (1, 3) is the required point.

(ii) Here, P divides AB externally in the ratio of 2 : 3.



$$\therefore\ P(x, y) = \left(\frac{2\times 4-3\times(-1)}{2-3}, \frac{2\times(-3)-3\times 7}{2-3}\right)$$

[by external division formula and $m_1 = 2$, $m_2 = 3$]

$$= \left(\frac{8+3}{-1}, \frac{-6-21}{-1}\right) \Rightarrow P(x, y) = (-11, 27)$$

Thus, $(-11, 27)$ is the required point.

EXAMPLE |6| Without using distance formula, show that the points $(-2, -1)$, $(4, 0)$, $(3, 3)$ and $(-3, 2)$ are the vertices of a parallelogram.

Sol. Let $A(x_1, y_1) \equiv A(-2, -1)$, $B(x_2, y_2) \equiv B(4, 0)$, $C(x_3, y_3) \equiv C(3, 3)$ and $D(x_4, y_4) \equiv (-3, 2)$

Now, mid-point of $AC = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

$$= \left(\frac{-2+3}{2}, \frac{-1+3}{2}\right) = \left(\frac{1}{2}, 1\right) \quad \text{...(i)}$$

and mid-point of $BD = \left(\frac{x_2+x_4}{2}, \frac{y_2+y_4}{2}\right)$

$$= \left(\frac{4-3}{2}, \frac{0+2}{2}\right) = \left(\frac{1}{2}, 1\right) \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

Mid-point of AC = Mid-point of BC

Thus, mid-points of both diagonals are coincide each other.

Hence, the points A, B, C and D are vertices of a parallelogram.

3. Points of Trisection

Trisection means a line is divided into three equal parts.

This can be done by finding two points P and Q on the line segments AB, such that $AP = PQ = QB$.

Let $\quad AP = PQ = QB = x$

Then, $\quad AP = x$ and $PB = PQ + QB = x + x = 2x$

$\therefore \quad AP : PB = x : 2x = 1 : 2$

Also, $\quad AQ = AP + PQ = 2x$ and $QB = x$

$\therefore \quad AQ : QB = 2x : x = 2 : 1$

Hence, to find points of trisection, we find two points P and Q which divides AB in the ratio 1 : 2 and 2 : 1, respectively.

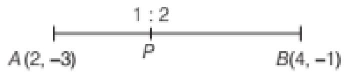
EXAMPLE |7| Find the coordinates of the points of trisection of the line segments joining $(2, -3)$ and $(4, -1)$.

Sol. Let $A(x_1, y_1) \equiv A(2, -3)$ and $B(x_2, y_2) \equiv B(4, -1)$

Again, let P and Q be two points of trisection as shown below

Then, $AP : PB = 1 : 2$ and $AQ : QB = 2 : 1$

(i) Here, P divides AB internally in the ratio of 1 : 2.

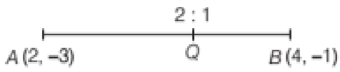


$$\therefore\ P \equiv \left(\frac{1\times 4+2\times 2}{1+2}, \frac{1\times(-1)+2\times(-3)}{1+2}\right)$$

[by internal division formula]

$$\Rightarrow P \equiv \left(\frac{4+4}{3}, \frac{-1-6}{3}\right) \Rightarrow P \equiv \left(\frac{8}{3}, \frac{-7}{3}\right)$$

(ii) Here, Q divides AB internally in the ratio of 2 : 1



$$\therefore \quad Q \equiv \left(\frac{2\times 4+1\times 2}{2+1}, \frac{2\times(-1)+1\times(-3)}{2+1}\right)$$

[by internal division formula]

$$\Rightarrow \quad Q \equiv \left(\frac{8+2}{3}, \frac{-2-3}{3}\right) \Rightarrow Q \equiv \left(\frac{10}{3}, \frac{-5}{3}\right)$$

Centroid of a Triangle

If G is the centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then

$$\textbf{Coordinates of } G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Note

The centroid divides each median in the ratio of 2 : 1.

EXAMPLE |8| If the vertices of a triangle are $P(1, 3)$, $Q(2, 5)$ and $R(3, -5)$, then find the centroid of a ΔPQR.

Sol. We know that, if the vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then centroid of a triangle is

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Here, $P(1, 3) \equiv P(x_1, y_1)$; $Q(2, 5) \equiv Q(x_2, y_2)$ and $R(3, -5) \equiv R(x_3, y_3)$

$\therefore$ Centroid of a triangle

$$= \left(\frac{1+2+3}{3}, \frac{3+5-5}{3}\right) = \left(\frac{6}{3}, \frac{3}{3}\right) = (2, 1)$$

Collinearity of Three Points

Three points A, B and C are collinear, if they lie on a same straight line, i.e. if $AB + BC = AC$.

EXAMPLE |9| Check whether the points $(1, -1)$, $(5, 2)$ and $(9, 5)$ are collinear or not.

Sol. Let $A = (1, -1)$, $B = (5, 2)$ and $C = (9, 5)$

Now, distance between A and B,

$$AB = \sqrt{(5-1)^2 + (2+1)^2} \quad \text{[by distance formula]}$$

$$= \sqrt{(4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

Distance between B and C, $BC = \sqrt{(5-9)^2 + (2-5)^2}$

$$= \sqrt{(-4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

Distance between A and C,

$$AC = \sqrt{(1-9)^2 + (-1-5)^2}$$

$$= \sqrt{(-8)^2 + (-6)^2}$$

$$= \sqrt{64+36} = 10$$

Clearly, $AC = AB + BC$

Hence, A, B and C are collinear points.

AREA OF A TRIANGLE

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a ΔABC, then, Area of ΔABC

$$= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$\Delta = \frac{1}{2}|x_1y_2 + x_2y_3 + x_3y_1 - (x_1y_3 + x_2y_1 + x_3y_2)|$$

EXAMPLE |10| Find the area of a ΔABC, whose vertices are $A(6, 3)$, $B(-3, 5)$ and $C(4, -2)$.

Sol. Area of ΔABC

$$= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Here, $(x_1, y_1) \equiv (6, 3)$, $(x_2, y_2) \equiv (-3, 5)$

and $(x_3, y_3) \equiv (4, -2)$

$\therefore$ Area of ΔABC

$$= \frac{1}{2}|6(5+2) + (-3)(-2-3) + 4(3-5)|$$

$$= \frac{1}{2}|6(7) - 3(-5) + 4(-2)|$$

$$= \frac{1}{2}|42 + 15 - 8| = \frac{1}{2}|49| = \frac{49}{2} \text{ sq units}$$

Hence, area of ΔABC is $\frac{49}{2}$ sq units.

EXAMPLE |11| For what value of k are points $(k, 2-2k)$, $(-k+1, 2k)$ and $(-4-k, 6-2k)$ are collinear?

Sol. Let three points be $A(x_1, y_1) \equiv (k, 2-2k)$,

$$B(x_2, y_2) \equiv (-k+1, 2k)$$

and $C(x_3, y_3) \equiv (-4-k, 6-2k)$

Condition for three points to be collinear is, area of triangle formed by these three points $= 0$

$$\therefore \; x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow k(2k - 6 + 2k) + (-k+1)(6 - 2k - 2 + 2k) + (-4-k)(2 - 2k - 2k) = 0$$

$$\Rightarrow k(4k-6) - 4(k-1) + (4+k)(4k-2) = 0$$

$$4k^2 - 6k - 4k + 4 + 4k^2 + 14k - 8 = 0$$

$$\Rightarrow 8k^2 + 4k - 4 = 0 \Rightarrow 2k^2 + k - 1 = 0$$

$$\Rightarrow (2k-1)(k+1) = 0 \Rightarrow k = \frac{1}{2} \text{ or } k = -1$$

Hence, the given points are collinear for $k = \frac{1}{2}$ or $k = -1$.

LOCUS OF A POINT

The curve described by a moving point under given geometrical condition(s), is called locus of that point.

Equation of the Locus of a Point

The equation of the locus of a point is the relation which is satisfied by the coordinates of every point on the locus of the point.

For finding the locus of a point, we may use the following steps

METHOD TO FIND THE LOCUS OF A POINT

***Step* I** First, assume the coordinates of the point, whose locus is to be found, say (h, k).

***Step* II** Write the given condition in mathematical form involving h, k and simplify it.

***Step* III** Eliminate the variable(s), if any.

***Step* IV** Replace h by x and k by y in the result obtained in Step III. The equation so obtained will be the required locus of the point.

EXAMPLE |12| The sum of the squares of the distances of a moving point from two fixed points $(a, 0)$ and $(-a, 0)$ is equal to a constant quantity $2b^2$. Find the equation to its locus.

Sol. Let $P(h, k)$ be the point whose locus is to be found.

Let given two fixed points be $A(a, 0)$ and $B(-a, 0)$.

According to the given condition, $PA^2 + PB^2 = 2b^2$

$$\Rightarrow (h-a)^2 + (k-0)^2 + (h+a)^2 + (k-0)^2 = 2b^2$$

[by distance formula]

$$\Rightarrow h^2 - 2ah + a^2 + k^2 + h^2 + 2ah + a^2 + k^2 = 2b^2$$

$$\Rightarrow 2h^2 + 2k^2 + 2a^2 = 2b^2$$

$\Rightarrow \quad h^2 + k^2 + a^2 = b^2$ [dividing both sides by 2] ...(i)

Here, in the Eq. (i), there is no other variable(s).

On putting $h = x$ and $k = y$, we get

$$x^2 + y^2 + a^2 = b^2$$

which is the required equation of locus of a given point.

EXAMPLE |13| A point moves, so that the sum of its distances from $(ae, 0)$ and $(-ae, 0)$ is $2a$, prove that the equation to its locus is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $b^2 = a^2(1 - e^2)$.

Sol. Let, $P(h, k)$ be the moving point such that the sum of its distances from $A(ae, 0)$ and $B(-ae, 0)$ is $2a$.

Then, $\quad PA + PB = 2a$

$\Rightarrow \sqrt{(h - ae)^2 + (k - 0)^2} + \sqrt{(h + ae)^2 + (k - 0)^2} = 2a$

[by distance formula]

$\Rightarrow \quad \sqrt{(h - ae)^2 + k^2} = 2a - \sqrt{(h + ae)^2 + k^2}$

$\Rightarrow \quad (h - ae)^2 + k^2 = 4a^2 + (h + ae)^2 + k^2 - 4a\sqrt{(h + ae)^2 + k^2}$

[squaring on both sides]

$h^2 + a^2e^2 - 2hae = 4a^2 + h^2 + a^2e^2 + 2hae - 4a\sqrt{(h + ae)^2 + k^2}$

$\Rightarrow \quad -4aeh - 4a^2 = -4a\sqrt{(h + ae)^2 + k^2}$

$\Rightarrow \quad (eh + a) = \sqrt{(h + ae)^2 + k^2}$

$\Rightarrow \quad (eh + a)^2 = (h + ae)^2 + k^2$

[again, squaring on both sides]

$\Rightarrow \quad e^2h^2 + 2aeh + a^2 = h^2 + a^2e^2 + 2aeh + k^2$

$\Rightarrow \quad h^2(1 - e^2) + k^2 = a^2(1 - e^2)$

$\Rightarrow \quad \frac{h^2}{a^2} + \frac{k^2}{a^2(1 - e^2)} = 1$

Hence, locus of point $P(h, k)$ is

$$\frac{x^2}{a^2} + \frac{y^2}{a^2(1 - e^2)} = 1 \text{ or } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

where $b^2 = a^2(1 - e^2)$

TOPIC PRACTICE 1

OBJECTIVE TYPE QUESTIONS

1 The study of coordinate geometry include
(a) coordinate axes and coordinate planes
(b) plotting of points in a plane
(c) distance between two points and section formulae
(d) All of the above

2 Distance between the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is
(a) $(x_2 - x_1) + (y_2 - y_1)$
(b) $(x_2 - x_1)^2 + (y_2 - y_1)^2$
(c) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
(d) $\sqrt[3]{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

3 The coordinates of a point dividing the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m : n$ are
(a) $\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}$ (b) $\frac{mx_1 + nx_2}{m + n}, \frac{my_1 + ny_2}{m + n}$
(c) $\frac{mx_2 - nx_1}{m + n}, \frac{my_2 - ny_1}{m + n}$ (d) $\frac{mx_1 - nx_2}{m + n}, \frac{my_1 - ny_2}{m + n}$

4 Coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2), are
(a) $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$ (b) $\frac{x_1 + x_2}{3}, \frac{y_1 + y_2}{3}$
(c) $\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}$ (d) $\frac{x_1 - x_2}{3}, \frac{y_1 - y_2}{3}$

VERY SHORT ANSWER Type Questions

5 Find the distance between the points
(i) $A(2, -3)$ and $B(6, -3)$
(ii) $A(1, -4)$ and $B(0, -3)$
(iii) $P(3, 4)$ and $Q(0, 0)$.

6 Find the point on the X-axis which is equidistant from the points (7, 6) and (3, 4).

7 Find a point on Y-axis which is equidistance from $A(-4, 3)$ and $B(5, 2)$.

8 If A is a point on the X-axis with abscissa -5 and B is a point on the Y-axis with ordinate 8. Find the distance AB.

9 Find the coordinates of the point which divides the join of $P(-5, 11)$ and $Q(4, -7)$ in the ratio 2 : 7.

SHORT ANSWER Type I Questions

10 Using the distance formula show that the points $A(3-2)$, $B(5, 2)$ and $C(8, 8)$ are collinear.

11 Show that the points $A(4, -1)$, $B(6, 0)$, $C(7, 2)$ and $D(5, 1)$ are the vertices of a rhombus.

12 If the points $A(-2, -1)$, $B(1, 0)$, $C(x, 3)$ and $D(1, y)$ are the vertices of a parallelogram, find the values of x and y (without using distance formula).

13 Find the area of ΔABC, the mid-points of whose sides AB, BC and CA are $D(3, -1)$, $E(5, 3)$ and $F(1, -3)$, respectively.

14 In what ratio is the line segment joining the points $A(-4, 2)$ and $B(8, 3)$ divided by the Y-axis? Also, find the point of intersection.

SHORT ANSWER Type II Questions

15 The base of an equilateral triangle with side $2a$ lies along the Y-axis such that the mid point of the base is at the origin. Find vertices of the triangle. **[NCERT]**

16 Show that the points $A(7, 10)$, $B(-2, 5)$ and $C(3, -4)$ are the vertices of an isosceles right angled triangle.

17 Draw the quadrilateral in the cartesian plane, whose vertices are $(-4, 5)$, $(0, 7)$, $(5, -5)$ and $(-4, -2)$. Also, find its area. **[NCERT]**

18 If four points $A(6, 3)$, $B(-3, 5)$, $C(4, -2)$ and $D(x, 3x)$ are given in such a way that $\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$, then find x.

19 The area of a triangle is 5 sq units and two of its vertices are $(2, 1)$ and $(3, -2)$. If third vertex is (x, y), where $y = x + 3$, then find the coordinates of the third vertex.

20 Find the locus of a point such that the sum of its distances from the point $(0, 2)$ and $(0, -2)$ is 6.

21 Find the locus of a point at which the angle subtended by the line segment joining $(1, 2)$ and $(-1, 3)$ is a right angle.

22 If the sum of the distance of a moving point in a plane from the axes is 1, then find the locus of the point. **[NCERT Exemplar]**

HINTS & ANSWERS

1. (d) The study of coordinate geometry include coordinate axes, coordinate plane, plotting of points in a plane, distance between two points and section formulae.

2. (c) Distance between the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

3. (a) The coordinates of a point dividing the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m : n$ are $\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$

4. (a) The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

5. (i) 4 (ii) $\sqrt{2}$ (iii) 5

6. Let the point be $P(x, 0)$.

Given, $(x - 7)^2 + (0 - 6)^2 = (x - 3)^2 + (0 - 4)^2$

Ans. $\left(\frac{15}{2}, 0\right)$

7. Similar as Q. 6. **Ans.** $(0, -2)$

8. Point $A = (-5, 0)$ and $B = (0, 8)$, then use distance formula.

Ans. $\sqrt{89}$

9. Solve as Example 5. **Ans.** $(-3, 7)$

10. Show that $AB + BC = AC$

11. Show that $AB = CD = BC = DA$

12. Mid-point of AC = Mid-point of BD **Ans.** $x = 4$, $y = 2$

13. Since, D, E and F are the mid-points of sides AB, BC and CA, respectively of a ΔABC.

$\therefore$ ar$(\Delta ABC) = 4 \times$ ar(ΔDEF) **Ans.** 8 sq units

14. Let the required ratio be $k : 1$. Then, the point of intersection are $\left(\frac{8k - 4}{k + 1}, \frac{3k + 2}{k + 1}\right)$.

Since, this point lies on Y-axis, therefore $\frac{8k - 4}{k + 1} = 0$

$\Rightarrow \quad k = \frac{1}{2}$ **Ans.** Ratio = 1 : 2, Point = $\left(0, \frac{7}{3}\right)$

15. Let ΔABC be the given equilateral triangle, with base BC on Y-axis. Then, cordinates of B and C are $(0, a)$ and $(0, -a)$ respectively. Let the coordinates of A be (h, k).

Since, ΔABC is an equilateral, therefore $AB = BC = AC$

$\Rightarrow \quad AB = AC \Rightarrow AB^2 = AC^2$

$\Rightarrow h^2 + (k - a)^2 = h^2 + (k + a)^2 \Rightarrow k = 0$

Also, $\quad AB^2 = BC^2$

$\therefore \quad (h - 0)^2 + (0 - a)^2 = (2a)^2 \Rightarrow h = \pm\sqrt{3}a$

Ans. $(\sqrt{3}a, 0), (0, a), (0, -a)$ or $(-\sqrt{3}a, 0), (0, a), (0, -a)$

16. First, find out the values of AB, BC, CA by distance formula and prove that ΔABC is an isosceles triangle, then prove that this triangle is also right angled triangle by converse of Pythagoras theorem.

17. Let the given points be $A(-4, 5)$, $B(0, 7)$, $C(5, -5)$ and $D(-4, -2)$. Area of quadrilateral $ABCD$

$$= \text{Area of } \Delta ADC + \text{Area of } \Delta ABC$$

Now, area of $\Delta ADC = \frac{1}{2}|[-4(-2 + 5) - 4(-5 - 5) + 5(5 + 2)]|$

Area of $\Delta ABC = \frac{1}{2}|[-4(7 + 5) + 0(-5 - 5) + 5(5 - 7)]|$

Ans. $\frac{121}{2}$ sq units

18. $\frac{11}{8}$

19. $\frac{1}{2}|x(1+2)+2(-2-y)+3(y-1)|=5 \Rightarrow |3x+y-7|=10$

$\Rightarrow \quad 3x+y-7=10$ or $3x+y-7=-10 \Rightarrow 3x+y=17$

or $\quad 3x+y=-3$

***Case* I** When $3x+y=17$ ...(i)

It is given that $y=x+3 \Rightarrow x-y=-3$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$x=\frac{7}{2} \text{ and } y=\frac{13}{2}$$

***Case* II** When $3x+y=-3$ ...(iii)

On solving Eqs. (ii) and (iii), we get

$$x=\frac{-3}{2} \text{ and } y=\frac{3}{2}$$

Ans. $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{-3}{2}, \frac{3}{2}\right)$

20. Let $P(h, k)$ be the moving and $A(0, 2)$ and $B(0, -2)$ be the given points. By the given condition, $PA+PB=6$

$\Rightarrow \quad \sqrt{(h-0)^2+(k-2)^2}+\sqrt{(h-0)^2+(k+2)^2}=6$

$\Rightarrow \quad (2k+9)=3\sqrt{h^2+(k+2)^2}$

On squaring both sides, we get

$$(2k+9)^2=9[h^2+(k+2)^2] \Rightarrow 9h^2+5k^2=45$$

Ans. $9x^2+5y^2=45$

21. Let the given points be $A(1, 2)$ and $B(-1, 3)$. Let $P(h, k)$ be the moving point. Then, $\angle APB=90°$.

Now, in ΔAPB, $AP^2+PB^2=AB^2$

$\Rightarrow (h-1)^2+(k-2)^2+(h+1)^2+(k-3)=(-1-1)^2+(3-2)^2$

Ans. $x^2+y^2-5y+5=0$

22. $|x|+|y|=1$

$\Rightarrow \pm x \pm y=1$, which forms a square.

Ans. The locus of the point is a square.

|TOPIC 2|

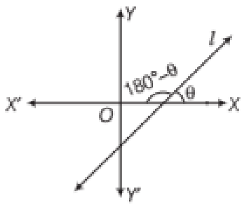
Slope of a Straight Line

A **straight line** is a curve, such that all the points on the line segment joining any two points of it lies on it.

Angle of Inclination of a Line

An angle 'θ' made by the line with positive X-axis in anti-clockwise direction is called angle of inclination of a line. A line in coordinate plane forms two angles with the X-axis, which are supplementary.

Thus, $$0° \leq \theta \leq 180°$$



Note

(i) When θ = 0°, then line is parallel to X-axis (horizontal line).

(ii) When θ = 90°, then line is perpendicular to X-axis i.e. parallel to Y-axis (vertical line).

Slope or Gradient of a Line

If θ is the angle of inclination of a line l, then tan θ is called the slope or gradient of the line l and it is denoted by m.

i.e. $$m=\tan\theta$$

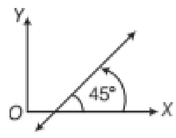
or

The slope of a line is the tangent of the angle made by the line in the anti-clockwise direction with the positive X-axis.

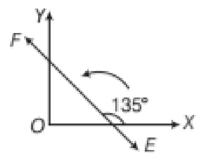
i.e. $$m=\tan\theta$$

e.g. For figure (i), $m=\tan 45°=1$

and for figure (ii), $m=\tan 135°=-1$



(i)



(ii)

Note

(i) The slope of X-axis is, $m=\tan 0°=0$.

(ii) The slope of a line, when θ = 90° is not defined, i.e. the slope of Y-axis is not defined.

EXAMPLE |1| Find the slope of line, whose inclination is 60° and 150°.

Sol. Let θ be the inclination of a line, then its slope $= \tan\theta$.

At $\theta = 60°$, slope of a line $= \tan 60° = \sqrt{3}$

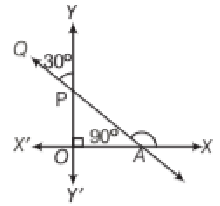
At $\theta = 150°$, slope of a line $= \tan(150°) = \tan(180° - 30°)$

$= -\tan 30° = -\frac{1}{\sqrt{3}}$ $[\because \tan(180° - \theta) = -\tan\theta]$

EXAMPLE |2| Find the slope of a line, which makes an angle of 30° with the positive direction of Y-axis measured anti-clockwise. **[NCERT]**

Sol. Given, $\angle YPQ = 30°$

To find $\angle PAX = ?$



Here, $\angle YPQ = \angle OPA$ [vertically opposite angle]

$\because$ $\angle OPA + \angle POA + \angle PAO = 180°$

[$\because$ sum of all angles of a triangle is 180°]

$\therefore$ $30° + 90° + \angle PAO = 180°$

$\Rightarrow$ $\angle PAO = 180° - 120° = 60°$

$\Rightarrow$ $\angle PAX = 180° - 60° = 120°$

[by linear pair axiom]

$\therefore$ Slope of line $AQ = m = \tan 120°$ $[\because m = \tan\theta]$

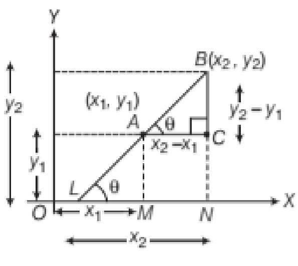
$= \tan(180° - 60°)$ $[\because \tan(180° - \theta) = -\tan\theta]$

$= -\tan 60° = -\sqrt{3}$

Slope of a Line Joining Two Points

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two given points and θ be the inclination of the line AB, so that $m = \tan\theta$.

From A and B, draw AM and BN perpendiculars to the axis and draw $AC \perp BN$. Let BA meets OX at L.



$\angle CAB = \angle XLB = \theta$ [corresponding angles]

$AC = MN = ON - OM = x_2 - x_1$

$BC = BN - CN$

$= BN - AM = y_2 - y_1$

In ΔACB, we have $\tan\theta = \frac{BC}{AC}$

$$\Rightarrow \quad \boxed{m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } \frac{y_1 - y_2}{x_1 - x_2}}$$

This relation is true in both the cases, whether θ is an acute angle or an obtuse angle.

EXAMPLE |3| Find the slope of a line joining following two points

(i) $A(1, 2)$ and $B(3, -4)$ (ii) $(3, -2)$ and $(7, -2)$.

Sol. We know that, slope of a line joining two points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$

(i) Given, $A(1, 2) \equiv (x_1, y_1)$ and $B(3, -4) \equiv (x_2, y_2)$

$\therefore$ Slope of line joining points A and B is

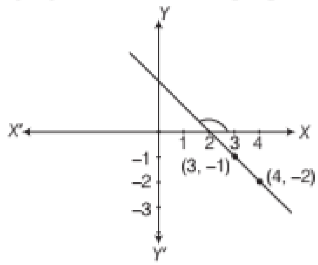
$$m = \frac{-4 - 2}{3 - 1} = \frac{-6}{2} = -3$$

(ii) Slope of line joining the points $(3, -2)$ and $(7, -2)$ is

$$m = \frac{-2 - (-2)}{7 - 3} = \frac{0}{4} = 0$$

EXAMPLE |4| Find the angle between the X-axis and the line joining the points $(3, -1)$ and $(4, -2)$. **[NCERT]**

Sol. Given, $(x_1, y_1) \equiv (3, -1)$ and $(x_2, y_2) \equiv (4, -2)$



Slope of line, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 + 1}{4 - 3}$

$\Rightarrow$ $\tan\theta = \frac{-1}{1} \Rightarrow \tan\theta = -1$

$\Rightarrow$ $\tan\theta = -\tan 45°$ $[\because 1 = \tan 45°]$

$\Rightarrow$ $\tan\theta = \tan(180° - 45°)$

$[\because \tan(180° - \theta) = -\tan\theta]$

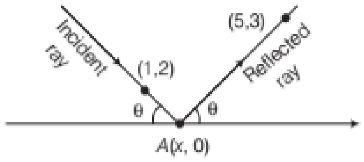
$\Rightarrow$ $\tan\theta = \tan 135° \Rightarrow \theta = 135°$

Hence, the required angle between X-axis and the line joining the points $(3, -1)$ and $(4, -2)$ is 135°.

EXAMPLE |5| A ray of light coming from the point (1, 2) is reflected at a point A on the X-axis and then passes through the point (5, 3). Find the coordinates of the point A. **[NCERT Exemplar]**

Sol. Let the coordinates of point A be $(x, 0)$. From the figure, the slope of the reflected ray is given by

$$\tan\theta = \frac{3}{5-x} \quad \text{...(i)}$$



Again, the slope of the incident ray is given by

$$\tan(\pi-\theta) = \frac{-2}{x-1} \qquad \left[\because m = \frac{y_2-y_1}{x_2-x_1}\right]$$

$$\Rightarrow \quad -\tan\theta = \frac{-2}{x-1} \qquad [\because \tan(\pi-\theta) = -\tan\theta]$$

$$\Rightarrow \quad \tan\theta = \frac{2}{x-1} \quad \text{...(ii)}$$

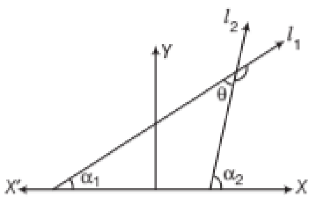
From Eqs. (i) and (ii), we get $\frac{3}{5-x} = \frac{2}{x-1}$

$$\Rightarrow \quad 3x-3 = 10-2x \Rightarrow x = \frac{13}{5}$$

Therefore, the required coordinates of the point A are $\left(\frac{13}{5}, 0\right)$.

Angle between Two Lines

Let l_1 and l_2 be two lines and their inclination are α_1 and α_2, respectively. Then, their slopes are $m_1 = \tan\alpha_1$ and $m_2 = \tan\alpha_2$.



Let θ be the angle between l_1 and l_2, then.

$$\tan\theta = \pm\frac{m_2-m_1}{1+m_1m_2}$$

For acute angle, we take $\tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right|$

or $\quad \theta = \tan^{-1}\left|\frac{m_2-m_1}{1+m_1m_2}\right|$

For obtuse angle, we take $\theta = \pi - \tan^{-1}\left|\frac{m_2-m_1}{1+m_1m_2}\right|$

Note

(i) If $\tan\theta$ is positive, then θ will be an acute angle.
(ii) If $\tan\theta$ is negative, then θ will be an obtuse angle.

EXAMPLE |6| Find the angle between the lines joining the points (0,0), (2,3) and the points (2, – 2), (3, 5).

Sol. Let θ be the angle between the given lines.

We have,

m_1 = Slope of the line joining (0, 0) and (2, 3) $= \frac{3-0}{2-0} = \frac{3}{2}$

m_2 = Slope of the line joining (2, – 2) and (3, 5)

$$= \frac{5+2}{3-2} = 7$$

Now, $\tan\theta = \pm\left(\frac{m_2-m_1}{1+m_1m_2}\right) = \pm\left(\frac{7-3/2}{1+7(3/2)}\right)$

$$= \pm\left(\frac{11/2}{23/2}\right) = \pm\left(\frac{11}{23}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{11}{23}\right) \text{ or } \pi - \tan^{-1}\left(\frac{11}{23}\right)$$

EXAMPLE |7| If the angle between two lines is $\frac{\pi}{4}$ and slope of one of the lines is $\frac{1}{2}$, then find the slope of the other line. **[NCERT]**

Sol. We know that, the acute angle θ between two lines with slopes m_1 and m_2 is given by

$$\tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right| \quad \text{...(i)}$$

Let $\quad m_1 = \frac{1}{2}, m_2 = m$ and $\theta = \frac{\pi}{4}$

Now, putting these values in Eq. (i), we get

$$\tan\frac{\pi}{4} = \left|\frac{m-\frac{1}{2}}{1+\frac{1}{2}m}\right| \Rightarrow 1 = \left|\frac{m-\frac{1}{2}}{1+\frac{1}{2}m}\right| \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$\Rightarrow \quad \pm 1 = \frac{m-\frac{1}{2}}{1+\frac{m}{2}} \Rightarrow \frac{m-\frac{1}{2}}{1+\frac{1}{2}m} = 1 \text{ or } \frac{m-\frac{1}{2}}{1+\frac{1}{2}m} = -1$$

$$\Rightarrow \quad m - \frac{1}{2} = 1 + \frac{1}{2}m \text{ or } m - \frac{1}{2} = -1 - \frac{1}{2}m$$

$$\Rightarrow \quad \left(1-\frac{1}{2}\right)m = 1 + \frac{1}{2} \text{ or } m\left(1+\frac{1}{2}\right) = -1 + \frac{1}{2}$$

$$\Rightarrow \quad m = 3 \text{ or } m = -\frac{1}{3}$$

EXAMPLE |8| The slope of a line is double of the slope of another line. If tangent of the angle between them is $\frac{1}{3}$, then find the slope of the lines. **[NCERT]**

Use $\tan\theta = \left|\frac{m_2-m_1}{1+m_2m_1}\right|$ and solve it.

Sol. If slope of one line is m. Then, the slope of the other line is $2m$. Let angle between these two lines be θ.

Then, $\quad \tan\theta = \frac{1}{3} \quad$ [given]

$$\Rightarrow \quad \left|\frac{2m-m}{1+2m\cdot m}\right| = \frac{1}{3} \qquad \left[\because \tan\theta = \left|\frac{m_2-m_1}{1+m_1\cdot m_2}\right|\right]$$

$$\Rightarrow \quad \frac{m}{1+2m^2} = \pm\frac{1}{3}$$

Here, two cases arise.

***Case* I** When $\frac{m}{1+2m^2} = \frac{1}{3}$, then $3m = 1 + 2m^2$

$$\Rightarrow \quad 2m^2 - 3m + 1 = 0$$

$$\Rightarrow \quad 2m^2 - 2m - m + 1 = 0$$

$$\Rightarrow \quad 2m(m-1) - 1(m-1) = 0$$

$$\Rightarrow \quad (m-1)(2m-1) = 0$$

$$\Rightarrow m - 1 = 0 \text{ or } 2m - 1 = 0 \Rightarrow m = 1 \text{ or } m = \frac{1}{2}$$

As the slopes of the lines are m and $2m$, therefore the slopes of the lines are 1 and 2 or $\frac{1}{2}$ and 1.

***Case* II** When $\frac{m}{1+2m^2} = \frac{-1}{3}$, then

$$3m = -1 - 2m^2 \Rightarrow 2m^2 + 3m + 1 = 0$$

$$\Rightarrow \quad 2m^2 + 2m + m + 1 = 0$$

$$\Rightarrow \quad 2m(m+1) + 1(m+1) = 0$$

$$\Rightarrow \quad (2m+1)(m+1) = 0$$

$$\Rightarrow \quad 2m + 1 = 0 \text{ or } m + 1 = 0$$

$$\Rightarrow \quad m = \frac{-1}{2} \text{ or } m = -1$$

As the slopes of the lines are m and $2m$, therefore the slopes of the lines are $\frac{-1}{2}$ and -1 or -1 and -2.

Condition of Parallelism of Lines

If two lines of slopes m_1 and m_2 are parallel, then the angle θ between them is 0°.

$$\therefore \quad \tan\theta = \tan 0° = 0 \quad [\because \tan 0° = 0]$$

$$\Rightarrow \quad \frac{m_2 - m_1}{1 + m_1 m_2} = 0$$

$$\Rightarrow \quad m_2 - m_1 = 0 \Rightarrow m_1 = m_2$$

Thus, two lines are parallel, if and only if their slopes are equal i.e. iff $m_1 = m_2$.

EXAMPLE |9| What is the value of y so that the line through (3, y) and (2, 7) is parallel to the line through (−1, 4) and (0, 6)?

Sol. Let $A(3, y)$, $B(2, 7)$, $C(-1, 4)$ and $D(0, 6)$ be the given points. Then,

$$m_1 = \text{Slope of the line } AB = \frac{7-y}{2-3} = (y-7)$$

and $m_2 = \text{Slope of the line } CD = \frac{6-4}{0-(-1)} = 2$

Since, AB and CD are parallel.

$$\therefore \quad m_1 = m_2 \Rightarrow y - 7 = 2 \Rightarrow y = 9$$

EXAMPLE |10| Line through the points (−2, 6) and (4, 8) is perpendicular to the line through the (8, 12) and (x, 24). Find the value of x. **[NCERT]**

Sol. Slope of the line through the points (−2, 6) and (4, 8) is

$$m_1 = \frac{8-6}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$$

Slope of the line through the points (8, 12) and (x, 24) is

$$m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$$

Since, two lines are perpendicular,

$$\therefore \quad m_1 m_2 = -1$$

$$\Rightarrow \quad \frac{1}{3} \times \frac{12}{x-8} = -1 \Rightarrow 4 = -(x-8) \Rightarrow x = 4$$

Condition of Perpendicularity of Two Lines

If two lines of slopes m_1 and m_2 are perpendicular, then the angle θ between them is 90°.

$$\therefore \quad \tan\theta = \tan 90° = \frac{\sin 90°}{\cos 90°} = \frac{1}{0}$$

$$\Rightarrow \quad \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{1}{0} \Rightarrow 1 + m_1 m_2 = 0$$

$$\therefore \quad m_1 m_2 = -1$$

Thus, two lines are perpendicular, if and only if their slopes m_1 and m_2 satisfy the following condition

$$\boxed{m_1 \cdot m_2 = -1} \quad \text{or} \quad \boxed{m_1 = -\frac{1}{m_2}}$$

Hence, two lines are perpendicular to each other, if and only if their slopes are negative reciprocals of each other.

EXAMPLE |11| Using slopes, prove that the points $A(-2, -1)$, $B(4, 0)$, $C(3, 3)$ and $D(-3, 2)$ are the vertices of a parallelogram.

Sol. Given points are $A(-2, -1)$, $B(4, 0)$, $C(3, 3)$ and $D(-3, 2)$.

Now, $\quad \text{slope of } AB = \frac{0-(-1)}{4-(-2)} = \frac{1}{6}$

$$\text{Slope of } DC = \frac{3-2}{3-(-3)} = \frac{1}{6}$$

$$\text{Slope of } BC = \frac{3-0}{3-4} = \frac{3}{-1} = -3$$

and $\quad \text{slope of } AD = \frac{2-(-1)}{-3-(-2)} = \frac{3}{-1} = -3$

$\because$ Slope of AB = Slope of DC, therefore $AB \parallel DC$

Slope of BC = Slope of AD, therefore $BC \parallel AD$

Thus, $ABCD$ is a parallelogram. **Hence proved.**

EXAMPLE |12| Without using Pythagoras theorem, show that $A(4, 4)$, $B(3, 5)$ and $C(-1, 1)$ are the vertices of a right angled triangle. **[NCERT]**

Sol. In ΔABC, we have m_1 = Slope of $AB = \frac{4-5}{4-3} = -1$

and m_2 = Slope of $BC = \frac{5-1}{3-(-1)} = \frac{4}{4} = 1$

Clearly, $m_1 m_2 = -1$

This shows that AB is perpendicular to BC.

i.e. $\angle ABC = \pi / 2$

Hence, the given points are the vertices of a right angled triangle. **Hence proved.**

Collinearity of Three Points

If A, B and C are three points in XY-plane, then they will be collinear i.e. will lie on a same line if and only if

Slope of AB = Slope of BC.

EXAMPLE |13| Prove that the points $A(1, 4)$, $B(3, -2)$ and $C(4, -5)$ are collinear.

Sol. Given points are $A(1, 4)$, $B(3, -2)$ and $C(4, -5)$

From the condition of collinearity of three points A, B and C, we should have

Slope of AB = Slope of BC

i.e. $$\frac{-2-4}{3-1} = \frac{-5+2}{4-3} \quad \left[\because \text{slope} = \frac{y_2 - y_1}{x_2 - x_1}\right]$$

$\Rightarrow \frac{-6}{2} = \frac{-3}{1} \Rightarrow -3 = -3$, which is true.

Thus, points A, B and C are collinear. **Hence proved.**

EXAMPLE |14| If three points $(h, 0)$, (a, b) and $(0, k)$ lie on a line, show that $\frac{a}{h} + \frac{b}{k} = 1$. **[NCERT]**

Sol. Let the given points be $A(h, 0)$, $B(a, b)$ and $C(0, k)$.

If A, B and C are collinear, then

Slope of AB = Slope of BC = Slope of CA

$$\Rightarrow \frac{b-0}{a-h} = \frac{k-b}{0-a} = \frac{0-k}{h-0}$$

On taking first two terms, we get

$$\frac{b}{a-h} = \frac{k-b}{-a}$$

$\Rightarrow -ab = (a-h)(k-b)$

$\Rightarrow -ab = ak - ab - hk + bh$

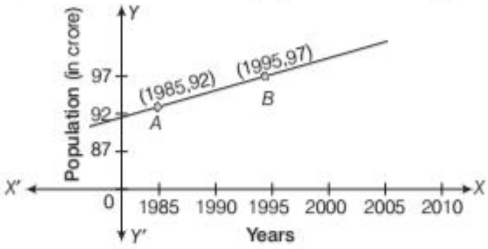
$\Rightarrow ak + bh = hk$

Dividing each term by hk, we get

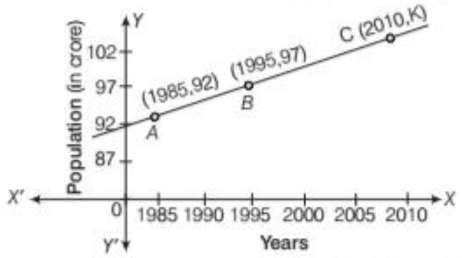
$$\frac{ak}{hk} + \frac{bh}{hk} = \frac{hk}{hk}$$

$\Rightarrow \frac{a}{h} + \frac{b}{k} = 1$ **Hence proved.**

EXAMPLE |15| Consider the following population and year graph (in figure). Find the slope of the line AB and using it, find what will be the population in the year 2010?



Sol. Let the population in the year 2010 will be k crore. Then, $C(2010, k)$ be on the year population graph.



Now, slope of $AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{97-92}{1995-1985} = \frac{5}{10}$

$[\because x_1 = 1985, x_2 = 1995, y_1 = 92 \text{ and } y_2 = 97]$

$\therefore$ Slope of $AB = \frac{1}{2}$ ...(i)

Since, A, B and C lie on the same line, i.e. A, B and C are collinear.

$\therefore$ Slope of AB = Slope of BC

$\Rightarrow \frac{1}{2} = \frac{k-97}{2010-1995}$

$\Rightarrow \frac{1}{2} = \frac{k-97}{15} \Rightarrow k - 97 = \frac{15}{2}$

$\Rightarrow k = 97 + \frac{15}{2} = 97 + 7.5 = 104.5$

Hence, slope of $AB = \frac{1}{2}$ and population in the year 2010 = 104.5 crore

TOPIC PRACTICE 2

OBJECTIVE TYPE QUESTIONS

1 If θ is the inclination of a line l, then the slope or gradient of the line l is
(a) $\sin\theta$ (b) $\cos\theta$ (c) $\tan\theta$ (d) $\cot\theta$

2 The slope of a line whose inclination is $90°$, is
(a) 1 (b) 0 (c) -1 (d) not defined

3 The slope of a line is denoted by m. Thus,
(a) $m = \tan\theta$ (b) $m = \tan\theta, \theta \neq 90°$
(c) $m = \tan\theta, \theta \neq 0°$ (d) None of these

4 The acute angle (θ) between lines L_1 and L_2 with slopes m_1 and m_2 respectively is given by
(a) $\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$, as $1 + m_1 m_2 \neq 0$
(b) $\tan\theta = \dfrac{m_2 - m_1}{1 + m_1 m_2}$, as $1 + m_1 m_2 \neq 0$
(c) $\tan\theta = \left|\dfrac{m_2 - m_1}{1 + m_1 m_2}\right|$, as $1 + m_1 m_2 \neq 0$
(d) None of the above

5. The value of x for which the points $(x, -1)$, $(2, 1)$ and $(4, 5)$ are collinear, will be
(a) 0 (b) 1 (c) 2
(d) 3

VERY SHORT ANSWER Type Questions

6 Find the slope of a line whose inclination is
(i) $30°$ (ii) $135°$

7 Find the inclination of a line whose slope is
(i) $\dfrac{1}{\sqrt{3}}$ (ii) $-\sqrt{3}$

8 Find the slope of a line which passes through the points
(i) $(0, -3)$ and $(2, 1)$ (ii) $(-2, 3)$ and $(4, -6)$
(iii) $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$

9 If the slope of the line joining the points $A(x, 2)$ and $B(6, -8)$ is $-5/4$, find the value of x.

10 Find the slope and inclination of line through pair of points $(1, 2)$ and $(5, 6)$.

11 Find the slope of a line perpendicular to the line, which passes through $(0, 8)$ and $(-5, 2)$.

12 What is the value of y, so that the line through $(3, y)$ and $(2, 7)$ is parallel to the line through $(-1, 4)$ and $(0, 6)$?

13 State whether the two lines in each of the following are parallel, perpendicular or neither.
(i) Through $(5, 6)$ and $(2, 3)$; through $(9, -2)$ and $(6, -5)$.
(ii) Through $(6, 3)$ and $(1, 1)$; through $(-2, 5)$ and $(2, -5)$
(iii) Through $(3, 15)$ and $(16, 6)$; through $(-5, 3)$ and $(8, 2)$.

14 Find the angle between the lines whose slopes are
(i) $\sqrt{3}$ and $\dfrac{1}{\sqrt{3}}$
(ii) $(2 - \sqrt{3})$ and $(2 + \sqrt{3})$

SHORT ANSWER Type Questions

15 Without using Pythagoras theorem, show that $A(12, 8)$, $B(-2, 6)$ and $C(6, 0)$ are the vertices of right angled triangle.

16 Find the coordinate of the orthocentre of the triangle whose vertices are $(-1, 3)$, $(2, -1)$ and $(0, 0)$.

17 By using slope method, show that the points $P(4, 8)$, $Q(5, 12)$ and $R(9, 28)$ are collinear.

18 Find the slope of a line, which passes through the origin and mid-point of the line segment joining the points $P(0, -4)$ and $Q(8, 0)$. **[NCERT]**

19 A line passes through the points $A(4, -6)$ and $B(-2, -5)$. Show that the line AB makes an obtuse angle with the X-axis.

20 Determine $\angle B$ of the triangle with vertices $A(-2, 1)$, $B(2, 3)$ and $C(-2, -4)$.

21 A quadrilateral has the vertices at the points $A(-4, 2)$, $B(2, 6)$, $C(8, 5)$ and $D(9, -7)$. Show that the mid-points of the sides of this quadrilateral are the vertices of a parallelogram.

22 If θ is the angle between the diagonals of a parallelogram $ABCD$ whose vertices are $A(0, 2)$, $B(2, -1)$, $C(4, 0)$ and $D(2, 3)$. Show that $\tan\theta = 2$.

23 Using slopes, show that the points $A(-4, -1)$, $B(-2, -4)$, $C(4, 0)$ and $D(2, 3)$ taken in order, are the vertices of a rectangle.

24 Two lines passing through the point $(2, 3)$ make an angle of $45°$. If the slope of one of the lines is 2, then find the slope of the other line.

HINTS & ANSWERS

1. (c) If θ is the inclination of a line l, then $\tan\theta$ is called the slope or gradient of the line l.

2. (d) The slope of a line whose inclination is 90°, is not defined.

3. (b) The slope of a line is denoted by m. Thus, $m = \tan\theta$, $\theta \neq 90°$.

4. (c) The acute angle (θ) between lines L_1 and L_2, with slopes m_1 and m_2 respectively, is given by

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|, \text{ as } 1 + m_1 m_2 \neq 0$$

5. (b) Let $x_1 = x$, $y_1 = -1$, $x_2 = 2$, $y_2 = 1$, $x_3 = 4$ and $y_3 = 5$

Slope of AB = Slope of BC

$$\Rightarrow \quad \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2}$$

$$\Rightarrow \quad \frac{1+1}{2-x} = \frac{5-1}{4-2} \quad \left(\because \text{ slope of line} = \frac{y_2 - y_1}{x_2 - x_1}\right)$$

$$\Rightarrow \quad x = 2 - 1 = 1$$

6. Use the formula, $m = \tan\theta$ **Ans.** (i) $\frac{1}{\sqrt{3}}$ (ii) -1

7. (i) 30° (ii) 120° **8.** (i) 2 (ii) $-\frac{3}{2}$ (iii) $\frac{2}{t_2 + t_1}$

9. $x = -2$ **10.** 1; 45° **11.** $-\frac{5}{6}$

12. $m_1 = \frac{7-y}{2-3}$, $m_2 = \frac{6-4}{0-(-1)}$

Also, $m_1 = m_2 \Rightarrow \frac{7-y}{-1} = 2 \Rightarrow y = 9$

13. (i) parallel (ii) perpendicular (iii) neither

14. (i) 30° or 150° (ii) 60° or 120°

15. Let $A(12, 8)$, $B(-2, 6)$ and $C(6, 0)$ are vertices of ΔABC

Slope of $AB = \frac{6-8}{-2-12} = \frac{-2}{-14} = \frac{1}{7}$

Slope of $BC = \frac{0-6}{6+2} = \frac{-6}{8} = \frac{-3}{4}$

Slope of $AC = \frac{0-8}{6-12} = \frac{-8}{-6} = \frac{4}{3}$

Now, as slope of $BC \times$ slope of $AC = \frac{-3}{4} \times \frac{4}{3} = -1$,

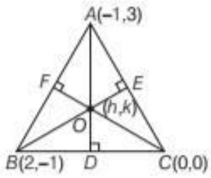
therefore $\angle C = \frac{\pi}{2}$.

16. The orthocentre is the point of intersection of the altitudes from the vertices to the opposite sides.

Slope of line $AO \times$ Slope of line $BC = -1$

$$\Rightarrow \quad \frac{k-3}{h+1} \times \left(-\frac{1}{2}\right) = -1 \quad [\because m_1 \times m_2 = -1]$$

$$\Rightarrow \quad \frac{k-3}{h+1} = 2 \Rightarrow 2h - k + 5 = 0 \quad \text{...(i)}$$



Also, $BO \perp AC$

$\therefore$ Slope of line BO $\times$ Slope of line $AC = -1$

$$\Rightarrow \quad \frac{k+1}{h-2} \times \frac{3-0}{-1-0} = -1$$

$$h - 3k - 5 = 0 \quad \text{...(ii)}$$

Ans. $(-4, -3)$

17. Slope of PQ = Slope of QR

18. Mid-point of $PQ = \left(\frac{0+8}{2}, \frac{-4+0}{2}\right) = (4, -2)$

Ans. Slope of required line $= -\frac{1}{2}$

19. Slope of AB is negative.

20. Slope of line, $AB = \frac{3-1}{2+2} = \frac{2}{4} = \frac{1}{2} = m_1$ (say)

Slope of line, $BC = \frac{-4-3}{-2-2} = \frac{7}{4} = m_2$

$$\therefore \quad \tan B = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{\frac{7}{4} - \frac{1}{2}}{1 + \frac{1}{2}\cdot\frac{7}{4}}\right|$$

Ans. $\angle B = \tan^{-1}\left(\frac{2}{3}\right)$

21. Coordinates of E = Mid-point of $A(-4, 2)$ and $B(2, 6)$

$$= \left(\frac{-4+2}{2}, \frac{2+6}{2}\right) = (-1, 4)$$

Coordinates of F = Mid-point of $B(2, 6)$ and $C(8, 5)$

$$= \left(\frac{2+8}{2}, \frac{6+5}{2}\right) = \left(5, \frac{11}{2}\right)$$

Coordinates of G = Mid-point of $C(8, 5)$ and $D(9, -7)$

$$= \left(\frac{8+9}{2}, \frac{5-7}{2}\right) = \left(\frac{17}{2}, -1\right)$$

Coordinates of H = Mid-point of $A(-4, 2)$ and $D(9, -7)$

$$= \left(\frac{9-4}{2}, \frac{-7+2}{2}\right) = \left(\frac{5}{2}, \frac{-5}{2}\right)$$

$$\text{Slope of } EF = \frac{\frac{11}{2} - 4}{5 - (-1)} = \frac{1}{4}$$

$$\text{Slope of } GH = \frac{-\frac{5}{2} - (-1)}{\frac{5}{2} - \frac{17}{2}} = \frac{1}{4}$$

$\therefore$ Slope of EF = Slope of $GH \Rightarrow EF \parallel GH$

Similarly, slope of FG = slope of $EH \Rightarrow FG \parallel EH$

22. Slope of AC, $m_1 = \frac{-2}{4} = -\frac{1}{2}$

Slope of BD, $m_2 = \frac{4}{0} = \tan 90°$. Let $m_1 = \tan\alpha = -\frac{1}{2}$.

Then, we have $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

$= \left|\frac{\tan 90° - \tan\alpha}{1 + \tan 90° \tan\alpha}\right| = |\tan(90° - \alpha)| = |(\cot\alpha)| = |-2| = 2$

$\Rightarrow \quad \tan\theta = 2$

23. Slope of AB = Slope of CD

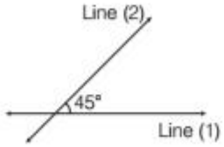
slope of BC = Slope of DA

Also, slope of $AB \times$ slope of $BC = -1$

24. Angle between these two lines is 45°.

$\therefore \quad 1 = \left|\frac{2 - m}{1 + 2m}\right|$

$\Rightarrow \quad 1 = \pm \frac{(2 - m)}{1 + 2m}$



Ans. $m = \frac{1}{3}$ or -3

|TOPIC 3|

Various Forms of Equation of Line

The equation of a straight line is the relation between x (the abscissa) and y (the ordinate) which is satisfied by the coordinates of each and every point on the line and is not satisfied by the coordinates of any point which does not lie on the line.

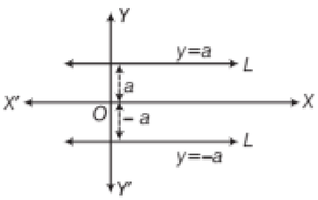
The various forms of the equation of a line under different conditions are given below

1. Equation of Line Parallel to X-axis (OR Equation of horizontal line)

Let L be a straight line parallel to X-axis at a distance a from it, then the ordinate of every point lying on the line is either a or $-a$. Equation of line parallel to X-axis (i.e. equation of horizontal line) is either

$$y = a \text{ or } y = -a$$

The choice of sign will depend upon the position of the line according as the line is on the positive or negative side of the Y-axis.



EXAMPLE |1| Write down the equation of a line parallel to the X-axis

(i) at a distance of 6 units above the X-axis.

(ii) at a distance of 3 units below the X-axis.

Sol. (i) The equation of a line parallel to the X-axis at a distance of 6 units above X-axis is $y = 6$.

(ii) The equation of a line parallel to the X-axis at a distance of 3 units below the X-axis is $y = -3$.

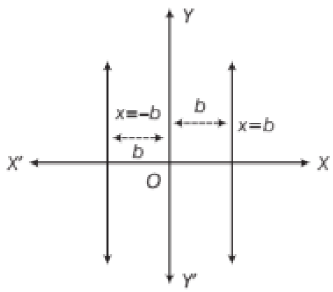
2. Equation of Line Parallel to Y-axis (OR Equation of vertical line)

Let L be a straight line parallel to Y-axis at a distance b from it, then the abscissa of every point lying on the line is either b or $-b$.

$\therefore$ Equation of line parallel to Y-axis (i.e. equation of vertical line) is either

$$x = b \text{ or } x = -b$$

Here, the choice of sign will depend upon the position of the line according as the line is on the positive or negative side of the X-axis.



Note

(i) Equation of X-axis is $y = 0$.

(ii) Equation of Y-axis is $x = 0$.

EXAMPLE |2| Write down the equation of a line parallel to the Y-axis

(i) at a distance of 5 units on left hand side of the Y-axis,

(ii) at a distance of 7 units on right hand side of the Y-axis.

Sol. (i) The equation of a line parallel to the Y-axis at a distance of 5 units on its left is $x = -5$.

(ii) The equation of a line parallel to the Y-axis at a distance of 7 units on its right is $x = 7$.

EXAMPLE |3| Find the equation of the lines parallel to the axes and passing through the point $(-3, 5)$.

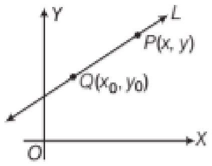
Sol. Clearly, the equation of a line parallel to the X-axis and passing through $(-3, 5)$ is $y = 5$.

The equation of a line parallel to the Y-axis and passing through $(-3, 5)$ is $x = -3$.

Point Slope Form

The equation of the straight line having slope m and passes through the point $Q(x_0, y_0)$ is

$$\boxed{y - y_0 = m(x - x_0)}$$



EXAMPLE |4| Find the equation of the line passing through $(-4, 3)$ and having slope $\frac{1}{2}$.

Sol. Equation of the line passing through the point (x_0, y_0) and having slope m is $\quad y - y_0 = m(x - x_0) \quad$ ...(i)

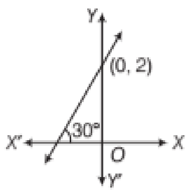
Given, m = slope of the line $= \frac{1}{2}$ and $x_0 = -4,\ y_0 = 3$

From Eq. (i), required equation of the line is

$$y - 3 = \frac{1}{2}(x + 4) \Rightarrow 2y - 6 = x + 4 \quad \Rightarrow x - 2y + 10 = 0$$

EXAMPLE |5| Find the equation of line intersecting the Y-axis at a distance of 2 units above the origin and making an angle of 30° with the positive direction of the X-axis. **[NCERT]**

Sol. Given, the line intersect the Y-axis at distance of 2 units above the origin. It shows that the line passes through $(0, 2)$.



$\because$ It makes an angle 30° with the positive direction of X-axis. So, the slope of the line,

$$m = \tan 30° = \frac{1}{\sqrt{3}} \qquad [\because m = \tan\theta]$$

Thus, the equation of straight line is

$$y - 2 = \frac{1}{\sqrt{3}}(x - 0) \quad [\because y - y_0 = m(x - x_0)]$$

$$\Rightarrow \frac{x}{\sqrt{3}} - y + 2 = 0 \Rightarrow x - \sqrt{3}\,y + 2\sqrt{3} = 0$$

EXAMPLE |6| If the line joining two points $A(2, 0)$ and $B(3, 1)$ is rotated about A in anti-clockwise direction through an angle of 15°. Find the equation of the line in new position. **[NCERT Exemplar]**

Sol. The slope of line AB is given by

$$m = \frac{1-0}{3-2} \qquad \left[\because m = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)\right]$$

$$\Rightarrow \tan\theta = 1 \qquad [\because m = \tan\theta]$$

$$\Rightarrow \tan\theta = \tan 45° \qquad [\because 1 = \tan 45°]$$

$$\Rightarrow \theta = 45°$$

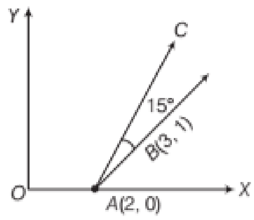
After rotation of the line AB about A is anticlockwise direction through an angle of 15°, the slope of the line AC in new position is given by

$$\Rightarrow m_1 = \tan(45° + 15°) = \tan 60°$$

$$= \sqrt{3} \qquad [\because \tan 60° = \sqrt{3}]$$

$\therefore$ The equation of the new line AC is

$$(y - y_1) = m_1(x - x_1) \Rightarrow (y - 0) = \sqrt{3}(x - 2)$$



$$\Rightarrow \sqrt{3}x - y - 2\sqrt{3} = 0$$

which is the required equation of the line.

EXAMPLE |7| Find the equation of the perpendicular bisector of the line segment joining the points $A(2, 3)$ and $B(6, -5)$.

Sol. The slope of AB is given by

$$m = \frac{-5-3}{6-2} = -2 \qquad \left[\because m = \frac{y_2 - y_1}{x_2 - x_1}\right]$$

$\therefore$ Slope of a line perpendicular to $AB = -\frac{1}{m} = \frac{1}{2}$

Let P be the mid-point of AB. Then, the coordinates of P are $\left(\frac{2+6}{2}, \frac{3-5}{2}\right)$ i.e. $(4, -1)$.

Thus, the required line passes through $P(4, -1)$ and has slope $\frac{1}{2}$. So, its equation is

$$y + 1 = \frac{1}{2}(x - 4) \qquad [\because y - y_0 = m(x - x_0)]$$

$$\Rightarrow \qquad x - 2y - 6 = 0$$

EXAMPLE |8| Two lines passing through the point (2, 3) intersect each other at an angle of 60°. If slope of one line is 2, then find the equation of the other line. **[NCERT]**

Sol. Let the slope of the other line be m.

It is given that the angle between the two lines is 60°.

$$\therefore \qquad \tan 60° = \left|\frac{m-2}{1+2m}\right| \qquad \left[\because \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|\right]$$

$$\Rightarrow \qquad \sqrt{3} = \left|\frac{m-2}{1+2m}\right| \Rightarrow \frac{m-2}{1+2m} = \pm\sqrt{3}$$

$$\Rightarrow \qquad \frac{m-2}{1+2m} = \sqrt{3} \text{ or } \frac{m-2}{1+2m} = -\sqrt{3}$$

$$\Rightarrow \qquad m - 2 = \sqrt{3} + 2\sqrt{3}\,m \text{ or } m - 2 = -\sqrt{3} - 2\sqrt{3}\,m$$

$$\Rightarrow \quad m(2\sqrt{3} - 1) = -(2 + \sqrt{3}) \text{ or } m(2\sqrt{3} + 1) = 2 - \sqrt{3}$$

$$\Rightarrow \qquad m = -\left(\frac{2+\sqrt{3}}{2\sqrt{3}-1}\right) \text{ or } m = \left(\frac{2-\sqrt{3}}{2\sqrt{3}+1}\right)$$

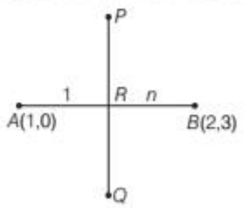
On substituting $x_1 = 2$, $y_1 = 3$ and the values of m in $y - y_1 = m(x - x_1)$, we obtain that the equation of the required line is

$$y - 3 = -\frac{2+\sqrt{3}}{2\sqrt{3}-1}(x - 2) \text{ or } y - 3 = \frac{2-\sqrt{3}}{2\sqrt{3}+1}(x - 2)$$

EXAMPLE |9| A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio $1 : n$. Find the equation of the line. **[NCERT]**

Sol. Let the given points be $A(1, 0)$ and $B(2, 3)$.

Let the line PQ divide AB in the ratio $1 : n$ at R internally.



Then, coordinates of R

$$= \left(\frac{1 \times x_2 + n \times x_1}{1+n}, \frac{1 \times y_2 + n \times y_1}{1+n}\right)$$

[by internal division formula]

$$= \left(\frac{1 \times 2 + n \times 1}{1+n}, \frac{1 \times 3 + n \times 0}{1+n}\right)$$

$$[\because x_1 = 1, y_1 = 0, x_2 = 2, y_2 = 3]$$

$$= \left(\frac{n+2}{n+1}, \frac{3}{1+n}\right)$$

Let slope of line PQ be m.

$$\because \qquad PQ \perp AB$$

$\therefore$ Slope of line $PQ \times$ Slope of line $AB = -1$

$$[\because m_1 m_2 = -1]$$

$$\Rightarrow \quad m \times \frac{y_2 - y_1}{x_2 - x_1} = -1 \Rightarrow m \times \frac{3-0}{2-1} = -1$$

$$[\because x_1 = 1, y_1 = 0, x_2 = 2, y_2 = 3]$$

$$\Rightarrow \qquad m \times 3 = -1 \Rightarrow m = -\frac{1}{3}$$

Now, equation of line PQ by using $y - y_0 = m(x - x_0)$ is

$$y - \frac{3}{1+n} = \frac{-1}{3}\left(x - \frac{n+2}{n+1}\right)$$

$$\left[\because R\left(\frac{n+2}{n+1}, \frac{3}{1+n}\right) = (x_0, y_0)\right]$$

$$\Rightarrow \quad \frac{3(n+1)\,y - 9}{1+n} = \frac{-x(n+1) + (n+2)}{n+1}$$

$$\Rightarrow \quad 3(n+1)\,y - 9 = -x(n+1) + (n+2)$$

$$\Rightarrow \quad x(n+1) + 3(n+1)\,y = n + 2 + 9$$

$$\Rightarrow \quad x(n+1) + 3(n+1)\,y = n + 11$$

which is the required equation of line.

EXAMPLE |10| If the slope of a line passing through the point A (3, 2) is $\frac{3}{4}$, then find the points on the line which are 5 units away from the point A. **[NCERT Exemplar]**

Sol. Equation of the line passing through (3, 2) having slope $\frac{3}{4}$ is given by

$$y - 2 = \frac{3}{4}(x - 3) \qquad [\because y - y_0 = m(x - x_0)]$$

$$\Rightarrow \qquad 4y - 3x + 1 = 0 \qquad \text{...(i)}$$

Let (h, k) be the required point on the line such that distance between (h, k) and (3, 2) is 5

$$\Rightarrow \qquad (h-3)^2 + (k-2)^2 = 25 \qquad \text{...(ii)}$$

[by distance formula]

Since, point (h, k) lies on the line (i)

$$4k - 3h + 1 = 0 \qquad \text{...(iii)}$$

$$\Rightarrow \qquad k = \frac{3h-1}{4} \qquad \text{...(iv)}$$

On putting the value of k in Eq. (ii) and simplifying, we get

$$25h^2 - 150h - 175 = 0 \Rightarrow h^2 - 6h - 7 = 0$$

$$\Rightarrow \qquad (h+1)(h-7) = 0 \Rightarrow h = -1 \text{ or } h = 7$$

On putting these values of h in Eq. (iv), we get

$$k = -1 \text{ or } k = 5$$

Therefore, the coordinates of the required points are either $(-1, -1)$ or $(7, 5)$.

EXAMPLE |11| Find the equation of line through the point (0, 2) making an angle $\frac{2\pi}{3}$ with the positive X-axis. Also, find the equation of the line parallel to it and crossing the Y-axis at a distance of 2 units below the origin. **[NCERT]**

Sol. $\because\ m = \tan\theta = \tan\frac{2\pi}{3}$

$$= \tan\left(\pi - \frac{\pi}{3}\right) = -\tan\frac{\pi}{3} \quad [\because \tan(\pi - \theta) = -\tan\theta]$$

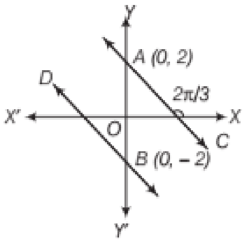
$\Rightarrow m = -\sqrt{3}$

Now, the equation of line passing through the point $A(0, 2)$ and having slope $-\sqrt{3}$ is

$$y - 2 = -\sqrt{3}(x - 0) \quad [\because y - y_1 = m(x - x_1)]$$

$$\Rightarrow \quad y - 2 = -\sqrt{3}x \Rightarrow \sqrt{3}x + y - 2 = 0$$

Since, the line BD, parallel to AC, intersect below Y-axis. It means that the y-coordinate of intersection point is negative and x-coordinate is 0, Thus, intersection point is $(0, -2)$.



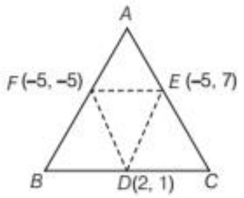
Hence, equation of line through the point $B(0, -2)$ and having slope $-\sqrt{3}$ is

$$y + 2 = -\sqrt{3}(x - 0)\ [\because y - y_1 = m(x - x_1)]$$

$$\Rightarrow \quad \sqrt{3}x + y + 2 = 0$$

EXAMPLE |12| The mid-points of the sides of a triangle are (2, 1), (– 5, 7) and (– 5, – 5). Find the equations of the sides of the triangle.

Sol. Let $D(2, 1)$, $E(-5, 7)$ and $F(-5, -5)$ be the mid-points of sides BC, CA and AB, respectively of ΔABC.



We know that the line joining the mid-points of two sides of a triangle is parallel to the third side.

$\therefore \quad DE \parallel AB$, $EF \parallel BC$ and $DF \parallel AC$

$\therefore$ Slope of AB = Slope of DE,

Slope of BC = Slope of EF and slope of AC = slope of DF

Let m_1, m_2 and m_3 be the slopes of AB, BC and CA, respectively.

Then, m_1 = Slope of AB = Slope of DE $= \frac{7-1}{-5-2} = \frac{-6}{7}$

m_2 = Slope of BC = Slope of EF $= \frac{7+5}{-5+5}$ (undefined)

m_3 = Slope of CA = Slope of DF $= \frac{1+5}{2+5} = \frac{6}{7}$

Since, side AB passes through $F(-5, -5)$ and has slope $m_1 = \frac{-6}{7}$. So, its equation is

$$y + 5 = \frac{-6}{7}(x + 5) \quad \text{[by using point slope form]}$$

$\Rightarrow 6x + 7y + 65 = 0$

Since, slope of side BC not defined, therefore it is parallel to Y-axis. Also, as it is passes through $D(2, 1)$. Therefore, its equation is $x = 2$,

Hence, equation of BC is $x = 2$.

Since, side CA passes through $E(-5, 7)$ and has slope $m_3 = \frac{6}{7}$.

So, its equation is $y - 7 = \frac{6}{7}(x + 5)$

$$\Rightarrow \quad 6x - 7y + 79 = 0$$

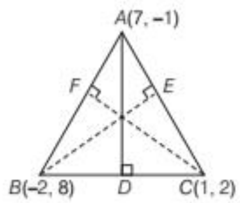
EXAMPLE |13| Find the equations of the altitudes of the triangle whose vertices are $A(7, -1)$, $B(-2, 8)$

Sol. Let AD, BE and CF be three altitudes of ΔABC. Let m_1, m_2 and m_3 be the slopes of AD, BE and CF, respectively.

$\because AD \perp BC$

$\therefore \quad$ Slope of AD × Slope of BC = – 1

$$\Rightarrow \quad m_1 \times \left(\frac{2-8}{1+2}\right) = -1 \Rightarrow m_1 = \frac{1}{2}$$



$\because \quad BE \perp AC$

$\therefore \quad$ Slope of BE × Slope of AC = – 1

$\Rightarrow m_2 \times \left(\frac{-1-2}{7-1}\right) = -1 \Rightarrow m_2 = 2$ and $CF \perp AB$

$\therefore$ Slope of CF × Slope of AB = – 1

$$\Rightarrow \quad m_3 \times \left(\frac{-1-8}{7+2}\right) = -1 \Rightarrow m_3 = 1$$

Since, AD passes through $A(7, -1)$ and has slope $m_1 = \frac{1}{2}$.

So, its equation is $y + 1 = \frac{1}{2}(x - 7)$

[by using point slope form]

$\Rightarrow \quad x - 2y - 9 = 0$

Similarly, equation of BE is

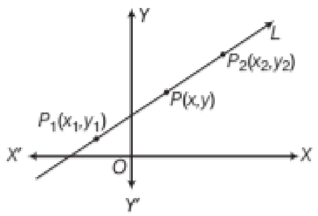
$$y - 8 = 2(x + 2) \Rightarrow 2x - y + 12 = 0$$

and equation of CF is $y - 2 = 1(x - 1)$

$\Rightarrow \quad x - y + 1 = 0$

Two Points Form

The equation of a line passing through the points (x_1, y_1) and $(x_2, y_2,)$ is given by



$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

EXAMPLE |14| Find the equation of line passing through the points $(-1, 1)$ and $(2, -4)$. **[NCERT]**

Sol. Let the given points are $A(x_1, y_1) \equiv A(-1, 1)$ and $B(x_2, y_2) \equiv B(2, -4)$, then equation of line AB is

$$y - 1 = \frac{-4 - 1}{2 + 1}(x + 1)$$

$$\left[\because y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right]$$

$\Rightarrow y - 1 = \frac{-5}{3}(x + 1) \Rightarrow 3y - 3 = -5x - 5$

$\Rightarrow \quad 5x + 3y + 2 = 0$

EXAMPLE |15| The length L (in centimetres) of a copper rod is a linear function of its celsius temperature C. In an experiment, if $L = 124.942$ when $C = 20$ and $L = 125.134$ when $C = 110$, express L in terms of C.

Sol. Given, $L = 124.942$ when $C = 20$ and $L = 125.134$ when $C = 110$

In coordinate form, $(20, 124.942)$ and $(110, 125.134)$ are two points. Now, the equation is given by

$$L - L_1 = \frac{L_2 - L_1}{C_2 - C_1}(C - C_1)$$

$$\left[\because y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right]$$

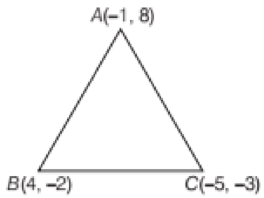
$\Rightarrow \quad L - 124.942 = \frac{125.134 - 124.942}{110 - 20}(C - 20)$

$\Rightarrow \quad L - 124.942 = \frac{0.192}{90}(C - 20)$

$\Rightarrow \quad L = \frac{0.192}{90}(C - 20) + 124.942$

EXAMPLE |16| Find the equations of the sides of a triangle whose vertices are $A(-1, 8)$, $B(4, -2)$ and $C(-5, -3)$.

Sol. Here, we use two points form to find the equation of sides.



Equation of AB is $y - 8 = \frac{-2 - 8}{4 + 1}(x + 1)$

$\Rightarrow 5(y - 8) + 10(x + 1) = 0 \Rightarrow 10x + 5y - 30 = 0$

$\Rightarrow \quad 2x + y - 6 = 0$ [dividing both sides by 5]

Equation of BC is

$$y + 2 = \frac{-3 + 2}{-5 - 4}(x - 4)$$

$\Rightarrow \quad -9(y + 2) + (x - 4) = 0 \Rightarrow x - 9y - 22 = 0$

Equation of AC is $y - 8 = \frac{-3 - 8}{-5 + 1}(x + 1)$

$\Rightarrow -4(y - 8) + 11(x + 1) = 0 \Rightarrow 11x - 4y + 43 = 0$

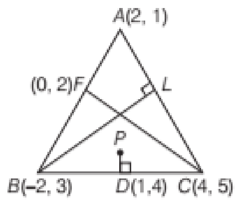
EXAMPLE |17| If $A(2, 1)$, $B(-2, 3)$ and $C(4, 5)$ are the vertices of a ΔABC, then find the equation of

(i) the median through C. (ii) the altitude through B.

(iii) the right bisector of side BC.

Sol. (i) Let F be the mid-point of side AB.

Then, CF is the median through C.



Coordinates of F are $F\left(\frac{2 + (-2)}{2}, \frac{1 + 3}{2}\right)$, i.e. $F(0, 2)$.

So, the equation of median CF is given by

$$y - 5 = \frac{2 - 5}{0 - 4}(x - 4)$$

[by using two points form]

$\Rightarrow \quad y-5=\frac{3}{4}(x-4)$

$\Rightarrow \quad 4(y-5)=3(x-4) \Rightarrow 3x-4y+8=0$

Hence, the equation of median CF is $3x-4y+8=0$.

(ii) Draw $BL \perp AC$. Then, BL is the altitude through B.

Slope of $AC=\frac{5-1}{4-2}=2$

Let the slope BL be m. Since, $BL \perp AC$, So, we have $2m=-1$ and therefore, $m=\frac{-1}{2}$.

Thus, the slope of BL is $\frac{-1}{2}$.

Now, the equation of BL is given by $y-3=\frac{-1}{2}(x+2)$

[by using point slope form]

$\Rightarrow \quad 2(y-3)=-(x+2) \Rightarrow x+2y-4=0.$

Hence, the equation of altitude BL is $x+2y-4=0$.

(iii) Let D be the mid-point of BC.

Then, the coordinates of D are $D\left(\frac{-2+4}{2}, \frac{3+5}{2}\right)$,

i.e. $D(1, 4)$.

Through D, draw $DP \perp BC$.

$$\text{Slope of } BC=\frac{5-3}{4+2}=\frac{2}{6}=\frac{1}{3}$$

Let the slope of PD be m.

Since, $PD \perp BC$. So, we have $m \times \frac{1}{3}=-1 \Rightarrow m=-3$

Thus, the slope of PD is -3.

Now, the equation of PD is given by

$$y-4=-3(x-1)$$

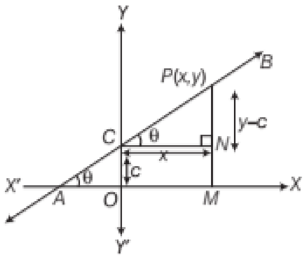
[by using point slope form]

$\Rightarrow \quad 3x+y-7=0$

Hence, the equation of the right bisector of BC is $3x+y-7=0$.

Slope Intercept Form

Suppose a line L with slope m cuts the Y-axis at a distance c from the origin (distance c is called the y-intercept of the line).



Then, equation of the line L is

$$\boxed{y=mx+c}$$

Suppose a line L with slope m cuts the X-axis at a distance d from the origin i.e. makes x-intercept, d.

Then, equation of line L is

$$\boxed{y=m(x-d)}$$

Note

(i) The value of C and d will be positive or negative according as the intercept is made on the positive or negative side of the axies, respectively.

(ii) If the line passes through origin, then $0=m(0)+c \Rightarrow c=0$

$\therefore$ Equation of line passing through the origin is $y=mx$, where m is the slope of the line.

(iii) If the line is parallel to X-axis, then $m=0$. Therefore, the equation of a line parallel to X-axis is $y=c$.

EXAMPLE |18| Find the equations of the line which have slope 1/2 and cuts-off an intercept

(i) -5 on Y-axis. (ii) 4 on X-axis.

Sol. Given, $m=$ Slope of the line $=\frac{1}{2}$

(i) Here, $c=$ Intercept of the line on Y-axis $=-5$

Hence, required equation of the line is

$$y=\frac{1}{2}x-5 \qquad [\because y=mx+c]$$

$\Rightarrow \quad x-2y-10=0$

(ii) Here, $d=$ Intercept of the line on X-axis $=4$.

Hence, required equation of the line is

$$y=\frac{1}{2}(x-4) \Rightarrow x-2y-4=0 \quad [\because y=m(x-d)]$$

EXAMPLE |19| Find the equation of the line intersecting the X-axis at a distance of 3 units to the left of origin with slope -2.

Sol. Given, the line intersecting the X-axis to the left of origin. It means it cuts the negative X-axis at a distance of 3 units from the origin.

$\therefore$ x-intercept of line on X-axis $(d)=-3$

Hence, required equation of the line is given by

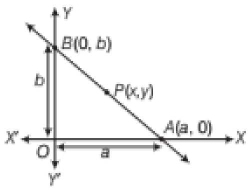
$$y=-2(x+3) \qquad [y=m(x-d)]$$

$\Rightarrow \quad y=-2x-6 \Rightarrow 2x+y+6=0$

Intercept Form

The equation of a line which cuts-off intercepts a and b on the X-axis and Y-axis respectively, is

$$\boxed{\frac{x}{a}+\frac{y}{b}=1}$$



EXAMPLE |20| Find the equation of the lines which cuts-off intercepts on the axes whose sum and product are 1 and −6, respectively.

Sol. Let a and b be the intercepts of the line on X and Y-axes, respectively.

Equation of the line will be $\frac{x}{a}+\frac{y}{b}=1$ ...(i)

Given, sum of intercepts, $a+b=1$ ...(ii)

and product of intercepts, $ab=-6$ ...(iii)

On putting the value of b from Eq. (ii) in Eq. (iii), we get

$$a(1-a)=-6$$

$$\Rightarrow \quad a^2-a-6=0$$

$$\Rightarrow \quad a^2-3a+2a-6=0$$

$$\Rightarrow \quad (a-3)(a+2)=0$$

$$\Rightarrow \quad a=3 \text{ or } -2$$

From Eq. (ii), when $a=3$, $b=-2$ and when $a=-2$, $b=3$.

On putting the values of a and b in Eq. (i), required equation of the lines are

$$\frac{x}{3}+\frac{y}{-2}=1 \text{ and } \frac{x}{-2}+\frac{y}{3}=1$$

$$\Rightarrow \quad 2x-3y-6=0 \text{ and } 3x-2y+6=0$$

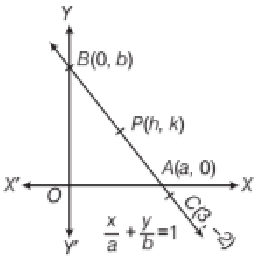
EXAMPLE |21| A line passes through the point (3, − 2). Find the locus of the middle point of the portion of the line intercepted between the axes.

Sol. Let the equation of the line be $\frac{x}{a}+\frac{y}{b}=1$ ...(i)

It passes through (3, − 2).

$$\therefore \quad \frac{3}{a}-\frac{2}{b}=1 \quad ...(ii)$$

The line (i) cuts the coordinate axes at $A(a, 0)$ and $B(0, b)$. Let $P(h, k)$ be the mid-point of the portion AB.



Then, $h=\frac{a+0}{2}$ and $k=\frac{0+b}{2}$

$$\Rightarrow \quad a=2h \text{ and } b=2k$$

On substituting the values of a and b in Eq. (ii), we get

$$\frac{3}{2h}-\frac{2}{2k}=1$$

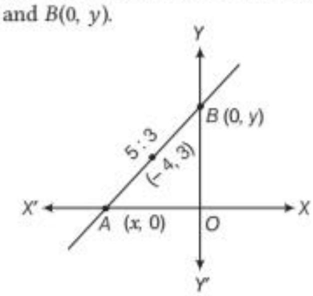
Hence, locus of $P(h, k)$ is $\frac{3}{2x}-\frac{1}{y}=1$

$$\Rightarrow \quad 3y-2x=2xy$$

EXAMPLE |22| Find the equation of the line which passes through the point (−4, 3) and the portion of the line intercepted between the axes is divided internally in the ratio 5 : 3 by this point. **[NCERT Exemplar]**

If the point (h, k) divides the join of $A(x, 0)$ and $B(0, y)$ internally, in the ratio $m_1 : m_2$. Then, first of all find the coordinates of A and B using section formula for internal division. Then, find the equation of required line.

Sol. Let the line intersects X and Y-axes respectively at $A(x, 0)$ and $B(0, y)$.



Then, $\quad -4=\frac{5\times 0+3x}{5+3} \Rightarrow -4=\frac{3x}{8} \Rightarrow x=\frac{-32}{3}$

and $\quad 3=\frac{5\times y+3\times 0}{5+3} \Rightarrow 3=\frac{5y}{8} \Rightarrow y=\frac{24}{5}$

Thus, the intercept on the X and Y-axes respectively are $a=\frac{-32}{3}$ and $b=\frac{24}{5}$.

Hence, equation of required line is

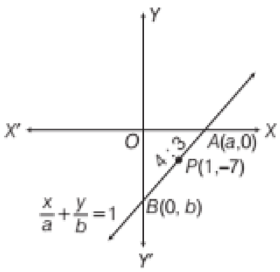
$$\frac{x}{-32/3}+\frac{y}{24/5}=1 \Rightarrow \frac{-3x}{32}+\frac{5y}{24}=1$$

$$\Rightarrow \quad -9x+20y=96 \Rightarrow 9x-20y+96=0$$

EXAMPLE |23| Find the equation of the line which passes through $P(1, -7)$ and meets the axes at A and B respectively so that $4AP-3BP=0$.

Sol. Let the equation of the required line be $\frac{x}{a}+\frac{y}{b}=1$ ... (i)

Since, it passes through $P(1, -7)$.



$$\therefore \quad \frac{1}{a}-\frac{7}{b}=1 \quad ...(ii)$$

It is given that the point $P(1, -7)$ divides segment AB in such a way that

$$4AP-3BP=0 \Rightarrow \frac{AP}{BP}=\frac{3}{4} \Rightarrow AP:BP=3:4$$

This means that P divides AB internally in the ratio 3 : 4.
So, the coordinates of P are

$$\left(\frac{3\times 0+4\times a}{3+4}, \frac{3\times b+4\times 0}{3+4}\right)=\left(\frac{4a}{7}, \frac{3b}{7}\right)$$

But, the coordinates of P are given as (1, – 7).

$\therefore \frac{4a}{7}=1$ and $\frac{3b}{7}=-7 \Rightarrow a=\frac{7}{4}$ and $b=-\frac{49}{3}$

On substituting the values of a and b in Eq. (i), we get

$\frac{4x}{7}-\frac{3y}{49}=1$ or $28x-3y=49$, which is the required equation.

TOPIC PRACTICE 3

OBJECTIVE TYPE QUESTIONS

1. The equation of X-axis is
(a) $x=0$ (b) $y=0$
(c) $x+y=0$ (d) $x-y=0$

2. The equation of the line through (– 2, 3) with slope – 4 is
(a) $x+4y-10=0$ (b) $4x+y+5=0$
(c) $x+y-1=0$ (d) $3x+4y-6=0$

3. Let the line L passes through two given points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. Let $P(x, y)$ be general point on L. Then, the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by
(a) $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x_1-x)$ (b) $y+y_1=\frac{y_1-y_2}{x_1-x_2}(x+x_1)$
(c) $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$ (d) $y-y_1=\frac{y_2+y_1}{x_2+x_1}(x-x_1)$

4. The equation of line passing through (0, 2) and (3, – 3), is
(a) $5x+3y-6=0$ (b) $5x-3y+6=0$
(c) $3x+5y-6=0$ (d) $3x-5y+6=0$

VERY SHORT ANSWER Type Questions

5 Find the equation of the straight line passing through the following pair of points.
(i) (0, 0) and (2, –2)
(ii) (a, b) and $(a+b, a-b)$

6 Find the equation of a line parallel to the Y-axis at a distance of 6 units to its right.

7 Find the equation of horizontal and vertical lines passing through the point (– 5, 6).

8 For specifying a straight line, how many geometrical parameters should be known? **[NCERT Exemplar]**

9 Write the equation of a line, parallel to X-axis and 5 units below it.

10 Find the equation of a line, which is parallel to Y-axis and passes through (– 4, 3).

11 A line cutting off intercept – 3 from the Y-axis and the tangent at angle to the X-axis is $\frac{3}{5}$. Find its equation.

12 Find the equation of a line passing through the point (– 4, 3) with slope –1/2.

13 Find the equation of a line, which passes through the point (2, 3) and makes an angle of 30° with the positive direction of X-axis. **[NCERT Exemplar]**

14 Find the equation of line joining the points (1, 1) and (2, 3).

15 If the line $\frac{x}{a}+\frac{y}{b}=1$ passes through the points (2, – 3) and (4, – 5), then find (a, b).

16 Find the equation of straight line, which passes through the point (5, 6) and has intercepts on the axes equal in magnitude but opposite in sign.

17 Find the equation of the straight line, which passes through the point (1, – 2) and cuts-off equal intercepts from axes.

18 Find the equation of the line for which $p=2$, $\sin\alpha=\frac{4}{5}$.

19 Find the equation of a line, which is equidistant from the lines $x=-2$ and $x=6$.

20 Find the equation of line, which intersect the Y-axis at a distance of 2 units above the origin and making an angle of 60° with positive direction of X-axis. **[NCERT]**

21 A straight line moves so that the sum of the reciprocals of its intercepts made on axes is constant. Show that the line passes through a fixed point. **[NCERT Exemplar]**

22 The perpendicular from the origin to a line meets it at the point (– 2, 9). Find the equation of the line.

23 Find the equation of the line passing through the point (5, 2) and perpendicular to the line joining the points (2, 3) and (3, – 1). **[NCERT Exemplar]**

24 Find the equation of line passing through $(2, 2\sqrt{3})$ and inclined with X-axis at an angle of 75°.

SHORT ANSWER Type I Questions

25 Find the equation of a line, whose inclination with the X-axis is 150° and which passes through the point (3, – 5).

26 The intercept cuts-off by a line from Y-axis is twice than that from X-axis and the line passes through the point (1, 2). Find the equation of the line.

27 If the coordinates of the middle point of the portion of a line intercepted between the coordinate axes is (3, 2), then find the equation of the line. **[NCERT Exemplar]**

28 A line passes through $P(1, 2)$ such that its intercept between the axes is bisected at P. Find the equation of line.

29 Find the equation of the straight line which bisects the distance between the points $A(a, b)$, $B(a', b')$ and also bisects the distance between the points $C(-a, b)$ and $D(a', -b')$.

30 Find the equation of lines passing through (1, 2) and making an angle 30° with positive Y-axis, measured clockwise.

31 $P(a, b)$ is the mid-point of a line segment between axes. Show that the equation of the line is $\frac{x}{a}+\frac{y}{b}=2$.

32 By using the concept of equation of a line, prove that the points $A(3, 0)$, $B(-2, -2)$ and $C(8, 2)$ are collinear. **[NCERT]**

33 The vertices of ΔPQR are $P(2, 1)$, $Q(-2, 3)$ and $R(4, 5)$. Find the equation of the median through the vertex Q.

SHORT ANSWER Type II Questions

34 If the intercept of a line between the coordinate axes is divided by the point (– 5, 4) in the ratio 1 : 2, then find the equation of the line.

35 Find the equation of straight line which passes through (3, 4) and the sum of whose intercepts on the coordinates axes is 14.

36 Find equation of the line passing through the point (2, 2) cutting off intercepts on the axes whose sum is 9. **[NCERT]**

37 The owner of a milk store finds that he can sell 980 L of milk each week at ₹ 14 per litre and 1220 L of milk each week at ₹ 16 per litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at ₹ 17 per litre.

38 Show that the perpendicular drawn from the point (4, 1) on the line segment joining (6, 5) and (2, – 1) divides it internally in the ratio 8 : 5.

39 The hypotenuse of a right angled triangle has its ends at the points (1, 3) and (– 4, 1). Find the equation of the legs (perpendicular sides) of the triangle.

40 Show that the points $(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$ and $(a, 0)$ are collinear, if $t_1t_2 = -1$.

LONG ANSWER Type Questions

41 If $A(1, 4)$, $B(2, -3)$ and $C(-1, -2)$ are the vertices of a ΔABC, find the equation of

(i) the median through A.

(ii) the altitude through A.

(iii) the perpendicular bisector of BC.

42 Find the equations of the altitudes of a ΔABC, whose vertices are $A(2, -2)$, $B(1, 1)$ and $C(-1, 0)$.

43 Find the equations of the medians of a ΔABC, whose vertices are $A(2, 5)$, $B(-4, 9)$ and $C(-2, -1)$.

HINTS & ANSWERS

1. (b) The equation of X-axis is $y = 0$
2. (b) Use formula: $(y - y_0) = m(x - x_0)$
   Ans: $4x + y + 5 = 0$
3. (c) By two points form, we get
   Ans: $(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
4. (a) Use $(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
   Ans: $5x + 3y - 6 = 0$
5. (i) $y = -x$
   (ii) $(a - 2b)x - by + b^2 + 2ab - a^2 = 0$
6. $x - 6 = 0$

7. Horizontal line $y-6=0$ and vertical line $x+5=0$

8. For (i) $y=mx+c$, number of parameters = 2

(ii) $\frac{x}{a}+\frac{y}{b}=1$, number of parameters = 2

(iii) $y-y_1=m(x-x_1)$, number of parameters = 2

(iv) $x\cos\alpha+y\sin\alpha=p$, parameter = 2

Ans. Two parameter should be known.

9. $y=-5$ **10.** $x=-4$

11. m = slope of line = $\tan\theta=\frac{3}{5}$.

Ans. $5y-3x+15=0$ **12.** $x+2y-2=0$

13. $m=\tan 30°=\frac{1}{\sqrt{3}}$ **Ans.** $x-\sqrt{3}y=2-3\sqrt{3}$

14. $2x-y-1=0$

15. According to given condition, we have.

$$\frac{2}{a}-\frac{3}{b}=1 \text{ and } \frac{4}{a}-\frac{5}{b}=1$$

Now, Let $\frac{1}{a}=u$ and $\frac{1}{b}=v$.

Then, we get $2u-3v=1$ and $4u-5v=1$

Ans. $(a, b)=(-1,-1)$

16. Let the equation of line in intercept form be $\frac{x}{a}+\frac{y}{b}=1$

We have, $a=-b$

Now, equation of line becomes $\frac{x}{-b}+\frac{y}{b}=1$

$\because$ It passes through (5, 6); $\therefore$ $\frac{5}{-b}+\frac{6}{b}=1 \Rightarrow b=1$

Ans. $x-y+1=0$

17. Let the intercepts along the X and Y-axes be a and a, respectively.

$\therefore$ Equation of the line is $\frac{x}{a}+\frac{y}{a}=1$ ...(i)

Since, the point $(1,-2)$ lies on the line,

$\therefore$ $\frac{1}{a}-\frac{2}{a}=1 \Rightarrow a=-1$

Ans. $x+y+1=0$

18. Now, $\cos\alpha=\pm\sqrt{1-\sin^2\alpha}=\pm\sqrt{1-\frac{16}{25}}=\pm\frac{3}{5}$

Equation of line in normal form, when $\cos\alpha=\frac{3}{5}$ is

$$x\cdot\left(\frac{3}{5}\right)+y\cdot\left(\frac{4}{5}\right)=2$$

Equation of line in normal form, when $\cos\alpha=-3/5$ is

$$x\cdot\left(-\frac{3}{5}\right)+y\cdot\left(\frac{4}{5}\right)=2$$

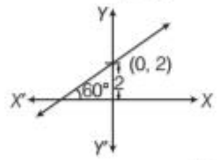
Ans. $3x+4y=10$ and $3x-4y+10=0$

19. Mid-point of $A(-2,0)$ and $B(6,0)$ is $M\left(\frac{-2+6}{2}, 0\right)$

i.e. $M(2,0)$. **Ans.** $x=2$

20. Since, the line intersect the Y-axis above the origin. It means that the line intersects the positive Y-axis.

Here, $c=2$



$$m=\tan 60°=\sqrt{3}$$

$$\sqrt{3}x-y+2=0$$

21. Given that, $\frac{1}{a}+\frac{1}{b}$ = constant = $\frac{1}{k}$ (say)

$\because$ $\frac{1}{a}+\frac{1}{b}=\frac{1}{k} \Rightarrow \frac{k}{a}+\frac{k}{b}=1$

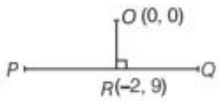
So, (k, k) lies on $\frac{x}{a}+\frac{y}{b}=1$.

22. Since, $PQ \perp OR$

$\therefore$ Slope of PQ × Slope of $OR=-1$

$$m\times\left(\frac{9-0}{-2-0}\right)=-1$$

$\Rightarrow$ $m=\frac{2}{9}$



Ans. $2x-9y+85=0$

23. Slope of line joining (2, 3) and (3, –1) is

$$m_1=\frac{-1-3}{3-2}=-4$$

Slope of the line perpendicular to it is $m_2=\frac{-1}{-4}=\frac{1}{4}$

Ans. $x-4y+3=0$

24. $m=\tan 75°=\tan(45°+30°)$;

$\therefore$ $m=2+\sqrt{3}$

Ans. $(2+\sqrt{3})x-y-4=0$

25. $m=\tan 150°=\tan(180°-30°)=-\tan 30°=-\frac{1}{\sqrt{3}}$

Ans. $x+\sqrt{3}y+(-3+5\sqrt{3})=0$

26. Here, equation of line is of the form $\frac{x}{a}+\frac{y}{2a}=1$. It passes through (1, 2).

So, we have $\frac{1}{a}+\frac{2}{2a}=1$

$\Rightarrow$ $a=2$ **Ans.** $2x+y=4$

27. Let equation of line is $\frac{x}{a}+\frac{y}{b}=1$

Since, the coordinates of the middle point are $P(3, 2)$.

$\therefore$ $3=\frac{0+a}{2} \Rightarrow 3=\frac{a}{2}$

$\Rightarrow$ $a=6$

Similarly, $b=4$ **Ans.** $2x+3y=12$

28. Let equation of line is $\frac{x}{a}+\frac{y}{b}=1$. Here, we have $1=\frac{a+0}{2}$ and $2=\frac{0+b}{2}$ **Ans.** $2x+y-4=0$

29. Let the line MN bisects the line AB at M and CD at N.

$\therefore$ Coordinates of $M=\left(\frac{a+a'}{2},\frac{b+b'}{2}\right)$

and coordinates of $N=\left(\frac{-a+a'}{2},\frac{b-b'}{2}\right)$

Now, equation of line MN is

$$\left(y-\frac{b+b'}{2}\right)=\left(\frac{\frac{b-b'}{2}-\frac{b+b'}{2}}{\frac{-a+a'}{2}-\frac{a+a'}{2}}\right)\left(x-\frac{a+a'}{2}\right)$$

Ans. $2b'x-2ay+ab-a'b'=0$

30. Given that, angle with Y-axis $=30°$

$\therefore$ Angle with X-axis $=60°$

$\Rightarrow$ Slope of the line, $m=\tan 60°=\sqrt{3}$

Ans. $y-\sqrt{3}x-2+\sqrt{3}=0$

31. Let the equation of line AB be

$\frac{x}{h}+\frac{y}{k}=1$, which intersect on X and Y-axes at $A(h, 0)$ and $B(0, k)$.

Mid-point of $AB=\left(\frac{h+0}{2},\frac{0+k}{2}\right)$

$\because P(a, b)$ is the mid-point of AB.

$\therefore \quad \frac{h+0}{2}=a$ and $\frac{0+k}{2}=b$

32. Equation of line joining the points $A(3, 0)$ and $B(-2,-2)$ is, $y-0=\frac{-2-0}{-2-3}(x-3)$

$\Rightarrow \quad 2x-5y-6=0 \quad$ ...(i)

On putting the point $C(8, 2)$ in Eq. (i), we get

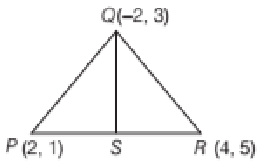
$2\times 8-5\times 2-6=0 \Rightarrow 0=0$, which is true.

Thus, point C also lies on the line.

Hence, the points are collinear.

33. Let QS be the median through Q. Since the median bisects the opposite side, i.e. S is the mid-point of PR.

$S=\left(\frac{2+4}{2},\frac{1+5}{2}\right)=(3, 3)$

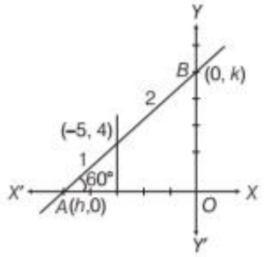


$\therefore$ Equation of line QS is $y-3=\frac{3-3}{3+2}(x+2)$

Ans. $y=3$

34. Let intercept of a line be (h, k).

The coordinates of A and B, where the line cuts the X-axis and and Y-axis, are $(h, 0)$ and $(0, k)$ respectively.



Then, $-5=\frac{1\times 0+2\times h}{1+2} \Rightarrow h=-\frac{15}{2}$

and $\quad 4=\frac{1\cdot k+0\cdot 2}{1+2} \Rightarrow k=12$

$\therefore \quad A=\left(-\frac{15}{2}, 0\right)$ and $B=(0, 12) \Rightarrow 5y=8x+60$

Ans. $8x-5y+60=0$

35. Let equation of line is $\frac{x}{a}+\frac{y}{b}=1 \quad$ ...(i)

$\because$ The sum of intercepts is 14, therefore $=a+b=14$

$\Rightarrow \quad b=14-a \Rightarrow \frac{x}{a}+\frac{y}{14-a}=1$

The line passes through (3, 4)

$\therefore \quad \frac{3}{a}+\frac{4}{14-a}=1 \Rightarrow a=6$ or 7

***Case* I** When $a=6$, then Eq. (i) becomes $\frac{x}{6}+\frac{y}{14-6}=1$

***Case* II** When $a=7$, then Eq. (i) becomes $\frac{x}{7}+\frac{y}{14-7}=1$

Ans. $4x+3y-24=0$ and $x+y-7=0$

36. Equation of the line in intercept form is

$$\frac{x}{a}+\frac{y}{b}=1 \quad \text{...(i)}$$

Given, $\quad a+b=9 \quad$ ...(ii)

and line (i) passes through the point (2, 2).

$\therefore \quad \frac{2}{a}+\frac{2}{b}=1 \quad$ ...(iii)

$\Rightarrow \quad \frac{2}{a}+\frac{2}{9-a}=1 \quad$ [from Eq. (ii)]

$\Rightarrow \quad a^2-9a+18=0$

$\Rightarrow \quad a=6$ or $3 \Rightarrow b=3$ or 6

Hence, Eq. (i) becomes $\frac{x}{6}+\frac{y}{3}=1$ or $\frac{x}{3}+\frac{y}{6}=1$

Ans. $x+2y=6$ or $2x+y=6$

37. Let x denotes the rupees per litre and y denotes the quantity of milk in litre. Then, we have the following ordered pairs in the cartesian plane.

(14, 980) and (16, 1220)

Now, the equation of line joining these two points is

$$y - 980 = \frac{1220 - 980}{16 - 14}(x - 14)$$

$$\Rightarrow \quad 120x - y = 700$$

Ans. He will sell weekly 1340 L milk at the rate of ₹ 17 per litre.

38. Let the points be A (4,1), B (6,5) and C (2, −1). Draw $AM \perp BC$. Slope of line $BC = \frac{-1-5}{2-6} = \frac{3}{2}$

Now, slope of line $AM \times$ slope of line $BC = -1$

Slope of line $AM \times \frac{3}{2} = -1$

$\Rightarrow$ Slope of line $AM = \frac{-2}{3}$

Equation of line AM is $y - 1 = \frac{-2}{3}(x - 4)$

$$\Rightarrow \quad 2x + 3y - 11 = 0 \quad \text{...(i)}$$

Let M divides BC in the ratio $k:1$ internally.

Then, Coordinates of M are $\left(\frac{2k+6}{k+1}, \frac{-k+5}{k+1}\right)$.

Since, M is on line AM, therefore the coordinates of M will satisfy the Eq. (i).

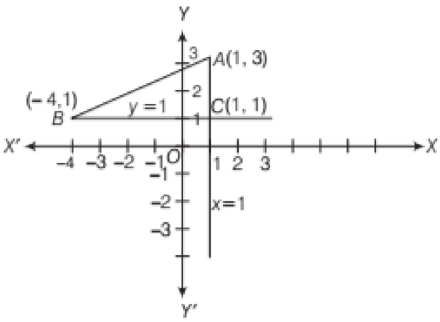
$\therefore \quad 2\left(\frac{2k+6}{k+1}\right) + 3\left(\frac{-k+5}{k+1}\right) - 11 = 0$ **Ans.** 8 : 5

39. First we plot the points $A(1, 3)$ and $B(-4, 1)$ in the XY-plane. From the point $A(1, 3)$, we draw a line parallel to Y-axis and from the point $B(-4, 1)$, we draw a line parallel to X-axis. The point of intersection of two lines is on C, which is right angled at C.

Clearly, the coordinate of C are (1, 1).

Now, Equation of line passing through $A(1, 3)$ and $C(1, 1)$ is

$$y - 3 = \frac{1-3}{1-1}(x - 1) \Rightarrow x = 1$$



Equation of line BC is $y - 1 = \frac{1-1}{1+4}(x - 1)$

$$\Rightarrow \quad y = 1$$

Ans. The legs of a triangle are $x = 1$ and $y = 1$.

40. $\because \quad y - 2at_1 = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2}(x - at_1^2)$

$$\Rightarrow \quad y - 2at_1 = \frac{2}{(t_2 + t_1)}(x - at_1^2)$$

$$\Rightarrow (t_1 + t_2)(y - 2at_1) = 2(x - at_1^2) \quad \text{...(i)}$$

Now, given three points are collinear, if $(a, 0)$ lies on Eq. (i). i.e.

$$(t_1 + t_2)(0 - 2at_1) = 2(a - at_1^2)$$

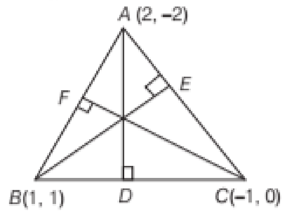
$$\Rightarrow \quad t_1 t_2 = -1$$

41. (i) $13x - y - 9 = 0$

(ii) $3x - y + 1 = 0$

(iii) $3x - y - 4 = 0$

42. Slope of $BC = \frac{1}{2}$, therefore slope of $AD = -2$

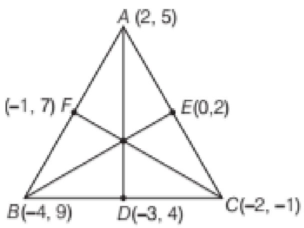


Equation of altitude AD, $y + 2 = -2(x - 2)$

Similarly, we can find the others equations.

Ans. $2x + y - 2 = 0$, $3x - 2y - 1 = 0$ and $x - 3y + 1 = 0$

43.



Equation of median AD is $\frac{y-5}{x-2} = \frac{4-5}{-3-2}$

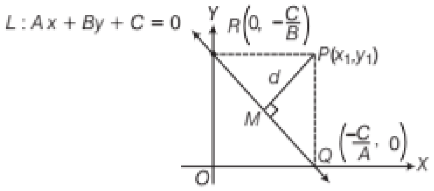
Equation of median BE is $\frac{y-9}{x+4} = \frac{2-9}{0+4}$

Equation of median CF is $\frac{y+1}{x+2} = \frac{7+1}{-1+2}$

Ans. $x - 5y + 23 = 0$, $7x + 4y - 8 = 0$ and $8x - y + 15 = 0$

|TOPIC 4|
Distance of a Point from a Line

The distance of a point from a line is the **length of perpendicular** drawn from the point to the line. Let $L: Ax + By + C = 0$ be a line, whose perpendicular distance from the point $P(x_1, y_1)$ is d. Then,



$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Note The distance of origin from the line $Ax + By + C = 0$ is

$$d = \frac{|C|}{\sqrt{A^2 + B^2}}$$

EXAMPLE |1| Find the distance of the point $(2, -3)$ from the line $2x - 3y + 6 = 0$.

Sol. Given equation of line is

$$2x - 3y + 6 = 0$$

$\therefore$ Required distance of the point from the line

= The perpendicular distance from point to the line

$$= \frac{|2 \times 2 - 3(-3) + 6|}{\sqrt{2^2 + (-3)^2}} = \frac{|4 + 9 + 6|}{\sqrt{4 + 9}} = \frac{19}{\sqrt{13}}$$

EXAMPLE |2| Find the length of perpendicular from the point (a, b) to the line $\frac{x}{a} + \frac{y}{b} = 1$.

Sol. The given point is $P(a, b)$ and the given line is $bx + ay - ab = 0$.

Let d be the length of perpendicular from $P(a, b)$ to the line $bx + ay - ab = 0$.

Then, $$d = \frac{|b \times a + a \times b - ab|}{\sqrt{b^2 + a^2}}$$

$$= \frac{|ab|}{\sqrt{a^2 + b^2}} \text{ units}$$

EXAMPLE |3| Find the points on X-axis whose perpendicular distance from the line $4x + 3y = 12$ is 4.

Sol. Given points are on X-axis, so let its coordinates be $(\alpha, 0)$.

Then, length of perpendicular from $(\alpha, 0)$ to the line $4x + 3y - 12 = 0$ is 4.

$$\therefore \quad \frac{|4\alpha + 3(0) - 12|}{\sqrt{(4)^2 + (3)^2}} = 4$$

$$\Rightarrow \quad \frac{|4\alpha - 12|}{\sqrt{16 + 9}} = 4$$

$$\Rightarrow \quad |4\alpha - 12| = 4 \times \sqrt{25} = 4 \times 5$$

$$\Rightarrow \quad \alpha - 3 = \pm 5$$

$$\Rightarrow \quad \alpha = 8 \text{ or } -2$$

Hence, the required points are $(8, 0)$ and $(-2, 0)$.

EXAMPLE |4| Find the coordinates of a point on the line $x + y + 3 = 0$, whose distance from the line $x + 2y + 2 = 0$ is $\sqrt{5}$.

Sol. Let the required point be $P(a, b)$.

Then, $$a + b + 3 = 0 \quad \text{...(i)}$$

Also, $$\frac{|a + 2b + 2|}{\sqrt{1^2 + 2^2}} = \sqrt{5}$$

$$\Rightarrow \quad |a + 2b + 2| = 5$$

$$\Rightarrow \quad a + 2b + 2 = 5 \text{ or } a + 2b + 2 = -5$$

$$\Rightarrow \quad a + 2b = 3 \text{ or } a + 2b = -7$$

Thus, $$a + 2b = 3 \quad \text{...(ii)}$$

or $$a + 2b = -7 \quad \text{...(iii)}$$

On solving Eqs. (i) and (ii), we get $a = -9$ and $b = 6$.

On solving Eqs. (i) and (iii), we get $a = 1$ and $b = -4$.

Hence, the required points are $(-9, 6)$ and $(1, -4)$.

EXAMPLE |5| Find the locus of a point which moves in such a way that the square of its distance from the point $(3, -2)$ is numerically equal to its distance from the line $5x - 12y = 13$. **[NCERT Exemplar]**

Sol. Let $P(h, k)$ be a variable point moving in such a way that the square of its distance from $A(3, -2)$ is numerically equal to its distance from the line $5x - 12y = 13$.

$$\therefore \quad (h - 3)^2 + (k + 2)^2 = \frac{|5h - 12k - 13|}{\sqrt{5^2 + (-12)^2}}$$

$$\Rightarrow \quad 13\{(h - 3)^2 + (k + 2)^2\} = \pm(5h - 12k - 13)$$

$$\Rightarrow \quad 13(h^2 + k^2) - 83h + 64k + 182 = 0$$

or $$13(h^2 + k^2) - 73h + 40k + 156 = 0$$

Hence, the locus of (h, k) is

$$13(x^2 + y^2) - 83x + 64y + 182 = 0$$

or $$13(x^2 + y^2) - 73x + 40y + 156 = 0$$

EXAMPLE |6| A variable line passes through a fixed point P. The algebraic sum of the perpendiculars drawn from the points (2, 0), (0, 2) and (1, 1) on the line is zero. Find the coordinates of the point P. **[NCERT Exemplar]**

Sol. Let slope of the line be m and the coordinates of fixed point P be (x_1, y_1).

Then, equation of line is $y - y_1 = m(x - x_1)$ ...(i)

Let the given points be $A(2, 0)$, $B(0, 2)$ and $C(1, 1)$.

Now, perpendicular distance from A

$$= \frac{0 - y_1 - m(2 - x_1)}{\sqrt{1 + m^2}}$$

Perpendicular distance from $B = \dfrac{2 - y_1 - m(0 - x_1)}{\sqrt{1 + m^2}}$

and perpendicular distance from $C = \dfrac{1 - y_1 - m(1 - x_1)}{\sqrt{1 + m^2}}$

Now, $\dfrac{\begin{bmatrix} -y_1 - 2m + mx_1 + 2 - y_1 \\ + mx_1 + 1 - y_1 - m + mx_1 \end{bmatrix}}{\sqrt{1 + m^2}} = 0$

$\Rightarrow \quad -3y_1 - 3m + 3mx_1 + 3 = 0$

$\Rightarrow \quad -y_1 - m + mx_1 + 1 = 0 \Rightarrow y_1 = -m + mx_1 + 1$

On substituting this value in Eq. (i), we get

$y + m - mx_1 - 1 = mx - mx_1 \;\Rightarrow\; y - 1 = m(x - 1)$

Thus, $\quad (x_1, y_1) = (1, 1)$

EXAMPLE |7| If p is the length of perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$ and a^2, p^2 and b^2 are in AP, then show that $a^4 + b^4 = 0$. **[NCERT Exemplar]**

Sol. Given equation of line is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad ...(i)$$

Perpendicular length from the origin to the line (i) is

$$p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = \frac{ab}{\sqrt{a^2 + b^2}} \Rightarrow p^2 = \frac{a^2b^2}{a^2 + b^2}$$

Since, a^2, p^2 and b^2 are in AP.

$\therefore \quad 2p^2 = a^2 + b^2 \Rightarrow \dfrac{2a^2b^2}{a^2 + b^2} = a^2 + b^2$

$\Rightarrow \quad 2a^2b^2 = (a^2 + b^2)^2 \Rightarrow 2a^2b^2 = a^4 + b^4 + 2a^2b^2$

$\Rightarrow \quad a^4 + b^4 = 0$ **Hence proved.**

EXAMPLE |8| Find the perpendicular distance from the origin of the line joining the points $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$. **[NCERT]**

Sol. Let the given points be $A(\cos\theta, \sin\theta)$ and $B(\cos\phi, \sin\phi)$.

Then, equation of line AB is

$$y - \sin\theta = \frac{\sin\phi - \sin\theta}{\cos\phi - \cos\theta}(x - \cos\theta)$$

$$\left[\because y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right]$$

$$\Rightarrow \quad y - \sin\theta = \frac{2\cos\left(\frac{\phi + \theta}{2}\right)\sin\left(\frac{\phi - \theta}{2}\right)}{-2\sin\left(\frac{\phi + \theta}{2}\right)\sin\left(\frac{\phi - \theta}{2}\right)}(x - \cos\theta)$$

$$\left[\because \sin C - \sin D = 2\cos\left(\frac{C + D}{2}\right)\sin\left(\frac{C - D}{2}\right)\right]$$

$$\Rightarrow \quad y\sin\left(\frac{\phi + \theta}{2}\right) - \sin\theta\sin\left(\frac{\phi + \theta}{2}\right) = -x\cos\left(\frac{\phi + \theta}{2}\right) + \cos\theta\cos\left(\frac{\theta + \phi}{2}\right)$$

$$\Rightarrow \quad x\cos\left(\frac{\theta + \phi}{2}\right) + y\sin\left(\frac{\theta + \phi}{2}\right) - \left\{\cos\theta\cos\left(\frac{\theta + \phi}{2}\right) + \sin\theta\sin\left(\frac{\theta+\phi}{2}\right)\right\} = 0$$

$$\Rightarrow \quad x\cos\left(\frac{\theta + \phi}{2}\right) + y\sin\left(\frac{\theta + \phi}{2}\right) - \cos\left(\theta - \frac{\theta + \phi}{2}\right) = 0$$

$$\Rightarrow \quad x\cos\left(\frac{\theta + \phi}{2}\right) + y\sin\left(\frac{\theta + \phi}{2}\right) - \cos\left(\frac{\theta - \phi}{2}\right) = 0$$

Now, perpendicular distance from origin

$$= \frac{\left|0 + 0 - \cos\left(\frac{\theta - \phi}{2}\right)\right|}{\sqrt{\cos^2\left(\frac{\theta + \phi}{2}\right) + \sin^2\left(\frac{\theta + \phi}{2}\right)}}$$

$$\left[\because \text{distance from } (x_1, y_1) \text{ to the line } ax + by + c = 0 \text{ is } \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}\right]$$

$$= \left|\cos\left(\frac{\theta - \phi}{2}\right)\right|$$

Distance between Two Parallel Lines

If two lines are parallel, then they have the same distance between them throughout. Therefore, the distance between two parallel lines can be find by the following two methods

METHOD 1

For finding the distance between two parallel lines, *we use following working steps*

***Step* I** First, find the coordinates of any point on one of the given line by putting $x = 0$ or $y = 0$, which is $(0, y_1)$ or $(x_1, 0)$, (say).

Step II Substitute the point $(0, y_1)$ or $(x_1, 0)$ in the equation of second line.

Step III Divide the result obtained in Step II by the square root of the sum of squares of the coefficient of x and y of second line.

Step IV Take the modulus of the result obtained in Step III and get the required distance.

EXAMPLE |9| Find the distance between parallel lines $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$.

Sol. Given lines are $15x + 8y - 34 = 0$ ...(i)

and $15x + 8y + 31 = 0$ ...(ii)

On putting $x = 0$ in Eq. (i), we get

$$15(0) + 8y - 34 = 0$$

$$\Rightarrow \quad 8y = 34$$

$$\Rightarrow \quad y = \frac{34}{8} = \frac{17}{4}.$$

$\therefore$ One point on the line (i) is $\left(0, \frac{17}{4}\right)$.

On putting $x = 0$, $y = \frac{17}{4}$ in the expression of line (ii), we get

$$15(0) + 8\left(\frac{17}{4}\right) + 31 = 34 + 31 = 65$$

From Eq. (ii), coefficient of $x = 15$

and coefficient of $y = 8$

$$\therefore \quad \sqrt{(\text{Coefficient of } x)^2 + (\text{Coefficient of } y)^2}$$

$$= \sqrt{(15)^2 + (8)^2} = \sqrt{225 + 64} = \sqrt{289} = 17$$

Now, required distance between two parallel lines $= \left|\frac{65}{17}\right|$

Hence, the required distance between given parallel lines is $\frac{65}{17}$ units.

METHOD 2

The distance between two parallel lines

$$y = m x + c_1 \quad \text{...(i)}$$

$$y = m x + c_2 \quad \text{...(ii)}$$

is given by

$$d = \frac{|c_1 - c_2|}{\sqrt{1 + m^2}}$$

If lines are given in general form i.e. given lines are $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, then distance between these lines,

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

EXAMPLE |10| Find the distance between following parallel lines.

(i) $3x - 4y + 9 = 0$ and $6x - 8y - 15 = 0$

(ii) $l(x + y) + h = 0$ and $l(x + y) - r = 0$ **[NCERT]**

Sol. (i) Given parallel lines are

$$3x - 4y + 9 = 0 \quad \text{...(i)}$$

and $6x - 8y - 15 = 0$

$$\Rightarrow \quad 3x - 4y - \frac{15}{2} = 0 \quad \text{...(ii)}$$

On comparing Eqs. (i) and (ii) with $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, we get

$$A = 3,\ B = -4,\ C_1 = 9 \text{ and } C_2 = \frac{-15}{2}$$

$$\therefore \text{ Required distance} = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{\left|9 + \frac{15}{2}\right|}{\sqrt{3^2 + (-4)^2}}$$

$$= \frac{\left|\frac{18 + 15}{2}\right|}{\sqrt{9 + 16}} = \frac{33}{2\sqrt{25}} = \frac{33}{2 \times 5} = \frac{33}{10} \text{ units}$$

Hence, the required distance is $\frac{33}{10}$ units.

(ii) Given parallel lines are

$$lx + ly + h = 0 \text{ and } lx + ly - r = 0$$

On comparing with $Ax + By + C_1 = 0$

and $Ax + By + C_2 = 0$, we get

$$A = l,\ B = l,\ C_1 = h,\ C_2 = -r$$

$$\therefore \text{Required distance} = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \left|\frac{h + r}{\sqrt{l^2 + l^2}}\right| = \left|\frac{h + r}{\sqrt{2}\, l}\right|$$

EXAMPLE |11| Find the equation of the line midway between the parallel lines $9x + 6y - 7 = 0$ and $3x + 2y + 6 = 0$.

Sol. Converting each of the given equations to the form $y = mx + C$, we get

$$9x + 6y - 7 = 0 \Rightarrow y = \frac{-3}{2}x + \frac{7}{6} \quad \text{...(i)}$$

and $$3x + 2y + 6 = 0 \Rightarrow y = \frac{-3}{2}x - 3 \quad \text{...(ii)}$$

Clearly, the slope of each one of the given lines is $\frac{-3}{2}$.

Let the given lines be $y = mx + C_1$ and $y = mx + C_2$.

Then, $m = \frac{-3}{2}, C_1 = \frac{7}{6}$ and $C_2 = -3$

Let L be the required line. Then, L is parallel to each one of lines (i) and (ii) and equidistant from each one of them.

$$\therefore \quad \text{Slope of } L = \frac{-3}{2}$$

Let the equation of L be $y = \frac{-3}{2}x + C$ ...(iii)

Then, distance between lines (i) and (iii) must be equal to the distance between lines (ii) and (iii).

$$\therefore \quad \frac{|C_1 - C|}{\sqrt{1+m^2}} = \frac{|C_2 - C|}{\sqrt{1+m^2}} \Rightarrow |C_1 - C| = |C_2 - C|$$

$$\Rightarrow \quad \left|\frac{7}{6} - C\right| = |-3 - C| \Rightarrow \left|\frac{7}{6} - C\right| = |3 + C|$$

$$\Rightarrow \quad \frac{7}{6} - C = \pm(3 + C)$$

$$\Rightarrow \quad 2C = \frac{-11}{6} \Rightarrow C = \frac{-11}{12}$$

$\therefore$ Equation of L is $y = \frac{-3}{2}x - \frac{11}{12}$, i.e. $18x + 12y + 11 = 0$

Hence, the line $18x + 12y + 11 = 0$ is midway between the parallel lines $9x + 6y - 7 = 0$ and $3x + 2y + 6 = 0$.

EXAMPLE |12| Find the area of a square, if two sides of a square are $x + 2y + 3 = 0$ and $x + 2y = 5$.

Sol. Given sides of a square are parallel lines.

$\therefore$ The distance between two parallel lines $x + 2y + 3 = 0$ and $x + 2y - 5 = 0$ is

$$d = \frac{|-5-3|}{\sqrt{1^2 + 2^2}} \qquad \left[\because d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}\right]$$

$$= \frac{8}{\sqrt{1+4}} = \frac{8}{\sqrt{5}}$$

$\therefore$ Length of side of a square, $a = d = \frac{8}{\sqrt{5}}$

$\therefore$ Area of a square $= a^2 = \left(\frac{8}{\sqrt{5}}\right)^2 = \frac{64}{5}$ sq units

TOPIC PRACTICE 4

OBJECTIVE TYPE QUESTIONS

1 The distance of the point $(3, -5)$ from the line $3x - 4y - 26 = 0$ is

(a) $\frac{3}{7}$ (b) $\frac{2}{5}$ (c) $\frac{7}{5}$ (d) $\frac{3}{5}$

2 If lines are given in general form, i.e. $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, then the distance d between them is given by $d = \frac{m}{n}$, where m and n respectively are

(a) $|C_1 - C_2|, \sqrt{A^2 + B^2}$ (b) $\sqrt{A^2 + B^2}, C_1 - C_2$

(c) $A^2 + B^2, C_2 - C_1$ (d) $C_2 - C_1, A^2 + B^2$

3 If $lx + ly + p = 0$ and $lx + ly - r = 0$ are two parallel lines, then distance between them is equal to $\left|\frac{m}{n}\right|$, where m and n respectively are

(a) $p - r, \sqrt{2}l$ (b) $r - p, \sqrt{2}l$

(c) $p + r, \sqrt{2}l$ (d) $p + r, \sqrt{2l}$

4 The distance between the parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$, is

(a) $\frac{3}{7}$ (b) $\frac{7}{5}$

(c) $\frac{2}{5}$ (d) $\frac{3}{5}$

5 The distance between two parallel lines $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$ is

(a) $\frac{65}{17}$ (b) $\frac{3}{17}$ (c) $\frac{2}{17}$ (d) $\frac{60}{17}$

VERY SHORT ANSWER Type Questions

6 Find the distance of the line $12x - 5y - 7 = 0$ from the origin.

7 Find the distance of the point $P(1, -3)$ from the line $2y - 3x = 4$. **[NCERT Exemplar]**

8 Find the distance between the lines.

$3x + 4y = 9$ and $6x + 8y = 15$ **[NCERT Exemplar]**

SHORT ANSWER Type I Questions

9 If p is the length of perpendicular from origin to the line whose intercept on the axes are a and b, then show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

10 The equation of two sides of a square are $5x - 12y - 65 = 0$ and $5x - 12y + 26 = 0$. Find the area of a square.

11 Find the values of k for which the length of perpendicular from the point $(4, 1)$ on the line $3x - 4y + k = 0$ is 2 units.

12 The perpendicular distance of a line from the origin is 5 units and its slope is -1. Find the equation of the line.

SHORT ANSWER Type II Questions

13 If p and q are the lengths of perpendiculars from the origin to the lines $x\cos\theta - y\sin\theta = k\cos 2\theta$ and $x\sec\theta + y\operatorname{cosec}\theta = k$, respectively. Prove that $p^2 + 4q^2 = k^2$.

14 Find the points on the Y-axis, whose perpendicular distance from the line $4x-3y-12=0$ is 3.

15 If sum of perpendicular distances of a variable point $P(x, y)$ from the lines $x+y-5=0$ and $3x-2y+7=0$ is always 10. Show that P must moves on a line. **[NCERT]**

16 Which of the lines $2x-y+3=0$ and $x-4y-7=0$, is farther from the origin?

17 Find the points on the line $x+y=4$ which lie at a unit distance from the line $4x+3y=10$. **[NCERT Exemplar]**

18 Find a point which is equidistant from the lines $4x+3y+10=0$, $5x-12y+26=0$ and $7x+24y-50=0$ **[NCERT Exemplar]**

19 Find the equation of two straight lines which are parallel to $x+7y+2=0$ and at unit distance from the point $(1,-1)$.

LONG ANSWER Type Question

20 Find the equation of the lines passing through the point $(1,0)$ and at a distance $\frac{\sqrt{3}}{2}$ from the origin. **[NCERT Exemplar]**

21 Find the ratio in which the line $3x+4y+2=0$ divides the distance between the lines $3x+4y+5=0$ and $3x+4y-5=0$. **[NCERT Exemplar]**

22 The points $A(2,3)$, $B(4,-1)$ and $C(-1,2)$ are the vertices of ΔABC. Find the length of perpendicular from C on AB and hence find the area of ΔABC.

23 Prove that the product of lengths of perpendicular drawn from the points $(\sqrt{a^2-b^2}, 0)$ and $(-\sqrt{a^2-b^2}, 0)$ to the line $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ is b^2.

HINTS & ANSWERS

1. (d) Using formula, $d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$, we get

$$d=\frac{|3\cdot 3+(-4)(-5)-26|}{\sqrt{3^2+(-4)^2}}=\frac{3}{5}$$

2. (a) If lines are given in general form, *i.e.*, $Ax+By+C_1=0$ and $Ax+By+C_2=0$, then the distance between them is given by

$$d=\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$$

3. (d) Given, lines $lx+ly+p=0$ and $lx+ly-r=0$ are parallel.

$\therefore$ Distance between two parallel lines

$$=\left|\frac{p-(-r)}{\sqrt{(l)^2+(l)^2}}\right|=\left|\frac{p+r}{\sqrt{2l^2}}\right|=\left|\frac{p+r}{\sqrt{2}l}\right|$$

4. (c) Here, $A=3$, $B=-4$, $C_1=7$, and $C_2=5$.

Therefore, the required distance is

$$d=\frac{|7-5|}{\sqrt{3^2+(-4)^2}}=\frac{2}{5}$$

5. (d) Required distance,

$$d=\left|\frac{C_1-C_2}{\sqrt{a^2+b^2}}\right|=\left|\frac{-34-31}{\sqrt{15^2+8^2}}\right|=\frac{65}{17}\text{ units}$$

6. Use the formula of distance of a point from a line.

Ans. $\frac{7}{13}$ units

7. Use the formula of distance of a point from a line.

Ans. $\sqrt{13}$ units

8. Given equations can be rewritten as

$$3x+4y-9=0 \text{ and } 3x+4y-\frac{15}{2}=0$$

Since, the slope of these lines are same and hence they are parallel to each other.

Therefore, the distance between them is given by

$$d=\left|\frac{9-\frac{15}{2}}{\sqrt{3^2+4^2}}\right| \quad \textbf{Ans. } \frac{3}{10}\text{ unit}$$

9. Equation of line in intercept form is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\Rightarrow \quad \frac{x}{a}+\frac{y}{b}-1=0$$

Its distance from origin is $\left|\frac{0+0-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right|=p$

10. Side of a square, $a=\frac{|26+65|}{\sqrt{5^2+12^2}}=\frac{91}{\sqrt{169}}$ units

Area of square $=a^2$ **Ans.** 49 sq units

11. $\frac{(3\times 4-4\times 1+k)}{\sqrt{3^2+(-4)^2}}=\pm 2$ **Ans.** $k=2$ or $k=-18$

12. Let the required equation be $y=-x+c$.

$\therefore \quad \dfrac{|0+0-c|}{\sqrt{1^2+1^2}}=5 \Rightarrow |c|=5\sqrt{2}$

$\Rightarrow \quad c=5\sqrt{2}$ or $c=-5\sqrt{2}$

Ans. $x+y+5\sqrt{2}=0$ or $x+y-5\sqrt{2}=0$

13. Here, $p=\dfrac{|0\cdot\cos\theta-0\cdot(-\sin\theta)-k\cos 2\theta|}{\sqrt{\cos^2\theta+\sin^2\theta}}$

$\Rightarrow \quad p=|k\cos 2\theta|$

and $q=\dfrac{|0\cdot\sec\theta+0\cdot\operatorname{cosec}\theta-k|}{\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}}$

$\therefore \quad q=\dfrac{|k|}{\sqrt{\dfrac{1}{\cos^2\theta}+\dfrac{1}{\sin^2\theta}}}=\dfrac{|k|}{\sqrt{\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta\cdot\cos^2\theta}}}$

$=\dfrac{|k|\cdot|\sin\theta\cos\theta|}{1}\times\dfrac{2}{2}$

$=\dfrac{1}{2}|k(2\sin\theta\cos\theta)|$

$\Rightarrow \quad q=\dfrac{1}{2}|k\sin 2\theta|$

$\Rightarrow \quad 2q=|k\sin 2\theta|$

$\therefore \quad p^2+4q^2=k^2\cos^2 2\theta+k^2\sin^2 2\theta$

14. $\dfrac{|4(0)-3(y_1)-12|}{\sqrt{4^2+(-3)^2}}=3$

$\Rightarrow \quad \dfrac{3y_1+12}{\sqrt{16+9}}=\pm 3$

Ans. $(0,1)$ and $(0,-9)$

15. $\dfrac{|x+y-5|}{\sqrt{(1)^2+(1)^2}}+\dfrac{|3x-2y+7|}{\sqrt{3^2+(-2)^2}}=10$

$\Rightarrow \sqrt{13}|x+y-5|+\sqrt{2}|3x-2y+7|-10\sqrt{26}=0$

$\Rightarrow x(\sqrt{13}+3\sqrt{2})+y(\sqrt{13}-2\sqrt{2})+7\sqrt{2}-5\sqrt{13}-10\sqrt{26}=0$

[assuming $(x+y-5)$ and $(3x-2y+7)$ as positive]

Similarly, we can obtain the equation of line for any signs of $(x+y-5)$ and $(3x-2y+7)$.

Thus, point P must move on a line.

16. $P_1=\dfrac{|0+0+3|}{\sqrt{(-2)^2+(1)^2}}=\dfrac{3}{\sqrt{5}}$

$P_2=\dfrac{|0+0-7|}{\sqrt{(1)^2+(-4)^2}}=\dfrac{7}{\sqrt{17}}$

Again, $P_1=\dfrac{3}{\sqrt{5}}\times\dfrac{\sqrt{17}}{\sqrt{17}}=\sqrt{\dfrac{153}{85}}$

and $P_2=\dfrac{7}{\sqrt{17}}\times\dfrac{\sqrt{5}}{\sqrt{5}}=\sqrt{\dfrac{245}{85}}$

$\because \quad \sqrt{\dfrac{245}{85}}>\sqrt{\dfrac{153}{85}}$

$\Rightarrow \quad P_2>P_1$

Ans. $x-4y-7=0$ is farther.

17. Let the required point be (h,k).

Since, point (h,k) lies on the line $x+y=4$,

therefore $\quad h+k=4 \quad$ ...(i)

Also, the distance of the point (h,k) from the line $4x+3y=10$ is $\left|\dfrac{4h+3k-10}{\sqrt{16+9}}\right|=1$

$\Rightarrow 4h+3k-10=\pm 5$

Ans. $(3,1)$ and $(-7,11)$

18. Let the point (h,k) is equidistant from the given lines.

Distance from line (i) $=\dfrac{|4h+3k+10|}{\sqrt{16+9}}$

Distance from line (ii) $=\dfrac{|5h-12k+26|}{\sqrt{25+144}}$

Distance from line (iii) $=\dfrac{|7h+24k-50|}{\sqrt{7^2+24^2}}$

Since, the point (h,k) is equidistant from lines (i), (ii) and (iii).

$\therefore \quad \dfrac{|4h+3k+10|}{\sqrt{16+9}}=\dfrac{|5h-12k+26|}{\sqrt{25+144}}=\dfrac{|7h+24k-50|}{\sqrt{49+576}}$

$\Rightarrow \quad \dfrac{|4h+3k+10|}{5}=\dfrac{|5h-12k+26|}{13}=\dfrac{|7h+24k-50|}{25}$

Clearly, if $h=0$, $k=0$, then $\dfrac{10}{5}=\dfrac{26}{13}=\dfrac{50}{25}=2$

Ans. (0, 0)

19. Slope of required lines $=\dfrac{-1}{7}$

Now, let the equation of required lines be $y=\dfrac{-1}{7}x+c$.

Then, we have

$\dfrac{\left|-1+\dfrac{1}{7}-c\right|}{\sqrt{1^2+\left(\dfrac{1}{7}\right)^2}}=1 \Rightarrow c=\dfrac{-6\pm 5\sqrt{2}}{7}$

Ans. $x+7y+6\pm 5\sqrt{2}=0$

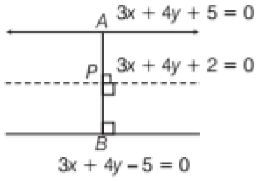
20. Here, $y-0=m(x-1) \Rightarrow y-mx+m=0$

Since, the distance from origin is $\dfrac{\sqrt{3}}{2}$.

Then, $\quad \dfrac{\sqrt{3}}{2}=\dfrac{|0-0+m|}{\sqrt{1+m^2}}$

Ans. $\sqrt{3}x-y-\sqrt{3}=0$ and $\sqrt{3}x+y-\sqrt{3}=0$

21. $3x+4y+5=0$



Now, distance between the lines $3x+4y+5=0$ and $3x+4y+2=0$ is $d_1=\frac{|5-2|}{\sqrt{9+16}}=\frac{3}{5}$

and distance between the lines $3x+4y+2=0$ and $3x+4y-5=0$ is $d_2=\frac{|-5-2|}{\sqrt{9+16}}=\frac{7}{5}$ **Ans.** 3 : 7

22. Equation of AB is $\frac{y-3}{x-2}=\frac{-4}{2}$ $\Rightarrow 2x+y-7=0$

Now, perpendicular distance from C on AB

$$=\frac{|2\times(-1)+2-7|}{\sqrt{2^2+1^2}}=\frac{7}{\sqrt{5}}\text{ units}$$

and ar $(\Delta ABC)=\frac{1}{2}\times AB\times\frac{7}{\sqrt{5}}=\left(\frac{1}{2}\times\sqrt{20}\times\frac{7}{\sqrt{5}}\right)$ sq units

Ans. $\frac{7}{\sqrt{5}}$ units, 7 sq units

23. The perpendicular distance from $(\sqrt{a^2-b^2},0)$ to the given line is $p_1=\frac{\left|\frac{\cos\theta}{a}\cdot\sqrt{a^2-b^2}+0-1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}$

and the perpendicular distance from $(-\sqrt{a^2-b^2},0)$ to the given line is $p_2=\frac{\left|\frac{\cos\theta}{a}(-\sqrt{a^2-b^2})-1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}$

Now, $p_1p_2=\frac{\left|\left(\frac{\sqrt{a^2-b^2}}{a}\cos\theta-1\right)\left(\frac{-\sqrt{a^2-b^2}}{a}\cos\theta-1\right)\right|}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}$

$$=\frac{\left|-[a^2(1-\cos^2\theta)+b^2\cos^2\theta]\right|b^2}{a^2\sin^2\theta+b^2\cos^2\theta}$$

SUMMARY

- A **straight line** is a curve, such that all the points on the line segment joining any two of it lies on it.
- The smallest angle 'θ' made by the line with positive X-axis in anti-clockwise direction, is called **angle of inclination** of a line.
- If θ is the angle of inclination of a line l, then $\tan\theta$ is called the **slope or gradient** (m) of the line i.e. $m = \tan\theta$.
- Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two given points and θ be the inclination of the line AB with X-axis, then slope of line joining two points is $m = \tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}$.
- Let m_1 and m_2 be the slopes of two lines, then angle between these lines $\theta = \tan^{-1}\left|\dfrac{m_2 - m_1}{1 + m_1 m_2}\right|$.

  (i) If $m_1 = m_2$, then two lines are **parallel**. (ii) If $m_1 \cdot m_2 = -1$, then two lines are **perpendicular**.
- Three points A, B and C are collinear, if slope of AB = slope of BC.
- Equation of **horizontal line** is $y = a$ or $y = -a$.
- Equation of **vertical line** is $x = b$ or $x = -b$.
- Equation of line in **point slope form** is $y - y_0 = m(x - x_0)$.
- Equation of line in **two point form** is $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$.
- Equation of line in **slope intercept form** is $y = mx + c$.
- Equation of line in **intercept form** is $\dfrac{x}{a} + \dfrac{y}{b} = 1$.
- Equation of line in **normal form** is $x\cos\alpha + y\sin\alpha = p$.
- The **distance of a point from a line** is the length of perpendicular drawn from the point to the line.

  Let $L: Ax + By + C = 0$, then distance of $P(x_1, y_1)$ from this line is $d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.
- Let $y = mx + c_1$ and $y = mx + c_2$ be two lines, the distance between **two parallel lines** is given by $d = \dfrac{|c_1 - c_2|}{\sqrt{1 + m^2}}$.

  and if two parallel lines are $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, then distance $d = \dfrac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$.
- If the equation of line is $Ax + By + C = 0$, then

  (i) parallel equation of line is $Ax + By + k = 0$. (ii) perpendicular equation of line is $Bx - Ay + k = 0$.

CHAPTER PRACTICE

OBJECTIVE TYPE QUESTIONS

1. The distance between the points $(6, -4)$ and $(3, 0)$ is
(a) 7 units (b) 25 units
(c) $\sqrt[3]{25}$ units (d) 5 units

2. The coordinates of a point which divides the line segment joining $A(1, -3)$ and $B(-3, 9)$ internally in the ratio 1 : 3, are given by
(a) $(-2, 6)$ (b) $(0, 0)$
(c) $\left(\frac{-6}{4}, \frac{18}{4}\right)$ (d) $\left(\frac{10}{4}, \frac{-30}{4}\right)$

3. Area of triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3), is
(a) $|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$
(b) $|x_1(y_2 - y_3) + x_2(y_1 - y_3) + x_3(y_1 - y_2)|$
(c) $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$
(d) $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_1 - y_3) + x_3(y_1 - y_2)|$

4. The angle between the X-axis and the line joining the points $(4, -2)$ and $(5, -3)$ is
(a) 45° (b) 135° (c) 90° (d) 180°

5. A line passes through (x_1, y_1) and (h, k). If slope of the line is m, then
(a) $y_1 - k = m(h - x_1)$ (b) $k - y_1 = m(h - x_1)$
(c) $h - x_1 = m(y_1 - k)$ (d) $h - x_1 = m(k - y_1)$

6. Suppose a line L make x-intercept a and y-intercept b on the axes. Then, the equation of the line L is
(a) $\frac{x}{a} + \frac{y}{b} = 1$ (b) $\frac{x}{a} - \frac{y}{b} = 1$ (c) $\frac{y}{b} - \frac{x}{a} = 1$ (d) $ax + by = 1$

7. The equation of the line which have slope -2 and cuts-off an intercept -6 on X-axis, is
(a) $2x - y + 12 = 0$ (b) $2x + y + 12 = 0$
(c) $-2x + y + 12 = 0$ (d) $2x + y - 12 = 0$

8. If the lines $x + q = 0$, $y - 2 = 0$ and $3x + 2y + 5 = 0$ are concurrent, then the value of q will be
(a) 1 (b) 2 (c) 3 (d) 5

9. A point moves such that its distance from the point $(4, 0)$ is half that of its distance from the line $x = 16$. The locus of the point is **[NCERT Exemplar]**
(a) $3x^2 + 4y^2 = 192$
(b) $4x^2 + 3y^2 = 192$
(c) $x^2 + y^2 = 192$
(d) None of the above

SHORT ANSWER Type I Questions

10. Without using Pythagoras theorem, show that the points $A(1,2)$, $B(1,5)$ and $C(5,2)$ are the vertices of a right angled triangle.

11. Prove that the line joining the mid-points of the two sides of a triangle is parallel to the third side.

12. If points $(a,0)$, $(0, b)$ and (x, y) are collinear using the concept of slope. Prove that $\frac{x}{a} + \frac{y}{b} = 1$.

SHORT ANSWER Type II Questions

13. A quadrilateral has the vertices at the points $(4,1)$, $(1,7)$, $(-6,0)$ and $(-1,-9)$. Show that the mid-points of the sides of this quadrilateral are the vertices of a parallelogram.

14. Prove that the points $A(2, 3)$, $B(6, 7)$, $C(6,9)$ and $D(2,5)$ are the vertices of a rectangle.

15. Prove that $A(4, 3)$, $B(6,4)$, $C(5,6)$ and $D(3,5)$ are the angular points of a square.

16. Find the equations of the bisectors of the angles between the coordinate axes.

17. If $A(2,0)$, $B(0,2)$ and $C(0,7)$ are three vertices taken in order of an isosceles trapezium $ABCD$ in which $AB \| DC$. Find the coordinates of D.

18. Find the equation of a line which is perpendicular to the line joining the points $(4, 2)$ and $(3, 5)$ and cuts-off an intercept of length 3 on Y-axis.

19. Find the equation of a line which divides the join of $(1, 0)$ and $(3, 0)$ in the ratio 2 : 1 and perpendicular to it.

20. Find the equation of the right bisector of the line segment joining the points $A(1,0)$ and $B(2,3)$.

LONG ANSWER Type Questions

21. Using the concept of slope, prove that medians of an equilateral triangle are perpendicular to the corresponding sides.

22. One side of a square makes an angle α with X-axis and one vertex of the square is at the origin. Prove that the equations of its diagonals are $x(\sin\alpha+\cos\alpha)=y(\cos\alpha-\sin\alpha)$ and $x(\cos\alpha-\sin\alpha)+y(\sin\alpha+\cos\alpha)=a$, where a is the length of the side of the square.

23. Find the equation of the line through the point $A(2,3)$ and making an angle of $45°$ with the X-axis. Also, determine the length of intercept on it between A and the line $x+y+1=0$.

24. Find the distance of the point $(2,3)$ from the line $2x-3y+9=0$ measured along a line $x-y+1=0$.

25. Find the distance of the point $(2, 5)$ from the line $3x+y+4=0$ measured parallel to the line $3x-4y+8=0$.

26. The line $\frac{x}{a}+\frac{y}{b}=1$ moves in such a way that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$, where c is a constant. Find the locus of the foot of the perpendicular from the origin on the given line.

27. Find the equation of the straight line passing through the origin and bisecting the portion of the line $ax+by+c=0$ intercepted between the coordinate axes.

28. Find the value of m for which the lines $mx+(2m+3)y+m+6=0$ and $(2m+1)x+(m-1)y+m-9=0$ intersect at a point on Y-axis.

29. Three sides AB, BC and CA of a ΔABC are $5x-3y+2=0$, $x-3y-2=0$ and $x+y-6=0$, respectively. Find the equation of the altitude through the vertex A.

30. Find the image of the point $(-8,12)$ with respect to the line mirror $4x+7y+13=0$.

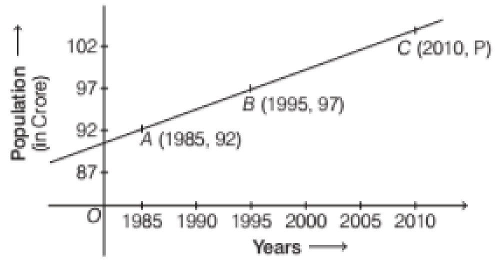
31. A straight line L is perpendicular to the line $5x-y=1$. The area of the triangle formed by the line L and the coordinate axes is 5 sq units. Find the equation of the line L.

32. The sides AB and AC of a ΔABC are $2x+3y=29$ and $x+2y=16$, respectively. If the mid-point of BC is (5, 6), then find the equation of BC.

33. The vertices of a triangle are $A(10,4)$, $B(-4,9)$ and $C(-2,-1)$. Find the equations of its altitude. Also, find the orthocentre.

CASE BASED Questions

34. Population *vs* Year graph given below.



Based on the above information answer the following questions.

(i) The slope of line AB is

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

(ii) The equation of line AB is

(a) $x+2y=1791$
(b) $x-2y=1801$
(c) $x-2y=1791$
(d) $x-2y+1801=0$

(iii) The population (in crores) in year 2010 is

(a) 104.5 (b) 119.5
(c) 109.5 (d) None of these

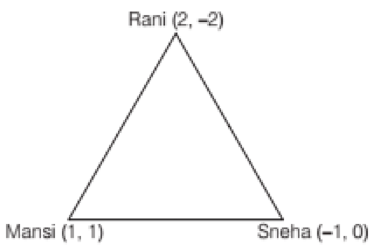
(iv) The equation of line perpendicular to line AB and passing through (1995, 97) is

(a) $2x-y=4087$ (b) $2x+y=4087$
(c) $2x+y=1801$ (d) None of these

(v) In which year the population becomes 110 crore is

(a) 2020 (b) 2019
(c) 2021 (d) 2022

35. Three girls, Rani, Mansi, Sneha are talking to each other while maintaining a social distance due to covid-19. They are standing on vertices of a triangle, whose coordinates are given.



Based on the above information answer the following questions.

(i) The equation of lines formed by Rani and Mansi is

(a) $3x - y = 4$ (b) $3x + y = 4$
(c) $x - 3y = 4$ (d) $x + 3y = 4$

(ii) Slope of equation of line formed by Rani and Sneha is

(a) $\frac{2}{3}$ (b) $\frac{-3}{2}$ (c) $\frac{-2}{3}$ (d) $\frac{1}{3}$

(iii) The equation of median of lines through Rani is

(a) $5x + 4y = 2$ (b) $5x - 4y = 2$
(c) $4x - 5y = 1$ (d) None of these

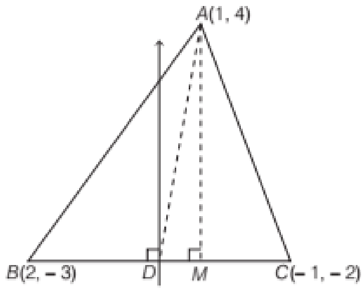
(iv) The equation of altitude through Mansi is

(a) $3x - 2y = 1$ (b) $2x + 3y = 5$
(c) $x + 2y = 3$ (d) None of these

(v) The equation of line passing through the Rani and parallel to line formed by Mansi and Sneha is

(a) $x - 2y = 4$ (b) $x + 2y = 6$
(c) $x - 2y = 6$ (d) $2x + y = 4$

36. Consider the ΔABC with vertices $A(1, 4)$, $B(2, -3)$ and $C(-1, -2)$ as shown in the given figure. AD is the median and AM is the altitude through A.



Based on the above information answer the following questions.

(i) Find the distance between A and C.

(a) $\sqrt{40}$ units (b) $\sqrt{53}$ units
(c) $\sqrt{41}$ units (d) $\sqrt{29}$ units

(ii) Find the slope of BC.

(a) $-\frac{4}{3}$ (b) $-\frac{1}{3}$ (c) $-\frac{3}{2}$ (d) $-\frac{3}{4}$

(iii) Find the equation of median through A.

(a) $x - 13y + 9 = 0$ (b) $x + 13y - 9 = 0$
(c) $13x - y - 9 = 0$ (d) $2x - 13y + 9 = 0$

(iv) Find the equation of the altitude through A.

(a) $3x - y + 1 = 0$ (b) $x + 2y - 3 = 0$
(c) $x - 3y + 2 = 0$ (d) $3x + 2y - 2 = 0$

(v) Find the equation of right bisector of side BC.

(a) $x + 3y - 3 = 0$ (b) $x - 3y + 3 = 0$
(c) $3x - y - 4 = 0$ (d) $3x + y - 2 = 0$

HINTS & ANSWERS

1. (d) The distance between the points $(6, -4)$ and $(3, 0)$

$$= \sqrt{(6-3)^2 + (-4-0)^2}$$
$$= \sqrt{(3)^2 + (4)^2}$$
$$= \sqrt{25} = 5 \text{ units}$$

2. (b) The coordinates of the point which divides the line segment joining $A\,(1, -3)$ and $B\,(-3, 9)$ internally in the ratio 1 : 3, are given by

$$x = \frac{1\cdot(-3) + 3\cdot 1}{1+3} = 0$$

and $$y = \frac{1\cdot 9 + 3\cdot(-3)}{1+3} = 0$$

3. (c) Area of the triangle, whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3), is

$$\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

4. (b) Given, $(x_1, y_1) = (4, -2)$ and $(x_2, y_2) = (5, -3)$

Slope of line $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3+2}{5-4}$

$$\tan\theta = \frac{-1}{1}$$
$\Rightarrow$ $\tan\theta = -1$
$\Rightarrow$ $\tan\theta = -\tan 45°$ $(\because 1 = \tan 45°)$
$\Rightarrow$ $\tan\theta = \tan(180° - 45°)$
$[\because \tan(180° - \theta) = -\tan\theta]$
$\Rightarrow$ $\tan\theta = \tan 135°$
$\Rightarrow$ $\theta = 135°$

5. (b) We know that, $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ $m = \frac{k - y_1}{h - x_1}$

$\Rightarrow$ $k - y_1 = m(h - x_1)$

6. (a) By intercept form, we get

$$\frac{x}{a} + \frac{y}{b} = 1$$

7. (b) Given, slope $(m) = -2$

and intercept on X-axis is -6

$\therefore$ $y = -2(x+6) \Rightarrow y = -2x - 12$

$\Rightarrow$ $2x + y + 12 = 0$

8. (c) The intersection point of $y - 2 = 0$ and $3x + 2y + 5 = 0$ is $(-3, 2)$.

Since the lines are concurrent, therefore $(-3, 2)$ lie on $x + q = 0$.

$\Rightarrow$ $-3 + q = 0 \Rightarrow q = 3$

9. (a) Let (h, k) be the coordinates of the moving point. Then, we have

$$\sqrt{(h-4)^2+k^2} = \frac{1}{2}\left(\frac{h-16}{\sqrt{1^2+0}}\right)$$

$$\Rightarrow \quad (h-4)^2+k^2 = \frac{1}{4}(h-16)^2$$

$$4(h^2-8h+16+k^2) = h^2-32h+256$$

$$\text{or} \quad 3h^2+4k^2 = 192$$

Hence, the required locus is given by $3x^2+4y^2=192$

10. Slope of AB = Not defined

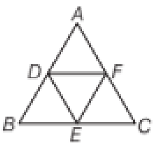
Slope of $BC = \frac{-3}{4}$ and slope of $AC = 0$

$\because$ AB is parallel to Y-axis and AC is parallel to X-axis.

Thus $\angle A = 90°$

[$\because$ angle between X-axis and Y-axis is 90°]

11. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of ΔABC. Also, let D, E and F are the mid-points of sides AB, BC and CA, respectively.



Then, coordinates of $D = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$,

Coordinates of $E = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

and coordinate of $F = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

Now, show that slope of DF = slope of BC,
slope of DE = slope of AC
and slope of EF = slope of AB

12. Let the given points are $A(a, 0)$, $B(0, b)$ and $C(x, y)$.

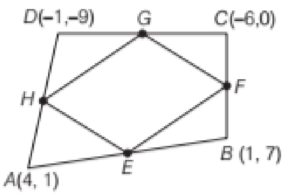
Now, we have

Slope of AB = Slope of BC

$$\Rightarrow \quad \frac{b}{-a} = \frac{y-b}{x} \Rightarrow xb = -ay+ab$$

$$\Rightarrow \quad ay+bx = ab$$

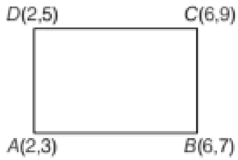
13. Let $ABCD$ be the given quadrilateral.



Also, let E, F, G and H are the mid-points of sides AB, BC, CD and DA, respectively.

Now, find the coordinates of E, F, G and H, and then show that slope of EF = slope of GH and slope of FG = slope of EH.

14. Prove that



(i) Slope of AB = Slope of DC

(ii) Slope of AD = Slope of BC

and (iii) Slope of $AB \times$ Slope of $BC = -1$.

15. Prove that

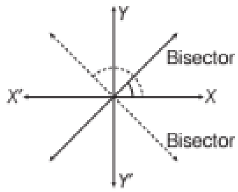
(i) Slope of AB = Slope of DC

(ii) Slope of AD = Slope of BC

(iii) Slope of $AC \times$ Slope of $BD = -1$

(iv) Slope of $AB \times$ Slope of $BC = -1$

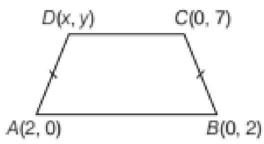
16. Bisectors of the angles between the coordinate axes, passes through origin and makes an angle of 45° or 135° with positive direction of X-axis.



Ans. $x \pm y = 0$

17. Given, $ABCD$ is an isosceles trapezium and $AB \parallel DC$.

We have, $AD = BC$ and Slope of AB = Slope of DC



Let the coordinates of D be (x, y).

Then, we have $x^2-4x+y^2=21$ and $x+y=7$

On solving these two equations, we get

$$(x, y) = (2, 5) \text{ or } (7, 0)$$

18. Slope of line joining the points (4, 2) and (3, 5) is -3.

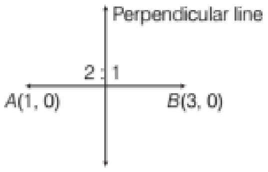
$\therefore$ Slope of line perpendicular to it is $\frac{1}{3}$.

$\because$ The perpendicular line cuts-off an intercept of length 3 on Y-axis.

$\therefore$ Equation of the line is $y = \frac{1}{3}x+3$

$$\Rightarrow \quad x-3y+9=0.$$

19. Slope of perpendicular line $= \dfrac{-1}{\text{Slope of } AB}$



$= \dfrac{-1}{0} =$ Not defined

$\Rightarrow$ Perpendicular line is parallel to Y-axis.

$\because$ The coordinates of C are $\left(\dfrac{7}{3}, 0\right)$

$\therefore$ Equation of the perpendicular line is $x = \dfrac{7}{3}$

i.e. $\quad 3x = 7.$

20. Mid-point of $AB = \left(\dfrac{3}{2}, \dfrac{3}{2}\right)$

Slope of right bisector $= \dfrac{-1}{\text{Slope of } AB} = \dfrac{-1}{3}$

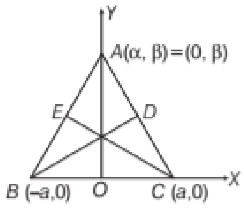
Equation of right bisector is given by

$$y - \frac{3}{2} = \frac{-1}{3}\left(x - \frac{3}{2}\right) \Rightarrow x + 3y - 6 = 0$$

21. Let ABC be an equilateral triangle of side $2a$.
For sake of convenience, consider BC along X-axis such that mid-point of BC be the origin. Then, the coordinates of B and C are $(-a, 0)$ and $(a, 0)$.

Let the coordinate of A be (α, β).

Since, ABC is an equilateral triangle.



$\therefore \quad AB = AC = BC$

$\Rightarrow \quad AB = AC \Rightarrow AB^2 = AC^2 \Rightarrow \alpha = 0$

Also, $AB^2 = BC^2 \Rightarrow a^2 + \beta^2 = (2a)^2 \Rightarrow \beta^2 = 3a^2$

$\Rightarrow \quad \beta = \pm\sqrt{3}a$

Thus, the coordinates of A are $(0, \pm\sqrt{3}a)$. Consider A on positive Y-axis, i.e. consider the coordinates of A as $(0, \sqrt{3}a)$. Now, let D and E are the mid-points of sides AC and AB, respectively.

Then, coordinates of $D = \left(\dfrac{a}{2}, \dfrac{\sqrt{3}a}{2}\right)$

and coordinates of $E = \left(\dfrac{-a}{2}, \dfrac{\sqrt{3}a}{2}\right)$

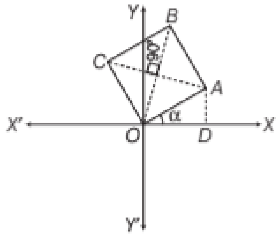
Now, prove that

(ii) Slope of $BD \times$ Slope of $AC = -1$

(iii) Slope of $CE \times$ Slope of $AB = -1$

22. Let $OABC$ be the given square, such that $OA = a$ and $\angle AOX = \alpha$.

Now, draw $AD \perp OX$.



In ΔODA, $\cos\alpha = \dfrac{OD}{OA} = \dfrac{OD}{a}$

$\Rightarrow \quad OD = a\cos\alpha$ and $\sin\alpha = \dfrac{AD}{OA} = \dfrac{AD}{a} \Rightarrow AD = a\sin\alpha$

Thus, the coordinates of A are $(a\cos\alpha, a\sin\alpha)$.

Clearly, the diagonal OB makes an angle $\dfrac{\pi}{4} + \alpha$ with positive X-axis and passes through $(0, 0)$.

So, the equation of OB is given by

$$y - 0 = \tan\left(\frac{\pi}{4} + \alpha\right)(x - 0)$$

$$\Rightarrow \quad y = \frac{1 + \tan\alpha}{1 - \tan\alpha} \cdot x \Rightarrow y = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} \cdot x$$

$$\Rightarrow \quad y(\cos\alpha - \sin\alpha) = (\cos\alpha + \sin\alpha)x$$

Now, as $OB \perp AC$

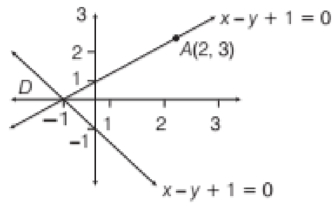
$\therefore$ Slope of $AC = \dfrac{-1}{\text{Slope of } OB} = \dfrac{-1}{\tan\left(\dfrac{\pi}{4} + \alpha\right)}$

Now, the equation of diagonal AC, having slope $\dfrac{-1}{\tan\left(\dfrac{\pi}{4} + \alpha\right)}$ and passes through $(a\cos\alpha, a\sin\alpha)$ is given

by $(y - a\sin\alpha) = \dfrac{-1}{\tan\left(\dfrac{\pi}{4} + x\right)}(x - a\cos\alpha)$

$$\Rightarrow \quad x(\cos\alpha - \sin\alpha) + y(\cos\alpha + \sin\alpha) = a$$

23. Equation of line, passing through $A(2, 3)$ and making an angle of 45° with X-axis, is given by

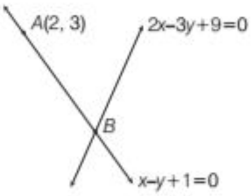


$$y - 3 = \tan 45°(x - 2) \Rightarrow x - y + 1 = 0$$

Let it intersect the line $x + y + 1$ at point D.

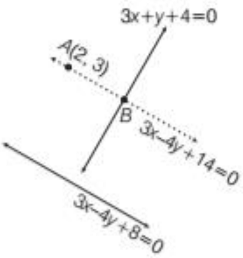
Then, solving these equations, we get $D(-1, 0)$
Now, length of intercept between A and the line $x + y + 1 = 0 =$ length of $AD = 3\sqrt{2}$.

24. Let the given point be $A(2, 3)$, which lie on the line $x - y + 1 = 0$.



Let $x - y + 1 = 0$ intersect $2x - 3y + 9 = 0$ at point B.
Then, required distance = length of $AB = 4\sqrt{3}$ units

25. Equation of the line parallel to $3x - 4y + 8 = 0$ is given by $3x - 4y + k = 0$



Now, required parallel line should pass through (2, 3).
We have, $3(2) - 4(5) + k = 0 \Rightarrow k = 20 - 6 = 14$
Thus, distance of the point (2, 5) from the line $3x + y + 4 = 0$ will be measured along the line

$$3x - 4y + 14 = 0.$$

Now, solving $3x + y + 4 = 0$ and $3x - 4y + 14 = 0$,
we get $x = -2$ and $y = 2$
Thus, the coordinates of B are $(-2, 2)$.
Hence, required distance = length of $AB = 5$ units.

26. Let $P(h, k)$ be the foot of perpendicular from the origin to the line. Then, $OP =$ Distance if origin from the line

$$\frac{x}{a} + \frac{y}{b} = 1 = \frac{|0+0-1|}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}}$$

$$\Rightarrow \sqrt{(h-0)^2 + (k-0)^2} = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \Rightarrow \sqrt{h^2 + k^2} = \frac{1}{\sqrt{\frac{1}{c^2}}}$$

$$\Rightarrow \sqrt{h^2 + k^2} = \sqrt{c^2}$$

On squaring both sides, we get

$$h^2 + k^2 = c^2$$

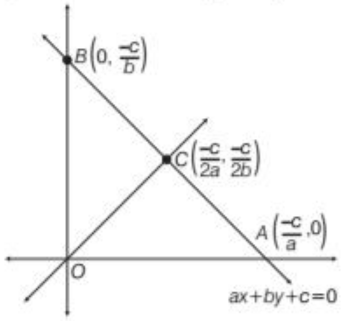
Thus, the locus of the foot of the perpendicular from the origin is $x^2 + k^2 = c^2$.

27. Clearly, the line $ax + by + c = 0$ intersect X-axis at

$$A\left(\frac{-c}{a}, 0\right) \text{ and } B\left(0, \frac{-c}{b}\right).$$

Mid-point of AB is $C\left(\frac{-c}{2a}, \frac{-c}{2b}\right)$.

Now, the equation of line OC is given by



$$y - 0 = \frac{-c/2b}{-c/2a}(x - 0) \Rightarrow ax - by = 0$$

28. Line (i) intersect Y-axis at $\left(0, \frac{-m+9}{2m+3}\right)$ and line (ii) intersect Y-axis at $\left(0, -\frac{(m-9)}{m-1}\right)$.

Now, for lines to be intersect at a point on Y-axis.

$$-\left(\frac{m+6}{2m+3}\right) = -\left(\frac{m-9}{m-1}\right) \Rightarrow \frac{m+6}{2m+3} = \frac{m-9}{m-1}$$

$$\Rightarrow m^2 - m + 6m - 6 = 2m^2 + 3m - 18m - 27$$

$$\Rightarrow \quad m^2 + 5m - 6 = 2m^2 - 15m - 27$$

$$\Rightarrow \quad m^2 - 20m - 21 = 0 \Rightarrow m = -1, 21$$

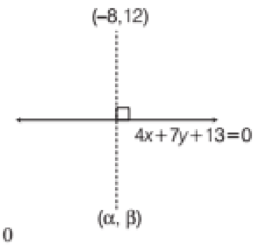
29. Let $AD \perp BC$. Then, slope of $AD = \frac{-1}{\text{Slope of } BC} = \frac{-1}{\left(\frac{1}{3}\right)} = -3$

Now, let the solve the equation of AB and AC, to get the coordinates of vertex A. This will give $A(2, 4)$.

Now, the equation of AD is given by

$$(y-4) = -3(x-2) \Rightarrow 3x + y - 10 = 0$$

30. Let the image of the point $P(-8, 12)$ be $Q(\alpha, \beta)$. Then,
(a) Mid-point of PQ lie on $4x + 7y + 13 = 0$.
(b) Slope of $PQ \times$ Slope of $(4x + 7y + 13 = 0) = -1$



Now, (a)

$$4\left(\frac{-8+\alpha}{2}\right) + 7\left(\frac{12+\beta}{2}\right) + 13 = 0$$

$$\Rightarrow \quad 4\alpha + 7\beta + 78 = 0 \quad \text{...(ii)}$$

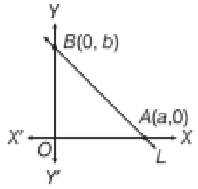
(b) $$\frac{\beta-12}{\alpha+8} \times \left(\frac{-4}{7}\right) = -1 \Rightarrow 7\alpha - 4\beta + 104 = 0 \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get $\alpha = -16$ and $\beta = -2$
Thus, the image of point $(-8, 12)$ is $(-16, -2)$.

31. Slope of line $L = \dfrac{-1}{\text{Slope of } 5x - y = 1} = \dfrac{-1}{5}$

Let line L intersect X-axis at $A(a, 0)$ and Y-axis at $B(0, b)$.

Then, area of $\Delta OAB = \dfrac{1}{2}ab$



$\Rightarrow \quad \dfrac{1}{2}ab = 5 \Rightarrow ab = 10$ sq units ...(i)

Also, slope of line $L = \dfrac{-b}{a} \Rightarrow \dfrac{-b}{a} = -\dfrac{1}{5} \Rightarrow b = \dfrac{a}{5}$ ...(ii)

On solving Eqs. (i), (ii) we get

$a = 5\sqrt{2}, b = \sqrt{2}$ or $a = -5\sqrt{2}, b = -\sqrt{2}$

Now, the equation of line L is

$$\frac{x}{5\sqrt{2}} + \frac{y}{\sqrt{2}} = 1 \text{ or } \frac{x}{-5\sqrt{2}} + \frac{y}{-\sqrt{2}} = 1$$

$\Rightarrow \quad x + 5y - 5\sqrt{2} = 0$ or $x + 5y + 5\sqrt{2} = 0$

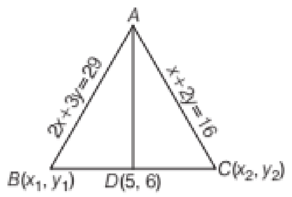
32. Let the coordinates of B and C are (x_1, y_1) and (x_2, y_2), respectively. Then, we have

$2x_1 + 3y_1 = 29$ ...(i)

$x_2 + 2y_2 = 16$ ...(ii)

$\dfrac{x_1 + x_2}{2} = 5 \Rightarrow x_1 + x_2 = 10$ ...(iii)

and $\dfrac{y_1 + y_2}{2} = 6 \Rightarrow y_1 + y_2 = 12$ ...(iv)



On substituting the values of x_2 and y_2 from Eqs. (iii) and (iv) in Eq. (ii), we get $(10 - x_1) + 2(12 - y_1) = 16$

$\Rightarrow \quad 10 - x_1 + 24 - 2y_1 = 16 \Rightarrow x_1 + 2y_1 = 18$ ...(v)

Now, solving Eqs. (i) and (v), we get

$x_1 = 4$ and $y_1 = 7 \Rightarrow B(x_1, y_1) = B(4, 7)$

Now, the equation of BC is given by

$$y - 7 = \frac{6-7}{5-4}(x - 4)$$

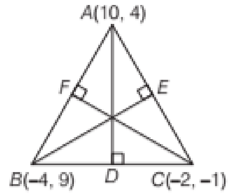
$\Rightarrow \quad x + y - 11 = 0$

33. Slope of $AD = \dfrac{-1}{\text{Slope of } BC} = \dfrac{-1}{-5} = \dfrac{1}{5}$

Slope of $BE = \dfrac{-1}{\text{Slope of } AC} = \dfrac{-12}{5}$

and slope of $CF = \dfrac{-1}{\text{Slope of } AD} = \dfrac{14}{5}$

Equation of AD is given by



$(y - 4) = \dfrac{1}{5}(x - 10) \Rightarrow x - 5y + 10 = 0$ ...(i)

Equation of BE is given by

$(y - 9) = -\dfrac{12}{5}(x + 4) \Rightarrow 12x + 5y + 3 = 0$ ...(ii)

and equation of CF is given by

$(y + 1) = \dfrac{14}{5}(x + 2) \Rightarrow 14x - 5y + 23 = 0$ ...(iii)

Now to find the orthocentre, let us solve any two equations.

34. (i) (c) Slope of $AB = \dfrac{97 - 92}{1995 - 1985} = \dfrac{5}{10} = \dfrac{1}{2}$

(ii) (b) Equation of line AB is

$y - 92 = \dfrac{1}{2}(x - 1985)$

$2y - 184 = x - 1985$

$x - 2y = 1801$

(iii) (a) Let the population in year 2010 is P.

Since, A, B, C are collinear

$\therefore$ Slope of AB = slope of BC

$\dfrac{97 - 92}{1995 - 1985} = \dfrac{P - 97}{2010 - 1995}$

$\Rightarrow \quad \dfrac{1}{2} = \dfrac{P - 97}{15}$

$P = 97 + 7.5 = 104.5$ crore

(iv) (b) Equation of line perpendicular to AB passing through (1995, 97).

$\because$ Slope of $AB = \dfrac{1}{2}$

Slope of line perpendicular to $AB = \dfrac{-1}{\frac{1}{2}} = -2$

$\therefore$ Equation of required line is

$y - 97 = -2(x - 1995)$

$\Rightarrow \quad y - 97 = -2x + 3990$

$\Rightarrow \quad 2x + y = 4087$

(v) (c) Equation of line AB is

$x - 2y = 1801$

Putting $y = 110$

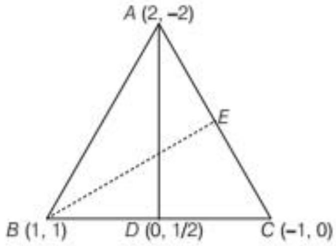
$\therefore \quad x = 1801 + 220$

$\Rightarrow \quad x = 2021$

$\therefore$ Population becomes 110 crore in 2021.

35. Let the point on Rani, Mansi and Sneha stand on a vertices of triangle be A, B, C

$\therefore$ $A(2, -2)$, $B(1, 1)$, $C(-1, 0)$



(i) (b) The equation of line AB is

$$y-1=\frac{-2-1}{2-1}(x-1)$$

$$y-1=-3x+3$$

$$\Rightarrow \quad 3x+y=4$$

(ii) (c) Slope of equation of line AC is $m=\dfrac{0+2}{-1-2}=\dfrac{2}{-3}$

(iii) (a) Let D be the mid-point of BC.

Coordinates of D are $\left(\dfrac{1-1}{2}, \dfrac{0+1}{2}\right)=\left(0, \dfrac{1}{2}\right)$

$\therefore$ Equation of AD is $y+2=\dfrac{\frac{1}{2}+2}{0-2}(x-2)$

$$y+2=\frac{-5}{4}(x-2)$$

$$4y+8=-5x+10$$

$$\Rightarrow \quad 5x+4y=2$$

(iv) (a) Slope of $AC=\dfrac{-2}{3}$

$\therefore$ Slope of $BE=\dfrac{3}{2}$ $\quad$ [$\because BE \perp AC$]

Equation of altitude through B is

$$y-1=\frac{3}{2}(x-1) \Rightarrow 3x-2y=1$$

(v) (c) Slope of line $BC=\dfrac{0-1}{-1-1}=\dfrac{1}{2}$

Equation of line passing through A and parallel to BC, is

$$y+2=\frac{1}{2}(x-2)$$

$$2y+4=x-2$$

$$\Rightarrow \quad x-2y=6$$

36. (i) (a) $AC=\sqrt{(-1-1)^2+(-2-4)^2}$ [using distance formula]

$$=\sqrt{(-2)^2+(-6)^2}$$

$$=\sqrt{4+36}=\sqrt{40} \text{ units}$$

(ii) (b) Slope of $BC=\dfrac{-2-(-3)}{-1-2}=\dfrac{-2+3}{-3}=-\dfrac{1}{3}$

(iii) (c) Since D is mid-point of BC

$\therefore$ Coordinates of D are $\left(\dfrac{2-1}{2}, \dfrac{-3-2}{2}\right)=\left(\dfrac{1}{2}, -\dfrac{5}{2}\right)$

$\therefore$ Slope of $AD=\dfrac{-\frac{5}{2}-4}{\frac{1}{2}-1}=\dfrac{-\frac{13}{2}}{-\frac{1}{2}}=13$

$\therefore$ Equation of the median AD is

$$y-4=13(x-1)$$

$$\Rightarrow \quad 13x-y-9=0$$

(iv) (a) Since AM is the altitude through A

$\therefore$ Slope of $AM=-\dfrac{1}{\text{slope of } BC}=-\dfrac{1}{-\frac{1}{3}}=3$

$\therefore$ Equation of the altitude through A is given by

$$y-4=3(x-1)$$

$$\Rightarrow \quad y-4=3x-3 \Rightarrow 3x-y+1=0$$

(v) (c) Equation of the right bisector of BC is a line which passes through D and having slope is 3.

$$\Rightarrow \quad y-\left(-\frac{5}{2}\right)=3\left(x-\frac{1}{2}\right)$$

$$\Rightarrow \quad y+\frac{5}{2}=3x-\frac{3}{2} \Rightarrow 3x-y-4=0$$