

Number System and its Operations

IIT Foundation Material

SECTION - I

Straight Objective Type

This section contains multiple choice questions. Each question has 4 choices (a), (b), (c), (d), out of which ONLY ONE is correct. Choose the correct option.

1. The square root of $14\sqrt{6} - 30$ is equal to
(a) $5^{1/2}(3 - \sqrt{5})$ (b) $5^{1/4}(3 - \sqrt{5})$
(c) $5^{1/2}(3 + \sqrt{5})$ (d) None of these
2. The value of $\frac{1}{\sqrt{5} - \sqrt{5} - \sqrt{24}} + \frac{1}{\sqrt{5} - \sqrt{5} - \sqrt{24}}$ is
(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{3}$ (d) $\frac{1}{\sqrt{3}}$
3. If $1 \leq a \leq 2$ then the expression $\sqrt{a + 2\sqrt{a-1}} + \sqrt{a - 2\sqrt{a-1}}$ is equal to
(a) 0 (b) 1 (c) 2 (d) $2\sqrt{a-1}$
4. If $[\sqrt{50} + \sqrt{48}]^{1/2} = K(\sqrt{3} + \sqrt{2})$ then K :
(a) $2^{1/2}$ (b) $2^{1/4}$ (c) 2 (d) None of these
5. $\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots a}}}}$ is nearly equal to
(a) 1 (b) -1 (c) 2 (d) -2
6. The value of $\sqrt[6]{2\sqrt{2} + 3} - \sqrt[3]{(1 - \sqrt{2})}$ is
(a) 1 (b) -1 (c) 2 (d) -2
7. If $(4 + \sqrt{15})^{3/2} - (4 - \sqrt{15})^{3/2} = K\sqrt{6}$ then K is
(a) 7 (b) 8 (c) 9 (d) None of these
8. If $x = 2 + \sqrt{3}$ and $xy = 1$ then $\frac{x}{\sqrt{2} + \sqrt{x}} + \frac{y}{\sqrt{2} - \sqrt{y}}$
(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 1 (d) None of these

9. If $\sqrt{b} + \sqrt{c}, \sqrt{c} + \sqrt{a}, \sqrt{a} + \sqrt{b}$, are in H.P. then a, b, c are in
 (a) A.P. (b) G.P. (c) H.P. (d) None
10. If $P = \sqrt{32} - \sqrt{24}, q = \sqrt{50} - \sqrt{48}$ then
 (a) $P < q$ (b) $P > q$
 (c) $P = q$ (d) Cannot be compared
11. If $\frac{4+3\sqrt{3}}{\sqrt{7+4\sqrt{3}}} = a + \sqrt{b}$ then $(a, b) =$
 (a) $(12, -1)$ (b) $(1, 12)$ (c) $(-1, 12)$ (d) $(-12, 1)$
12. A rationalizing factor of $\sqrt[3]{16} + \sqrt[3]{4} + 1$ is
 (a) $\sqrt[3]{4} + 1$ (b) $\sqrt[3]{4} + 2$ (c) $\sqrt[3]{4} - 1$ (d) $\sqrt[3]{4} - 2$
13. If $x = \sqrt[3]{11}, y = \sqrt[4]{12}, z = \sqrt[6]{13}$ then
 (a) $z > y > x$ (b) $x > y > z$ (c) $y > x > z$ (d) None
14. The smallest number among $\sqrt[3]{4}, \sqrt[4]{5}, \sqrt[4]{6}, \sqrt[3]{8}$
 (a) $\sqrt[3]{4}$ (b) $\sqrt[4]{5}$ (c) $\sqrt[4]{6}$ (d) 2
15. If $x = \sqrt{5}, y = \sqrt[4]{10}, z = \sqrt[3]{6}$, then
 (a) $x > y > z$ (b) $y > z > x$
 (c) $x > z > y$ (d) $y > x > z$
16. If $x = \sqrt[3]{9}, y = \sqrt[4]{11}, z = \sqrt[6]{17}$, then
 (a) $x > y > z$ (b) $x > y > z$
 (c) $y > z > x$ (d) None
17. The square root of $a + \sqrt{2ax + x^2}$ is
 (a) $\sqrt{x} + \sqrt{2a + x}$ (b) $\frac{1}{2}(\sqrt{2} + \sqrt{2a + x})$
 (c) $\frac{1}{\sqrt{2}}[\sqrt{2} + \sqrt{2a + x}]$ (d) None

18. $\sqrt[4]{17+\sqrt{288}} =$
 (a) $1+\sqrt{3}$ (b) $2+\sqrt{3}$ (c) $3+\sqrt{2}$ (d) $1+\sqrt{2}$
19. $(26+15\sqrt{3})^{2/3} + (26-15\sqrt{3})^{2/3}$ is
 (a) 11 (b) 12 (c) 13 (d) 14
20. If $x = 2 + 2^{1/2} + 4^{1/3}$ then $x^3 - 6x^2 + 6x$
 (a) 4 (b) 5 (c) 2 (d) 7
21. Simplify $\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32+\sqrt{50}}}$
 (a) $\sqrt{3}$ (b) $\frac{3+\sqrt{6}}{\sqrt{3}+\sqrt{2}}$ (c) $2\sqrt{2}$ (d) $2+\sqrt{3}$
22. If $x = \frac{\sqrt{a+2b} + \sqrt{a-2b}}{\sqrt{a+2b} - \sqrt{a-2b}}$ then $bx^2 - ax + b$ is
 (a) $\sqrt{a+2b}$ (b) $\frac{1}{2b}$ (c) a (d) 0
23. The value of $(20+14\sqrt{2})^{-1/3} + (20-14\sqrt{2})^{-1/3}$
 (a) 2 (b) 20 (c) $14\sqrt{2}$ (d) 40
24. If $x = 2\sqrt{2} + \sqrt{7}$ then the value of $\frac{1}{2}\left(x + \frac{1}{x}\right)$ is
 (a) 3 (b) $4\sqrt{2}$ (c) $\sqrt{8}$ (d) $\sqrt{8} - \sqrt{7}$
25. If $x = 3 + 3^{1/3} + 3^{2/3}$ then $x^3 - 9x^2 + 18x$ is
 (a) 3 (b) 12 (c) 10 (d) 8
26. The square root of $12 - \sqrt{68 + 48\sqrt{2}}$ is
 (a) $2 + \sqrt{2}$ (b) $2 - \sqrt{3}$ (c) $8 + 2\sqrt{2}$ (d) $\sqrt{2} + \sqrt{9}$

- 27.** The cube root of $38+17\sqrt{5}$
 (a) $3+\sqrt{5}$ (b) $2+\sqrt{5}$ (c) $6+\sqrt{5}$ (d) $4+\sqrt{5}$
- 28.** If $\left(4+\sqrt{15}\right)^{3/2} + \left(4-\sqrt{15}\right)^{3/2} = P\sqrt{10}$ then the value of P is
 (a) 5 (b) 6 (c) 7 (d) 4
- 29.** The value of $\sqrt{12+\sqrt{12+\sqrt{12+\sqrt{12}}}}$ is
 (a) 12 (b) 4^2 (c) 4 (d) 6
- 30.** The smallest number among $\sqrt{2}, \sqrt[2]{3}, \sqrt[4]{5}$ is
 (a) $\sqrt{2}$ (b) $\sqrt[3]{5}$ (c) $\sqrt[4]{5}$ (d) None

SECTION - II

Assertion - Reason Questions

This section contains certain number of questions. Each question contains STATEMENT-1 (Assertion) and STATEMENT - 2 (Reason). Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct. Choose the correct option.

- 31.** STATEMENT-1 : $\sqrt{a^2 - b^2} = \sqrt{a-b}$
because

$$\text{STATEMENT-2 : } \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

- (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
- (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
- (c) Statement-1 True, Statement-2 is False
- (d) Statement-1 is False, Statement-2 is True

- 32.** STATEMENT-1 : $\sqrt{15}, \sqrt{13}, \sqrt[3]{11}$ are pure surds
because

STATEMENT-2 : $\sqrt[5]{320}$ in simplest form is $2 \cdot \sqrt[5]{10}$

- (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 Is NOT a correct explanation for Statement-1
(c) Statement-1 True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

- 33.** STATEMENT-1 : The ascending order of $\sqrt[4]{10}, \sqrt[3]{6}, \sqrt{3}$ is $\sqrt{3}, \sqrt[4]{10}, \sqrt[3]{6}$
because

STATEMENT-2 : $\sqrt[4]{12} - \sqrt{50} - \sqrt[3]{48} = -20\sqrt{3} - 5\sqrt{2}$

- (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

- 34.** STATEMENT-1 : $\sqrt{4} \times \sqrt{21} = \sqrt{294}$
Because

STATEMENT-2 : $\sqrt[n]{x} + \sqrt[n]{y} = \sqrt[n]{xy}$

- (A) Statement-1 is True, Statement-2 is True; Statement-2 Is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

- 35.** STATEMENT-1 : The conjugate of $\sqrt{5} - \sqrt{3}$ is $\sqrt{5} + \sqrt{3}$
because

STATEMENT-2 : If the product of two surd is rational then the surds are said to be consing etc. surds.

- (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

- 36.** STATEMENT-1 : If $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$ then $a=11$

because

STATEMENT-2 : $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}} = -0.213$

- (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

- 37.** STATEMENT-1 : If $x = 2\sqrt{6} + 5$ then $x + \frac{1}{x}$ is 10

because

STATEMENT-2 : $(a+b)(a-b) = a^2 - b^2$

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

38. STATEMENT-1 : R.F of a given surd is not unique
because

STATEMENT-2 : The simplest R.F of $\sqrt{75}$ is $\sqrt{3}$

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

39. STATEMENT-1 : If $\sqrt{10} = 3.162$ (approximately). Then $\frac{1}{\sqrt{10}} = 0.316$
because

STATEMENT-2 : Simplest R.F of $\sqrt[3]{36}$ is $\sqrt[3]{6}$

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

40. STATEMENT-1 : $\sqrt[4]{3} - \sqrt[3]{12} - \sqrt[2]{75} = \sqrt[8]{3}$

because

STATEMENT-2 : $\sqrt[4]{81} - \sqrt[8.3]{216} + \sqrt[15.5]{32} + \sqrt{625} = 0$

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

SECTION - III

Linked Comprehension Type

This section contains paragraphs. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct. Choose the correct option.

Paragraph for Question Nos. 41 to 43

If the product of two surds is rational, then each of the two surds is called rationalising factor of the other. The rationalising factor of a given surd is not unique. If one R.F. of a surd is known the product of this factor by a non-zero rational number is also R.F. of the given surd. It is convenient to use the simplest of all R.F. of a given surd.

41. If $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$ the values of a and b are

- (a) 11, -6 (b) 12, -6 (c) 6, -3 (d) 4, -5

42. If $x = 2\sqrt{6} + 5$ then the value of $x + \frac{1}{x}$ is

- (a) 12 (b) 10 (c) 24 (d) $10\sqrt{6}$

- 43.** $2\sqrt{3}-\sqrt{5}$ is a rationalizing factor of
 (a) 7 (b) $112+\sqrt{5}$ (c) $2\sqrt{3}+\sqrt{5}$ (d) $2\sqrt{15}$

Paragraph for Question No. 44 to 46

Let $\sqrt{x}, \sqrt{y}$ be two dissimilar surds. Hence $\sqrt{xy}$ is a surd also $x+y$ is a positive rational number. Consider $(\sqrt{x} + \sqrt{y})^2 = x + y + 2\sqrt{xy} = a + \sqrt{b}$ (say)

Where $a = x + y$ and $b = 4xy$

$\therefore a^2 - b = (x + y)^2 - 4xy = (x - y)^2$ is a positive real number

$$\Rightarrow \sqrt{a^2 - b} = x - y$$

Solving $x = a + \sqrt{a^2 - b}, y = a - \sqrt{a^2 - b}$

$\therefore \sqrt{a + \sqrt{b}} = \sqrt{x} + \sqrt{y}$, where $a, b, \sqrt{a^2 - b}$ are positive rational numbers and $\sqrt{b}$ is a surd.

- 44.** The square root of $11 - 4\sqrt{7}$ is
 (a) $\sqrt{7} - 2$ (b) $\sqrt{7} - 4$ (c) $\sqrt{7} - \sqrt{2}$ (d) $2 + \sqrt{7}$
- 45.** If $\sqrt{15 - x\sqrt{14}} = \sqrt{18} - \sqrt{7}$ then the value of x is
 (a) 1 (b) 3 (c) 4 (d) 5
- 46.** The value of square root of $(2x - 3) - 2\sqrt{x^2 - 3x + 2}$
 (a) $\sqrt{x - 1} - \sqrt{x - 2}$ (b) $\sqrt{x - 1} - \sqrt{2x - 3}$
 (c) $\sqrt{x + 1} - \sqrt{2x + 3}$ (d) None

Paragraph for Question Nos. 47 to 49

If a and x are rational numbers and $\sqrt{b}$ and $\sqrt{y}$ are surds.

$$\text{Then } \sqrt[3]{a + \sqrt{b}} = x + \sqrt{y} \Leftrightarrow \sqrt[3]{a - b} = x - \sqrt{y}.$$

$$a + \sqrt{b} = (x + \sqrt{y})^3 = (x^3 + 3xy) + \sqrt{y}(y + 3x^2)$$

$$\Rightarrow a = x^3 + 3xy \text{ and } \sqrt{b} = \sqrt{y} = (y + 3x^2)$$

- 47.** The value of cube root of $37 - 30\sqrt{3}$ is
(a) $1 - 2\sqrt{3}$ (b) $7 - 5\sqrt{3}$ (c) $3 - \sqrt{3}$ (d) None
- 48.** If $(4 + \sqrt{15})^{3/2} + (4 - \sqrt{15})^{3/2} = P\sqrt{10}$ then the value of P is
(a) 3 (b) 4 (c) 7 (d) 8
- 49.** The value of cube root of $9\sqrt{3} - 11\sqrt{2}$ is
(a) $\sqrt{3} - \sqrt{2}$ (b) $\sqrt{3} - \sqrt{11}$ (c) $\sqrt{9} - \sqrt{2}$ (d) $\sqrt{18} - \sqrt{11}$

Paragraph for Question Nos. 50 to 52

Comparison of surds is possible only when they are of the same order. The radicals are then to be compared. In order to compare the surds of different order and different base we first reduce them to the same order.

- 50.** The smallest among $\sqrt[4]{10}, \sqrt[3]{6}, \sqrt{3}$ is
(a) $\sqrt[3]{6}$ (b) $\sqrt[4]{10}$ (c) $\sqrt{3}$ (d) None
- 51.** The greatest surd among $-(11 + 56\sqrt{3})$ is
(a) $\sqrt[3]{2}$ (b) $\sqrt[6]{5}$ (c) $\sqrt[9]{6}$ (d) None
- 52.** The smallest surd among $\sqrt[3]{3}, \sqrt[6]{8}, \sqrt[9]{25}$ is

- (a) $\sqrt[3]{3}$ (b) $\sqrt[6]{8}$ (c) $\sqrt[9]{25}$ (d) None

Paragraph for Question Nos. 53 to 55

If the sum and the product of two binomial surds are rational numbers, then they are called Conjugate Surds. Two surds of the form $a + \sqrt{b}$ and $a - \sqrt{b}$ are called conjugate surds. Each surd is said to be the conjugate of the other.

- 53.** If $x = \frac{\sqrt{2}+1}{\sqrt{2}-1}$, $y = \frac{\sqrt{2}-1}{\sqrt{2}+1}$ then $x + y$ is
 (a) 3 (b) 4 (c) 6 (d) 5
- 54.** The value of $\frac{2}{\sqrt{10+2\sqrt{21}}} - \frac{1}{\sqrt{12-2\sqrt{35}}} + \frac{1}{\sqrt{8-2\sqrt{15}}}$ is
 (a) 1 (b) 0 (c) 2 (d) 4
- 55.** If $x = \frac{\sqrt{3}}{2}$ then $\frac{1+x}{1+\sqrt{1+x}}$ is
 (a) $\frac{3+\sqrt{3}}{6}$ (b) 2 (c) 4 (d) $\frac{3}{\sqrt{6}}$

SECTION - IV

Matrix - Match Type

This section contains Matrix-Match type questions. Each question contains statements given in two columns which have to be matched. Statements (a, b, c, d) in Column I have to be matched with statements (p, q, r, s) in Column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are a-p, a-s, b-q, b-r, c-p, c-q and d-s, then the correctly bubbled 4 x 4 matrix should be as follows:

	p	q	r	s
A	p	q	r	s
B	p	q	r	s
C	p	q	r	s
D	p	q	r	s

- 56.** Rationalizing factor of
Column I

a) $3\sqrt{3}$

b) $\sqrt{12}$

c) $\sqrt{108}$

d) $\sqrt[3]{192}$

Column II

p) $\sqrt{12}$

q) $\sqrt{3}$

r) $\sqrt{108}$

s) $\sqrt[3]{3}$

- 57.** The square root of
Column I

a) $11 - 4\sqrt{7}$

b) $12\sqrt{5} + 2\sqrt{55}$

c) $5\sqrt{2} + 4\sqrt{3}$

d) $7 - 3\sqrt{5}$

Column II

p) $2^{1/4}(\sqrt{3} + \sqrt{2})$

q) $5^{1/4}(\sqrt{11} + 1)$

r) $\sqrt{7} - 2$

s) $\frac{1}{2}(3\sqrt{2} - \sqrt{10})$

- 58.** The cube root of
Column I

a) $37 - 30\sqrt{3}$

b) $38 - 17\sqrt{5}$

c) $9\sqrt{3} - 11\sqrt{2}$

Column II

p) $\sqrt{3} + \sqrt{1}$

q) $\sqrt{3} - \sqrt{2}$

r) $2 + \sqrt{5}$

d) $10 + 6\sqrt{3}$

s) $1 - 2\sqrt{3}$

59. Simplify

Column I

Column II

a) $\sqrt{6 + \sqrt{6} - \sqrt{26 + 4\sqrt{30}}}$

p) $\sqrt{5} - 1$

b) $\sqrt{6 - \sqrt{7} + \sqrt{27 + 4\sqrt{35}}}$

q) $\sqrt{5} + 1$

c) $\sqrt{12 - \sqrt{68 + 48\sqrt{2}}}$

r) $2 - \sqrt{2}$

d) $\sqrt{4 + \sqrt{5} + \sqrt{17 - 4\sqrt{15}}}$

s) $1 + \sqrt{3}$

60. Find the value of

Column I

Column II

a) $x = \frac{1}{2 + \sqrt{3}}, y = \frac{1}{2 - \sqrt{3}}$

p) 18

then $7x^2 - 11xy - 7y^2$

b) $x = \frac{\sqrt{5} - 2}{\sqrt{5} + 2}$ then the value

q) 194

of $x + \frac{1}{x}$

c) $x = 7 + 4\sqrt{3}, y = 7 - 4\sqrt{3}$

r) $-(1 + 56\sqrt{3})$

then the value of $\frac{1}{x^2} + \frac{1}{y^2}$

d) If $x = \frac{\sqrt{2}+1}{\sqrt{2}-1}, y = \frac{\sqrt{2}-1}{\sqrt{2}+1}$ s) 6

then is $x + y$