Ex 24.1

Scalar or Dot Product Ex 24.1 Q1

.

(i)

$\vec{a} . \vec{b}$

$= (\hat{i} - 2\hat{j} + \hat{k}) . (4\hat{i} - 4\hat{j} + 7\hat{k})$

$= (1)(4) + (-2) . (-4) + (1)(7)$

$= 4 + 8 + 7$

$= 19$

$\vec{a} . \vec{b} = 19$

(ii)

$\vec{a} . \vec{b} = (\hat{j} + 2\hat{k}) . (2\hat{i} + \hat{k})$

$= (0 \times \hat{i} + \hat{j} + 2\hat{k})(2\hat{i} + 0 \times \hat{j} + \hat{k})$

$= (0)(2) + (1)(0) + (2)(1)$

$= 0 + 0 + 2$

$\vec{a} . \vec{b} = 2$

(iii)

$\vec{a} . \vec{b} = (\hat{j} - \hat{k}) . (2\hat{i} + 3\hat{j} - 2\hat{k})$

$= (0 \times \hat{i} + \hat{j} - \hat{k})(2\hat{i} + 3\hat{j} - 2\hat{k})$

$= (0)(2) + (1)(3) + (-1)(-2)$

$= 0 + 3 + 2$

$\vec{a} . \vec{b} = 5$

Scalar or Dot Product Ex 24.1 Q2

(i)

$\vec{a}$ and $\vec{b}$ are prependicular

$\Rightarrow \vec{a}\vec{b} = 0$

$\Rightarrow (\lambda\hat{i} + 2\hat{j} + \hat{k}) . (4\hat{i} - 9\hat{j} + 2\hat{k}) = 0$

$\Rightarrow (\lambda)(4) + (2)(-9) + (1)(2) = 0$

$\Rightarrow 4\lambda - 18 + 2 = 0$

$\Rightarrow 4\lambda - 16 = 0$

$\Rightarrow 4\lambda = 16$

$\Rightarrow \lambda = \frac{16}{4}$

$\Rightarrow \lambda = 4$

(ii)

$\vec{a}$ and $\vec{b}$ are prependicular

$\Rightarrow \vec{a}\vec{b} = 0$

$\Rightarrow (\lambda\hat{i} + 2\hat{j} + \hat{k}) . (5\hat{i} - 9\hat{j} + 2\hat{k}) = 0$

$\Rightarrow (\lambda)(5) + (2)(-9) + (1)(2) = 0$

$\Rightarrow 5\lambda - 18 + 2 = 0$

$\Rightarrow 5\lambda - 16 = 0$

$\Rightarrow 5\lambda = 16$

$\Rightarrow \lambda = \frac{16}{5}$

(iii)

$\vec{a}$ and $\vec{b}$ are prependicular

$\Rightarrow \vec{a}.\vec{b} = 0$

$\Rightarrow (2\hat{i} + 3\hat{j} + 4\hat{k})(3\hat{i} + 2\hat{j} - \lambda\hat{k}) = 0$

$\Rightarrow (2)(3) + (3)(2) + (4)(-\lambda) = 0$

$\Rightarrow 6 + 6 - 4\lambda = 0$

$\Rightarrow 12 - 4\lambda = 0$

$\Rightarrow -4\lambda = -12$

$\Rightarrow \lambda = \frac{-12}{-4}$

$\Rightarrow \lambda = 3$

(iv)

$\vec{a}$ and $\vec{b}$ are prependicular

$\Rightarrow \vec{a}.\vec{b} = 0$

$\Rightarrow (\lambda\hat{i} + 3\hat{j} + 2\hat{k})(\hat{i} - \hat{j} + 3\hat{k}) = 0$

$\Rightarrow (\lambda)(1) + (3)(-1) + (2)(3) = 0$

$\Rightarrow \lambda - 3 + 6 = 0$

$\Rightarrow \lambda + 3 = 0$

$\Rightarrow \lambda = -3$

Scalar or Dot Product Ex 24.1 Q3

We know that, if θ is the angle between $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{6}{4\times 3}$$

$$= \frac{6}{12}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\theta = \frac{\pi}{3}$$

Angle between $\vec{a}$ and $\vec{b}$ = $\frac{\pi}{3}$

Scalar or Dot Product Ex 24.1 Q4

$$\left(\vec{a}-2\vec{b}\right) = \left(\hat{i}-\hat{j}\right) - 2\left(-\hat{j}+2\hat{k}\right)$$

$$= \left(\hat{i}-\hat{j}\right) + 2\hat{j} - 4\hat{k}$$

$$= \left(\hat{i}+\hat{j}-4\hat{k}\right)$$

$$\left(\vec{a}+\vec{b}\right) = \left(\hat{i}-\hat{j}\right) + \left(-\hat{j}+2\hat{k}\right)$$

$$= \hat{i}-\hat{j}-\hat{j}+2\hat{k}$$

$$= \left(\hat{i}-2\hat{j}+2\hat{k}\right)$$

Now,

$$\left(\vec{a}-2\vec{b}\right).\left(\vec{a}+\vec{b}\right)$$

$$= \left(\hat{i}+\hat{j}-4\hat{k}\right)\left(\hat{i}-2\hat{j}+2\hat{k}\right)$$

$$= (1)(1)+(1)(-2)+(-4)(2)$$

$$= 1-2-8$$

$$= -9$$

$$\left(\vec{a}-2\vec{b}\right).\left(\vec{a}+\vec{b}\right) = -9$$

Scalar or Dot Product Ex 24.1 Q5(i)

Let θ be the angle between $\vec{a}$ and $\vec{b}$, ther

$$\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{|\vec{a}||\vec{b}|} \qquad ---(i)$$

$$\vec{a}\,.\,\vec{b} = (\hat{i}-\hat{j})(\hat{j}+\hat{k})$$
$$= (\hat{i}-\hat{j}+0\times\hat{k})(0\times\hat{i}+\hat{j}+\hat{k})$$
$$= (1)(0)+(-1)(1)+(0)(1)$$
$$= 0-1+0$$
$$\vec{a}\,.\,\vec{b} = -1$$

$$|\vec{a}| = |\hat{i}-\hat{j}|$$
$$= |\hat{i}-\hat{j}+0\times\hat{k}|$$
$$= \sqrt{(1)^2+(-1)^2+(0)^2}$$
$$= \sqrt{1+1+0}$$
$$|\vec{a}| = \sqrt{2}$$

$$|\vec{b}| = |\hat{j}+\hat{k}|$$
$$= |0\times\hat{i}+\hat{j}+\hat{k}|$$
$$= \sqrt{(0)^2+(1)^2+(1)^2}$$
$$= \sqrt{0+1+1}$$
$$|\vec{b}| = \sqrt{2}$$

Put $\vec{a}\times\vec{b}, |\vec{a}|$ and $|\vec{b}|$ in equation (i)

$$\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{|\vec{a}||\vec{b}|}$$
$$= \frac{-1}{\sqrt{2}\times\sqrt{2}}$$
$$\cos\theta = \frac{-1}{2}$$
$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$
$$\theta = \pi - \frac{\pi}{3}$$
$$\theta = \frac{2\pi}{3}$$

Angle between $\vec{a}$ and $\vec{b} = \frac{2\pi}{3}$

Scalar or Dot Product Ex 24.1 Q5(ii)

Let θ be the angle between two vactor $\vec{a} = 3\hat{i}-2\hat{j}-6\hat{k}$ and $\vec{b} = 4\hat{i}-\hat{j}+8\hat{k}$

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} \cdots(1)$$

$$\vec{a}.\vec{b} = (3\hat{i}-2\hat{j}-6\hat{k})(4\hat{i}-\hat{j}+8\hat{k})$$
$$= 3*4+(-2)(-1)+(-6)8$$
$$= 12+2-48$$
$$= -34$$

$$|\vec{a}| = \sqrt{3^2+(-2)^2+(-6)^2}$$
$$= \sqrt{49}$$
$$= 7$$

$$|\vec{b}| = \sqrt{4^2+(-1)^2+8^2}$$
$$= \sqrt{81}$$
$$= 9$$

Putting value of $|\vec{a}|, |\vec{b}|$ and $\vec{a}.\vec{b}$ in equation (1)

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$
$$= \frac{-34}{7*9}$$
$$= \frac{-34}{63}$$
$$\theta = \cos^{-1}\left(\frac{-34}{63}\right)$$
$$= 122.66^0$$

Scalar or Dot Product Ex 24.1 Q5(iii)

Let the angle between $\vec{a}$ and $\vec{b}$ be θ, then

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \qquad ---(i)$$

$$\vec{a}.\ \vec{b} = \left(2\hat{i} - \hat{j} + 2\hat{k}\right)\left(4\hat{i} + 4\hat{j} - 2\hat{k}\right)$$
$$= (2)(4) + (-1)(4) + (2)(-2)$$
$$= 8 - 4 - 4$$
$$\vec{a}.\ \vec{b} = 0$$

$$|\vec{a}| = \left|2\hat{i} - \hat{j} + 2\hat{k}\right|$$
$$= \sqrt{(2)^2 + (-1)^2 + (2)^2}$$
$$= \sqrt{4+1+4}$$
$$= \sqrt{9}$$
$$|\vec{a}| = 3$$

$$|\vec{b}| = \left|4\hat{i} + 4\hat{j} - 2\hat{k}\right|$$
$$= \sqrt{(4)^2 + (4)^2 + (-2)^2}$$
$$= \sqrt{16+16+4}$$
$$= \sqrt{36}$$
$$|\vec{b}| = 6$$

Put $\vec{a}.\ \vec{b}$, $|\vec{a}|$ and $|\vec{b}|$ in equation (i)

$$\cos\theta = \frac{\vec{a}\ .\ \vec{b}}{|\vec{a}||\vec{b}|}$$
$$= \frac{0}{3\times 6}$$
$$= \frac{0}{18}$$
$$\cos\theta = 0$$
$$\theta = \cos^{-1}(0)$$

Angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$

Scalar or Dot Product Ex 24.1 Q5(iv)

Let θ be the angle between vector $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a} : \vec{b}}{|\vec{a}||\vec{b}|} \qquad ---(i)$$

$$\vec{a}\quad \vec{b} = \left(2\hat{i} - 3\hat{j} + \hat{k}\right)\left(\hat{i} + \hat{j} - 2\hat{k}\right)$$
$$= (2)(1) + (-3)(1) + (1)(-2)$$
$$= 2 - 3 - 2$$
$$\vec{a}\cdot\vec{b} = -3$$

$$|\vec{a}| = \left|2\hat{i} - 3\hat{j} + \hat{k}\right|$$
$$= \sqrt{(2)^2 + (-3)^2 + (-1)^2}$$
$$= \sqrt{4+9+1}$$
$$= \sqrt{14}$$

$$|\vec{b}| = \left|\hat{i} + \hat{j} - 2\hat{k}\right|$$
$$|\vec{b}| = \sqrt{(1)^2 + (1)^2 + (-2)^2}$$
$$= \sqrt{1+1+4}$$
$$|\vec{b}| = \sqrt{6}$$

Put $\vec{a}.\ \vec{b}$, $|\vec{a}|$ and $|\vec{b}|$ in equation (i),

$$\cos\theta = \frac{\vec{a}\ .\ \vec{b}}{|\vec{a}||\vec{b}|}$$
$$= \frac{-3}{\sqrt{14}\times\sqrt{6}}$$
$$\cos\theta = \frac{-3}{\sqrt{84}}$$
$$\theta = \cos^{-1}\left(\frac{-3}{\sqrt{84}}\right)$$

Angle between vector $\vec{a}$ and $\vec{b}$ $= \cos^{-1}\left(\frac{-3}{\sqrt{84}}\right)$

Scalar or Dot Product Ex 24.1 Q5(v)

Let θ be the angle between vector $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \qquad ---(i)$$

$$\vec{a}\cdot\vec{b} = \left(\hat{i}+2\hat{j}-\hat{k}\right)\left(\hat{i}-\hat{j}+\hat{k}\right)$$
$$= (1)(1)+(2)(-1)+(-1)(1)$$
$$= 1-2-1$$
$$\vec{a}\cdot\vec{b} = -2$$

$$|\vec{a}| = \left|\hat{i}+2\hat{j}-\hat{k}\right|$$
$$= \sqrt{(1)^2+(2)^2+(-1)^2}$$
$$= \sqrt{1+4+1}$$
$$= \sqrt{6}$$

$$|\vec{b}| = \left|\hat{i}-\hat{j}+\hat{k}\right|$$
$$|\vec{b}| = \sqrt{(1)^2+(-1)^2+(1)^2}$$
$$= \sqrt{1+1+1}$$
$$|\vec{b}| = \sqrt{3}$$

Put $\vec{a}\cdot\vec{b}$, $|\vec{a}|$, $|\vec{b}|$ in equation (i),

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$
$$= \frac{-2}{\sqrt{6}\sqrt{3}}$$
$$= \frac{-2}{\sqrt{18}}$$
$$= \frac{-2\times\sqrt{2}}{3\sqrt{2}\times\sqrt{2}}$$
$$= \frac{-2\sqrt{2}}{3\times 2}$$
$$\cos\theta = \frac{-\sqrt{2}}{3}$$
$$\theta = \cos^{-1}\left(\frac{-\sqrt{2}}{3}\right)$$

Angle between vector $\vec{a}$ and $\vec{b}$ $= \cos^{-1}\left(\frac{-\sqrt{2}}{3}\right)$

Scalar or Dot Product Ex 24.1 Q6

Component along x-, y- and z-axis are $\hat{i}, \hat{j}$ and $\hat{k}$ respectively.

Let θ_1 be the angle between $\vec{a}$ and $\hat{i}$.

$$\cos\theta_1 = \frac{\vec{a}.\hat{i}}{|\vec{a}||\hat{i}|}$$
$$= \frac{\left(\hat{i}-\hat{j}+\sqrt{2}\hat{k}\right)\left(\hat{i}+0.\hat{j}+0.\hat{k}\right)}{\left|\hat{i}-\hat{j}+\sqrt{2}\hat{k}\right|\left|\hat{i}+0.\hat{j}+0.\hat{k}\right|}$$
$$= \frac{(1)(1)+(-1)(0)+\left(\sqrt{2}\right)(0)}{\sqrt{(1)^2+(-1)^2+\left(\sqrt{2}\right)^2}.\sqrt{(1)^2+(0)^2+(0)^2}}$$
$$= \frac{1+0+0}{\sqrt{4}\sqrt{1}}$$
$$\cos\theta_1 = \frac{1}{2}$$
$$\theta_1 = \frac{\pi}{3}$$

Let θ_2 be the angle between $\vec{a}$ and $\hat{j}$.

$$\cos\theta_2 = \frac{\vec{a}.\hat{j}}{|\vec{a}||\hat{j}|}$$
$$= \frac{\left(\hat{i}-\hat{j}+\sqrt{2}\hat{k}\right)\left(0.\hat{i}+\hat{j}+0\,\hat{k}\right)}{\sqrt{(1)^2+(-1)^2+\left(\sqrt{2}\right)^2}.\sqrt{(0)^2+(1)^2+(0)^2}}$$
$$= \frac{(1)(0)+(-1)(1)+\left(\sqrt{2}\right)(0)}{\sqrt{1+1+2}\;\sqrt{1}}$$
$$= \frac{-1}{\sqrt{4}\sqrt{1}}$$
$$= \frac{-1}{2}$$
$$\cos\theta_2 = -\frac{1}{2}$$
$$\theta_2 = \pi-\frac{\pi}{3}$$
$$\theta_2 = \frac{2\pi}{3}$$

Let θ_3 be the angle between $\vec{a}$ and $\hat{k}$, then

$$\cos\theta_3 = \frac{\vec{a}.\hat{k}}{|\vec{a}||\hat{k}|}$$
$$= \frac{\left(\hat{i}-\hat{j}+\sqrt{2}\hat{k}\right)\left(0.\hat{i}+0.\hat{j}+\hat{k}\right)}{\sqrt{(1)^2+(-1)^2+\left(\sqrt{2}\right)^2}.\sqrt{(0)^2+(0)^2+(1)^2}}$$
$$= \frac{(1)(0)+(-1)(0)+\left(\sqrt{2}\right)(1)}{\sqrt{1+1+2}\;\sqrt{1}}$$
$$= \frac{\sqrt{2}}{\sqrt{4}\;\sqrt{1}}$$
$$\cos\theta_3 = \frac{1}{\sqrt{2}}$$
$$\theta_3 = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$
$$\theta_3 = \frac{\pi}{4}$$

So, the angle between vector $\vec{a}$ and x-axis is $\frac{\pi}{3}$, vector $\vec{a}$ and y-axis is $\frac{2\pi}{3}$, vector $\vec{a}$ and z-axis is $\frac{\pi}{4}$.

Scalar or Dot Product Ex 24.1 Q7(i)

Let the requird vector be $x\hat{i}+y\hat{j}+z\hat{k}$

According to question,

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(\hat{i}+\hat{j}-3\hat{k}\right)=0$$
$$(x)(1)+(y)(1)+(z)(-3)=0$$
$$x+y-3z=0 \qquad ---(i)$$

And,

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(\hat{i}+3\hat{j}-2\hat{k}\right)=5$$
$$(x)(1)+(y)(3)+(z)(-2)=5$$
$$x+3y-2z=5 \qquad ---(ii)$$

And,

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(2\hat{i}+\hat{j}+4\hat{k}\right)=8$$
$$(x)(2)+(y)(1)+(z)(4)=8$$
$$2x+y+4z=8 \qquad ---(iii)$$

Subtracting (i) from (ii),

$$x+3y-2z=5$$
$$\underset{(-)}{x}\ \underset{(-)}{+y}\ \underset{(+)}{-3z}=0$$
$$2y+z=5 \qquad ---(iv)$$

Subtracting $2\times$(ii) from (iii),

$$2x+y+4z=8$$
$$\underset{(-)}{2x}\ \underset{(-)}{+6y}\ \underset{(+)}{-4z}=\underset{(-)}{10}$$
$$-5y+8z=-2 \qquad ---(v)$$

Subtracting $8\times$(iv) from (v),

$$-5y+8z=-2$$
$$\underset{(-)}{16y}\ \underset{(-)}{+8z}=\underset{(-)}{40}$$
$$-21y=-42 \qquad ---(vi)$$
$$y=\frac{-42}{-21}$$
$$y=2$$

Put $y=2$ in equation (iv),

$$2y+z=5$$
$$2(2)+z=5$$
$$4+z=5$$
$$z=5-4$$
$$z=1$$

Put $y=2$ and $z=1$ in equation (i),

$$x+y-3z=0$$
$$x+(2)-3(1)=0$$
$$x+2-3=0$$
$$x-1=0$$
$$x=1$$

The required vector $=x\hat{i}+y\hat{j}+z\hat{k}$

The required vector $=\hat{i}+2\hat{j}+\hat{k}$

Scalar or Dot Product Ex 24.1 Q8(i)

Here, $\hat{a}$ and $\hat{b}$ are unit vectors, then

$$\left|\hat{a}\right| = \left|\hat{b}\right| = 1$$

$$\left|\hat{a}+\hat{b}\right|^2 = \left(\hat{a}+\hat{b}\right)^2$$

$$= \left(\hat{a}\right)^2 + \left(\hat{b}\right)^2 + 2\hat{a}.\hat{b}$$

$$= \left|\hat{a}\right|^2 + \left|\hat{b}\right|^2 + 2\hat{a}.\hat{b}$$

$$= (1)^2 + (1)^2 + 2\hat{a}.\hat{b}$$

$$\left|\hat{a}+\hat{b}\right|^2 = 2 + 2\hat{a}\times\hat{b}$$

$$\left|\hat{a}+\hat{b}\right|^2 = 2 + 2\times\left|\hat{a}\right|\left|\hat{b}\right|\cos\theta \qquad \left[\text{Since } \vec{a}.\vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\cos\theta\right]$$

$$\left|\hat{a}+\hat{b}\right|^2 = 2 + 2\times1\times1\times\cos\theta$$

$$= 2 + 2\cos\theta$$

$$\left|\hat{a}+\hat{b}\right|^2 = 2\left(1+\cos\theta\right)$$

$$\left|\hat{a}+\hat{b}\right|^2 = 2\left(2\cos^2\frac{\theta}{2}\right) \qquad \left[\text{Since } 1+\cos\theta = 2\cos^2\frac{\theta}{2}\right]$$

$$\left|\hat{a}+\hat{b}\right|^2 = 4\cos^2\frac{\theta}{2}$$

$$\left|\hat{a}+\hat{b}\right| = \sqrt{4\cos^2\frac{\theta}{2}}$$

$$\left|\hat{a}+\hat{b}\right| = 2\cos\frac{\theta}{2}$$

$$\cos\frac{\theta}{2} = \frac{1}{2}\left|\hat{a}+\hat{b}\right|$$

Scalar or Dot Product Ex 24.1 Q8(ii)

Here, $\hat{a}$ and $\hat{b}$ are unit vectors

$$\left|\hat{a}\right| = \left|\hat{b}\right| = 1$$

$$\frac{\left|\hat{a}-\hat{b}\right|^2}{\left|\hat{a}+\hat{b}\right|^2} = \frac{\left(\hat{a}-\hat{b}\right)^2}{\left(\hat{a}+\hat{b}\right)^2}$$

$$= \frac{\left(\hat{a}\right)^2 + \left(\hat{b}\right)^2 - 2\hat{a}.\hat{b}}{\left(\hat{a}\right)^2 + \left(\hat{b}\right)^2 + 2\hat{a}.\hat{b}}$$

$$= \frac{\left|\hat{a}\right|^2 + \left|\hat{b}\right|^2 - 2\hat{a}.\hat{b}}{\left|\hat{a}\right|^2 + \left|\hat{b}\right|^2 + 2\hat{a}.\hat{b}}$$

$$\frac{\left|\hat{a}-\hat{b}\right|^2}{\left|\hat{a}+\hat{b}\right|^2} = \frac{(1)^2 + (1)^2 - 2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta}{(1)^2 + (1)^2 + 2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta} \qquad \left[\text{Since } \vec{a}.\vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\cos\theta\right]$$

$$\frac{\left|\hat{a}-\hat{b}\right|^2}{\left|\hat{a}+\hat{b}\right|^2} = \frac{1+1-2(1)(1)\cos\theta}{1+1+2(1)(1)\cos\theta}$$

$$= \frac{2-2\cos\theta}{2+2\cos\theta}$$

$$= \frac{2\left(1-\cos\theta\right)}{2\left(1+\cos\theta\right)}$$

$$= \frac{2\times\sin^2\frac{\theta}{2}}{2\times\cos^2\frac{\theta}{2}} \qquad \left[\text{Since } 1-\cos\theta = 2\sin^2\frac{\theta}{2},\ 1+\cos\theta = 2\cos^2\frac{\theta}{2}\right]$$

$$\frac{\left|\hat{a}-\hat{b}\right|^2}{\left|\hat{a}+\hat{b}\right|^2} = \tan^2\frac{\theta}{2}$$

$$\tan\frac{\theta}{2} = \frac{\left|\hat{a}-\hat{b}\right|}{\left|\hat{a}+\hat{b}\right|}$$

Scalar or Dot Product Ex 24.1 Q9

Let $\hat{a}$ and $\hat{b}$ are two unit vectors

Then, $|\hat{a}| = |\hat{b}| = 1$

And sum of $\hat{a}$ and $\hat{b}$ is a unit vector, then

$$|\hat{a} + \hat{b}| = 1$$

Taking square of both the sides,

$$|\hat{a} + \hat{b}|^2 = (1)^2$$

$$(\hat{a} + \hat{b})^2 = 1$$

$$(\hat{a})^2 + (\hat{b})^2 + 2\hat{a}\,.\,\hat{b} = 1$$

$$|\hat{a}|^2 + |\hat{b}|^2 + 2\hat{a}.\,\hat{b} = 1$$

$$(1)^2 + (1)^2 + 2\hat{a}\,.\,\hat{b} = 1$$

$$2 + 2\hat{a}\quad\hat{b} = 1$$

$$2\hat{a}\,.\,\hat{b} = 1 - 2$$

$$2\hat{a}\,.:\hat{b} = -1$$

$$\hat{a}\,.\,\hat{b} = \frac{-1}{2} \qquad ---(i)$$

$$|\hat{a} - \hat{b}|^2 = (\hat{a} - \hat{b})^2$$

$$= (\hat{a})^2 + (\hat{b})^2 - 2\hat{a}\,.\,\hat{b}$$

$$= |\hat{a}|^2 + |\hat{b}|^2 - 2 \times \hat{a}\,.\,\hat{b}$$

$$= (1)^2 + (1)^2 - 2 \times \left(\frac{-1}{2}\right)$$

Using equation (i)

$$= 1 + 1 + \frac{2}{2}$$

$$= 1 + 1 + 1$$

$$|\hat{a} - \hat{b}|^2 = 3$$

$$|\hat{a} - \hat{b}| = \sqrt{3}$$

Scalar or Dot Product Ex 24.1 Q10

Given that $\vec{a}, \vec{b}, \vec{c}$ are mutually prependicular, so,

$$\vec{a}\,.\,\vec{b} = \vec{b}\cdot\,\vec{c} = \vec{c}\,.\,\vec{a} = 0$$

and $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors, so

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

Now,

$$|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c})^2$$

$$= (\vec{a})^2 + (\vec{b})^2 + (\vec{c})^2 + 2\vec{a}\vec{b} + 2\vec{b}\vec{c} + 2\vec{c}\vec{a}$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(0) + 2(0) + 2(0)$$

$$= (1)^2 + (1)^2 + (1)^2 + 0$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 1 + 1 + 1$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 3$$

$$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$$

Scalar or Dot Product Ex 24.1 Q11

Here, $\left|\vec{a}+\vec{b}\right| = 60$

Squaring both the sides,

$$\left|\vec{a}+\vec{b}\right|^2 = (60)^2$$

$$\left(\vec{a}+\vec{b}\right) = (60)^2$$

$$\left(\vec{a}\right)^2 + \left(\vec{b}\right)^2 + 2\vec{a}\vec{b} = 3600$$

$$\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\vec{a}\vec{b} = 3600 \qquad ---(i)$$

Now, $\left|\vec{a}-\vec{b}\right| = 40$

Squaring both the sides,

$$\left|\vec{a}-\vec{b}\right|^2 = (40)^2$$

$$\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 - 2\vec{a}\vec{b} = 1600 \qquad ---(ii)$$

Adding (i) and (ii),

$$2\left|\vec{a}\right|^2 + 2\left|\vec{b}\right|^2 + 2\vec{a}\vec{b} - 2\vec{a}\vec{b} = 3600 - 1600$$

$$2\left|\vec{a}\right|^2 + 2(46)^2 = 5200$$

$$2\left|\vec{a}\right|^2 = 5200 - 4232$$

$$2\left|\vec{a}\right|^2 = 968$$

$$\left|\vec{a}\right|^2 = \frac{968}{2}$$

$$\left|\vec{a}\right|^2 = 484$$

$$\left|\vec{a}\right| = \sqrt{484}$$

$$\left|\vec{a}\right| = 22$$

Scalar or Dot Product Ex 24.1 Q12

Let θ be the angle between $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}$

Then,

$$\cos\theta = \frac{\left(\hat{i}+\hat{j}+\hat{k}\right).\left(\hat{i}\right)}{\left|\hat{i}+\hat{j}+\hat{k}\right|\left|\hat{i}\right|}$$

$$= \frac{1}{\frac{1}{\sqrt{3}}}$$

$$= \sqrt{3}$$

Similarly, if α and γ are angles that $\hat{i}+\hat{j}+\hat{k}$ make with $\hat{j}$ and $\hat{k}$

Then,

$$\cos\alpha = \sqrt{3}$$

and $$\cos\gamma = \sqrt{3}$$

Therefore, $\hat{i}+\hat{j}+\hat{k}$ is equally inclined the three axes.

Scalar or Dot Product Ex 24.1 Q13

We have,

$$\vec{a} = \frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$$

$$\vec{b} = \frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right)$$

$$\vec{c} = \frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$$

Then,

$$\vec{a}.\vec{b} = \frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right)\times\frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right)$$

$$= \frac{1}{49}(6-18+12) = 0$$

Similarly,

$$\vec{b}.\vec{c} = \vec{a}.\vec{c} = 0$$

$\therefore$ $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular

Scalar or Dot Product Ex 24.1 Q14

Let $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 0$

$$|\vec{a}|^2 - |\vec{b}|^2 = 0$$

$$|\vec{a}|^2 = |\vec{b}|^2$$

$$|\vec{a}| = |\vec{b}|$$

Let $|\vec{a}| = |\vec{b}|$

Squaring both the sides.

$$|\vec{a}|^2 = |\vec{b}|^2$$

$$|\vec{a}|^2 - |\vec{b}|^2 = 0$$

$$(\vec{a})^2 - (\vec{b})^2 = 0$$

$$(\vec{a}+\vec{b}) . (\vec{a}-\vec{b}) = 0$$

Thus,

$$(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 0 \Leftrightarrow |\vec{a}| = |\vec{b}|$$

Scalar or Dot Product Ex 24.1 Q15

If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - \hat{k}$, find λ

Given that $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$

$\therefore\ \vec{a} \bullet (\lambda\vec{b} + \vec{c}) = 0$

$\lambda\vec{a} \bullet \vec{b} + \vec{a} \bullet \vec{c} = 0$

$\lambda(2\hat{i} - \hat{j} + \hat{k}) \bullet (\hat{i} + \hat{j} - 2\hat{k}) + (2\hat{i} - \hat{j} + \hat{k}) \bullet (\hat{i} + 3\hat{j} - \hat{k}) = 0$

$\lambda(2 - 1 - 2) + (2 - 3 - 1) = 0$

$-\lambda - 2 = 0$

$\lambda = 2$

Scalar or Dot Product Ex 24.1 Q16

$\vec{p} = 5\hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{q} = \hat{i} + 3\hat{j} - 5\hat{k}$

$\vec{p} + \vec{q}$

$= 5\hat{i} + \lambda\hat{j} - 3\hat{k} + \hat{i} + 3\hat{j} - 5\hat{k}$

$= 6\hat{i} + (\lambda+3)\hat{j} - 8\hat{k}$

$\vec{p} - \vec{q}$

$= 5\hat{i} + \lambda\hat{j} - 3\hat{k} - \hat{i} - 3\hat{j} + 5\hat{k}$

$= 4\hat{i} + (\lambda-3)\hat{j} + 2\hat{k}$

$(\vec{p}+\vec{q}).(\vec{p}-\vec{q}) = 0$

$\Rightarrow [6\hat{i} + (\lambda+3)\hat{j} - 8\hat{k}].[4\hat{i} + (\lambda-3)\hat{j} + 2\hat{k}] = 0$

$\Rightarrow 24 + (\lambda^2 - 9) - 16 = 0$

$\Rightarrow \lambda^2 - 9 + 8 = 0$

$\Rightarrow \lambda^2 - 1 = 0$

$\therefore \lambda = \pm 1$

Scalar or Dot Product Ex 24.1 Q17

According to question $\overline{\beta}_1$ is parallel to $\overline{\alpha}$. So

$$\overline{\beta}_1 = \gamma\overline{\alpha}$$
$$= \gamma\left(3\hat{i}+4\hat{j}+5\hat{k}\right)$$

$$\overline{\beta} = \overline{\beta_1} + \overline{\beta_2}$$
$$2\hat{i}+\hat{j}-4\hat{k} = \gamma\left(3\hat{i}+4\hat{j}+5\hat{k}\right)+\overline{\beta_2} \qquad \left(\text{putting } \overline{\beta} \text{ and } \overline{\beta_1}\right)$$
$$\overline{\beta_2} = (2-3\gamma)\hat{i}+(1-4\gamma)\hat{j}-(4+5\gamma)\hat{k}$$

Again $\overline{\beta}_2$ is perpendicular to $\overline{\alpha}$. So

$$\overline{\beta}_2.\overline{\alpha} = 0$$
$$\left[(2-3\gamma)\hat{i}+(1-4\gamma)\hat{j}-(4+5\gamma)\hat{k}\right].\left(3\hat{i}+4\hat{j}+5\hat{k}\right) = 0$$
$$6-9\gamma+4-16\gamma-20-25\gamma = 0$$
$$-50\gamma = 10$$
$$\gamma = -\frac{1}{5}$$

$$\overline{\beta}_1 = -\frac{1}{5}\left(3\hat{i}+4\hat{j}+5\hat{k}\right)$$
$$\overline{\beta} = \overline{\beta_1} + \overline{\beta_2}$$
$$2\hat{i}+\hat{j}-4\hat{k} = -\frac{1}{5}\left(3\hat{i}+4\hat{j}+5\hat{k}\right)+\overline{\beta_2} \qquad \left(\text{putting } \overline{\beta} \text{ and } \overline{\beta_1}\right)$$
$$\overline{\beta_2} = \frac{1}{5}\left(13\hat{i}+9\hat{j}-15\hat{k}\right)$$
$$\overline{\beta} = -\frac{1}{5}\left(3\hat{i}+4\hat{j}+5\hat{k}\right)+\frac{1}{5}\left(13\hat{i}+9\hat{j}-15\hat{k}\right)$$

Scalar or Dot Product Ex 24.1 Q18

Consider $\vec{a} = 2\hat{i}+4\hat{j}+3\hat{k}$ and $\vec{b} = 3\hat{i}+3\hat{j}-6\hat{k}$.

Then,

$$\vec{a}\cdot\vec{b} = 2.3+4.3+3(-6) = 6+12-18 = 0$$

We now observe that:

$$|\vec{a}| = \sqrt{2^2+4^2+3^2} = \sqrt{29}$$
$$\therefore \vec{a} \neq \vec{0}$$
$$|\vec{b}| = \sqrt{3^2+3^2+(-6)^2} = \sqrt{54}$$
$$\therefore \vec{b} \neq \vec{0}$$

Hence, the converse of the given statement need not be true.

Scalar or Dot Product Ex 24.1 Q19

Here,

$\vec{b} + \vec{c} = \left(\hat{i} - 3\hat{j} + 5\hat{k}\right)\left(2\hat{i} + \hat{j} - 4\hat{k}\right)$

$= 3\hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} + \vec{c} = \vec{a}$

$\therefore$ $\vec{a}, \vec{b}, \vec{c}$ are represents the sides of a triangle.

$\left|\vec{a}\right| = \sqrt{(3)^2 + (-2)^2 + (1)^2}$

$= \sqrt{9 + 4 + 1}$

$= \sqrt{14}$

$\left|\vec{b}\right| = \sqrt{(1)^2 + (-3)^2 + (5)^2}$

$= \sqrt{1 + 9 + 25}$

$\left|\vec{b}\right| = \sqrt{35}$

$\left|\vec{c}\right| = \sqrt{(2)^2 + (1)^2 + (-4)^2}$

$= \sqrt{4 + 1 + 16}$

$= \sqrt{21}$

$\left(\sqrt{21}\right)^2 + \left(\sqrt{14}\right)^2 = \left(\sqrt{35}\right)^2$

$21 + 14 = 35$

$35 = 35$

$\left|\vec{c}\right|^2 + \left|\vec{a}\right|^2 = \left|\vec{b}\right|^2$

$\therefore$ By the pythagorous theorem,

Triangle formed by $\vec{a}, \vec{b}, \vec{c}$ is a right angled triangled.

Scalar or Dot Product Ex 24.1 Q20

The given vectors are $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, and $\vec{c} = 3\hat{i} + \hat{j}$.

Now,

$\vec{a} + \lambda\vec{b} = \left(2\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(-\hat{i} + 2\hat{j} + \hat{k}\right) = (2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}$

If $\left(\vec{a} + \lambda\vec{b}\right)$ is perpendicular to $\vec{c}$, then

$\left(\vec{a} + \lambda\vec{b}\right) \cdot \vec{c} = 0.$

$\Rightarrow \left[(2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}\right] \cdot \left(3\hat{i} + \hat{j}\right) = 0$

$\Rightarrow (2 - \lambda)3 + (2 + 2\lambda)1 + (3 + \lambda)0 = 0$

$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$

$\Rightarrow -\lambda + 8 = 0$

$\Rightarrow \lambda = 8$

Hence, the required value of λ is 8.

Scalar or Dot Product Ex 24.1 Q21

$$\vec{A} = 0.\hat{i} - \hat{j} - 2\hat{k}$$
$$\vec{B} = 3\hat{i} + \hat{j} + 4\hat{k}$$
$$\vec{C} = 5\hat{i} + 7\hat{j} + \hat{k}$$

$$\overrightarrow{AB} = \vec{B} - \vec{A}$$
$$= \left(3\hat{i} + \hat{j} + 4\hat{k}\right) - \left(0.\hat{i} - \hat{j} - 2\hat{k}\right)$$
$$= 3\hat{i} + \hat{j} + 4\hat{k} - 0.\hat{i} + \hat{j} + 2\hat{k}$$
$$\overrightarrow{AB} = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\overrightarrow{BC} = \vec{C} - \vec{B}$$
$$= \left(5\hat{i} + 7\hat{j} + \hat{k}\right) - \left(3\hat{i} + \hat{j} + 4\hat{k}\right)$$
$$= 5\hat{i} + 7\hat{j} + \hat{k} - 3\hat{i} - \hat{j} - 4\hat{k}$$
$$\overrightarrow{BC} = 2\hat{i} + 6\hat{j} - 3\hat{k}$$

$$\overrightarrow{AC} = \vec{C} - \vec{A}$$
$$= \left(5\hat{i} + 7\hat{j} + \hat{k}\right) - \left(-\hat{j} - 2\hat{k}\right)$$
$$= 5\hat{i} + 7\hat{j} + \hat{k} + \hat{j} + 2\hat{k}$$
$$\overrightarrow{AC} = 5\hat{i} + 8\hat{j} + 3\hat{k}$$

Angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$,

$$\cos A = \frac{\overrightarrow{AB}.\overrightarrow{AC}}{\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|}$$
$$= \frac{\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)\left(5\hat{i} + 8\hat{j} + 3\hat{k}\right)}{\sqrt{(3)^2 + (2)^2 + (6)^2}\sqrt{(5)^2 + (8)^2 + (3)^2}}$$
$$= \frac{(3)(5) + (2)(8) + (6)(3)}{\sqrt{9+4+36}\sqrt{25+64+9}}$$
$$= \frac{15+16+18}{\sqrt{49}\sqrt{98}}$$
$$= \frac{49}{\sqrt{49}\sqrt{49 \times 2}}$$
$$\cos A = \frac{49}{49\sqrt{2}}$$
$$\cos A = \frac{1}{\sqrt{2}}$$
$$A = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$
$$= \frac{\pi}{4}$$
$$\angle A = \frac{\pi}{4}$$

Angle between $\overrightarrow{BC}$ and $\overrightarrow{BA}$

$$\cos B = \frac{\overrightarrow{BC}.\overrightarrow{BA}}{\left|\overrightarrow{BC}\right|\left|\overrightarrow{BA}\right|}$$
$$= \frac{\left(2\hat{i} + 6\hat{j} - 3\hat{k}\right)\left(-3\hat{i} - 2\hat{j} - 6\hat{k}\right)}{\sqrt{(2)^2 + (6)^2 + (-3)^2}\sqrt{(-3)^2 + (-2)^2 + (-6)^2}}$$
$$= \frac{(2)(-3) + (6)(-2) + (-3)(-6)}{\sqrt{4+36+9}\sqrt{9+4+36}}$$
$$= \frac{-6-12+18}{\sqrt{49}\sqrt{98}}$$
$$\cos B = \frac{-18+18}{49}$$
$$= \frac{0}{49}$$
$$\cos B = 0$$
$$B = \cos^{-1}(0)$$
$$\angle B = \frac{\pi}{2}$$

We know that,

$$\angle A + \angle B + \angle C = \pi$$

$$\frac{\pi}{4} + \frac{\pi}{2} + \angle C = \pi$$

$$\frac{3\pi}{4} + \angle C = \pi$$

$$\angle C = \frac{\pi}{1} - \frac{3\pi}{4}$$

$$\angle C = \frac{4\pi - 3\pi}{4}$$

$$\angle C = \frac{\pi}{4}$$

$$\angle A = \frac{\pi}{4}$$

$$\angle B = \frac{\pi}{2}$$

Scalar or Dot Product Ex 24.1 Q22

Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$.

It is given that $|\vec{a}| = |\vec{b}|$, $\vec{a} \cdot \vec{b} = \frac{1}{2}$, and $\theta = 60°$. ...(1)

We know that $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

$$\therefore \frac{1}{2} = |\vec{a}||\vec{a}|\cos 60° \quad \text{[Using (1)]}$$

$$\Rightarrow \frac{1}{2} = |\vec{a}|^2 \times \frac{1}{2}$$

$$\Rightarrow |\vec{a}|^2 = 1$$

$$\Rightarrow |\vec{a}| = |\vec{b}| = 1$$

Scalar or Dot Product Ex 24.1 Q23

Given

$$\vec{a} = 4\hat{i} - 3\hat{j} + \hat{k}$$

$$\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$$

$$\vec{c} = \hat{i} - \hat{j}$$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right) - \left(4\hat{i} - 3\hat{j} + \hat{k}\right)$$

$$= 2\hat{i} - 4\hat{j} + 5\hat{k} - 4\hat{i} + 3\hat{j} - \hat{k}$$

$$\overrightarrow{AB} = -2\hat{i} - \hat{j} + 4\hat{k}$$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$$= \left(\hat{i} - \hat{j}\right) - \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right)$$

$$= \hat{i} - \hat{j} - 2\hat{i} + 4\hat{j} - 5\hat{k}$$

$$= -\hat{i} + 3\hat{j} - 5\hat{k}$$

$\overrightarrow{CA}$ = Position vector of A – Position vector of C

$$= \left(4\hat{i} - 3\hat{j} + \hat{k}\right) - \left(\hat{i} - \hat{j}\right)$$

$$= 4\hat{i} - 3\hat{j} + \hat{k} - \hat{i} + \hat{j}$$

$$= 3\hat{i} - 2\hat{j} + \hat{k}$$

Now, $\overrightarrow{AB}.\overrightarrow{CA}$

$$= \left(-2\hat{i} - \hat{j} + 4\hat{k}\right).\left(3\hat{i} - 2\hat{j} + \hat{k}\right)$$

$$= (-2)(3) + (-1)(-2) + (4)(1)$$

$$= -6 + 2 + 4$$

$$= -6 + 6$$

$$= 0$$

So, $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{CA}$

$\angle A$ is right angle.

Hence, ABC is a right triangle

Scalar or Dot Product Ex 24.1 Q24

Given,

$A=(1,2,3)$
$B=(-1,0,0)$
$C=(0,1,2)$

Position vector of $A=\hat{i}+2\hat{j}+3\hat{k}$

Position vector of $B=-\hat{i}+0\hat{j}+0\hat{k}$

Position vector of $C=0\hat{i}+\hat{j}+2\hat{k}$

$\overline{AB}$ = Position vector of B - Position vector of A

$$=\left(-\hat{i}+0\hat{j}+0\hat{k}\right)-\left(\hat{i}+2\hat{j}+3\hat{k}\right)$$
$$=-2\hat{i}-2\hat{j}-3\hat{k}$$

$\overline{BC}$ = Position vector of C - Position vector of B

$$=\left(0\hat{i}+\hat{j}+2\hat{k}\right)-\left(-\hat{i}+0\hat{j}+0\hat{k}\right)$$
$$=\hat{i}+\hat{j}+2\hat{k}$$

$\overline{AC}$ = Position vector of C - Position vector of A

$$=\left(0\hat{i}+\hat{j}+2\hat{k}\right)-\left(1\hat{i}+2\hat{j}+3\hat{k}\right)$$
$$=-\hat{i}-\hat{j}-\hat{k}$$

$$\overline{AB}.\overline{BC}=\left(-2\hat{i}-2\hat{j}-3\hat{k}\right).\left(\hat{i}+\hat{j}+2\hat{k}\right)$$
$$=-2-2-6$$
$$=-10$$

$$\angle ABC=\frac{\overrightarrow{AB}.\overrightarrow{BC}}{\left|\overrightarrow{AB}\right|\left|\overrightarrow{BC}\right|}$$

$$=\frac{-10}{\sqrt{(-2)^2+(-2)^2+(-3)^2}\sqrt{1^2+1^2+2^2}}$$

$$=\frac{-10}{\sqrt{17}\sqrt{6}}$$

$$=\frac{-10}{\sqrt{102}}$$

$$\angle ABC=\cos^{-1}\left(\frac{-10}{\sqrt{102}}\right)$$

Scalar or Dot Product Ex 24.1 Q25

Given

$A=(0,1,1)$
$B=(\ 3,1,5)$
$C=(0,3,3)$

Position vector of $A=0\hat{i}+\hat{j}+\hat{k}$
Position vector of $B=3\hat{i}+\hat{j}+5\hat{k}$
Position vector of $C=0.\hat{i}+3\hat{j}+3\hat{k}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$=\left(3\hat{i}+\hat{j}+5\hat{k}\right)-\left(0.\hat{i}+\hat{j}+\hat{k}\right)$$
$$=3\hat{i}+\hat{j}+5\hat{k}-\hat{j}-\hat{k}$$

$\overrightarrow{AB}=3\hat{i}+4\hat{k}$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$$=\left(0.\hat{i}+3\hat{j}+3\hat{k}\right)-\left(3\hat{i}+\hat{j}+5\hat{k}\right)$$

$\overrightarrow{BC}=3\hat{j}+3\hat{k}-3\hat{i}-\hat{j}-5\hat{k}$

$$=-3\hat{i}+2\hat{j}-2\hat{k}$$

$\overrightarrow{AC}$ = Position vector of C – Position vector of A

$$=\left(-3\hat{j}+3\hat{k}\right)-\left(\hat{j}+\hat{k}\right)$$
$$=3\hat{j}+3\hat{k}-\hat{j}-\hat{k}$$
$$=2\hat{j}+2\hat{k}$$

$\overrightarrow{BC}.\overrightarrow{AC}$

$= \left(-3\hat{i} + 2\hat{j} - 2\hat{k}\right)\left(2\hat{j} + 2\hat{k}\right)$

$= (-3)(0) + (2)(2) + (-2)(+2)$

$= 0 + 4 - 4$

$= 0$

So, $\overrightarrow{BC}$ and $\overrightarrow{AC}$ is perpendicular

$\Rightarrow$ $\angle C$ is right angle.

Scalar or Dot Product Ex 24.1 Q26

Projection of $\left(\vec{b} + \vec{c}\right)$ on $\vec{a}$

$$= \frac{\left(\vec{b} + \vec{c}\right).\vec{a}}{|\vec{a}|}$$

$$= \frac{\vec{b}.\vec{a} + \vec{c}.\vec{a}}{\sqrt{(2)^2 + (-2)^2 + (1)^2}}$$

$$= \frac{\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)\left(2\hat{i} - 2\hat{j} + \hat{k}\right)\left(2\hat{i} - \hat{j} + 4\hat{k}\right)\left(2\hat{i} - 2\hat{j} + \hat{k}\right)}{\sqrt{4 + 4 + 1}}$$

$$= \frac{(1)(2) + (2)(-2) + (-2)(1) + (2)(2) + (-1)(-2) + (4)(1)}{\sqrt{9}}$$

$$= \frac{2 - 4 - 2 + 4 + 2 + 4}{3}$$

$$= \frac{12 - 6}{3} = \frac{6}{3} = 2$$

Projection of $\left(\vec{b} + \vec{c}\right) = 2$

Scalar or Dot Product Ex 24.1 Q27

$\vec{a} + \vec{b} = \left(5\hat{i} - \hat{j} - 3\hat{k}\right) + \left(\hat{i} + 3\hat{j} - 5\hat{k}\right)$

$= 5\hat{i} - \hat{j} - 3\hat{k} + \hat{i} + 3\hat{j} - 5\hat{k}$

$\vec{a} + \vec{b} = 6\hat{i} + 2\hat{j} - 8\hat{k}$ --- (i)

$\vec{a} - \vec{b} = \left(5\hat{i} - \hat{j} - 3\hat{k}\right) - \left(\hat{i} + 3\hat{j} - 5\hat{k}\right)$

$= 5\hat{i} - \hat{j} - 3\hat{k} - \hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{a} - \vec{b} = 4\hat{i} - 4\hat{j} + 2\hat{k}$ --- (ii)

Now, $\left(\vec{a} + \vec{b}\right).\left(\vec{a} - \vec{b}\right)$

$= \left(6\hat{i} + 2\hat{j} - 8\hat{k}\right).\left(4\hat{i} - 4\hat{j} + 2\hat{k}\right)$

$= (6)(4) + (2)(-4) + (-8)(2)$

$= 24 - 8 - 16$

$= 0$

So, $\left(\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - \vec{b}\right)$ are perpendicular.

Scalar or Dot Product Ex 24.1 Q28

Let unit vector $\vec{a}$ have (a_1, a_2, a_3) components.

$\Rightarrow \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Since $\vec{a}$ is a unit vector, $|\vec{a}| = 1$.

Also, it is given that $\vec{a}$ makes angles $\frac{\pi}{4}$ with $\hat{i}$, $\frac{\pi}{3}$ with $\hat{j}$, and an acute angle θ with $\hat{k}$.

Then, we have:

$\cos\frac{\pi}{4} = \frac{a_1}{|\vec{a}|}$

$\Rightarrow \frac{1}{\sqrt{2}} = a_1$ $\quad [|\vec{a}| = 1]$

$\cos\frac{\pi}{3} = \frac{a_2}{|\vec{a}|}$

$\Rightarrow \frac{1}{2} = a_2$ $\quad [|\vec{a}| = 1]$

Also, $\cos\theta = \frac{a_3}{|\vec{a}|}$.

$\Rightarrow a_3 = \cos\theta$

Now,

$|a| = 1$

$\Rightarrow \sqrt{a_1^2 + a_2^2 + a_3^2} = 1$

$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$

$\Rightarrow \frac{3}{4} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

$\therefore a_3 = \cos\frac{\pi}{3} = \frac{1}{2}$

Hence, $\theta = \frac{\pi}{3}$ and the components of $\vec{a}$ are $\left(\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}\right)$.

Scalar or Dot Product Ex 24.1 Q29

$\left(3\vec{a} - 5\vec{b}\right) \cdot \left(2\vec{a} + 7\vec{b}\right)$

$= 3\vec{a} \cdot 2\vec{a} + 3\vec{a} \cdot 7\vec{b} - 5\vec{b} \cdot 2\vec{a} - 5\vec{b} \cdot 7\vec{b}$

$= 6\vec{a} \cdot \vec{a} + 21\vec{a} \cdot \vec{b} - 10\vec{a} \cdot \vec{b} - 35\vec{b} \cdot \vec{b}$

$= 6|\vec{a}|^2 + 11\vec{a} \cdot \vec{b} - 35|\vec{b}|^2$

$= \mathbf{6 * 2^2 + 11 * 1 - 35 * 1^2}$

$= \mathbf{35 - 35}$

$= \mathbf{0}$

Scalar or Dot Product Ex 24.1 Q30(i)

We have,

$(\bar{x} - \bar{a}).(\bar{x} + \bar{a}) = 8$

$\Rightarrow |\bar{x}|^2 - |\bar{a}|^2 = 8$

$\Rightarrow |\bar{x}|^2 - 1^2 = 8 \qquad \text{since } |\bar{a}| = 1$

$\Rightarrow |\bar{x}|^2 = 8 + 1$

$\Rightarrow |\bar{x}|^2 = 9$

$\Rightarrow |\bar{x}| = 3$

Scalar or Dot Product Ex 24.1 Q30(ii)

We have,

$(\bar{x} - \bar{a}).(\bar{x} + \bar{a}) = 12$

$\Rightarrow |\bar{x}|^2 - |\bar{a}|^2 = 12$

$\Rightarrow |\bar{x}|^2 - 1^2 = 12 \qquad \text{since } |\bar{a}| = 1$

$\Rightarrow |\bar{x}|^2 = 12 + 1$

$\Rightarrow |\bar{x}|^2 = 13$

$\Rightarrow |\bar{x}| = \sqrt{13}$

Scalar or Dot Product Ex 24.1 Q31(i)

Here, $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 12$

$|\vec{a}|^2 - |\vec{b}|^2 = 12$

$(2|\vec{b}|)^2 - |\vec{b}|^2 = 12$ $\quad\left[\text{Using } |\vec{a}| = 2|\vec{b}|\right]$

$4|\vec{b}|^2 - |\vec{b}|^2 = 12$

$3|\vec{b}|^2 = 12$

$|\vec{b}|^2 = \frac{12}{3}$

$|\vec{b}|^2 = 4$

$|\vec{b}| = 2$

$|\vec{a}| = 2|\vec{b}| = 2(2)$

$|\vec{a}| = 4$

$|\vec{b}| = 2$

Scalar or Dot Product Ex 24.1 Q31(ii)

$(\vec{a}\cdot\vec{b}).(\vec{a}-\vec{b}) = 8$

$\Rightarrow \vec{a}\cdot\vec{a} - \vec{a}.\vec{b} + \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b} = 8$

$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8$

$\Rightarrow (8|\vec{b}|)^2 - |\vec{b}|^2 = 8$ $\quad\left[|\vec{a}| = 8|\vec{b}|\right]$

$\Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8$

$\Rightarrow 63|\vec{b}|^2 = 8$

$\Rightarrow |\vec{b}|^2 = \frac{8}{63}$

$\Rightarrow |\vec{b}| = \sqrt{\frac{8}{63}}$ $\quad$[Magnitude of a vector is non-negative]

$\Rightarrow |\vec{b}| = \frac{2\sqrt{2}}{3\sqrt{7}}$

$|\vec{a}| = 8|\vec{b}| = \frac{8\times 2\sqrt{2}}{3\sqrt{7}} = \frac{16\sqrt{2}}{3\sqrt{7}}$

Scalar or Dot Product Ex 24.1 Q31(iii)

Here, $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 3$

$|\vec{a}|^2 - |\vec{b}|^2 = 3$

$(2|\vec{b}|)^2 - |\vec{b}|^2 = 3$ $\quad\left[\text{Using } |\vec{a}| = 2|\vec{b}|\right]$

$4|\vec{b}|^2 - |\vec{b}|^2 = 3$

$3|\vec{b}|^2 = 3$

$|\vec{b}|^2 = \frac{3}{3}$

$|\vec{b}|^2 = 1$

$|\vec{b}| = 1$

$|\vec{a}| = 2|\vec{b}|$

$= 2(1)$

$|\vec{a}| = 2$

$|\vec{b}| = 1$

Scalar or Dot Product Ex 24.1 Q32(i)

$$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$$
$$= (2)^2 + (5)^2 - 2(8)$$
$$= 4 + 25 - 16$$
$$|\vec{a}-\vec{b}|^2 = 13$$

$$|\vec{a}-\vec{b}| = \sqrt{13}$$

Scalar or Dot Product Ex 24.1 Q32(ii)

$$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$$
$$= (3)^2 + (4)^2 - 2.(1)$$
$$= 9 + 16 - 2$$
$$|\vec{a}-\vec{b}|^2 = 23$$

$$|\vec{a}-\vec{b}| = \sqrt{23}$$

Scalar or Dot Product Ex 24.1 Q32(iii)

We have

$$|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b}).(\vec{a}-\vec{b})$$
$$= \vec{a}.\vec{a} - \vec{a}.\vec{b} - \vec{b}.\vec{a} + \vec{b}.\vec{b}$$
$$= |\vec{a}|^2 - 2(\vec{a}.\vec{b}) + |\vec{b}|^2 = (2)^2 - 2(4) + (3)^2 = 5$$
$$\therefore \quad |\vec{a}-\vec{b}| = \sqrt{5}$$

Scalar or Dot Product Ex 24.1 Q33(i)

We have,

$|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$ and $\vec{a}.\vec{b} = \sqrt{6}$

Let θ be the angle between $\vec{a}$ and $\vec{b}$. Then

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$
$$= \frac{\sqrt{6}}{\sqrt{3}\times 2}$$
$$= \frac{1}{\sqrt{2}}$$
$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$
$$= \frac{\pi}{4}$$

Scalar or Dot Product Ex 24.1 Q33(ii)

Let the angle between $\vec{a}$ and $\vec{b}$ is θ, then

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$
$$= \frac{1}{3.3}$$
$$\cos\theta = \frac{1}{9}$$

$$\theta = \cos^{-1}\left(\frac{1}{9}\right)$$

Scalar or Dot Product Ex 24.1 Q34

Let $\vec{a} = \vec{u} + \vec{v}$

$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v}$ --- (i)

Such that $\vec{u}$ is parallel to $\vec{b}$ and $\vec{v}$ is perpendicular to $\vec{b}$.

Now, $\vec{u}$ is parallel to $\vec{b}$

$\vec{u} = \lambda\vec{b}$

$= \lambda\left(3\hat{i} + \hat{k}\right)$

$\vec{u} = 3\lambda\hat{i} + \lambda\hat{k}$ --- (ii)

Put value of $\vec{u}$ in equation (i),

$5\hat{i} - 2\hat{j} + 5\hat{k} = \left(3\lambda\hat{i} + \lambda\hat{k}\right) + \vec{v}$

$\vec{v} = 5\hat{i} - 2\hat{j} + 5\hat{k} - 3\lambda\hat{i} - \lambda\hat{k}$

$\vec{v} = (5 - 3\lambda)\hat{i} + (-2)\hat{j} + (5 - \lambda)\hat{k}$

$\vec{v}$ is perpendicular to $\vec{b}$

Then, $\vec{v}.\vec{b} = 0$

$\left[(5 - 3\lambda)\hat{i} + (-2)\hat{j} + (5 - \lambda)\hat{k}\right].\left(3\hat{i} + 0 \times \hat{j} + \hat{k}\right) = 0$

$(5 - 3\lambda)(3) + (-2)(0) + (5 - \lambda)(1) = 0$

$15 - 9\lambda + 0 + 5 - \lambda = 0$

$20 - 10\lambda = 0$

$-10\lambda = -20$

$\lambda = \frac{-20}{-10}$

$\lambda = 2$

Put λ in equation (ii)

$\vec{u} = 3\lambda\hat{i} + \lambda\hat{k}$

$= 3(2)\hat{i} + (2)\hat{k}$

$\vec{u} = 6\hat{i} + 2\hat{k}$

Put the value of $\vec{u}$ in equation (i)

$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v}$

$5\hat{i} - 2\hat{j} + 5\hat{k} = \left(6\hat{i} + 2\hat{k}\right) + \vec{v}$

$\vec{v} = 5\hat{i} - 2\hat{j} + 5\hat{k} - 6\hat{i} - 2\hat{k}$

$\vec{v} = -\hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{a} = \left(6\hat{i} + 2\hat{k}\right) + \left(-\hat{i} - 2\hat{j} + 3\hat{k}\right)$

Scalar or Dot Product Ex 24.1 Q35

Vectors $\vec{a}$ and $\vec{b}$ have same magnitude, then

$|\vec{a}| = |\vec{b}| = x$ (Say)

Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$\cos\theta = \frac{\vec{a}\vec{b}}{|\vec{a}||\vec{b}|}$

$\cos 30° = \frac{3}{x.x}$

$\frac{\sqrt{3}}{2} = \frac{3}{x^2}$

$\sqrt{3}x^2 = 6$

$x^2 = \frac{6}{\sqrt{3}}$

Rationalizing the denominator,

$x^2 = \frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$x^2 = \frac{6\sqrt{3}}{3}$

$x^2 = 2\sqrt{3}$

$x = \sqrt{2\sqrt{3}}$

$|\vec{a}| = |\vec{b}| = \sqrt{2\sqrt{3}}$

Scalar or Dot Product Ex 24.1 Q36

Let $\left(2\hat{i} - \hat{j} + 3\hat{k}\right) = \vec{a} + \vec{b}$ $\quad$ ---(i)

Such that $\vec{a}$ is a vector parallel to vector $\left(2\hat{i} + 4\hat{j} - 2\hat{k}\right)$ and $\vec{b}$ is a vector perpendicular to the vector $\left(2\hat{i} + 4\hat{j} - 2\hat{k}\right)$.

Since, $\vec{a}$ is parallel to $\left(2\hat{i} + 4\hat{j} - 2\hat{k}\right)$

$$\vec{a} = \lambda\left(2\hat{i} + 4\hat{j} - 2\hat{k}\right)$$

$$\vec{a} = 2\lambda\hat{i} + 4\lambda\hat{j} - 2\lambda\hat{k} \quad \text{---(ii)}$$

Put value of $\vec{a}$ in equation (i),

$$\left(2\hat{i} - \hat{j} + 3\hat{k}\right) = \left(2\lambda\hat{i} + 4\lambda\hat{j} - 2\lambda\hat{k}\right) + \vec{b}$$

$$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k} - 2\lambda\hat{i} - 4\lambda\hat{j} + 2\lambda\hat{k}$$

$$\vec{b} = (2 - 2\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} + (3 + 2\lambda)\hat{k}$$

$\vec{b}$ is a vector perpendicular to the vector $\left(2\hat{i} + 4\hat{j} - 2\hat{k}\right)$, then

$$\vec{b}.\left(2\hat{i} + 4\hat{j} - 2\hat{k}\right) = 0$$

$$\left[(2 - 2\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} + (3 + 2\lambda)\hat{k}\right]\left(2\hat{i} + 4\hat{j} - 2\hat{k}\right) = 0$$

$$(2 - 2\lambda)(2) + (-1 - 4\lambda)(4) + (3 + 2\lambda)(-2) = 0$$

$$4 - 4\lambda - 4 - 16\lambda - 6 - 4\lambda = 0$$

$$-6 - 24\lambda = 0$$

$$-24\lambda = 6$$

$$\lambda = -\frac{1}{4}$$

Put λ in equation (ii),

$$\vec{a} = 2\lambda\hat{i} + 4\lambda\hat{j} - 2\lambda\hat{k}$$

$$= 2\left(-\frac{1}{4}\right)\hat{i} + 4\left(-\frac{1}{4}\right)\hat{j} - 2\left(-\frac{1}{4}\right)\hat{k}$$

$$\vec{a} = -\frac{1}{2}\hat{i} - \hat{j} + \frac{1}{2}\hat{k}$$

Put the value of $\vec{a}$ in equation (i),

$$\left(2\hat{i} - \hat{j} + 3\hat{k}\right) = \left(-\frac{1}{2}\hat{i} - \hat{j} + \frac{1}{2}\hat{k}\right) + \vec{b}$$

$$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k} + \frac{1}{2}\hat{i} + \hat{j} - \frac{1}{2}\hat{k}$$

$$= \frac{4\hat{i} - 2\hat{j} + 6\hat{k} + \hat{i} + 2\hat{j} - \hat{k}}{2}$$

$$= \frac{5\hat{i} + 5\hat{k}}{2}$$

$$\vec{b} = \frac{5}{2}\left(\hat{i} + \hat{k}\right)$$

$$\left(2\hat{i} - \hat{j} + 3\hat{k}\right) = \left(-\frac{1}{2}\hat{i} - \hat{j} + \frac{1}{2}\hat{k}\right) + \frac{5}{2}\left(\hat{i} + \hat{k}\right)$$

Scalar or Dot Product Ex 24.1 Q37

Let $\left(6\hat{i}-3\hat{j}-6\hat{k}\right)=\vec{a}+\vec{b}$ $\quad ---(i)$

Such that $\vec{a}$ is parallel to $\left(\hat{i}+\hat{j}+\hat{k}\right)$ and $\vec{b}$ is perpendicular to $\left(\hat{i}+\hat{j}+\hat{k}\right)$.

Since, $\vec{a}$ is parallel to $\left(\hat{i}+\hat{j}+\hat{k}\right)$

$$\vec{a}=\lambda\left(\hat{i}+\hat{j}+\hat{k}\right) \quad ---(ii)$$

Put $\vec{a}$ in equation (i),

$$\left(6\hat{i}-3\hat{j}-6\hat{k}\right)=\left(\lambda\hat{i}+\lambda\hat{j}+\lambda\hat{k}\right)+\vec{b}$$
$$\vec{b}=6\hat{i}-\lambda\hat{i}-3\hat{j}-\lambda\hat{j}-6\hat{k}-\lambda\hat{k}$$
$$\vec{b}=(6-\lambda)\hat{i}+(-3-\lambda)\hat{j}+(-6-\lambda)\hat{k}$$

$\vec{b}$ is a vector perpendicular to the vector $\left(\hat{i}+\hat{j}+\hat{k}\right)$, then

$$\vec{b}.\left(\hat{i}+\hat{j}+\hat{k}\right)=0$$
$$\left[(6-\lambda)\hat{i}+(-3-\lambda)\hat{j}+(-6-\lambda)\hat{k}\right]\left(\hat{i}+\hat{j}+\hat{k}\right)=0$$
$$(6-\lambda)(1)+(-3-\lambda)(1)+(-6-\lambda)(1)=0$$
$$6-\lambda-3-\lambda-6-\lambda=0$$
$$-3-3\lambda=0$$
$$-3=3\lambda$$
$$\lambda=\frac{-3}{3}$$
$$\lambda=-1$$

Put value of λ in (ii),

$$\vec{a}=-1.\left(\hat{i}+\hat{j}+\hat{k}\right)$$
$$\vec{a}=-\hat{i}-\hat{j}-\hat{k}$$

Using $\vec{a}$ in equation (i),

$$\left(6\hat{i}-3\hat{j}-6\hat{k}\right)=\left(-\hat{i}-\hat{j}-\hat{k}\right)+\vec{b}$$
$$\vec{b}=6\hat{i}+\hat{i}-3\hat{j}+\hat{j}-6\hat{k}+\hat{k}$$
$$\vec{b}=7\hat{i}-2\hat{j}-5\hat{k}$$

Thus,

Vector $\vec{a}=-\hat{i}-\hat{j}-\hat{k}$ and

$\vec{b}=7\hat{i}-2\hat{j}-5\hat{k}$

are required vectors.

Scalar or Dot Product Ex 24.1 Q38

Here, $\left(\vec{a}+\vec{b}\right)$ is orthogonal to $\left(\vec{a}-\vec{b}\right)$

Then, $\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)=0$

$$\left|\vec{a}\right|^2=\left|\vec{b}\right|^2=0$$
$$\left\{\sqrt{(5)^2+(-1)^2+(7)^2}\right\}^2-\left\{\sqrt{(1)^2+(-1)^2+(\lambda)^2}\right\}^2=0$$
$$(25+1+49)-\left(1+1+\lambda^2\right)=0$$
$$75-\left(2+\lambda^2\right)=0$$
$$75-2-\lambda^2=0$$
$$-\lambda^2=-73$$
$$\lambda=\sqrt{73}$$

Scalar or Dot Product Ex 24.1 Q39

It is given that $\vec{a}\cdot\vec{a}=0$ and $\vec{a}\cdot\vec{b}=0$.

Now,

$\vec{a}\cdot\vec{a}=0 \Rightarrow |\vec{a}|^2=0 \Rightarrow |\vec{a}|=0$

$\therefore \vec{a}$ is a zero vector.

Hence, vector $\vec{b}$ satisfying $\vec{a}\cdot\vec{b}=0$ can be any vector

Scalar or Dot Product Ex 24.1 Q40

Given that $\vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, so,

$\vec{a}\,.\,\vec{c}=0$ and $\vec{b}\,.\,\vec{c}=0$

Now, $\vec{c}\,.\left(\vec{a}+\vec{b}\right)$

$=\vec{c}\,.\,\vec{a}+\vec{c}\,.\,\vec{b}$

$=0+0$

$=0$

$\therefore \vec{c}$ is perpendicular to $\left(\vec{a}+\vec{b}\right)$

$\vec{c}\,.\left(\vec{a}-\vec{b}\right)$

$=\vec{c}\,.\,\vec{a}-\vec{c}\,.\,\vec{b}$

$=0-0$

$=0$

$\therefore \vec{c}$ is perpendicular to $\left(\vec{a}-\vec{b}\right)$

Scalar or Dot Product Ex 24.1 Q41

Here $|\vec{a}|=a,\ |\vec{b}|=b$

$$\text{LHS} = \left(\frac{\vec{a}}{a^2}-\frac{\vec{b}}{b^2}\right)^2$$

$$= \left(\frac{\vec{a}}{a^2}\right)^2+\left(\frac{\vec{b}}{b^2}\right)^2-2\frac{\vec{a}}{a^2}\cdot\frac{\vec{b}}{b^2}$$

$$= \frac{|\vec{a}|^2}{a^4}+\frac{|\vec{b}|^2}{b^4}-\frac{2\vec{a}\vec{b}}{a^2b^2}$$

$$= \frac{a^2}{a^4}+\frac{b^2}{b^4}-\frac{2\vec{a}\vec{b}}{a^2b^2} \qquad \left[\text{Since } |\vec{a}|=a,\ |\vec{b}|=b\right]$$

$$= \frac{1}{a^2}+\frac{1}{b^2}-\frac{2\vec{a}\vec{b}}{a^2b^2}$$

$$= \frac{b^2+a^2-2\vec{a}\vec{b}}{a^2b^2}$$

$$= \frac{|\vec{a}|^2+|\vec{b}|^2-2\vec{a}\vec{b}}{a^2b^2}$$

$$= \frac{\left(\vec{a}-\vec{b}\right)^2}{a^2b^2}$$

$$= \left(\frac{\vec{a}-\vec{b}}{ab}\right)^2$$

= RHS

Hence proved

$$\therefore \quad \left(\frac{\vec{a}}{a^2}-\frac{\vec{b}}{b^2}\right)^2=\left(\frac{\vec{a}-\vec{b}}{ab}\right)^2$$

Scalar or Dot Product Ex 24.1 Q42

Given that

$\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors such that

$\vec{d}.\vec{a} = \vec{d}.\vec{b} = \vec{d}.\vec{c} = 0$

Given that

$\vec{d}.\vec{a} = 0$

$\Rightarrow$ $\vec{d}$ perpendicular to $\vec{a}$

or $\vec{d} = 0$ $\quad ---(i)$

$\vec{d}.\vec{b} = 0$

$\Rightarrow$ $\vec{d}$ is perpendicular to $\vec{b}$ or $\vec{d} = 0$ $\quad ---(ii)$

$\vec{d}.\vec{c} = 0$

$\Rightarrow$ $\vec{d}$ is perpendicular to $\vec{c}$ or $\vec{d} = 0$ $\quad ---(iii)$

From (i), (ii), (iii), we get

$\vec{d}$ is perpendicular to $\vec{a},\vec{b},\vec{c}$ or $\vec{d} = 0$, but $\vec{d}$ can not be perpendicular to $\vec{a},\vec{b}$ and $\vec{c}$ because $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors, so

$\vec{d} = 0$

Scalar or Dot Product Ex 24.1 Q43

Given that

$\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$

It means,

$\vec{a}.\vec{b} = 0$ and $\vec{a}.\vec{c} = 0$ $\quad ---(i)$

Let $\vec{r}$ be some vector in the plane of $\vec{b}$ and $\vec{c}$

Then, $\vec{r},\vec{b},\vec{c}$ are coplanar

We know that,

Three vectors are coplanar if one of them is expressible as linear combination of other two vectors.

Let $\vec{r} = x\vec{b} + y\vec{c}$

where x and y are same scalar

$\vec{r}.\vec{a} = \left(x\vec{b} + y\vec{c}\right).\vec{a}$ $\quad$ [Taking dot product with $\vec{a}$ on both the side]

$\vec{r}.\vec{a} = x\vec{b}.\vec{a} + y\vec{c}.\vec{a}$

$= x.0 + y.0$ $\quad$ [Using (i)]

$\vec{r}.\vec{a} = 0 + 0$

$\vec{r}.\vec{a} = 0$

So, $\vec{r}$ is perpendicular to $\vec{a}$

Thus,

$\vec{a}$ is perpendicular to every vector in the plane of $\vec{b}$ and $\vec{c}$

Scalar or Dot Product Ex 24.1 Q44

We have,

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\vec{b} + \vec{c} = -\vec{a}$

Squaring both the sides.

$\left(\vec{b} + \vec{c}\right)^2 = \left(-\vec{a}\right)^2$

$\left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 + 2\vec{b}.\vec{c} = \left|\vec{a}\right|^2$

$2\vec{b}.\vec{c} = \left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 - \left|\vec{c}\right|^2$

$2\left|\vec{b}\right|\left|\vec{c}\right|\cos\theta = \left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 - \left|\vec{c}\right|^2$ $\quad$ [Since $\vec{b}.\vec{c} = \left|\vec{b}\right|\left|\vec{c}\right|\cos\theta$]

$$\cos\theta = \frac{\left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 - \left|\vec{c}\right|^2}{2\left|\vec{b}\right|\left|\vec{c}\right|}$$

Scalar or Dot Product Ex 24.1 Q45

Here, $\vec{u}+\vec{v}+\vec{w}=0$

Squaring both the sides,

$$\left(\vec{u}+\vec{v}+\vec{w}\right)^2=(0)^2$$

$$\left|\vec{u}\right|^2+\left|\vec{v}\right|^2+\left|\vec{w}\right|^2+2\vec{u}\vec{v}+2\vec{v}\vec{w}+2\vec{w}\vec{u}=0$$

$$(3)^2+(4)^2+(5)^2+2\left(\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}\right)=0$$

$$9+16+25+2\left(\vec{u}\vec{v}+\vec{v}\vec{w}+\vec{w}\vec{u}\right)=0$$

$$2\left(\vec{u}\vec{v}+\vec{v}\vec{w}+\vec{w}\vec{u}\right)=-50$$

$$\vec{u}\vec{v}+\vec{v}\vec{w}+\vec{w}\vec{u}=\frac{-50}{2}$$

$$\vec{u}\vec{v}+\vec{v}\vec{w}+\vec{w}\vec{u}=-25$$

Scalar or Dot Product Ex 24.1 Q46

Given

$$\vec{a}=x^2\hat{i}+2\hat{j}-2\hat{k}$$

$$\vec{b}=\hat{i}-\hat{j}+\hat{k}$$

$$\vec{c}=x^2\hat{i}+5\hat{j}-4k$$

Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\vec{a}.\vec{b}=\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta$$

$$\cos\theta=\frac{\vec{a}\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}$$

$$=\frac{\left(x^2\hat{i}+2\hat{j}-2\hat{k}\right)\left(\hat{i}-\hat{j}+\hat{k}\right)}{\sqrt{\left(x^2\right)^2+(2)^2+(-2)^2}\sqrt{(1)^2+(-1)^2+(1)^2}}$$

$$=\frac{\left(x^2\right)(1)+(2)(-1)+(-2)(1)}{\sqrt{x^4+4+4}\sqrt{1+1+1}}$$

$$=\frac{x^2-2-2}{\sqrt{8+x^2}\sqrt{3}}$$

$$\cos\theta=\frac{x^2-4}{\sqrt{3}\sqrt{8+x^4}}$$

Since θ is an acute angle, so

$$\cos\theta>0$$

$$\frac{x^2-4}{\sqrt{3}\sqrt{8+x^4}}>0$$

$$x^2-4>0$$

$$x^2>4$$

$\Rightarrow \quad x<-2$ or $x>2$ --- (i)

Again, let ϕ be the angle between $\vec{b}$ and $\vec{c}$,

$$\cos\phi=\frac{\vec{b}\vec{c}}{\left|\vec{b}\right|\left|\vec{c}\right|}$$

$$=\frac{\left(\hat{i}-\hat{j}+\hat{k}\right)\left(x^2\hat{i}+5\hat{j}-4k\right)}{\sqrt{(1)^2+(-1)^2+(1)^2}\sqrt{\left(x^2\right)^2+(5)^2+(-4)^2}}$$

$$\cos\phi=\frac{(1)\left(x^2\right)+(-1)(5)+(1)(-4)}{\sqrt{3}\sqrt{x^4+25+16}}$$

$$\cos\phi=\frac{x^2-5-4}{\sqrt{3}\sqrt{x^2+41}}$$

$$\cos\phi=\frac{x^2-9}{\sqrt{3}\sqrt{x^2+41}}$$

Since ϕ is an obtuse angle, so

$$\cos\phi<0$$

$$\frac{x^2-9}{\sqrt{3}\sqrt{x^2+41}}<0$$

$$x^2-9<0$$

$$x^2<9$$

$\Rightarrow \quad x>-3$ and $x<3$ --- (ii)

From

$$-3<x<-2 \text{ and } 2<x<3$$

$$x\in(-3,-2)\cup(2,3)$$

Scalar or Dot Product Ex 24.1 Q47

Here, $\vec{a}$ and $\vec{b}$ are mutually perpenducular, then

$$\vec{a}.\vec{b} = 0$$
$$\left(3\hat{i} + x\hat{j} - \hat{k}\right)\left(2\hat{i} + \hat{j} + y\hat{k}\right) = 0$$
$$(3)(2) + (x)(1) + (-1)(y) = 0$$
$$6 + x - y = 0$$
$$x - y = -6 \qquad ---(i)$$

Also, $\vec{a}$ and $\vec{b}$ have equal magnitude,

$$|\vec{a}| = |\vec{b}|$$
$$\sqrt{(3)^2 + (x)^2 + (-1)^2} = \sqrt{(2)^2 + (1)^2 + (y)^2}$$
$$9 + x^2 + 1 = 4 + 1 + y^2$$
$$x^2 + 10 = 5y^2$$
$$x^2 - y^2 = 5 - 10$$
$$x^2 - y^2 = -5$$
$$(x + y)(x - y) = -5$$
$$(x + y)(-6) = -5 \qquad [\text{Using } (i)]$$
$$-6x - 6y = -5$$
$$-(6x + 6y) = -5$$
$$6x + 6y = 5 \qquad ---(ii)$$

Solving (i) and (ii),

$$6x + 6y = 5$$
$$\underline{6x - 6y = -36} \qquad [(i) \times 6]$$
$$12x \quad = -31$$
$$x = \frac{-31}{12}$$

Put value of x in equation (i),

$$x - y = -6$$
$$\frac{-31}{12} - y = -6$$
$$-y = \frac{-6}{1} + \frac{31}{12}$$
$$-y = \frac{-72 + 31}{12}$$
$$y = \frac{41}{12}$$

Scalar or Dot Product Ex 24.1 Q48

Given

$\vec{a}$ and $\vec{b}$ are unit vectors

Then, $|\vec{a}| = |\vec{b}| = 1$

$$|\vec{a} + \vec{b}| = \sqrt{3}$$

Squaring both the sides,

$$|\vec{a} + \vec{b}|^2 = \left(\sqrt{3}\right)^2$$
$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = 3$$
$$1 + 1 + 2\vec{a}.\vec{b} = 3$$
$$2 + 2\vec{a}.\vec{b} = 3$$
$$2\vec{a}.\vec{b} = 3 - 2$$
$$2\vec{a}.\vec{b} = 1$$
$$\vec{a}.\vec{b} = \frac{1}{2}$$

Now, $\left(2\vec{a} - 5\vec{b}\right).\left(3\vec{a} + \vec{b}\right)$

$$= 2\vec{a}.3\vec{a} + 2\vec{a}\vec{b} - 5\vec{b}.3\vec{a} - 5\vec{b}.\vec{b}$$
$$= 6\left(\vec{a}\right)^2 + 2\vec{a}.\vec{b} - 15\vec{a}.\vec{b} - 5\left(\vec{b}\right)^2$$
$$= 6|\vec{a}|^2 - 13\vec{a}\vec{b} - 5|\vec{b}|^2$$
$$= 6(1)^2 - 13\left(\frac{1}{2}\right) - 5(1)^2$$
$$= \frac{6}{1} - \frac{13}{2} - \frac{5}{1}$$
$$= \frac{12 - 13 - 10}{2}$$
$$= \frac{12 - 23}{2}$$
$$= -\frac{11}{2}$$

$$\left(2\vec{a} - 5\vec{b}\right).\left(3\vec{a} + \vec{b}\right) = -\frac{11}{2}$$

Scalar or Dot Product Ex 24.1 Q49

$|\vec{a}+\vec{b}|^2 = |\vec{b}|^2$

$\Rightarrow (\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = \vec{b}.\vec{b}$

$\Rightarrow \vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{b}.\vec{a} + \vec{b}.\vec{b} = \vec{b}.\vec{b}$

$\Rightarrow \vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{b} + \vec{b}.\vec{b} = \vec{b}.\vec{b}$

$\Rightarrow \vec{a}.\vec{a} + 2\vec{a}.\vec{b} = 0$

$\Rightarrow \vec{a}.(\vec{a} + 2\vec{b}) = 0$

$\therefore \vec{a} + 2\vec{b}$ is perpendicular to $\vec{a}$.

Let $\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right) = 0$

$$\left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 = 0$$

$$\left|\vec{a}\right|^2 = \left|\vec{b}\right|^2$$

$$\left|\vec{a}\right| = \left|\vec{b}\right|$$

Let $\left|\vec{a}\right| = \left|\vec{b}\right|$

Squaring both the sides.

$$\left|\vec{a}\right|^2 = \left|\vec{b}\right|^2$$

$$\left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 = 0$$

$$\left(\vec{a}\right)^2 - \left(\vec{b}\right)^2 = 0$$

$$\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right) = 0$$

Thus,

$$\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right) = 0 \Leftrightarrow \left|\vec{a}\right| = \left|\vec{b}\right|$$

Ex 24.2

Scalar or Dot Product Ex 24.2 Q1

Let $\vec{o}$, $\vec{a}$ and $\vec{b}$ be the position vector of the O, A and B.

P and Q are points of trisection of AB.

Position vector of point P = $\dfrac{2\vec{a}+\vec{b}}{3}$

Position vector of point Q = $\dfrac{\vec{a}+2\vec{b}}{3}$

$$OP = \frac{2\vec{a}+\vec{b}}{3} - \vec{o} = \frac{2\vec{a}+\vec{b}-3\vec{o}}{3} = \frac{2OA+OB}{3}$$

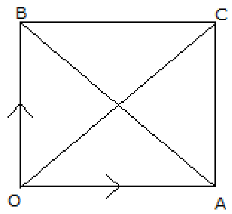
$$OQ = \frac{\vec{a}+2\vec{b}}{3} - \vec{o} = \frac{\vec{a}+2\vec{b}-3\vec{o}}{3} = \frac{OA+2OB}{3}$$

$$OP^2 + OQ^2 = \left(\frac{2OA+OB}{3}\right)^2 + \left(\frac{OA+2OB}{3}\right)^2$$

$$= \frac{5\left(OA^2+OB^2\right)+8(OA)(OB)\cos 90^\circ}{9}$$

$$= \frac{5}{9}AB^2 \ldots\ldots\ldots \left[\because OA^2+OB^2 = AB^2 \text{ and } \cos 90^\circ = 0\right]$$

Scalar or Dot Product Ex 24.2 Q2



Let OACB be a quadrilateral such that its diagonal bisect each other at right angles.
We know that if the diagonals of a quadrilateral bisect each other then its a parallelogram.
∴ OACB is a parallelogram.
⇒ OA = BC and OB = AC.

Taking O as origin let $\vec{a}$ and $\vec{b}$ be the position vector of the A and B.

AB and OC be the diagonals of quadrilateral which bisect each other at right angles.

$\therefore \overrightarrow{OC} \bullet \overrightarrow{AB} = 0$

$\Rightarrow \left(\vec{a}+\vec{b}\right) \bullet \left(\vec{b}-\vec{a}\right) = 0$

$\Rightarrow \left|\vec{b}\right|^2 = \left|\vec{a}\right|^2$

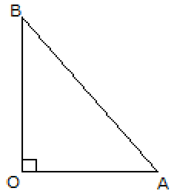
⇒ OB = OA

Simillarly we can show that

OA = OB = BC = CA

Hence OACB is a rhombus.

Scalar or Dot Product Ex 24.2 Q3



Let OAC be a right triangle, right angled at O.

Taking O as origin let $\vec{a}$ and $\vec{b}$ be the position vector of the $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

$\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$

$\therefore \overrightarrow{OA} \bullet \overrightarrow{OB} = 0$

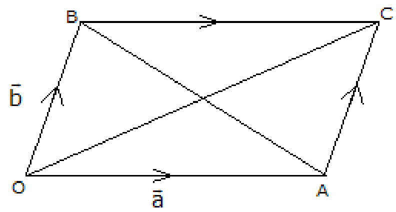
$\vec{a} \bullet \vec{b} = 0$

Now,

$\overrightarrow{AB}^2 = (\vec{b}-\vec{a})^2 = (\vec{a})^2 + (\vec{b})^2 - 2\vec{a}\bullet\vec{b} = (\vec{a})^2 + (\vec{b})^2 - 0 = (\overrightarrow{OA})^2 + (\overrightarrow{OB})^2$

Hence proved.

Scalar or Dot Product Ex 24.2 Q4



Let OAC be a right triangle, right angled at O.

Taking O as origin let $\vec{a}$ and $\vec{b}$ be the position vector of the $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

$\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$

$\therefore \overrightarrow{OA} \bullet \overrightarrow{OB} = 0$

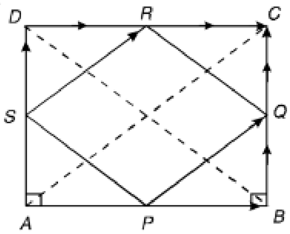
$\vec{a} \bullet \vec{b} = 0$

Now,

$\overrightarrow{AB}^2 = (\vec{b}-\vec{a})^2 = (\vec{a})^2 + (\vec{b})^2 - 2\vec{a}\bullet\vec{b} = (\vec{a})^2 + (\vec{b})^2 - 0 = (\overrightarrow{OA})^2 + (\overrightarrow{OB})^2$

Hence proved.

Scalar or Dot Product Ex 24.2 Q5



ABCD be a rectangle.
Let P, Q, R and S be the midpoints of the sides AB, BC, CD and DA respectively.
Now,

$\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BQ} = \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{BC}\right) = \frac{1}{2}\overrightarrow{AC}$.........(i)

$\overrightarrow{SR} = \overrightarrow{SD} + \overrightarrow{DR} = \frac{1}{2}\left(\overrightarrow{AD} + \overrightarrow{DC}\right) = \frac{1}{2}\overrightarrow{AC}$.........(ii)

From (i) and (ii), we have

$\overrightarrow{PQ} = \overrightarrow{SR}$ i.e. sides PQ and SR are equal and parallel.

∴ PQRS is a parallelogram.

$(PQ)^2 = \overrightarrow{PQ} \bullet \overrightarrow{PQ} = \left(\overrightarrow{PB} + \overrightarrow{BQ}\right) \bullet \left(\overrightarrow{PB} + \overrightarrow{BQ}\right) = |PB|^2 + |BQ|^2$........(iii)

$(PS)^2 = \overrightarrow{PS} \bullet \overrightarrow{PS} = \left(\overrightarrow{PA} + \overrightarrow{PS}\right) \bullet \left(\overrightarrow{PA} + \overrightarrow{PS}\right) = |PA|^2 + |AS|^2 = |PB|^2 + |BQ|^2$........(iv)

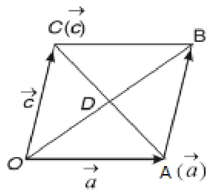
From (iii) and (iv) we get,

$(PQ)^2 = (PQ)^2$ i. e. PQ = PS

⇒ The adjacent sides of PQRS are equal.

∴ PQRS is a rhombus.

Scalar or Dot Product Ex 24.2 Q6



Let OABC be a rhombus, whose diagonals OB and AC intersect at point D.
Let O be the origin.
Let the position vector of A and C be $\vec{a}$ and $\vec{c}$ respectively then,

$\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OC} = \vec{c}$

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{OC} = \vec{a} + \vec{c}$............$\left[\because \overrightarrow{AB} = \overrightarrow{OC}\right]$

Position vector of mid-point of $\overrightarrow{OB} = \frac{1}{2}\left(\vec{a} + \vec{c}\right)$

Position vector of mid-point of $\overrightarrow{AC} = \frac{1}{2}\left(\vec{a} + \vec{c}\right)$

∴ Midpoints of OB and AC coincide.

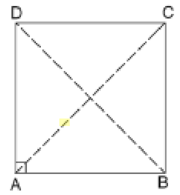
∴ Diagonal OB and AC bisect each other.

$\overrightarrow{OB} \bullet \overrightarrow{AC} = \left(\vec{a} + \vec{c}\right) \bullet \left(\vec{c} - \vec{a}\right) = \left(\vec{c} + \vec{a}\right) \bullet \left(\vec{c} - \vec{a}\right) = \left|\vec{c}\right|^2 - \left|\vec{a}\right|^2 = \overrightarrow{OC} - \overrightarrow{OA} = 0$

$\left[\because \text{OC and OA are sides of the rhombus}\right]$

$\Rightarrow \overrightarrow{OB} \perp \overrightarrow{AC}$

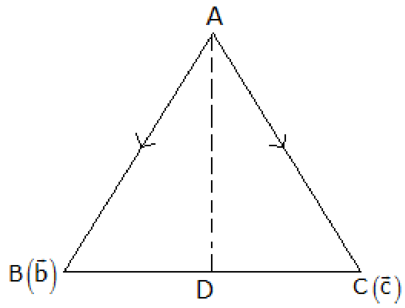
Scalar or Dot Product Ex 24.2 Q7



Let ABCD be a rectangle.
Take A as origin.
Let position vectors of point B, D be $\vec{a}$ and $\vec{b}$ respectively.
By parallelogram law,
$\overrightarrow{AC} = \vec{a} + \vec{b}$ and $\overrightarrow{BD} = \vec{a} - \vec{b}$
As ABCD is a rectangle, AB ⊥ AD
$\Rightarrow \vec{a} \bullet \vec{b} = 0$.........(i)

Now, diagonals AC and BD are perpendicular iff $\overrightarrow{AC} \bullet \overrightarrow{BD} = 0$
$\Rightarrow (\vec{a} + \vec{b})(\vec{a} - \vec{b}) = 0$
$\Rightarrow (\vec{a})^2 - (\vec{b})^2 = 0$
$\Rightarrow |\overrightarrow{AB}|^2 = |\overrightarrow{AD}|^2$
$\Rightarrow |AB| = |AD|$
Hence ABCD is a square.

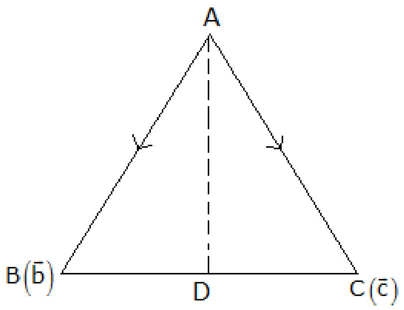
Scalar or Dot Product Ex 24.2 Q8



Take A as origin, let the position vectors of B and C are $\vec{b}$ and $\vec{c}$ respectively.
Position vector of D = $\frac{\vec{b} + \vec{c}}{2}$, $\overrightarrow{AB} = \vec{b}$ and $\overrightarrow{AC} = \vec{c}$.
$\overrightarrow{AD} = \frac{\vec{b} + \vec{c}}{2} - \vec{0} = \frac{\vec{b} + \vec{c}}{2}$

Consider, $2\left(AD^2 + CD^2\right)$
$= 2\left[\left(\frac{\vec{b} + \vec{c}}{2}\right)^2 + \left(\frac{\vec{b} + \vec{c}}{2} - \vec{c}\right)^2\right]$
$= 2\left[\left(\frac{\vec{b} + \vec{c}}{2}\right)^2 + \left(\frac{\vec{b} - \vec{c}}{2}\right)^2\right]$
$= \frac{1}{2}\left[(\vec{b} + \vec{c})^2 + (\vec{b} - \vec{c})^2\right]$
$= (\vec{b})^2 + (\vec{c})^2$
$= (\overrightarrow{AB})^2 + (\overrightarrow{AC})^2$
$= AB^2 + AC^2$
Hence proved.

Scalar or Dot Product Ex 24.2 Q9



Take A as origin, let the position vectors of B and C are $\vec{b}$ and $\vec{c}$ respectively.

Position vector of D = $\frac{\vec{b}+\vec{c}}{2}$, $\overrightarrow{AB} = \vec{b}$ and $\overrightarrow{AC} = \vec{c}$.

$\overrightarrow{AD} = \frac{\vec{b}+\vec{c}}{2} - \vec{0} = \frac{\vec{b}+\vec{c}}{2}$

AD is perpendicular to BC

$\Rightarrow \overrightarrow{AD} \bullet \overrightarrow{BC} = 0$

$\Rightarrow \left(\frac{\vec{b}+\vec{c}}{2}\right) \bullet (\vec{c} - \vec{b}) = 0$

$\Rightarrow (\vec{b}+\vec{c}) \bullet (\vec{c} - \vec{b}) = 0$

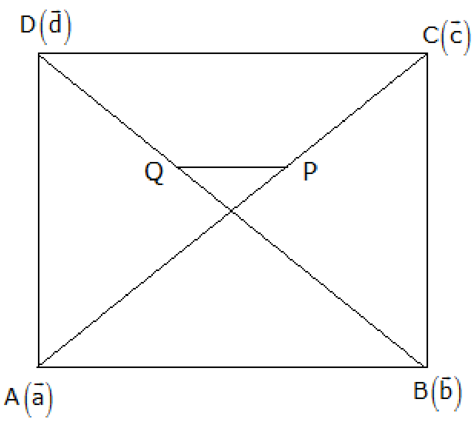
$\Rightarrow |\vec{c}|^2 = |\vec{b}|^2$

$\Rightarrow |\vec{c}| = |\vec{b}|$

$\Rightarrow AC = AB$

Hence ΔABC is an isoscales triangle.

Scalar or Dot Product Ex 24.2 Q10



Take O as origin, let the position vectors of A, B C and D are $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ respectively.

Position vector of P = $\frac{\vec{a}+\vec{c}}{2}$

Position vector of Q = $\frac{\vec{a}+\vec{d}}{2}$

$LHS = AB^2 + BC^2 + CD^2 + DA^2$

$= (\vec{b}-\vec{a})^2 + (\vec{c}-\vec{b})^2 + (\vec{d}-\vec{c})^2 + (\vec{d}-\vec{a})^2$

$= 2\left[(\vec{a})^2 + (\vec{b})^2 + (\vec{c})^2 + (\vec{d})^2 - \vec{a}\vec{b}\cos\theta_1 - \vec{b}\vec{c}\cos\theta_2 - \vec{d}\vec{c}\cos\theta_3 - \vec{c}\vec{a}\cos\theta_4\right]$

$RHS = AC^2 + BD^2 + 4PQ^2$

$= (\vec{c}-\vec{a})^2 + (\vec{d}-\vec{b})^2 + 4\left(\frac{\vec{a}+\vec{d}}{2} - \frac{\vec{a}+\vec{c}}{2}\right)^2$

$= 2\left[(\vec{a})^2 + (\vec{b})^2 + (\vec{c})^2 + (\vec{d})^2 - \vec{a}\vec{b}\cos\theta_1 - \vec{b}\vec{c}\cos\theta_2 - \vec{d}\vec{c}\cos\theta_3 - \vec{c}\vec{a}\cos\theta_4\right]$

$= LHS$

Hence proved.