Class XII Session 2024-25
Subject - Applied Mathematics
Sample Question Paper - 5

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.
3. **Section – A:** It comprises of 20 MCQs of 1 mark each.
4. **Section – B:** It comprises of 5 VSA type questions of 2 marks each.
5. **Section – C:** It comprises of 6 SA type of questions of 3 marks each.
6. **Section – D:** It comprises of 4 LA type of questions of 5 marks each.
7. **Section – E:** It has 3 case studies. Each case study comprises of 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.
8. Internal choice is provided in 2 questions in Section - B, 2 questions in Section – C, 2 questions in Section - D. You have to attempt only one of the alternatives in all such questions.

Section A

1. If A is a non-singular matrix, then **[1]**

a) $|A| \neq |A'|$ b) $|AA'| \neq |A^2|$

c) $|A| + |A'| \neq 0$ d) $|A^{-1}| \neq |A|^{-1}$

2. A machine makes car wheels and in a random sample of 26 wheels, the test statistic is found to be 3.07. As per t-distribution test (of 5% level of significance), what can you say about the quality of wheels produced by the machine? (Use t_{25} (0.05) = 2.06) **[1]**

a) Different quality b) Inferior quality

c) Same quality d) Superior quality

3. A person invested ₹180000 in a mutual fund in year 2016. If the value of mutual fund increased to ₹ 225000 in year 2020, then compound annual growth rate of his investment is [use $(1.25)^{1/4}$ = 1.057) **[1]**

a) 57% b) 10.57%

c) 5.7% d) 57.57%

4. The position of points O(0, 0) and P(2, -2) in the region of graph of in equation 2x - 3y < 5 will be **[1]**

a) O outside and P inside

b) O and P both inside

c) O and P both outside

d) O inside and P outside

5. If the total cost function is given by $C(x) = 10x - 7x^2 + 3x^3$, then the marginal average cost function (MAC) is given by **[1]**

a) $-14 + 18x$

b) $10 - 7x + 3x^2$

c) $-7 + 6x$

d) $10 - 14x + 9x$

6. A fair coin is tossed 100 times. The probability of getting tails an odd number of times is **[1]**

a) $\frac{5}{8}$

b) $\frac{3}{8}$

c) $\frac{1}{2}$

d) $\frac{1}{8}$

7. If X is a binomial variate with parameters n and p, where 0 < p < 1 such that $\frac{P(X=r)}{P(X=n-r)}$ is independent of n and r, then p equals **[1]**

a) $\frac{1}{2}$

b) $\frac{1}{4}$

c) $\frac{1}{3}$

d) $\frac{1}{5}$

8. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents: **[1]**

a) parabolas

b) straight lines

c) ellipses

d) circles

9. In a 400 m race, A gives B a start of 5 seconds and beats him by 15 m. In another race of 400 m, A beats B by $7\frac{1}{7}$ seconds. Their respective speeds are: **[1]**

a) 8 m/sec, 7 m/sec

b) 5 m/sec, 7 m/sec

c) 9 m/sec, 7 m/sec

d) 6 m/sec, 7 m/sec

10. The solution of the differential equation $\frac{dy}{dx} = \frac{2y}{x} = 0$ with y(1) = 1 is given by: **[1]**

a) $x = \frac{1}{y}$

b) $y = \frac{1}{x^2}$

c) $x = \frac{1}{y^2}$

d) $y = \frac{1}{x}$

11. (49 + 57) (mod 50) is **[1]**

a) 6

b) 4

c) 5

d) 7

12. If B > A, then which expression will have the highest values given that A and B are positive integers? **[1]**

a) A + B

b) $A \times B$

c) can't say

d) A - B

13. A pipe can fill a tank in n_1 hours and another pipe can empty it in n_2 ($n_2 > n_1$) hours. If both the pipes are opened together, the tank will be filled in **[1]**

a) $(n_1 - n_2)$ hours

b) $\frac{n_1 n_2}{n_2 - n_1}$ hours

c) $\frac{n_1 n_2}{n_1 + n_1}$ hours

d) $\frac{n_1 n_2}{n_1 - n_2}$ hours

14. Feasible region in the set of points which satisfy **[1]**

a) Non objective functions

b) Some the given constraints

c) The objective functions

d) All of the given constraints

15. Given that x, y and b are real numbers and x < y, b > 0, then **[1]**

a) $\frac{x}{b} \geq \frac{y}{b}$

b) $\frac{x}{b} \leq \frac{y}{b}$

c) $\frac{x}{b} > \frac{y}{b}$

d) $\frac{x}{b} < \frac{y}{b}$

16. For the purpose of t-test of significance, a random sample of size (n) 34 is drawn from a normal population, then the degree of freedom (v) is **[1]**

a) 35

b) $\frac{1}{34}$

c) 33

d) 34

17. $\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$ is equal to **[1]**

a) $-\frac{e^x}{1+x^2} + C$

b) $-\frac{e^x}{\left(1+x^2\right)^2} + C$

c) $\frac{e^x}{1+x^2} + C$

d) $\frac{e^x}{\left(1+x^2\right)^2} + C$

18. Time series analysis helps to **[1]**

a) predict the future behaviour of a variable.

b) understand the behaviour of a variable in the past.

c) all of these.

d) plan future operations.

19. **Assertion (A):** Scalar matrix A = $[a_{ij}] = \begin{cases} k, & i = j \\ 0, & i \neq j \end{cases}$, where k is a scalar, is an identity matrix when k = 1. **[1]**

Reason (R): Every identity matrix is not a scalar matrix.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. Let f(x) = $2x^3 - 15x^2 + 36x + 1$ **[1]**

Assertion (A): f is strictly decreasing in [2, 3]

Reason (R): f is strictly increasing in $(-\infty$, 2] ∪ [3, (0).

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Assuming a four yearly cycle, calculate the trend by the method of moving averages from the following data: **[2]**

Year	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
Value	12	25	39	54	70	87	105	100	82	65

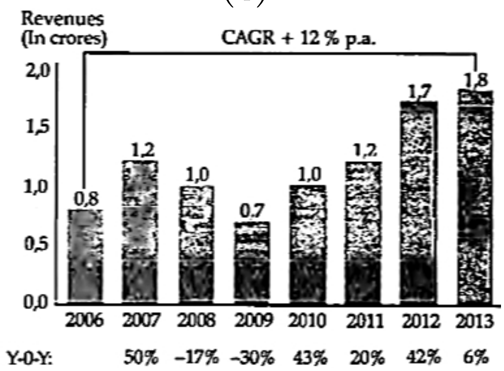
22. Mr. Y has two investment options - either at 10% per annum compounded semi-annually or 9.5 % per annum compounded continuously. Which option is preferable and why? **[2]**

OR

A person has set up a sinking fund so that he can accumulate ₹ 100000 in 10 years for his children's higher education. How much amount should he deposit every six months if interest is 5% per annum compounded semi-annually?

23. By using property of definite integrals, evaluate $\int_0^2 x\sqrt{2-x}dx$ **[2]**

24. An interviewer gives the following graph on a client's sales in the last 7 years to candidate and said find the CAGR. Given that $\left(\frac{9}{4}\right)^{\frac{1}{7}}$ = 1.1228. **[2]**



OR

Find the declared rate of return compounded semiannually which is equivalent to 6% effective rate of return [Use $(1.06)^{\frac{1}{2}}$ = 1.0296]

25. Solve: 12x $\equiv$ 44 (mod 59) **[2]**

Section C

26. Solve the initial value problem: x$\frac{dy}{dx}$ + y = x log x, y(1) = $\frac{1}{4}$ **[3]**

OR

Obtain the differential equation of all circles of radius r.

27. The cost of a washing machine depreciates by ₹720 during the second year and by ₹648 during the third year. Calculate: **[3]**

i. the rate of depreciation per annum.
ii. the original cost of the machine.
iii. the value of the machine at the end of third year.

28. The demand and supply functions for a commodity are p = x^2 - 6x + 16 and p = $\frac{1}{3}x^2 + \frac{4}{3}x + 4$ respectively. Find each of the following assuming x $\leq$ 5: **[3]**

i. The equilibrium point.
ii. The consumer's surplus at the equilibrium point.
iii. The producer's surplus at the equilibrium point.

29. Find the mean, variance and standard deviation of the number of tails in three tosses of a coin. **[3]**

OR

A random variable X has the following probability distribution:

x_i	-2	-1	0	1	2	3
p_i	0.1	k	0.2	2k	0.3	k

i. Find the value of k.
ii. Calculate the mean of X.
iii. Calculate the variance of X.

30. i. Obtain the three year moving averages for the following series of observations: **[3]**

Year	1995	1996	1997	1998	1999	2000	2001	2002

Annual Sales (in 000 ₹)	3.6	4.3	4.3	3.4	4.4	5.4	3.4	2.4

ii. Obtain the five year moving average.

iii. Construct also the 4-year centred moving average.

31. A random sample of 10 boys had the following I.Q's: 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q. of 100? Find a reasonable range in which most of the mean I.Q. values of samples of 10 boys lie. (Given to (0.05) = 2.262) **[3]**

Section D

32. A firm produces three products P_1, P_2 and P_3 requiring the mix-up of three materials M_1, M_2 and M_3. The per-unit requirement of each product for each material is as follows: **[5]**

	M_1	M_2	M_3
P_1	2	4	5
P_2	3	2	4
P_3	1	3	2

Using matrix algebra, find:

i. The total requirement of each material if the firm produces 100, 200 and 300 units of products P_1, P_2 and P_3 respectively.

ii. The per-unit cost of production of each product if the per-unit costs of materials M_1, M_2 and M_3 are ₹10, ₹15 and ₹12 respectively.

iii. The total cost of production.

OR

If A = $\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1} Using A^{-1} solve the system of linear equations

x - 2y = 10, 2x - y - z = 8, -2y + z = 7.

33. A person can row a boat at 5 km/h in still water. It takes him thrice as long to row upstream as to row downstream. Find the rate at which the stream is flowing. **[5]**

34. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Compute the variance of the number of aces. **[5]**

OR

Two positive numbers are selected at random (without replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean and the variance of the distribution.

35. The cost of a car purchased 2 years ago, depreciates at the rate of 20 % every year. If its present worth is ₹ 315600, find: **[5]**

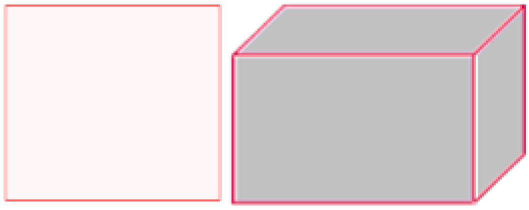
i. its purchase price

ii. its value after 3 years.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Yash wants to prepare a handmade gift box for his friend's birthday at his home. For making lower part of the box, he took a square piece of paper of each side equal to 10 cm.



(a) If x cm be the size of the square piece cut from each corner of the paper of size 10cm, then a possible value of x will be given by interval:

(b) Volume of the open box formed by folding up the cutting corner can be expressed as:

(c) Find the value of x for which $\frac{dV}{dx}$ = 0?

OR

Yash is interested to maximise the volume of the box, So what will be the side of the square to be cut to maximise the volume?

37. **Read the text carefully and answer the questions:** **[4]**

Loans are an integral part of our lives today. We take loans for a specific purpose - for buying a home, or a car, or sending kids abroad for education - loans help us achieve some significant life goals. That said, when we talk about loans, the word “EMI", eventually crops up because the amount we borrow has to be returned to the lender with interest.

Suppose a person borrows ₹1 lakh for one year at the fixed rate of 9.5 percent per annum with a monthly rest. In this case, the EMI for the borrower for 12 months works out to approximately ₹8,768.

Example:

In year 2000, Mr. Tanwar took a home loan of ₹3000000 from State Bank of India at 7.5% p.a. compounded monthly for 20 years.

(a) Find the equated monthly installment paid by Mr. Tanwar.

(b) Find interest paid by Mr. Tanwar in 150th payment.

(c) Find Principal paid by Mr. Tanwar in 150th payment.

OR

Find principal outstanding at the beginning of 193th month.

38. **Read the following text carefully and answer the questions that follow:** **[4]**

The feasible region for an L.P.P. is shown in the adjoining figure. The line CB is parallel to OA.

i. Find the equation of the line OA. (1)
ii. Find the equation of the line BC. (1)
iii. Find the constraints for the L.P.P (2)

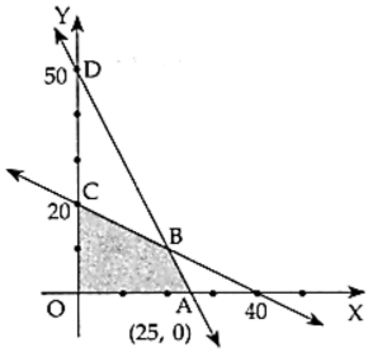
OR

Find the minimum value of the objective function Z = 3x - 4y. (2)

OR

Read the following text carefully and answer the questions that follow:

The feasible region for an L.P.P is shown in the figure given below:



i. What is equation of the line AD? (1)
ii. What is the equation of the line BC? (1)
iii. What are the coordinates of the points B and C? (2)

OR

What are the constraints for the L.P.P.? (2)

Solution

Section A

1.

(c) $|\mathrm{A}| + |\mathrm{A}'| \neq 0$

Explanation: $\because$ | A | $\neq$ 0 and | A | = |A'|.

So, |A| + |A'| = |A| + |A| = 2|A| $\neq$ 0.

$\therefore$ Option (d) is the correct answer.

2.

(b) Inferior quality

Explanation: Given n = 26 and t = 3.0

$\Rightarrow$ degree of freedom = n - 1 = 26 - 1 = 25

Now, level of significance = 5% = 0.05

$t_{25}(0.05) = 2.06$

$\because$ t = 3.07 > 2.06

So, the wheels produced by the machine are of inferior quality.

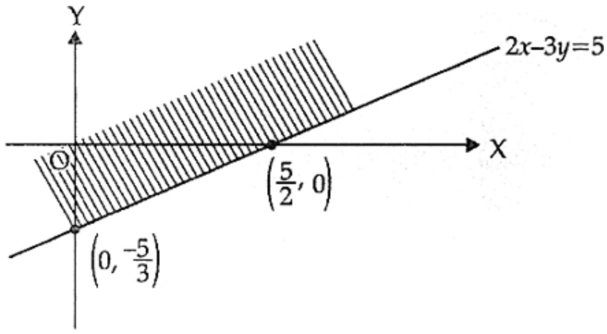
3.

(c) 5.7%

Explanation: 5.7%

4.

(d) O inside and P outside

Explanation: 

5.

(c) -7 + 6x

Explanation: $C(x) = 10x - 7x^2 + 3x^3$

AC = $\frac{C(x)}{x}$ = $10 - 7x + 3x^2$

Now, MAC = $\frac{d}{dx}$(AC) = -7 + 6x

6.

(c) $\frac{1}{2}$

Explanation: A fair coin tossed 100 times then probability of odd or even numbers are same and equals $= \frac{1}{2}$

$\Rightarrow$ The probability of getting tails an odd number of times is also $\frac{1}{2}$

7. **(a)** $\frac{1}{2}$

Explanation: $\because$ $P(X = r) = {}^nC_r(P)^r(q)^{n-r}$

$= \frac{n!}{(n-r)!r!}(P)^r(1 - p)^{n-r}$ [$\because$ q = 1 - p] ...(i)

$P(X = 0) = (1 - p)^n$

And $P(X = n - r) = {}^nC_{n-r}(P)^{n-r}(q)^{n-(n-r)}$

$= \frac{n!}{(n-r)!r!}$ $(p)^{n-r}(1 - p)^{-r}$ [$\because$ q = 1 - p] [$\because$ ${}^nC_r = {}^nC_{n-r}$] ...(ii)

Now, $\frac{P(x=r)}{P(x=n-r)} = \frac{\frac{n!}{(n-r)!r!}p^r(1-p)^{n-r}}{\frac{n!}{(n-r)!r!}p^{n-r}(1-p)^{+r}}$ [using Eqs. (i) and (ii)]

$= \left(\frac{1-p}{p}\right)^{n-\gamma} \times \frac{1}{\left(\frac{1-p}{p}\right)^r}$

The above expression is independent of n and r, if $\frac{1-p}{p} = 1 \Rightarrow \frac{1}{p} = 2 \Rightarrow p = \frac{1}{2}$

8. **(a)** parabolas

Explanation: 2x$\frac{dy}{dx}$ = y + 3 $\Rightarrow \frac{dy}{dx} = \frac{y+3}{2x} \Rightarrow \frac{2\frac{dy}{dx}}{y+3} = \frac{1}{x}$

Integration both sides

$\int \frac{2\frac{dy}{dx}}{y+3} = \int \frac{1}{x}$

$\Rightarrow$ 2 log (y + 3) = log x + c

$\Rightarrow$ $(y + 3)^2$ = x + c

9. **(a)** 8 m/sec, 7 m/sec

Explanation: Suppose A covers 400 m in t seconds

Then, B covers 385 m in (t + 5) seconds

$\therefore$ B covers 400 m = $\left\{\frac{(t+5)}{385} \times 400\right\}$sec

= $\frac{80(t+5)}{77}$ sec

Also, B covers 400 m = $\left(t + 7\frac{1}{7}\right)$sec

= $\frac{(7t+50)}{7}$sec

$\therefore \frac{80(t+5)}{77} = \frac{7t+50}{7}$

$\therefore$ 80(t + 5) = 11(7t + 50)

$\Rightarrow$ (80t - 77t) = (550 - 400)

$\Rightarrow$ 3t = 150

$\Rightarrow$ t = 50

$\therefore$ A's speed

= $\frac{400}{50}$ m/sec

= 8 m/sec

$\therefore$ B's speed

= $\frac{385}{55}$ m/sec

= 7 m/sec

10.

(b) y = $\frac{1}{x^2}$

Explanation: We have,

$\frac{dy}{dx} + \frac{2y}{x}$ = 0

$\Rightarrow \frac{dy}{dx} = -\frac{2y}{x}$

q

$\Rightarrow \frac{dy}{2y} = -\frac{dx}{x}$

$\Rightarrow \int \frac{dy}{2y} = -\int \frac{dx}{x}$

$\Rightarrow \frac{1}{2}$log |y| = -log |x| + log c

$\Rightarrow \sqrt{y}x$ = c

$\Rightarrow$ yx^2 = c

Given that y(1) = 1 $\Rightarrow$ x = y = 1

$\Rightarrow$ c = 1

$\Rightarrow$ yx^2 = 1

$\Rightarrow$ y = $\frac{1}{x^2}$

11. **(a)** 6

Explanation: 6

12.

(c) can't say

Explanation: Cannot say, because when A = 1, B = 3, then

A $\times$ B = 1 $\times$ 3 = 3 and A + B = 4

Here, A + B > A $\times$ B

and when A = 2, B = 3, then

A $\times$ B = 2 $\times$ 3 = 6 and A + B = 2 + 3 = 5.

Here, A $\times$ B > A + B

13.

(b) $\frac{n_1 n_2}{n_2 - n_1}$ hours

Explanation: $\frac{n_1 n_2}{n_2 - n_1}$ hours

14.

(d) All of the given constraints

Explanation: All of the given constraints

15.

(c) $\frac{x}{b} > \frac{y}{b}$

Explanation: x < y and b < 0

$\Rightarrow \frac{x}{b} > \frac{y}{b}$

16.

(c) 33

Explanation: Given n = 34

$\Rightarrow$ degree of freedom (v) = 34 - 1 = 33

17.

(c) $\frac{e^x}{1+x^2} + C$

Explanation: Given $\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$

$\Rightarrow \int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx = \int e^x \left(\frac{1+x^2-2x}{\left(1+x^2\right)^2}\right) dx$

$\Rightarrow \int e^x \left(\frac{1+x^2-2x}{\left(1+x^2\right)^2}\right) dx = \int e^x \left\{\left(\frac{1+x^2}{\left(1+x^2\right)^2}\right) + \left(\frac{-2x}{\left(1+x^2\right)^2}\right)\right\} dx$

$= \int e^x \left\{\left(\frac{1}{\left(1+x^2\right)}\right) + \left(\frac{-2x}{\left(1+x^2\right)^2}\right)\right\} dx$

Now using the property: $\int \mathrm{e^x} \left(\mathrm{f(x)} + \mathrm{f'(x)}\right) \mathrm{dx} = \mathrm{e^x f(x)}$

Now in $\int e^x \left\{\left(\frac{1}{\left(1+x^2\right)}\right) + \left(\frac{-2x}{\left(1+x^2\right)^2}\right)\right\} dx$

$\Rightarrow$ f(x) = $\frac{1}{\left(1+x^2\right)}$

$\Rightarrow$ f'(x) = $\frac{-2x}{\left(1+x^2\right)^2}$

$\Rightarrow \int \mathrm{e^x} \left\{\left(\frac{1}{\left(1+\mathrm{x}^2\right)}\right) + \left(\frac{-2\mathrm{x}}{\left(1+\mathrm{x}^2\right)^2}\right)\right\} \mathrm{dx} = \frac{\mathrm{e^x}}{1+\mathrm{x}^2} + \mathrm{C}$

$\Rightarrow \int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx = \frac{e^x}{1+x^2} + C$.

18.

(c) all of these.

Explanation: all of these.

19.

(c) A is true but R is false.

Explanation: A scalar matrix A = $[a_{ij}] = \begin{cases} k; & i = j \\ 0; & i \neq j \end{cases}$ is an identity matrix when k = 1.

But every identity matrix is clearly a scalar matrix.

20.

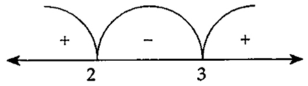
(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Given $f(x) = 2x^3 - 15x^2 + 36x + 1$

$\Rightarrow f'(x) = 6x^2 - 30x + 36$

$= 6(x^2 - 5x + 6) = 6(x - 2)(x - 3)$

$\therefore$ f is strictly decreasing in [2, 3].



f is strictly increasing in $(-\infty, 2] \cup [3, \infty)$.

Assertion and Reason both are true.

Reason is not the correct explanation of Assertion.

Section B

21. **Calculation of 4-year centred moving average:**

Year	Value	4-yearly moving total	4-yearly moving average	4-yearly centre moving average
1984	12			
1985	25	130	32.5	
1986	39			39.75
		188	47	
1987	54			54.75
		250	62.5	
1988	70			70.75
		316	79	
1989	87			84.75
		362	90.5	
1990	105			92
		374	93.5	
1991	100			90.75
		352	88	
1992	82			
1993	65			

22. When compounded semi-annually we have r = 0.10, m = 2

Now, $r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$

$= \left(1 + \frac{0.10}{2}\right)^2 - 1$

= 0.1025 or 10.25 %

when compounded continuously

$r_{eff} = e^r - 1 = e^{0.095} - 1$

= 0.0996 = 9.96%

Thus, the first investment is preferable.

OR

Given A = ₹ 100000, r = $\frac{5}{2}$% per half year $\Rightarrow$ i = $\frac{2.5}{100}$ = 0.025 and n = 20 half year

Using formula

A = R $\left[\frac{(1+i)^n - 1}{i}\right] \Rightarrow$ 100,000 = R $\left[\frac{(1.025)^{20} - 1}{0.025}\right]$

$\Rightarrow$ R = $\frac{100000 \times 0.025}{(1.025)^{20} - 1}$

$\Rightarrow$ R = $\frac{2500}{1.637 - 1} = \frac{2500}{0.637}$

$\Rightarrow$ R = ₹ 3924.64

Let $x = (1.025)^{20}$

Taking logarithm on both sides, we get

log x = 20 log 1.025

$\Rightarrow$ log x = 20 $\times$ 0.0107

$\Rightarrow$ log x = 0.2140

$\Rightarrow$ x = antilog 0.2140

$\Rightarrow$ x = 1.637

23. $\int_0^2 x\sqrt{2-x}dx = \int_0^2 (2-x)(2-(2-x))^{\frac{1}{2}}dx$ (by property P_4)

$= \int_0^2 (2-x)x^{\frac{1}{2}}dx = \int_0^2 \left(2x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)dx$

$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^2 = \frac{4}{3}\cdot 2\sqrt{2} - \frac{2}{5}\cdot 4\sqrt{2} = \frac{16}{15}\sqrt{2}.$

24. Sales in 2006 were 0.8 crores (beginning value). In 2013, after 7 years, sales increased to 1.8 crores.

CAGR $= \left(\frac{\text{End value}}{\text{Beginning value}}\right)^{\frac{1}{n}} - 1$

$= \left(\frac{1.8}{0.8}\right)^{\frac{1}{7}} - 1 = \left(\frac{9}{4}\right)^{\frac{1}{7}}$ -1

= 0.1228

CAGR% = 12.28%

OR

Let declared rate of interest be r % p.a. compounded half yearly.

Given effective rate of return (per rupee) = $\frac{6}{100}$ = 0.06 (per-rupee), p = 2 half years.

$\therefore$ 0.06 = $\left(1 + \frac{r}{200}\right)^2$ - 1

$\Rightarrow \left(1 + \frac{r}{200}\right)^2$ = 1.06 $\Rightarrow$ 1 + $\frac{r}{200}$ = $(1.06)^{\frac{1}{2}}$

$\Rightarrow$ 1 + $\frac{r}{200}$ = 1.0296 $\Rightarrow$ $\frac{r}{200}$ = 0.0296

$\Rightarrow$ r = 0.0296 $\times$ 200 $\Rightarrow$ r = 5.92

Hence, the declared rate of return = 5.92%

25. We know that a $\equiv$ b (mod m) $\Rightarrow \frac{a}{x} \equiv \frac{b}{x}\ (mod \frac{m}{d})$, where d = (x, m).

$\therefore$ 12x $\equiv$ 44 (mod 59)

$\Rightarrow$ 3x $\equiv$ 11 (mod 59) [$\because$ (4, 59) = 1]

We find that (3, 59) = 1, hence it has unique solution (mod 59). Using division algorithm, we obtain

59 = 19 $\times$ 3 + 2

3 = 2 $\times$ 1 + 1

Using back substitution, we obtain

1 = 3 - 2 $\times$ 1

$\Rightarrow$ 1 = 3 - (59 - 19 $\times$ 3) $\times$ 1

$\Rightarrow$ 1 = 59 $\times$ (-1) + 20 $\times$ 3

The coefficient of 3 i.e. 20 is the inverse of 3 (mod 59)

Now,

3x $\equiv$ 11 (mod 59)

$\Rightarrow$ 20 $\times$ 3x $\equiv$ 20 $\times$ 11 (mod 59) [Multiplying throughout by inverse of 3 i.e. 20]

$\Rightarrow$ (20 $\times$ 3)x $\equiv$ 220 (mod 59)

$\Rightarrow$ x $\equiv$ 43 (mod 59)

Hence x $\equiv$ 43 (mod 59) is the solution of the given linear congruence.

Section C

26. We have, x$\frac{dy}{dx}$ + y = x log x

$\Rightarrow \frac{dy}{dx} + \frac{1}{x}y$ = log x ...(i)

This is linear differential equation of the form $\frac{dy}{dx}$ + Py = Q with P = $\frac{1}{x}$ and Q = log x

$\therefore$ I.F. = $e^{\int \frac{1}{x}dx}$ = $e^{\log x}$ = x [$\because$ x > 0]

Multiplying both sides of (i) by I.F. = x, we get

x$\frac{dy}{dx}$ + y = x log x

Integrating with respect to x, we ge

yx = $\int \underset{II}{x} \underset{I}{\log x}\, dx$ [Using: y (I.F.) = $\int$ Q (I.F.) dx + C]

$\Rightarrow$ yx = $\frac{x^2}{2}$(log x)$\frac{1}{2}\int$ x dx

$\Rightarrow$ yx = $\frac{x^2}{2}$(log x) - $\frac{x^2}{4}$ + C ...(ii)

It is given that y (1) = $\frac{1}{4}$ i.e. y = $\frac{1}{4}$ where x = 1. Putting x = 1 and y = $\frac{1}{4}$ in (ii), we get

$\frac{1}{4}$ = 0 - $\frac{1}{4}$ + C ⇒ C = $\frac{1}{2}$

Putting C = $\frac{1}{2}$ in (ii), we get

xy = $\frac{x^2}{2}$(log x) - $\frac{x^2}{2}$ + $\frac{1}{2}$ ⇒ y = $\frac{1}{2}$x log x - $\frac{x}{4} + \frac{1}{2x}$

Hence, y = $\frac{1}{2}$ x log x - $\frac{x}{4} + \frac{1}{2x}$ is the solution of the given differential equation.

OR

The equation of the family of circles of radius r is

(x - a)^2 + (y - b)^2 = 2 ...(i)

where a and bare a parameters.

Clearly equation (i) contains two arbitrary constants. So, let us differentiate it two times with respect to x.

Differentiating (i) with respect to x, we get

2 (x - a) + 2 (y - b)$\frac{dy}{dx}$ = 0

⇒ (x - a) + (y - b)$\frac{dy}{dx}$ = 0 ...(ii)

Differentiating (ii) with respect to x, we get

1 + (y - b) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2$ = 0 ...(iii)

⇒ y - b = $-\frac{1+(\frac{dy}{dx})^2}{\frac{d^2y}{dx^2}}$ ...(iv)

Putting this value of (y - b) in (ii), we obtain

x - a = $\frac{\left|1+\left(\frac{dy}{dx}\right)^2\right\}\frac{dy}{dx}}{\frac{d^2y}{dx^2}}$ ...(v)

Substituting the values of (x - a) and (y - b) in (i), we get

$\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right|^2\left(\frac{dy}{dx}\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2} + \frac{\left|1+\left(\frac{dy}{dx}\right)^2\right\}^2}{\left(\frac{d^2y}{dx^2}\right)^2}$ = r^2 ⇒ $\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^3 = r^2\left(\frac{d^2y}{dx^2}\right)^2$

This is the required differential equation.

27. i. Let the original cost of the washing machine be ₹P and the rate of depreciation be r % p.a. Then the value of machine (in ₹) after one year, two years and 3 years are P(1 - i), P(1 - i)^2 and P(1 - i)^3 respectively, where i = $\frac{r}{100}$.

According to given,

P(1 - i) - P(1 - i)^2 = 720 and P(1 - i)^2 - P(1 - i)^3 = 648

⇒ P(1 - i)[1 - (1 - i)] = 720 and P(1 - i)^2[1 - (1 - i)] = 648 ...(ii)

⇒ P(1 - i) i = 720 ...(i) and P(1 - i)^2·i = 648

Dividing (ii) by (i), we get

1 - i = $\frac{648}{720}$ ⇒ 1 - i = $\frac{9}{10}$

⇒ i = 1 - $\frac{9}{10}$ ⇒ i = $\frac{1}{10}$ ⇒ $\frac{r}{100} = \frac{1}{10}$

⇒ r = 10

Hence, the rate of depreciation = 10 % p.a.

ii. Putting i = $\frac{1}{10}$ in equation (i), we get

P$\left(1-\frac{1}{10}\right) \times \frac{1}{10}$ = 720 ⇒ P × $\frac{9}{100}$ = 720 ⇒ P = 8000

Hence, the original cost of the machine = ₹ 8000

iii. The value of machine at the end of third year = P(1 - i)^3

= 8000$\left(1-\frac{1}{10}\right)^3$ = 8000(0.9)^3

= 8000 × 0.729 = 5832

Hence, the value of the machine at the end of the third year = ₹5832

28. The demand and supply functions are p = D(x) and p = S(x), where D(x) = x^2 - 6x + 16 and S(x) = $\frac{1}{3}x^2 + \frac{4}{3}x + 4$

i. The equilibrium point (x_0, p_0) is the point at which the demand-supply curves intersect. Therefore, the equilibrium point is obtained by setting D(x) = S(x).

Now, D(x) = S(x)

$\Rightarrow$ x^2 - 6x + 16 = $\frac{1}{3}x^2 + \frac{4}{3}x + 4$

$\Rightarrow$ $\frac{2}{3}x^2 - \frac{22}{3}x + 12 = 0$ $\Rightarrow$ x^2 - 11x + 18 = 0 $\Rightarrow$ (x - 2)(x - 9) $\Rightarrow$ x = 2 [$\because$ x $\leq$ 5]

Putting x = 2 either in p = D (x) or in p = S(x), we obtain p = 8. Thus, x_0 = 2 and p_0 = 8. Hence, (2, 8) is the equilibrium point.

ii. The consumer's surplus (CS) at the equilibrium point (2, 8) is given by

CS = $\int_0^{x_0} D(x)dx - p_0x_0$

$\Rightarrow$ CS = $\int_0^2 \left(x^2 - 6x + 16\right) dx - 8 \times 2$

$\Rightarrow$ CS = $\left[\frac{x^3}{3} - 3x^2 + 16x\right]_0^2 - 16$ = $\left(\frac{8}{3} - 12 + 32\right) - 16$ = $\frac{20}{3}$

iii. The producer's surplus (PS) at the equilibrium point (2, 8) is given by

PS = $p_0x_0 - \int_0^{x_0} S(x)dx$

$\Rightarrow$ PS = $8 \times 2 - \int_0^2 \left(\frac{1}{3}x^2 + \frac{4}{3}x + 4\right) dx$

$\Rightarrow$ PS = $16 - \left[\frac{x^3}{9} + \frac{2}{3}x^2 + 4x\right]_0^2$ = $16 - \left(\frac{8}{9} + \frac{8}{3} + 8\right)$ = $\frac{40}{9}$

29. Let X be a random variable denoting the number of tails in three tosses of a coin. Then, X can take the values 0, 1, 2 and 3

Now, we have,

P(X = 0) = P(HHH) = $\frac{1}{8}$

P(X = 1) = P(THH or HHT or HTH) = $\frac{3}{8}$

P(X = 2) = P(TTH or THT or HTT) = $\frac{3}{8}$

P(X = 3) = P(TTT) = $\frac{1}{8}$

Thus, the probability distribution of X is as follows:

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Computation of mean and variance:

x_i	p_i	p_ix_i	$p_ix_i^2$
0	$\frac{1}{8}$	0	0
1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$
		$\sum p_ix_i = \frac{3}{2}$	$\sum p_ix_i^2 = 3$

Therefore, mean = $\sum p_ix_i = \frac{3}{2}$

Variance = $\sum p_ix_i^2$ - $(\text{Mean})^2$

= 3 - $\left(\frac{3}{2}\right)^2$

= 3 - $\frac{9}{4}$

= $\frac{3}{4}$

Standard deviation = $\sqrt{\text{Variance}}$

= $\sqrt{\frac{3}{4}}$

= 0.87

OR

We know that sum of the probabilities in a probability distribution is always unity.

$\therefore$ 0.1 + k + 0.2 + 2k + 0.3 + k = 1

$\Rightarrow$ 0.6 + 4k = 1 $\Rightarrow$4k = 0.4 $\Rightarrow$ k = 0.1

Calculation of mean and variance:

x_i	p_i	P_i x_i	$p_ix_i^2$
- 2	0.1	- 0.2	0.4
- 1	0.1	- 0.1	0.1

0	0.2	0	0
1	0.2	0.2	0.2
2	0.3	0.6	1.2
3	0.1	0.3	0.9
		$\Sigma p_i x_i$ = 0.8	$\Sigma p_i x_i^2$ = 2.8

Therefore, $\Sigma p_i x_i$ = 0.8 and $\Sigma p_i x_i^2$ = 2.8

$\therefore$ Mean = $\Sigma p_i x_i$ = 0.8

and Variance = $\Sigma p_i x_i^2$ - $(\Sigma p_i x_i)^2$ = 2.8 - $(0.8)^2$ = 2.8 - 0.64 = 2.16

30. i. First 3-year moving average is $\frac{3.6+4.3+4.3}{3} = \frac{12.2}{3}$ = 4.067, and is placed against 2nd year i.e. 1996; second 3-year moving average is $\frac{4.3+4.3+3.4}{3} = \frac{12.0}{3}$ = 4.0, and is placed against 3rd year i.e. 1997, and so on. Thus, we have:

Calculation of 3-year moving averages:

Year	Annual sale	3-year moving total		3-year moving average
1995	3.6	–	1/3	–
1996	4.3	12.2	→	4.067
1997	4.3	12.0	→	4.00
1998	3.4	12.1	→	4.03
1999	4.4	13.2	→	4.40
2000	5.4	13.2	→	4.40
2001	3.4	11.2	→	3.73
2002	2.4	–		–

ii. First 5-yearly moving average is $\frac{3.6+4.3+4.3+3.4+4.4}{5} = \frac{20.0}{5}$ = 4.00, and is placed against 3rd year i.e. 1997. Second 5-yearly moving average is $\frac{4.3+4.3+3.4+4.4+5.4}{5} = \frac{21.8}{5}$ = 4.36, and is placed against 4th year i.e. 1998, and so on. Thus, we have:

Calculation of 5-year moving averages:

Year	Annual sale	5-year moving total		5-year moving average
1995	3.6	–		–
1996	4.3	–	1/5	–
1997	4.3	20.0	→	4.00
1998	3.4	21.8		4.36
1999	4.4	20.9		4.18
2000	5.4	19.0		3.80
2001	3.4	–		–
2002	2.4	–		–

iii. In the 4-year moving averages, the first step of averaging of 4 values each results in placing these in between years — so we take averages of each two successive moving averages to synchronise them with given time frame. Thus, we have the following table:

Construction of 4-year centred moving averages

Year	Annual Sale total	4-yearly moving total	4-yearly moving average	4-year centred moving average
1995	3.6			
1996	4.3			
		15.6	3.9	
1997	4.3			4.0
		16.4	4.1	
1998	3.4			4.2375
		17.5	4.375	
1999	4.4			4.2652
		16.6	4.15	
2000	5.4			4.025
		15.6	3.9	
2001	3.4			
2002	2.4			

Note that values of 4th column are not synchronised with first column, but values of 5th column are synchronised.

31. We have,

μ = Population mean = 100, n = Sample size = 10

We define

Null Hypothesis H_0: The data are consistent with the assumption of a mean I.Q. of 100 in the population.

Alternate hypothesis H_1: The mean I.Q. of population $\neq$ 100

Let the sample statistic t be given by

t = $\frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}}$, where $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{X}\right)^2$

Let us now compute $\bar{X}$ and S^2.

Computation of $\bar{X}$ and S

x_i	$d_1 = x_i - 90$	d_i^2
70	-20	400
120	30	900
110	20	400
101	11	121
88	-2	4
83	-7	49
95	5	25
98	8	64
107	17	289
100	10	100
	$\sum d_i = 72$	$\sum d_i^2 = 2352$

Here, $d_i = x_i - 90$

$\therefore \bar{X} = 90 + \frac{1}{10}\sum d_i = 90 + \frac{72}{10} = 972$ [Using : $\bar{X} = A + \frac{1}{n}\sum d_i$]

$S^2 = \frac{1}{n-1}\left\{\sum d_i^2 - \frac{1}{n}(\sum d_i)^2\right\} = \frac{1}{9}\left\{2352 - \frac{(72)^2}{10}\right\} = \frac{1833.6}{9} = 203.73$

$\therefore t = \frac{\bar{X}-\mu}{S/\sqrt{n}} \Rightarrow t = \frac{97.2-100}{\sqrt{\frac{203.73}{10}}} = \frac{-2.8}{\sqrt{20.37}} = \frac{-2.8}{4514} = -0.62$

$\Rightarrow |t| = 0.62$

The sample statistict follows student's t -distribution with v = (10 - 1) = 9 degrees of freedom. It is given that $t_9(0.05) = 2.262$

$\therefore$ Calculated $|t| <$ tabulated $t_9(0.05)$

So, the null hypothesis may be accepted at 5% level of significance.

Hence, the assumption of a population mean I.Q. of 100 is valid.

The 95% confidence limits within which the mean I.Q. values of samples of 10 boys will lie are

$\bar{X} - \frac{S}{\sqrt{n}} t_9(0.05)$ and $\bar{X} + \frac{S}{\sqrt{n}} t_9(0.05)$

or $97.2 - \sqrt{\frac{203.73}{10}} \times 2.262$ and $97.2 + \sqrt{\frac{203.73}{10}} \times 2.262$

or, $97.2 - 4514 \times 2.262$ and $97.2 + 4.514 \times 2.262$

or, 97.2 - 10.21 and 97.2 + 10.21

or, 86.99 and 107.41

Hence, the required 95% confidence interval is [86.99, 107.41]

Section D

32. The matrix showing per unit requirement of materials M_1, M_2 and M_3 in producing three products P_1, P_2 and P_3 is

$$A = \begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \end{array} \begin{array}{c} \begin{array}{ccc} P_2 & P_3 & P_3 \end{array} \\ \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 3 \\ 5 & 4 & 2 \end{bmatrix} \end{array}$$

i. (i) The matrix representing the requirements of products P_1, P_2 and P_3 is

$$B = \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$$

So, the requirements of each material for producing the given quantities of three products is given by the product

$$AB = \begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \end{array} \begin{array}{c} \begin{array}{ccc} P_2 & P_3 & P_3 \end{array} \\ \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 3 \\ 5 & 4 & 2 \end{bmatrix} \end{array} \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$$

$$\Rightarrow AB = \begin{array}{c} M_1 \\ M_2 \\ M_3 \end{array}\begin{bmatrix} 200+600+300 \\ 400+400+900 \\ 500+800+600 \end{bmatrix} = \begin{array}{c} M_1 \\ M_2 \\ M_3 \end{array}\begin{bmatrix} 1100 \\ 1700 \\ 1900 \end{bmatrix}$$

Thus, 1100 units of material M_1, 1700 emits of material M_2 and 1900 units of material M_3 are required to produce 100 units of P_1, 200 units of P_2 and 300 units of P_3.

ii. The matrix representing per-unit costs of materials M_1, M_2 and M_3 is as given below:

$$C = \begin{array}{c} M_1 \\ M_2 \\ M_3 \end{array}\begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix}$$

The matrix exhibiting the materials M_1, M_2 and M_3 in three products P_1, P_2 and P_3 is

$$D = \begin{array}{cc} & \begin{array}{ccc} M_1 & M_2 & M_3 \end{array} \\ \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} & \begin{bmatrix} 2 & 4 & 5 \\ 3 & 2 & 4 \\ 1 & 3 & 2 \end{bmatrix} \end{array}$$

So, the per emit cost of each product is given by the matrix product

$$DC = \begin{array}{cc} & \begin{array}{ccc} M_1 & M_2 & M_3 \end{array} \\ \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} & \begin{bmatrix} 2 & 4 & 5 \\ 3 & 2 & 4 \\ 1 & 3 & 2 \end{bmatrix} \end{array} \begin{array}{c} M_1 \\ M_2 \\ M_3 \end{array}\begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix} = \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array}\begin{bmatrix} 20+60+60 \\ 30+30+48 \\ 10+45+24 \end{bmatrix} = \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array}\begin{bmatrix} 140 \\ 108 \\ 79 \end{bmatrix}$$

Hence, per unit cost of production of products P_1, P_2 and P_3 are ₹ 140, ₹ 180 and ₹ 79 respectively.

iii. The total cost of product is given by the matrix product

$$\begin{array}{c} \begin{array}{ccc} P_1 & P_2 & P_3 \end{array} \\ \begin{bmatrix} 100 & 200 & 300 \end{bmatrix} \end{array} \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array}\begin{bmatrix} 140 \\ 108 \\ 79 \end{bmatrix} = (14,000 + 21,600 + 23,700) = ₹\ 59,300$$

Hence, the total cost of product is ₹ 59,300

OR

Here, $|A| = \begin{vmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{vmatrix} = 1(-1 - 2) - 2(-2 - 0)$

$= -3 + 4 = 1 \neq 0$

$\Rightarrow A^{-1}$ exists and $A^{-1} = \frac{1}{|A|}$ adj A

Here, $A_{11} = \begin{vmatrix} -1 & -2 \\ -1 & 1 \end{vmatrix} = -3$, $A_{12} = -\begin{vmatrix} -2 & -2 \\ 0 & 1 \end{vmatrix} = 2$, $A_{13} = \begin{vmatrix} -2 & -1 \\ 0 & -1 \end{vmatrix} = 2$;

$A_{21} = -\begin{vmatrix} 2 & 0 \\ -1 & 1 \end{vmatrix} = -2$, $A_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$, $A_{23} = -\begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} = 1$;

$A_{31} = \begin{vmatrix} 2 & 0 \\ -1 & -2 \end{vmatrix} = -4$, $A_{32} = -\begin{vmatrix} 1 & 0 \\ -2 & -2 \end{vmatrix} = 2$, $A_{33} = \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} = 3$

$$\therefore \text{adj } A = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}^t = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{1}\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

The given system of equations can be written as

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

or A'X = B, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

Now $|A'| = |A| = 1 \neq 0 \Rightarrow (A')^{-1}$ exists

$\Rightarrow$ the given system has a unique solution $X = (A')^{-1} B$

or X = $(A^{-1})'$ B ($\because$ (A')-1 = $(A^{-1})'$)

$\Rightarrow$ X = $\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}' \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ = $\begin{bmatrix} -30+16+14 \\ -20+8+7 \\ -40+16+21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$ $\Rightarrow$ x = 0, y = -5, z = -3

Hence, the solution of the given system of equations is x = 0, y = -5, z = -3.

33. Let the distance covered be d km. and y be speed of stream

speed of boat = 5 km/h

speed of stream = y km/h

speed of boat in upstram(u): x - y km/h

= 5 - y km/h

speed of boat in downstream (v) = x + y km/h

= 5 + y km/h

ATQ.

$\frac{d}{5-y} = 3\left(\frac{d}{5+y}\right)$ $\left[\because T = \frac{D}{S}\right]$

$\frac{1}{5-y} = \frac{3}{5+y}$

5 + y = 3(5 - y)

5 + y = 15 - 3y

y + 3y = 15 - 5

4y = 10

y = $\frac{10}{4}$

y = $\frac{5}{2}$ km/h

y = $2\frac{1}{2}$ km/h

speed of stream is 2.5 km/h

34. Let A_i(i = 1, 2) denote the event of getting an ace in ith draw. Since the cards are drawn with replacement. Therefore,

$P(A_i)$ = Probability of getting an ace in i^{th} draw = $\frac{^4C_1}{^{52}C_1} = \frac{4}{52} = \frac{1}{13}$

and $P\left(\overline{A_i}\right)$ = 1 - $P(A_i)$ = $1 - \frac{1}{13} = \frac{12}{13}$, i = 1, 2

Let X denote the number of aces in two draws. Then, X can take values 0, 1, 2.

Now, P(X = 0) = Probability of getting no ace in two draws

$\Rightarrow$ P(X = 0) = $P\left(\overline{A_1} \cap \overline{A_2}\right) = P\left(\overline{A_1}\right) P\left(\overline{A_2}\right)$ = $\frac{12}{13} \times \frac{12}{13} = \frac{144}{169}$

$\Rightarrow$ P(X = 1) = Probability of getting an ace in either of the two draws

$\Rightarrow$ P(X = 1) = $P\left(\left(A_1 \cap \overline{A_2}\right) \cup \left(\overline{A_1} \cap A_2\right)\right)$

$\Rightarrow$ P (X = 1) = $P\left(A_1 \cap \overline{A_2}\right) + P\left(\overline{A_1} \cap A_2\right)$

$\Rightarrow$ P (X = 1) = $P(A_1) P\left(\overline{A_2}\right) + P\left(\overline{A_1}\right) P(A_2)$ = $\frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$

Thus, the probability distribution of X is given by:

X	0	1	2
P(X)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

$\therefore \Sigma p_i x_i = 0 \times \frac{144}{169} + 1 \times \frac{24}{169} + 2 \times \frac{1}{169} = \frac{26}{169}$

and, $\Sigma p_i x_i{}^2 = 0 \times \frac{144}{169} + 1 \times \frac{24}{169} + 4 \times \frac{1}{169} = \frac{28}{169}$

Hence, $\bar{X}$ = Mean = $\Sigma p_i x_i = \frac{26}{169} = \frac{2}{13}$

and, Var(X) = $\sum p_i x_i^2 - (\Sigma p_i x_i)^2 = \frac{28}{169} - \left(\frac{2}{13}\right)^2 = \frac{24}{169}$

$\therefore$ S.D. = $\sqrt{\operatorname{Var}(X)} = \sqrt{\frac{24}{169}} = \frac{2\sqrt{6}}{13}$

Hence, Mean = $\frac{2}{13}$ and S.D. = $\frac{2\sqrt{6}}{13}$

OR

The number of ways of selecting two numbers from the first five positive integers $= {}^5C_2 = 10$.
So, the sample space S of the random experiment has 10 equally likely outcomes.
The outcomes are:
1, 2; 1, 3; 1, 4; 1, 5; 2, 3; 2, 4; 2, 5; 3, 4; 3, 5; 4, 5.
As the random variable X denote the larger of the two numbers. X can take values 2, 3, 4, 5. Note that in a sample space S, we have

Larger than any number	**Number of outcomes**
2	1
3	2
4	3
5	4

$P(X=2)=\frac{1}{10}, P(X=3)=\frac{2}{10},\ P(X=4)=\frac{3}{10}, P(X=5)=\frac{4}{10}$
$\therefore$ Probability distribution of X is

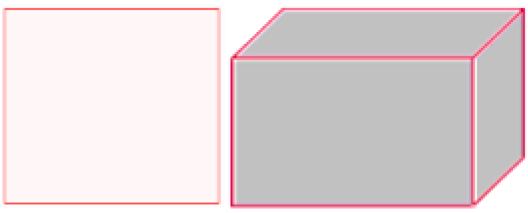
X	2	3	4	5
P(X)	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

Mean $= \Sigma p_i x_i = \frac{1}{10} \times 2 + \frac{2}{10} \times 3 + \frac{3}{10} \times 4 + \frac{4}{10} \times 5 = 4$
Variance $= \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2$
$= \frac{1}{10}$ $(1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + 4 \times 5^2) - (4)^2$
$= \frac{170}{10}$ - 16 = 17 - 16 = 1.

35. It is given that,
Present value of car = 315600
Rate of depreciation (r) = 20%
   i. We know that
   Value of car 2 years ago = A $\div(1 - \frac{r}{100})^n$
   Substituting the values
   = 315600 $\div(1 - \frac{20}{100})^2$
   By further calculation
   = 315600 $\times \frac{5}{4} \times \frac{5}{4}$
   = 493125
   ii. We know that
   Value of car after 3 years = 315600 $\times(1 - \frac{20}{100})^3$
   By further calculation
   = 315600 $\times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$
   = 161587.20

Section E

36. **Read the text carefully and answer the questions:**
Yash wants to prepare a handmade gift box for his friend’s birthday at his home. For making lower part of the box, he took a square piece of paper of each side equal to 10 cm.



(i) (0,10)
(ii) V = x(10 - 2x)(10 - 2x)
(iii) $\frac{5}{3}$, 5

OR

$\frac{5}{3}$ cm

37. **Read the text carefully and answer the questions:**

Loans are an integral part of our lives today. We take loans for a specific purpose - for buying a home, or a car, or sending kids abroad for education - loans help us achieve some significant life goals. That said, when we talk about loans, the word “EMI", eventually crops up because the amount we borrow has to be returned to the lender with interest.

Suppose a person borrows ₹1 lakh for one year at the fixed rate of 9.5 percent per annum with a monthly rest. In this case, the EMI for the borrower for 12 months works out to approximately ₹8,768.

Example:

In year 2000, Mr. Tanwar took a home loan of ₹3000000 from State Bank of India at 7.5% p.a. compounded monthly for 20 years.

(i) ₹ 24167.82

(ii) ₹ 10458.69

(iii) ₹ 13709.13

OR

₹ 410293.41

38. i. O(0, 0), A(6, 12), so equation of OA is

$y - 0 = \frac{12-0}{6-0}(x - 0) \Rightarrow y - 2x = 0$.

ii. Since BC is parallel to OA, so slope of BC = slope of OA.

$\therefore$ Equation of BC is $Y - 4 = 2(x - 0) \Rightarrow y - 2x = 4$.

iii. Constraints for the L.P.P. are

$y \geq 2x$, $y - 2x \leq 4$, $x \leq 6$, $x \geq 0$, $y \geq 0$

OR

The corner points of the feasible region are (0, 0), (6, 12), (6, 16) and (0, 4). The values of Z = 3x - 4y at these corner points are 0, -30, - 46 and -16. Hence, the minimum value of Z = - 46.

OR

i. From the given figure, OA = 25 and OD = 50

The equation of the line AD is $\frac{x}{25} + \frac{y}{50} = 1$ ...(intercept form)

i.e. 2x + y = 50

ii. The equation of the line BC is $\frac{x}{40} + \frac{y}{20} = 1$ i.e. x + 2y = 40

iii. Solving equations 2x + y = 50 and x + 2y = 40 simultaneously, we get x = 20, y - 10

$\therefore$ The coordinates of point B are (20, 10).

From the given figure, the coordinates of point C are (0, 20).

OR

As (0, 0) lies in the region $2x + y \leq 50$ and (0,0) also lies in the region $x + 2y \leq 40$, therefore, the constraints for the L.P.P. are $2x + y \leq 50$, $x + 2y \leq 40$, $x \geq 0$, $y \geq 0$