Assignment Problem and Sequencing

EXERCISE 7.1 [PAGES 118 - 119]

Exercise 7.1 | Q 1 | Page 118

A job production unit has four jobs A, B, C, D which can be manufactured on each of the four machines P, Q, R and S. The processing cost of each job for each machine is given in the following table:

Jobs	Machines (Processing Cost in ₹)			
	P	**Q**	**R**	**S**
A	31	25	33	29
B	25	24	23	21
C	19	21	23	24
D	38	36	34	40

Find the optimal assignment to minimize the total processing cost.

Solution: Step 1: Row minimum
Subtract the smallest element in each row from every element in its row. The matrix obtained is given below:

Jobs	Machines (Processing Cost in ₹)			
	P	**Q**	**R**	**S**
A	6	0	8	4
B	4	3	2	0
C	0	2	4	5
D	4	2	0	6

Step 2: Column minimum
Subtract the smallest element in each column of assignment matrix obtained in step 1 from every element in its column.

Jobs	Machine (Processing Cost in ₹)			
	P	**Q**	**R**	**S**

A	6	0	8	4
B	4	3	2	0
C	0	2	4	5
D	4	2	0	6

Step 3:
Draw minimum number of vertical and horizontal lines to cover all zeros.
First cover all rows and columns which have maximum number of zeros.

Jons	**Machines (Processing Cost in ₹)**			
	P	**Q**	**R**	**S**
A	~~6~~	~~0~~	~~8~~	~~4~~
B	~~4~~	~~3~~	~~2~~	~~0~~
C	~~0~~	~~2~~	~~4~~	~~5~~
D	~~4~~	~~2~~	~~0~~	~~6~~

Step 4:
From step 3, minimum number of lines covering all the zeros are 4, which is equal to order of the matrix, i.e., 4
∴ Select a row with exactly one zero, enclose that zero in (□) and cross out all zeros in its respective column.
Similarly, examine each row and column and mark the assignment (□).
∴ The matrix obtained is as follows:

Jobs	**Machines (Processing Cost in ₹)**			
	P	**Q**	**R**	**S**
A	6	**0**	8	4
B	4	3	2	**0**
C	**0**	2	4	5
D	4	2	**0**	6

Step 5:
The matrix obtained in step 4 contains exactly one assignment for each row and column.
∴ Optimal assignment schedule is as follows:

Jobs	**Machines**	**Processing cost (₹)**
A	Q	25
B	S	21
C	P	19
D	R	34

∴ Total minimum processing cost = 25 + 21 + 19 + 34 = ₹ 99.

Exercise 7.1 | Q 2 | Page 118

Five wagons are available at stations 1, 2, 3, 4 and 5. These are required at 5 stations I, II, III, IV and V. The mileage between various stations are given in the table below. How should the wagons be transported so as to minimize the mileage covered?

	I	**II**	**III**	**IV**	**V**
1	10	5	9	18	11
2	13	9	6	12	14
3	3	2	4	4	5
4	18	9	12	17	15
5	11	6	14	19	10

Solution: Step 1: Row minimum

Subtract the smallest element in each row from every element in its row.

The matrix obtained is given below

	I	**II**	**III**	**IV**	**V**
1	5	0	4	13	6
2	7	9	0	6	8
3	1	0	2	2	3
4	9	0	3	8	6
5	5	0	8	13	4

Step 2: Column minimum

Subtract the smallest element in each column of assignment matrix obtained in step 1 from every element in its column.

	I	II	III	IV	V
1	4	0	4	11	3
2	6	3	0	4	5
3	0	0	2	0	0
4	8	0	3	6	3
5	4	0	8	11	1

Step 3:

Draw minimum number of vertical and horizontal lines to cover all zeros.

First cover all rows and columns which have maximum number of zeros.

	I	II	III	IV	V
1	4	~~0~~	4	11	3
2	~~6~~	~~3~~	~~0~~	~~4~~	~~5~~
3	~~0~~	~~0~~	~~2~~	~~0~~	~~0~~
4	8	~~0~~	3	6	3
5	4	~~0~~	8	11	1

Step 4:

From step 3, minimum number of lines covering all the zeros are 3, which is less than order of matrix, i.e., 5.

∴ Select smallest element from all the uncovered elements, i.e., 1 and subtract it from all the uncovered elements and add it to the elements which lie at the intersection of two lines.

	I	II	III	IV	V
1	3	0	3	10	2
2	6	4	0	4	5
3	0	1	2	0	0
4	7	0	2	5	2
5	3	0	7	10	0

Step 5:

Draw minimum number of vertical and horizontal lines to cover all zeros.

	I	II	III	IV	V

1	3	~~0~~	3	10	2
2	~~6~~	~~4~~	~~0~~	~~4~~	~~5~~
3	~~0~~	~~1~~	~~2~~	~~0~~	~~0~~
4	7	~~0~~	2	5	2
5	~~3~~	~~0~~	~~7~~	~~10~~	~~0~~

Step 6:

From step 5, minimum number of lines covering all the zeros are 4, which is less than order of matrix, i.e., 5.

∴ Select smallest element from all the uncovered elements, i.e., 2 and subtract it from all the uncovered elements and add it to the elements which lie at the intersection of two lines.

	I	**II**	**III**	**IV**	**V**
1	1	0	1	8	0
2	6	6	0	4	5
3	0	3	2	0	0
4	5	0	0	3	0
5	3	2	7	10	0

Step 7:

Draw minimum number of vertical and horizontal lines to cover all zeros.

	I	**II**	**III**	**IV**	**V**
1		1	0	1	8
2	6	6	0	4	5
3	0	3	2	0	0
4	5	0	0	3	0
5	3	2	7	10	0

Step 8:

From step 7, minimum number of lines covering all the zeros are 4, which is less than order of matrix, i.e.,5.

∴ Select smallest element from all the uncovered elements, i.e., 1 and subtract it from all the uncovered elements and add it to the elements which lie at the intersection of two lines.

	I	**II**	**III**	**IV**	**V**
1	0	0	1	7	0
2	5	6	0	3	5
3	0	4	3	0	1
4	4	0	0	2	0
5	2	2	7	9	0

Step 9:

Draw minimum number of vertical and horizontal lines to cover all zeros.

	I	II	III	IV	V
1	~~0~~	~~0~~	~~1~~	~~7~~	~~0~~
2	5	6	~~0~~	3	~~5~~
3	~~0~~	~~4~~	~~3~~	~~0~~	~~1~~
4	~~4~~	~~0~~	~~0~~	~~2~~	~~0~~
5	2	2	~~7~~	9	~~0~~

Step 10:

From step 9, minimum number of lines covering all the zeros are 5, which is equal to order of the matrix, i.e., 5.

∴ Select a row with exactly one zero, enclose that zero in ( ) and cross out all zeros in its respective column.

Similarly, examine each row and column and mark the assignment ( ).

∴ The matrix obtained is as follows:

	I	**II**	**III**	**IV**	**V**
1	**0**	~~0~~	1	7	~~0~~
2	5	6	**0**	3	5

3	~~0~~	4	3	**0**	1
4	4	**0**	~~0~~	2	~~0~~
5	2	2	7	9	**0**

Step 11:

The matrix obtained in step 10 contains exactly one assignment for each row and column.

∴ Optimal assignment schedule is as follows:

Jobs	**Wagons**	**Mileage**
1	I	10
2	II	6
3	III	4
4	IV	9
5	V	10

∴ Total minimum mileage = 10 + 6 = 4 + 9 + 10 = 39.

Exercise 7.1 | Q 3 | Page 118

Five different machines can do any of the five required jobs, with different profits resulting from each assignment as shown below:

Job	**Machines (Profit in ₹)**				
	A	**B**	**C**	**D**	**E**
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

Find the optimal assignment schedule.

Solution: Step 1:
Since it is a maximization problem, subtract each of the elements in the table from the largest element, i.e., 62

Jobs	Machines (Profit in ₹)				
	A	B	C	D	E
1	32	25	22	34	22
2	22	38	35	41	26
3	22	30	29	32	27
4	37	24	22	26	26
5	33	0	21	28	23

Step 2:
Row minimum Subtract the smallest element in each row from every element in its row. The matrix obtained is given below:

Jobs	Machines (Profit in ₹)				
	A	B	C	D	E
1	10	3	0	12	0
2	0	16	13	19	4
3	0	8	7	10	5
4	15	2	0	4	4
5	33	0	21	28	23

Step 3:
Column minimum Subtract the smallest element in each column of assignment matrix obtained in step 2 from every element in its column.

Jobs	Machines (Profit in ₹)				
	A	B	C	D	E

1	10	3	0	8	0
2	0	16	13	15	4
3	0	8	7	6	5
4	15	2	0	0	4
5	33	0	21	24	23

Step 4:
Draw minimum number of vertical and horizontal lines to cover all zeros.
First cover all rows and columns which have maximum number of zeros.

Jobs	**Machines (Profit in ₹)**				
	A	**B**	**C**	**D**	**E**
1	~~10~~	~~3~~	~~0~~	~~8~~	~~0~~
2	~~0~~	16	13	15	4
3	~~0~~	8	7	6	5
4	~~15~~	~~2~~	~~0~~	~~0~~	~~4~~
5	~~33~~	~~0~~	~~21~~	~~24~~	~~23~~

Step 5:
From step 4, minimum number of lines covering all the zeros are 4, which is less than order of matrix, i.e., 5.
∴ Select smallest element from all the uncovered elements, i.e., 4 and subtract it from all the uncovered elements and add it to the elements which lie at the intersection of two lines.

Jobs	**Machines (Profit in ₹)**				
	A	**B**	**C**	**D**	**E**
1	14	3	0	8	0
2	0	12	9	11	0
3	0	4	3	2	1
4	19	2	0	0	4
5	37	0	21	24	23

Step 6:
Draw minimum number of vertical and horizontal lines to cover all zeros.

Jobs	Machines (Profit in ₹)				
	A	B	C	D	E
1	~~14~~	~~3~~	~~0~~	~~8~~	~~0~~
2	~~0~~	12	9	11	0
3	~~0~~	4	3	2	1
4	~~19~~	~~2~~	~~0~~	~~0~~	~~4~~
5	37	0	21	24	23

Step 7:

From step 6, minimum number of lines covering all the zeros are 5, which is equal to order of the matrix, i.e., 5.

∴ Select a row with exactly one zero, enclose that zero in (□) and cross out all zeros in its respective column.

Similarly, examine each row and column and mark the assignment (□).

∴ The matrix obtained is as follows:

Jobs	Machines (Profit in ₹)				
	A	B	C	D	E
1	14	3	**0**	8	~~0~~
2	~~0~~	12	9	11	**0**
3	**0**	4	3	2	1
4	19	2	~~0~~	**0**	4
5	37	**0**	21	24	23

Step 8:

The matrix obtained in step 7 contains exactly one assignment for each row and column.

∴ Optimal assignment schedule is as follows:

Jobs	Machines	Profit (in ₹)
1	C	40
2	E	36
3	A	40
4	D	36

5	B	62

∴ Total maximum profit = 40 + 36 + 40 + 36 + 62 = ₹ 214.

Exercise 7.1 | Q 4 | Page 119

Four new machines M1, M2, M3 and M4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M2 cannot be placed at C and M3 cannot be placed at A. The cost matrix is given below.

Machines	**Places**				
	A	**B**	**C**	**D**	**E**
M1	4	6	10	5	6
M2	7	4	–	5	4
M3	–	6	9	6	2
M4	9	3	7	2	3

Find the optimal assignment schedule.

Solution: Step 1:
Here, number of rows and columns are not equal.
∴ The problem is unbalanced.
∴ It is balanced by introducing dummy machine M5 with zero cost.
Also, M2 cannot be placed at C and M3 cannot be placed at A.
Let a very high cost (∞) be assigned to corresponding elements.
∴ Matrix obtained is as follows:

Machines	**Places**				
	A	**B**	**C**	**D**	**E**
M1	4	6	10	5	6
M2	7	4	∞	5	4
M3	∞	6	9	6	2
M4	9	3	7	2	3
M5	0	0	0	0	0

Step 2:
Row minimum Subtract the smallest element in each row from every element in its row. The matrix obtained is given below:

Machines	Places				
	A	**B**	**C**	**D**	**E**
M1	0	2	6	1	2
M2	3	0	∞	1	0
M3	∞	4	7	4	0
M4	7	1	5	0	1
M5	0	0	0	0	0

Step 3:
Column minimum
Here, each column contains element zero.
∴ Matrix obtained by column minimum is same as above matrix.

Step 4:
Draw minimum number of vertical and horizontal lines to cover all zeros.
First cover all rows and columns which have maximum number of zeros.

Machines	Places				
	A	**B**	**C**	**D**	**E**
M1	~~0~~	~~2~~	6	~~1~~	~~2~~
M2	~~3~~	~~0~~	∞	~~1~~	~~0~~
M3	~~∞~~	~~4~~	7	~~4~~	~~0~~
M4	~~7~~	~~1~~	5	~~0~~	~~1~~
M5	~~0~~	~~0~~	0	~~0~~	~~0~~

Step 5:
From step 4, minimum number of lines covering all the zeros are 5, which is equal to order of the matrix, i.e., 5.
∴ Select a row with exactly one zero, enclose that zero in (□) and cross out all zeros in its respective column.
Similarly, examine each row and column and mark the assignment (□).
∴ The matrix obtained is as follows:

Machines	Places				
	A	**B**	**C**	**D**	**E**

M1	**0**	2	6	1	2
M2	3	**0**	∞	1	~~0~~
M3	∞	4	7	4	**0**
M4	7	1	5	**0**	1
M5	~~0~~	~~0~~	**0**	~~0~~	~~0~~

Step 6:
The matrix obtained in step 5 contains exactly one assignment for each row and column.
∴ Optimal assignment schedule is as follows:

Machines	Place	Cost
M1	A	4
M2	B	4
4M3	E	2
M4	D	2
M5	C	0

∴ The total minimum cost = 4 + 4 + 2 + 2 + 0 = ₹ 12.

Exercise 7.1 | Q 5 | Page 119

A company has a team of four salesmen and there are four districts where the company wants to start its business. After taking into account the capabilities of salesmen and the nature of districts, the company estimates that the profit per day in rupees for each salesman in each district is as below:

Salesman	District			
	1	**2**	**3**	**4**
A	16	10	12	11
B	12	13	15	15
C	15	15	11	14
D	13	14	14	15

Find the assignment of salesman to various districts which will yield maximum profit.

Solution: Step 1:
Since it is a maximization problem, subtract each of the elements in the table from the largest element, i.e., 16

Salesman	District			
	1	**2**	**3**	**4**
A	0	6	4	5
B	4	3	1	1
C	1	1	5	2
D	3	2	2	1

Step 2:
Row minimum Subtract the smallest element in each row from every element in its row. The matrix obtained is given below:

Salesman	District			
	1	**2**	**3**	**4**
A	0	6	4	5
B	3	2	0	0
C	0	0	4	1
D	2	1	1	0

Step 3:
Column minimum Here, each column contains element zero.
∴ Matrix obtained by column minimum is same as above matrix.

Step 4:
Draw minimum number of vertical and horizontal lines to cover all zeros.
First cover all rows and columns which have maximum number of zeros.

Salesman	District			
	1	**2**	**3**	**4**
A	~~0~~	6	4	~~5~~
B	~~3~~	~~2~~	~~0~~	~~0~~
C	~~0~~	~~0~~	~~4~~	~~1~~
D	~~2~~	1	1	~~0~~

Step 5:
From step 4, minimum number of lines covering all the zeros are 4, which is equal to order of the matrix, i.e., 4.
∴ Select a row with exactly one zero, enclose that zero in (□) and cross out all zeros in its respective column.
Similarly, examine each row and column and mark the assignment (□).
∴ The matrix obtained is as follows:

Salesman	**District**			
	1	**2**	**3**	**4**
A	**0**	6	4	5
B	3	2	**0**	~~0~~
C	~~0~~	**0**	4	1
D	2	1	1	**0**

Step 6:
The matrix obtained in step 5 contains exactly one assignment for each row and column.
∴ Optimal assignment schedule is as follows:

Salesman	**District**	**Profit (in ₹)**
A	1	16
B	2	15
C	3	15
D	4	15

∴ The maximum profit = 16 + 15 + 15 + 15 = ₹ 61.

Exercise 7.1 | Q 6 | Page 119

In the modification of a plant layout of a factory four new machines M_1, M_2, M_3 and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M_2 cannot be placed at C and M_3 cannot be placed at A. The cost of locating a machine at a place (in hundred rupees) is as follows.

Machines	**Location**				
	A	**B**	**C**	**D**	**E**

M_1	9	11	15	10	11
M_2	12	9	–	10	9
M_3	–	11	14	11	7
M_4	14	8	12	7	8

Find the optimal assignment schedule.

Solution: Here, number of rows and columns are not equal.

∴ The problem is unbalanced.

∴ It is balanced by introducing dummy job machine M_5 with zero cost.

Also, M_2 cannot be placed at C and M_3 cannot be placed at A.

Let a very high cost (∞) be assigned to corresponding elements.

∴ Matrix obtained is as follows:

Machines	**Location**				
	A	**B**	**C**	**D**	**E**
M_1	9	11	15	10	11
M_2	12	9	∞	10	9
M_3	∞	11	14	11	7
M_4	14	8	12	7	8
M_5	0	0	0	0	0

Step 2: Row minimum

Subtract the smallest element in each row from every element in its row.

The matrix obtained is given below:

Machines	**Location**				
	A	**B**	**C**	**D**	**E**
M_1	0	2	6	1	2
M_2	3	0	∞	1	0
M_3	∞	4	7	4	0
M_4	7	1	5	0	1

M5	0	0	0	0	0

Step 3: Column minimum

Here, each column contains element zero.

∴ Matrix obtained by column minimum is same as above matrix.

Step 4:

Draw minimum number of vertical and horizontal lines to cover all zeros.

First cover all rows and columns which have maximum number of zeros.

Machines	**Location**				
	A	**B**	**C**	**D**	**E**
M1	~~0~~	~~2~~	6	~~1~~	~~2~~
M2	~~3~~	~~0~~	∞	~~1~~	~~0~~
M3	~~∞~~	4	7	4	~~0~~
M4	~~7~~	~~1~~	5	~~0~~	~~1~~
M5	~~0~~	~~0~~	0	~~0~~	~~0~~

Step 5:

From step 4, minimum number of lines covering all the zeros are 5, which is equal to order of the matrix i.e., 5

∴ Select a row with exactly one zero, enclose that zero in (□) and cross out all zeros in its respective column.

Similarly, examine each row and column and mark the assignment (□).

∴ The matrix obtained is as follows:

Machines	**Location**				
	A	**B**	**C**	**D**	**E**
M1	**0**	2	6	1	2
M2	3	**0**	∞	1	~~0~~
M3	∞	4	7	4	**0**

M_4	7	1	5	**0**	1
M_5	~~0~~	~~0~~	**0**	~~0~~	~~0~~

Step 6:

The matrix obtained in step 5 contains exactly one assignment for each row and column.

∴ Optimal assignment schedule is as follows:

Machines	**Location**	**Cost (in hundred rupees)**
M_1	A	9
M_2	B	9
M_3	C	7
M_4	D	7
M_5	E	0

The total minimum cost (in hundred rupees) = 9 + 9 + 7 + 7 + 0 = 32.

EXERCISE 7.2 [PAGES 125 - 126]

Exercise 7.2 | Q 1 | Page 125

A machine operator has to perform two operations, turning and threading on 6 different jobs. The time required to perform these operations (in minutes) for each job is known. Determine the order in which the jobs should be processed in order to minimize the total time required to complete all the jobs. Also find the total processing time and idle times for turning and threading operations.

Job	**1**	**2**	**3**	**4**	**5**	**6**
Time of turning	3	12	5	2	9	11
Time for threading	8	10	9	6	3	1

Solution:

Job	1	2	3	4	5	6
Time of turning	3	12	5	2	9	11
Time for threading	8	10	9	6	3	1

Observe that Min (turning, threading) = 1, which corresponds to job 6 on threading machine.

Therefore, job 6 is placed last in sequence.

					6

Then the problem reduces to

Job	1	2	3	4	5
Time of turning	3	12	5	2	9
Time for threading	8	10	9	6	3

Now, Min (turning, threading) = 2, which corresponds to job 4 on turning machine

∴ Job 4 is placed first in sequence.

4					6

Then the problem reduces to

Job	1	2	3	5
Time of turning	3	12	5	9
Time for threading	8	10	9	3

Now, Min (turning, threading) = 3, which corresponds to job 1 on turning and job 5 on threading.

∴ Job 1 is placed second and job 5 is placed second last.

4	1			5	6

Then the problem reduces to

Job	2	3
Time of turning	12	5
Time for threading	10	9

Now, Min (turning, threading) = 5, which corresponds to job 3 on turning.

∴ Job 3 is placed third on sequence and job 2 on remaining.

∴ Optimal solution is

4	1	3	2	5	6

Total elapsed time

Job	Turning		Threading	
	In	Out	In	Out
4(2, 6)	0	2	2	8
1(3, 8)	2	5	8	16
3(5, 9)	5	10	16	25
2(12, 10)	10	22	25	35
5(9, 3)	22	31	35	**38**
6(11, 1)	31	42	**42**	43

∴ Total elapsed time = 43 mins

Idle time for turning = 43 – 42 = 1 min

Idle time for threading = 2 + 4 = 6 mins.

Exercise 7.2 | Q 2 | Page 125

A company has three jobs on hand. Each of these must be processed through two departments, in the order AB where

Department A: Press shop and

Department B: Finishing

The table below gives the number of days required by each job in each department

Job	I	II	III
Department A	8	6	5
Department B	8	3	4

Find the sequence in which the three jobs should be processed so as to take minimum time to finish all the three jobs. Also find idle time for both the departments.

Solution:

Job	**I**	**II**	**III**
Department A	8	6	5
Department B	8	3	4

Observe that Min(A, B) = 3, corresponds to job II on depatment B.

∴ Job II is placed last in sequence

		II

Then the problem reduces to

Job	**I**	**III**
Department A	8	5
Department B	8	4

Now, Min (A, B) = 4, corresponds to job III on department B.

∴ Job III is placed second in sequence and job I on remaining.

∴ Optimal sequence is

I	III	II

Total elapsed time.

Job	**Dept. A**		**Dept. B**	
	In	**Out**	**In**	**Out**
I (8, 8)	0	8	8	16
III (5, 4)	8	13	16	20
II (6, 3)	13	19	20	23

∴ Total elapsed time = 23 days

Idle time for Department A = 23 – 19 = 4 days

Idle time for Department B = 8 days.

Exercise 7.2 | Q 3 | Page 125

An insurance company receives three types of policy application bundles daily from its head office for data entry and tiling. The time (in minutes) required for each type for these two operations is given in the following table:

Policy	1	2	3
Data Entry	90	120	180
Filing	140	110	100

Find the sequence that minimizes the total time required to complete the entire task.

Solution:

Policy	1	2	3
Data Entry	90	120	180
Filing	140	110	100

Observe that Min (Data entry, filing) = 90, corresponds to policy 1 on data entry
∴ Policy 1 is placed first in sequence.

1		

Then the problem reduces to

Policy	2	3
Data Entry	120	180
Filing	110	100

Now, Min (Data entry, filing) = 100, corresponds to policy 3 on filing.
∴ Policy 3 is placed last on sequence and policy 2 on remaining.
∴ Optimal sequence is

1	2	3

Total elapsed time

Policy	Data entry		Filling	
	In	**Out**	**In**	**Out**
1 (90, 140)	0	90	90	230
2 (120, 110)	90	210	230	**340**

3 (180, 100)	210	390	**390**	490

∴ Total elapsed time = 490 mins
Idle time for data entry = 490 – 390 = 100 mins
Idle time for filling = 90 + 50 = 140 mins.

Exercise 7.2 | Q 4 | Page 125

There are five jobs, each of which must go through two machines in the order XY. Processing times (in hours) are given below. Determine the sequence for the jobs that will minimize the total elapsed time. Also find the total elapsed time and idle time for each machine.

Job	**A**	**B**	**C**	**D**	**E**
Machine X	10	2	18	6	20
Machine Y	4	12	14	16	8

Solution:

Job	**A**	**B**	**C**	**D**	**E**
Machine X	10	2	18	6	20
Machine Y	4	12	14	16	8

Observe that Min(X, Y) = 2, corresponding to job B on machine X.

∴ Job B is placed first in sequence

B			

Then the problem reduces to

Job	**A**	**C**	**D**	**E**
Machine X	10	18	6	20
Machine Y	4	14	16	8

Now, Min(X, Y) = 4, corresponds to job A on machine Y.

∴ Job A is placed last in sequence.

B			A

Then the problem reduces to

Job	C	D	E
Machine X	18	6	20
Machine Y	14	16	8

Now, Min(X, Y) = 6, corresponds to job D on machine X.

∴ Job D is placed second in sequence.

B	D			A

Then the problem reduces to

Job	C	E
Machine X	18	20
Machine Y	14	8

Now, Min(X, Y) = 8, corresponds to job E on machine Y.

∴ Job E is placed fourth and job C on remaining in sequence.

∴ The optimal sequence is

B	D	C	E	A

Total elapsed time,

Job	Machine X		Machine Y	
	In	**Out**	**In**	**Out**
B (2, 12)	0	2	2	14
D (6, 16)	2	8	14	30
C (18, 14)	8	26	30	**44**
E (20, 8)	26	46	**46**	**54**
A (10, 4)	46	56	**56**	60

∴ Total elapsed time = 60 hrs.

Idle time for machine X = 60 – 56 = 4 hrs

Idle time for machine Y = 2 + 2 + 2 = 6 hrs.

Exercise 7.2 | Q 5 | Page 125

Find the sequence that minimizes the total elapsed time to complete the following jobs in the order AB. Find the total elapsed time and idle times for both the machines.

Job	I	II	IIII	IV	V	VI	VII
Machine A	7	16	19	10	14	15	5
Machine B	12	14	14	10	16	5	7

Solution: Observe that Min(A, B) = 5, corresponds to job VI on machine B and job VII on machine A.
∴ Job VI is placed last and job VII is placed first in sequence.

VII					VI

Then the problem reduces to

Job	I	II	IIII	IV	V
Machine A	7	16	19	10	14
Machine B	12	14	14	10	6

Now, Min(A, B) = 7, corresponds to job I on machine A.
∴ Job I is placed second in sequence.

VII	I					VI

Then the problem reduces to

Job	II	III	IV	V
Machine A	16	19	10	14
Machine B	14	14	10	16

Now, Min(A, B) = 10, corresponds to job IV on machine A as well as on machine B.
∴ Job IV is placed second or second last in sequence.

VII	I	IV				VI

OR

VII	I				IV	VI

Then the problem reduces to

Job	II	III	V
Machine A	16	19	14
Machine B	14	14	16

Now, Min (A, B) = 14, corresponds to job II and job III on machine B and job V on machine A.
∴ These three jobs can be placed in the sequence in order: V – III – II or V – II – III
∴ The optimal sequence can be

VII	I	IV	V	III	II	VI

OR

VII	I	V	III	II	IV	VI

OR

VII	I	IV	V	II	III	VI

OR

VII	I	V	II	III	IV	VI

∴ We consider the optimal sequence as VII – I – IV – V – III – II – VI
Total Elapsed Time

Job	**Machine A**		**Machine B**	
	In	**Out**	**In**	**Out**
VII (5, 7)	0	5	5	12
I (7, 12)	5	12	12	24
IV (10, 10)	12	22	24	**34**
V (14, 16)	22	36	**36**	**52**
III (19, 14)	36	55	**55**	**69**
II (16, 14)	55	71	**71**	**85**
V (15, 5)	71	86	**86**	91

∴ Total elapsed time = 91 units
Idle time for Machine A = 91 – 86 = 5 units
Idle time for Machine B = 5 + 2 + 3 + 2 + 1 = 13 units.

Exercise 7.2 | Q 6.1 | Page 125

Find the optimal sequence that minimizes total time required to complete the following jobs in the order ABC. The processing times are given in hrs.

Job	I	II	III	IV	V	VI	VII
Machine A	6	7	5	11	6	7	12
Machine B	4	3	2	5	1	5	3
Machine C	3	8	7	4	9	8	7

Solution:

Job	I	II	III	IV	V	VI	VII
Machine A	6	7	5	11	6	7	12
Machine B	4	3	2	5	1	5	3
Machine C	3	8	7	4	9	8	7

Here min A = 5, max B = 5, min C = 3
Since min A ≥ max B is satisfied, the problem can be converted into a two machine problem.
Let G and H be two fictitious machines such that G = A + B and H = B + C
Then the problem can be written as

Job	I	II	III	IV	V	VI	VII
Machine G	10	10	7	16	7	12	15
Machine H	7	11	9	9	10	1	10

Observe that Min (G, H) = 7, corresponds to job I on machine H, job III on machine G and job V on machine G.
∴ Job I is placed last in sequence, Job III and V are placed either first or second in sequence.

V	III					I

OR

III	V					I

Then the problem reduces to

Job	II	IV	VI	VII
Machine G	10	16	12	15
Machine H	11	9	13	10

Now, Min (G, H) = 9, corresponds to Job IV on machine H.
∴ Job IV is placed second last in the sequence.

V	III				IV	I

OR

III	V				IV	I

Then the problem reduces to

Job	**II**	**VI**	**VII**
Machine G	10	12	15
Machine H	11	13	10

Now, Min (G, H) = 10, corresponds to Job II on machine G and Job VII on machine H.
∴ Job II is placed third in the sequence, Job VII is placed third last in the sequence and remaining Job VI is placed fourth in the sequence.

V	III	II	VI	VII	IV	I

OR

III	V	II	VI	VII	IV	I

∴ We consider the optimal sequence as V – III – II – VI – VII – IV – I
Total elapsed time

Job	**Machine A**		**Machine B**		**Machine C**	
	In	**Out**	**In**	**Out**	**In**	**Out**
V (6, 1, 9)	0	6	6	**7**	7	16
III (5, 2, 7)	6	11	**11**	**13**	16	23
II (7, 3, 8)	11	18	**18**	**21**	23	31
VI (7, 5, 8)	18	25	**25**	**30**	31	**39**
VII(12, 3, 7)	25	37	**37**	**40**	**40**	**47**
IV (11, 5, 4)	37	48	**48**	**53**	**53**	**57**
I (6, 4, 3)	48	54	**54**	58	**58**	61

Total elapsed time = 61 hrs
Idle time for Machine A = 61 – 54 = 7 hrs

Idle time for Machine B = (61 – 58) + 6 + 4 + 5 + 4 + 7 + 8 + 1 = 38 hrs
Idle time for Machine C = 7 + 1 + 6 + 1 = 15 hrs.

Exercise 7.2 | Q 6.2 | Page 126

Find the optimal sequence that minimizes total time required to complete the following jobs in the order ABC. The processing times are given in hrs.

Job	1	2	3	4	5
Machine A	5	7	6	9	5
Machine B	2	1	4	5	3
Machine C	3	7	5	6	7

Solution:

Job	1	2	3	4	5
Machine A	5	7	6	9	5
Machine B	2	1	4	5	3
Machine C	3	7	5	6	7

Here min A = 5, max B = 5, min C = 3
Since min A ≥ max B is satisfied, the problem can be converted into a two machine problem.
Let G and H be two fictitious machines such that G = A + B and H=B+C
Then the problem can be written as

Job	1	2	3	4	5
Machine G	7	8	10	14	8
Machine H	5	8	9	11	10

Observe that Min (G, H) = 5, corresponds to Job 1 on machine H.
∴ Job 1 is placed last in sequence.

				1

Then the problem reduces to

Job	2	3	4	5
Machine G	8	10	14	8
Machine H	8	9	11	10

Now, Min (G, H) = 8, corresponds to Job 2 on machine G as well as on machine H and Job 5 on machine G.

∴ Job 2 and 5 are placed either first or second in the sequence OR Job 5 is placed first and Job 2 is placed second last in the sequence.

2	5			1

OR

5	2			1

OR

5			2	1

Then the problem reduces to

Job	**3**	**4**
Machine G	10	14
Machine H	9	11

Now, Min (G, H) = 9, corresponds to job 3 on machine H.
∴ Job 3 is placed either second last or third last and job 4 on remaining in the sequence.

2	5	4	3	1

OR

5	2	4	3	1

OR

5	4	3	2	1

∴ We consider the optimal sequence as 2 – 5 – 4 – 3 – 1
Total time elapsed

Job	**Machine A**		**Machine B**		**Machine C**	
	In	**Out**	**In**	**Out**	**In**	**Out**
2 (7, 1, 7)	0	7	7	**8**	8	15
5 (5, 3, 7)	7	12	**12**	**15**	15	**22**
4 (9, 5, 6)	12	21	**21**	**26**	**26**	32
3 (6, 4, 5)	21	27	**27**	**31**	32	37
1 (5, 2, 3)	27	32	**32**	34	37	40

Total elapsed time = 40 hrs
Idle time for Machine A = 40 – 32 = 8 hrs

Idle time for Machine B = (40 – 34) + 7 + 4 + 6 + 1 + 1 = 25 hrs
Idle time for Machine C = 8 + 4 = 12 hrs.

Exercise 7.2 | Q 7 | Page 126

A publisher produces 5 books on Mathematics. The books have to go through composing, printing and binding done by 3 machines P, Q, R. The time schedule for the entire task in proper unit is as follows.

Book	A	B	C	D	E
Machine P	4	9	8	6	5
Machine Q	5	6	2	3	4
Machine R	8	10	6	7	11

Determine the optimum time required to finish the entire task.

Solution:

Book	A	B	C	D	E
Machine P	4	9	8	6	5
Machine Q	5	6	2	3	4
Machine R	8	10	6	7	11

Here min P = 4, max Q = 6, min R = 6
Since min R ≥ max Q is satisfied, the problem can be converted into a two machine problem.
Let G and H be two fictitious machines such that G = P + Q and H=Q+R
Then the problem can be written as

Job	A	B	C	D	E
Machine G	9	15	10	9	9
Machine H	13	16	8	10	15

Observe that Min (G, H) = 8, corresponds to Job C on machine H.
∴ Job C is placed last in sequence.

				C

Then the problem reduces to

Job	A	B	D	E

Machine G	9	15	9	9
Machine H	13	16	10	15

Now, Min (G, H) = 9, corresponds to job A, D and E on machine G.
∴ Job A, D and E are placed either first, second or third and job B on remaining in the sequence.

A	D	E	B	C

OR

A	E	D	B	C

OR

D	A	E	B	C

OR

D	E	A	B	C

OR

E	D	A	B	C

OR

E	A	D	B	C

∴ We consider the optimal sequence as A – D – E – B – C
Total elapsed time

Job	**Machine P**		**Machine Q**		**Machine R**	
	In	**Out**	**In**	**Out**	**In**	**Out**
A (4, 5, 8)	0	4	4	**9**	9	17
D (6, 3, 7)	4	10	**10**	**13**	17	24
E (5, 4, 11)	10	15	**15**	**19**	24	35
B (9, 6, 10)	15	24	**24**	**30**	35	45
C (8, 2, 6)	24	32	**32**	34	45	51

Total elapsed time = 51 units
Idle time for machine P = 51 – 32 = 19 units

Idle time for machine Q = (51 – 34) + 4 + 1 + 2 + 5 + 2 = 31 units
Idle time for machine R = 9 units.

MISCELLANEOUS EXERCISE 7 [PAGES 126 - 128]

Miscellaneous Exercise 7 | Q 1.01 | Page 126

Choose the correct alternative :

In sequencing, an optimal path is one that minimizes ________

1. Elapsed time
2. Idle time
3. **Both (a) and (b)**
4. Ready time

Solution: In sequencing, an optimal path is one that minimizes **Both (a) and (b)**.

Miscellaneous Exercise 7 | Q 1.02 | Page 126

Choose the correct alternative :

If job A to D have processing times as 5, 6, 8, 4 on first machine and 4, 7, 9, 10 on second machine then the optimal sequence is :

1. CDAB
2. **DBCA**
3. BCDA
4. ABCD

Solution: If job A to D have processing times as 5, 6, 8, 4 on first machine and 4, 7, 9, 10 on second machine then the optimal sequence is : **DBCA**.

Miscellaneous Exercise 7 | Q 1.03 | Page 126

Choose the correct alternative :

The objective of sequencing problem is

1. to find the order in which jobs are to be made
2. to find the time required for the completing all the job on hand
3. **to find the sequence in which jobs on hand are to be processed to minimize the total time required for processing the jobs**
4. to maximize the cost

Solution: The objective of sequencing problem is **to find the sequence in which jobs on hand are to be processed to minimize the total time required for processing the jobs**.

Miscellaneous Exercise 7 | Q 1.04 | Page 126

Choose the correct alternative :

If there are n jobs and m machines, then there will be________ sequences of doing the jobs.

1. mn
2. m(n!)
3. n^m
4. **$(n!)^m$**

Solution: If there are n jobs and m machines, then there will be **$(n!)^m$** sequences of doing the jobs.

Miscellaneous Exercise 7 | Q 1.05 | Page 126

Choose the correct alternative :

The Assignment Problem is solved by

1. Simplex method,
2. **Hungarian method**
3. Vector method,
4. Graphical method,

Solution: The Assignment Problem is solved by **Hungarian method**.

Miscellaneous Exercise 7 | Q 1.06 | Page 126

Choose the correct alternative :

In solving 2 machine and n jobs sequencing problem, the following assumption is wrong

1. No passing is allowed
2. Processing times are known
3. Handling time is negligible
4. **The time of passing depends on the order of machining**

Solution: In solving 2 machine and n jobs sequencing problem, the following assumption is wrong **The time of passing depends on the order of machining**.

Miscellaneous Exercise 7 | Q 1.07 | Page 126

Choose the correct alternative :

To use the Hungarian method, a profit maximization assignment problem requires

1. **Converting all profits to opportunity losses**
2. A dummy person or job
3. Matrix expansion
4. Finding the maximum number of lines to cover all the zeros in the reduced matrix

Solution: To use the Hungarian method, a profit maximization assignment problem requires **Converting all profits to opportunity losses**.

Miscellaneous Exercise 7 | Q 1.08 | Page 126

Choose the correct alternative :

Using Hungarian method the optimal assignment obtained for the following assignment problem to minimize the total cost is:

Agent	**Job**			
	A	**B**	**C**	**D**
1	10	12	15	25
2	14	11	19	32
3	18	21	23	29
4	15	20	26	28

1. **1 – C, 2 – B, 3 – D, 4 – A**
2. 1 – B, 2 – C, 3 – A, 4 – D
3. 1 – A, 2 – B, 3 – C, 4 – D
4. **1 – D, 2 – A, 3 – B, 4 – C**.

Solution: 1 – C, 2 – B, 3 – D, 4 – A

Miscellaneous Exercise 7 | Q 1.09 | Page 127

Choose the correct alternative :

The assignment problem is said to be unbalance if

1. Number of rows is greater than number of columns
2. Number of rows is lesser than number of columns

3. Number of rows is equal to number of columns
4. **Both (a) and (b)**

Solution: The assignment problem is said to be unbalance if **Both (a) and (b)**.

Miscellaneous Exercise 7 | Q 1.1 | Page 127

Choose the correct alternative :

The assignment problem is said to be balanced if

1. Number of rows is greater than number of columns
2. Number of rows is lesser than number of columns
3. **Number of rows is equal to number of columns**
4. If the entry of row is zero

Solution: The assignment problem is said to be balanced if **Number of rows is equal to number of columns**.

Miscellaneous Exercise 7 | Q 1.11 | Page 127

Choose the correct alternative :

The assignment problem is said to be balanced if it is a

1. **Square matrix**
2. Rectangular matrix
3. Unit matrix
4. Triangular matrix

Solution: The assignment problem is said to be balanced if it is a **Square matrix**.

Miscellaneous Exercise 7 | Q 1.12 | Page 127

Choose the correct alternative :

In an assignment problem if number of rows is greater than number of columns then

1. **Dummy column is added**
2. Dummy row is added
3. Row with cost 1 is added
4. Column with cost 1 is added

Solution: In an assignment problem if number of rows is greater than number of columns then **Dummy column is added**.

Miscellaneous Exercise 7 | Q 1.13 | Page 127

Choose the correct alternative :

In a 3 machine and 5 jobs problem, the least of processing times on machine A, B and C are 5, 1 and 3 hours and the highest processing times are 9, 5 and 7 respectively, then it can be converted to a 2 machine problem if order of the machines is:

1. B-A-C,
2. A-B-C
3. C - B - A
4. **Both (B) and (C)**

Solution: In a 3 machine and 5 jobs problem, the least of processing times on machine A, B and C are 5, 1 and 3 hours and the highest processing times are 9, 5 and 7 respectively, then it can be converted to a 2 machine problem if order of the machines is: **Both (B) and (C)**.

Miscellaneous Exercise 7 | Q 1.14 | Page 127

Choose the correct alternative :

The objective of an assignment problem is to assign

1. Number of jobs to equal number of persons at maximum cost.
2. **Number of jobs to equal number of persons at minimum cost**
3. Only the maximize cost
4. Only to minimize cost

Solution: The objective of an assignment problem is to assign **Number of jobs to equal number of persons at minimum cost**.

Miscellaneous Exercise 7 | Q 2.01 | Page 127

Fill in the blank :

An assignment problem is said to be unbalanced when _______.

Solution: An assignment problem is said to be unbalanced when **number of rows is not equal to the number of columns**.

Miscellaneous Exercise 7 | Q 2.02 | Page 127

Fill in the blank :

When the number of rows is equal to the number of columns then the problem is said to be _______ assignment problem.

Solution: When the number of rows is equal to the number of columns then the problem is said to be **balanced** assignment problem.

Miscellaneous Exercise 7 | Q 2.03 | Page 127

Fill in the blank :

For solving an assignment problem the matrix should be a _______ matrix.

Solution: For solving an assignment problem the matrix should be a **square** matrix.

Miscellaneous Exercise 7 | Q 2.04 | Page 127

Fill in the blank :

If the given matrix is not a _______ matrix, the assignment problem is called an unbalanced problem.

Solution: If the given matrix is not a **square** matrix, the assignment problem is called an unbalanced problem.

Miscellaneous Exercise 7 | Q 2.05 | Page 127

Fill in the blank :

A dummy row(s) or column(s) with the cost elements as _______ is added to the matrix of an unbalanced assignment problem to convert into a square matrix.

Solution: A dummy row(s) or column(s) with the cost elements as **zero** is added to the matrix of an unbalanced assignment problem to convert into a square matrix.

Miscellaneous Exercise 7 | Q 2.06 | Page 127

Fill in the blank :

The time interval between starting the first job and completing the last job including the idle time (if any) in a particular order by the given set of machines is called _______.

Solution: The time interval between starting the first job and completing the last job including the idle time (if any) in a particular order by the given set of machines is called **Total elapsed time**.

Miscellaneous Exercise 7 | Q 2.07 | Page 127

Fill in the blank :

The time for which a machine j does not have a job to process to the start of job is called _______.

Solution: The time for which a machine j does not have a job to process to the start of job is called **Idle time**.

Miscellaneous Exercise 7 | Q 2.08 | Page 127

Fill in the blank :

Maximization assignment problem is transformed to minimization problem by subtracting each entry in the table from the _______ value in the table.

Solution: Maximization assignment problem is transformed to minimization problem by subtracting each entry in the table from the **maximum** value in the table.

Miscellaneous Exercise 7 | Q 2.09 | Page 127

Fill in the blank :

When an assignment problem has more than one solution, then it is _______ optimal solution.

Solution: When an assignment problem has more than one solution, then it is **multiple** optimal solution.

Miscellaneous Exercise 7 | Q 2.1 | Page 127

Fill in the blank :

The time required for printing of four books A, B, C and D is 5, 8, 10 and 7 hours while its data entry requires 7, 4, 3 and 6 hrs respectively. The sequence that minimizes total elapsed time is _______

Solution: The time required for printing of four books A, B, C and D is 5, 8, 10 and 7 hours while its data entry requires 7, 4, 3 and 6 hrs respectively. The sequence that minimizes total elapsed time is **A – D – B – C**.

Miscellaneous Exercise 7 | Q 2.11 | Page 127

Fill in the blank :

In Hungarian Method, only _______ task can be assigned to each facility.

Solution: In Hungarian Method, only **one** task can be assigned to each facility.

Miscellaneous Exercise 7 | Q 2.12 | Page 127

Fill in the blank :

In an assignment problem, a solution having _______ total cost is an optimum solution.

Solution: In an assignment problem, a solution having **zero** total cost is an optimum solution.

Miscellaneous Exercise 7 | Q 2.13 | Page 127

Fill in the blank :

In maximization type, all the elements in the matrix are subtracted from the _______ element in the matrix.

Solution: In maximization type, all the elements in the matrix are subtracted from the **largest** element in the matrix.

Miscellaneous Exercise 7 | Q 2.14 | Page 127

Fill in the blank :

In a sequencing problem, all machines are of _______ types.

Solution: In a sequencing problem, all machines are of **different** types.

Miscellaneous Exercise 7 | Q 2.15 | Page 127

Fill in the blank :

An _______ is a special type of linear programming problem.

Solution: An **assignment problem** is a special type of linear programming problem.

Miscellaneous Exercise 7 | Q 3.01 | Page 127

State whether the following is True or False :

One machine - one job is not an assumption in solving sequencing problems.

1. True
2. **False**

Solution: One machine - one job is not an assumption in solving sequencing problems **False**.

Miscellaneous Exercise 7 | Q 3.02 | Page 128

State whether the following is True or False :

If there are two least processing times for machine A and machine B, priority is given for the processing time which has lowest time of the adjacent machine.

1. **True**
2. False

Solution: If there are two least processing times for machine A and machine B, priority is given for the processing time which has lowest time of the adjacent machine **True**.

Miscellaneous Exercise 7 | Q 3.03 | Page 128

State whether the following is True or False :

To convert the assignment problem into a maximization problem, the smallest element in the matrix is deducted from all other elements.

1. True
2. **False**

Solution: To convert the assignment problem into a maximization problem, the smallest element in the matrix is deducted from all other elements **False**.

Miscellaneous Exercise 7 | Q 3.04 | Page 128

State whether the following is True or False :

The Hungarian method operates on the principle of matrix reduction, whereby the cost table is reduced to a set of opportunity costs.

1. **True**
2. False

Solution: The Hungarian method operates on the principle of matrix reduction, whereby the cost table is reduced to a set of opportunity costs **True**.

Miscellaneous Exercise 7 | Q 3.05 | Page 128

State whether the following is True or False :

In a sequencing problem, the processing times are dependent of order of processing the jobs on machines.

1. True
2. **False**

Solution: In a sequencing problem, the processing times are dependent of order of processing the jobs on machines **False**.

Miscellaneous Exercise 7 | Q 3.06 | Page 128

State whether the following is True or False :

Optimal assignments are made in the Hungarian method to cells in the reduced matrix that contain a zero.

1. **True**
2. False

Solution: Optimal assignments are made in the Hungarian method to cells in the reduced matrix that contain a zero **True**.

Miscellaneous Exercise 7 | Q 3.07 | Page 128

State whether the following is True or False :

Using the Hungarian method, the optimal solution to an assignment problem is found when the minimum number of lines required to cover the zero cells in the reduced matrix equals the no of persons.

1. **True**
2. False

Solution: Using the Hungarian method, the optimal solution to an assignment problem is found when the minimum number of lines required to cover the zero cells in the reduced matrix equals the no of persons **True**.

Miscellaneous Exercise 7 | Q 3.08 | Page 128

State whether the following is True or False :

In an assignment problem, if number of column is greater than number of rows, then a dummy column is added.

1. True
2. **False**

Solution: In an assignment problem, if number of column is greater than number of rows, then a dummy column is added **False**.

Miscellaneous Exercise 7 | Q 3.09 | Page 128

State whether the following is True or False :

The purpose of dummy row or column in an assignment problem is to obtain balance between total number of activities and total number of resources.

1. **True**
2. False

Solution: The purpose of dummy row or column in an assignment problem is to obtain balance between total number of activities and total number of resources **True**.

Miscellaneous Exercise 7 | Q 3.1 | Page 128

State whether the following is True or False :

One of the assumptions made while sequencing n jobs on 2 machines is: two jobs must be loaded at a time on any machine.

1. True
2. **False**

Solution: One of the assumptions made while sequencing n jobs on 2 machines is: two jobs must be loaded at a time on any machine **False**.

Miscellaneous Exercise 7 | Q 3.11 | Page 128

State whether the following is True or False :

In assignment problem, each facility is capable of performing each task.

1. **True**
2. False

Solution: In assignment problem, each facility is capable of performing each task **True**.

Miscellaneous Exercise 7 | Q 3.12 | Page 128

State whether the following is True or False

In number of lines (horizontal on vertical) > order of matrix then we get optimal solution.

1. True
2. **False**

Solution: In number of lines (horizontal on vertical) > order of matrix then we get optimal solution **False**.

Miscellaneous Exercise 7 | Q 3.13 | Page 128

State whether the following is True or False :

It is not necessary to express an assignment problem into n x n matrix.

1. True

2. **False**

Solution: It is not necessary to express an assignment problem into n x n matrix **False**.

Miscellaneous Exercise 7 | Q 3.14 | Page 128

State whether the following is True or False :

In a sequencing problem, a machine can process more than one job at a time.

1. True
2. **False**

Solution: In a sequencing problem, a machine can process more than one job at a time **False**.

Miscellaneous Exercise 7 | Q 3.15 | Page 128

State whether the following is True or False :

The time involved in moving a job from one machine to another is negligibly small.

1. **True**
2. False

Solution: The time involved in moving a job from one machine to another is negligibly small **True**.

PART I [PAGES 128 - 129]

Part I | Q 1 | Page 128

Solve the following problem :

A plant manager has four subordinates, and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. This estimate of the time each man would take to perform each task is given in the effectiveness matrix below.

	I	**II**	**III**	**IV**
A	7	25	26	10
B	12	27	3	25
C	37	18	17	14
D	18	25	23	9

How should the tasks be allocated, one to a man, as to minimize the total man hours?

Solution: Step 1: Row minimum
Subtract the smallest element in each row from every element in its row.
The matrix obtained is given below:

	I	**II**	**III**	**IV**
A	0	18	19	3
B	9	24	0	22
C	23	4	3	0
D	9	16	14	0

Step 2: Column minimum
Subtract the smallest element in each column of assignment matrix obtained in step 1 from every element in its column.

	I	**II**	**III**	**IV**
A	0	14	19	3
B	9	20	0	22
C	23	0	3	0
D	9	12	14	0

Step 3:
Draw minimum number of vertical and horizontal lines to cover all zeros.
First cover all rows and columns which have maximum number of zeros.

	I	**II**	**III**	**IV**
A	~~0~~	14	~~19~~	~~3~~
B	~~9~~	20	~~0~~	~~22~~
C	~~23~~	~~0~~	~~3~~	~~0~~
D	~~9~~	12	~~14~~	~~0~~

Step 4:
From step 3, minimum number of lines covering all the zeros are 4, which is equal to order of the matrix, i.e., 4.
∴ Select a row with exactly one zero, enclose that zero in (□) and cross out all zeros in its respective column.
Similarly, examine each row and column and mark the assignment (□).
∴ The matrix obtained is as follows:
Assigning through zeroes, we get,

	I	**II**	**III**	**IV**

A	**0**	14	19	3
B	9	20	**0**	22
C	23	**0**	3	~~0~~
D	9	12	1	**0**

Step 5:
The matrix obtained in step 4 contains exactly one assignment for each row and column.
∴ Optimal assignment schedule is as follows:

Job	Subordination	Time (hrs)
A	I	7
B	III	3
C	II	18
D	IV	9

∴ Total minimum time = 7 + 3 + 18 + 9 = 37 hrs.

Part I | Q 2 | Page 128

Solve the following problem :

A dairy plant has five milk tankers, I, II, III, IV & V. These milk tankers are to be used on five delivery routes A, B, C, D & E. The distances (in kms) between the dairy plant and the delivery routes are given in the following distance matrix.

	I	II	III	IV	V
A	150	120	175	180	200
B	125	110	120	150	165
C	130	100	145	160	175
D	40	40	70	70	100
E	45	25	60	70	95

How should the milk tankers be assigned to the chilling center so as to minimize the distance travelled?

Solution: Step 1: Row minimum
Subtract the smallest element in each row from every element in its row.
The matrix obtained is given below:

	I	**II**	**III**	**IV**	**V**
A	30	0	55	60	80
B	15	0	10	40	55
C	30	0	45	60	75
D	0	0	30	30	60
E	20	0	35	45	70

Step 2: Column minimum
Subtract the smallest element in each column of assignment matrix obtained in step 1 from every element in its column.

	I	**II**	**III**	**IV**	**V**
A	30	0	45	30	25
B	15	0	0	10	0
C	30	0	35	30	20
D	0	0	20	0	5
E	20	0	25	15	15

Step 3:
Draw minimum number of vertical and horizontal lines to cover all zeros. F
First cover all rows and columns which have maximum number of zeros.

	I	**II**	**III**	**IV**	**V**
A	30	~~0~~	45	0	25
B	~~15~~	~~0~~	~~0~~	~~10~~	~~0~~
C	30	~~0~~	20	0	5
D	~~0~~	~~0~~	~~20~~	~~0~~	~~5~~
E	20	~~0~~	25	15	15

Step 4:
From step 3, minimum number of lines covering all the zeros are 3, which is less than order of matrix, i.e., 5.
∴ Select smallest element from all the uncovered elements, i.e., 15 and subtract it from

all the uncovered elements and add it to the elements which lie at the intersection of two lines.

	I	II	III	IV	V
A	15	0	30	15	10
B	15	15	0	10	10
C	15	0	20	15	5
D	0	15	20	0	5
E	5	0	10	0	0

Step 5:
Draw minimum number of vertical and horizontal lines to cover all zeros.

	I	II	III	IV	V
A	15	~~0~~	30	15	10
B	~~15~~	~~15~~	~~0~~	~~10~~	~~0~~
C	15	~~0~~	20	15	5
D	~~0~~	~~15~~	~~20~~	~~0~~	~~5~~
E	~~5~~	~~0~~	~~10~~	~~0~~	~~0~~

Step 6:
From step 5, minimum number of lines covering all the zeros are 4, which is less than order of matrix, i.e., 5.
∴ Select smallest element from all the uncovered elements, i.e., 5 and subtract it from all the uncovered elements and add it to the elements which lie at the intersection of two lines.

	I	II	III	IV	V
A	10	0	25	10	5
B	15	20	0	10	0
C	10	0	15	10	0
D	0	20	20	0	5
E	5	5	10	0	0

Step 7:
Draw minimum number of vertical and horizontal lines to cover all zeros.

	I	II	III	IV	V
A	10	~~0~~	25	10	~~5~~
B	~~15~~	~~20~~	~~0~~	~~10~~	~~0~~
C	10	~~0~~	15	10	~~0~~
D	~~0~~	~~20~~	~~20~~	~~0~~	~~5~~
E	~~5~~	~~5~~	~~10~~	~~0~~	~~0~~

Step 8:
From step 7, minimum number of lines covering all the zeros are 5, which is equal to order of the matrix, i.e., 5.
∴ Select a row with exactly one zero, enclose that zero in (□) and cross out all zeros in its respective column.
Similarly, examine each row and column and mark the assignment (□).
∴ The matrix obtained is as follows:

	I	**II**	**III**	**IV**	**V**
A	10	**0**	25	10	5
B	15	20	**0**	10	0
C	10	~~0~~	15	10	~~0~~
D	**0**	20	20	~~0~~	5
E	5	5	10	**0**	~~0~~

Step 9:
The matrix obtained in step 8 contains exactly one assignment for each row and column.
∴ Optimal assignment schedule is as follows:

Routes	**Dairy Plant**	**Distance (kms)**
A	II	120
B	III	120
C	V	175
D	I	40
E	IV	70
		525

∴ Minimum distance travelled = 120 + 120 + 175 + 40 + 70 = 525 kms.

Part I | Q 3 | Page 128

Solve the following problem :

Solve the following assignment problem to maximize sales:

Salesman	Territories				
	I	**II**	**III**	**IV**	**V**
A	11	16	18	15	15
B	7	19	11	13	17
C	9	6	14	14	7
D	13	12	17	11	13

Solution: Step 1:

The given problem is maximization problem. This can be converted to minimization problem by subtracting all the elements from the largest element which is 19.

Also, the number of rows is not equal to number of columns.

∴ It is an unbalanced assignment problem. It can be balanced by introducing a dummy salesman E with zero sales.

The resulting matrix is

Salesman	Territories				
	I	**II**	**III**	**IV**	**V**
A	8	3	1	4	4
B	12	0	8	6	2
C	10	13	5	5	12
D	6	7	2	8	6
E	0	0	0	0	0

Step 2: Row minimum

Subtract the smallest element in each row from every element in its row. The matrix obtained is given below:

Salesman	Territories				
	I	**II**	**III**	**IV**	**V**
A	7	2	0	3	3
B	12	0	8	6	2
C	5	8	0	0	7

D	4	5	0	6	4
E	0	0	0	0	0

Step 3: Column minimum

Here, each column contains element zero.

∴ Matrix obtained by column minimum is same as above matrix.

Step 4:

Draw minimum number of vertical and horizontal lines to cover all zeros.

First cover all rows and columns which have maximum number of zeros.

Salesman	**Territories**				
	I	**II**	**III**	**IV**	**V**
A	7	~~2~~	~~0~~	3	3
B	12	~~0~~	~~8~~	6	2
C	~~5~~	~~8~~	~~0~~	~~0~~	~~7~~
D	4	~~5~~	~~0~~	6	4
E	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~

Step 5:

From step 4, minimum number of lines covering all the zeros are 4, which is less than order of matrix, i.e., 5.

∴ Select smallest element from all the uncovered elements, i.e., 2 and subtract it from all the uncovered elements and add it to the elements which lie at the intersection of two lines.

Salesman	**Territories**				
	I	**II**	**III**	**IV**	**V**
A	5	2	0	1	1
B	10	0	8	4	0
C	5	10	2	0	7
D	2	5	0	4	2
E	0	2	2	0	0

Step 6:

Draw minimum number of vertical and horizontal lines to cover all zeros.

Salesman	Territories				
	I	II	III	IV	V
A	5	2	~~0~~	~~1~~	1
B	~~10~~	~~0~~	~~8~~	~~4~~	~~0~~
C	5	10	~~2~~	~~0~~	7
D	2	5	~~0~~	~~4~~	2
E	~~0~~	~~2~~	~~2~~	~~0~~	~~0~~

Step 7:

From step 6, minimum number of lines covering all the zeros are 4, which is less than order of matrix, i.e., 5.

∴ Select smallest element from all the uncovered elements, i.e., 1 and subtract it from all the uncovered elements and add it to the elements which lie at the intersection of two lines.

Salesman	Territories				
	I	II	III	IV	V
A	4	1	0	1	0
B	10	0	9	5	0
C	4	9	2	0	6
D	1	4	0	4	1
E	0	2	3	1	0

Step 8:

Draw minimum number of vertical and horizontal lines to cover all zeros.

Salesman	Territories				
	I	II	III	IV	V
A	4	1	~~0~~	~~1~~	~~0~~
B	~~10~~	~~0~~	~~9~~	~~5~~	~~0~~
C	4	9	~~2~~	~~0~~	~~6~~

D	1	4	~~0~~	~~4~~	~~1~~
E	~~0~~	~~2~~	~~3~~	~~1~~	~~0~~

Step 9:

From step 8, minimum number of lines covering all the zeros are 5, which is equal to order of the matrix, i.e., 5.

∴ Select a row with exactly one zero, enclose that zero in (□) and cross out all zeros in its respective column.

Similarly, examine each row and column and mark the assignment (□).

∴ The matrix obtained is as follows:

Salesman	**Territories**				
A	**I**	**II**	**III**	**IV**	**V**
B	4	1	~~0~~	1	**0**
C	10	**0**	9	5	~~0~~
D	4	9	2	**0**	6
E	1	4	**0**	4	1
	0	2	3	1	~~0~~

∴ The optimal solution is

Salesman	**Territories**	**Sales**
A	V	15
B	II	19
C	IV	14
D	III	17
E	I	0

∴ Maximum sales = 15 + 19 + 14 + 17 + 0 = 65 units.

Part I | Q 4 | Page 128

Solve the following problem :

The estimated sales (tons) per month in four different cities by five different managers are given below:

Manager	Cities			
	P	**Q**	**R**	**S**
I	34	36	33	35
II	33	35	31	33
III	37	39	35	35
IV	36	36	34	34
V	35	36	35	33

Find out the assignment of managers to cities in order to maximize sales.

Solution: Step 1:

The given problem is maximization problem. This can be converted to minimization problem by subtracting all the elements from the largest element which is 39.

Also, the number of rows is not equal to number of columns.

∴ It is an unbalanced assignment problem. It can be balanced by introducing a dummy city T with zero sales.

The resulting matrix is

Manager	Cities				
	P	**Q**	**R**	**S**	**T**
I	5	3	6	4	0
II	6	4	8	6	0
III	2	0	4	4	0
IV	3	3	5	5	0
V	4	3	4	6	0

Step 2: Row minimum

Here, each row contains element zero.

∴ Matrix obtained by row minimum is same as above matrix

Step 3: Column minimum

Subtract the smallest element in each column of assignment matrix obtained in step 2 from every element in its column.

Manager	Cities

	P	Q	R	S	T
I	3	3	2	0	0
II	4	4	4	2	0
III	0	0	0	0	0
IV	1	3	1	1	0
V	2	3	0	2	0

Step 4:
Draw minimum number of vertical and horizontal lines to cover all zeros.
First cover all rows and columns which have maximum number of zeros.

Manager	**Cities**				
	P	**Q**	**R**	**S**	**T**
I	3	3	~~2~~	~~0~~	~~0~~
II	4	4	~~4~~	~~2~~	~~0~~
III	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~
IV	1	3	~~1~~	~~1~~	~~0~~
V	2	3	~~0~~	~~2~~	~~0~~

Step 5:
From step 4, minimum number of lines covering all the zeros are 4, which is less than order of matrix, i.e., 5.
∴ Select smallest element from all the uncovered elements, i.e., 1 and subtract it from all the uncovered elements and add it to the elements which lie at the intersection of two lines.

Manager	**Cities**				
	P	**Q**	**R**	**S**	**T**
I	2	2	2	0	0
II	3	3	4	2	0
III	0	0	1	1	1
IV	0	2	1	1	0
V	1	2	0	2	0

Step 6:
Draw minimum number of vertical and horizontal lines to cover all zeros.

Manager	Cities				
	P	Q	R	S	T
I	~~2~~	~~2~~	~~2~~	~~0~~	~~0~~
II	~~3~~	3	4	2	~~0~~
III	~~0~~	~~0~~	~~1~~	~~1~~	~~1~~
IV	~~0~~	2	1	1	~~0~~
V	~~1~~	~~2~~	~~0~~	~~2~~	~~0~~

Step 7:
From step 6, minimum number of lines covering all the zeros are 5, which is equal to order of the matrix, i.e., 5.
∴ Select a row with exactly one zero, enclose that zero in (□) and cross out all zeros in its respective column.
Similarly, examine each row and column and mark the assignment (□).
∴ The matrix obtained is as follows:

Manager	**Cities**				
	P	**Q**	**R**	**S**	**T**
I	2	2	2	**0**	~~0~~
II	3	3	4	2	**0**
III	~~0~~	**0**	1	1	1
IV	**0**	2	1	1	~~0~~
V	1	2	**0**	2	~~0~~

∴ The optimal solution is

Manager	**Cities**	**Sales (tons)**
I	S	35
II	T	0
III	Q	39
IV	P	36
V	R	35

∴ Maximum sales 35 + 0 + 39 + 36 + 35 = 145 tons.

Part I | Q 5 | Page 129

Solve the following problem :

Consider the problem of assigning five operators to five machines. The assignment costs are given in following table.

Operator	Machine				
	1	**2**	**3**	**4**	**5**
A	6	6	–	3	7
B	8	5	3	4	5
C	10	4	6	–	4
D	8	3	7	8	3
E	7	6	8	10	2

Operator A cannot be assigned to machine 3 and operator C cannot be assigned to machine 4. Find the optimal assignment schedule.

Solution: Step 1:

Observe that the given problem is a restricted assignment problem. So we assign high cost '∞' to the prohibited cells.

Operator	Machine				
	1	**2**	**3**	**4**	**5**
A	6	6	∞	3	7
B	8	5	3	4	5
C	10	4	6	∞	4
D	8	3	7	8	3
E	7	6	8	10	2

Step 2: Row minimum

Subtract the smallest element in each row from every element in its row.

The matrix obtained is given below:

Operator	Machine				
	1	**2**	**3**	**4**	**5**
A	3	3	∞	0	4
B	5	2	0	1	2
C	6	0	2	∞	0
D	5	0	4	5	0

E	5	4	6	8	0

Step 3: Column minimum
Subtract the smallest element in each column of assignment matrix obtained in step 2 from every element in its column.

Operator	**Machine**				
	1	**2**	**3**	**4**	**5**
A	0	3	∞	0	4
B	2	2	0	1	2
C	3	0	2	`oo	0
D	2	0	4	5	0
E	2	4	6	8	0

Step 4:
Draw minimum number of vertical and horizontal lines to cover all zeros.
First cover all rows and columns which have maximum number of zeros.

Operator	**Machine**				
	1	**2**	**3**	**4**	**5**
A	~~0~~	~~3~~	~~∞~~	~~0~~	~~4~~
B	~~2~~	~~2~~	~~0~~	~~1~~	~~2~~
C	3	~~0~~	2	`oo	~~0~~
D	2	~~0~~	4	5	~~0~~
E	2	~~4~~	6	8	~~0~~

Step 5:
From step 4, minimum number of lines covering all the zeros are 4, which is less than order of matrix, i.e., 5.
∴ Select smallest element from all the uncovered elements, i.e., 2 and subtract it from all the uncovered elements and add it to the elements which lie at the intersection of two lines.

Operator	**Machine**				
	1	**2**	**3**	**4**	**5**

A	0	5	∞	0	6
B	2	4	0	1	4
C	1	0	0	∞	0
D	0	0	2	3	0
E	0	4	4	6	0

Step 6:
Draw minimum number of vertical and horizontal lines to cover all zeros.

Operator	**Machine**				
	1	**2**	**3**	**4**	**5**
A	~~0~~	~~5~~	~~∞~~	~~0~~	~~6~~
B	~~2~~	~~4~~	~~0~~	1	~~4~~
C	~~1~~	~~0~~	~~0~~	∞	~~0~~
D	~~0~~	~~0~~	~~2~~	3	~~0~~
E	~~0~~	~~4~~	~~4~~	6	~~0~~

Step 7:
From step 6, minimum number of lines covering all the zeros are 5, which is equal to order of the matrix, i.e., 5.
∴ Select a row with exactly one zero, enclose that zero in (□) and cross out all zeros in its respective column.
Similarly, examine each row and column and mark the assignment (□).
∴ The matrix obtained is as follows:

Operator	**Machine**				
	1	**2**	**3**	**4**	**5**
A	~~0~~	5	∞	**0**	6
B	2	4	**0**	1	4
C	1	**0**	~~0~~	∞	~~0~~
D	~~0~~	~~0~~	2	3	**0**
E	**0**	4	4	6	~~0~~

OR

Operator	Machine				
	1	**2**	**3**	**4**	**5**
A	~~0~~	5	∞	**0**	6
B	2	4	**0**	1	4
C	1	**0**	~~0~~	∞	~~0~~
D	**0**	~~0~~	2	3	~~0~~
E	~~0~~	4	4	6	**0**

OR

Operator	Machine				
	1	**2**	**3**	**4**	**5**
A	~~0~~	5	∞	**0**	6
B	2	4	**0**	1	4
C	1	~~0~~	~~0~~	∞	**0**
D	~~0~~	**0**	2	3	~~0~~
E	**0**	4	4	6	~~0~~

Step 8:
The matrix obtained in step 7 contains exactly one assignment for each row and column.
∴ Optimal assignment schedule is as follows:

Operator	Machine	Cost
A	4	3
B	3	3
C	2	4
D	5	3
E	1	7

Total	20

OR

Operator	Machine	Cost
A	4	3
B	3	3
C	2	4
D	1	8
E	5	2
Total		20

OR

Operator	Machine	Cost
A	4	3
B	3	3
C	5	4
D	2	3
E	1	7
Total		20

∴ Minimum cost = 20 units.

Part I | Q 6 | Page 129

Solve the following problem :

A chartered accountant's firm has accepted five new cases. The estimated number of days required by each of their five employees for each case are given below, where - means that the particular employee cannot be assigned the particular case. Determine the optimal assignment of cases of the employees so that the total number of days required to complete these five cases will be minimum. Also find the minimum number of days.

Employee	Cases				
	I	II	III	IV	V

E_1	6	4	5	7	8
E_2	7	–	8	6	9
E_3	8	6	7	9	10
E_4	5	7	–	4	6
E_5	9	5	3	10	–

Solution: Step 1:
Observe that the given problem is a restricted assignment problem. So we assign a very high number of days ∞ to the prohibited cells.

Employee	**Cases**				
	I	**II**	**III**	**IV**	**V**
E_1	6	4	5	7	8
E_2	7	∞	8	6	9
E_3	8	6	7	9	10
E_4	5	7	∞	4	6
E_5	9	5	3	10	∞

Step 2: Row minimum
Subtract the smallest element in each row from every element in its row.
The matrix obtained is given below:

Employee	**Cases**				
	I	**II**	**III**	**IV**	**V**
E_1	2	0	1	3	4
E_2	1	∞	2	0	3
E_3	2	0	1	3	4
E_4	1	3	∞	0	2
E_5	6	2	0	7	∞

Step 3: Column minimum
Subtract the smallest element in each column of assignment matrix obtained in step 2 from every element in its column.

Employee	**Cases**				
	I	**II**	**III**	**IV**	**V**
E_1	1	0	1	3	2

E_2	0	∞	2	0	1
E_3	1	0	1	3	2
E_4	0	3	∞	0	0
E_5	5	2	0	7	∞

Step 4:
Draw minimum number of vertical and horizontal lines to cover all zeros.
First cover all rows and columns which have maximum number of zeros.

Employee	**Cases**				
	I	**II**	**III**	**IV**	**V**
E_1	1	~~0~~	1	3	2
E_2	~~0~~	~~∞~~	~~2~~	~~0~~	~~1~~
E_3	1	~~0~~	1	3	2
E_4	~~0~~	~~3~~	~~∞~~	~~0~~	~~0~~
E_5	~~5~~	~~2~~	~~0~~	~~7~~	~~∞~~

Step 5:
From step 4, minimum number of lines covering all the zeros are 4, which is less than order of matrix, i.e., 5.
∴ Select smallest element from all the uncovered elements, i.e., 1 and subtract it from all the uncovered elements and add it to the elements which lie at the intersection of two lines.

Employee	**Cases**				
	I	**II**	**III**	**IV**	**V**
E_1	0	0	0	2	1
E_2	0	∞	2	0	1
E_3	0	0	0	2	1
E_4	0	4	∞	0	0
E_5	5	3	0	7	∞

Step 6:
Draw minimum number of vertical and horizontal lines to cover all zeros.

Employee	**Cases**				
	I	**II**	**III**	**IV**	**V**
E_1	~~0~~	~~0~~	~~0~~	~~2~~	1

E2	~~0~~	~~∞~~	~~2~~	~~0~~	1
E3	~~0~~	~~0~~	~~0~~	~~2~~	1
E4	~~0~~	4	~~∞~~	~~0~~	~~0~~
E5	~~5~~	~~3~~	~~0~~	~~7~~	∞

Step 7:

From step 6, minimum number of lines covering all the zeros are 5, which is equal to order of the matrix, i.e., 5.

∴ Select a row with exactly one zero, enclose that zero in (□) and cross out all zeros in its respective column.

Similarly, examine each row and column and mark the assignment (□).

∴ The matrix obtained is as follows:

Employee	**Cases**				
	I	**II**	**III**	**IV**	**V**
E1	**0**	~~0~~	~~0~~	2	1
E2	~~0~~	∞	2	**0**	1
E3	~~0~~	**0**	~~0~~	2	1
E4	~~0~~	4	∞	~~0~~	**0**
E5	5	3	**0**	7	∞

OR

Employee	**Cases**				
	I	**II**	**III**	**IV**	**V**
E1	~~0~~	**0**	~~0~~	2	1
E2	~~0~~	∞	2	**0**	1
E3	**0**	~~0~~	~~0~~	2	1
E4	~~0~~	4	∞	~~0~~	**0**
E5	5	3	**0**	7	∞

Step 8:

The matrix obtained in step 7 contains exactly one assignment for each row and

column.
∴ Optimal assignment schedule is as follows:

Employee	Cases	Time (days)
E_1	I	6
E_2	II	6
E_3	III	6
E_4	IV	6
E_5	V	3
Total		27

OR

Employee	Cases	Time (days)
E_1	I	4
E_2	II	6
E_3	III	8
E_4	IV	6
E_5	V	3
Total		27

∴ Minimum Time = 27 days.

PART II [PAGES 129 - 130]

Part II | Q 1 | Page 129

Solve the following problem :

A readymade garments manufacturer has to process 7 items through two stages of production, namely cutting and sewing. The time taken in hours for each of these items in different stages are given below:

Items	1	2	3	4	5	6	7
Time for Cutting	5	7	3	4	6	7	12

Time for Sewing	2	6	7	5	9	5	8

Find the sequence in which these items are to be processed through these stages so as to minimize the total processing time. Also find the idle time of each machine.

Solution:

Items	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Time for Cutting	5	7	3	4	6	7	12
Time for Sewing	2	6	7	5	9	5	8

Observe that Min (cutting, sewing) = 2, corresponds to item 1 on sewing.
∴ Item 1 is placed last in sequence.

						1

Then the problem reduces to

Items	**2**	**3**	**4**	**5**	**6**	**7**
Time for Cutting	7	3	4	6	7	12
Time for Sewing	6	7	5	9	5	8

Now, Min (cutting, sewing) = 3, corresponds to item 3 on cutting.
∴ Item 3 is placed first in sequence.

3						1

Then the problem reduces to

Items	**2**	**4**	**5**	**6**	**7**
Time for Cutting	7	4	6	7	12
Time for Sewing	6	5	9	5	8

Now, Min (cutting, sewing) = 4, corresponds to item 4 on cutting.
∴ Item 4 is placed after 3 in sequence.

3	4						1

Item 4 is placed after 3 in sequence.

Items	2	5	6	7
Time for Cutting	7	6	7	12
Time for Sewing	6	9	5	8

Now, Min (cutting, sewing) = 5, corresponds to item 6 on sewing
∴ Item 6 is placed before 1 in sequence.

3	4				6	1

Then the problem reduces to

Items	2	5	7
Time for Cutting	7	6	12
Time for Sewing	6	9	8

Now, Min (cutting, sewing) = 6, corresponds to item 2 on sewing and item 5 on cutting.
∴ Item 2 is placed before 6 and item 5 is placed after 4 and item 7 on remaining in sequence.
∴ Optimal sequence is

3	4	5	7	2	6	1

Total elapsed time

Item	Cutting		Sewing	
	In	Out	In	Out
3 (3, 7)	0	3	3	10
4 (4, 5)	3	7	10	15

5 (6, 9)	7	13	15	**24**
7 (12, 8)	13	25	**25**	33
2 (7, 6)	25	32	33	39
6 (7, 5)	32	39	39	44
1 (5, 2)	39	44	44	46

∴ Total elapsed time = 46 hrs
Idle time for cutting = 46 – 44 = 2 hrs
Idle time for sewing = 3 + 1 = 4 hrs.

Part II | Q 2 | Page 129

Solve the following problem :

Five jobs must pass through a lathe and a surface grinder, in that order. The processing times in hours are shown below. Determine the optimal sequence of the jobs. Also find the idle time of each machine.

Job	**I**	**II**	**III**	**IV**	**V**
Lathe	4	1	5	2	5
Surface grinder	3	2	4	3	6

Solution:

Job	**I**	**II**	**III**	**IV**	**V**
Lathe	4	1	5	2	5
Surface grinder	3	2	4	3	6

Observe that Min (lathe, grinder) = 1, corresponds to job II on lathe.
∴ Job II is placed first in sequence.

II				

Then the problem reduces to

Job	**I**	**III**	**IV**	**V**
Lathe	4	5	2	5
Surface grinder	3	4	3	6

Now, Min (lathe, grinder) = 2, corresponds to job IV on lathe.
∴ Job IV is placed after II in sequence.

II	IV			

Then the problem reduces to

Job	**I**	**III**	**V**
Lathe	4	5	5
Surface grinder	3	4	6

Now, Min (lathe, grinder) = 3, corresponds to job I on grinder.
∴ Job I is placed last in sequence.

II	IV			I

Then the problem reduces to

Job	**III**	**V**
Lathe	5	5
Surface grinder	4	6

Now, Min (lathe, grinder) = 4, corresponds to job III on grinder.
∴ Job III is placed before I and V on remaining in sequence.

∴ Optimal sequence is

II	IV	V	III	I

Total elapsed time

Job	**Lath**		**Surface Grinder**	
	In	**Out**	**In**	**Out**
II (1, 2)	0	1	1	3
IV (2, 3)	1	3	3	**6**
V (5, 6)	3	8	**8**	14
III (5, 4)	8	13	14	18
I (4, 3)	13	17	18	21

∴ Total elapsed time = 21 hrs
Idle time for lathe = 21 – 17 = 4 hrs
Idle time for surface grinder = 1 + 2 = 3 hrs.

Part II | Q 3 | Page 129

Solve the following problem :

Find the sequence that minimizes the total elapsed time to complete the following jobs. Each job is processed in order AB.

	Jobs (Processing times in minutes)						
	I	**II**	**III**	**IV**	**V**	**Vi**	**VII**
Machine A	12	6	5	11	5	7	6
Machine B	7	8	9	4	7	8	3

Determine the sequence for the jobs so as to minimize the processing time. Find the total elapsed time and the idle times for both the machines.

Solution:

	Jobs (Processing times in minutes)						
	I	**II**	**III**	**IV**	**V**	**Vi**	**VII**
Machine A	12	6	5	11	5	7	6
Machine B	7	8	9	4	7	8	3

Observe that Min (A, B) = 3, corresponds to job VII on machine B.
∴ Job VII is placed last in sequence.

						VII

Then the problem reduces to

	Jobs (Processing times in minutes)					
	I	**II**	**III**	**IV**	**V**	**Vi**
Machine A	12	6	5	11	5	7
Machine B	7	8	9	4	7	8

Now, Min (A, B) = 4, corresponds to job IV on machine B
∴ Job IV is placed before VII in sequence.

					IV	VII

Then the problem reduces to

	Jobs (Processing times in minutes)				
	I	**II**	**III**	**V**	**VI**
Machine A	12	6	5	5	7
Machine B	7	8	9	7	8

Now, Min (A, B) = 5, corresponds to job III and V on machine A.
∴ Job III and V is placed either first or second in sequence.

III	V				IV	VII

OR

V	III				IV	VII

Then the problem reduces to

	Jobs (Processing times in minutes)		
	I	II	VI
Machine A	12	6	7
Machine B	7	8	8

Now, Min (A, B) = 6, corresponds to job II on machine A
∴ Job II is placed on third place in sequence.

III	V	II			IV	VII

OR

V	III	II			IV	VII

Then the problem reduces to

	Jobs (Processing times in minutes)	
	I	**VI**
Machine A	12	7
Machine B	7	8

Now, Min (A, B) = 7, corresponds to job I on machine B and VI on machine A.
∴ Job I is placed before IV and job VI on remaining in sequence.

III	V	II	VI	I	IV	VII

OR

V	III	II	VI	I	IV	VII

We take the optimal sequence as,

III	V	II	Vi	I	IV	VII

Total elapsed time

Job	Machine A		Machine B	
	In	Out	In	Out
III (5, 9)	0	5	5	14
V (5, 7)	5	10	14	21
II (6, 8)	10	16	21	29
VI (7, 8)	16	23	29	37
I (12, 7)	23	25	37	**44**
IV (11, 4)	35	46	**46**	**50**
VII (6, 3)	56	52	**52**	55

∴ Total elapsed time = 55 mins
Idle time for machine A = 55 – 52 = 3 mins
Idle time for machine B = 5 + 2 + 2 = 9 mins.

Part II | Q 4 | Page 129

Solve the following problem :

A toy manufacturing company produces five types of toys. Each toy has to go through three machines A, B, C in the order ABC. The time required in hours for each process is given in the following table.

Type	1	2	3	4	5
Machine A	16	20	12	14	22
Machine B	10	12	4	6	8
Machine C	8	18	16	12	10

Solve the problem for minimizing the total elapsed time.

Solution:

Type	1	2	3	4	5
Machine A	16	20	12	14	22
Machine B	10	12	4	6	8
Machine C	8	18	16	12	10

Here min A = 12, max B = 12, min C = 8.
Since min A ≥ max B is satisfied, the problem can be converted into a two machine problem.
Let G and H be two fictitious machines such that G = A + B and H = B + C
Then the problem can be written as

Type	**1**	**2**	**3**	**4**	**5**
Machine G	26	32	16	20	30
Machine H	18	30	20	18	18

Observe that Min (G, H) = 16, corresponds to Type 3 on machine G.
∴ Type 3 is placed first in sequence.

3				

Then the problem reduces to

Type	**1**	**2**	**4**	**5**
Machine G	26	32	20	30
Machine H	18	30	18	18

Now, Min (G, H) = 18, corresponds to Type 1, Type 4 and Type 5 on machine H.
∴ Type 1, 4 and 5 are placed either last, second last or third last and Type 2 on remaining in the sequence.

3	2	5	4	1

OR

3	2	5	1	4

OR

3	2	1	4	5

OR

3	2	1	5	4

OR

3	2	4	1	5

OR

3	2	4	5	1

We take the optimal sequence as, 3 – 2 – 5 – 4 – 1
Total elapsed time

Job	Machine A		Machine B		Machine C	
	In	**Out**	**In**	**Out**	**In**	**Out**
3 (12, 4, 16)	0	12	12	16	16	**32**
2 (20, 12, 18)	12	32	32	**44**	**44**	62
5 (22, 8, 10)	32	54	**54**	**62**	62	**72**
4 (14, 6, 12)	54	68	**68**	**74**	**74**	**86**
1 (16, 10, 8)	68	84	**84**	94	**94**	102

∴ Total elapsed time = 102 hrs
Idle time for machine A = 102 – 84 = 18 hrs
Idle time for machine B = (102 – 94) + 12 + 16 + 10 + 6 + 10 = 62
Idle time for machine C = 16 + 12 + 2 + 8 = 38 hrs.

Part II | Q 5 | Page 130

Solve the following problem :

A foreman wants to process 4 different jobs on three machines: a shaping machine, a drilling machine and a tapping machine, the sequence of operations being shaping-drilling-tapping. Decide the optimal sequence for the four jobs to minimize the total elapsed time. Also find the total elapsed time and the idle time for every machine.

Job	Shaping (Minutes)	Drilling (Minutes)	Trapping (Minutes)
1	13	3	18
2	18	8	4
3	8	6	13
4	23	6	8

Solution:

job	1	2	3	4

Shaping (Minutes)	13	18	8	23
Drilling (Minutes)	3	8	6	6
Trapping (Minute)	18	4	13	8

Here, Min Shaping = 8, Max Drilling = 8, Min Trapping = 4.
Since Min Shaping ≥ Max Drilling is satisfied, the problem can be converted into a two machine problem.
Let G and H be two fictitious machines such that G = Shaping + Drilling and H = Drilling + Trapping
Then the problem can be written as

Job	**1**	**2**	**3**	**4**
Machine G	16	26	14	29
Machine H	21	12	19	14

Observe that Min (G, H) = 12, corresponds to Job 2 on machine H.
∴ Job 2 is placed last in sequence.

			2

Then the problem reduces to

Job	**1**	**3**	**4**
Machine G	16	14	29
Machine H	21	19	14

Now, Min (G, H) = 14, corresponds to Job 3 on machine G and Job 4 on machine H.
∴ Job 3 is placed first in the sequence, Job 4 is placed second last in the sequence and remaining Job 1 is placed second in the sequence.
∴ Optimal sequence is

3	1	4	2

Total elapsed time

Job	**Shaping**		**Drilling**		**Trapping**	
	In	**Out**	**In**	**Out**	**In**	**Out**
3 (8, 6, 13)	0	8	8	**14**	14	27

1(13, 3, 18)	8	21	**21**	**24**	27	**45**
4 (23, 6, 8)	21	44	**44**	**50**	**50**	**58**
2 (18, 8, 4)	44	62	**62**	70	**70**	74

∴ Total elapsed time = 74 mins
Idle time for Shaping = 74 – 62 = 12 mins
Idle time for Drilling = (74 – 70) + 8 + 7 + 20 + 12 = 51 mins
Idle time for Trapping = 14 + 5 +12 = 31 mins.