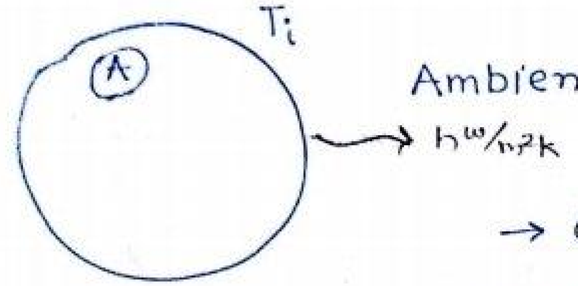
Unsteady state (OR) Transient Conduction H.T.

$T = f(\text{time})$



A - Contact Area

m, V, ρ, C_p

m = mass
V = Volume
ρ = density
C_p = Specific heat

→ Consider a solid body of mass m, Volume V, density ρ, specific heat C_p, which is at an initial temp. of T_i (having been heated in a furnace) is suddenally exposed to an ambiant fluid which is at a Temp of T_∞.

Since the body keeps on loosing heat by Convection to the ambient Fluid (a thermal resorvior) with a Convective H.T. Coeff. of $h\ W/m^2K$. The internal energy of body keeps on decreasing as the time progresses which is manifested by decreasing temp. of body w.r.t. time

let T_i = Initial temp. of body at the instant of time $\tau = 0$ sec. i.e. when body just exposed to ambient Fluid $\quad T = f(\tau)$

T = temp. of body at any instant of time 'τ' sec later

→ writing the energy balance for the body at any instant of time 'τ' sec.

the rate of Convective H.T. b/w body and Fluid = The rate of decrease of I.E. of body w.r.t. time

$$\Rightarrow \quad hA(T - T_\infty) = -mC_p \frac{dT}{d\tau} \quad \text{Joule/se.}$$

$$hA(T - T_\infty) = -\rho V C_p \frac{dT}{d\tau} \quad \text{J/sec.}$$

treating all other parameter including h as Constant and seprating the variable time and temp. we get

$$\int_0^\tau \left(\frac{hA}{\rho V C_p}\right) d\tau = \int_{T_i}^{T} \frac{-dT}{(T - T_\infty)}$$

$$\frac{hA}{\rho V C_p}\left[\tau\right]_0^\tau = \left[-\ln(T - T_\infty)\right]_{T_i}^{T}$$

$$\frac{hA}{\rho V C_p}\tau = -\ln(T - T_\infty) + \ln(T_i - T_\infty)$$

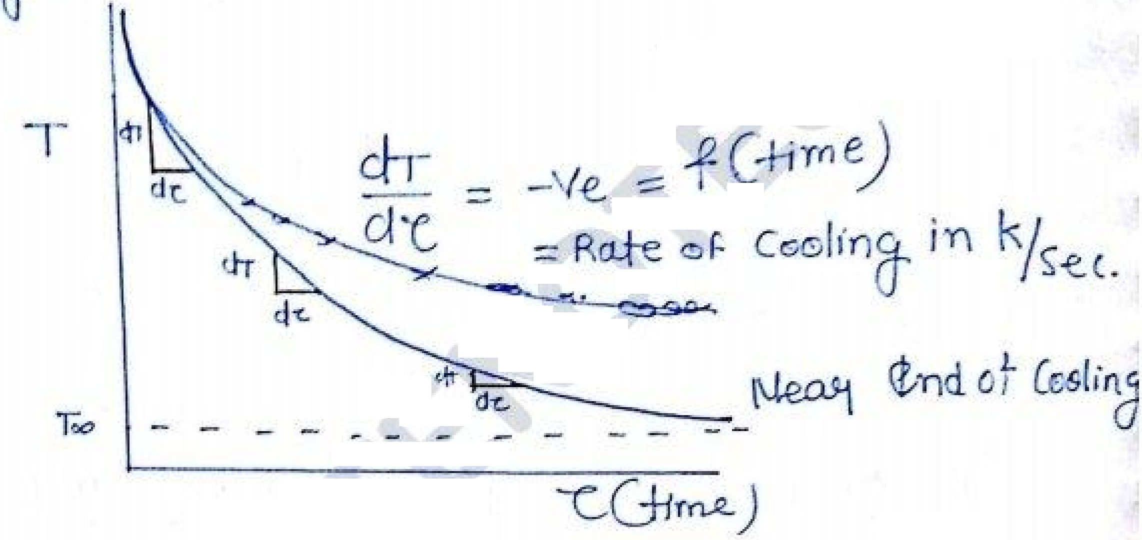
$$\ln\left(\frac{T - T_\infty}{T_i - T_\infty}\right) = -\frac{hA}{\rho V C_p}\tau$$

$$\boxed{\frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA}{\rho V C_p}\tau}}$$

$\frac{hA}{\rho V C_p} \rightarrow \text{sec}^{-1}$

$\frac{\rho V C_p}{hA}$ has got the unit of sec.

Hence in any unsteady H.T., the temp. of body changes exponentially w.r.t. time as shown in the fig.



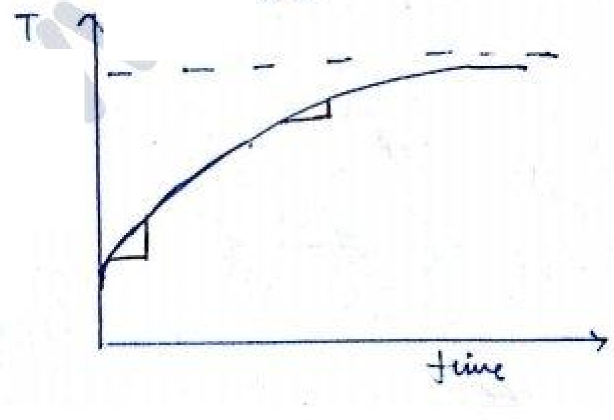
$\frac{dT}{d\tau} = -ve = f(\text{time})$

= Rate of Cooling in K/sec.

* During unsteady state H.T. the rate of Cooling or heating of body it self becomes a function of time initially the rate of Cooling is very high due to high convection H.T. Rate between body and fluid because of large temp. diff. ~~the~~ b/w them. but as the time progresses this rate of Cooling decreases due to reducing H.T. rate b/w body and fluid because of lesser temp. ~~bet~~ diff. between them.

Heating Case :- $\frac{dT}{d\tau} > 0$ → Rate of heating

when body piking up Heat



Rate of heating also ↓ as temp. diff ↓

Time Constant

$$\frac{hA}{\rho V C_p} \rightarrow sec^{-1}$$

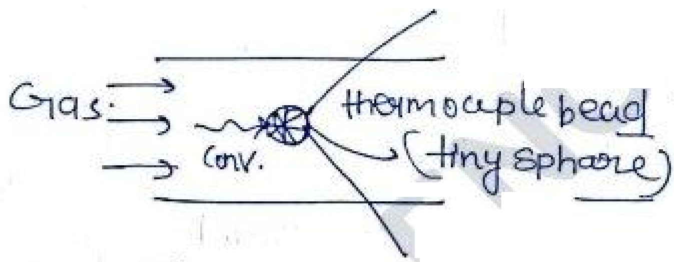
$\left(\frac{\rho V C_p}{hA}\right)$ has got the unit of sec.

This group $\left(\frac{\rho V C_p}{hA}\right)$ is called time constant.

This time constant of body signifies how much time will be taken by the body in approaching the temp. of the thermal ambient [hotter or cooler] when the body suddenly exposed to it.

⇒

Thermocouple



→ time const very small ≃ 2 sec

Time constant of thermocouple is very small because

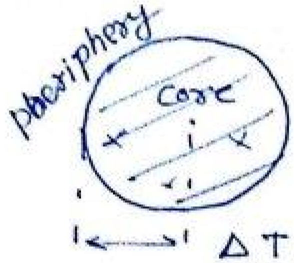
① h value is very high

② $V/A = \frac{\frac{4}{3}\pi R^3}{4\pi R^2}$ The size of bead is very small

③ heat capacity (ρC_p) is also small

thermometer time constant = 90 sec.

In the above analysis done it is assumed that the temp. of body is uniform throughout it's mass at any instant of time i.e. internal temp. gradient within the body are neglected.



$\Delta T = 0$ (At any instant)

$T = f(\text{time})$

$T \neq f(\text{location/space})$

$\Rightarrow$ Such Analysis is called **Lumped heat capacity Analysis**

* Criteria for Lumped Heat capacity Analysis:-

Biot No < 0.1

where $\boxed{\text{Biot No.} = \left(\dfrac{hs}{k_{Solid}}\right)}$

where $S = \dfrac{\text{Volume of body}}{\text{surface area}} = \dfrac{V}{A}$

$$\text{Biot No.} = \frac{\left(S/_{kA}\right)}{\left(1/_{hA}\right)}$$

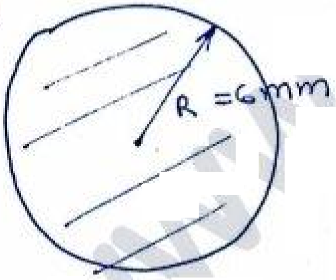
$$\text{Biot No.} = \frac{\text{Internal Conductive Resistance offered by body}}{\text{External Convecting Resistance.}}$$

* low biot no. value signifies that the body offers very little conduction resistance for any internal heat transfer within the body as compared to surface convective resistance, thereby leveling the temp. differances that may exist between any two location within the body.

Eg. Any metallic body reasonably small size

→ (very rapid conduction, slow convection)

Q.36d



$k = 20\ W/mK$

$h = 5\ W/m^2K$

$S_{sphere} = R/3$

$$\text{Biot No.} = \frac{hS}{k_{stee}} = \frac{5 \times 6}{20 \times 1000 \times 3}$$

$$\text{Biot No.} = 0.0005 < 0.1$$

$$ICR << ECR$$

Q.15

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-\frac{hA}{\rho V C_p}\tau}$$

$$\frac{430 - 30}{1030 - 30} = e^{-\left(\frac{20 \times 3 \times 1000}{7800 \times 30 \times 600}\right)\tau}$$

$$\frac{400}{100} = e^{-\tau/2340} \Rightarrow \tau = 2144\ \text{sec}$$

Q.53

$$\frac{T-T_\infty}{T_0-T_\infty} = e^{-\frac{hA}{\rho V C_p}\tau} \qquad \tau_c$$

$$\frac{1}{\tau_c} = \frac{hA}{\rho V C_p} = \frac{1}{16} \Rightarrow \tau_c = 16 \text{ Sec.}$$

$T = 350\text{K}$
$T_\infty = 300\text{ K}$
$T_0 = 1000\text{ K}$

$$\frac{350-300}{1000-300} = e^{-t/16}$$

$$\frac{50}{700} = e^{-t/16}$$

$$\frac{1}{14} = e^{-t/16}$$

$$\frac{t}{16} = \ln(14) \Rightarrow t = 16\ln(14)$$

$$t = 42.22 \text{ Sec.}$$

Q.52

$$\frac{T-T_\infty}{T_0-T_\infty} = e^{-\frac{hA}{\rho V C_p}\tau}$$

$$\tau_c = \frac{\rho V C_p}{hA} = \frac{7801 \times 473}{250} \times \frac{1}{410}$$

$$\frac{V}{A} = \frac{\pi R^2 H}{2\pi R^2 + 2\pi R H}$$

$$V/A = \frac{RH}{2(R+H)}$$

$$V/A = \frac{0.01 \times 0.2}{2 \times 2\left(\frac{0.01}{2}+0.2\right)} = \frac{1}{410}$$

$$\frac{300-100}{750-100} = e^{-t/\tau_c}$$

$$t = \tau_c \ln 13/4$$

$$t = 42.43 \text{ Sec}$$

Q. 39

$$\tau_c = \frac{\rho V C_p}{hA}$$

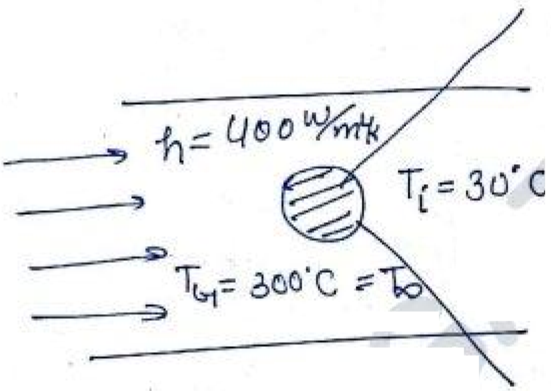
$$\tau_c = \frac{8500 \times 0.706 \times 400}{2 \times 400 \times 3 \times 1000} = \frac{2}{2} = 1 \text{ sec.}$$

$$\tau_c = 1 \text{ sec.}$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-\tau/1} \Rightarrow e^{-\tau} = \frac{298-300}{30-300}$$

$$e^{-\tau} = \frac{2}{270} \Rightarrow \tau = \ln 135$$

$$\tau = 4.9 \text{ sec.}$$



T= 298°C

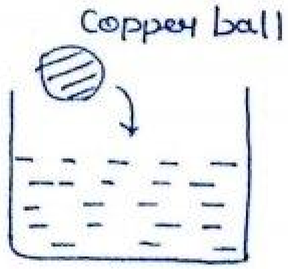
~~V.16~~ workbook Ans

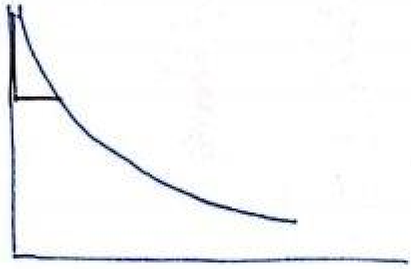
(10)

① c
③ d
⑥ b
⑦ a
⑧ d
⑨ c (

solid ice > Kwater > Kwater vapour

Q.16





$C_p \rightarrow J/kgK$

Rate of fall of temp. of ball at the very begining of cooling $\left(\frac{dT}{d\tau}\right)_{\tau = 0\,sec} = ?$

writing the energy balance at the very begining

$$hA(T_i - T_\infty) = -mC_p\left(\frac{dT}{d\tau}\right)_{at\,\tau=0}$$

$$-\left(\frac{dT}{d\tau}\right)_{at\,\tau=0} = \frac{hA}{\rho V C_p}(T_i - T_\infty)$$

$$-\left(\frac{dT}{d\tau}\right)_{\tau=0} = \frac{200}{11.55} = 17.3\ K/s$$

Q.57

$$\tau_c = \frac{\rho V C_p}{hA} = 1$$

$$\frac{8500 \times R \times 200}{400 \times 3} = 1$$

$$R = 0.7058\ mm$$

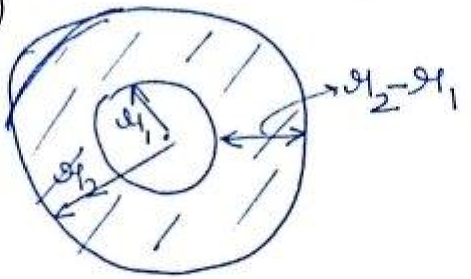
$$D = 1.41\ mm$$

Q.1 c   3 b   4 b $\alpha = \frac{k}{\rho C_p}$ 7. a   8 d

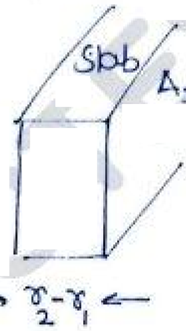
9 c $\left(k_{\substack{\text{solid}\\ \text{ice}}} > k_{\text{water}} > k_{\substack{\text{water}\\ \text{vaper}}} \right)$ 11 c

10 a $\left(\left(\frac{dT}{dx} \right)_{\text{in } I} \right)$ is higher   12 d

13



thermally equivalent



$A_{slab} = A_m$
= Equivlant mean area of hollow sphere

$$R_{th} = \frac{r_2 - r_1}{4\pi k r_1 r_2} \qquad R_{th} = \frac{(r_2 - r_1)}{k A_m} \ K/watt$$

A hollow sphere and slab are said to be thermally equivalent only if both of them are having same total thickness and off same thermal resistance

Equivalating R_{th}

$$\Rightarrow \frac{\cancel{r_2 - r_1}}{4\pi r_1 r_2} = \frac{\cancel{r_2 - r_1}}{k A_m}$$

$$A_m = 4\pi r_1 r_2$$

$$A_1 = 4\pi r_1^2 = 2\, m^2$$

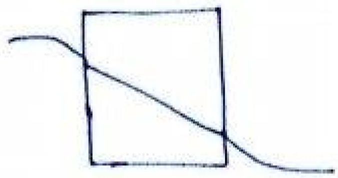
$$r_1 = \sqrt{\frac{2}{4\pi}}$$

$$A_2 = 4\pi r_2^2 = 8$$

$$r_2 = \sqrt{\frac{8}{4\pi}}$$

$$A_m = 4\pi \sqrt{\frac{8}{4\pi}} \times \sqrt{\frac{2}{4\pi}} \qquad A_m = \underline{4\, m^2}$$

(20) b



(25) A $R_{th} = \frac{b}{KA} = \frac{0.1}{200 \times 5} = 10^{-4}\ K/W$

(29) c

(24) Since k varies linearly wrt temp.

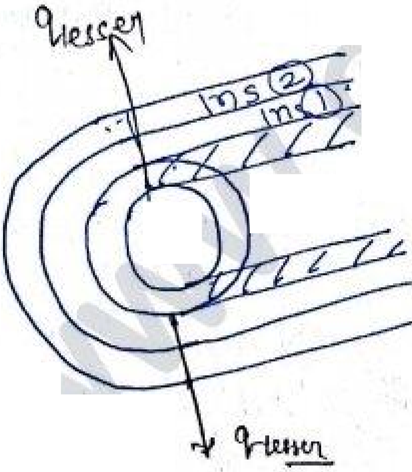
we can take $K_{mean} = \frac{25+15}{2} = 20\ W/mK$

$$\left(\frac{q}{A}\right) = \frac{K_{mean}\,\Delta T}{L}$$

$$= \frac{20 \times (300-200)}{0.5}$$

$$\left(\frac{q}{A}\right) = 4000\ W/m^2$$

(31)



To Reduce the heat transfer rate in a better way from a steam pipe Always keep better insulation having lesser k immediately Next to pipe

Reason:- In such arrangment thermal resistance $(R_{th})_{total}$ is higher

Q.38 D During unsteady state H.T. :

if Biot No < 0.1

i.e. when lumped heat analysis is valid

$T = f(\text{time})$ but $T \neq f(\text{space/Location})$

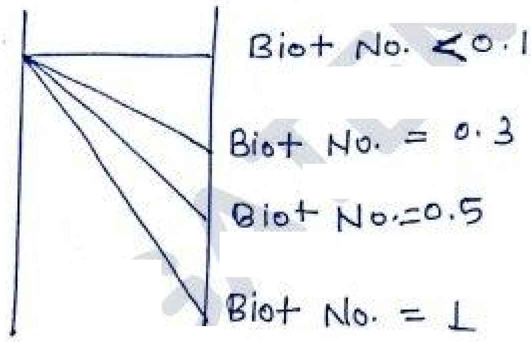
But Biot No > 0.1

Then lumped heat analysis NOT Valid

since $T = f(\text{space, time})$

i.e. At different location within the body, we observe different temprature at a given instant of time

i.e. To get temprature at a perticular location at a given instant of time Heisher chart must use.



Put Biot No. = 1

$$\underset{\text{Cond.}}{\underset{\downarrow}{ICR}} = \underset{\text{Conv.}}{\underset{\downarrow}{ECR}}$$

Both the resistance become equal significance.

42
q

$$R_{conv.} = \frac{1}{h A_{cond.}}$$

when insulation keept inside, Contact area b/w fluid & pipe ↓,

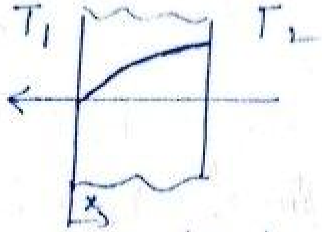
Internal → Rconv (Surface Resistance

Both $R_{cond} \uparrow$ & $R_{Conv} \uparrow$ → ~~at~~ surface

H.T Rate alway ↓

(45) b

Q(58)



$T_2 > T_1$

$q = \text{constant}$

$$q = kA\left(\frac{dT}{dx}\right)$$

$$\left(\frac{q}{A}\right) = (k_0 + bT)\frac{dT}{dx}$$

steady state

$$\frac{dT}{dx} = \frac{1}{k_0 + bT}$$

$x\uparrow \rightarrow T\uparrow \rightarrow (k_0 + bT)\uparrow$

$\left(\frac{dT}{dx}\right) \rightarrow \downarrow$