CHAPTER – 9

DIRECT AND INVERSE VARIATION

EXERCISE 9.1

1. Observe the following tables and find if x and y are directly proportional:

(i)

x	**5**	**8**	**12**	**15**	**18**	**20**
y	**15**	**24**	**36**	**60**	**72**	**100**

(ii)

x	**3**	**5**	**7**	**9**	**10**
y	**9**	**15**	**21**	**27**	**30**

Solution:

(i) It is given that,

$\frac{x}{y} = \frac{5}{15} = \frac{1}{3}$

Similarly,

$\frac{x}{y} = \frac{8}{24} = \frac{1}{3}$

$\frac{x}{y} = \frac{12}{36} = \frac{1}{3}$

But, $\frac{x}{y} = \frac{15}{60} = \frac{1}{4}$

$\frac{x}{y} = \frac{18}{72} = \frac{1}{4}$

$\frac{x}{y} = \frac{20}{100} = \frac{1}{5}$ Which is not equal to $\frac{1}{3}$

so, $\frac{x}{y}$ is not constant.

Hence, x and y are not directly proportional.

(ii) It is given that,

$\frac{x}{y} = \frac{3}{9} = \frac{1}{3}$

Similarly,

$\frac{x}{y} = \frac{5}{15} = \frac{1}{3}$

$\frac{x}{y} = \frac{7}{21} = \frac{1}{3}$

$\frac{x}{y} = \frac{9}{27} = \frac{1}{3}$

$\frac{x}{y} = \frac{10}{30} = \frac{1}{3}$

so, $\frac{x}{y}$ is constant.

Hence, x and y are directly proportional.

2. If x and y are in direct variation, complete the following tables:

(i)

x	**3**	**5**	**…**	**…**	**10**
y	**45**	**…**	**90**	**120**	**…**

(ii)

x	**4**	**8**	**…**	**20**	**28**
y	**7**	**…**	**21**	**…**	**…**

Solution:

(i) It is given that,

$\frac{x}{y} = \frac{3}{45} = \frac{1}{15}$

$\frac{x}{y}$ is constant with $\frac{1}{15}$.

Hence, x and y are directly proportional.

Now,

$\frac{x_1}{y_1} = \frac{1}{15} \Rightarrow \frac{5}{y_1} = \frac{1}{15}$

$y_1 = 5 \times 15 = 75$

$\frac{x_2}{y_2} = \frac{x_2}{90} = \frac{1}{15}$

$x_2 = \frac{90}{15} = 6$

$\frac{x_3}{y_3} = \frac{x_3}{120} = \frac{1}{15}$

$x_3 = \frac{120}{15} = 8$

$\frac{x_4}{y_4} = \frac{10}{y_4} = \frac{1}{15}$

$y_4 = 10 \times 15 = 150$

Here is the complete table:

x	3	5	6	8	10
y	45	75	90	120	150

(ii) It is given that,

$\frac{x}{y} = \frac{4}{7}$

$\frac{x}{y}$ is constant with $\frac{4}{7}$.

Hence, x and y are directly proportional.

Now,

$\frac{x_1}{y_1} = \frac{8}{y_1} = \frac{4}{7}$

$y_1 = \frac{(8 \times 7)}{4} = 14$

$\frac{x_2}{y_2} = \frac{x_2}{21} = \frac{4}{7}$

$x_2 = \frac{(21 \times 4)}{7} = 12$

$\frac{x_3}{y_3} = \frac{21}{y_3} = \frac{4}{7}$

$y_3 = \frac{(21 \times 7)}{4} = 35$

$\frac{x_4}{y_4} = \frac{28}{y_4} = \frac{4}{7}$

$y_4 = \frac{(28 \times 7)}{4} = 49$

Here is the complete table:

x	4	8	12	20	28
y	7	14	21	35	49

3. If 8 meters cloth costs ₹250, find the cost of 5.8 meters of the same cloth.

Solution:

Let the cost of 5.8 m cloth be ₹ x

So,

Length (in m)	8	5.8
Cost of cloth (in ₹)	250	x

We know that it is a direct variation.

So, 8: 250 = 5.8: x

$$\frac{8}{250} = \frac{5.8}{x}$$

$$x = \frac{(5.8 \times 250)}{8}$$

= ₹181.25

4. If a labourer earns ₹672 per week, how much will he earn in 18 days?

Solution:

Let the labourer earn ₹ x in 18 days

So,

Days	7	18
Money earned (in ₹)	672	x

We know that it is a direct variation.

So, 7: 672 = 18: x

$$\frac{7}{672} = \frac{18}{x}$$

$$x = \frac{(18 \times 672)}{7}$$

= ₹1728

5. If 175 dollars cost ₹7350, how many dollars can be purchased in ₹24024?

Solution:

Let ‘x’ dollars be purchased in ₹24024

So,

Cost (in ₹)	7350	24024
Dollars	175	x

We know that it is a direct variation.

So, 7350: 175 = 24024: x

$$\frac{7350}{175} = \frac{24024}{x}$$

$$x = \frac{(24024 \times 175)}{7350}$$

= 572 Dollars

6. If a car travels 67.5 km in 4.5 liters of petrol, how many kilometers will it travel in 26.4 liters of petrol?

Solution:

Let car travels ‘x’ km in 26.4 liters of petrol.

So,

Distance (in km)	x	67.5
Petrol (in L)	26.4	4.5

We know that it is a direct variation.

So, x: 26.4 = 67.5: 4.5

$$\frac{x}{26.4} = \frac{67.5}{4.5}$$

$$x = \frac{(67.5 \times 26.4)}{4.5} = 396 \text{ km}$$

7. If the thickness of a pile of 12 cardboard sheets is 45 mm, then how many sheets of the same cardboard would be 90 cm thick?

Solution:

It is given that,

Thickness of 12 cardboard = 45 mm

Let 'x' be the number of cardboard whose thickness is 90 cm = 900 m

So, 12: x = 45: 900

$$\frac{12}{x} = \frac{45}{900}$$

$$x = \frac{(12 \times 900)}{45}$$

$$= 240 \text{ mm}$$

∴ Thickness of 90 cardboard is 240 mm.

8. In a model of a ship, the mast (flagstaff) is 6 cm high, while the mast of the actual ship is 9 m high. If the length of the ship is 33 m, how long is the model of the ship?

Solution:

Given:

Height of a model of ship = 6 cm

But height of actual ship = 9 m

If length of ship = 33 m

Let length of model be 'x'

So, 6: x = 9 m: 33

$\frac{6}{x} = \frac{9}{33}$

x = $\frac{(6 \times 33)}{9}$

∴ Length of model is 22 cm.

9. The mass of an aluminium rod varies directly with its length. If a 16 cm long rod has a mass of 192 g, find the length of the rod whose mass is 105 g.

Solution:

Length of rod = 16 cm and mass = 192 g

If mass is 105 g, then

Let length of rod = x cm

So, 16: x = 192: 105

$\frac{16}{x} = \frac{192}{105}$

x = $\frac{(16 \times 105)}{192}$

= $\frac{35}{4}$

= 8.75 cm

∴ Length of rod = 8.75cm.

10. Anita has to drive from village A to village B. She measures a distance of 3.5 cm between these villages on the map. What is the actual distance between the villages if the map scale is 1 cm = 20 km?

Solution:

Given:

Distance from village A to B on the map = 3.5 cm

Scale of map 1 cm = 20 km

Let actual distance = x km

So, 1: 3.5 = 20: x

$$\frac{1}{3.5} = \frac{20}{x}$$

$$x = \frac{(20 \times 3.5)}{1}$$

x = 70 km

∴ Distance between village A and village B is 70 km.

11. A 23 m 75 cm high water tank casts a shadow 20 m long. Find at the same time;

(i) The length of the shadow cast by a tree 9 m 50 cm high.

(ii) The height of the tree if the length of the shadow is 12 m.

Solution:

Given:

Height of a water tank = 23 m 75 cm

$$= 23\frac{3}{4}\text{ m} = \frac{95}{4}\text{ m.}$$

Its shadow = 20 m

(i) If height of a tree = 9 m 50 cm

= $9\frac{1}{2}$ m

= $\frac{19}{2}$ m

Now, let its shadow be 'x' m

So,

$\frac{95}{4} : \frac{19}{2} :: 20 : x$

$\left(\frac{95}{4}\right) \times \left(\frac{2}{19}\right) = \frac{20}{x}$

$\frac{5}{2} = \frac{20}{x}$

$x = \frac{(2 \times 20)}{5}$

= 8 m

∴ its shadow = 8 m

(ii) Let the height of a tree be 'x' m and its shadow = 12 m

$\frac{95}{4} : x :: 20 : 12$

$\frac{95}{4x} = \frac{20}{12}$

$4x = \frac{(95 \times 12)}{20}$

x = $\frac{57}{4}$ m

x =14 m 25 cm

∴ Height of the tree is14 m 25 cm.

12. If 5 men or 7 women can earn ₹525 per day, how much 10 men and 13 women would earn per day.

Solution:

Given:

In a day, 5 men can earn = ₹ 525

So, in a day, 1 men can earn = ₹$\frac{525}{5}$ = ₹105

In a day, 10 men will earn = ₹105 × 10 = ₹1050

In a day 7 women can earn = ₹525

So, in a day, 1 women can earn = ₹ $\frac{525}{7}$ = ₹75

In a day 13 women will earn = ₹75 × 13 = ₹975

∴ Total earning of 7 men and 13 women in a day = ₹ (1050 + 975) = ₹2025

EXERCISE 9.2

1. Which of the following are in inverse variation?

(i) Number of students in a hostel and consumption of food.

(ii) Time taken by a train to cover a fixed distance and the speed of the train.

(iii) Area of land and its cost.

(iv) The number of people working and the time to complete the work.

(v) The quantity of rice and its cost.

Solution:

(i), (ii) and (iv) are inverse variations as xy = constant.

2. Observe the following tables and find which pair of variables (here x and y) are in inverse variation:

(i)

x	**90**	**60**	**45**	**30**	**20**
y	**10**	**15**	**20**	**30**	**45**

(ii)

x	**75**	**45**	**30**	**20**	**10**
y	**10**	**30**	**25**	**35**	**65**

Solution:

We know that, x and y is in inverse variation.

So, xy is constant.

(i) Now,

$xy = 90 \times 10 = 900$

$xy = 60 \times 15 = 900$

$xy = 45 \times 20 = 900$

$xy = 30 \times 30 = 900$

$xy = 20 \times 45 = 900$

We see that in every case, xy is constant.
∴ It is in inverse variation.

(ii) Now,

$xy = 75 \times 10 = 750$

$xy = 45 \times 30 = 1350$

$xy = 30 \times 25 = 750$

$xy = 20 \times 35 = 700$

$xy = 10 \times 65 = 650$

We see that in each case xy is not same.

∴ It is not in inverse variation.

3. Under the condition that the temperature remains constant, the volume of gas is inversely proportional to its pressure. If the volume of gas is 630 cubic centimeters at a pressure of 360 mm of mercury,

then what will be the pressure of the gas, if its volume is 720 cubic centimeters at the same temperature?

Solution:

Given:

Temperature remains constant and volume of gas is inversely proportional to its pressure.

So, let 'x' = Volume of gas = 630 cubic cm

and 'y' = Pressure = 360 mm

Let pressure of gas be an 'mm'

and their volume = 720 cubic cm

So, 630 × 360 = a × 720

$a = \frac{(630 \times 360)}{720} = 315$

∴ Pressure of gas is 315 mm of mercury.

4. A packet of sweets was distributed among 20 children and each of them received 4 sweets. How many sweets will each child get, if the number of children is reduced by 4?

Solution:

Given:

$x_1 = 20$ and $y_1 = 4$

$x_2 = 16$ and $y_2 = ?$

$x_1y_1 = x_2y_2$

$20 \times 4 = 16 \times y_2$

$y_2 = \frac{(20 \times 4)}{16} = 5$

∴ each child will get 5 sweets.

5. Pooja has enough money to buy 36 oranges at the rate of ₹4.50 per orange. How many oranges she can buy if the price of each orange is increased by 90 paisa?

Solution:

Given:

Cost of one orange = ₹4.50

Cost of 36 oranges = ₹4.50 × 36 = ₹162

so new price of one orange = ₹ 4.50 + 0.90 = ₹5.40

With ₹ 162, the number of oranges that can be bought = $\frac{162}{5.40}$ = 30

∴ the number of oranges that is available are 30.

6. It takes 8 days for 12 men to construct a wall. How many men should be put on the job if it is required to be constructed in 6 days?

Solution:

Given:

12 men construct a wall in = 8 days.

A man construct a wall in = 12 × 8 days

∴ 6 men construct a wall in = $\frac{(12 \times 8)}{6}$ days = 16 days

7. Eight taps through which water flows at the same rate can fill a tank in 27 minutes. If two taps go out of order, how long will the remaining taps take to fill the tank?

Solution:

Given:

Eight taps through which water flows at same rate can fill a tank in = 27 minutes.

So, taps through which water flows can fill a tank in = 27 × 8 minutes

Two taps go out of order then, remaining taps = 8 – 2 = 6 taps

∴ 6 taps through which water flows can fill a tank in = $\frac{(27 \times 8)}{6}$ minutes

= 36 minutes

8. A contractor undertook a contract to complete a part of a stadium in 9 months with a team of 560 persons. Later on, it was required to complete the job in 5 months. How many extra persons should he employ to complete the work?

Solution:

Given:

To complete a work, a team of 560 persons were employed for = 9 months

When the same work to be completed in 5 months, the number of persons required will be = $\frac{(560 \times 9)}{5}$ = 1008 person

We know that number of persons already working are = 560

∴ More number of persons to be required are = 1008 – 560 = 648 persons

9. A batch of bottles was packed in 30 boxes with 10 bottles in each box. If the same batch is packed using 12 bottles in each box, how many boxes would be filled?

Solution:

Given:

A batch of bottles were packed in 30 boxes with 10 bottles in each box.

∴ Number of boxes required to fill 12 bottles in each box = $\frac{(30 \times 10)}{12}$

= 25 boxes

10. Vandana takes 24 minutes to reach her school if she goes at a speed of 5 km/h. If she wants to reach school in 20 minutes, what should be her speed?

Solution:

Given:

With a speed of 5 km/h Vandana takes 24 minutes to reach her school.

∴ To reach the school in 20 minutes, she will go with a speed of = $\frac{(24\times5)}{20}$

= 6 km/h

11. A fort is provided with food for 80 soldiers to last for 60 days. Find how long would the food last if 20 additional soldiers join after 15 days.

Solution:

Given:

A fort is provided with food for 80 soldiers to last for 60 days.

After 15 days, the food is sufficient for 80 soldiers for (60 – 15) days = 45 days.

After 15 days, 20 additional soldiers join the fort.

So number of soldiers in the fort = 80 + 20 = 100

We know that for 80 soldiers, the food is sufficient for = 45 days

for 1 soldiers, the food is sufficient for = (80 × 45) days

∴ for 100 soldiers, the food is sufficient for = $\frac{(80 \times 45)}{100}$ days = 36 days.

12. 1200 soldiers in a fort had enough food for 28 days. After 4 days, some soldiers were sent to another fort and thus, the food lasted for 32 more days. How many soldiers left the fort?

Solution:

Given:

1200 soldiers in a fort had enough food for 28 days.

So, total number of days = 28

Days spent = 4 days

Number of days left = 28 – 4 = 24

Number of soldiers = 1200

and the food lasted for = 32 days

In 24 days, the food is sufficient for = 1200 soldiers

In 1 day, the food will be sufficient for = 1200×24

In 32 days, the food will be sufficient for = $(1200 \times 24) = 900$ persons

$\therefore$ Number of soldiers sent to another fort = 1200 – 900 = 300 persons

EXERCISE 9.3

1. A farmer can reap a field in 10 days while his wife can do it in 8 days (she does not waste time in smoking). If they work together, in how much time can they reap the field?

Solution:

Given:

A farmer can reap a field in = 10 days

In 1 day farmer can reap field = $\frac{1}{10}$ days

The farmer's wife can reap a field in = 8 days

In 1 day farmer's wife can reap field = $\frac{1}{8}$ days

Both farmer and his wife can reap field in 1 day = $\frac{1}{10} + \frac{1}{8}$

= $\frac{(4+5)}{40}$

= $\frac{9}{40}$

∴ Farmer and his wife reap field in $\frac{40}{9}$ days i.e. $4\frac{4}{9}$ days.

2. A can do $\frac{1}{5}$th of a certain work in 2 days and B can do $\frac{2}{3}$rd of it in 8 days. In how much time can they together complete the work?
Solution:

Given:

A can do $\frac{1}{5}$th of a certain work in = 2 days

So, A's, 1 day work = $\frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$

B can do $\frac{2}{3}$rd of a certain work in = 8 days

So, B's, 1 day work = $\frac{2}{3} \times \frac{1}{8} = \frac{1}{12}$

(A + B)'s 1 day work = $\frac{1}{10} + \frac{1}{12}$

= $\frac{(6+5)}{60}$

= $\frac{11}{60}$

∴ (A + B) can do the complete work in = $\frac{1}{\frac{11}{60}}$ days

= $\frac{60}{11}$ days

= $5\frac{5}{11}$ days

3. One tap fills a tank in 20 minutes and another tap fills it in 12 minutes. The tank being empty and if both taps are opened together, in how many minutes the tank will be full?

Solution:

Given:

First tap fill a tank in 20 minutes

Second tap fill a tank in 12 minutes

In 1 minute first tank fills = $\frac{1}{20}$ parts

In 1 minute 2nd tank fills = $\frac{1}{12}$ parts

In 1 minute both 1st and 2nd tank fills = $\frac{1}{20} + \frac{1}{12}$

= $\frac{(3+5)}{60}$

= $\frac{8}{60}$ parts

= $\frac{2}{15}$ parts

∴ Both first and 2nd tank fills in = $\frac{1}{\frac{2}{15}}$ minutes

= $\frac{15}{2}$ minutes

= $7\frac{1}{2}$ minutes

4. A can do a work in 6 days and B can do it in 8 days. They worked together for 2 days and then B left the work. How many days will A require to finish the work?

Solution:

Given:

A can do a work in = 6 days

A's, 1 day work = $\frac{1}{6}$

B can do a work in = 8 days

B's, 1 day work = $\frac{1}{8}$

Both (A + B)'s 1 day work = $\frac{1}{6} + \frac{1}{8}$

$= \frac{(4+3)}{24}$

$= \frac{7}{24}$

Both (A+B)'s 2 day work = $2 \times \left(\frac{7}{24}\right)$

$= \frac{7}{12}$

So, the remaining work = $1 - \frac{7}{12}$

$= \frac{(12-7)}{12}$

$= \frac{5}{12}$

Now,

It is given that, A can do a work in = 6 days

A can do $\frac{5}{12}$ work in = $6 \times \left(\frac{5}{12}\right)$ days

$= \frac{5}{2}$ days

$= 2\,\frac{1}{2}$ days

∴ A can finish the work in $2\,\frac{1}{2}$ days.

5. A can do a piece of work in 40 days. He works at it for 8 days and then B finishes the remaining work in 16 days. How long will they take to complete the work if they do it together?

Solution:

Given:

A can do a piece of work in = 40 days

A's, 1 day work = $\frac{1}{40}$

A's, 8 day work $= \frac{8}{40} = \frac{1}{5}$

Remaining work = $1 - \frac{1}{5}$

$= \frac{(5-1)}{5}$

$= \frac{4}{5}$

B can do $\frac{4}{5}$ piece of work in = 16 days

B's, 1 day work = $\frac{\left(\frac{4}{5}\right)}{16}$

$= \frac{4}{5\times16}$

$= \frac{1}{20}$

Now, (A + B)'s 1 day work = $\frac{1}{40} + \frac{1}{20}$

$= \frac{(1+2)}{40}$

$= \frac{3}{40}$

∴ Both (A+B) can do a piece of work in = $\frac{1}{\frac{3}{40}}$ days

= $\frac{40}{3}$ days

= $13\frac{1}{3}$ days

6. A and B separately do a work in 10 and 15 Solution: days respectively. They worked together for some days and then A completed the remaining work in 5 days. For how many days had A and B worked together?

Solution:

Given:

A can do a work in = 10 days

B can do a work in = 15 days

A's, 1 day work = $\frac{1}{10}$ …. (1)

B's, 1 day work = $\frac{1}{15}$

(A + B)'s 1 day work = $\frac{1}{10} + \frac{1}{15}$

= $\frac{(3+2)}{30}$

= $\frac{5}{30}$

= $\frac{1}{6}$

Now, let us consider (A+B) worked together for ‘x’ days.

So, (A + B)’s, x day work = $\frac{x}{6}$

Remaining work = $1 - \frac{x}{6}$

It is given that,

A can do the remaining work, $\left(1 - \frac{x}{6}\right)$ in = 5 days

So, A’s 1 day work = $\frac{\left(1 - \frac{x}{6}\right)}{5}$

$= \frac{6 - x}{6 \times 5}$

$= \frac{(6 - x)}{30}$ ….. (2)

From (1) and (2), we get

$\frac{1}{10} = \frac{(6 - x)}{30}$

$\frac{30}{10} = 6 - x$

$3 = 6 - x$

$x = 6 - 3$

$= 3$

∴ Both A and B can do work together in 3 days.

7. If 3 women or 5 girls take 17 days to complete a piece of work, how long will 7 women and 11 girls working together take to complete the work?

Solution:

Given:

3 women’s work = 5 girl’s work

1 women’s work = $\frac{5}{3}$ girl’s work

7 women’s work = $\left(\frac{5}{3}\right) \times 7$ girl’s work = $\frac{35}{3}$ girl’s work

7 women and 11 girls works = $\left(\frac{35}{3}\right) + 11$ girl’s work = $\frac{68}{3}$ girl’s work

Since 5 girls can do the work in 17 days

1 girl can do the work in = 17 × 5 days

$\frac{68}{3}$ girls can do the work in = $\frac{17 \times 5}{\frac{68}{3}}$ days

= $\frac{(17 \times 5 \times 3)}{68}$ days

= $\frac{15}{4}$ days

= $3\frac{3}{4}$ days

∴ 7 women and 11 girls can do the complete work in $3\frac{3}{4}$ days.

8. A can do a job in 10 days while B can do it in 15 days. If they work together and earn ₹3500, how should they share the money?

Solution:

Given:

A can do a job in = 10 days

A's 1 day job is = $\frac{1}{10}$

B can do a job in 15 days

B's 1 day job is = $\frac{1}{15}$

Both A and B can do job in one day = $\frac{1}{10} + \frac{1}{15}$

$= \frac{(3+2)}{30}$

$= \frac{5}{30}$

$= \frac{1}{6}$

For $\frac{1}{6}$ part of work the earnings is = ₹3500

For 1 part of work the earnings is = ₹3500 × 6

For $\frac{1}{10}$ part of work earning = ₹ $\frac{(3500\times 6)}{10}$ = ₹2100

For $\frac{1}{15}$ part of work earning = ₹ $\frac{(3500\times 6)}{15}$ = ₹1400

∴ A gets ₹2100 and B gets ₹ 1400

9. A, B and C can separately do a work in 2, 6 and 3 days respectively. Working together, how much time would they require to do it? If the work earns them ₹960, how should they divide the money?

Solution:

Given:

A’s one day work = $\frac{1}{2}$

B’s one day work = $\frac{1}{6}$

C’s one day work = $\frac{1}{3}$

A + B + C when working together = $\frac{1}{2} + \frac{1}{6} + \frac{1}{3}$

$= \frac{(3+1+2)}{6}$

$= \frac{6}{6}$

= 1day

∴ A, B and C can finish the work by working together in 1 day.

So they should divide the money in the ratio $\frac{1}{2} : \frac{1}{6} : \frac{1}{3}$

$\left(\frac{1}{2}\right) \times 12 : \left(\frac{1}{6}\right) \times 12 : \left(\frac{1}{3}\right) \times 12 = 6 : 2 : 4$

So, sum of terms of ratio = 6 + 2 + 4 = 12

∴ A’s share = $\left(\frac{6}{12}\right)$ × ₹960 = ₹480

B’s share = $\left(\frac{2}{12}\right)$ × ₹960 = ₹160

C’s share = $\left(\frac{4}{12}\right)$ × ₹960 = ₹320

10. A, B and C together can do a piece of work in 15 days, B alone can do it in 30 days and C alone can do it in 40 days. In how many days will A alone do the work?

Solution:

Given:

(A + B + C) can do a piece of work in = 15 days

(A + B + C)'s 1 day work = $\frac{1}{15}$

B alone can do a piece of work in = 30 days

B's 1 day work = $\frac{1}{30}$

C alone can do a piece of work in = 40 days

C's 1 day work = $\frac{1}{40}$

So, A's 1 day work = $\left(\frac{1}{15}\right) - \left[\frac{1}{30} + \frac{1}{40}\right]$

= $\left(\frac{1}{15}\right) - \left[\frac{(4+3)}{120}\right]$

= $\left(\frac{1}{15}\right) - \left(\frac{7}{120}\right)$

= $\frac{(8-7)}{120}$

= $\frac{1}{120}$

∴ A can do a piece of work in = 120 days.

11. A, B and C working together can plough a field in 4 4/5 days. A And C together can do it in 8 days. How long would B working alone take to plough the field?

Solution:

Given:

(A + B + C) can plough a field in = $4\frac{4}{5}$ days = $\frac{24}{5}$ days

(A + B + C)'s 1 day work = $\frac{5}{24}$ days

(A + C) can plough a field in = 8 days

(A + C)'s 1 day work = $\frac{1}{8}$ days

B's 1 day work = $\left(\frac{5}{24}\right) - \frac{1}{8}$

$= \frac{(5-3)}{24}$

$= \frac{2}{24}$

$= \frac{1}{12}$

∴ B can do the work in 12 days.

12. A and B together can build a wall in 10 days; B and C working together can do it in 15 days; C and A together can do it in 12 days. How long will they take to finish the work, working altogether? Also find the number of days taken by each to do the same work, working alone.

Solution:

Given:

(A + B)'s can build a wall in = 10 day

(A + B)'s 1 day work = $\frac{1}{10}$

(B + C)'s can build a wall in = 15 days

(B + C)'s 1 day work = $\frac{1}{15}$

(C + A)'s can build a wall in = 12 days

(C + A)'s 1 day work = $\frac{1}{12}$

[(A+B) + (B+C) + (C+A)]'s 1 day work = $\frac{1}{10} + \frac{1}{15} + \frac{1}{12}$

2 (A+B+C)'s 1 day work = $\frac{(6 + 4 + 5)}{60}$

$= \frac{15}{60}$

$= \frac{1}{4}$

(A+B+C)'s 1 day work = $\frac{1}{2\times4}$

$= \frac{1}{8}$

So, (A+B+C) can build the wall in = 8 days

Now let us find the number of days taken by each to do the same work, working alone:

⇒ Also, [(A+B+C) – (B + C)]'s 1 day work = $\frac{1}{8} - \frac{1}{15}$

$= \frac{(15 - 8)}{120}$

$= \frac{7}{120}$

A’s 1 day work = $\frac{7}{120}$

So, A can build the wall in = $\frac{120}{7}$ days

= $17\frac{1}{7}$ days

⇒ Also, [(A+B+C) – (A + C)]’s 1 day work = $\frac{1}{8} - \frac{1}{12}$

= $\frac{(3-2)}{24}$

= $\frac{1}{24}$

B’s 1 day work = $\frac{1}{24}$

So, B can build the wall in = 24 days

⇒ Also, [(A+B+C) – (A + B)]’s 1 day work = $\frac{1}{8} - \frac{1}{10}$

= $\frac{(5-4)}{40}$

= $\frac{1}{40}$

C’s 1 day work = $\frac{1}{40}$

So, C can build the wall in = 40 days

∴ A can complete the work in $17\frac{1}{7}$ days.

B can complete the work in 24 days.

C can complete the work in 40 days.

13. A pipe can fill a tank in 12 hours. By mistake, a waste pipe in the bottom is left opened and the tank is filled in 16 hours. If the tank is full, how much time will the waste pipe take to empty it?

Solution:

Given:

Pipe can fill a tank in = 12 hours

In 1 hour a tank can fill = $\frac{1}{12}$ part

By mistake pipe can fill a tank in = 16 hours

By mistake in 1 hour a tank can fill = $\frac{1}{16}$ part

Portion of the tank emptied by the waste pipe in 1 hour = $\frac{1}{12} - \frac{1}{16}$

$= \frac{(3-2)}{48}$

$= \frac{1}{48}$

$\therefore$ When the tank is full, time required by the waste pipe to empty the tank is 48 hours.

Mental Maths

Question 1: Fill in the blanks:

(i) Two quantities are said to be in direct variation if increase (or decrease) in one quantity causes in ………….. other quantity.
(ii) Two quantities x andy are said to be in inverse variation if xy is …………..
(iii) The total cost of articles varies ………….. to the number of articles purchased.
(iv) More work is done in ………….. time.
(v) The time taken to finish a work varies ………….. to the number of men at work.
(vi) The speed of a moving object varies inversely to the ………….. to cover a certain distance.
(vii) The number of articles varies ………….. with the cost per article, if a fixed amount is available.
(viii)Remuneration is in ………….. of work done.
Solution:
(i) Two quantities are said to be in direct variation if increase (or decrease)
in one quantity causes increase or decrease in other quantity.
(ii) Two quantities x and y are said to be in inverse variation if xy is constant.
(iii) The total cost of articles varies directly to the number of articles purchased.
(iv) More work is done in more time.
(v) The time taken to finish a work varies inversely to the number of men at work.
(vi) The speed of a moving object varies inversely
to the time taken to cover a certain distance.
(vii) The number of articles varies inversely with the cost per article,

if a fixed amount is available.
(viii) Remuneration is in a proportion of work done.

Question 2: State whether the following statements are true (T) or false (F):
(i) Two quantities x andy are said to be in inverse variation if $\frac{x}{y}$ is constant.
(ii) Number of days needed to complete the work = $\frac{1}{\text{one day's work}}$
(iii) Two quantities x and y are said to be in direct variation if x = ky, where k is constant of variation.
(iv) The work done varies inversely to the number of men at work.
(v) In the given time, the distance covered by a moving object varies directly to its speed.
(vi) If A can complete a work in n days, then A's one day's work is $\frac{1}{n}$ of the work, n
(vii) More the money deposited in a bank, more is the interest earned.
(viii) If the number of articles purchased increases the total cost decreases.
(ix) At the same time length of shadow is in direct variation with length of the object.
(x) The distance covered varies inversely to the consumption of petrol.
Solution:
(i) Two quantities x and y are said to be in inverse variation if $\frac{x}{y}$ is constant. False
Correct:
Two quantities x and y are said to be in inverse variation if xy is constant.
(ii) Number of days needed to complete the work = $\frac{1}{\text{one day's work}}$ True
(iii) Two quantities x andy are said to be in direct variation

if x = ky, where k is constant of variation. True
(iv) The work done varies inversely to the number of ipen at work. False
Correct:
The work done varies directly to the number of men at work.
(v) In the given time, the distance covered by a moving object varies directly to its speed. True
(vi) If A can complete a work in n days,
then A's one day's work is $\frac{1}{n}$ of the work. True
(vii) more the money deposited in a bank, more is the interest earned. True
(viii) if the number of articles purchased increases the total cost decreases. False
Correct:
It more articles increases, more cost increase.
(ix) At the same time length of shadow is in the direct variation with length of the object.
True
(x) the distance covered varies inversely to the consumption of petrol.
False
Correct:
It varies directly not inversely.

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 13):

Question 3: Two quantities x and y are said to be in inverse variation if
(a) xy = k
(b) $x \propto \frac{1}{y}$

(c) x = $\frac{k}{y}$

(d) all of these

Solution:

x and y vary inversely

∴ xy = constant or x $\propto \frac{1}{y}$

x = $\frac{\text{constant}}{y}$ are correct

All of these. (d)

Question 4: If 12 metre wire costs ₹24, then the cost of 8 metre wire is

(a) ₹16

(b) ₹20

(c) ₹12

(d) ₹18

Solution:

Cost of 12 metre wire = ₹24

Cost of 8 metre wire = x

∴ 12 : 8 :: 24 : x ⇒ x = $\frac{8\times24}{12}$ = 16

∴ Cost of 8 metre wire = ₹16

Question 5: If 5 kg wheat cost ₹60, then cost of 20 kg wheat is

(a) ₹200

(b) ₹210

(c) ₹220

(d) ₹240

Solution:

Cost of 5 kg of wheat = ₹60

Then cost of 20 kg of wheat

5 : 20 :: 60 : x (more wheat, more cost)

$\frac{5}{20} = \frac{60}{x} \Rightarrow x = \frac{20 \times 60}{5}$ = ₹240(d)

Question 6: If 10-men can complete a work in 6 days, then 30 men can complete the same work in

(a) 2 days

(b) 3 days

(c) 4 days

(d) 5 days

Solution:

10 men can complete a work in = 6 days

30 men will complete it in less days (say × day)

$\therefore \frac{30}{10} = \frac{6}{x} \Rightarrow x = \frac{6 \times 10}{30}$ = 2 days (a)

Question 7: A car travels 80 km in 5 litres of petrol, then the distance covered by it in 15 litres of petrol is

(a) 400 km

(b) 240 km

(c) 200 km

(d) 100 km

Solution:

In 5 litres, a car travels = 80 km

In 15’litres, the car will travel x km

∴ 5 : 15 :: 80 : x

$\Rightarrow$ x = $\frac{15 \times 80}{5}$ = 240

∴ 240 km distance (b)

Question 8: In a mess, there was enough food for 200 students for 20 days. If 50 new students joined them, then the food will last for

(a) 15 days
(b) 16 days
(c) 17 days
(d) 18 days
Solution:
Food lasts for 200 students for = 20 days
Let first will last for 200 + 50 = 250 students = x days
More students, less days
200 : 250 :: 20 : x
By inverse variation,
250 : 200 :: 20 : x
x = $\frac{200 \times 20}{250}$ = 16 days (b)

Question 9: 3 persons can paint a house in 8 days, then 4 persons can paint the same house in

(a) 5 days
(b) 6 days
(c) 7 days
(d) none of these
Solution:
3 persons can paint a house in 8 days.
4 persons will paint it in less days.
$\therefore \frac{4}{3} = \frac{8}{x} \Rightarrow x = \frac{3 \times 8}{4}$ = 6 days (b)

Question 10: A photograph of bacteria is enlarged 100000 times attains a length of 5 cm, then actual length of the bacteria is

(a) 0.00005 cm
(b) 5×10^{-5}
(c) 5×10^{-7}
(d) all of these
Solution:
Photograph of bacteria is enlarged 100000 times attain a length of 5 cm.
∴ Actual length = $\frac{5}{100000}$ = 5.0×10^{-5} (d)
(∵ 0.00005 cm = 5.0×10^{-5} cm = 5×10^{-7} m)

Question 11: A tree 12 metre high casts a shadow of length 8 metre. Height of the tree whose shadow is 6 metre in length is

(a) 6 m
(b) 9 m
(c) 15 m
(d) none of these
Solution:
Shadow of 12 m high tree = 8 m
Shadow of another tree = 6 m
Let is height = x m
∴ 8 : 6 :: 12 : x
∴ x = $\frac{12 \times 6}{8} = \frac{72}{8}$ = 9 m (b)

Question 12: If 5 pipes can fill the tank in 1 hour, then 4 pipes will fill the tank in
(a) 75 minutes
(b) 70 minutes
(c) 65 minutes
(d) none of these
Solution:

5 pipes can fill a tank in 1 hour
Then 4 pipes will fill in more time
$\therefore\ \frac{4}{5} = \frac{1}{x} \Rightarrow x = \frac{5}{4}$ hours = 75 minutes (a)

Question 13: A tap fills a tank in 8 hours and another tap at the bottom empties it in 10 hours. If both work together, the tank will be filled in

(a) 18 hours
(b) 24 hours
(c) 36 hours
(d) 40 hours
Solution:
First tap's 1 hour work for filling = $\frac{1}{8}$
Second tap's 1 hour work for emptying = $\frac{1}{10}$
Both work for 1 hour = $\frac{1}{8} - \frac{1}{10} = \frac{1}{40}$
$\therefore$ The tank will be filled in 40 hours. (d)

Value-Based Questions

Question 1: The cost of fuel for running a train is proportional to the speed generated in km/h. It costs ₹40 per hour when train is moving with 20 km/h. What would be the cost of fuel per hour, if the train is moving with 60 km/h?

Keeping the safety and fuel prices in mind, state the values promoted in the question.

Solution:

$\because$ Cost of fuel is proportional to the speed of the train.

$\Rightarrow \frac{20 \text{ km/h}}{60 \text{ km/h}} = \frac{₹40}{x}$

(Suppose cost of fuel = x)

$x = \frac{40 \times 60}{20} = ₹120$

∴ Cost of fuel = ₹120

Save the fuel, save the Nation.

Question 2: A pipe can fill a tank in 9 hours. There is a leakage in the bottom of the tank due to which tank is filled in 12 hours. If the tank is full, how much time will leakage take to empty the tank? Should we repair the leakage tank? Should we repair the leakage of the tank immediately? What values are being promoted?

Solution:

A pipe can fill a tank in = 9 hours

Due to leakage at the bottom it is filled in = 12 hours

∴ Leakage empty the tank $= \frac{1}{9} - \frac{1}{12} = \frac{4-3}{36} = \frac{1}{36}$

∴ The leakage will empty the fill tank in 36 hours.

Save the water, save the world.

Higher Order Thinking Skills (Hots)

Question 1: If 8 labourers can earn ₹9000 in 15 days, how many labourers can earn ₹6300 in 7 days?

Solution:

8 labourers can earn ₹9000 in 15 days

To find : How many labourers will earn ₹6300 in 7 days

Less amount, less labourers

Less days, more labours

∴ Number of labourers = $\frac{8\times6300\times15}{9000\times7}$ = 12

OR

8 labourers earn in 15 days = ₹9000

120 labour days = ₹9000

1 labour day = $\frac{9000}{120}$ = ₹7.5

Hence, the number of labourers day required to earn ₹6300 in 7 days = $\frac{6300}{75}$ = 84 days

∴ The number of labourers required = $\frac{84}{7}$ = 12

Question 2: Three typists working 8 hours a day type a document in 10 days. If only 2 typists are working, how many hours a day should they work to finish the job in 12 days?

Solution:

3 typist in 8 hours a day can type a document in = 10 days

2 typists in 12 days will finish the work by working x hour a day less typists, more hours a day

More days, less hours

∴ x = $\frac{8\times3\times10}{2\times12}$ = 10 hours a day

∴ They will work for 10 hours a day.

Check Your Progress

Question 1: Match each of the entries in column I with the appropriate entry in column II.

Column I	Column II
i. x and y very inversely to each other	**a. $\frac{x}{y}$ = constant**
ii. Mathematical representation of direct variation of quantities *m* and n.	**b. y will increase in proportion**
iii. Mathematical representation of inverse variation of quantities p and q	**c. xy = constant**
iv. If x and y vary inversely then on decreasing x	**d. $p \propto \frac{1}{q}$**
v. x and y vary directly to each other	**e. y will decrease in proportion**
vi. If x and y vary directly then on decreasing x.	**f. $m \propto n$**
	g. $p \propto q$
	h. $m \propto \frac{1}{n}$

Solution:

Column I	Column II
i. x and y very inversely to each other	a. xy = constant
ii. Mathematical representation of direct variation of quantities m and n.	b. $m \propto n$

iii. Mathematical representation of inverse variation of quantities p and q	c. $p \propto \frac{1}{q}$
iv. If x and y vary inversely then on decreasing x	d. y will increase in proportion
v. x and y vary directly to each other	e. $\frac{x}{y}$ = constant
vi. If x and y vary directly then on decreasing x.	f. y will decrease in proportion

Question 2: (i) From the following table, determines and q if x and y vary directly:

x	**6**	**p**	**15**	**20**	**25**
y	**18**	**27**	**q**	**60**	**75**

(ii) From the following table, determine a and b if x and y vary inversely:

x	**2**	**a**	**6**	**10**
y	**45**	**12.5**	**b**	**9**

Solution:

(i)

x	6	p	15	20	25
y	18	27	q	60	75

$\because \frac{x}{y} = \frac{6}{18} = \frac{1}{3}, \frac{20}{60} = \frac{1}{3}, \frac{25}{75} = \frac{1}{3}$

$\therefore$ It is direct variation.

Now, $\frac{p}{27} = \frac{1}{3} \Rightarrow p = \frac{27}{3} = 9$

$\frac{15}{q} = \frac{1}{3} \Rightarrow q = 15 \times 3 = 45$

$\therefore p = 9, q = 45$

(ii)

x	2	a	6	10
y	45	12.5	b	9

$xy = 2 \times 45 = 90, 10 \times 9 = 90$

$\therefore x, y$ Are inverse variation

$\therefore a \times 12.5 = 90 \Rightarrow a = \frac{90}{12.5} = \frac{90 \times 10}{125}$

$a = \frac{36}{5} = 7.2$

$6 \times b = 90 \Rightarrow b = \frac{90}{6} = 15$

$\therefore a = 7.2, b = 15$

Question 3: It rained 80 mm in first 20 days of April. What would be the total rainfall in April?

Solution:

Rained 80 mm in first 20 days of April.
We can't calculate the total rainfall in April
as there is no information about the rest 10 days.

Question 4: Mamta earns ₹540 for a working week of 48 hours. If she was absent for 6 hours, how much did she earn?

Solution:

Mamta earns ₹540 for a working week of 48 hours

Let Mamta earns ₹x for a working week of 42 hours

No. 80 hours	48	42
Cost	540	x

∴ It is a direct variation

$\Rightarrow \frac{48}{540} = \frac{42}{x}$

$\Rightarrow x = \frac{42 \times 540}{48} = 472.50$

Hence, Mamta earns ₹472.50 for a working week of 42 hours.

Question 5: Navjot can do a piece of work in 6 days working 10 hours per day. In how many days can he do the same work if he increases his working hours by 2 hours per day?

Solution:

Working hours per day = 10

∴ Working hours in 6 days = 6 × 10 = 60

When increase the working hours by 2 hours per day

Then new working hours = 10 + 2 = 12

Hence, Number of days to complete the work = $\frac{60}{12}$ = 5 days.

Question 6: Sharmila has enough money to buy 24 bananas at the rate of ₹ 1·50 per banana. How many bananas she can buy if the price of each orange is decreased by 30 paise?

Solution:

Amount of money to buy 24 bananas at the rate of ₹ 1·50 per banana

= ₹24 × 1·50 = ₹36
When price of each banana is decreased by 30 paise
then the new decreased price of each banana
= ₹(1·50 – 0·30) = ₹1·20
In ₹1 ·20 buy one banana
∴ In ₹1 buy = $\frac{1}{1.20}$ banana
∴ In ₹36 buy = $\frac{36}{1.20}$ bananas = 30 bananas
i.e. 30 bananas
Hence Sharmila can buy 30 bananas.

Question 7: A fort has rations for 180 soldiers for 40 days. After 10 days, 30 soldiers leave the fort. Find the total number of days for which the food will last.

Solution:
No. of soldiers in the beginning = 180
No. of soldiers left = 30
∴ No. of soldiers now = 180 – 130 = 150
No. of total days = 40
No of days passed = 10
∴ No. of days left = 40 – 10 = 30
Now, for 180 soldiers the ration is sufficient for = 30 days
For 1 soldier, the ration will be for = 30 × 180 days
and for 150 soldiers, the ratio will be for = $\frac{30 \times 180}{150}$ = 36 days
∴ Total period = 36 + 10 = 46 days

Question 8: There are 100 students in a hostel. Food provision for them is for 20 days. How long will these provisions last, if 25 more students join the group?

Solution:

In a hostel, there are 100 students and food
is sufficient for them for 20 days By joining 25 more students,
the number of students now = 100 + 25 = 125 students
100 : 125 = 20 days : x
More students, less days
∴ By inverse variation
125 : 100 = 20 : x
x = $\frac{100\times20}{125}$ = 16
The food will be sufficient for 16 days.

Question 9: If 4 goats or 6 sheep can graze a field in 40 days, how many days will 4 goats and 14 sheep take to graze the same field?

Solution:

∵ 4 goat's work = 6 sheep's work

∴ 4goat'sand 14 sheep's work = (6 + 14) sheep's work = 20 sheep's work

∴ 6 sheep's can graze a field in = 40 days
∴ 1 sheep can graze a field in = 6 × 40 days
20 sheep can graze a field in = $\frac{6\times40}{20}$ days = 12 days.

Question 10: A tap can fill a tank in 20 hours, while the other can empty it in 30 hours. The tank being empty and if both taps are opened together, how long will it take for the tank to be half full?

Solution:

∵ Time for tap can fill a tank in = 20 hours
∴ In time 1 hour tap can fill a tank = $\frac{1}{20}$
∵ Time for another can empty a tank in = 36 hours

$\therefore$ In time 1 hour tap can empty a tank = $\frac{1}{30}$

In time 1 hour the tank to be full $= \frac{1}{20} - \frac{1}{30} = \frac{3-2}{60} = \frac{1}{60}$

In time 1 hour the tank to be half full = $\frac{2}{60} = \frac{1}{30}$

$\therefore$ Time take for the tank to be half fill = 30 hours

Question 11: Three ants separately can gobble a grasshopper in 3, 4, and 6 days respectively. How many days will they take together to finish off the poor chap? If the grasshopper weighs 63 gram, find the share of each.

Solution:

$\because$ First ant can gobble a grasshopper in = 3 days

$\because$ Second ant can gobble a grasshopper in = 4 days

$\because$ Third ant can gobble a grasshopper in = 6 days

$\therefore$ First ant's do work in 1 day = $\frac{1}{3}$

2nd ant's do work in 1 day = $\frac{1}{4}$

3rd ant's do work in 1 day = $\frac{1}{6}$

Three ant's combined work in 1 day

$= \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{6}\right) = \left(\frac{4+3+2}{12}\right)$

$= \frac{9}{12} = \frac{3}{4}$

Hence three ant's take time finish off the poor chap = $\frac{4}{3}$ days = $1\frac{1}{3}$ days.

Ratio of works of First, Second and Third ant in 1 day = $\frac{1}{3} : \frac{1}{4} : \frac{1}{6}$

$= \frac{1}{3} \times 12 : \frac{1}{4} \times 12 : \frac{1}{6} \times 12$

$= 4 : 3 : 2$

Sum of ratio = 4 + 3 + 2 = 9

Shares of first ant = $\frac{4}{9} \times 63$ gm = 28 gm

Shares of second ant = $\frac{3}{9} \times 63$ gm = 21 gm

Shares of third ant = $\frac{2}{9} \times 63$ gm = 14 gm

Question 12: A and B together can do a piece of work in 12 days; B and C together can do it in 15 days. If A is twice as good a workman as C, in how many days A alone will do the same work?

Solution:

(A + B)'s can do work in = 12 days
(B + C)'s can do work in = 15 days
Since A is twice as good a workman as C,
Let C can do work in = x days

$\therefore$ A can do work in = $\frac{x}{2}$ days

$\because$ (A + B)'s, 1 day work = $\frac{1}{12}$

A's 1 day work = $\frac{2}{x}$

$\therefore$ [(A + B) – A]'s, 1 day work = $\frac{1}{12} - \frac{2}{x}$

$\Rightarrow$ B's, 1 day work = $\frac{1}{12} - \frac{2}{x}$ …….(i)

Also (B + C)'s 1 day work = $\frac{1}{15}$

C's 1 day work = $\frac{1}{x}$

$\therefore$ [(B + C) – C]'s, 1 day work = $\frac{1}{15} - \frac{1}{x}$

$\Rightarrow$ B's 1 day work = $\frac{1}{15} - \frac{1}{x}$ …..(ii)

From (i) and (ii)

$$\frac{1}{12} - \frac{2}{x} = \frac{1}{15} - \frac{1}{x}$$

$$\Rightarrow \frac{1}{x} - \frac{2}{x} = \frac{1}{15} - \frac{1}{12}$$

$$\Rightarrow \frac{1-2}{x} = \frac{4-5}{60} \Rightarrow -\frac{1}{x} = -\frac{1}{60}$$

$$\Rightarrow x = 60$$

Hence A can do work in = $\frac{x}{2}$ days = $\frac{60}{2}$ days = 30 days