

Ex 18.1

Q1(i)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $(2x+3y)^5$ has 6 terms.

Using binomial theorem, we have

$$\begin{aligned}(2x+3y)^5 &= {}^5C_0(2x)^5(3y)^0 + {}^5C_1(2x)^4(3y)^1 + {}^5C_2(2x)^3(3y)^2 + {}^5C_3(2x)^2(3y)^3 \\ &\quad + {}^5C_4(2x)(3y)^4 + {}^5C_5(2x)^0(3y)^5 \\ &= 2^5x^5 + 5 \times 2^4 \times 3 \times x^4 \times y + 10 \times 2^3 \times 3^2 \times x^3 \times y^2 + 10 \times 2^2 \times 3^3 \times x^2 \times y^3 \\ &\quad + 5 \times 2 \times 3^4 \times x \times y^4 + 3^5y^5 \\ &= 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5\end{aligned}$$

Q1(ii)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $(2x-3y)^4$ has 5 terms.

Using binomial theorem, we have

$$\begin{aligned}(2x-3y)^4 &= {}^4C_0(2x)^4(3y)^0 - {}^4C_1(2x)^3(3y)^1 + {}^4C_2(2x)^2(3y)^2 - {}^4C_3(2x)^1(3y)^3 + {}^4C_4(2x)^0(3y)^4 \\ &= 2^4x^4 - 4 \times 2^3 \times 3 \times x^3y + 6 \times 2^2 \times 3^2 \times x^2y^2 - 4 \times 2 \times 3^3 \times xy^3 + 3^4y^4 \\ &= 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4\end{aligned}$$

Q1(iii)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(x - \frac{1}{x}\right)^6$ has 7 terms.

Using binomial theorem, we get

$$\begin{aligned}\left(x - \frac{1}{x}\right)^6 &= {}^6C_0x^6\left(\frac{1}{x}\right)^0 - {}^6C_1x^5\left(\frac{1}{x}\right)^1 + {}^6C_2x^4\left(\frac{1}{x}\right)^2 - {}^6C_3x^3\left(\frac{1}{x}\right)^3 + {}^6C_4x^2\left(\frac{1}{x}\right)^4 - {}^6C_5x\left(\frac{1}{x}\right)^5 + {}^6C_6x^0\left(\frac{1}{x}\right)^6 \\ &= x^6 - 6x^4 + 15x^2 - 20 + \frac{15}{x^2} - \frac{6}{x^4} + \frac{1}{x^6}\end{aligned}$$

Q1(iv)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $(1-3x)^7$ has 8 terms.

Using binomial theorem to expand, we get

$$\begin{aligned}(1-3x)^7 &= {}^7C_0(1)^7(3x)^0 + {}^7C_1(3x) + {}^7C_2(3x)^2 + {}^7C_3(3x)^3 + {}^7C_4(3x)^4 + {}^7C_5(3x)^5 + {}^7C_6(3x)^6 + {}^7C_7(3x)^7 \\ &= 1 - 21x + 21 \times 9x^2 - 35 \times 3^3 x^3 + 35 \times 3^4 x^4 - 21 \times 3^5 x^5 + 7 \times 3^6 x^6 - 3^7 x^7 \\ &= 1 - 21x + 189x^2 - 945x^3 + 2835x^4 - 5103x^5 + 5103x^6 - 2187x^7\end{aligned}$$

Q1(v)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(ax - \frac{b}{x}\right)^6$ has 7 terms.

Using binomial theorem to expand, we get

$$\begin{aligned}\left(ax - \frac{b}{x}\right)^6 &= {}^6C_0(ax)^6\left(\frac{b}{x}\right)^0 + {}^6C_1(ax)^5\left(\frac{b}{x}\right) + {}^6C_2(ax)^4\left(\frac{b}{x}\right)^2 + {}^6C_3(ax)^3\left(\frac{b}{x}\right)^3 + {}^6C_4(ax)^2\left(\frac{b}{x}\right)^4 + {}^6C_5(ax)\left(\frac{b}{x}\right)^5 \\ &\quad + {}^6C_6(ax)^0\left(\frac{b}{x}\right)^6 \\ &= a^6x^6 - 6a^5x^5\frac{b}{x} + 15a^4x^4\frac{b^2}{x^2} - 20a^3x^3\frac{b^3}{x^3} + 15a^2\frac{b^4}{x^5} - 6a\frac{b^5}{x^7} + \frac{b^6}{x^6} \\ &= a^6x^6 - 6a^5x^4b + 15a^4b^2x^2 - 20a^3b^3 + 15\frac{a^2b^4}{x^2} - 6\frac{ab^5}{x^4} + \frac{b^6}{x^6}\end{aligned}$$

Q1(vi)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}}\right)^6$ has 7 terms.

Using binomial theorem to expand, we get

$$\begin{aligned}\left(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}}\right)^6 &= {}^6C_0\left(\sqrt{\frac{x}{a}}\right)^6\left(\sqrt{\frac{a}{x}}\right)^0 + {}^6C_1\left(\sqrt{\frac{x}{a}}\right)^5\left(\sqrt{\frac{a}{x}}\right)^1 + {}^6C_2\left(\sqrt{\frac{x}{a}}\right)^4\left(\sqrt{\frac{a}{x}}\right)^2 + {}^6C_3\left(\sqrt{\frac{x}{a}}\right)^3\left(\sqrt{\frac{a}{x}}\right)^3 \\ &\quad + {}^6C_4\left(\sqrt{\frac{x}{a}}\right)^2\left(\sqrt{\frac{a}{x}}\right)^4 + {}^6C_5\left(\sqrt{\frac{x}{a}}\right)\left(\sqrt{\frac{a}{x}}\right)^5 + {}^6C_6\left(\sqrt{\frac{x}{a}}\right)^0\left(\sqrt{\frac{a}{x}}\right)^6 \\ &= \left(\frac{x}{a}\right)^{\frac{1}{2} \times 6} - 6\left(\frac{x}{a}\right)^{\frac{1}{2} \times 5}\left(\frac{a}{x}\right)^{\frac{1}{2}} + 15\left(\frac{x}{a}\right)^{\frac{1}{2} \times 4}\left(\frac{a}{x}\right)^{\frac{2}{2}} - 20\left(\frac{x}{a}\right)^{\frac{1}{2} \times 3}\left(\frac{a}{x}\right)^{\frac{3}{2}} + 15\left(\frac{x}{a}\right)^{\frac{1}{2} \times 2}\left(\frac{a}{x}\right)^{\frac{4}{2}} \\ &\quad - 6\left(\frac{x}{a}\right)^{\frac{1}{2}}\left(\frac{a}{x}\right)^{\frac{5}{2}} + \left(\frac{a}{x}\right)^{\frac{6}{2}} \\ &= \frac{x^3}{a^3} - 6\frac{x^{\frac{3}{2}}a^{\frac{1}{2}}}{a^{\frac{3}{2}}x^{\frac{1}{2}}} + 15x\frac{x^2 \times a}{a^2 \times x} - 20 \times \frac{x^{\frac{3}{2}} \times a^{\frac{3}{2}}}{a^{\frac{3}{2}}x^{\frac{3}{2}}} + 15 \times \frac{x \times a^2}{a \times x^2} - 6 \times \frac{x^{\frac{1}{2}} \times a^{\frac{5}{2}}}{a^{\frac{1}{2}}x^{\frac{5}{2}}} + a^3 \\ &= \frac{x^3}{a^3} - \frac{6x^2}{a^2} + \frac{15x}{a} - 20 + \frac{15a}{x} - \frac{6a^2}{x^2} + \frac{a^3}{x^3}\end{aligned}$$

Q1(vii)

$$\begin{aligned}
 & (\sqrt[3]{x} - \sqrt[3]{a})^6 \\
 &= \binom{6}{0} (\sqrt[3]{x})^6 (\sqrt[3]{a})^0 + \binom{6}{1} (\sqrt[3]{x})^5 (\sqrt[3]{a})^1 + \binom{6}{2} (\sqrt[3]{x})^4 (\sqrt[3]{a})^2 \\
 &+ \binom{6}{3} (\sqrt[3]{x})^3 (\sqrt[3]{a})^3 + \binom{6}{4} (\sqrt[3]{x})^2 (\sqrt[3]{a})^4 + \binom{6}{5} (\sqrt[3]{x})^1 (\sqrt[3]{a})^5 \\
 &+ \binom{6}{6} (\sqrt[3]{x})^0 (\sqrt[3]{a})^6 \\
 &= x^2 - 6x^{\frac{5}{3}}a^{\frac{1}{3}} + 15x^{\frac{4}{3}}a^{\frac{2}{3}} - 20x a + 5x^{\frac{2}{3}}a^{\frac{4}{3}} - 6x^{\frac{1}{3}}a^{\frac{5}{3}} + a^2
 \end{aligned}$$

Q1(viii)

Let $y = 1 + 2x$, then

$$(1 + 2x - 3x^2)^5 = (y - 3x^2)^5$$

The expansion of $(x + y)^n$ has $n + 1$ terms so the expansion of $(y - 3x^2)^5$ has 6 terms.

Using binomial theorem to expand, we get

$$\begin{aligned}
 (y - 3x^2)^5 &= {}^5C_0 y^5 (3x^2)^0 - {}^5C_1 y^4 (3x^2)^1 + {}^5C_2 y^3 (3x^2)^2 - {}^5C_3 y^2 (3x^2)^3 + {}^5C_4 y (3x^2)^4 - {}^5C_5 y^0 (3x^2)^5 \\
 &= y^5 - 5y^4 \cdot 3x^2 + 10y^3 \cdot 9x^4 - 10y^2 (27x^6) + 5y(81x^8) - 243x^{10}
 \end{aligned}$$

Now,

$$\begin{aligned}
 y^5 &= (1 + 2x)^5 = {}^5C_0 + {}^5C_1(2x) + {}^5C_2(2x)^2 + {}^5C_3(2x)^3 + {}^5C_4(2x)^4 + {}^5C_5(2x)^5 \\
 y^4 &= (1 + 2x)^4 = {}^4C_0 + {}^4C_1(2x) + {}^4C_2(2x)^2 + {}^4C_3(2x)^3 + {}^4C_4(2x)^4 \\
 y^3 &= (1 + 2x)^3 = {}^3C_0 + {}^3C_1(2x) + {}^3C_2(2x)^2 + {}^3C_3(2x)^3 \\
 y^2 &= (1 + 2x)^2 = {}^2C_0 + {}^2C_1(2x) + {}^2C_2(2x)^2 \\
 y &= (1 + 2x)
 \end{aligned}$$

Substituting the values of powers of y in the equation above, we get,

$$\begin{aligned}
 (1 + 2x - 3x^2)^5 &= [{}^5C_0 + {}^5C_1(2x) + {}^5C_2(2x)^2 + {}^5C_3(2x)^3 + {}^5C_4(2x)^4 + {}^5C_5(2x)^5] \\
 &\quad - 15x^2 [{}^4C_0 + {}^4C_1(2x) + {}^4C_2(2x)^2 + {}^4C_3(2x)^3 + {}^4C_4(2x)^4] \\
 &\quad + 90x^4 [{}^3C_0 + {}^3C_1(2x) + {}^3C_2(2x)^2 + {}^3C_3(2x)^3] - 270x^6 \\
 &\quad [{}^2C_0 + {}^2C_1(2x) + {}^2C_2(2x)^2] + 5 \times 81x^8 (1 + 2x) - 243x^{10} \\
 &= 10 + 10x + 10 \times 4x^2 + 10 \times 8x^3 + 5 \times 16x^4 + 32x^5 - 15x^2 - 120x^3 \\
 &\quad - 180x^4 + 480x^5 - 240x^6 + 90x^4 + 540x^5 + 1080x^6 + 720x^7 - 270x^6 \\
 &\quad - 1080x^7 - 1080x^8 + 405x^8 + 810x^9 - 243x^{10} \\
 &= 1 + 10x + 25x^2 - 40x^3 - 190x^4 + 92x^5 + 570x^6 - 360x^7 - 675x^8 + 810x^9 - 243x^{10}
 \end{aligned}$$

Q1(ix)

Let $y = x + 1$, then

$$\left(x + 1 - \frac{1}{x}\right)^3 = \left(y - \frac{1}{x}\right)^3$$

The expansion of $(x + y)^n$ has $n + 1$ terms so the expansion of $\left(y - \frac{1}{x}\right)^3$ has 4 terms.

Using binomial theorem to expand, we get

$$\begin{aligned}\left(y - \frac{1}{x}\right)^3 &= {}^3C_0 y^3 \left(\frac{1}{x}\right)^0 - {}^3C_1 y^2 \left(\frac{1}{x}\right) + {}^3C_2 y \left(\frac{1}{x}\right)^2 - {}^3C_3 y^0 \left(\frac{1}{x}\right)^3 \\ &= y^3 - 3y^2 \times \frac{1}{x} + 3y \times \frac{1}{x^2} - \frac{1}{x^3}\end{aligned}$$

Putting $y = x + 1$, we get

$$\begin{aligned}\left(x + 1 - \frac{1}{x}\right)^3 &= (x + 1)^3 - 3(x + 1)^2 \times \frac{1}{x} + 3(x + 1) \times \frac{1}{x^2} - \frac{1}{x^3} \\ &= x^3 + 1 + 3x^2 + 3x - 3x - \frac{3}{x} - 6 + \frac{3}{x} + \frac{3}{x^2} - \frac{1}{x^3} \\ &= x^3 + 3x^2 - 5 + \frac{3}{x^2} - \frac{1}{x^3}\end{aligned}$$

Q1(x)

Let $y = 1 - 2x$, then

$$(1 - 2x + 3x^2)^3 = (y + 3x^2)^3$$

The expansion of $(x + y)^n$ has $n + 1$ terms so the expansion of $(y + 3x^2)^3$ has 4 terms.

Using binomial theorem to expand, we get

$$\begin{aligned}(y + 3x^2)^3 &= {}^3C_0 y^3 (3x^2)^0 + {}^3C_1 y^2 (3x^2)^1 + {}^3C_2 y (3x^2)^2 + {}^3C_3 y^0 (3x^2)^3 \\ &= y^3 + 3y^2 (3x^2) + 3y (9x^2) + (27x^6)\end{aligned}$$

Substituting $y = 1 - 2x$, we get,

$$\begin{aligned}(1 - 2x + 3x^2)^3 &= (1 - 2x)^3 + 3(1 + 4x^2 - 4x)(3x^2) + 3(1 - 2x)(9x^2) + (27x^6) \\ &= 1 - 8x^3 - 6x + 12x^2 + 9x^2 + 36x^4 - 36x^3 + 27x^2 - 54x^3 + 27x^6 \\ &= 1 - 6x + 21x^2 - 44x^3 + 63x^4 - 54x^5 + 27x^6\end{aligned}$$

Q2(i)

$$\begin{aligned}
 & (\sqrt{x+1} + \sqrt{x-1})^6 + (\sqrt{x+1} - \sqrt{x-1})^6 \\
 &= {}^6C_0(\sqrt{x+1})^6 + {}^6C_1(\sqrt{x+1})^5(\sqrt{x-1}) + {}^6C_2(\sqrt{x+1})^4(\sqrt{x-1})^2 - {}^6C_3(\sqrt{x+1})^3(\sqrt{x-1})^3 \\
 &+ {}^6C_4(\sqrt{x+1})^2(\sqrt{x-1})^4 + {}^6C_5(\sqrt{x+1})(\sqrt{x-1})^5 + {}^6C_6(\sqrt{x-1})^6 + {}^6C_0(\sqrt{x+1})^6 - \\
 &{}^6C_1(\sqrt{x+1})^5(\sqrt{x-1}) + {}^6C_2(\sqrt{x+1})^4(\sqrt{x-1})^2 - {}^6C_3(\sqrt{x+1})^3(\sqrt{x-1})^3 + \\
 &{}^6C_4(\sqrt{x+1})^2(\sqrt{x-1})^4 - {}^6C_5(\sqrt{x+1})(\sqrt{x-1})^5 + {}^6C_6(\sqrt{x-1})^6 \\
 &= 2[(x+1)^3 + 15(x+1)^2(x-1) + 15(x+1)(x-1)^2 + (x-1)^3] \\
 &= 2 \left[\begin{array}{l} x^3 + 1 + 3x + 3x^2 + 15x^3 - 15x^2 + 15x - 15 + 30x^2 - 30x \\ + 15x^3 + 15x^2 + 15x + 15 - 30x^2 - 30x + x^3 - 1 - 3x^2 + 3x \end{array} \right] \\
 &= 64x^3 - 48x \\
 &= 16x(4x^2 - 3)
 \end{aligned}$$

Q2(ii)

$$\begin{aligned}
 & (x + \sqrt{x^2-1})^6 + (x - \sqrt{x^2-1})^6 \\
 &= 2 \left[{}^6C_0x^6 + {}^6C_2x^4(\sqrt{x^2-1})^2 + {}^6C_4x^2(\sqrt{x^2-1})^4 + {}^6C_6(\sqrt{x^2-1})^6 \right] \\
 &= 2 \left[x^6 + 15x^4(x^2-1) + 15x^2(x^2-1)^2 + (x^2-1)^3 \right] \\
 &= 2 \left[x^6 + 15x^6 - 15x^4 + 15x^6 + 15x^2 - 30x^4 + x^6 - 1 - 3x^4 + 3x^2 \right] \\
 &= 64x^6 - 96x^4 + 36x^2 - 2
 \end{aligned}$$

Q2(iii)

$$\begin{aligned} & (1+2\sqrt{x})^5 + (1-2\sqrt{x})^5 \\ &= 2 \left[{}^5C_0 + {}^5C_2(2\sqrt{x})^2 + {}^5C_4(2\sqrt{x})^4 \right] \\ &= 2 \left[1 + 10 \times 4 \times x + 16 \times x^2 \times 5 \right] \\ &= 2 + 80x + 160x^2 \end{aligned}$$

Q2(iv)

$$\begin{aligned} & (\sqrt{2}+1)^6 + (\sqrt{2}-1)^6 \\ &= {}^6C_0(\sqrt{2})^6 + {}^6C_1(\sqrt{2})^5 + {}^6C_2(\sqrt{2})^4 + {}^6C_3(\sqrt{2})^3 + {}^6C_4(\sqrt{2})^2 + {}^6C_5(\sqrt{2}) + {}^6C_6 + {}^6C_0(\sqrt{2})^6 - \\ & \quad {}^6C_1(\sqrt{2})^5 + {}^6C_2(\sqrt{2})^4 - {}^6C_3(\sqrt{2})^3 + {}^6C_4(\sqrt{2})^2 - {}^6C_5(\sqrt{2}) + {}^6C_6(\sqrt{2})^0 \\ &= 2 \left[2^3 + 15 \times 2^2 + 15 \times 2 + 1 \right] \\ &= 2 \left[8 + 60 + 30 + 1 \right] = 2(99) = 198 \end{aligned}$$

Q2(v)

$$\begin{aligned} & (3+\sqrt{2})^5 - (3-\sqrt{2})^5 \\ &= 2 \left[{}^5C_1(3)^4(\sqrt{2})^1 + {}^5C_3(3)^2(\sqrt{2})^3 + {}^5C_5(\sqrt{2})^5 \right] \\ &= 2 \left[5 \times 81 \times \sqrt{2} + 10 \times 9 \times 2\sqrt{2} + 4\sqrt{2} \right] \\ &= 2 \left[405\sqrt{2} + 180\sqrt{2} + 4\sqrt{2} \right] \\ &= 2 \left[589\sqrt{2} \right] \\ &= 1178\sqrt{2} \end{aligned}$$

Q2(vi)

$$\begin{aligned} & (2+\sqrt{3})^7 + (2-\sqrt{3})^7 \\ &= 2 \left[{}^7C_0 2^7 + {}^7C_2 2^5 (\sqrt{3})^2 + {}^7C_4 (2)^3 (\sqrt{3})^4 + {}^7C_6 2 (\sqrt{3})^6 \right] \\ &= 2 [128 + 21 \times 32 \times 3 + 35 \times 8 \times 9 + 7 \times 2 \times 27] \\ &= 2 [128 + 2016 + 2520 + 378] \\ &= 2 [5042] \\ &= 10084 \end{aligned}$$

Q2(vii)

$$\begin{aligned} & (\sqrt{3}+1)^5 - (\sqrt{3}-1)^5 \\ &= 2 \left[{}^5C_1 (\sqrt{3})^4 + {}^5C_3 (\sqrt{3})^2 + {}^5C_5 \right] \\ &= 2 [5 \times 9 + 10 \times 3 + 1] \\ &= 2 [45 + 30 + 1] \\ &= 2 [76] \\ &= 152 \end{aligned}$$

Q2(viii)

$$\begin{aligned} & (0.99)^5 + (1.01)^5 \\ &= (1-0.01)^5 + (1+0.01)^5 \\ &= 2 \left[{}^5C_1 + {}^5C_3 (0.01)^2 + {}^5C_5 (0.01)^4 \right] \\ &= 2 \left[5 + 10 \times \frac{1}{10^4} + \frac{1}{10^{10}} \right] \\ &= 2 \left[5 + \frac{1}{1000} + \frac{1}{10^{10}} \right] \\ &= 2.0020001 \end{aligned}$$

Q2(ix)

$$\begin{aligned}
 & (\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6 \\
 &= 2 \left[{}^6C_1 (\sqrt{3})^5 (\sqrt{2}) + {}^6C_3 (\sqrt{3})^3 (\sqrt{2})^3 + {}^6C_5 (\sqrt{3}) (\sqrt{2})^5 \right] \\
 &= 2 \left[6 \times \sqrt{6} \times 9 + 20 \times 3\sqrt{3} \times 2\sqrt{2} + 6 \times \sqrt{3} \times 4\sqrt{2} \right] \\
 &= 2 \left[54\sqrt{6} + 120\sqrt{6} + 24\sqrt{6} \right] \\
 &= 2 \left[198\sqrt{6} \right] \\
 &= 396\sqrt{6}
 \end{aligned}$$

Q2(x)

$$\begin{aligned}
 & \left(a^2 - \sqrt{a^2 - 1} \right)^4 + \left(a^2 + \sqrt{a^2 - 1} \right)^4 \\
 \text{Let: } a^2 &= A, \quad \sqrt{a^2 - 1} = B \\
 & (A + B)^4 + (A - B)^4 \\
 &= B^4 + {}^4C_1 AB^3 + {}^4C_2 A^2 B^2 + {}^4C_3 A^3 B + A^4 + B^4 - {}^4C_1 AB^3 + {}^4C_2 A^2 B^2 - {}^4C_3 A^3 B + A^4 \\
 &= 2 \left(A^4 + {}^4C_2 A^2 B^2 + B^4 \right) \\
 &= 2 \left(A^4 + 6A^2 B^2 + B^4 \right) \\
 &= 2 \left(a^8 + 6a^4 (a^2 - 1) + (a^2 - 1)^2 \right) \\
 &= 2 \left[a^8 + 6a^6 - 6a^4 + a^4 - 1 - 2a^2 \right] \\
 & \left(a^2 - \sqrt{a^2 - 1} \right)^4 + \left(a^2 + \sqrt{a^2 - 1} \right)^4 = 2a^8 - 12a^6 - 10a^4 - 4a^2 + 2
 \end{aligned}$$

Q3

We have,

$$\begin{aligned}
 & (a+b)^4 - (a-b)^4 \\
 &= \left[{}^4C_0 a^4 b^0 + {}^4C_1 a^3 b^1 + {}^4C_2 a^2 b^2 + {}^4C_3 a^1 b^3 + {}^4C_4 a^0 b^4 \right] \\
 & \quad - \left[{}^4C_0 a^4 b^0 - {}^4C_1 a^3 b^1 + {}^4C_2 a^2 b^2 - {}^4C_3 a^1 b^3 + {}^4C_4 a^0 b^4 \right] \\
 &= \left[{}^4C_0 a^4 (-b)^0 + {}^4C_1 a^3 (-b)^1 + {}^4C_2 a^2 (-b)^2 + {}^4C_3 a^1 (-b)^3 + {}^4C_4 a^0 (-b)^4 \right] \\
 & \quad - \left[{}^4C_0 a^4 (-b)^0 + {}^4C_1 a^3 (-b)^1 + {}^4C_2 a^2 (-b)^2 + {}^4C_3 a^1 (-b)^3 + {}^4C_4 a^0 (-b)^4 \right] \\
 &= \left[{}^4C_0 a^4 + {}^4C_1 a^3 b + {}^4C_2 a^2 b^2 + {}^4C_3 a b^3 + {}^4C_4 a^0 b^4 \right] - \left[{}^4C_0 a^4 - {}^4C_1 a^3 b - {}^4C_2 a^2 b^2 - {}^4C_3 a b^3 + {}^4C_4 b^4 \right] \\
 &= {}^4C_0 a^4 + {}^4C_1 a^3 b + {}^4C_2 a^2 b^2 + {}^4C_3 a b^3 + {}^4C_4 a^0 b^4 - {}^4C_0 a^4 + {}^4C_1 a^3 b - {}^4C_2 a^2 b^2 + {}^4C_3 a b^3 - {}^4C_4 b^4 \\
 &= 2 \left[{}^4C_1 a^3 b + {}^4C_3 a b^3 \right] \\
 &= 2 \left[4a^3 b + 4ab^3 \right] \\
 &= 8 \left[a^3 b + ab^3 \right] \\
 \therefore (a+b)^4 - (a-b)^4 &= 8(a^3 b + ab^3) \quad \text{--- (i)}
 \end{aligned}$$

Putting $a = \sqrt{3}$ and $b = \sqrt{2}$ in equation (i), we get

$$\begin{aligned}
 & (\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 = 8 \left[(\sqrt{3})^3 \times \sqrt{2} + (\sqrt{3}) \times (\sqrt{2})^3 \right] \\
 &= 8 \left[3\sqrt{6} + 2\sqrt{6} \right] \\
 &= 8 \times 5\sqrt{6} \\
 &= 40\sqrt{6}
 \end{aligned}$$

$$\therefore (\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 = 40\sqrt{6}.$$

Q4

We have,

$$\begin{aligned} & (x+1)^6 - (x-1)^6 \\ &= \left[{}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x^1 + {}^6C_6x^0 \right] \\ &+ \left[{}^6C_0x^6(-1)^0 + {}^6C_1x^5(-1)^1 + {}^6C_2x^4(-1)^2 + {}^6C_3x^3(-1)^3 + {}^6C_4x^2(-1)^4 + {}^6C_5x^1(-1)^5 + {}^6C_6x^0(-1)^6 \right] \\ &= \left[{}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 - {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x + {}^6C_6 - {}^6C_0x^6 - {}^6C_1x^5 + {}^6C_2x^4 - {}^6C_3x^3 + {}^6C_4x^2 \right. \\ &\quad \left. - {}^6C_5x + {}^6C_6 \right] \\ &= 2 \left[{}^6C_0x^6 - {}^6C_2x^4 + {}^6C_4x^2 + {}^6C_6 \right] \\ &= 2 \left[x^6 + 15x^4 + 15x^2 + 1 \right] \end{aligned}$$

$$\therefore (x+1)^6 + (x-1)^6 = 2 \left[x^6 + 15x^4 + 15x^2 + 1 \right] \quad \text{---(i)}$$

Putting $x = \sqrt{2}$ in equation (i), we get:

$$\begin{aligned} (x+1)^6 + (x-1)^6 &= 2 \left[(\sqrt{2})^6 + 15(\sqrt{2})^4 + 15(\sqrt{2})^2 + 1 \right] \\ &= 2[8 + 60 + 30 + 1] \\ &= 2[99] \\ &= 198 \end{aligned}$$

$$\therefore (x+1)^6 + (x-1)^6 = 198$$

Q5(i)

We have,

$$\begin{aligned} (96)^3 &= (100 - 4)^3 \\ &= {}^3C_0 \times 100^3 + {}^3C_1 \times 100^2 \times (-4) + {}^3C_2 \times 100 \times (-4)^2 + {}^3C_3 \times (-4)^3 \\ &= 100^3 - 3 \times 100^2 \times 4 + 3 \times 100 \times 4^2 - 4^3 \\ &= 1000000 - 120000 + 4800 - 64 \\ &= 1004800 - 120064 \\ &= 884736 \end{aligned}$$

$$\therefore (96)^3 = 884736$$

Q5(ii)

We have,

$$\begin{aligned}(102)^5 &= (100 + 2)^5 \\ &= {}^5C_0 \times 100^5 + {}^5C_1 \times 100^4 \times 2 + {}^5C_2 \times 100^3 \times 2^2 + {}^5C_3 \times 100^2 \times 2^3 + {}^5C_4 \times 100 \times 2^4 + {}^5C_5 \times 2^5 \\ &= 100^5 + 5 \times 100^4 \times 2 + 10 \times 100^3 \times 2^2 + 10 \times 100^2 \times 2^3 + 5 \times 100 \times 2^4 + 2^5 \\ &= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32 \\ &= 11040808032\end{aligned}$$

$$\therefore (102)^5 = 11040808032$$

Q5(iii)

We have,

$$\begin{aligned}(101)^4 &= (100 + 1)^4 \\ &= {}^4C_0 \times 100^4 + {}^4C_1 \times 100^3 + {}^4C_2 \times 100^2 + {}^4C_3 \times 100 + {}^4C_4 \\ &= 100^4 + 4 \times 100^3 + 6 \times 100^2 + 4 \times 100 + 1 \\ &= 100000000 + 4000000 + 60000 + 400 + 1 \\ &= 104060401\end{aligned}$$

$$\therefore (101)^4 = 104060401$$

Q5(iv)

We have,

$$\begin{aligned}(98)^5 &= (100 - 2)^5 \\ &= {}^5C_0 \times 100^5 - {}^5C_1 \times 100^4 \times (-2) + {}^5C_2 \times 100^3 \times (-2)^2 - {}^5C_3 \times 100^2 \times (-2)^3 + {}^5C_4 \times 100 \times (-2)^4 - {}^5C_5 \times (-2)^5 \\ &= {}^5C_0 \times 100^5 - {}^5C_1 \times 100^4 \times 2 + {}^5C_2 \times 100^3 \times 4 - {}^5C_3 \times 100^2 \times 8 + {}^5C_4 \times 100 \times 16 - {}^5C_5 \times 32 \\ &= 100^5 - 10 \times 100^4 + 40 \times 100^3 - 80 \times 100^2 + 80 \times 100 - 32 \\ &= 1000000000 - 100000000 + 40000000 - 8000000 + 80000 - 32 \\ &= 1004000000 - 100000000 \\ &= 9039207968\end{aligned}$$

$$\therefore (98)^5 = 9039207968$$

Q6

$$\begin{aligned}2^{3n} - 7n - 1 &= 2^{3(n)} - 7(n) - 1 \\&= 8^n - 7n - 1 \\&= (1+7)^n - 7n - 1 \\&= \left({}^n C_0 + {}^n C_1(7)^1 + {}^n C_2(7)^2 + \dots + {}^n C_n(7)^n \right) - 7n - 1 \\&= \left(1 + 7n + 49 {}^n C_2 + \dots + 49(7)^{n-2} \right) - 7n - 1 \\&= 49 \left({}^n C_2 + \dots + 7^{n-2} \right)\end{aligned}$$

$\therefore 2^{3n} - 7n - 1$ is divisible by 49

Hence, proved

Q7

$$\begin{aligned}3^{2n+2} - 8n - 9 &= 3^{2(n+1)} - 8n - 9 \\&= 9^{n+1} - 8n - 9 \\&= (1+8)^{n+1} - 8n - 9 \\&= \left({}^{n+1} C_0 + {}^{n+1} C_1 8^1 + {}^{n+1} C_2 8^2 + \dots + {}^{n+1} C_{n+1} 8^{n+1} \right) - 8n - 9 \\&= \left(1 + 8(n+1) + 64 {}^{n+1} C_2 + \dots + 64(8)^{n-1} \right) - 8n - 9 \\&= 64 \left({}^{n+1} C_2 + \dots + 8^{n-1} \right)\end{aligned}$$

Thus, $3^{2n+2} - 8n - 9$ is divisible by 64.

Q8

$$\begin{aligned} & 3^{3n} - 26n - 1 \\ &= (3^3)^n - 26n - 1 \\ &= 27^n - 26n - 1 \\ &= (1+26)^n - 26n - 1 \\ &= \left({}^n C_0 + {}^n C_1 (26)^1 + {}^n C_2 (26)^2 + \dots + {}^n C_n (26)^n \right) - 26n - 1 \\ &= \left(1 + 26n + 676 {}^n C_2 + \dots + 676 (26)^{n-2} \right) - 26n - 1 \\ &= 676 \left({}^n C_2 + \dots + (26)^{n-2} \right) \end{aligned}$$

$\therefore 3^{3n} - 26n - 1$ is divisible for $n \in \mathbb{N}$.

Hence, proved

Q9

We have,

$$\begin{aligned} (1.1)^{10000} &= (1+0.1)^{10000} \\ &= {}^{10000} C_0 + {}^{10000} C_1 (0.1) + {}^{10000} C_2 (0.1)^2 + \dots + {}^{10000} C_{10000} (0.1)^{10000} \\ &= 1 + 10000 \times (0.1) + \text{other positive terms} \\ &= 1 + 1000 + \text{other positive terms} \\ &= 1001 + \text{other positive terms} > 1000 \end{aligned}$$

$\therefore (1.1)^{10000} > 1000$

Q10

$$\begin{aligned} (1.2)^{4000} &= (1+0.2)^{4000} \\ &= {}^{4000} C_0 (0.2)^0 (1)^{4000} + {}^{4000} C_1 \times (0.2)^1 \times 1^{3999} + \dots + {}^{4000} C_{4000} (0.2)^{4000} 1^0 \\ &= 1 + 4000 \times 0.2 \times 1 + \dots + (0.2)^{4000} \\ &= 1 + 800 + \dots + (0.2)^{4000} \end{aligned}$$

Here, we clearly observe $(1.2)^{4000}$ is less than 801 thus, $(1.2)^{4000} < 800$.

Q11

$$\begin{aligned}(1.01)^{10} + (1 - 0.01)^{10} &= (1 + 0.01)^{10} + (1 - 0.01)^{10} \\ &= \left({}^{10}C_1 + {}^{10}C_2 \frac{1}{10^2} + {}^{10}C_3 \frac{1}{10^3} + \dots + {}^{10}C_{10} \frac{1}{10^{10}} \right) + \left({}^{10}C_1 - {}^{10}C_2 \frac{1}{10^2} + {}^{10}C_3 \frac{1}{10^3} - {}^{10}C_4 \frac{1}{10^4} + \dots \right) \\ &= 2 \left({}^{10}C_1 - {}^{10}C_3 \frac{1}{10^3} + {}^{10}C_5 \frac{1}{10^5} + {}^{10}C_7 \frac{1}{10^7} + {}^{10}C_9 \frac{1}{10^9} \right) \\ &= 2 \left(10 + \frac{10!}{3!7!} \frac{1}{1000} + \frac{10!}{5!5!} \frac{1}{(10)^5} + \frac{10!}{7!3!} \times \frac{1}{10^7} + \frac{10!}{9!1!} \frac{1}{10^9} \right) \\ &= 2 \left(10 + \frac{9 \times 8}{3 \times 2 \times 1000} + \frac{9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 10^5} + \frac{9 \times 8}{3 \times 2 \times 10^7} + \frac{1}{10^8} \right) \\ &= 2.0090042\end{aligned}$$

Q12

$$\begin{aligned}2^{4n+4} - 15n - 16 &= 2^{4(n+1)} - 15n - 15 - 1 \\ &= (16)^{(n+1)} - 15(n+1) - 1 \\ &= (1+15)^{n+1} - 15(n+1) - 1 \\ &= \left[{}^{n+1}C_0 + {}^{n+1}C_1(15) + {}^{n+1}C_2(15)^2 + \dots + {}^{n+1}C_{n+1}(15)^{n+1} \right] - 15(n+1) - 1 \\ &= \left[1 + 15(n+1) + {}^{n+1}C_2(15)^2 + \dots + {}^{n+1}C_{n+1}(15)^{n+1} \right] - 15(n+1) - 1 \\ &= 225 \left[{}^{n+1}C_2 + \dots + {}^{n+1}C_{n+1}(15)^{n-1} \right] \\ &= 225 \times \text{natural number}\end{aligned}$$

Ex 18.2

Q1

$$T_{r+1} = T_n = (-1)^r {}^n C_r x^{n-r} y^r$$

$$T_{11} = T_{10+1} = (-1)^{10} {}^{25} C_{10} (2x)^{15} \left(\frac{1}{x^2}\right)^{10} = {}^{25} C_{10} \left(\frac{2^{15}}{x^5}\right) = \frac{25!}{10!15!} 2^{15} x^{15} \times x^{-20}$$

11th term from the end = (26 - 11 + 1) = 16th from beginning.

$$\Rightarrow T_{16} = T_{15+1} = (-1)^{15} {}^{25} C_{15} (2x)^{10} \left(\frac{1}{x^2}\right)^{15} = -{}^{25} C_{15} \frac{2^{10}}{x^{20}}$$

Q2

$$T_n = T_{r+1} = (-1)^r x^{n-r} y^r \times {}^{10} C_r$$

$$n = 7, r = 6, x = 3x^2, y = \frac{1}{x^3}$$

$$T_7 = T_{6+1} = (-1)^6 {}^{10} C_6 (3x^2)^4 \left(\frac{1}{x^3}\right)^6 = {}^{10} C_6 3^4 x^8 \times \frac{1}{x^{18}} = {}^{10} C_6 \times \frac{81}{x^{10}} = \frac{210 \times 81}{x^{10}} = \frac{17010}{x^{10}}$$

Q3

Fifth term from the end is

(11 - 5 + 1) = 7th term from beginning

$$T_7 = T_{6+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$-(-1)^6 {}^{11} C_6 (3x)^4 \left(\frac{1}{x^2}\right)^6 = {}^{11} C_6 \times 3^4 \times \frac{x^4}{x^{12}} = \frac{210 \times 81}{x^8} = \frac{17010}{x^8}$$

Q4

$$T_N = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$N = 8, r = 7, x = x^{3/2} y^{1/2}, y = x^{1/2} y^{3/2}, n = 10$$

$$T_8 = T_{7+1} = (-1)^7 {}^{10} C_7 (x^{3/2} y^{1/2})^3 (x^{1/2} y^{3/2})^7 = -{}^{10} C_7 x^{9/2} \times x^{7/2} \times y^{3/2} y^{21/2} = -120 x^8 y^{12}$$

Q5

$$T_N = T_{r+1} = {}^n C_r x^{n-r} y^r$$

$$N = 7, r = 6, n = 8, x = \frac{4x}{5}, y = \frac{5}{2x}$$

$$T_7 = T_{6+1} = {}^8 C_6 \left(\frac{4x}{5}\right)^2 \left(\frac{5}{2x}\right)^6 = 28 \times \frac{4^2}{5^2} \times x^4 \times \frac{5^6}{2^6 \times x^6} = \frac{28}{4} \times \frac{5^4}{x^4} = \frac{7 \times 5 \times 125}{x^4} = \frac{4375}{x^4}$$

Q6

Term from the beginning

$$T_N = T_{r+1} = {}^n C_r x^{n-r} y^r \quad \text{---(i)}$$

$$N = 4, r = 3, n = 9, x = x, y = \frac{2}{x}$$

$$T_4 = T_{3+1} = {}^9 C_3 x^6 \left(\frac{2}{x}\right)^3 = \frac{9 \times 7 \times 8}{3 \times 2} x^3 \times 8 = 672x^3$$

4th term from the end = 7th term from beginning

Using (i)

$$N = 7, r = 6, n = 9, x = x, y = \frac{2}{x}$$

$$T_7 = T_{6+1} = {}^9 C_6 x^3 \left(\frac{2}{x}\right)^6 = \frac{9 \times 8 \times 7}{3 \times 2} x \frac{2^6}{x^3} = \frac{5376}{x^3}$$

Q7

$$T_N = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

4th term from the end = 7th term from beginning

$$N = 7, r = 6, n = 9, x = \frac{4x}{5}, y = \frac{5}{2x}$$

$$T_7 = T_{6+1} = (-1)^6 {}^9 C_6 \left(\frac{4x}{5}\right)^3 \left(\frac{5}{2x}\right)^6 = \frac{9 \times 8 \times 7}{3 \times 2} x \frac{4^3 \times 5^6}{5^3 \times 2^6} \times \frac{x^3}{x^6} = \frac{9 \times 8 \times 7 \times 5^3}{6 \times x^3} = \frac{9 \times 8 \times 7 \times 125}{6 \times x^3} = \frac{10500}{x^3}$$

Q8

7th term from the end = 3rd term from beginning

$$T_N = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$N = 3, r = 2, n = 8, x = 2x^2, y = \frac{3}{2x}$$

$$T_3 = T_{2+1} = (-1)^2 {}^8 C_2 (2x^2)^6 \left(\frac{3}{2x}\right)^2 = \frac{8 \times 7}{2} x \frac{2^6 \times 3^2 \times x^{12}}{2^2 \times x^2} = 8 \times 7 \times 9 \times 8 \times x^{10} = 4032x^{10}$$

Q9(i)

$$x^{10} \text{ in } \left(2x^2 - \frac{1}{x}\right)^{20}$$

$$T_n = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$(-1)^r {}^{20} C_r (2x^2)^{20-r} \left(\frac{1}{x}\right)^r$$

Coefficient of x^{10} is

$$(-1)^r {}^{20} C_r 2^{20-r} x^{40-2r} x^{-r}$$

—(i)

$$\Rightarrow x^{40-3r} = x^{10}$$

$$\Rightarrow 10 - 3r = 10$$

$$3r = 30$$

$$r = 10$$

Substituting $r = 10$ in (i)

$$(-1)^{10} {}^{20} C_{10} 2^{10}$$

$$= {}^{20} C_{10} 2^{10}$$

Q9(ii)

$$x^7 \text{ in } \left(x - \frac{1}{x^2}\right)^{40}$$

$$T_n = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$= (-1)^r {}^{40} C_r x^{40-r} \left(\frac{1}{x^2}\right)^r$$

$$= (-1)^r {}^{40} C_r x^{40-r-2r}$$

$$\Rightarrow x^7 = x^{40-3r}$$

$$7 = 40 - 3r$$

$$3r = 33$$

$$r = 11$$

$$= (-1)^{11} {}^{40} C_{11} \text{ is coeff of } x^7$$

$$= -{}^{40} C_{11}$$

Q9(iii)

$$\begin{aligned}x^{-15} \text{ in } \left(3x^2 - \frac{a}{3x^3}\right)^{10} \\& (-1)^r {}^{10}C_r (3x^2)^{10-r} \left(\frac{a}{3x^3}\right)^r \\& (-1)^r {}^{10}C_r \frac{3^{10-r} a^r}{3^r} x^{20-2r-3r} \\ \Rightarrow x^{20-5r} &= x^{-15} \\ 20-5r &= -15 \\ 35 &= 5r \\ r &= 7 \\ (-1)^7 {}^{10}C_7 \frac{3^3 a^7}{3^7} \\ &= -\frac{40}{27} a^7\end{aligned}$$

Q9(iv)

$$\begin{aligned}x^9 \text{ in expansion of } \left(x^2 - \frac{1}{3x}\right)^9 \\ T_n = T_{r+1} &= (-1)^r {}^9C_r x^{n-r} y^r \\ &= (-1)^r {}^9C_r (x^2)^{9-r} \left(\frac{1}{3x}\right)^r \\ &= (-1)^r {}^9C_r x \frac{1}{3^r} x x^{18-2r-r} \\ \Rightarrow x^{18-3r} &= x^9 \\ 18-3r &= 9 \\ 3r &= 9 \\ r &= 3 \\ &= (-1)^3 {}^9C_3 \frac{1}{3^3} \\ &= -\frac{9 \times 8 \times 7}{3 \times 2 \times 9 \times 3} \\ &= -\frac{28}{9}\end{aligned}$$

Q9(v)

x^m in expansion of $\left(x + \frac{1}{x}\right)^n$

$$T_r = {}^n C_r x^{n-r} y^r$$

$$= {}^n C_r x^{n-r} \left(\frac{1}{x}\right)^r$$

$$x^{n-2r} = x$$

$$n - 2r = m$$

$$r = \frac{n-m}{2}$$

$${}^n C_{\frac{n-m}{2}} = \frac{n!}{\left(\frac{n-m}{2}\right)! \left(\frac{n+m}{2}\right)!}$$

Q9(vi)

$$\begin{aligned} (1-2x^2+3x^4)\left(1+\frac{1}{x}\right)^4 &= (1-2x^2+3x^4) \left({}^4 C_0 + {}^4 C_1 \frac{1}{x} + {}^4 C_2 \left(\frac{1}{x}\right)^2 + {}^4 C_3 \left(\frac{1}{x}\right)^3 + {}^4 C_4 \left(\frac{1}{x}\right)^4 + \right. \\ &\quad \left. {}^4 C_2 \left(\frac{1}{x}\right)^2 + {}^4 C_3 \left(\frac{1}{x}\right)^3 + {}^4 C_4 \left(\frac{1}{x}\right)^4 + {}^4 C_4 \left(\frac{1}{x}\right)^4 \right) \\ &= -(2x^2) \left({}^4 C_2 \left(\frac{1}{x}\right)^2 \right) + \left(3x^4 \times {}^4 C_4 \left(\frac{1}{x}\right)^4 \right) \\ &= -(56) + (210) \\ &= -112 + 168 \\ &= 154 \end{aligned}$$

Q9(vii)

$$\begin{aligned} (a-2b)^{12} &= {}^{12} C_0 a^{12} - {}^{12} C_1 a^{11} (2b)^1 + {}^{12} C_2 a^{10} (2b)^2 - {}^{12} C_3 a^9 (2b)^3 + \dots - {}^{12} C_7 a^5 (2b)^7 + \dots \\ &= -\frac{12!}{7!5!} \times 128 \\ &= -\frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2} \times 128 \\ &= -101376 \end{aligned}$$

Q9(viii)

$$(1 - 3x + 7x^2)(1 - x)^{16} = (1 - 3x + 7x^2)({}^{16}C_0 - {}^{16}C_1x + {}^{16}C_2x^2 + \dots + {}^{16}C_{16}x^{16})$$

∴ Coefficient of x in $(1 - 3x + 7x^2)(1 - x)^{16}$

$$= 1 \times (-{}^{16}C_1) - 3 \times (-{}^{16}C_0)$$

$$= -16 - 3$$

$$= -19$$

Q10

$$\begin{aligned} T_n = T_{r+1} &= {}^n C_r x^{n-r} y^r \\ &= {}^{21} C_r \left(\frac{x}{\sqrt{y}} \right)^{21-r} \left(\frac{y}{x^3} \right)^r \\ &= {}^{21} C_r \left(\frac{x^{\frac{7-r}{3}}}{y^{\frac{7-r}{6}}} \right) \frac{y^{\frac{r}{2}}}{x^{\frac{r}{6}}} \\ &\quad \frac{x^{\frac{7-r}{3 \cdot 6}}}{y^{\frac{7-r}{6 \cdot 2}}} \\ \Rightarrow x^{\frac{42-2r-r}{6}} &= y^{\frac{21-r-3r}{6}} \end{aligned}$$

Since x and y have same power

$$\frac{42-3r}{6} = \frac{-(21-4r)}{6}$$

$$42+21 = 4r+3r$$

$$63 = 7r$$

$$r = 9$$

Term is 10th

$$(t_n = t_{r+1})$$

Q11

$$(-1)^r {}^{20}C_r (2x^2)^{20-r} \left(\frac{1}{x}\right)^r$$

$$x^{40-2r} x^{-r} = x^9$$

$$40 - 3r = 9$$

$$31 = 3r$$

$$r = \frac{31}{3}$$

r can not be in fraction

∴ There is no term involving x^9 .

Q12

Any term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$ is

$$T_{r+1} = {}^nC_r x^{n-r} Y^r$$

$$= {}^{12}C_r (x^2)^{12-r} \left(\frac{1}{x}\right)^r$$

$$= {}^{12}C_r x^{24-2r} x^{-r}$$

$$x^{12-2r} = x^{-4}$$

$$12 - 2r = -4$$

$$2r = 16$$

$$r = \frac{16}{2}$$

r can not be a fraction, therefore there is no term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$ having the term x^{-4} .

Q13(i)

$$\left(\frac{2}{3}x - \frac{3}{2x}\right)^{20}$$

Here, $n = 20$ which is an even number so, $\left(\frac{20}{2} + 1\right)^{\text{th}}$ i.e., 11th term is the middle term.

We know that,

$$T_{r+1} = (-1)^r {}^nC_r x^{n-r} Y^r$$

$$n = 20, r = 10, x = \frac{2}{3}x, Y = \frac{3}{2x}$$

$$T_{11} = T_{10+1} = (-1)^{10} {}^{20}C_{10} \left(\frac{2}{3}x\right)^{10} \left(\frac{3}{2x}\right)^{10}$$

$$= {}^{20}C_{10} \frac{2^{10}}{3^{10}} x^{10} \frac{3^{10}}{2^{10}} x^{-10}$$

$$= {}^{20}C_{10}$$

Q13(ii)

Here, $n = 12$, which is even number.

∴, $\left(\frac{12}{2} + 1\right)$ th term i.e., 7th term is the middle term.

Hence, the middle term = $T_7 = T_{6+1}$

$$\begin{aligned} \therefore T_7 - T_{6+1} &= {}^{12}C_6 \times \left(\frac{a}{x}\right)^{12-6} \times (bx)^6 \\ &= {}^{12}C_6 \left(\frac{a}{x}\right)^6 \times (bx)^6 \\ &= \frac{12!}{(12-6)!6!} \times \frac{a^6}{x^6} \times b^6 x^6 \\ &= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6!}{(6 \times 5 \times 4 \times 3 \times 2 \times 1)} \times a^6 b^6 \\ &= 524 \times a^6 b^6 \end{aligned}$$

∴ The middle term = $524 \times a^6 b^6$.

Q13(iii)

$$\left(x^2 - \frac{2}{x}\right)^{10}$$

Here, $n = 10$

∴ $\left(\frac{n}{2} + 1\right) = \left(\frac{10}{2} + 1\right) = 6^{\text{th}}$ term is the middle term.

The term formula is

$$\begin{aligned} T_r - T_{r+1} &= (-1)^r {}^nC_r x^{n-r} y^r \\ T_6 - T_{5+1} &= (-1)^5 {}^{10}C_5 (x^2)^{10-5} \left(\frac{2}{x}\right)^5 \\ &= -{}^{10}C_5 x^{20-10} \frac{2^5}{x^5} \\ &= \frac{-10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \times 2^5 \times x^3 \\ &= -8064x^3 \end{aligned}$$

Q13(iv)

$$\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$$

Here $n = 10$, which is even, therefore it has 11 terms

∴ middle term is $\left(\frac{n}{2} + 1\right) = 6^{\text{th}}$ term

$$\begin{aligned} T_r - T_{r+1} &= (-1)^r {}^nC_r x^{n-r} y^r \\ T_5 - T_{5+1} &= (-1)^5 {}^{10}C_5 \left(\frac{x}{a}\right)^{10-5} \left(\frac{a}{x}\right)^5 \\ &= -\frac{10!}{5!5!} \times \frac{x^5}{a^5} \times a^5 \times x^{-5} \\ &= -252 \end{aligned}$$

Q14(i)

$$\left(3x - \frac{x^3}{6}\right)^9$$

Here, $n = 9$, which is odd number

$\therefore \left(\frac{9+1}{2}\right)^{\text{th}}$ and $\left(\frac{9+1}{2} + 1\right)^{\text{th}}$ i.e., 5th, 6th term are the middle term.

Here, the term formula is:

$$T_5 = T_{r+1} = (-1)^r {}^9C_r (3x)^9 \left(\frac{x^3}{6}\right)^r$$

$$= {}^9C_4 \frac{3^5}{6^4} x x^5 x x^{12}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 3^5}{4 \times 3 \times 2 \times 3^4 \times 2^4} x^{17}$$

$$= \frac{189}{8} x^{17}$$

$$T_6 = T_{5+1} = (-1)^5 {}^9C_5 (3x)^4 \left(\frac{x^3}{6}\right)^5$$

$$= -\frac{9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \times \frac{3^4}{6^5} x x^4 x x^{15}$$

$$= -\frac{9 \times 8 \times 7 \times 6 \times 3^4}{5 \times 4 \times 3 \times 2 \times 3^5 \times 2^5} x^{19}$$

$$= -\frac{21}{16} x^{19}$$

Q14(ii)

$$\left(3x^2 - \frac{1}{x}\right)^7$$

Here, $n = 7$, which is odd

$\therefore \left(\frac{7+1}{2}\right)^{\text{th}}$ and $\left(\frac{7+1}{2} + 1\right)^{\text{th}} = 4^{\text{th}}$, 5th term are middle term or $\left(2x^2 - \frac{1}{x}\right)^7$

$$T_4 = T_{r+1} = (-1)^r {}^7C_r x^{2r} y^r$$

$$T_4 = T_{3+1} = (-1)^3 {}^7C_3 (2x^2)^{7-3} \left(\frac{1}{x}\right)^3$$

$$= -{}^7C_3 \frac{2^4 x^8}{x^3}$$

$$= -560x^5$$

$$T_5 = T_{4+1} = (-1)^4 {}^7C_4 (2x^2)^{7-4} \left(\frac{1}{x}\right)^4$$

$$= {}^7C_4 \frac{2^3 x^6}{x^4}$$

$$= {}^7C_4 \frac{7 \times 6 \times 5 \times 8}{3 \times 2} x^2$$

$$= 280x^2$$

Q14(iii)

$$\left(3x - \frac{2}{x^2}\right)^{15}$$

7th and 8th terms are middle terms

$$\begin{aligned} & \binom{15}{7} (3x)^8 \left(-\frac{2}{x^2}\right)^7, \binom{15}{8} (3x)^7 \left(-\frac{2}{x^2}\right)^8 \\ & \frac{-6435 \times 3^8 \times 2^7}{x^6}, \frac{6437 \times 3^7 \times 2^8}{x^9} \end{aligned}$$

Q14(iv)

$$\left(x^4 - \frac{1}{x^3}\right)^{11}$$

Here, $n = 11$, which is odd number

$\therefore \left(\frac{11+1}{2}\right)^{\text{th}}$ and $\left(\frac{11+1}{2} + 1\right)^{\text{th}} = 6^{\text{th}}, 7^{\text{th}}$ term are the middle terms in $\left(x^4 - \frac{1}{x^3}\right)^{11}$

The term formula is

$$T_r = T_{r+1} = (-1)^r {}^{n}C_r x^{n-r} y^r$$

$$\begin{aligned} T_6 = T_{5+1} &= (-1)^5 {}^{11}C_5 (x^4)^{11-5} \left(\frac{1}{x^3}\right)^5 \\ &= -{}^{11}C_5 x^{24} \frac{1}{x^{15}} \end{aligned}$$

$$= \frac{-11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} x^9$$

$$= -11 \times 3 \times 2 \times 7 x^9$$

$$= -462 x^9$$

$$T_7 = T_{6+1} = (-1)^6 {}^{11}C_6 (x^4)^{11-6} \left(\frac{1}{x^3}\right)^6$$

$$= 462 \frac{x^{20}}{x^{18}}$$

$$= 462 x^2$$

Q15(i)

$$\left(x - \frac{1}{x}\right)^{10}$$

Here, $n = 10$, which is even, \therefore it has 11 terms

\therefore middle term is $\left(\frac{n}{2} + 1\right) = 6^{\text{th}}$ term

$$T_n = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$\begin{aligned} T_6 = T_{5+1} &= (-1)^5 {}^{10} C_5 (x)^{10-5} \left(\frac{1}{x}\right)^5 \\ &= \frac{-10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} x \frac{x^5}{x^5} \\ &= -3 \times 2 \times 7 \times 6 \\ &= -252 \end{aligned}$$

Q15(ii)

$$(1 - 2x + x^2)^n$$

Here, n is odd, $\therefore (1 - 2x + x^2)$ has $n + 1 =$ even term

\therefore middle term is $\left(\frac{n+1}{2}\right)^{\text{th}}$ term

$$T_n = T_{r+1} = {}^n C_r x^{n-r} y^r$$

$$\begin{aligned} \frac{T_{\frac{n+1}{2}}}{2} = \frac{T_n}{2} &= \frac{{}^n C_{\frac{n}{2}} (1 - 2x)^{\frac{n}{2}} (x^2)^{\frac{n}{2}}}{2} \\ &= \frac{n!}{\frac{n!}{2!} \frac{n!}{2!}} (1 - 2x)^{\frac{n}{2}} x^{\frac{2n}{2}} \\ &= \frac{(2n)!}{(n!)^2} (-1)^{\frac{n}{2}} x^n \quad \left[\because (1-x)^n = 1 - nx \right] \end{aligned}$$

Q15(iii)

$$(1 + 3x + 3x^2 + x^3)^{2n}$$

This expansion is $\left((1+x)^3\right)^{2n} = (1+x)^{6n}$

Since $6n$ is even \therefore it has $6n + 1 =$ odd terms has middle term is

$$\left(\frac{6n}{2} + 1\right)^{\text{th}} = (4n)^{\text{th}} \text{ term}$$

$$T_n = T_{r+1} = {}^n C_r x^{n-r} y^r$$

$$\begin{aligned} T_{4n} = T_{3n+1} &= \frac{{}^{6n} C_{3n} (1)^{6n-3n} (x)^{3n}}{(3n)!(3n)!} x^{3n} \quad \left[\because 1^{6n-3n} = 1 \right] \\ &= \frac{(6n)!}{(3n)!(3n)!} x^{3n} \end{aligned}$$

Q15(iv)

$$\left(2x - \frac{x^2}{4}\right)^9$$

4th and 5th terms are middle terms

$$\binom{9}{4}(2x)^5\left(-\frac{x^2}{4}\right)^4 + \binom{9}{5}(2x)^4\left(-\frac{x^2}{4}\right)^5$$

$$\frac{63}{4}x^{13}, -\frac{63}{32}x^{14}$$

Q15(v)

$$\left(x - \frac{1}{x}\right)^{2n+1}$$

$2n + 1$ is odd hence this expansion will have $2n + 2 =$ even terms.

Hence, middle terms is $\frac{2n+1}{2} = n+1, n+2$

Term formula is

$$T_n = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$T_{n+1} = T_{n+1} = (-1)^n {}^{2n+1} C_n (x)^{2n+1-n} \left(\frac{1}{x}\right)^n$$

$$= (-1)^n {}^{2n+1} C_n x^{n+1-n}$$

$$= (-1)^n {}^{2n+1} C_n x$$

$$T_{n+2} = T_{n+1+1} = (-1)^{n+1} {}^{2n+1} C_{n+1} (x)^{2n+1-n-1} \left(\frac{1}{x}\right)^{n+1}$$

$$= (-1)^{n+1} {}^{2n+1} C_{n+1} x^{-1}$$

$$= (-1)^{n+1} {}^{2n+1} C_{n+1} \frac{1}{x}$$

$$= (-1)^{n+1} {}^{2n+1} C_n \frac{1}{x}$$

$$[\because {}^n C_r = {}^n C_{n-r}]$$

Q15(vi)

$$\left(3 - \frac{x^3}{6}\right)^7$$

Here $n = 7$, which is odd

\therefore middle term is $\binom{7+1}{2}$ and $\binom{7+1}{2} + 1 = 4^{\text{th}}, 5^{\text{th}}$ terms

$$T_r = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$T_4 = T_{5+1} = (-1)^4 {}^7 C_4 (3)^{7-4} \left(\frac{x^3}{6}\right)^4$$

$$= -\frac{7!}{3!4!} \times 3^4 \times \frac{x^9}{6^4}$$

$$= -\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times 81 \times \frac{x^9}{216}$$

$$= -\frac{105}{8} x^9$$

And

$$T_r = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$T_5 = T_{4+1} = (-1)^4 {}^7 C_4 (3)^{7-4} \left(\frac{x^3}{6}\right)^4$$

$$= \frac{7!}{4!3!} \times 3^3 \times \frac{x^{12}}{6^4}$$

$$= \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times 27 \times \frac{x^{12}}{1296}$$

$$= \frac{35}{48} x^{12}$$

Q15(vii)

$$\left(\frac{x}{3} + 9y\right)^{10}$$

Here $n = 10$, which is even, therefore it has 11 terms

\therefore middle term is $\binom{n}{2} + 1 = 6^{\text{th}}$ term

$$T_r = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$T_6 = T_{6+1} = (-1)^5 {}^{10} C_5 \left(\frac{x}{3}\right)^{10-5} (9y)^5$$

$$= -\frac{10!}{5!5!} \times \frac{x^5}{3^5} \times 9^5 \times y^5$$

$$= 61236 x^5 y^5$$

Q15(viii)

For the given binomial expansion $n = 12$

So middle term is $\binom{12}{2} = 7^{\text{th}}$ term.

$$T_7 = {}^{12}C_2 (2dx)^{12-6} \left(-\frac{b}{x^2}\right)^6$$

$$T_7 = {}^{12}C_2 (2dx)^6 \left(\frac{b}{x^2}\right)^6$$

$$T_7 = {}^{12}C_2 (2^6 d^6 x^6) \left(\frac{b^6}{x^{12}}\right)$$

$$T_7 = {}^{12}C_2 \left(\frac{2^6 d^6 b^6}{x^6}\right)$$

$$\text{Middle term} = {}^{12}C_2 \left(\frac{2^6 d^6 b^6}{x^6}\right)$$

Q15(ix)

For the given binomial expansion $n = 9$.

So middle terms are $\binom{9+1}{2} = 5^{\text{th}}$ term and $\binom{9+3}{2} = 6^{\text{th}}$ term.

$$T_5 = {}^9C_4 \left(\frac{p}{x}\right)^{9-4} \left(\frac{x}{p}\right)^4$$

$$T_5 = {}^9C_4 \left(\frac{p}{x}\right)^5 \left(\frac{x}{p}\right)^4$$

$$T_5 = {}^9C_4 \left(\frac{p}{x}\right)$$

$$T_6 = {}^9C_5 \left(\frac{p}{x}\right)^{9-5} \left(\frac{x}{p}\right)^5$$

$$T_6 = {}^9C_5 \left(\frac{p}{x}\right)^4 \left(\frac{x}{p}\right)^5$$

$$T_6 = {}^9C_5 \left(\frac{x}{p}\right)$$

The middle terms are ${}^9C_4 \left(\frac{p}{x}\right)$ and ${}^9C_5 \left(\frac{x}{p}\right)$.

Q15(x)

For the given binomial expansion $n = 10$.

So middle term is $\left(\frac{10}{2} + 1\right) = 6^{\text{th}}$ term.

$$T_6 = {}^{10}C_5 \left(\frac{x}{a}\right)^{10-5} \left(-\frac{a}{x}\right)^5$$

$$T_6 = - {}^{10}C_5 \left(\frac{x}{a}\right)^5 \left(\frac{a}{x}\right)^5$$

$$T_6 = - {}^{10}C_5 = -252$$

Middle term is -252 .

Q16(i)

$$\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$$

In expansion

$$\begin{aligned} T_{r+1} &= {}^9C_r \left(\frac{3x^2}{2}\right)^{9-r} \left(\frac{-1}{3x}\right)^r \\ &= {}^9C_r \left(\frac{3}{2}\right)^{9-r} (x^{18-2r}) \left(\frac{-1}{3}\right)^r x^{-r} \end{aligned}$$

Let T_{r+1} be independent of x

$$18 - 3r = 0 \text{ or } r = 6$$

\therefore Required term

$$\begin{aligned} \Rightarrow T_{r+1} &= T_{6+1} = T_7 = {}^9C_6 \left(\frac{3}{2}\right)^{9-6} \left(\frac{-1}{3}\right)^6 x^{18-3(6)} \\ &= 84 \left(\frac{27}{8}\right) \left(\frac{1}{179}\right) x^0 = \frac{7}{18} \end{aligned}$$

Q16(ii)

$$\left(2x + \frac{1}{3x^2}\right)^9$$

4th term is independent of x

$$\binom{9}{3} (2x)^6 \left(\frac{1}{3x^2}\right)^3 = \binom{9}{3} \frac{64}{27}$$

Q16(iii)

$$T_{r+1} = (-1)^r {}^nC_r (2x^2)^{25-r} \left(\frac{3}{x^3}\right)^r = (-1)^r {}^nC_r 2^{25-r} 3^r x^{50-2r-3r}$$

Term independent of $x = x^0$

$$\Rightarrow x^{50-5r} = x^0 \Rightarrow 50 - 5r = 0 \Rightarrow r = 10$$

$$\therefore t_{11} = (-1)^{10} {}^{25}C_{10} 2^{15} \times 3^{10} = {}^{25}C_{10} 2^{15} 3^{10}$$

Q16(iv)

$$\left(3x - \frac{2}{x^2}\right)^{15}$$

$$T_{r+1} = (-1)^r {}^{15}C_r (3x)^{15-r} \left(\frac{2}{x^2}\right)^r$$

$$= (-1)^r {}^{15}C_r 3^{15-r} 2^r x^{15-r-2r}$$

Term independent of $x \Rightarrow x^0$

$$\Rightarrow x^{15-3r} = x^0$$

$$15 - 3r = 0 \Rightarrow r = 5$$

$$\therefore t_6 = (-1)^5 {}^{15}C_5 3^{10} 2^5$$

$$= -\frac{15!}{5!10!} 3^{10} 2^5 = -\frac{15 \times 14 \times 13 \times 12 \times 11}{120} 3^{10} 2^5$$

$$= -3003 \times 3^{10} \times 2^5$$

Q16(v)

$$\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$$

$$T_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\frac{3}{2x^2}\right)^r$$

$$= {}^{10}C_r x^{\frac{5-r}{2}} 3^{\frac{r}{2}} \times 3^{-\frac{5+r}{2}} \times 2^{-r}$$

Independent of $x \Rightarrow x^0$

$$x^{\frac{10-r-4r}{2}} = x^0$$

$$10 - 5r = 0$$

$$r = 2$$

$$t_3 = {}^{10}C_2 3^{2-5} 2^{-2}$$

$$= {}^{10}C_2 3^{-3} 2^{-2}$$

$$= \frac{10!}{2!8!} \times \frac{1}{36} = \frac{10 \times 9}{2 \times 36} = \frac{5}{4}$$

Q16(vi)

$$\left(x - \frac{1}{x^2}\right)^{3n}$$

$$T_{r+1} = (-1)^r {}^{3n}C_r x^{3n-r} \left(\frac{1}{x^2}\right)^r$$

$$= (-1)^r {}^{3n}C_r x^{3n-r-2r}$$

Independent of $x \Rightarrow x^0$

$$x^{3n-3r} = x^0 \Rightarrow r = n$$

$$= (-1)^n {}^{3n}C_n$$

Q16(vii)

We have,

$$\left(\frac{1}{2}x^{\frac{1}{3}} + x^{\frac{1}{5}}\right)^8$$

Let $(r+1)^{\text{th}}$ term be independent of x ,

$$\therefore T_{r+1} = {}^8C_r \left(\frac{1}{2}x^{\frac{1}{3}}\right)^{8-r} \left(x^{\frac{1}{5}}\right)^r$$

$$= {}^8C_r \left(\frac{1}{2}\right)^{8-r} \times \left(x^{\frac{1}{3}}\right)^{8-r} \times \left(\frac{1}{x^{\frac{1}{5}}}\right)^r$$

$$= {}^8C_r \left(\frac{1}{2}\right)^{8-r} \times (x)^{\frac{8-r}{3}} \times \left(\frac{1}{x^{\frac{1}{5}}}\right)^r$$

$$= {}^8C_r \left(\frac{1}{2}\right)^{8-r} \times (x)^{\frac{8-r}{3} - \frac{r}{5}}$$

$$= {}^8C_r \left(\frac{1}{2}\right)^{8-r} \times (x)^{\frac{40-5r-3r}{15}}$$

$$= {}^8C_r \left(\frac{1}{2}\right)^{8-r} \times (x)^{\frac{40-8r}{15}}$$

If it is independent of x , we must have

$$\frac{40-8r}{15} = 0$$

$$\Rightarrow 8r = 40$$

$$\Rightarrow r = 5$$

\therefore The term independent of $x = T_6$

Now,

$$T_6 = {}^8C_5 \left(\frac{1}{2}x^{\frac{1}{3}}\right)^{8-5} \left(x^{\frac{1}{5}}\right)^5$$

$$= 56 \times \left(\frac{1}{2}\right)^3$$

$$= 56 \times \frac{1}{8}$$

$$= 7$$

Hence, required term = 7

Q16(viii)

$$\begin{aligned} & (1+x+2x^2)^3 \left(\frac{3}{2}x^2 - \frac{1}{3x} \right)^9 \\ &= (1+x+2x^2)^3 \left[\left(\frac{3}{2}x^2 \right)^9 - {}^9C_1 \left(\frac{3}{2}x^2 \right)^8 \frac{1}{3x} - \dots - {}^9C_6 \left(\frac{3}{2}x^2 \right)^3 \left(\frac{1}{3x} \right)^6 - {}^9C_7 \left(\frac{3}{2}x^2 \right)^2 \left(\frac{1}{3x} \right)^7 \right] \end{aligned}$$

In the second bracket, we have to search the term so x^n and $\frac{1}{x^8}$ which when multiplying

by 1 and $2x^2$ is first bracket will give the term independent of x . The term containing $\frac{1}{x^8}$ will not occur in second bracket.

The term independent of x

$$\begin{aligned} &= 1 \left[{}^9C_0 \left(\frac{3}{2} \right)^9 \times \frac{1}{3^9} \right] - 2x^2 \left[{}^9C_7 \left(\frac{3}{2} \right)^2 \times \frac{1}{3^7} \times \frac{1}{x^8} \right] \\ &= \left[\frac{9 \times 8 \times 7}{1 \times 2 \times 3} \times \frac{1}{8 \times 27} \right] - 2 \left[\frac{9 \times 8}{1 \times 2} \times \frac{1}{4 \times 243} \right] \\ &= \frac{7}{11} - \frac{2}{27} \\ &= \frac{17}{81} \end{aligned}$$

Required term = $\frac{17}{81}$

Q16(ix)

We have,

$$\left(\sqrt[3]{x} + \frac{1}{2\sqrt[3]{x}} \right)^{18}, x > 0$$

Let $(r+1)^{\text{th}}$ term be independent of x .

$$\begin{aligned} \therefore T_{r+1} &= {}^{18}C_r (\sqrt[3]{x})^{18-r} \times \left(\frac{1}{2\sqrt[3]{x}} \right)^r \\ &= {}^{18}C_r (x)^{\frac{18-r}{3}} \times \left(\frac{1}{2} \right)^r \times \left(\frac{1}{x^{\frac{1}{3}}} \right)^r \\ &= {}^{18}C_r (x)^{\frac{18-r}{3}} \times \left(\frac{1}{x^{\frac{1}{3}}} \right)^r \times \left(\frac{1}{2} \right)^r \\ &= {}^{18}C_r (x)^{\frac{18-r}{3}} \times \left(\frac{1}{2} \right)^r \\ &= {}^{18}C_r (x)^{\frac{18-r}{3}} \times \left(\frac{1}{2} \right)^r \end{aligned}$$

If it is independent of x , we must have

$$\frac{18-r}{3} = 0$$

$$\Rightarrow 18 = 2r$$

$$\Rightarrow r = 9$$

\therefore Term independent of $x = T_{9+1} = T_{10}$

Now,

$$\begin{aligned} T_{10} &= {}^{18}C_9 (\sqrt[3]{x})^{18-9} \left(\frac{1}{2\sqrt[3]{x}} \right)^9 \\ &= {}^{18}C_9 (\sqrt[3]{x})^9 \times \frac{1}{2^9} \times \left(\frac{1}{\sqrt[3]{x}} \right)^9 \\ &= \frac{{}^{18}C_9}{2^9} \end{aligned}$$

Hence, required term = $\frac{{}^{18}C_9}{2^9}$

Q16(x)

$$\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^6$$

In expansion

$$\begin{aligned}T_{r+1} &= {}^6C_r \left(\frac{3x^2}{2}\right)^{6-r} \left(-\frac{1}{3x}\right)^r \\ &= {}^6C_r \left(\frac{3}{2}\right)^{6-r} (x^{12-3r}) \left(-\frac{1}{3}\right)^r\end{aligned}$$

Let T_{r+1} be independent of x ,

$$12 - 3r = 0 \text{ or } r = 4$$

∴ Required term

$$\begin{aligned}\Rightarrow T_{r+1} = T_{4+1} = T_5 &= {}^6C_4 \left(\frac{3}{2}\right)^{6-4} \left(-\frac{1}{3}\right)^4 x^{12-3(4)} \\ &= 15 \left(\frac{9}{4}\right) \left(\frac{1}{81}\right) x^0 = \frac{5}{12}\end{aligned}$$

Q17

We know that the coefficient of r th term in the expansion of $(1+x)^n$ is ${}^nC_{r-1}$

∴ Coefficient of $(2r+4)$ th term of the expansion $(1+x)^{18} = {}^{18}C_{2r+4-1} = {}^{18}C_{2r+3}$

and, coefficient of $(r-2)$ th term of the expansion $(1+x)^{18} = {}^{18}C_{r-2-1} = {}^{18}C_{r-3}$

It is given that these coefficients are equal.

$$\therefore {}^{18}C_{2r+3} = {}^{18}C_{r-3}$$

$$\Rightarrow 2r+3 = r-3 \text{ or, } 2r+3+r-3 = 18$$

$$\Rightarrow r = -6 \text{ or, } 3r = 18$$

$$\Rightarrow r = -6 \text{ or, } r = 6$$

$$\Rightarrow r = 6$$

$$\left[\begin{array}{l} \because {}^nC_r = {}^nC_s \\ \Rightarrow r = s \text{ or, } r+s = n \end{array} \right]$$

[∵ $r = -6$ is not possible]

Q18

$$(1+x)^{43}$$

$$\binom{43}{2r} = \binom{43}{r+1}$$

$$2r + r + 1 = 43$$

$$3r = 42$$

$$r = 14$$

Q19

Now, Coefficient of $(r+1)$ th term in the expansion of $(1+x)^{n+1} = {}^{n+1}C_{r+1-1} = {}^{n+1}C_r$

and, Coefficient of r th term in $(1+x)^n$ + Coefficient of $(r+1)$ th term in $(1+x)^n$

$$= {}^nC_{r-1} + {}^nC_{r+1-1}$$

$$= {}^nC_{r-1} + {}^nC_r$$

$$= \frac{n!}{\{n-(r-1)\}!(r-1)!} + \frac{n!}{(n-r)!r!}$$

$$= \frac{n!}{(n-r+1)!(r-1)!} + \frac{n!}{(n-r)!r!}$$

$$= \frac{n!}{(n-r+1)(n-r)!(r-1)!} + \frac{n!}{(n-r)!r(r-1)!}$$

$$= \frac{n!}{(n-r+1)(n-r)!(r-1)!} + \frac{n!}{(n-r)!(r-1)!r}$$

$$= \frac{n!}{(n-r)!(r-1)!} \left[\frac{1}{n-r+1} + \frac{1}{r} \right]$$

$$= \frac{n!}{(n-r)!(r-1)!} \left[\frac{r+n-r+1}{(n-r+1)r} \right]$$

$$= \frac{n!}{(n-r)!(r-1)!} \left[\frac{n+1}{(n-r+1)r} \right]$$

$$= \frac{n!(n+1)}{(n-r)!(n-r+1)(r-1)!r}$$

$$= \frac{(n+1)!}{(n-r+1)!r!}$$

$$= \frac{(n+1)!}{(n+1-r)!r!}$$

$$= {}^{n+1}C_r$$

$$\therefore {}^{n+1}C_r = {}^nC_{r-1} + {}^nC_r$$

The coefficient of $(r+1)$ th term in the expansion of $(1+x)^{n+1}$ is equal to the sum of the coefficients of r th and $(r+1)$ th terms in the expansion of $(1+x)^n$.

Q20

We have,

$$\left(x + \frac{1}{x}\right)^{2n}$$

Let $(r+1)^{\text{th}}$ term be independent of x .

$$\begin{aligned}\therefore T_{r+1} &= {}^{2n}C_r (x)^{2n-r} \left(\frac{1}{x}\right)^r \\ &= {}^{2n}C_r (x)^{2n-r-r} \\ &= {}^{2n}C_r x^{2n-2r}\end{aligned}$$

If it is independent of x , we must have,

$$2n - 2r = 0$$

$$\Rightarrow 2n = 2r$$

$$\Rightarrow r = n$$

\therefore Term independent of $x = T_{n+1}$

Now,

$$\begin{aligned}T_{n+1} &= {}^{2n}C_n (x-1)^{2n-n} \left(\frac{1}{x}\right)^n \\ &= {}^{2n}C_n \\ &= \frac{(2n)!}{(2n-n)!n!} \\ &= \frac{(2n)!}{n!n!} \\ &= \frac{(2n)(2n-1)(2n-2)\dots 5 \times 4 \times 3 \times 2 \times 1}{n!n!} \\ &= \frac{\{1 \times 3 \times 5 \times \dots (2n-1)\} \{2 \times 4 \times 6 \times \dots 2n\}}{n!n!} \\ &= \frac{\{1 \times 3 \times 5 \times \dots (2n-1)\} \times 2^n \{1 \times 2 \times 3 \times \dots n\}}{n!n!} \\ &= \frac{\{1 \times 3 \times 5 \times \dots (2n-1)\} \times 2^n \times n!}{n!n!} \\ &= 2^n \times \frac{\{1 \times 3 \times 5 \times \dots (2n-1)\}}{n!}\end{aligned}$$

\therefore The term independent to $x = \frac{\{1 \times 3 \times 5 \times \dots (2n-1)\}}{n!} \times 2^n$ Hence proved.

Q21

We have,

$$(1+x)^n$$

Now,

$$\text{Coefficient of 5th term} = {}^nC_{5-1} = {}^nC_4$$

$$\text{Coefficient of 5th term} = {}^nC_{6-1} = {}^nC_5$$

$$\text{and, Coefficient of 5th term} = {}^nC_{7-1} = {}^nC_6$$

It is given that these coefficients are in A.P.

$$\therefore 2{}^nC_5 = {}^nC_4 + {}^nC_6$$

$$\Rightarrow 2 \left[\frac{n!}{(n-5)!5!} \right] = \frac{n!}{(n-4)!4!} + \frac{n!}{(n-6)!6!}$$

$$\Rightarrow \frac{2}{(n-5)!5!} = \frac{1}{(n-4)!4!} + \frac{1}{(n-6)!6!}$$

$$\Rightarrow \frac{2}{(n-5)(n-6)!5 \times 4!} = \frac{1}{(n-4)(n-5)(n-6)!4!} + \frac{1}{(n-6)!6 \times 5 \times 4!}$$

$$\Rightarrow \frac{2}{(n-5) \times 5} = \frac{1}{(n-4)(n-5)} + \frac{1}{6 \times 5}$$

$$\Rightarrow \frac{2}{5(n-5)} - \frac{1}{30} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \frac{12 - (n-5)}{30(n-5)} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \frac{12 - n + 5}{30} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \frac{17 - n}{30} = \frac{1}{n-4}$$

$$\Rightarrow 17n - 68 - n^2 + 4n = 30$$

$$\Rightarrow 21n - 68 - n^2 - 30 = 0$$

$$\Rightarrow 21n - n^2 - 98 = 0$$

$$\Rightarrow n^2 - 21n + 98 = 0$$

$$\Rightarrow n^2 - 7n - 14n + 98 = 0$$

$$\Rightarrow n(n-7) - 17(n-7) = 0$$

$$\Rightarrow (n-7)(n-14) = 0$$

$$\Rightarrow n = 7 \text{ or, } n = 14$$

Q22

We have,

$$(1+x)^{2n}$$

Now,

$$\text{Coefficient 2nd term} = {}^{2n}C_{2-1} = {}^{2n}C_1$$

$$\text{Coefficient 3rd term} = {}^{2n}C_{3-1} = {}^{2n}C_2$$

$$\text{and, Coefficient 4th term} = {}^{2n}C_{4-1} = {}^{2n}C_3$$

It is given that these coefficients are in A.P.

$$\therefore 2^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$$

$$\Rightarrow 2 = \frac{{}^{2n}C_1}{{}^{2n}C_2} + \frac{{}^{2n}C_3}{{}^{2n}C_2}$$

$$\Rightarrow 2 = \frac{2}{2n-2+1} + \frac{2n-3+1}{3}$$

$$\Rightarrow 2 = \frac{2}{2n-1} + \frac{2n-2}{3}$$

$$\Rightarrow 2 = \frac{6 + (2n-1)(2n-2)}{3(2n-1)}$$

$$\Rightarrow 6(2n-1) = 6 + 4n^2 - 4n - 2n + 2$$

$$\Rightarrow 12n - 6 = 8 + 4n^2 - 6n$$

$$\Rightarrow 4n^2 - 6n - 12n + 8 + 6 = 0$$

$$\Rightarrow 4n^2 - 18n + 14 = 0$$

$$\Rightarrow 2(2n^2 - 9n + 7) = 0$$

$$\Rightarrow 2n^2 - 9n + 7 = 0 \quad \text{Hence proved.}$$

$$\left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r} \right]$$

Q23

We have,

$$(1+x)^n$$

Let the three consecutive terms are r th $(r+1)^{\text{th}}$ and $(r+2)^{\text{th}}$ i.e., T_r, T_{r+1} and T_{r+2}

$$\therefore \text{Coefficients of } r\text{th term} = {}^n C_{r-1} = 220$$

$$\text{Coefficients of } (r+1)^{\text{th}} \text{ term} = {}^n C_{r+1-1} = {}^n C_r = 495$$

$$\text{and, Coefficients of } (r+2)^{\text{th}} \text{ term} = {}^n C_{r+2-1} = {}^n C_{r+1} = 792$$

Now,

$$\frac{{}^n C_{r+1}}{{}^n C_r} = \frac{792}{495}$$

$$\Rightarrow \frac{n - (r+1) + 1}{r+1} = \frac{792}{495} \quad \left[\because \frac{{}^n C_r}{{}^n C_{r-1}} = \frac{n-r+1}{r} \right]$$

$$\begin{aligned} \Rightarrow \frac{n-r}{r+1} &= \frac{792}{495} \\ &= \frac{72}{45} \\ &= \frac{8}{5} \end{aligned}$$

$$\Rightarrow \frac{n-r}{r+1} = \frac{8}{5}$$

$$\Rightarrow 5n - 5r = 8r + 8$$

$$\Rightarrow 5n - 5r - 8r = 8$$

$$\Rightarrow 5n - 13r = 8$$

—(i)

$$\text{and, } \frac{{}^n C_r}{{}^n C_{r-1}} = \frac{495}{220}$$

$$\begin{aligned} \Rightarrow \frac{n-r+1}{r} &= \frac{495}{220} \\ &= \frac{45}{20} \\ &= \frac{9}{4} \end{aligned}$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{9}{4}$$

$$\Rightarrow 4n - 4r + 4 = 9r$$

$$\Rightarrow 4n - 4r - 9r = -4$$

$$\Rightarrow 4n - 13r = -4$$

—(ii)

Subtracting equation (ii) from equation (i),

$$n = 8 + 4$$

$$\Rightarrow n = 12$$

Q24

We have,

$$(1+x)^n$$

$$\therefore \text{Coefficients of 2nd term} = {}^n C_{2-1} = {}^n C_1$$

$$\text{Coefficients of 3rd term} = {}^n C_{3-1} = {}^n C_2$$

$$\text{and, Coefficients of 4th term} = {}^n C_{4-1} = {}^n C_3$$

It is given that these coefficients are in A.P.

$$\therefore 2{}^n C_2 = {}^n C_1 + {}^n C_3$$

$$\Rightarrow 2 = \frac{{}^n C_1}{{}^n C_2} + \frac{{}^n C_3}{{}^n C_2}$$

$$\Rightarrow 2 = \frac{2}{n-2+1} + \frac{n-3+1}{3}$$

$$\left[\because \frac{{}^n C_{r+1}}{{}^n C_r} = \frac{n-r}{r+1} \right]$$

$$\Rightarrow 2 = \frac{2}{n-1} + \frac{n-2}{3}$$

$$\Rightarrow 2 = \frac{6 + (n-1)(n-2)}{3(n-1)}$$

$$\Rightarrow 6(n-1) = 6 + n^2 - 2n - n + 2$$

$$\Rightarrow 6n - 6 = 8 + n^2 - 3n$$

$$\Rightarrow n^2 - 3n - 6n + 8 + 6 = 0$$

$$\Rightarrow n^2 - 9n + 14 = 0$$

$$\Rightarrow n^2 - 7n - 2n + 14 = 0$$

$$\Rightarrow n(n-7) - 2(n-7) = 0$$

$$\Rightarrow (n-2)(n-7) = 0$$

$$[\because n-2 \neq 0]$$

$$\Rightarrow n = 7$$

Q25

We have,

$$(1+x)^n$$

$$\text{Coefficients of } p\text{th term} = {}^n C_{p-1}$$

$$\text{and, Coefficients of } q\text{th term} = {}^n C_{q-1}$$

It is given that, these coefficients are equal.

$$\therefore {}^n C_{p-1} = {}^n C_{q-1}$$

$$\Rightarrow p-1 = q-1 \text{ or, } p-1+q-1 = n$$

$$\Rightarrow p-q = 0 \text{ or, } p+q = n+2$$

$$\therefore p+q = n+2 \text{ Hence proved.}$$

$$\left[\begin{array}{l} \because {}^n C_r = {}^n C_s \\ \Rightarrow r = s \text{ or, } r+s = n \end{array} \right]$$

Q26

We have,

$$(1+x)^n$$

Let the three consecutive terms are T_r, T_{r+1} and T_{r+2}

$$\therefore \text{Coefficients of } T_r = {}^n C_{r-1} = 56$$

$$\text{Coefficients of } T_{r+1} = {}^n C_{r+1-1} = {}^n C_r = 70$$

$$\text{and, Coefficients of } T_{r+2} = {}^n C_{r+2-1} = {}^n C_{r+1} = 56$$

Now,

$$\frac{{}^n C_{r+1}}{{}^n C_r} = \frac{56}{70}$$

$$\Rightarrow \frac{n - (r+1) + 1}{r+1} = \frac{4}{5} \quad \left[\because \frac{{}^n C_r}{{}^n C_{r-1}} = \frac{n-r+1}{r} \right]$$

$$\Rightarrow \frac{n-r}{r+1} = \frac{4}{5}$$

$$\Rightarrow 5n - 5r = 4r + 4$$

$$\Rightarrow 5n - 9r = 4 \quad \text{---(i)}$$

and,

$$\frac{{}^n C_r}{{}^n C_{r-1}} = \frac{70}{56}$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{5}{4}$$

$$\Rightarrow 4n - 4r + 4 = 5r$$

$$\Rightarrow 4n - r = -4 \quad \text{---(ii)}$$

Subtracting equation (ii) from (i), we get

$$n = 4 + 4 = 8$$

Put $n = 8$ in equation (i), we get

$$5 \times 8 - 9r = 4$$

$$\Rightarrow -9r = 4 - 40$$

$$\Rightarrow r = 4$$

\therefore Three consecutive terms are 4th, 5th and 6th.

Q27

We are given,

$$T_3 = a, T_4 = b, T_5 = c, T_6 = d$$

We have to prove that

$$\begin{aligned} & \frac{b^2 - ac}{c^2 - bd} = \frac{5a}{3c} \\ \Rightarrow & \frac{b^2 - ac}{a} = \frac{5}{3} \left[\frac{c^2 - bd}{c} \right] \\ \Rightarrow & \frac{1}{b} \left[\frac{b^2 - ac}{a} \right] = \frac{5}{3} \left[\frac{c^2 - bd}{bc} \right] \\ \Rightarrow & \frac{b}{a} - \frac{c}{b} = \frac{5}{3} \left[\frac{c}{b} - \frac{d}{c} \right] \quad \text{---(i)} \end{aligned}$$

Now we know,

$$a = {}^n C_2 x^{n-2} \alpha^2$$

$$b = {}^n C_3 x^{n-3} \alpha^3$$

$$c = {}^n C_4 x^{n-4} \alpha^4$$

$$d = {}^n C_5 x^{n-5} \alpha^5$$

Putting these values in equation (i), we get

$$\begin{aligned} & \frac{{}^n C_3 x^{n-3} \alpha^3 - {}^n C_4 x^{n-4} \alpha^4}{{}^n C_2 x^{n-2} \alpha^2 - {}^n C_3 x^{n-3} \alpha^3} = \frac{5}{3} \left[\frac{{}^n C_4 x^{n-4} \alpha^4 - {}^n C_5 x^{n-5} \alpha^5}{{}^n C_3 x^{n-3} \alpha^3 - {}^n C_4 x^{n-4} \alpha^4} \right] \\ \Rightarrow & \left[\frac{{}^n C_3 - {}^n C_4}{{}^n C_2 - {}^n C_3} \right] \frac{\alpha}{x} = \frac{5\alpha}{3x} \left[\frac{{}^n C_4 - {}^n C_5}{{}^n C_3 - {}^n C_4} \right] \end{aligned}$$

We know that,

$$\frac{{}^n C_r}{{}^n C_{r-1}} = \frac{n-r+1}{r}$$

\(\therefore\) The given equation above becomes,

$$\begin{aligned} & \left[\frac{n-2}{3} - \frac{n-3}{4} \right] = \frac{5}{3} \left[\frac{n-3}{4} - \frac{n-4}{5} \right] \\ \Rightarrow & \frac{4n-8-3n+9}{3 \times 4} = \frac{5n-15-4n+16}{3 \times 4} \\ \Rightarrow & \frac{n+1}{12} = \frac{n+1}{12} \end{aligned}$$

Which is true.

Hence proved.

Q28

Suppose the binomial is $(x+a)^n$

We are given,

$$T_6 = a, T_7 = b, T_8 = c, T_9 = d$$

We have to prove that

$$\begin{aligned} & \frac{b^2 - ac}{c^2 - bd} = \frac{4a}{3c} \\ \Rightarrow & \frac{b^2 - ac}{a} = \frac{4}{3} \left[\frac{c^2 - bd}{c} \right] \\ \Rightarrow & \frac{1}{b} \left[\frac{b^2 - ac}{a} \right] = \frac{4}{3} \left[\frac{c^2 - bd}{bc} \right] \\ \Rightarrow & \frac{b}{a} - \frac{c}{b} = \frac{4}{3} \left[\frac{c}{b} - \frac{d}{c} \right] \quad \text{---(i)} \end{aligned}$$

Now we know,

$$a = {}^n C_5 x^{n-5} a^5$$

$$b = {}^n C_6 x^{n-6} a^6$$

$$c = {}^n C_7 x^{n-7} a^7$$

$$d = {}^n C_8 x^{n-8} a^8$$

Putting these values in equation (i), we get

$$\begin{aligned} & \frac{{}^n C_6 x^{n-6} a^6}{{}^n C_5 x^{n-5} a^5} - \frac{{}^n C_7 x^{n-7} a^7}{{}^n C_6 x^{n-6} a^6} = \frac{4}{3} \left[\frac{{}^n C_7 x^{n-7} a^7}{{}^n C_6 x^{n-6} a^6} - \frac{{}^n C_8 x^{n-8} a^8}{{}^n C_7 x^{n-7} a^7} \right] \\ \Rightarrow & \left[\frac{{}^n C_6}{{}^n C_5} - \frac{{}^n C_7}{{}^n C_6} \right] \frac{a}{x} = \frac{4a}{3x} \left[\frac{{}^n C_7}{{}^n C_6} - \frac{{}^n C_8}{{}^n C_7} \right] \end{aligned}$$

We know that,

$$\frac{{}^n C_r}{{}^n C_{r-1}} = \frac{n-r+1}{r}$$

\therefore The given equation above becomes,

$$\begin{aligned} & \left[\frac{n-5}{6} - \frac{n-6}{7} \right] = \frac{4}{3} \left[\frac{n-6}{7} - \frac{n-7}{8} \right] \\ \Rightarrow & \frac{7n - 35 - 6n + 36}{6 \times 7} = \frac{8n - 48 - 7n + 49}{3 \times 7 \times 2} \\ \Rightarrow & \frac{n+1}{42} = \frac{n+1}{42} \end{aligned}$$

Which is true.

Hence proved.

Q29

We have,

$$(1+x)^n$$

Let the three consecutive terms are T_r, T_{r+1} and T_{r+2}

$$\therefore \text{Coefficients of } r\text{th term} = {}^n C_{r-1} = 76$$

$$\text{Coefficients of } (r+1)\text{th term} = {}^n C_{r+1-1} = {}^n C_r = 95$$

$$\text{and, Coefficients of } (r+2)\text{th term} = {}^n C_{r+2-1} = {}^n C_{r+1} = 76$$

Now,

$$\frac{{}^n C_{r+1}}{{}^n C_r} = \frac{76}{95}$$

$$\Rightarrow \frac{n - (r+1) + 1}{r+1} = \frac{76}{95} \quad \left[\because \frac{{}^n C_r}{{}^n C_{r-1}} = \frac{n-r+1}{r} \right]$$

$$\Rightarrow \frac{n-r-1}{r+1} = \frac{4}{5}$$

$$\Rightarrow \frac{n-r}{r+1} = \frac{4}{5}$$

$$\Rightarrow 5n - 5r = 4r + 4$$

$$\Rightarrow 5n - 5r - 4r = 4$$

$$\Rightarrow 5n - 9r = 4 \quad \text{---(i)}$$

and,

$$\frac{{}^n C_r}{{}^n C_{r-1}} = \frac{95}{76}$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{5}{4}$$

$$\Rightarrow 4n - 4r + 4 = 5r$$

$$\Rightarrow 4n - 9r = -4 \quad \text{---(ii)}$$

Subtracting equation (ii) from (i), we get

$$n = 4 + 4$$

$$\Rightarrow n = 8$$

Q30

It is given that,

$$T_6 = 112, T_7 = 7, T_8 = \frac{1}{4}$$

$$\therefore T_6 = {}^n C_{n-5} X^{n-5} \times a^5 = 112$$

$$T_7 = {}^n C_{n-6} X^{n-6} \times a^6 = 7$$

$$\text{and, } T_8 = {}^n C_{n-7} X^{n-7} \times a^7 = \frac{1}{4}$$

Now,

$$\frac{T_7}{T_6} = \frac{{}^n C_{n-6} X^{n-6} \times a^6}{{}^n C_{n-5} X^{n-5} \times a^5} = \frac{7}{112}$$

$$\Rightarrow \frac{{}^n C_{n-6}}{{}^n C_{n-5}} \times \frac{a}{X} = \frac{1}{16}$$

$$\Rightarrow \frac{n-6+1}{n-(n-5)+1} \times \frac{a}{X} = \frac{1}{16}$$

$$\left[\because \frac{{}^n C_r}{{}^n C_{r-1}} = \frac{n-r+1}{r} \right]$$

$$\Rightarrow \frac{n-5}{6} \times \frac{a}{X} = \frac{1}{16}$$

$$\Rightarrow \frac{a}{X} = \frac{6}{16} \times \frac{1}{n-5}$$

$$\Rightarrow \frac{a}{X} = \frac{3}{8} \times \frac{1}{(n-5)}$$

---(i)

and,

$$\frac{T_8}{T_7} = \frac{{}^n C_{n-7} X^{n-7} \times a^7}{{}^n C_{n-6} X^{n-6} \times a^6} = \frac{1}{4 \times 7}$$

$$\Rightarrow \frac{T_8}{T_7} = \frac{{}^n C_{n-7}}{{}^n C_{n-6}} \times \frac{a}{X} = \frac{1}{28}$$

$$\Rightarrow \frac{{}^n C_{n-7}}{{}^n C_{n-6}} \times \frac{a}{X} = \frac{1}{28}$$

$$\Rightarrow \frac{n-7+1}{n-(n-6)+1} \times \frac{a}{X} = \frac{1}{28}$$

$$\Rightarrow \frac{n-6}{7} \times \frac{a}{X} = \frac{1}{28}$$

$$\Rightarrow \frac{a}{X} = \frac{1}{4(n-6)}$$

---(ii)

Comparing equation (i) and (ii), we get

$$\frac{3}{8} \times \frac{1}{(n-5)} = \frac{1}{4(n-6)}$$

$$\Rightarrow \frac{3}{2} \times \frac{1}{(n-5)} = \frac{1}{(n-6)}$$

$$\Rightarrow 3(n-6) = 2(n-5)$$

$$\Rightarrow 3n - 18 = 2n - 10$$

$$\Rightarrow 3n - 2n = 18 - 10$$

$$\Rightarrow n = 8$$

Putting $n = 8$ in equation (ii), we get

$$\frac{a}{x} = \frac{1}{4(8-6)}$$

$$\Rightarrow \frac{a}{x} = \frac{1}{8}$$

$$\Rightarrow x = 8a$$

Now,

$$76 = 112$$

$$\Rightarrow {}^n C_{n-5} \times x^{n-5} \times a^5 = 112$$

$$\Rightarrow {}^8 C_3 \times x^3 \times a^5 = 112$$

$$[\because n = 8]$$

$$\Rightarrow {}^8 C_3 \times (8a)^3 \times a^5 = 112$$

$$[\because x = 8a]$$

$$\Rightarrow \frac{8!}{(8-3)!3!} \times 8^3 \times a^8 = 112$$

$$\Rightarrow \frac{8!}{5!3!} \times 512 \times a^8 = 112$$

$$\Rightarrow \frac{8 \times 7 \times 6 \times 5!}{5!3!} \times 512 \times a^8 = 112$$

$$\Rightarrow a^8 = \frac{112}{56 \times 512}$$

$$\Rightarrow a^8 = \frac{2}{512}$$

$$\Rightarrow a^8 = \frac{1}{256}$$

$$\Rightarrow a^8 = \left(\frac{1}{2}\right)^8$$

$$\Rightarrow a = \frac{1}{2}$$

Putting $a = \frac{1}{2}$ in $x = 8a$, we get

$$x = 8 \times \frac{1}{2} = 4$$

Hence, $x = 4$, $a = \frac{1}{2}$ and $n = 8$.

Q31

It is given that

$$T_2 = 240$$

$$T_3 = 720$$

$$T_4 = 1080$$

$$\therefore T_2 = {}^n C_1 \times X^{n-1} \times a = 240$$

$$T_3 = {}^n C_2 \times X^{n-2} \times a^2 = 720$$

and, $T_4 = {}^n C_3 \times X^{n-3} \times a^3 = 1080$

Now,

$$\frac{T_4}{T_3} = \frac{{}^n C_3 \times X^{n-3} \times a^3}{{}^n C_2 \times X^{n-2} \times a^2} = \frac{1080}{720}$$

$$\Rightarrow \frac{{}^n C_3 a}{{}^n C_2 X} = \frac{3}{2}$$

$$\Rightarrow \frac{n-3+1}{2+1} \times \frac{a}{X} = \frac{3}{2}$$

$$\Rightarrow \frac{n-2}{3} \times \frac{a}{X} = \frac{3}{2}$$

$$\Rightarrow \frac{a}{X} = \frac{9}{2(n-2)} \quad \text{---(i)}$$

and,

$$\frac{T_3}{T_2} = \frac{{}^n C_2 \times X^{n-2} \times a^2}{{}^n C_1 \times X^{n-1} \times a} = \frac{720}{240}$$

$$\Rightarrow \frac{{}^n C_2 \times a}{{}^n C_1 \times X} = 3$$

$$\Rightarrow \frac{n-2+1}{2} \times \frac{a}{X} = 3$$

$$\Rightarrow \frac{n-1}{2} \times \frac{a}{X} = 3$$

$$\Rightarrow \frac{a}{X} = \frac{6}{n-1} \quad \text{---(ii)}$$

Comparing equation (i) and equation (ii), we get

$$\frac{6}{n-1} = \frac{9}{2(n-2)}$$

$$\Rightarrow 12(n-2) = 9(n-1)$$

$$\Rightarrow 12n - 24 = 9n - 9$$

$$\Rightarrow 3n = 24 - 9$$

$$\Rightarrow 3n = 15$$

$$\Rightarrow n = 5$$

Putting $n = 5$ in equation (ii), we get

$$\frac{a}{x} = \frac{6}{5-1}$$

$$\Rightarrow \frac{a}{x} = \frac{6}{4}$$

$$\Rightarrow \frac{a}{x} = \frac{3}{2}$$

$$\Rightarrow a = \frac{3}{2}x$$

Now,

$$T_2 = {}^n C_1 \times x^{n-1} \times a = 240$$

$$\Rightarrow {}^5 C_1 \times x^4 \times \left(\frac{3}{2}x\right) = 240$$

$$\left[\because n = 5 \text{ and } a = \frac{3}{2}x \right]$$

$$\Rightarrow x^5 = \frac{240 \times 2}{5 \times 3}$$

$$\Rightarrow x^5 = 32$$

$$\Rightarrow x^5 = 2^5$$

$$\Rightarrow x = 2$$

Putting $x = 2$ in $a = \frac{3}{2}x$, we get

$$a = \frac{3}{2} \times 2 = 3$$

Hence, $x = 2$, $a = 3$ and $n = 5$.

Q32

It is given that

$$T_1 = 729$$

$$T_2 = 7290$$

and, $T_3 = 30375$

$$\therefore T_1 = {}^n C_0 \times a^n = 729$$

$$T_2 = {}^n C_{n-1} \times a^{n-1} \times b = 7290$$

and, $T_3 = {}^n C_{n-2} \times a^{n-2} \times b^2 = 30375$

Now,

$$\frac{T_2}{T_1} = \frac{{}^n C_{n-1} \times a^{n-1} \times b}{{}^n C_0 \times a^n} = \frac{7290}{729}$$

$$\Rightarrow \frac{{}^n C_{n-1} \times a^{n-1} \times b}{{}^n C_0 \times a^n} = 10$$

$$\Rightarrow \frac{{}^n C_{n-1} \times b}{1 \times a} = 10$$

$$\Rightarrow \frac{n!}{(n-n+1)!(n-1)!} \times \frac{b}{a} = 10$$

$$\Rightarrow \frac{n!}{(n-1)!} \times \frac{b}{a} = 10$$

$$\Rightarrow \frac{n(n-1)!}{(n-1)!} \times \frac{b}{a} = 10$$

$$\Rightarrow \frac{b}{a} = \frac{10}{n} \quad \text{---(i)}$$

and,

$$\frac{T_3}{T_2} = \frac{{}^n C_{n-2} \times a^{n-2} \times b^2}{{}^n C_{n-1} \times a^{n-1} \times b} = \frac{30375}{7290}$$

$$\Rightarrow \frac{{}^n C_{n-2} \times b}{{}^n C_{n-1} \times a} = \frac{25}{6}$$

$$\Rightarrow \frac{n-2+1}{n-(n-1)+1} \times \frac{b}{a} = \frac{25}{6}$$

$$\Rightarrow \frac{n-1}{2} \times \frac{b}{a} = \frac{25}{6}$$

$$\Rightarrow \frac{b}{a} = \frac{25}{6} \times \frac{2}{(n-1)}$$

$$\left[\because \frac{{}^n C_r}{{}^n C_{r-1}} = \frac{n-r+1}{r} \right]$$

Q33

We have,

$$(3+ax)^9 = {}^9C_0 \times 3^9 + {}^9C_1 \times 3^8 \times (ax)^1 + {}^9C_2 \times 3^7 \times (ax)^2 + {}^9C_3 \times 3^6 \times (ax)^3 + \dots$$

$$\therefore \text{Coefficient of } x^2 = {}^9C_2 \times 3^7 \times a^2$$

$$\text{and, Coefficient of } x^3 = {}^9C_3 \times 3^6 \times a^3$$

Now, Coefficient of $x^2 =$ Coefficient of x^3

$$\Rightarrow {}^9C_2 \times 3^7 \times a^2 = {}^9C_3 \times 3^6 \times a^3$$

$$\Rightarrow 36 \times 3^7 \times a^2 = 84 \times 3^6 \times a^3$$

$$\Rightarrow a = \frac{36 \times 3^7}{84 \times 3^6} = \frac{9}{7}$$

Q34

We have,

$$(1+2a)^4 (z-a)^5$$

Now,

$$(1+2a)^4 = {}^4C_0 + {}^4C_1(2a) + {}^4C_2(2a)^2 + {}^4C_3(2a)^3 + {}^4C_4(2a)^4$$

$$\text{and, } (z-a)^5 = {}^5C_0 \times z^5 + {}^5C_1 \times z^4 \times (-a) + {}^5C_2 \times z^3 \times (-a)^2 + {}^5C_3 \times z^2 \times (-a)^3 + {}^5C_4 \times z \times (-a)^4 + {}^5C_5 \times (-a)^5$$

$$= {}^5C_0 \times z^5 - {}^5C_1 \times z^4 \times a + {}^5C_2 \times z^3 \times a^2 - {}^5C_3 \times z^2 \times a^3 + {}^5C_4 \times z \times a^4 - {}^5C_5 \times a^5$$

$$\therefore (1+2a)^4 (z-a)^5 = \left[{}^4C_0 + {}^4C_1(2a) + {}^4C_2(2a)^2 + {}^4C_3(2a)^3 + {}^4C_4(2a)^4 \right] \left[{}^5C_0 \times z^5 - {}^5C_1 \times z^4 \times a + {}^5C_2 \times z^3 \times a^2 - {}^5C_3 \times z^2 \times a^3 + {}^5C_4 \times z \times a^4 - {}^5C_5 \times a^5 \right]$$

$$\therefore \text{Coefficients of } a^4 = {}^2^5C_4 - {}^4C_1 \times 2 \times {}^5C_3 \times 2^2 + {}^4C_2 (2)^2 \times {}^5C_2 \times 2^2 - {}^4C_3 (2)^3 \times {}^5C_1 \times 2^4 + {}^4C_4 (2)^4 \times {}^5C_0 \times 2^5$$

$$= 2^5 \times 5 - 8 \times 4 \times 10 + 32 \times 6 \times 15 - 128 \times 4 \times 5 + 512 \times 1 \times 1$$

$$= 10 - 320 + 1920 - 2560 + 512$$

$$= 742 - 2880$$

$$= -1138$$

$$\therefore \text{Coefficients of } a^4 = -1138.$$

Q35

$$\left(\sqrt{x} - \frac{k}{x^2} \right)^{10}$$

$$\binom{10}{2} \left(\sqrt{x} \right)^8 \left(-\frac{k}{x^2} \right)^2 = 405$$

$$45k^2 = 405$$

$$k^2 = 9$$

$$k = 3$$

Q36

$$(y^{1/2} + x^{1/3})^n$$

$$\binom{n}{n-2} (y^{1/2})^2 (x^{1/3})^{n-2}$$

$$\binom{n}{n-2} = 45$$

$$n(n-1) = 90$$

$$n^2 - 10n + 9n - 90$$

$$n(n-10) + 9(n-10) = 0$$

$$n = -9 \text{ or } 10$$

n cannot be negative. So, n = 10

$$6\text{th term} \binom{10}{5} (y^{1/2})^5 (x^{1/3})^5 = 252y^{\frac{5}{2}}x^{\frac{5}{3}}$$

Q37

$$\left(\frac{p}{2} + 2\right)^8$$

$$\binom{8}{4} \left(\frac{p}{2}\right)^4 2^4 = 1120$$

$$70p^4 = 1120$$

$$p^4 = 16$$

$$p = 2$$

Q38

$$\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$$

7th term from beginning is

$$\binom{n}{6} (\sqrt[3]{2})^{n-6} \left(\frac{1}{\sqrt[3]{3}}\right)^6$$

7th term from end is

$$\binom{n}{n-6} (\sqrt[3]{2})^6 \left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}$$

$$\text{Given } \frac{\text{7th term from beginning}}{\text{7th term from end}} = \frac{\binom{n}{6} (\sqrt[3]{2})^{n-6} \left(\frac{1}{\sqrt[3]{3}}\right)^6}{\binom{n}{n-6} (\sqrt[3]{2})^6 \left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}}$$

$$= \frac{\binom{n}{6} (\sqrt[3]{2})^{n-6} (\sqrt[3]{3})^{n-6}}{\binom{n}{n-6}}$$

$$= \frac{\binom{n}{6} (6)^{\frac{n-6}{3}}}{\binom{n}{n-6}} = \frac{1}{6}$$

$$\frac{n-6}{3} = -1$$

$$n = 6 - 3 = 3$$

Q39

Seventh term from the beginning and one in the binomial expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{2}}\right)^n$ are equal.

$$\Rightarrow T_7 = T_{n-6}$$

$$\Rightarrow {}^n C_6 (\sqrt[3]{2})^6 \left(\frac{1}{\sqrt[3]{2}}\right)^{n-6} = {}^n C_{n-6} (\sqrt[3]{2})^{n-6} \left(\frac{1}{\sqrt[3]{2}}\right)^6$$

$$\Rightarrow (\sqrt[3]{2})^6 \left(\frac{1}{\sqrt[3]{2}}\right)^{n-6} = (\sqrt[3]{2})^{n-6} \left(\frac{1}{\sqrt[3]{2}}\right)^6$$

$$\rightarrow \left(\frac{1}{\sqrt[3]{2}}\right)^{2n-12} = \left(\frac{1}{\sqrt[3]{2}}\right)^{12}$$

$$\Rightarrow 2n - 12 = 12$$

$$\Rightarrow n = 12$$