Chapter 6

List of Formulae

CHAPTER HIGHLIGHTS

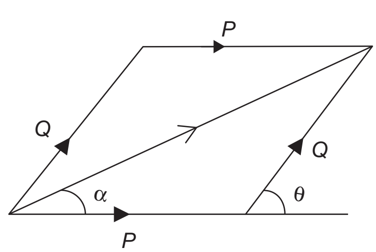
☞ *List of formulae*

List of Formulae

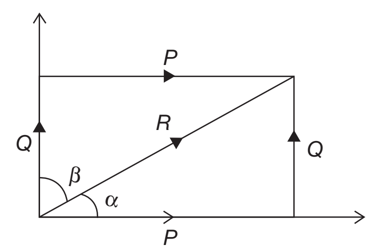
- **Resultant of forces acting in a straight line:**
  (a) Same direction: $R = P + Q$
  (b) Opposite direction: $R = P - Q$
- **Resultant of two concurrent forces:**

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\alpha = \tan^{-1}\frac{Q\sin\theta}{P + Q\cos\theta}$$



- **Components of a given force in two given directions:**



- **Resultant of number of co-planar concurrent forces:**

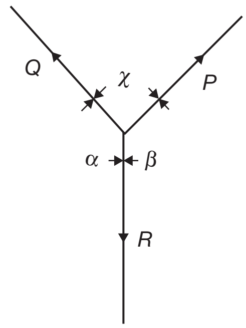
$$R\cos\theta = \Sigma\, pi\, \cos\theta_I = \Sigma H = X$$
$$R\sin\theta = \Sigma\, pi\, \sin\theta_I = \Sigma V = Y$$
$$R\sqrt{X^2 + Y^2}$$
$$\theta = \tan^{-1}\frac{y}{x}$$

- **Resultant of co-planar parallel forces:**

$$R = \Sigma P_1$$
$$\bar{x} = \frac{\Sigma P_1 X_1}{\Sigma P_1}$$

where, x_1 are distances of various forces from a reference axis, $\bar{x}$ is the distance (perpendicular) of R from reference axis.

- Mechanical advantage of a lever $= \dfrac{\text{Power arm}}{\text{Load arm}}$
- **Lami's theorem:**



$$\frac{P}{\sin\alpha}\frac{Q}{\sin\beta}\frac{R}{\sin\chi}$$

- **Friction force:** $F = \mu N$; N = normal reaction

Total reaction; $R = \sqrt{F^2 + N^2}$

$(F' = F_{\max})$

Angle of friction; $\theta = \tan^{-1}\dfrac{F}{N}$

Also, $\mu = \tan\theta$

- **Angle of repose:** $\alpha = \phi$
- **Work done by a varying force:**

$$w = \int_o^s F \cdot \delta S$$

S = Total distance covered

- If a body freely falls from a height H, then velocity on reaching the ground;

$$v = \sqrt{2gH}$$

- If a body is projected vertically with initial velocity u, then the maximum height attained by it is: $H = \dfrac{U^2}{2g}$
- **Distance covered in the nth second**

$$D_n = u + \frac{a}{2}(2n-1)$$

- **Motion of a particle in a plane**

$$u = \frac{dx}{dt}; \quad v = \frac{dy}{dt}$$

$$v_R = \sqrt{u^2 + v^2}$$

$$a_x = \frac{du}{dt} = \frac{d^2x}{dt^2}; \qquad a_y = \frac{dv}{dt} = \frac{d^2y}{dt^2}$$

- **Work done by a force field $F(x, y, z)$ along path 1–2 is given by:**

$$W_{1-2} = \int_1^2 F\,dr = V_1(x, y, z) - V_2(x, y, z)$$

where, $V(x, y, z)$ is the potential energy function (scalar function).

- **For a conservation force field:**

$$\int F \cdot dr = 0, \text{ and}$$

$$F = -\nabla V$$

- **Equation of trajectory:**

$$Y = x\tan\alpha - \frac{1}{2g} \cdot \frac{x^2}{u^2\cos^2\alpha}$$

(a) Motion of projectile up an inclined plane:

Time of flight, $T = \dfrac{2u\sin(\alpha-\beta)}{g\cos\beta}$

[β: Inclination of plane]
Range up of the place:

$$R = \frac{2u^2\cos\alpha\sin(\alpha-\beta)}{g\cos^2\beta}$$

$$R_{\max} = \frac{u2}{g(1+\sin\beta)} \text{ at } \alpha = \frac{\pi}{4} + \frac{\beta}{2}$$

(c) Motion of a projectile down a plane:

$$= \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

$$R = \frac{2u^2\cos\alpha\sin(\alpha-\beta)}{g\cos^2\beta}$$

$$R_{\max} = \frac{u^2}{g(1+\sin\beta)}$$

(d) Motion of a projectile projected horizontally at a height above the ground:

$$x = ut$$

$$y = \frac{1}{2}gt^2$$

$$\frac{x^2}{y} = \frac{2u^2}{g} = \text{constant}$$

- **Elastic collision:** Both momentum and (Kinetic energy) are conserved

$$m_1u_1 + m_2u_2 = \frac{1}{2}m_1v_1 + m_2v^2{}_2$$

$$\frac{1}{2}m_1u_1{}^2 + \frac{1}{2}m^2u^2{}_2 = \frac{1}{2}m_1v_1{}^2 + \frac{1}{2}m_2v^2{}_2$$

- **Inelastic collision:** Only momentum is conserved

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

- **Coefficient of elasticity or restitution:**

$$e = \frac{v_2 - v_1}{u_2 - u_1}$$

$e = 1$ for perfectly elastic bodies, $e = 0$ for plastic impact

- **Apparent weight in a lift:**

Upward moving lift:

$$w_{eq} = mg\left(1 + \frac{a}{g}\right)n$$

Downward moving Lift:

$$w_{eq} = mg\left(1 - \frac{a}{g}\right)n$$

- **Total kinetic energy of a body:**

$$\text{TE} = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2$$

- **Momentum of inertia of a thin circular ring:**
  (a) About an axis perpendicular to plane of ring:
  $$I = MR^2$$
  (b) About any diameter:
  $$I = \frac{1}{2}MR^2$$
  (c) About a tangent in the plane of ring:
  $$I = \frac{3}{2}MR^2$$
  (d) About a tangent perpendicular to the plane of ring:
  $$I = \frac{1}{2}MR^2$$

- **Momentum of inertia of a uniform circular disc:**

$$I = \frac{1}{2}MR^2$$

(About an axis perpendicular to plane of disc.)

- **Moment of inertia of a thin uniform rod:**
  (a) About an axis passing through the centre of length and perpendicular to the length:
  $$I = \frac{1}{2}Ml^2$$
  (b) About its axis:
  $$I = \frac{1}{2}MR^2$$
  (c) Hollow rod:
  $$I = MR^2$$

- **Moment of inertia of hollow sphere:**

$$I = \frac{2}{3}MR^2$$

- **Moment of inertia of solid sphere**

$$I = \frac{2}{5}MR^2$$

- **Motion of a cylinder rolling without slipping on an inclined plane:**

$$a = \frac{mg\sin\theta}{m + \frac{1}{r^2}} = \frac{2}{3}g\sin\theta[a < g]$$

$$F = \frac{1}{3}mg\sin\theta; \quad F < mg$$

$$\mu = \frac{1}{3}\tan\theta$$