10. FIELD ASTRONOMY

RELATIONSHIP BETWEEN COORDINATES

If Z = Zenith

δ = Declination (-ve-south + ve North equation)

θ = Latitude of the observer.

Then $\theta = \delta + Z$

$\theta = \alpha - p$.....(If the star is north of zenith but above the pole)

$\theta = \alpha + p$......(If the star is north of zenith but below the pole)

α = meridian altitude of star

P = Polar distance

- Latitude of Pole = Latitude of observer (always)
- Hour Angle of Equinox = Hour Angle of star + R.A. of star where, R.A.= Right Ascension
- 1 Tropical year = 365.24422 mean solar days
- 1 SIDEREAL days = 366.2422 sidereal days.

Remember

- 1 Solar day = $1 + \frac{1}{365.2422}$ Sidereal days

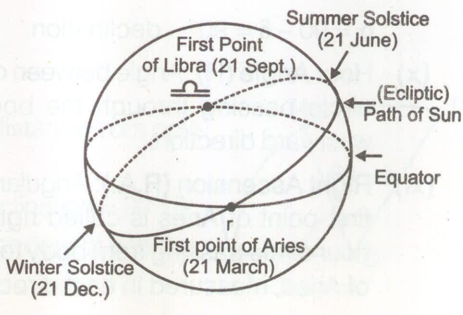
1 Solar days = 24h 3m 56.56s Sidereal time.

To convert the mean solar time to the sidereal time, we will have to add a correction of 9.8565 second per hour of mean time this correction is called the acceleration.

(i) **First point of Aries and Libra:** First point of Aries is the point where sun crosses the equator from south to north on 21st March. On this day and nights are of equal direction.

First point of Libra is point where sun crosses the equator from north to south on 21/22 Sept.

- 1st point of Aries – also known as – vernal equinox.
- 1st point of Libra - also Autumnal equinox.



- Points on the ecliptic at which north and south declination is max. are known as - Solstices.

  21st June → Summer solstices

  21/22 December → Winter solstice

  r → first point of Aries (Vernal Equinox)

  Ω → first point of Libra (Autumnal Equinox)

(ii) **Latitude (θ) :** Angular distance of any plane north or south of the Equator. Angle between zenith and celestial equator is called latitude.

(iii) **Co-Latitude :** Measured angle between pole to zenith point for any place is called colatitude.

colatitude (c) = 90 – latitude = 90 – θ.

(iv) **Longitude:** Angle between meridian of a place from a fixed (prime) meridian is called longitude universally adopted prime meridian is Greenwich angle is measured 0° to 180° East or west of prime meridian.

(v) **Altitude (α):** Altitude of any celestial body or star is angular distance from horizon, measures on the vertical circle passing through the body.

(vi) **Co-Altitude (z):** Angular distance of body from zenith also called zenith distance.

$z = 90 - \alpha = 90$ – Altitude.

(vii) **Azimuth (A):** Angle between vertical circle passing through the body from observer's meridian (z-p line).

(viii) **Declination (δ):** Angular distance of a body from the plane of celestial equator, measured along declination circle, declination circle is great circle passing through body and celestial pole. Varies from 0° to 90° (Nors)

(ix) **Co-declination (ρ) or Polar distance:** Angular distance of heavenly body from Pole.

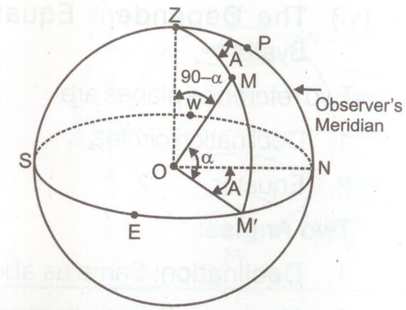
$\rho = 90 - \delta = 90°$ – declination.

(x) **Hour Angle (H):** Angle between observer's meridian and declination circle passing through the body. But measured from south in westward direction.

(xi) **Right Ascension (R.A.):** Angular distance measured eastward from first point of Aries is called right ascension. It is angle between hour circle passing from body to hour circle passing from first point of Aries, measured in east direction.

- **Coordinate System**

(i) **Horizon System (Altitude and Azimuth system):** In this system, zenith is the reference point, and plane of reference is horizon.



- **Angle**

(i) **Azimuth:** Horizontal angle between two great circle (1) observer's meridian and great circle passing from the point M. Angle NOM′ = A is called Azimuth.

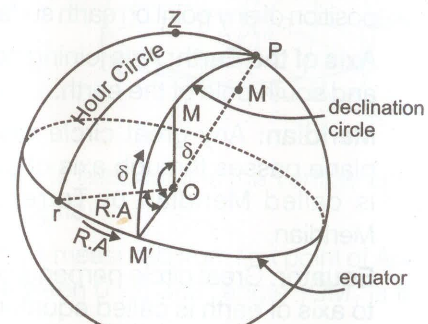
(ii) **Altitude:** Angle above or below the horizon (LM′ OM = α) is called altitude.

- Azimuth is the horizontal angle, whereas altitude is the vertical angle.
- This system of measurement is dependent on the position of the observer.
- Zenith distance (ZM) or ZOM, is the angular distance of the object form zenith.

  Zenith distance = ZM = 90 – α

(iii) **Declination:** Angle above or below the equator wrt to pole is called declination.

(δ) = ∠ MOM.

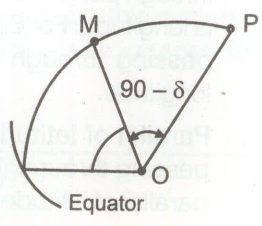


(iv) **Right Ascension:** Right ascension is the angle measured along the equator w.r.t. first point of Aries (r) going towards east

R. A. = Angle γOM′.

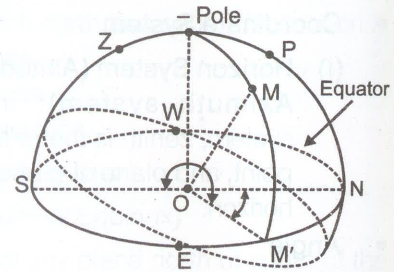
(v) **Co-declination:** Angular distance from pole is called co-declination.

angle POM is called co-declination = 90 – δ.



- This system is independent of position of observer.

(vi) The Dependent Equatorial System

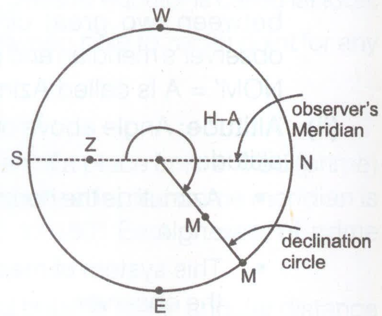


Two reference planes are

1. Declination circle
2. Equator.

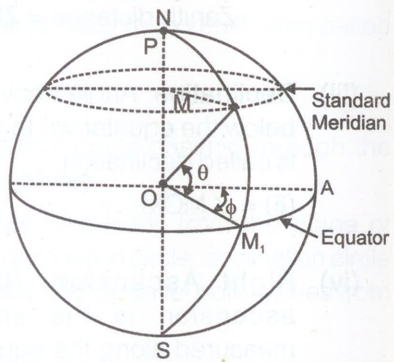
Two Angles:

1. **Declination:** Same as above
2. **Hour angle:** Angle measured between two great circles, 1. observer's meridian and 2. great circle passing from the point and pole is called hour angle. Hour angle is measured from south going towards west upto the declination circle angle SOM is Hour angle.



TERRESTRIAL LATITUDE AND LONGITUDE

This system is used for locating position of any point on earth surface.



Axis of the Earth: Axis joining north and south pole of the earth.

Meridian: Any great circle whose plane passes through axis of earth is called Meridian or Terrestrial Meridian.

Equator: Great circle perpendicular to axis of earth is called equator.

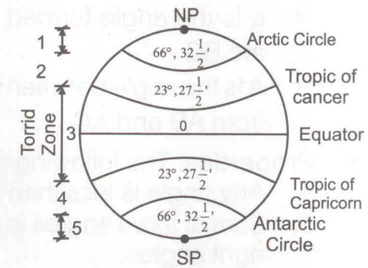
Latitude(θ): Vertical angle above or below equator is called latitude (angle MOM_1).

Longitude(ϕ): Horizontal angle between great circle (Meridian) passing through place and standard meridian is called longitude (ϕ). Angle AOM_1 is longitude. For Earth, prime meridian or standard meridian is meridian passing through Greenwich. All points on a meridian have same longitude.

Parallel of latitude: Parallel of latitude through a point is small circle passing through that point and parallel to equator. All the points on a parallel of latitude hour have same latitude.

Zones of the Earth:

1. North frigid zone
2. North Temperate zone
3. Torrid zone
4. South temperate zone
5. South frigid zone



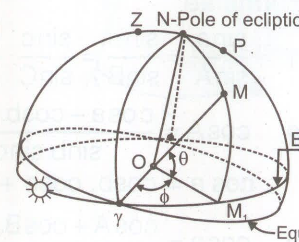
NAUTICAL MILE

. Nautical Mile = distance on arc of great circle by 1 minute angle at centre of earth.

$$= \frac{2\pi R}{360} \times \frac{1}{60} = \frac{2\pi \times 6370}{360 \times 60} = 1.852\text{km}$$

Celestial Latitude and Longitude System: Primary plane of reference are:

1. Plane of ecliptic - Horizontal plane.
2. Great circle passing through first point of Aries and perpendicular to plane of ecliptic.
   - Vertical plane



Coordinates are :

(i) Celestial latitude

(ii) Celestial longitude

Celestial latitude: Angle $MOM_1 = \theta$ is latitude. It is the vertical angle measured above or below arc of ecliptic.

Celestial longitude: Horizontal angle measured from first point of Aries to the east. This angle may be between 0° to 360° angle γOM_1 is the longitude.

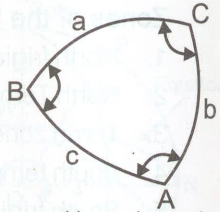
North Pole of Ecliptic: Point on great circle that is perpendicular to ecliptic and passes through first point of Aries. It is the point where all great circles perpendicular to ecliptic meets in north of ecliptic.

As per this system latitude of sun is always zero. So this system is very useful to fix the position of sun. This system is also independent of place of observation.

SPHERICAL TRIANGLE

- a, b, and c are the sides of spherical triangle.

- a is the angle formed at centre of sphere by arc BC.
- A is the angle between two great circle passing from AB and AC.



- **Properties:** The following are the properties:
  1. Any angle is less than $2 \times 90°(\pi)$
  2. Sum of three angles is more than two right angles and less than six right angles.

$$2\times\frac{\pi}{2}<(A+B+C)\leq 6\times\frac{\pi}{2}$$

$$\pi<(A+B+C)\leq 3\pi$$

  3. Sum of any two sides > third side.
  4. If some of any two sides is equal to two right angle (π), Sum of angles opposite them is also equal to $2 \times 90°$ or π

If $\quad a + b = 2 \times 90° = \pi$

then $\quad A + B = 2 \times 90° = \pi$

  5. The smaller angle is opposite the smaller side.

- **Formulae**

1. $$\frac{\sin a}{\sin A}=\frac{\sin b}{\sin B}=\frac{\sin c}{\sin C}$$

2. $$\cos A=\frac{\cos a-\cos b.\cos c}{\sin b.\sin c}$$

$\cos a = \cos b.\ \cos c + \sin b.\ \sin c.\cos A$

3. $$\cos a=\frac{\cos A+\cos B.\cos C}{\sin B.\sin C}$$

$\cos A = -\cos B.\ \cos C + \sin B.\ \sin C.\cos a$

4. $$\sin\frac{A}{2}=\sqrt{\frac{\sin(S-b)\sin(S-c)}{\sin b.\ \sin c}}$$

$$\cos\frac{A}{2}=\sqrt{\frac{\sin S.\sin(S-a)}{\sin b.\sin C}}\qquad S=\frac{a+b+c}{2}$$

$$\tan\frac{A}{2}=\sqrt{\frac{\sin(S-b)\sin(S-c)}{\sin S.\sin(S-a)}}$$

5. $$\sin\frac{a}{2}=\sqrt{\frac{-\cos S\cos(S-A)}{\sin B.\sin C}}$$

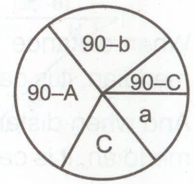
$$\cos\frac{a}{2}=\sqrt{\frac{\cos(S-B)(S-C)}{\sin B.\sin C}}\qquad S=\frac{A+B+C}{2}$$

$$\tan\frac{a}{2}=\sqrt{\frac{-\cos S.\cos(S-A)}{\cos(S-B)\cos(S-C)}}$$

6. $$\tan\frac{1}{2}(a+b)=\frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)}\tan\frac{1}{2}C$$

$$\tan\frac{1}{2}(a-b)=\frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)}\cot\frac{1}{2}C$$

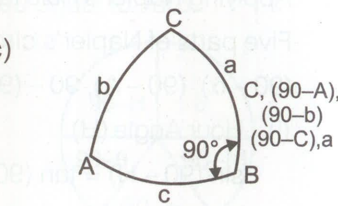
Napier's Rule (Applicable for right angled spherical triangle):



sin (middle) = tan (adjacent$_1$) × tan (adjacent$_2$)

sin (middle) = cos (app.$_1$) × cos (app.$_2$)

1. sin c = tan (90 – A). tan a
   = cot A. tan a
2. sin c = cos (90 – b). cos (90 – c)
   = sec. b sec.C



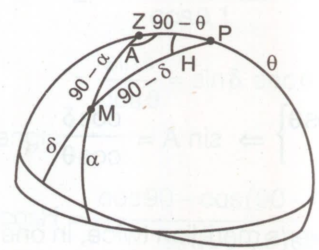
- **Spherical excesses**

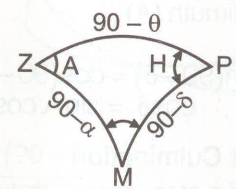
$E = (A + B + C) - 180$

area of spherical triangle

$$\Delta\text{area} = \frac{\pi R^2\times E}{180}$$

- **Astronomical Triangle**

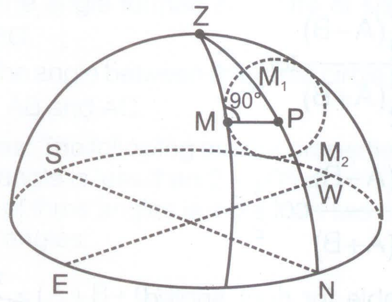




DIFFERENT POSITION OF STAR W.R.T TO OBSERVER'S MERIDIAN

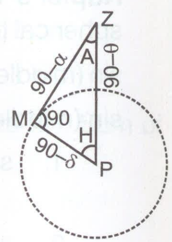
- **Star at Elongation**

A star is said to be at Elongation, where it is at greatest distance from standard meridian (Z–P line). Azimuth of the star is maximum in this position.



When distance is maximum to the east of observer's meridian, it is called star at eastern elongation.

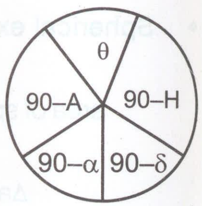
And when distance is maximum to west of observer's meridian, it is called star at western elongation.



Applying Napier's Rule for Astrotriangle ZMP.

Five parts of Napier's circle will

(90 – α), (90 – A), 90 – (90 – θ) = θ (90 – H), (90 – δ)



(i) Hour Angle (H)

$$\sin(90-H) = \tan(90-\delta).\tan\theta$$

$$\cos H = \cot\delta.\tan\theta$$

(ii) Altitude (α)

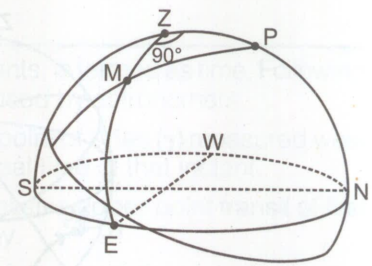
$$\sin\theta = \cos(90-\alpha).\cos(90-\delta) = \sin\alpha.\sin\delta$$

$$\text{so } \sin\alpha = \frac{\sin\theta}{\sin\delta} = \sin\theta\times\operatorname{cosec}\delta$$

(iii) Azimuth (A)

$$\left.\begin{aligned}\sin(90-\delta) &= \cos(90-A)\cos\theta\\ \cos\delta &= \sin A.\cos\theta\end{aligned}\right\} \Rightarrow \sin A = \frac{\cos\delta}{\cos\theta}$$

- **Star at Culmination**
  Path of a star crosses the observer's meridian twice, in one revolution around the pole. A star is said to be at culmination, where it crosses the observer's meridian in above figure. M_1 is position of upper culmination and M_2 is position of lower culmination.
- **Star at Prime Vertical**
  A star is said to be at prime vertical when it crosses the prime vertical. At this position Azimuth of star i.e. angle at zenith is equal to 90°.



Five parts of Napier's circle will be

(90 – θ), (90 – H), δ, (90 – M), (90 – α)

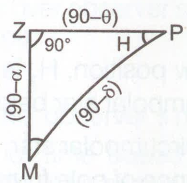
Given are – δ and θ

(i) Calculation of Hour angle (H)

$$\sin(90-H) = \tan\delta.\tan(90-\theta)$$

$$\cos H = \tan\delta.\cot\theta$$

(ii) Altitude (α)



$$\sin\delta = \cos(90-\theta).\cos(90-\alpha)$$

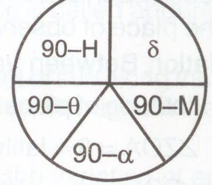
$$\sin\delta = \sin\theta.\sin\alpha$$

$$\sin\alpha = \sin\theta.\operatorname{cosec}\delta.$$

- **Star at Horizon**

When star is at Horizon, it's altitude will be equal to zero.

α = 0 so ZM = 90°



PM = 90 – δ, ZP = 90 – θ

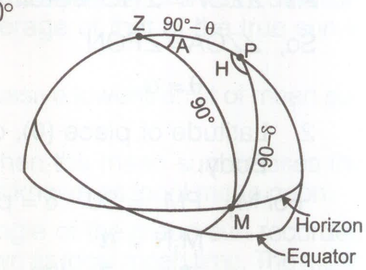
∠P = H ∠Z = A.

Azimuth.

$$\cos A = \frac{\cos(90-\delta)-\cos(90-\theta).\cos 90^\circ}{\sin(90-\theta).\sin 90^\circ}$$

$$= \frac{\cos(90-\delta)-0}{\cos\theta.1}$$

$$= \frac{\sin\delta}{\cos\theta} = \sin\delta.\sec\theta$$

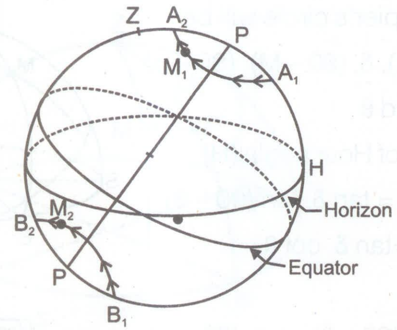


Hour angle

$$\cos H = \frac{\cos 90-\cos(90-\theta).\cos(90-\delta)}{\sin(90-\theta)\sin(90-\delta)}$$

$$= -\tan\theta.\tan\delta$$

- **Circumpolar star**
  The stars which remain always above the horizon (or below the horizon) and also do not set. Any time (when above the horizon) or do not rise any time (when below the horizon), are called circumpolar star.



Arrow position, H_1 is a circumpolar star above the horizon, and M_2 is circumpolar star below the horizon.

For circumpolar star, distance of star from pole PA_1 should be less than distance of pole from horizon.

$PA_1 < PH$ and or $(90 - \delta) < \theta$ so $\delta > 90 - \theta$.

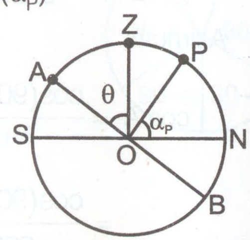
So declination of a circumpolar star is always greater than the colatitude of the place of observation.

- **Relation Between various coordinate**

1. Latitude of place (θ) and altitude of Pole (α_P)

   $\angle ZOA = \theta$ = latitude of place

   $\angle PON = \alpha_P$ = Altitude of pole

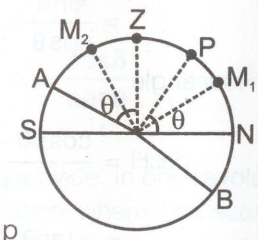


As $\angle ZOP = \angle NOB$. (from geometry)

So, $\angle ZOA = \angle PON$

$\theta = \alpha_P$

2. Latitude of place (θ), declination (δ) and Altitude (α) of celestial Body:



For M_1, $PM_1 = 90 - \delta = p$

$M_1N = \alpha$

$ZM_1 = Z = (90 - \alpha)$

$M_1B = \delta$

$M_1B = M_1N + NB$

$\delta = a + (90 - \delta)$ or $\theta = \alpha + (90 - d) = \alpha + p$

$\theta = \alpha + p$

For star M_2

$M_2A = \delta . M_2Z = Z$ $ZA = \theta$

$ZA = ZM_2 + M_2A$

$\Rightarrow$ $\theta = \delta + Z$ This equation covers all cases.

TIME

Interval which lapses between any two instants, is termed as time. Following type of time measurements are generally used by astronomers.

(i) **Sidereal Time:** Hour angle of first point of Aries (γ) measured westward at any instant is called sidereal time of that instant.

Interval of time between two successive upper point transit of first point of Aries is called sidereal day.

Local Sidereal Time (LST): The interval of time which elapses since the upper transit of first point of Aries over observer's meridian is known as local sidereal time of the place.

LST = RA of a star + HA of star

= Right Ascension (RA) of the observer's meridian.

(ii) **Apparent Solar Time :** Measurement of time based on daily apparent motion of the sun round the earth, is known as apparent solar time.

Interval of time between two successive lower transit (culmination) of centre of sun over meridian of the place is called apparent solar day.

(iii) **Mean Solar Time:** As the rate of movement of sun along the ecliptic is not uniform, length of apparent solar day, throughout the year is also not uniform.

To overcome this difficulty of recording the variation of apparent solar time by a clock, a fictitious sun is assumed to move at uniform rate along the equator so that to have a solar day of uniform duration. Motion of this mean sun is the average of that of the true sun in right ascension.

Interval of time between two successive lower transit of mean sun is called mean solar day

Local Mean Noon: The instant when the mean sun crosses the local meridian at its upper transit is known as local mean noon.

Local Mean time: (LMT): Hour angle of the mean sun recorded westward from 0 to 24 hours, is known as local mean time. The mean solar day begins at mid night and completes at next mid-night.

(iv **Standard Time:** As local mean time of any place is taken from lower transit of mean sun at the meridian of that place. So local mean time of different meridian will also be different for a country having difference of meridian of different places, to avoid confusion, a standard time is taken as per a central meridian of the country called standard meridian.

Standard meridian of a country is generally selected such that it lies at an exact number of hours from Greenwich. But Indian Standard Meridian is at $5^{1/2}$ hours (80° 30' longitude) east of

Greenwich. All watches in a country shows the same standard time irrespective of the place.

Standard time (ST) = LMT ± difference of longitude converted to time.

- **Equation of Time**

  Difference between apparent solar time and mean solar time at any instant is known as the equation of time.

  Equation of time = Apparent solar time – Mean solar time.

- **Conversion of Time**

Longitude	360°	15°	1°C	15′	1′	15″
Time	24 hours	1 hour	4 minute	1 minute	4 second	1 second

- **Conversions**

1. Conversion of local time to standard time

   LMT = MT ± diff. in longitude $\left(\frac{E}{W}\right)$

2. Conversion of local time to Greenwich time

   LMT = GMT ± longitude of place $\left(\frac{E}{W}\right)$

3. Local Apparent time = Local mean time + equation of time.

4. Mean solar time

   =sidereal time – retardation

   =sidereal time – 9.8296 seconds per hour of given sidereal time

5. Sidereal time

   = Mean solar time + Acceleration

   = Mean solar time + 9.8565 sec per hour of given mean solar time.

6. LST at LMM = GST at GMM

   ±9.8565 sec. per hour of longitude $\left(\frac{W}{E}\right)$.

   LST at LMN = GST at GMN

   ± 9.8565 sec. per hour of longitude $\left(\frac{W}{E}\right)$.

7. LST at LMT = LST at LMM + SI from LMM

   SI (sidereal time interval) = LST at LMT – LST at LMM

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