OLYMPIAD EXCELLENCE BOOK MATHEMATICS

QUESTIONS

1. **The L.C.M. and H.C. F. of marks scored by Supravin & Kumar in a test are 1489645 and 1 respectively. If Supravin's score is 1145, what is Kumar's score?**

(a) 68 (b) 666 (c) 1295 (d) 1301

2. **$5.6\overline{723}$**

(a) An integer

(b) A rational number

(c) An irrational number

(d) A natural number

3. **The value of $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}$ is**

(a) 0 (b) 1 (c) 2 (d) 4

4. **The value of $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots\ldots\frac{1}{\sqrt{34}+\sqrt{35}}+\frac{1}{\sqrt{35}+\sqrt{36}}$ is**

(a) 0 (b) 2 (c) 3 (d) 5

5. **'P' is the remainder obtained when a perfect square is divided by 3. What is the value of 'p'?**

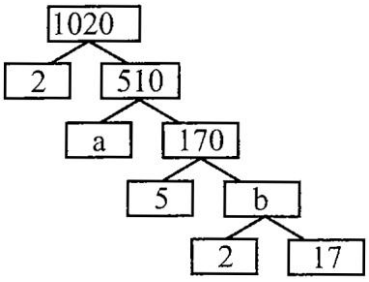
(a) 1 (b) 0 (c) Either (a) or (b) (d) Neither (a) or (b)

6. **If $\sqrt{p}<\sqrt{q}<\sqrt{r}<\sqrt{s}$ where p, q, r, s are consecutive natural numbers, then**

(a) $\sqrt{s}-\sqrt{r}>\sqrt{q}-\sqrt{p}$ (b) $\sqrt{q}-\sqrt{p}>\sqrt{s}-\sqrt{r}$

(c) $\sqrt{s}-\sqrt{q}>\sqrt{r}-\sqrt{p}$ (d) $\sqrt{s}-\sqrt{r}=\sqrt{q}-\sqrt{p}$

7. **The factor tree shows the prime factorization of 1020. Then (a, b) is**



(a) 2, 17 (b) 3, 34 (c) 34, 3 (d) 5, 17

8. **If $m = \left(1 - 2^{\frac{1}{4}}\right)^{-1}$, then m can also be written as,**

(a) $\left(1-\sqrt[4]{2}\right)\left(2-\sqrt{2}\right)$ (b) $\left(1-\sqrt[4]{2}\right)\left(2+\sqrt{2}\right)$

(c) $-\left(1+\sqrt[4]{2}\right)\left(1+\sqrt{2}\right)$ (d) $\left(1+\sqrt[4]{2}\right)\left(2+\sqrt{2}\right)$

9. **Euclid's division lemma: For any two positive integers 'a' and 'b', there exist unique integers 'q' and 'r' such that a = bq + r.**

What is the condition that 'r' must satisfy?

(a) $0 \le r \le b$ (b) $0 < r \le b$ (c) $0 \le r < b$ (d) $0 < r < b$

10. **The 1000th root of $10^{(10^{10})}$ is**

(a) 10^{77} (b) $10^{(10^7)}$ (c) $\left(\sqrt[3]{10}\right)^{10^5}$ (D) $10^{(\sqrt[3]{10})^{10}}$

11. **The following are the first and last step in finding the H.C.F. of 36 and 56 using Euclid's algorithm.**

Step 1: $56 = 36 \times 1 + 20$

Step 2: _____

Step 3: _____

Step 4: $16 = 4 \times 4 + 0$

Choose the steps 2 and 3.

(i) $36 = 20 \times 1 + 16$ (ii) $24 = 20 \times 1 + 4$

(iii) $20 = 16 \times 1 + 4$ (iv) $56 = 18 \times 2 + 20$

(a) (i) and (ii) (b) (i) and (iii) (c) (ii) and (iii) (d) (iii) and (iv)

12. **The difference between $10^{(10)^2}$ and $\left(10^{10}\right)^2$ is the order of**

(a) 0 (b) 10^{80} (c) 10^{20} (D) None of these

13. **For what value of 'x' does 3^x end with 5?**

(a) 0 (b) 5 (c) 500 (d) Never ends with 5

14. **When a number is divided by 19, its remainder is always**

(a) Greater than 19

(b) Lies between 19 and 57

(c) Greater or equal to zero but less than 19

(d) Less than zero

15. If 9 divides 6561, which of the following statements is true?

(a) 9 divides 81 (b) 7 divides 243 (c) 7 divides 2178 (d) 9 divides 2189

16. $\mathbf{X_1, X_2, X_3 \ldots, X_{12}}$ are integers none of which are divisible by 3. The remainder when $\mathbf{X_1^2, X_2^2, X_3^2 + \ldots + X_{12}^2}$ is divided by 3 is

(a) 0 (b) 0 or 2 (c) 1 or 2 (d) 1

17. Diagonals of a rhombus are $\mathbf{\left(2^5 \times 7\right)}$ **cm and** $\mathbf{\left(2 \times 5^2 \times 7^3\right)}$ **cm. Express the area of the rhombus in prime factorization form.**

(a) $2 \times 5 \times 7\ cm^2$ (b) $2^2 \times 5^2 \times 7^2 cm^2$ (c) $2^5 \times 5^2 \times 7^4 cm^2$ (d) $2^6 \times 5^2 \times 7^4 cm^2$

18. Set of natural number is a subset of

(a) Set of even numbers (b) Set of odd numbers

(c) Set of composite numbers (d) Set of real numbers.

19. Choose the irrational number.

(a) $3 - \sqrt{9}$ (b) $\left(\sqrt{12}\right)^2$ (c) $\sqrt{625} - \sqrt{576}$ (d) $\sqrt{125} - \sqrt{64}$

20. $\mathbf{1 / \left(\sqrt{3} - \sqrt{2}\right)}$ **is not equal to**

(a) $\sqrt{3} + \sqrt{2}$ (b) $\dfrac{\sqrt{3}}{\left(3 - \sqrt{6}\right)}$

(c) $\left(\sqrt{3} - \sqrt{2}\right) / \left(5 - 2\sqrt{6}\right)$ (d) $\left(\dfrac{\sqrt{4}}{\sqrt{10} - \sqrt{8}}\right)$

21. Given $\mathbf{a = p - \sqrt{q}}$ **and** $\mathbf{b = p + \sqrt{q}}$ **which of the following is correct. Where q is a prime number.**

(a) a + b is irrational (b) a - b is rational. (c) 2ab is rational (d) $\dfrac{a}{b}$ is rational

22. According to the fundamental theorem of arithmetic, if p(a prime number) divides $\mathbf{a^2}$ **and a is positive, then**

(a) a divides p (b) a^2 divides p (c) p^2 divides a^2 (d) p divides a

23. Which of the following is a non-terminating repeating decimal?

(a) $\dfrac{72}{6000}$ (b) $\dfrac{1771}{8000}$ (c) $\dfrac{123}{4^2 \times 5^4}$ (d) $\dfrac{145}{4^3 \times 5^2 \times 7^2}$

24. The number in the form of 4K + 3 where K is whole number, is always;

(a) An odd number (b) An even number (c) A perfect square (d) Divisible by 3

25. Choose the terminating decimal.

(a) $\frac{641}{8000}$ (b) $\frac{29}{66}$ (c) $\frac{283}{120}$ (d) $\frac{617}{81}$

26. The number of subsets of A = {0, 1, 2} will be

(a) 3 (b) 5 (c) 6 (d) 8

27. Find the number which when divided by 87 leaves a remainder 49 and gives a quotient 50.

(a) 3997 (b) 4399 (c) 4301 (d) 4019

28. By what number must 1587 be divided to get a quotient 27 and remainder 21?

(a) 58 (b) 57 (c) 59 (d) 63

29. At an event on Saraswati Puja for students, 1643 Calendars and 1060 sweets were to be distributed amongst students of class X such that each student gets the same number of calendar and also same number of sweets, what is the maximum number of students in class X?

(a) 53 (b) 93 (c) 79 (d) 69

30. Which of the following is true for two co-prime numbers?

(a) Then- H.C.F. is 1.

(b) Their L.C.M. is 1.

(c) Their H.C.F. is equal to their product.

(d) Their L.C.M. is twice their H.C.F.

31. Choose the methods that can be used to find the H.C.F. of any two numbers.

(i) Euclid's division lemma

(ii) Prime factorization

(iii) Division of the numbers

(iv) Product of numbers

(a) (i) and (iv) only (b) (i), (ii) and (iii) only

(c) (i), (iii) and (iv) only (d) (ii), (iii) and (iv) only

32. A positive number 'n' when divided by 9 leaves a remainder 6 what is the remainder when 3n + 2 is divided by 3?

(a) 0 (b) 1 (c) 2 (d) 3

33. The remainder when a number is divided by 165 is 21. What is the remainder, when the same number is divided by 11?

(a) 4 (b) 5 (c) 11 (d) 10

34. Three distances are 8 m, 9 m 20 cm and 10 m 80 cm long. What is the greatest possible length which can be used to measure these ropes?

(a) 40 cm (b) 50 cm (c) 60 cm (s) 1 m 20 cm

35. Sohan purchased 98 fruits out of which 35 are oranges and the remaining 63 are mangoes. Oranges and mangoes are to be packed in separate bundles and each bundle must contain the same number of fruit. Find the least number of bundles which can be made of these 98 fruit.

(a) 13 (b) 7 (c) 14 (d) 9

36. Two candidates during a physical test start running around a circular path. First candidate takes 22 minutes and second candidates takes 24 minutes to complete one round of the path. If both of them start at the same point, then find after how many minutes they will meet again at the same starting point.

(a) 46 minutes (b) 1 hour 22 minutes

(c) 92 minutes (d) 4 hours 24 minutes

37. The sum of $1^2 + 2^2 + ...n^2$ is

(a) $\frac{n(n+1)^3}{3}$ (b) $\frac{(n+1)n^2}{6}$ (c) $\frac{n(n-1)(2n+1)}{12}$ (d) $\frac{n(n+1)(2n+1)}{6}$

38. The sum of $1^3 + 2^3 + ...n^3$ is

(a) $\frac{n^2(2n+1)(3n+2)}{6}$ (b) $\frac{n^2(n+1)^2}{4}$ (c) $\frac{n^3(n-1)}{3}$ (d) $\frac{(n-3)^2(n+5)}{9}$

39. Find the sum of $2^2 + 3^2 + ...100^2$.

(a) 338349 (b) 99999 (c) 103245 (d) 563000

40. Find the sum of $2^2 + 4^2 + 6^2 .....(2n)^2$.

(a) $\frac{n(n+1)^3}{3}$ (b) $\frac{2n(n+1)(2n+1)}{3}$ (c) $\frac{n(n-3)(2n+3)}{12}$ (d) $\frac{n(n+1)(2n+4)}{6}$

ANSWER - KEY				
1. D	**2.** B	**3.** B	**4.** D	**5.** C
6. B	**7.** B	**8.** C	**9.** C	**10.** B
11. B	**12.** D	**13.** D	**14.** C	**15.** A
16. A	**17.** C	**18.** D	**19.** D	**20.** D
21. C	**22.** D	**23.** D	**24.** A	**25.** A
26. D	**27.** B	**28.** A	**29.** A	**30.** A
31. B	**32.** C	**33.** D	**34.** A	**35.** C
36. D	**37.** D	**38.** B	**39.** A	**40.** B

SOLUTIONS

1. (D): $1145 \times b = 1489645 \times 1$

$\Rightarrow b = 1301$

2. (B): $5.67\overline{23} = 5.67 + .00\overline{23}$

$\therefore 5.67\overline{23} = \frac{567}{100} + \frac{23}{9900}$ which is a rational number.

$\frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} + \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} \times \frac{\sqrt{4}-\sqrt{3}}{\sqrt{4}-\sqrt{3}} ..... = 1$

3. (B): Not Available

4. (D): Follow the same procedure as in previous question.

(Multiplying successive term by $\left(\sqrt{2}-1\right), \left(\sqrt{3}-\sqrt{2}\right), ..., \left(\sqrt{36}-\sqrt{35}\right)$ lead to rationalization of each terms.

Middle terms cancel. What remain is $\sqrt{36}-1 = 6-1 = 5$)

5. (C) Not Available

6. (B) Not Available

7. (B): $\frac{510}{a} = 170 \Rightarrow a = 3$

$\frac{170}{5} = b \Rightarrow b = 34$

8. (C): $\frac{1}{1-\sqrt[4]{2}} = \frac{1+\sqrt[4]{2}}{1^2 - \left(\sqrt[4]{2}\right)^2} =$

$\frac{1+\sqrt[4]{2}}{1-\sqrt{2}} = \frac{\left(1+\sqrt[4]{2}\right)\left(1+\sqrt{2}\right)}{-1}$

9. (C) Not Available

10. (B): $\left\{10^{\left(10^{10}\right)}\right\}^{\frac{1}{1000}} = \left\{10^{\left(10^{10} \div 1000\right)}\right\} = 10^{\left(10^7\right)}$

11. (B): Step 2: $36 = 20 \times 1 + 16$

Step 3: $20 = 16 \times 1 + 4$

12. (D): $\left(a^m\right)^n = a^{mn} \therefore (10)^2 = 100 \Rightarrow 10^{(10)^2} = 10^{100}$

$\left(10^{10}\right)^2 = 10^{10\times 2} = 10^{20}$

$\therefore$ Difference $1 - 10^{20} = 10^{20}\left(10^{80} - 1\right)$: so it is of the order of 10^{100}

13. (D): 3^x always ands with 3, 9, 7. 1. Try it with some TRIVIAL Values of x e.g. x= 1, 2, 3, 4, 5

14. (C): This is as per Euclid's division lemma.

15. (A) Not Available

16. (A): When a perfect square is divided by 3, remainder is 1.

$\therefore$ Each time remainder is 1 in all twelve terms.

$\therefore$ Sum of remainders = 1 + 1 +.....(12 times).

= 12

Which is divisible by 3

$\therefore$ Net remainder = zero

17. (C): Area $= \frac{1}{2} \times$ Product of diagonals

$= \frac{1}{2} \times \left(2^5 \times 7\right) \times \left(2 \times 5^2 \times 7^3\right)$

$= 2^5 \times 7^4 \times 5^2$

18. (D) Not Available

19. (D) Not Available

20. (D): From $\frac{1}{\sqrt{3} - \sqrt{2}}$, (A) is obtained by multiplying numerator and denominator by $\left(\sqrt{3} + \sqrt{2}\right)$

(B) is obtained by multiplying numerator and denominator by $\sqrt{3}$

(C) is obtained by multiplying numerator and denominator by $\left(\sqrt{3} - \sqrt{2}\right)$ But (D) cannot be obtained.

21. (C): $2ab = 2\left(p - \sqrt{q}\right)\left(p + \sqrt{q}\right) = 2\left(p^2 - q\right)$

22. (D) Not Available

23. (D): A non - terminating repeating decimal is one is which denominator has a factor other than 2 and 5.

24. (A) Not Available

25. (A): Terminating decimal is one in which its fractional form (in its lowest term) has a denominator which can be expressed as product of only 2 (or its powers) and / or 5 (or its powers). This is satisfied in (A) only as denominator $= 2^3 \times 1000 = 2^6 \times 5^3$

26. (D): subsets are $\phi, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}$ i.e. 8 is total

27. (B): Number = Dividend = Divisor $\times$ Quotient + Remainder

$= 87 \times 50 + 49$

$= 4399$

28. (A): $1587 = n \times 27 + 21$

(Solve for 'n')

29. (A): Mind of a mathematician

Just by careful study of question, we see that 53 should fully divide both 1643 and 1060 i.e. $53 \times x_1 = 1643$ and $53 \times x_2$

30. (A) Not Available

31. (B) Not Available

32. (C): According to statement of problem, $n = 9k + 6$

$\therefore 3n + 2 = 27k + 20$

Dividing $27k + 20$ *by* $3; \frac{27k + 20}{3} = (9k + 6) + \frac{2}{3}$

$\Rightarrow$ Remainder = 2

33. (D): Let number be $'n' \Rightarrow n = 165 \times k + 21$ where $k \in N$

$= 11 \times 15 \times k + 11 + 10$

The indicated part is clearly divisible by 11

$\therefore$ Remainder = 10

34. (A): Greatest length will be HCF of 800, 920 and 1080 Consider 80, 92 and 180

By prime factorization method,

$$\left.\begin{array}{l} 80 = 2^4 \times 5 \\ 92 = 2^2 \times 23 \\ 108 = 2^2 \times 3^3 \end{array}\right] \text{Common factor} = 2^2 = 4$$

Hence, HCF = 40

35. (C): Let number of bundles of oranges be B_1 and bundle of mangoes be B_2. Let bundles B_1 have n oranges and bundles B_2 have n mangoes each. Then $35 = B_1 \times n$ and $63 = B_2 \times n$.

Since B_1, n_1, B_2, n_2 all have to be natural numbers.

$\therefore$ Possibilities are $B_1 = (1,5,7,35)$: $B_2 = (1,3,7,9,63)$

Common factor is $7 \Rightarrow$ no of fruits in each bundle = 7

$\Rightarrow B_1 = \dfrac{35}{7} = 5$ and $B_2 = \dfrac{63}{7} = 9$

$\therefore B_1 + B_2 = 14$

36. (D): It will be LCM of 22 min and 24 min

37. (D) Consider the identify $n^3 - (\text{n}-1)^3 - 3n^2 - 3n + 1$

Now, RHS

$$\sum\left(3n^2 - 3n + 1\right) = 3 \times \sum n^2 - 3\sum n + \sum 1$$

$$= 3S - \frac{3n(\text{n}+1)}{2} + n$$

Taking the summation of LHS

$$\sum_1^n n^3 - (\text{n}-1)^3 = n^3$$

$$\therefore n^3 = 3S - \frac{3n(\text{n}+1)}{2} + n$$

$$\therefore 3S = n^3 + \frac{3n(\text{n}+1)}{2} - n$$

$$\therefore 6S = 2n^3 + 3n^2 + 3n - 2n$$

$$= 2n^3 + 3n^2 + n = n[2n^2 + 3n + 1]$$

$$= n(2\text{n}+1)(\text{n}+1)$$

$$= \frac{n(\text{n}+1)(2\text{n}+1)}{6}$$

38. (B): Consider $n^4-(n-1)^4=4n^3-6n^2+4n-1$

$$\sum n^4-(n-1)^4=4\sum n^3-6\sum n^2+4\sum n-\sum 1$$

$$n^4=4s-n(n+1)(2n+1)+2n(n+1)-n$$

$$4s=n^4+n(n+1)(2n+1)-2n(n+1)+n$$

$$4s=\frac{n^2(n+1)^2}{4}$$

$$s=\frac{n^2(n+1)^2}{4}$$

39. (A): Not Available

40. (B): $S=2^2+4^2+6^2+.....$

$$=2^2\left[1^2+2^2+3^2+.....+n^2\right]$$

$$=2^2\times\frac{n(n+1)(2n+1)}{6}$$

$$=\frac{2n(n+1)(2n+1)}{3}$$