Binomial Theorem

Binomial Expansions (2 terms) = (a+b)

$(a+b)^0 = 1$

$(a+b)^1 = 1.a + 1.b$

$(a+b)^2 = 1.a^2 + 2ab + 1.b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a+b)^5 = 1.a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

$(a+b)^6 = 1.a^6 + 6.a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6.ab^5 + 1.b^6$

Term = coefficient × variable

Variables.

coefficients.

Coeff. ← Pascal's Triangle

Powers.	Coeff.
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 $\boxed{6}$ 4 1
5	1 5 $\boxed{10}$ 10 5 1
6	1 6 15 20 15 6 1

For Better Clarity.

$$(a+b)^3 = (a+b)(a+b)(a+b) = 1.a^3 + 3a^2b + 3ab^2 + 1.b^3$$

$$\rightarrow a^3 + a^2b + a^2b + a^2b + ab^2 + ab^2 + ab^2 + b^3$$

$(a+\ )(a+\ )(\ +b)$

$(a+\_)(\_+b)(a+\_)$

$(\ +b)(+b)(\_+b)$

$(a+\_)(\_+b)(\_+b)$

$a < a \cdot b \rightarrow$ P&C

Chapter 7

$^nC_0 = 1$

$$^nC_r = \text{select 'r' out of 'n'} = \frac{n!}{(n-r)!\,r!}$$

$$(a+b)^3 = (a+b)(a+b)(a+b) = 1\,a^3 + 3\,a^2b + 3\,ab^2 + 1\,b^3$$

$$= {}^3C_0 . a^3 . b^0 + {}^3C_1 . a^2 b^1 + {}^3C_2 . ab^2 + {}^3C_3 . b^3 . a^0$$

way way way way

Binomial Theorem (for Binomial Expansion)

$n =$ whole No.

$$(a+b)^n = {}^nC_0 \, a^n \cdot b^0 + {}^nC_1 \, a^{n-1} \cdot b^1 + {}^nC_2 \cdot a^{n-2} \cdot b^2 + {}^nC_3 \cdot a^{n-3} \cdot b^3 \; \cdots\cdots + {}^nC_{n-1} \cdot a^1 \cdot b^{n-1} + {}^nC_n \cdot a^0 \cdot b^n$$

$n-n$

$(a+b)^n \leftarrow$ Power $=$ index $=$ Exponent

$n \in W =$ whole No.

$${}^nC_r = \frac{n!}{(n-r)!\, r!}$$

${}^nC_0, \; {}^nC_1, \; {}^nC_2, \; {}^nC_3 \; \cdots \; {}^nC_{n-1}, \; {}^nC_n =$ Binomial Coefficients.

${}^nC_0 \downarrow 1$, ${}^nC_1 \downarrow n$, ${}^nC_{n-1} \downarrow n$, ${}^nC_n \downarrow 1$

$\longleftarrow \qquad \longrightarrow$

Symmetry.

sigma

$\Sigma \rightarrow +$ → Summation $= +$

$$\sum_{r=5}^{10} 2^r = 2^5 + 2^6 + 2^7 + 2^8 + 2^9 + 2^{10}$$

Upper limit → 10

lower limit → $r=5$

Observations:

(1) $$(a+b)^n = \underbrace{\sum_{r=0}^{r=n} \boxed{{}^nC_r \cdot a^{n-r} \cdot b^r}}_{\text{Short form}}$$

Observations

(2) No. of terms in the expansion of $(a+b)^n = n+1$

$= $ one more than index (n)

(3) In subsequent terms,

power of $a = \downarrow$ decreases.

Power of $b = \uparrow$ increases.

(4) In each term, sum of powers of a & $b = n$.

$(a+b)^n$

Some Special Expansion.

$$(a+b)^n = {}^nC_0 \cdot a^n \cdot b^0 + {}^nC_1 \cdot a^{n-1} \cdot b^1 + {}^nC_2 \cdot a^{n-2} \cdot (b)^2 + \dots + {}^nC_n \cdot a^0 b^n$$

(I) Replace $\underbrace{a \to a}$, $\underbrace{b \to -b}$

$$(a-b)^n = \underline{{}^nC_0 \cdot a^n} - \underline{{}^nC_1 \cdot a^{n-1} \cdot b} + \underline{{}^nC_2 \cdot a^{n-2} \cdot b^2} - \dots + (-1)^n \cdot {}^nC_n \cdot a^0 \cdot b^n$$

(II) Replace $\underbrace{a \to 1}$, $\underbrace{b \to x}$

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 \cdot x^2 + \dots + {}^nC_n \cdot x^n$$

(III) Replace $\underbrace{a \to 1}$, $\underbrace{b \to -x}$

$$(1-x)^n = {}^nC_0 - {}^nC_1 \cdot x + {}^nC_2 \cdot x^2 - \dots + {}^nC_n (-x)^n$$

(IV) Replace $\underbrace{a \to 1}$ $\underbrace{b \to 1}$

$$\boxed{2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n}$$

Revision: $(a+b)^n = {}^nC_0\, a^n\, b^0 + {}^nC_1 . a^{n-1} . b^1 + {}^nC_2 . a^{n-2} . b^2 + \dots + {}^nC_{n-1} . a^1 . b^{n-1} + {}^nC_n . a^0 . b^n$

$n =$ whole No.

$n =$ index

Binomial coefficients $= {}^nC_r = \dfrac{n!}{(n-r)!\, r!}$, $\quad ({}^nC_r = {}^nC_{n-r})$

Exercise 7.1

$\boxed{Q.1}$ $(1-2x)^5$ $\quad \cancel{(a+b)^5}$

$(-1)^{\text{Even}} = +1$

$(-1)^{\text{odd}} = -1$

${}^5C_0 = 1 = {}^5C_5$

${}^5C_1 = 5 = {}^5C_4$

${}^5C_2 = 10 = {}^5C_3$

By Binomial theorem:

$[\underset{a}{1} + \underset{b}{(-2x)}]^5$

$(1-2x)^5 = {}^5C_0 (1)^5 (-2x)^0 + {}^5C_1 (1)^4 (-2x)^{\overset{\text{odd}}{1}} + {}^5C_2 (1)^3 (-2x)^2 + {}^5C_3 (1)^2 (-2x)^{\overset{\text{odd}}{3}} + {}^5C_4 (1)^1 (-2x)^4$
$+ {}^5C_5 (1)^0 (-2x)^{\overset{\text{odd}}{5}}$

$= [1.1.1] - [5.1.2x] + [10.1.4x^2] - [10.1.8x^3] + [5.1.16.x^4]$
$- [1.1.32.x^5]$

$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

$\boxed{Q.2}$ $\left(\frac{2}{x} - \frac{x}{2}\right)^{5 \to n}$

$a \leftarrow$, $\rightarrow b$

By Binomial Theorem:

$$= {}^5C_0\left(\frac{2}{x}\right)^5\left(-\frac{x}{2}\right)^0 + {}^5C_1\left(\frac{2}{x}\right)^4\left(-\frac{x}{2}\right)^1 + {}^5C_2\left(\frac{2}{x}\right)^3\left(-\frac{x}{2}\right)^2 + {}^5C_3\left(\frac{2}{x}\right)^2\left(-\frac{x}{2}\right)^3$$

$$+ {}^5C_4\left(\frac{2}{x}\right)^1\left(-\frac{x}{2}\right)^4 + {}^5C_5\left(\frac{2}{x}\right)^0\left(-\frac{x}{2}\right)^5$$

$$= 1 \cdot \frac{32}{x^5} \cdot 1 - 5 \cdot \frac{\cancel{16}^{\,8}}{x^4} \cdot \frac{x}{\cancel{2}} + 10 \cdot \frac{\cancel{8}^{\,2}}{x^3} \cdot \frac{x^2}{\cancel{4}} - \cancel{10}^{\,5} \cdot \frac{\cancel{4}}{x^2} \cdot \frac{x^3}{\cancel{8}_{\,\cancel{2}}}$$

$$+ 5 \cdot \frac{\cancel{2}}{x} \cdot \frac{x^4}{\cancel{16}_{\,8}} - 1 \cdot 1 \cdot \frac{x^5}{32}$$

$$= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5x^3}{8} - \frac{x^5}{32}$$

Q.3 $(2x-3)^6 \longrightarrow n=6$

$a=2x \quad b=-3$

$^6C_0 = 1 = {^6C_6}$

$^6C_1 = 6 = {^6C_5}$

$^6C_2 = {^6C_4} = \frac{6!}{4!2!} = \frac{6.5.\cancel{4!}}{\cancel{4!}\,2\times1} = 15$

$$= {^6C_0}\cdot(2x)^6\cdot(-3)^0 + {^6C_1}\cdot(2x)^5\cdot(-3)^1 + {^6C_2}\cdot(2x)^4\cdot(-3)^2 + {^6C_3}(2x)^3\cdot(-3)^3$$

$$+ {^6C_4}(2x)^2\cdot(-3)^4 + {^6C_5}(2x)^1(-3)^5 + {^6C_6}\cdot(2x)^0(-3)^6$$

$^6C_3 = \frac{6!}{3!\,3!} = \frac{6.5.4.\cancel{3!}}{\cancel{3!}\,3.2.} = 20$

$$= 1.64.x^6.1 + 6.32.x^5.(-3) + (15.16.x^4.9) + (20.8.x^3.(-27))$$

$$+ (15.4x^2.81) + (6.2x.(-243)) + (1.1.729)$$

$$= 64x^6 - 586x^5 + 2160x^4 - 4320.x^3$$

$$+ 4860x^2 - 2916x + 729$$

243
×3
729

18
×32
36
54
586.

729
×4
216
432

729
×4
2916

240
×9
2160

6
81
486

Q. 4 $\left(\frac{x}{3}+\frac{1}{x}\right)^5 \longrightarrow n=5$

$a=\frac{x}{3}$ $\qquad$ $b=\frac{1}{x}$

By Binomial Theorem;

$$= {}^5C_0\cdot\left(\frac{x}{3}\right)^5\cdot\left(\frac{1}{x}\right)^0 + {}^5C_1\cdot\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right)^1 + {}^5C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2 + {}^5C_3\cdot\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3$$
$$+ {}^5C_4\left(\frac{x}{3}\right)^1\cdot\left(\frac{1}{x}\right)^4 + {}^5C_5\cdot\left(\frac{x}{3}\right)^0\cdot\left(\frac{1}{x}\right)^5$$

$$= 1\cdot\frac{x^5}{243}\cdot 1 + 5\cdot\frac{x^4}{81}\cdot\frac{1}{x} + 10\cdot\frac{x^3}{27}\cdot\frac{1}{x^2} + 10\cdot\frac{x^2}{9}\cdot\frac{1}{x^3}$$
$$+ 10\times\frac{x}{3}*\frac{1}{x^4} + 1\cdot 1\cdot\frac{1}{x^5}$$

$$= \frac{x^5}{243} + \frac{5x^3}{81} + \frac{10\cdot x}{27} + \frac{10}{9x} + \frac{10}{3x^3} + \frac{1}{x^5}$$

$\boxed{Q.5}$ $\left(\underset{a}{x} + \underset{b}{\frac{1}{x}}\right)^6 \longrightarrow n = 6$

By Binomial Theorem:

$$= {}^6C_0 \cdot (x)^6 \cdot \left(\frac{1}{x}\right)^0 + {}^6C_1 \cdot (x)^5 \cdot \left(\frac{1}{x}\right)^1 + {}^6C_2 \cdot (x)^4 \left(\frac{1}{x}\right)^2 + {}^6C_3 \cdot \cancel{(x)^3}\left(\frac{1}{\cancel{x}}\right)^3$$

$$+ {}^6C_4 \, (x)^2 \left(\frac{1}{x}\right)^4 + {}^6C_5 \, (x)^1 \left(\frac{1}{x}\right)^5 + {}^6C_6 \, (x)^0 \left(\frac{1}{x}\right)^6$$

$$= 1 \cdot x^6 \cdot 1 + 6 \cdot x^4 + 15 \cdot x^2 + 20 \cdot + \frac{15}{x^2} + 6 \cdot \frac{1}{x^4} + 1 \cdot 1 \cdot \frac{1}{x^6}$$

$$= x^6 + 6 \cdot x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

$\boxed{Q.6}$ $(96)^3 = \underset{\times}{(95+1)^3} = \underset{\checkmark}{(100-4)^3}$

$$(a \pm b)^n = {}^nC_0 \cdot a^n \cdot b^0 \pm {}^nC_1 \cdot a^{n-1} \cdot b^1 + {}^nC_2 \cdot a^{n-2} \cdot b^2 \pm \cdots + {}^nC_n \cdot a^0 \cdot b^n$$

$$(96)^3 = (\underset{\substack{\uparrow \\ a=100}}{100} - \underset{\substack{\uparrow \\ b=-4}}{4})^3 = {}^3C_0 \cdot 100^3 \cdot \underbrace{(-4)^0}_{1} + {}^3C_1 \cdot 100^2 \cdot (-4)^1 + \underbrace{{}^3C_2}_{3} \cdot 100^1 \cdot \underbrace{(-4)^2}_{16} + \underbrace{{}^3C_3}_{1} \cdot \underbrace{100^0}_{1} \underbrace{(-4)^3}_{-64}$$

$$= 1.1000000 - 120000 + 4800 - 64$$

$$\begin{array}{r} +1004800 \\ -120064 \\ \hline 884736 \end{array}$$

$$(96)^3 = 884736$$

[Q.7] $(102)^5 = (100+2)^5$

Binomial Theorem :

$^5C_2 = {^5C_3} = 10$
$^5C_0 = 1 = {^5C_5}$
$^5C_1 = {^5C_4} = 5$

$^nC_r = \dfrac{n!}{(n-r)!\,r!}$

$(100+2)^5 = {^5C_0}\cdot 100^5\cdot 2^0 + {^5C_1}\cdot 100^4\cdot 2^1 + {^5C_2}\cdot 100^3\cdot 2^2 + {^5C_3}\cdot 100^2\cdot 2^3 + \underset{5}{{^5C_4}}\cdot 100^1\cdot \underset{16}{2^4}$
$+ {^5C_5}\cdot 100^0\cdot 2^5$

$= 10000000000 + 1000000000 + 40000000 + 800000 + 8000$
$+ 32$

$= 11040808032$

[Q.8] $(101)^4 = (100+1)^4$

$= \underset{1}{{^4C_0}}\cdot 100^4\cdot \underset{1}{1^0} + \overset{4}{{^4C_1}}\cdot 100^3\cdot 1^1 + \overset{6}{{^4C_2}}\cdot 100^2\cdot 1^2 + \overset{4}{{^4C_3}}\cdot 100^1\cdot 1^3 + \overset{1}{{^4C_4}}\cdot 100^0\cdot 1^4$

$= 100000000 + 4000000 + 60000 + 400 + 1$

$= 104060401$

(9) $(99)^5 = (100-1)^5 = [\underset{a}{100} + \underset{b}{(-1)}]^5$

$= {}^5C_0 \cdot (100)^5 (-1)^0 + {}^5C_1 \cdot 100^4 \cdot (-1)^1 + {}^5C_2 \cdot 100^3 \cdot \overbrace{(-1)^2}^{+1} + {}^5C_3 \cdot 100^2 \cdot (-1)^3$

$+ {}^5C_4 \cdot (100)^1 \cdot (-1)^4 + {}^5C_5 \cdot 100^0 \cdot (-1)^5$

$= 10000000000 - 500000000 + 1000000 - 100000$

$+ 500 - 1$

$$\begin{array}{r} 10010000500 \\ -\ 500100001 \\ \hline 9509900499 \end{array}$$

$= 9509900499$

$$(a+b)^n = {}^nC_0 \cdot a^n \cdot b^0 + {}^nC_1 \cdot a^{n-1} \cdot b^1 + {}^nC_2 \cdot a^{n-2} \cdot b^2 + \cdots + {}^nC_n \cdot a^0 \cdot b^n$$

$${}^nC_r = \frac{n!}{(n-r)!\, r!}$$

$\boxed{Q.10}$ Find larger Number. $\boxed{(1.1)^{10000}}$ or $\boxed{1000}$

$${}^nC_1 = n$$

$$(1.1)^{10000} = (1+0.1)^{10000}$$

$$= {}^{10000}C_0 \cdot 1^{10000} \cdot (0.1)^0 + {}^{10000}C_1 \cdot \underbrace{1^{9999}} \cdot \underbrace{(0.1)^1} + \text{other positive Terms.}$$

$$1 \cdot 1 \cdot 1 \qquad 10000 \times 1 \times 0.1 = 1000$$

$$= 1 + 1000 + \cancel{001} \text{ other positive Terms.}$$

$$= \cancel{1000}\ 1001 + \text{other positive Terms} > 1000$$

$$\boxed{(1.1)^{10000} > 1000}$$

$\boxed{Q.11}$ $(a+b)^4 - (a-b)^4 = ?$ $\quad (\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4 = ?$

$$(a+b)^4 = \cancel{{}^4C_0 \cdot a^4 \cdot b^0} + {}^4C_1 \cdot a^3 \cdot b^1 + \cancel{{}^4C_2 \cdot a^2 b^2} + {}^4C_3 \cdot a^1 b^3 + \cancel{{}^4C_4 \cdot a^0 b^4}$$

$$-\quad (a-b)^4 = \cancel{{}^4C_0 \cdot a^4 \cdot b^0} - {}^4C_1 \cdot a^3 \cdot b^1 + \cancel{{}^4C_2 \cdot a^2 b^2} - {}^4C_3 \cdot a^1 \cdot b^3 + \cancel{{}^4C_4 \cdot a^0 \cdot b^4}$$

$$\quad - \quad + \quad - \quad + \quad -$$

$$(a+b)^4 - (a-b)^4 = 2\left({}^4C_1 \cdot a^3 \cdot b^1 + {}^4C_3\, a^1 \cdot b^3\right)$$

${}^4C_1 = 4$

${}^4C_3 = 4$

$$(a+b)^4 - (a-b)^4 = 2(4a^3 b + 4a \cdot b^3)$$

$$\boxed{(a+b)^4 - (a-b)^4 = 8ab(a^2+b^2)}$$

put $a = \sqrt{3}$, $b = \sqrt{2}$

$$(\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4 = 8 \cdot \sqrt{3} \cdot \sqrt{2}\left[(\sqrt{3})^2 + (\sqrt{2})^2\right]$$

$$= 8\sqrt{6} \cdot (\underbrace{3+2}_{5})$$

$$= 40\sqrt{6}$$

$\boxed{Q.12}$ $(x+1)^6 + (x-1)^6 = ?$ $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$

$(x+1)^6 = {}^6C_0 . x^6 . 1^0 + \cancel{{}^6C_1 . x^5 . 1^1} + {}^6C_2 . x^4 . 1^2 + \cancel{{}^6C_3 . x^3 . 1^3} + {}^6C_4 . x^2 . 1^4 + \cancel{{}^6C_5 . x^1 . 1^5} + {}^6C_6 . x^0 . 1^6$

$(x-1)^6 = {}^6C_0 . x^6 . 1^0 - \cancel{{}^6C_1 . x^5 . 1^1} + {}^6C_2 . x^4 . 1^2 - \cancel{{}^6C_3 . x^3 . 1^3} + {}^6C_4 . x^2 . 1^4 - \cancel{{}^6C_5 . x^1 . 1^5} + {}^6C_6 . x^0 . 1^6$

+

$(x+1)^6 + (x-1)^6 = 2\left(\underbrace{{}^6C_0}_{1} . x^6 + \underbrace{{}^6C_2}_{15} . x^4 + \underbrace{{}^6C_4}_{15} . x^2 + \underbrace{{}^6C_6}_{1} .\right)$

${}^6C_2 = \frac{6!}{4!\,2!} = 15$

$= \frac{\cancel{6}^{3} \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{2} \times 1}$

$\boxed{(x+1)^6 + (x-1)^6 = 2(x^6 + 15x^4 + 15x^2 + 1)}$

$\left((2)^{\frac{1}{\cancel{2}}}\right)^{\cancel{6}\,3} = 2^3$

Put $x = \sqrt{2}$

$(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6 = 2\left((\sqrt{2})^6 + 15(\sqrt{2})^4 + 15(\sqrt{2})^2 + 1\right)$

$= 2(2^3 + 15 . 2^2 + 15 . 2 + 1)$

$= 2(8 + \underline{60} + \underline{30} + 1)$

$= 2 \times 99 = 198$

$$(a+b)^n = {}^nC_0 \cdot a^n \cdot b^0 + {}^nC_1 \cdot a^{n-1} \cdot b^1 + {}^nC_2 \cdot a^{n-2} \cdot b^2 + \dots + {}^nC_n \cdot a^0 \cdot b^n$$

$$= \sum_{r=0}^{r=n} \boxed{{}^nC_r \cdot a^{n-r} \cdot b^r}$$

variable = r

Division Algorithm ⟹ Dividend = Divisor × quotient + Remainder.

Q.13 Dividend = $9^{n+1} - 8n - 9$, Divisor = 64,

Prove ⤵
Remainder = 0

$$\text{Dividend} = 9^{n+1} - 8n - 9 = 64 \times \boxed{} + \boxed{0}$$

$64 = 8^2$

9

100

$9 \to 10$

$$= (1+8)^{n+1} - 8n - 9$$

Expand → By Binomial Theorem.

$$\text{Dividend} = 9^{n+1} - 8n - 9$$

$$^{n+1}C_0 = 1 \qquad ^{n+1}C_1 = n+1$$

$$64 \times 8$$

$$= (1+8)^{n+1} - 8n - 9$$

$$64 = 8^2$$

$$8^3 = 512$$

$$= {}^{n+1}C_0 \cdot 1^{n+1} \cdot 8^0 + {}^{n+1}C_1 \cdot 1^n \cdot 8^1 + {}^{n+1}C_2 \cdot 1^{n-1} \cdot 8^2 + {}^{n+1}C_3 \cdot 1^{n-2} \cdot 8^3 + \dots + {}^{n+1}C_{n+1} \cdot 1^0 \cdot 8^{n+1}$$

$$- 8n - 9$$

$$= 1.1.1 + (n+1).1.8 + \underbrace{{}^{n+1}C_2 . 1.64 + {}^{n+1}C_3 . 1.64.8 + \dots + {}^{n+1}C_{n+1} . 1.64.8^{n-1}}_{\text{64 Common.}}$$

$$-8n-9$$

$$= \cancel{1} + \cancel{8n} + \cancel{8} + 64.\left({}^{n+1}C_2 + {}^{n+1}C_3 . 8 + \dots + {}^{n+1}C_{n+1} . 8^{n-1}\right)$$

$$- \cancel{8n} - \cancel{9}$$

$$= 64.\overbrace{\left({}^{n+1}C_2 + {}^{n+1}C_3 \cdot 8 + \dots + {}^{n+1}C_{n+1} \cdot 8^{n-1}\right)}^{\text{Integer}} + 0$$

Dividend = $\underset{\uparrow\ \text{Divisor}}{64} \times$ Quotient + $\underset{\uparrow\ \text{Remainder}}{0}$

$\therefore 9^{n+1} - 8n - 9$ is divisible by 64 ($\because$ remainder = 0)

Basics of Exercise 7.2

$a^0 = a^{n-n}$

$$(a+b)^n = \underbrace{{}^nC_0 \cdot a^n \cdot b^0}_{1st} + \underbrace{{}^nC_1 \cdot a^{n-1} \cdot b^1}_{2^{nd}} + \underbrace{{}^nC_2 \cdot a^{n-2} \cdot b^2}_{3^{rd}} + \cdots + \underbrace{{}^nC_n \cdot a^0 \cdot b^n}_{(n+1)^{th}}$$

Total no. of Terms $= n+1$

$$r^{th} \text{ Term} = {}^nC_{r-1} \cdot a^{n-(r-1)} \cdot b^{r-1} = {}^nC_{r-1} \cdot a^{n-r+1} \cdot b^{r-1}$$

complicated expression.

$$\text{General Term} = T_{r+1} = \boxed{(r+1)^{th} \text{ term} = {}^nC_r \times a^{n-r} \times b^r}$$

use → coeff. = ?

→ particular Term =

$$T_7 = T_{6+1} = {}^nC_6 \cdot a^{n-6} \cdot b^6$$

coeff.

Middle Term:

Tough → 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 → Even

easy → $\underbrace{1, 2, 3, 4, 5}_{5 \text{ terms}}, \underset{\text{middle}}{6}, \underbrace{7, 8, 9, 10, 11}_{5 \text{ terms}}$ → odd No. of Terms

Middle Term in $(a+b)^n$ ← power

	No. of Terms $=(n+1)$	No. of Middle terms	which terms are the middle terms. (Rank) ← seat No.
$n =$ Even	$n+1 =$ odd	1	$\left(\frac{(n+1)+1}{2}\right)^{th}$ term $= \left(\frac{n}{2}+1\right)^{th}$ term
$n =$ odd	$n+1 =$ even	2	$\left(\frac{n+1}{2}\right)^{th}$ term, $\left(\frac{n+1}{2}+1\right)^{th}$ term

e.g. middle term in $(a+b)^{11}$ $n=11$ No. of midde terms $= 2$

middle Term $\Rightarrow T_{\frac{11+1}{2}} = T_6 = {}^{11}C_5 \cdot a^{11-5} \cdot b^5$

Middle term $\Rightarrow T_{\left(\frac{11+1}{2}+1\right)} = T_7 = T_{6+1} = {}^{11}C_6 \cdot a^{11-6} \cdot b^6$

Note:

Middle Term in the expansion of $\left(x+\frac{1}{x}\right)^{2n}$, $x \neq 0$ → $a = x,\ b = \frac{1}{x}$

Power = $2n$ = Even.

Middle Term = $\left(\frac{2n}{2}+1\right)^{th}$ term = $(n+1)^{th}$ term = T_{n+1}

$$T_{n+1} = {}^{2n}C_n \cdot \underbrace{a^{2n-n} \cdot b^{n} = {}^{2n}C_n \cdot a^n \cdot b^n}$$

middle term in $\left(x+\frac{1}{x}\right)^{2n}$ $= T_{n+1} = {}^{2n}C_n \cdot \cancel{(x)^n} \cdot \cancel{\left(\frac{1}{x}\right)^n}$

$$= {}^{2n}C_n \cdot x^0$$

$$= \underbrace{{}^{2n}C_n = \text{term independent of '}x\text{'}}_{x^0}$$

Power of variable $x = 0$

General Term in the expansion of $(a+b)^n$

$$T_{r+1} = (r+1)^{th} \text{ Term} = {}^nC_r \cdot a^{n-r} \cdot b^r$$

$r=0$, $T_1 = {}^nC_0 \cdot a^n \cdot b^0$

$r=1$, $T_2 = {}^nC_1 \cdot a^{n-1} \cdot b^1$

use → Coefficient ✓
→ General Term ✓
→ Particular Term ✓

Q.1 $\underbrace{\text{coefficient of } x^5}_{\text{II}}$ in $\underbrace{(\overset{a}{x}+\overset{b}{3})}_{\text{I}}^{8} \to n=8$

I General Term

General Term = $\boxed{T_{r+1} = {}^nC_r \cdot a^{n-r} \cdot b^r}$

$T_{r+1} = {}^8C_r \cdot \underset{\text{variable}}{x^{8-r}} \cdot 3^r$

$T_{r+1} = \underbrace{{}^8C_r \cdot 3^r}\cdot x^{8-r}$

$T_{r+1} = {}^8C_r \cdot 3^r \cdot x^{8-r}$

For x^5 $\quad 5 = 8-r$

Comparing $x^{(8-r)}$ & $x^{(5)}$

$\Rightarrow r = 8-5$

$\boxed{r=3}$

Put $r=3$ in General Term

$T_{3+1} = {}^8C_3 \cdot 3^3 \cdot x^{8-3}$

$T_4 = \underbrace{{}^8C_3 \cdot 3^3} \cdot x^5$

Coeff. of $x^5 = {}^8C_3 \cdot 3^3$

$= \frac{8!}{5!\,3!} \times 27$

$=$ ______

Q.2 Coefficient of $a^5 b^7$ in $(a-2b)^{12}$

$(A+B)^n$

General Term in the expansion of $(a-2b)^{12}$ =

$T_{r+1} = {}^nC_r \cdot A^{n-r} \cdot B^r$

$T_{r+1} = {}^{12}C_r \cdot a^{12-r} \cdot (-2b)^r$

$$\boxed{T_{r+1} = \underbrace{{}^{12}C_r \cdot (-2)^r}_{\text{coeff.}} \cdot \underbrace{a^{12-r} \cdot b^r}_{\text{Variable}}}$$ General Term

~~For~~ For $a^5 . b^7$

$a^{12-r} \cdot b^{\boxed{r}} = a^5 \cdot b^{\boxed{7}}$

(a) $12 - r = 5$

$12 - 5 = r$

$r = 7$

(b) $r = 7$

Put $r = 7$ in General Term:

$T_{r+1} = T_{7+1} = {}^{12}C_7 \cdot (-2)^7 \cdot a^{12-7} \cdot b^7$

$T_8 = \underbrace{{}^{12}C_7 \cdot (-128)}\cdot a^5 \cdot b^7$

Coeff. of $a^5 . b^7 = {}^{12}C_7 \times (-128)$

Coeff of $a^1 . b^2 = 0$

$0 \times a^1 b^2$

③ General Term in the expansion of $(x^2 - y)^6$

$(x^2-y)^6$: $x^2 \to a$, $-y \to b$, $6 \to n$

General

$$T_{r+1} = {}^nC_r \cdot a^{n-r} \cdot b^r$$

$-y = -1 \times y$

$$T_{r+1} = {}^6C_r \cdot (x^2)^{6-r} \cdot (-y)^r$$

$$T_{r+1} = {}^6C_r \cdot x^{12-2r} \cdot [(-1) \cdot y]^r$$

$$\boxed{T_{r+1} = {}^6C_r \cdot (-1)^r \cdot x^{12-2r} \cdot y^r}$$

$\cancel{x^{24-2r+r}}$

④ General Term in the expansion of $(x^2 - yx)^{12}$, $x \neq 0$

$x^2 \to a$, $-yx \to b$, $12 \to n$

$$T_{r+1} = {}^nC_r \cdot a^{n-r} \cdot b^r$$

General Term

$$T_{r+1} = {}^{12}C_r \cdot (x^2)^{12-r} \cdot (-yx)^r$$

$$T_{r+1} = {}^{12}C_r \cdot x^{24-2r} \cdot (-1)^r \cdot y^r \cdot x^r$$

$$\boxed{T_{r+1} = (-1)^r \cdot {}^{12}C_r \cdot x^{24-r} \cdot y^r}$$

General Term $T_{r+1} = {}^{n}C_{r} \cdot a^{n-r} \cdot b^{r}$

in $(a+b)^n$

Q.5 4^{th} Term in $(x-2y)^{12}$ $\rightarrow n=12$

$a = x$, $b = -2y$

$T_4 = T_{3+1} = {}^{12}C_3 \cdot (x)^{12-3} \cdot (-2y)^3$

$4 = r+1$
$r = 3$

$T_4 = {}^{12}C_3 \cdot x^9 \cdot (-2)^3 \cdot y^3$

$T_4 = \left(\frac{12!}{9!\,3!}\right) \times (-8) \cdot x^9 \cdot y^3$

$T_4 = \frac{\cancel{12}^{2} \times 11 \times 10 \times \cancel{9!}}{\cancel{9!} \times \cancel{3 \times 2 \times 1}} \times (-8) \cdot x^9 \cdot y^3$

$T_4 = -1760\, x^9 \cdot y^3$

Q.6 13^{th} term in $\left(9x - \frac{1}{3\sqrt{x}}\right)^{n}$, $x \neq 0$ $\rightarrow 18$

$a = 9x$, $b = -\frac{1}{3\sqrt{x}}$

$T_{13} = T_{12+1} = {}^{18}C_{12} \cdot (9x)^{18-12} \cdot \left(\frac{-1}{3\sqrt{x}}\right)^{12}$

$r = 12$

$T_{13} = {}^{18}C_{12} \cdot 9^6 \cdot x^6 \cdot \frac{1}{3^{12} \cdot (\sqrt{x})^{12}}$

$T_{13} = {}^{18}C_{12} \cdot \cancel{9^6}^{\,12} \cdot \cancel{x^6} \cdot \frac{1}{\cancel{3^{12}} \cdot \cancel{x^6}}$

$(\sqrt{x})^{12} = x^6$

$T_{13} = {}^{18}C_{12}$

$T_{13} = \frac{18!}{6! \times 12!}$

$\boxed{Q.7}$ Middle Term in the expansion of $\left(3-\frac{x^3}{6}\right)^7$

Index = 7 = odd
(Power)
(Exponent)

No. of terms = 8 = Even

$\underbrace{T_1+T_2+T_3+T_4}+\underbrace{T_5+T_6+T_7+T_8}$

middle Terms. = T_4, T_5

Middle Terms → $T_{\frac{n+1}{2}}, \; T_{\frac{n+1}{2}+1}$

~~Index~~ Index = n = odd

n = 7

middle terms → $T_{\frac{7+1}{2}}, \; T_{\frac{7+1}{2}+1}$

$= T_4, T_5$

$$T_4 = T_{3+1} = {}^7C_3 \cdot (3)^{7-3} \cdot \left(-\frac{x^3}{6}\right)^3$$

$$= -{}^7C_3 \cdot 3^4 \cdot \frac{x^9}{6^3}$$

$(6)^3 = (2\times3)^3$

$$T_4 = -{}^7C_3 \cdot \cancel{3^4}^{\,3} \cdot \frac{x^9}{2^3 \cdot \cancel{3^3}}$$

$$T_4 = -\frac{{}^7C_3}{2^3} \cdot 3x^9$$

$2^3 = 8$

~~Q.8 Middle Term in $\left(\frac{x}{3}+9y\right)^{10}$~~

$$T_5 = T_{4+1} = {}^7C_4 \cdot 3^{7-4} \cdot \left(-\frac{x^3}{6}\right)^4$$

$$= +\,{}^7C_4 \cdot 3^3 \cdot \frac{x^{12}}{6^4}$$

$$= {}^7C_4 \cdot \cancel{3^3} \cdot \frac{x^{12}}{2^4 \cdot \cancel{3^4}_{\,3}}$$

$$= \frac{{}^7C_4 \cdot x^{12}}{2^4 \cdot 3}$$

$2^4 = 16$

${}^7C_4 = {}^7C_3 = \frac{7!}{3!\,4!}$

${}^nC_r = {}^nC_{n-r}$

Q.8 middle Term in $\left(\frac{x}{3} + 9y\right)^{10}$

$n = 10 = \text{index} = \text{Even}$.

No. of Terms $= 11 = \text{odd}$

middle term.

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5 \quad T_6 \quad T_7 \quad T_8 \quad T_9 \quad T_{10} \quad T_{11}$

middle term $= T_{\frac{n}{2}+1} = T_{\frac{10}{2}+1} = T_{5+1} = T_6$

$n = \text{Even}$

$T_6 = T_{5+1} = {}^{10}C_5 \cdot \left(\frac{x}{3}\right)^{10-5} \cdot (9y)^5$

r

${}^{10}C_5 = \frac{10!}{5!\,5!}$

$T_6 = {}^{10}C_5 \cdot \left(\frac{x}{3}\right)^5 \cdot 9^5 \cdot y^5$

$T_6 = {}^{10}C_5 \cdot \frac{x^5}{\cancel{3^5}} \cdot \cancel{9}^{\,3^5 \, 10} \cdot y^5$

$3^5 = 243$

$\boxed{T_6 = {}^{10}C_5 \cdot 3^5 \cdot x^5 \cdot y^5}$

General Term in $(A+B)^n$

$(r+1)^{th}$ term $= T_{r+1} = {}^nC_r \cdot A^{n-r} \cdot B^r$

Q.9 $(1+a)^{m+n}$

To prove $\boxed{\text{Coeff. of } a^m = \text{coeff. of } a^n}$

Proof:

General Term of $(1+a)^{m+n}$

$$T_{r+1} = {}^{m+n}C_r \cdot (1)^{m+n-r} \cdot (a)^r$$

$$\boxed{T_{r+1} = {}^{m+n}C_r \; a^r}$$ ← General Term

coeff. of $a^r = {}^{m+n}C_r$

Similarly coeff. of $a^m = {}^{m+n}C_m$ ✓

coeff. of $a^n = {}^{m+n}C_n$ ✓

Coeff. of $a^m = {}^{m+n}C_m$

Property

$\boxed{{}^nC_r = {}^nC_{n-r}}$

$= {}^{m+n}C_{(m+n)-(m)}$

$= {}^{m+n}C_n$

$=$ Coeff. of a^n

Q.10 (coeff. of $(r-1)^{th}$ term) : (coeff. of r^{th} Term) : (coeff. of $(r+1)^{th}$ Term) $= 1:3:5$

in $(x+1)^n$, $x \to a$, $1 \to b$

$\Rightarrow$ $(r+1)^{th}$ Term $= T_{r+1} = {}^nC_r \cdot a^{n-r} \cdot b^r$

(General Term)

$$\Rightarrow T_{r+1} = {}^nC_r \cdot x^{n-r} \cdot 1^r = \underbrace{{}^nC_r}_{\text{coeff.}} \cdot \underbrace{x^{n-r}}_{\text{variable}}$$

~~r^{th} Term $= T_r = {}^nC_{r-1}$~~

Coeff. of $(r+1)^{th}$ Term $= {}^nC_r$ ✓

coeff. of r^{th} term $= {}^nC_{r-1}$ ✓

coeff. of $(r-1)^{th}$ term $= {}^nC_{r-2}$ ✓

According to Question:

$${}^nC_{r-2} : {}^nC_{r-1} : {}^nC_r = 1:3:5$$

to be continued.

Q.11

To Prove

$\underbrace{\text{coeff. of } x^n \text{ in } (1+x)^{2n}} = 2 \times \underbrace{\text{coeff. of } x^n \text{ in } (1+x)^{2n-1}}$

$${}^{2n}C_n = 2 \times {}^{2n-1}C_n$$

General Term $= T_{r+1} = {}^{n}C_r \cdot a^{n-r} \cdot b^r$

$(a+b)^n$

$a=1,\ b=x$

General Term of $(1+x)^{2n}$

$T_{r+1} = {}^{2n}C_r \cdot (1)^{2n-r} \cdot x^r$

$T_{r+1} = {}^{2n}C_r \cdot x^r$

coeff. of $x^r = {}^{2n}C_r$

Coeff. of $x^n = {}^{2n}C_n$ ✓

General Term of $(1+x)^{2n-1}$

$t_{r+1} = {}^{2n-1}C_r \cdot (1)^{2n-1-r} \cdot x^r$

$t_{r+1} = {}^{2n-1}C_r \cdot x^r$

Coeff. of $x^r = {}^{2n-1}C_r$

Coeff. of $x^n = {}^{2n-1}C_n$

... to be continued

$^nC_{r-2} : {}^nC_{r-1} = 1:3$

$\Rightarrow \dfrac{^nC_{r-2}}{^nC_{r-1}} = \dfrac{1}{3}$

$\Rightarrow \dfrac{\dfrac{\cancel{n!}}{(n-r+2)\cancel{(n-r+1)!}\,\cancel{(r-2)!}}}{\dfrac{\cancel{n!}}{\cancel{(n-r+1)!}\,(r-1)\cancel{(r-2)!}}} = \dfrac{1}{3}$

$(r-1)! = (r-1).\cancel{(r-2)!}$

$(n-r+2)! = (n-r+2).(n-r+1)!$

$\Rightarrow \dfrac{\dfrac{1}{(n-r+2)}}{\dfrac{1}{(r-1)}} = \dfrac{1}{3}$

$\Rightarrow \dfrac{r-1}{n-r+2} = \dfrac{1}{3}$

$\Rightarrow 3r - 3 = n - r + 2$

$\Rightarrow \boxed{4r - 5 = n}$ —(1)

similarly,

$^nC_{r-1} : {}^nC_r = 3:5$

$\Rightarrow \dfrac{^nC_{r-1}}{^nC_r} = \dfrac{3}{5}$

$\Rightarrow \dfrac{\dfrac{\cancel{n!}}{(n-r+1)\cancel{(n-r)!}\,\cancel{(r-1)!}}}{\dfrac{\cancel{n!}}{\cancel{(n-r)!}\,\cancel{(r-1)!}\,r}} = \dfrac{3}{5}$

$\Rightarrow \dfrac{\dfrac{1}{n-r+1}}{\dfrac{1}{r}} = \dfrac{3}{5}$

$\Rightarrow \dfrac{r}{n-r+1} = \dfrac{3}{5} \Rightarrow 5r = 3n - 3r + 3$

$\Rightarrow 8r - 3 = 3n$ —(2)

By eq^n (1) & (2)

$\cancel{8r} - 3 = 3n$

$\underline{-\cancel{8r} \mp 10 = -2n}$

$\boxed{7 = n}$

$n = 7$ ✓

$r = 3$ ✓

Now we have to prove that $\boxed{{}^{2n}C_n = 2 \cdot {}^{2n-1}C_n}$

$$\frac{(2n)!}{n!\, n!}$$

$$\text{RHS} = 2 \cdot {}^{2n-1}C_n$$

$$= 2 \times \frac{(2n-1)!}{(n-1)!\, n!} \times \frac{n}{n}$$

$\cancel{2n-1-n = n-1}$

$(n-1)! \times n = n!$

$$= \frac{(2n) \cdot (2n-1)!}{n!\, n!}$$

$$= \frac{(2n)!}{n!\, n!}$$

$$= {}^{2n}C_n = \text{LHS}.$$

$\boxed{Q.12}$ Find m

coeff. of x^2 in $(1+x)^m = 6$ $\quad m=n$

$\quad a \quad b$

General Term $T_{r+1} = {}^nC_r \cdot a^{n-r} \cdot b^r$

$T_{r+1} = {}^mC_r \cdot (1)^{m-r} \cdot x^r$

$\quad 1$

$$\boxed{T_{r+1} = {}^mC_r \cdot x^r}$$

coeff. of $x^r = {}^mC_r$

Coeff. of $x^2 = {}^mC_2 = 6$

$\Rightarrow \dfrac{m!}{(m-2)!\,2!} = 6$

$\Rightarrow \dfrac{m \times (m-1) \times \cancel{(m-2)!}}{\cancel{(m-2)!} \times 2 \times 1} = 6$

$\Rightarrow \dfrac{m^2-m}{2} = 6$

$m^2 - m = 12$

$\Rightarrow m^2 - m - 12 = 0$

$\Rightarrow m^2 - 4m + 3m - 12 = 0$

$\Rightarrow m(m-4) + 3(m-4) = 0$

$\Rightarrow (m-4)(m+3) = 0$

$m = 4$ ✓ $\qquad m = -3$ ✗

Positive value of $m = 4$

Miscellaneous Exercise 7.3

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

$$\boxed{{}^nC_0 = 1} \qquad \boxed{{}^nC_1 = n} \qquad {}^nC_2 = \frac{n!}{\cancel{(n-2)!}\,2!} = \frac{n(n-1)}{2}$$

$n! \to n.(n-1).\cancel{(n-2)!}$

Q.1 In the expansion of $(a+b)^n = \underbrace{{}^nC_0 . a^n . b^0}_{T_1} + \underbrace{{}^nC_1 . a^{n-1} . b^1}_{T_2} + \underbrace{{}^nC_2 . a^{n-2} . b^2}_{T_3} + \ldots$

$$\boxed{T_1 = a^n} \qquad \boxed{T_2 = n . a^{n-1} . b} \qquad \boxed{T_3 = \frac{n(n-1)}{2} . a^{n-2} . b^2}$$

First Term $= T_1 = 729 = a^n$ —(1)

second Term $= T_2 = 7290 = n.a^{n-1}.b$ —(2)

Third Term $= T_3 = 30375 = \underbrace{\frac{n(n-1)}{2} . a^{n-2} . b^2}$ —(3)

By $\frac{eq^n (1)}{eq^n (2)}$; $\frac{\cancel{729}}{\cancel{7290}_{10}} = \frac{\cancel{a^n}^{\,a}}{n . \cancel{a^{n-1}} . b}$

$\Rightarrow \frac{1}{10} = \frac{a}{nb}$

$\Rightarrow \boxed{nb = 10.a}$ —(4)

By $\frac{eq^n (2)}{eq^n (3)}$: $\frac{7290}{30375} = \frac{\cancel{n} . \cancel{a^{n-1}}^{\,a} . \cancel{b}}{\frac{\cancel{n}(n-1)}{2} . \cancel{a^{n-2}} . \cancel{b^2}^{\,b}}$

$\Rightarrow \frac{\cancel{3.3.3.3.3.3}.3.\cancel{2}\times\cancel{5}}{\cancel{3.3.3.3.3}.\cancel{5}.5.5} = \frac{\cancel{2}a}{(n-1)b}$

$\Rightarrow \frac{3}{25} = \frac{a}{(n-1)b}$

$\Rightarrow \boxed{3(n-1)b = 25\,a}$ —(5)

By $\dfrac{eq^n (4)}{eq^n (5)}$:→ $\dfrac{n\cancel{b}}{3(n-1)\cancel{b}} = \dfrac{^{2}\cancel{10}.\cancel{a}}{_{5}\cancel{25}.\cancel{a}}$

$\Rightarrow \dfrac{n}{3(n-1)} = \dfrac{2}{5}$

$\Rightarrow 5n = 6n - 6$

$\Rightarrow$

$6 = 6n - 5n$

$\Rightarrow \boxed{n = 6}$

By $eq^n (1)$: $a^n = 729$

$\Rightarrow a^6 = 3^6$

$\Rightarrow \boxed{a = 3}$

$a = 3$

$n = 6$

By $eq^n (2)$:

$7290 = n \cdot a^{n-1} \cdot b$

$\Rightarrow 7290 = 6 \cdot 3^5 \cdot b$

$\Rightarrow \cancel{3}^{6} \times \cancel{2} \times 5 = \cancel{2} \times \cancel{3} \times \cancel{3}^{5} \cdot b$

$\boxed{b = 5}$

$\boxed{Q.2}$ $(3+ax)^{9} \leftarrow n$ coeff. of x^2 = coeff. of x^3 — ③

Given.

General Term:

$$T_{r+1} = {}^{n}C_{r}\cdot a^{n-r}\cdot b^{r} \leftarrow (a+b)^{n}$$

$$T_{r+1} = {}^{9}C_{r}\cdot 3^{9-r}\cdot (ax)^{r}$$

$$T_{r+1} = \underbrace{{}^{9}C_{r}\cdot 3^{9-r}\cdot a^{r}}\cdot x^{r}$$

${}^{9}C_{2}$

coeff. of x^{r} = ${}^{9}C_{r}\cdot 3^{9-r}\cdot a^{r}$

coeff. of x^{2} = ${}^{9}C_{2}\cdot 3^{7}\cdot a^{2}$ — ①

coeff. of x^{3} = ${}^{9}C_{3}\cdot 3^{6}\cdot a^{3}$ — ②

$a = \frac{9}{7}$

By Question: ${}^{9}C_{2}\cdot \cancel{3^{7}}^{\,3}\cdot \cancel{a^{2}} = {}^{9}C_{3}\cdot \cancel{3^{6}}\cdot \cancel{a^{3}}^{\,a}$

$$\Rightarrow \frac{{}^{9}C_{2}\times 3}{{}^{9}C_{3}} = a = \frac{\frac{9!}{7!\,2!}\times 3}{\frac{9!}{6!\,3!}} = \frac{\frac{3}{7}}{\frac{1}{3}} = \frac{9}{7}$$

$\boxed{Q.3}$ Coeff. of x^5 in the product $(1+2x)^6 . (1-x)^7$

$(1+2x)^6 = [{}^6C_0 \cdot (2x)^0 + {}^6C_1 \cdot (2x)^1 + {}^6C_2 \cdot (2x)^2 + {}^6C_3 \cdot (2x)^3 + {}^6C_4 (2x)^4 + {}^6C_5 (2x)^5 + {}^6C_6 (2x)^6]$

$(1-x)^7 = [{}^7C_0 (-x)^0 + {}^7C_1 (-x)^1 + {}^7C_2 (-x)^2 + {}^7C_3 (-x)^3 + {}^7C_4 (-x)^4 + {}^7C_5 (-x)^5 + {}^7C_6 (-x)^6 + {}^7C_7 (-x)^7]$

$\times$

$(1+2x)^6 \times (1-x)^7 = [\quad - \quad] \times [\quad - \quad]$

Coeff. of x^5 in $(1+2x)^6 . (1-x)^7$ = coeff. of x^5 in $[\quad] \times [\quad]$

$= \underbrace{({}^6C_0 . 2^0)(-{}^7C_5)} + \underbrace{({}^6C_1 . 2)({}^7C_4)} + \underbrace{({}^6C_2 . 2^2)(-{}^7C_3)} + \underbrace{({}^6C_3 . 2^3).({}^7C_2)}$

$+ \underbrace{({}^6C_4 . 2^4) . (-{}^7C_1)} + \underbrace{({}^6C_5 . 2^5).({}^7C_0)} = 171$ Answer

---

${}^nC_r \to \frac{n!}{(n-r)!\,r!}$

${}^6C_0 = 1$, $\underset{{}^6C_5}{{}^6C_1 = 6}$, $\underset{{}^6C_4}{{}^6C_2 = 15}$, ${}^6C_3 = 20$

${}^7C_0 = 1$

${}^7C_1 = 7$

${}^7C_2 = 21 = {}^7C_5$

${}^7C_3 = 35 = {}^7C_4$

${}^nC_r = {}^nC_{n-r}$

Q.4 Prove that $(a-b)$ is a factor of $a^n - b^n$

2-methods

Binomial.

PMI

ch-4

$a = a-b+b$

$$a^n - b^n = (a-b+b)^n - b^n$$

$$= [\underset{A}{(a-b)} + \underset{B}{(b)}]^n - b^n$$

${}^nC_n = 1 = {}^nC_0$

$$= [{}^nC_0 \cdot (a-b)^n \cdot b^0 + {}^nC_1 \cdot (a-b)^{n-1} \cdot b^1 + {}^nC_2 \cdot (a-b)^{n-2} \cdot b^2 \dots$$
$$\dots + {}^nC_{n-1} \cdot (a-b)^1 \cdot b^{n-1} + {}^nC_n \cdot (a-b)^0 \cdot b^n] - b^n$$

$$= [(a-b)^n + {}^nC_1 \cdot (a-b)^{n-1} \cdot b + \dots + {}^nC_{n-1} \cdot (a-b)^1 \cdot b^{n-1} + \cancel{b^n}] - \cancel{b^n}$$

$$= (a-b)\{(a-b)^{n-1} + {}^nC_1 \cdot (a-b)^{n-2} \cdot b + \dots + {}^nC_{n-1} \cdot b^{n-1}\}$$

$a, b, n \rightarrow$ integers

$$a^n - b^n = (a-b) \cdot \{\text{Integer}\}$$

N

factor

$\boxed{Q.5}$ Evaluate $(\sqrt{3}+\sqrt{2})^6 - (\sqrt{3}-\sqrt{2})^6$

$$(\sqrt{3}+\sqrt{2})^6 = {}^6C_0 \cdot \cancel{(\sqrt{3})^6}(\sqrt{2})^0 + {}^6C_1(\sqrt{3})^5(\sqrt{2})^1 + \cancel{{}^6C_2(\sqrt{3})^4(\sqrt{2})^2} + {}^6C_3(\sqrt{3})^3(\sqrt{2})^3 + \cancel{{}^6C_4(\sqrt{3})^2(\sqrt{2})^4}$$

$$+ {}^6C_5(\sqrt{3})^1(\sqrt{2})^5 + \cancel{{}^6C_6(\sqrt{3})^0(\sqrt{2})^6}$$

$$(\sqrt{3}-\sqrt{2})^6 = +\,\cancel{{}^6C_0(\sqrt{3})^6(\sqrt{2})^0} - {}^6C_1(\sqrt{3})^5(\sqrt{2})^1 + \cancel{{}^6C_2(\sqrt{3})^4(\sqrt{2})^2} - {}^6C_3(\sqrt{3})^3(\sqrt{2})^3 + \cancel{{}^6C_4(\sqrt{3})^2(\sqrt{2})^4}$$

$$-\ {}^6C_5(\sqrt{3})^1(\sqrt{2})^5 + \cancel{{}^6C_6(\sqrt{3})^0(\sqrt{2})^6}$$

$$- \quad - \quad + \quad - \quad + \quad - \quad + \quad -$$

Subtract

$$(\sqrt{3}+\sqrt{2})^6 - (\sqrt{3}-\sqrt{2})^6 = 2 \times \left({}^6C_1(\sqrt{3})^5(\sqrt{2})^1 + {}^6C_3(\sqrt{3})^3(\sqrt{2})^3 + {}^6C_5(\sqrt{3})^1(\sqrt{2})^5\right)$$

$$= 2 \cdot \left[6 \cdot 9\sqrt{3} \cdot \sqrt{2} + 20 \cdot 3\sqrt{3} \cdot 2\sqrt{2} + 6 \cdot \sqrt{3} \cdot 4\sqrt{2}\right)$$

$$= 2\left[54 \cdot \sqrt{6} + 120 \cdot \sqrt{6} + 24\sqrt{6}\right]$$

$$= 2 \times 198\sqrt{6} = 396 \cdot \sqrt{6}$$

$$\begin{array}{r} 54 \\ 120 \\ +\ 24 \\ \hline 198 \end{array}$$

Q.6 $(a^2+\sqrt{a^2-1})^4+(a^2-\sqrt{a^2-1})^4 = ?$

$^4C_2 = 6$, $^4C_0 = 1$, $^4C_4 = 1$

$$(a^2+\sqrt{a^2-1})^4 = {}^4C_0 (a^2)^4 \cdot (\sqrt{a^2-1})^0 + \cancel{{}^4C_1 \cdot (a^2)^3 \cdot (\sqrt{a^2-1})^1} + {}^4C_2 (a^2)^2 (\sqrt{a^2-1})^2 + \cancel{{}^4C_3 (a^2)^1 (\sqrt{a^2-1})^3}$$
$$+ {}^4C_4 (a^2)^0 (\sqrt{a^2-1})^4$$

$$(a^2-\sqrt{a^2-1})^4 = {}^4C_0 (a^2)^4 (\sqrt{a^2-1})^0 - \cancel{{}^4C_1 (a^2)^3 (\sqrt{a^2-1})^1} + {}^4C_2 (a^2)^2 (\sqrt{a^2-1})^2 - \cancel{{}^4C_3 (a^2)^1 (\sqrt{a^2-1})^3}$$
$$+ {}^4C_4 (a^2)^0 (\sqrt{a^2-1})^4$$

$+$ add

$$(a^2+\sqrt{a^2-1})^4 + (a^2-\sqrt{a^2-1})^4 = 2\{a^8 + 6 \cdot a^4 \cdot (a^2-1) + (a^2-1)^2\}$$
$$= 2\{a^8 + 6 \cdot a^6 - 6a^4 + a^4 - 2a^2 + 1\}$$
$$= 2\{a^8 + 6 \cdot a^6 - 5 \cdot a^4 - 2a^2 + 1\}$$
$$= 2 \cdot a^8 + 12a^6 - 10a^4 - 4a^2 + 2$$

$\boxed{Q.7}$ $(0.99)^5$ by only first 3 terms

(Approximate)

$(0.99)^5 = (1-0.01)^5$

$\approx \underbrace{{}^5C_0 \cdot (1)^5 (-0.01)^0}_{1^{st}} + \underbrace{{}^5C_1 \cdot (1)^4 (-0.01)^1}_{2^{nd}} + \underbrace{{}^5C_2 \cdot (1)^3 (-0.01)^2}_{3^{rd}}$ term (only) $\times \times$

$\approx$

$= 1 \cdot 1 \cdot 1 - 5 \times 1 \times 0.01 + (10 \times (1) \times (0.0001))$

$= 1 - 0.05 + 0.001$

$= 1.001 - 0.05$

$= 0.951$

$$\begin{array}{r} 1.001 \\ -\ 0.05 \\ \hline 0.951 \end{array}$$

Q. 8 $\left[\sqrt[4]{2} + \dfrac{1}{\sqrt[4]{3}}\right]^n \approx (a+b)^n$ $\quad \underline{a} = (2)^{\frac{1}{4}}$, $\underline{b} = \left(\dfrac{1}{3}\right)^{\frac{1}{4}} = \dfrac{1}{3^{1/4}}$

Index = power = n

No. of Terms = n+1

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5 \quad T_6 \; - - - - \quad T_{n-3} \quad T_{n-2} \quad T_{n-1} \quad T_n \quad T_{n+1}$

Fifth from beginning $\qquad$ Fifth from end.

ATQ: $\dfrac{T_5}{T_{n-3}} = \dfrac{\sqrt{6}}{1}$

General Term $T_{r+1} = {}^nC_r \cdot a^{n-r} \cdot b^r$

one less

$$\Rightarrow \frac{{}^nC_4 \cdot a^{n-4} \cdot b^4}{{}^nC_{n-4} \cdot a^{n-(n-4)} \cdot b^{n-4}} = \sqrt{6}$$

$$\Rightarrow \frac{\cancel{\left(\dfrac{n!}{(n-4)!\,4!}\right)} \times a^{n-4} \times b^4}{\cancel{\left(\dfrac{n!}{4!\,(n-4)!}\right)} \times a^4 \times b^{n-4}} = \sqrt{6}$$

$$\Rightarrow a^{n-8} \times b^{4-(n-4)} = \sqrt{6}$$

$$\Rightarrow a^{n-8} \times b^{8-n} = \sqrt{6}$$

$$\Rightarrow \frac{a^{n-8}}{b^{n-8}} = \sqrt{6}$$

$$\Rightarrow \frac{a^{n-8}}{b^{n-8}} = \sqrt{6}$$

$$\Rightarrow \left(\frac{a}{b}\right)^{n-8} = \sqrt{6}$$

$$\Rightarrow \left[\frac{\left(2^{1/4}\right)}{\left(\frac{1}{3^{1/4}}\right)}\right]^{n-8} = \sqrt{6}$$

$$\Rightarrow \left(2^{\frac{1}{4}} \cdot 3^{\frac{1}{4}}\right)^{n-8} = \sqrt{6}$$

$$\Rightarrow \left[(6)^{\frac{1}{4}}\right]^{n-8} = (6)^{1/2}$$

$$\Rightarrow 6^{\frac{n-8}{4}} = 6^{\frac{1}{2}}$$

$$\therefore \frac{n-8}{\cancel{4}_{2}} = \frac{1}{\cancel{2}} \Rightarrow n-8 = 2$$

$$\boxed{n = 10}$$

$\boxed{Q.9}$ $\left(\underbrace{1}_{a}+\underbrace{\frac{x}{2}-\frac{2}{x}}_{b}\right)^4$; $x \neq 0$

$= \underline{{}^4C_0 \cdot 1^4 \cdot \left(\frac{x}{2}-\frac{2}{x}\right)^0} + \underline{{}^4C_1 \cdot 1^3 \cdot \left(\frac{x}{2}-\frac{2}{x}\right)^1} + \underline{{}^4C_2 \cdot 1^2 \cdot \left(\frac{x}{2}-\frac{2}{x}\right)^2} + {}^4C_3 \cdot 1^1 \cdot \underline{\left(\frac{x}{2}-\frac{2}{x}\right)^3} + {}^4C_4 \cdot 1^0 \cdot \underline{\left(\frac{x}{2}-\frac{2}{x}\right)^4}$

$\left(\frac{x}{2}-\frac{2}{x}\right)^3 = {}^3C_0\left(\frac{x}{2}\right)^3\left(\frac{2}{x}\right)^0 - {}^3C_1\left(\frac{x}{2}\right)^2\left(\frac{2}{x}\right)^1 + {}^3C_2\left(\frac{x}{2}\right)^1\left(\frac{2}{x}\right)^2 - {}^3C_3\left(\frac{x}{2}\right)^0\left(\frac{2}{x}\right)^3$

$= 1 \cdot \frac{x^3}{8} \cdot 1 - 3 \cdot \frac{x^2}{\cancel{4}_2} \cdot \frac{\cancel{2}}{x} + 3 \cdot \frac{x}{\cancel{2}} \cdot \frac{\cancel{4}^2}{x^2} - 1 \cdot 1 \cdot \frac{8}{x^3}$

$= \frac{x^3}{8} - \frac{3x}{2} + \frac{6}{x} - \frac{8}{x^3}$

$\left(\frac{x}{2}-\frac{2}{x}\right)^4 = {}^4C_0\left(\frac{x}{2}\right)^4\left(\frac{2}{x}\right)^0 - {}^4C_1 \cdot \left(\frac{x}{2}\right)^3 \cdot \left(\frac{2}{x}\right)^1 + {}^4C_2 \cdot \left(\frac{x}{2}\right)^2 \cdot \left(\frac{2}{x}\right)^2 - {}^4C_3\left(\frac{x}{2}\right)^1 \cdot \left(\frac{2}{x}\right)^3$

$+ {}^4C_4 \left(\frac{x}{2}\right)^0 \cdot \left(\frac{2}{x}\right)^4$

$= 1 \cdot \frac{x^4}{16} \cdot 1 - \cancel{4} \cdot \frac{x^3}{\cancel{8}} \cdot \frac{\cancel{2}}{x} + 6 \cdot \frac{\cancel{x^2}}{\cancel{4}} \cdot \frac{\cancel{4}}{\cancel{x^2}} - \cancel{4}^2 \cdot \frac{x}{\cancel{2}} \cdot \frac{8}{x^3} + 1 \cdot 1 \cdot \frac{16}{x^4}$

$= \frac{x^4}{16} - x^2 + 6 - \frac{16}{x^2} + \frac{16}{x^4}$

$$\left(1+\frac{x}{2}-\frac{2}{x}\right)^4 = {}^4C_0 \cdot 1^4 \cdot \left(\frac{x}{2}-\frac{2}{x}\right)^0 + {}^4C_1 \cdot 1^3 \cdot \left(\frac{x}{2}-\frac{2}{x}\right)^1 + {}^4C_2 \cdot 1^2 \cdot \left(\frac{x}{2}-\frac{2}{x}\right)^2 + {}^4C_3 \cdot 1^1 \cdot \left(\frac{x}{2}-\frac{2}{x}\right)^3 + {}^4C_4 \cdot 1^0 \cdot \left(\frac{x}{2}-\frac{2}{x}\right)^4$$

$$= 1.1.1 + 4.1.\left(\frac{x}{2}-\frac{2}{x}\right) + 6.1.\left(\frac{x^2}{4}+\frac{4}{x^2}-2\right) + 4.\left(\frac{x^3}{8}-\frac{3x}{2}+\frac{6}{x}-\frac{8}{x^3}\right)$$

$$+ 1.1.\left\{\frac{x^4}{16}-x^2+6-\frac{16}{x^2}+\frac{16}{x^4}\right\}$$

$$= 1 + 2x - \frac{8}{x} + \frac{3}{2}x^2 + \frac{24}{x^2} - 12 + \frac{x^3}{2} - 6x + \frac{24}{x} - \frac{32}{x^3}$$

$$+ \frac{x^4}{16} - x^2 + 6 - \frac{16}{x^2} + \frac{16}{x^4}$$

$$= \frac{x^4}{16} + \frac{x^3}{3} + \frac{x^2}{2} - 4x - 5 + \frac{16}{x} + \frac{8}{x^2} - \frac{32}{x^3} + \frac{16}{x^4}$$

$\boxed{Q.10}$

$$\left(\underbrace{3x^2-2ax}_{A}+\underbrace{3a^2}_{B}\right)^3$$

$$= {}^3C_0\cdot(3x^2-2ax)^3\cdot(3a^2)^0 + {}^3C_1\cdot(3x^2-2ax)^2\cdot(3a^2)^1 + {}^3C_2\cdot(3x^2-2ax)^1\cdot(3a^2)^2$$
$$+ {}^3C_3\cdot(3x^2-2ax)^0\cdot(3a^2)^3$$

$$(3x^2-2ax)^3 = (3x^2)^3 - 3(3x^2)^2\cdot(2ax) + 3(3x^2)(2ax)^2 - (2ax)^3$$
$$= 27x^6 - 54a\cdot x^5 + 36a^2x^4 - 8a^3\cdot x^3$$

$$= 1\cdot(27x^6 - 54a\cdot x^5 + 36a^2x^4 - 8a^3\cdot x^3)\cdot 1 + 3\cdot(9x^4 - 12\cdot a\cdot x^3 + 4a^2x^2)\cdot 3a^2$$
$$+ 3\cdot(3x^2-2ax)\cdot(9a^4) + 1\cdot 1\cdot 27\cdot a^6$$

$$= 27x^6 - 54a\cdot x^5 + 36\cdot a^2x^4 - 8a^3x^3 + 81a^2x^4 - 108\cdot a^3x^3 + 36a^4x^2$$
$$+ 81a^4x^2 - 54a^5x + 27\cdot a^6$$

$$= 27x^6 - 54\cdot ax^5 + 117a^2x^4 - 116a^3x^3 + 117\cdot a^4x^2 - 54\cdot a^2x + 27\cdot a^6$$