TRIGONOMETRIC EQUATIONS [JEE ADVANCED PREVIOUS YEAR SOLVED PAPER]

JEE ADVANCED

Single Correct Answer Type

1. The equation $2\cos^2\frac{x}{2}\sin^2 x = x^2 + x^{-2};\ 0 < x \le \frac{\pi}{2}$ has
   a. no real solution **b.** one real solution
   c. more than one solution **d.** none of these
   (IIT-JEE 1980)

2. The general solution of the trigonometric equation $\sin x + \cos x = 1$ is given by
   a. $x = 2n\pi;\ n = 0, \pm 1, \pm 2, \ldots$
   b. $x = 2n\pi + \pi/2;\ n = 0, \pm 1, \pm 2, \ldots$
   c. $x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}\ n = 0, \pm 1, \pm 2, \ldots$
   d. none of these **(IIT-JEE 1981)**

3. The general solution of the equation $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$ is $(n \in Z)$
   a. $n\pi + \frac{\pi}{8}$ **b.** $\frac{n\pi}{2} + \frac{\pi}{8}$
   c. $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$ **d.** $2n\pi + \cos^{-1}\frac{2}{3}$
   (IIT-JEE 1989)

4. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$ in the variable x has real roots. Then p can take any value in the interval
   a. $(0, 2\pi)$ **b.** $(-\pi, 0)$
   c. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ **d.** $(0, \pi)$ **(IIT-JEE 1990)**

5. The number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is
   a. 0 **b.** 1 **c.** 2 **d.** 3
   (IIT-JEE 1993)

6. The general values of θ satisfying the equation $2\sin^2\theta - 3\sin\theta - 2 = 0$ is $(n \in Z)$
   a. $n\pi + (-1)^n \pi/6$ **b.** $n\pi + (-1)^n \pi/2$
   c. $n\pi + (-1)^n 5\pi/6$ **d.** $n\pi + (-1)^n 7\pi/6$
   (IIT-JEE 1995)

7. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is
   a. 0 **b.** 2 **c.** 1 **d.** 3
   (IIT-JEE 2001)

8. The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution is
   a. 4 **b.** 8 **c.** 10 **d.** 12
   (IIT-JEE 2002)

9. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$, where $\alpha, \beta \in [-\pi, \pi]$. Number of pairs of α, β which satisfy both the equations is
   a. 0 **b.** 1 **c.** 2 **d.** 4
   (IIT-JEE 2005)

10. The value of $\theta \in (0, 2\pi)$ for which the equation $2\sin^2\theta - 5\sin\theta + 2 > 0$ is
    a. $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ **b.** $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$
    c. $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ **d.** $\left(\frac{41\pi}{48}, \pi\right)$
    (IIT-JEE 2006)

11. The number of solutions of the pair of equations
    $2\sin^2\theta - \cos 2\theta = 0$
    $2\cos^2\theta - 3\sin\theta = 0$
    in the interval $[0, 2\pi]$ is
    a. 0 **b.** 1 **c.** 2 **d.** 4
    (IIT-JEE 2007)

12. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in R$. Then the set of all x satisfying $(fogogof)(x) = (gogof)(x)$, where $(fog)(x) = f(g(x))$, is

a. $\pm\sqrt{n\pi}$, $n \in \{0, 1, 2, \ldots\}$

b. $\pm\sqrt{n\pi}$, $n \in \{1, 2, \ldots\}$

c. $\frac{\pi}{2} + 2n\pi$, $n \in \{\ldots, -2, -1, 0, 1, 2\ldots\}$

d. $2n\pi$, $n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ **(IIT-JEE 2011)**

13. For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has

a. infinitely many solutions

b. three solutions

c. one solution

d. no solution **(JEE Advanced 2014)**

Multiple Correct Answers Type

1. The number of all the possible triplets (a_1, a_2, a_3) such that $a_1 + a_2\cos(2x) + a_3\sin^2(x) = 0$ for all x is

a. 0 b. 1 c. 3 d. infinite

(IIT-JEE 1987)

2. The values of θ lying between $\theta = 0$ and $\theta = \pi/2$ and satisfying the equation

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0 \text{ are}$$

a. $7\pi/24$ b. $5\pi/24$ c. $11\pi/24$ d. $\pi/24$

(IIT-JEE 1988)

3. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is

a. 0 b. 5 c. 6 d. 10

(IIT-JEE 1998)

4. Let $2\sin^2 x + 3\sin x - 2 > 0$ and $x^2 - x - 2 < 0$ (x is measured in radians). Then x lies in the interval

a. $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ b. $\left(-1, \frac{5\pi}{6}\right)$ c. $(-1, 2)$ d. $\left(\frac{\pi}{6}, 2\right)$

(IIT-JEE 1994)

5. For $0 < \theta < \frac{\pi}{2}$, the solution(s) of

$$\sum_{m=1}^{6} \text{cosec}\left(\theta + \frac{(m-1)}{4}\pi\right)\text{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$$

is (are)

a. $\pi/4$ b. $\pi/6$ c. $\pi/12$ d. $5\pi/12$

(IIT-JEE 2009)

6. Let $\theta, \varphi \in [0, 2\pi]$ be such that $2\cos\theta(1 - \sin\varphi) = \sin^2\theta\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\varphi - 1$, $\tan(2\pi - \theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$. Then φ cannot satisfy

a. $0 < \varphi < \frac{\pi}{2}$ b. $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$

c. $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$ d. $\frac{3\pi}{2} < \varphi < 2\pi$

(IIT-JEE 2012)

Linked Comprehension Type

1. Let $f(x) = (1 - x)^2\sin^2 x + x^2$ for all $x \in R$.

Consider the statements:

P: There exists some $x \in R$ such that $f(x) + 2x = 2(1 + x^2)$.

Q: There exists some $x \in R$ such that $2f(x) + 1 = 2x(1 + x)$.

Then

a. both P and Q are true b. P is true and Q is false

c. P is false and Q is true d. both P and Q are false

(IIT-JEE 2012)

Matching Column Type

1. Match the statements/expressions in Column I with the statements/expressions in Column II.

Column I	Column II
(a) The minimum value of $\frac{x^2+2x+4}{x+2}$ is	(p) 0
(b) Let A and B be 3×3 matrices of real numbers, where A is symmetric, B is skew symmetric, and $(A + B)(A - B) = (A - B)(A + B)$. If $(AB)' = (-1)^k AB$, where $(AB)'$ is the transpose of the matrix AB, then the possible values of k are	(q) 1
(c) Let $a = \log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k+3^{-a})} < 2$, must be less than	(r) 2
(d) If $\sin\theta = \cos\phi$, then the possible values of $\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right)$ are	(s) 3

(IIT-JEE 2008)

2. Match the statements/expressions in Column I with the statements/expressions in Column II.

Column I	Column II
(a) Root(s) of the equation $2\sin^2\theta + \sin^2 2\theta = 2$	(p) $\frac{\pi}{6}$
(b) Points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$, where $[y]$ denotes the largest integer less than or equal to y	(q) $\frac{\pi}{4}$

(c) Volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$	(r) $\frac{\pi}{3}$
(d) Angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\,\vec{c}=\vec{0}$	(s) $\frac{\pi}{2}$
	(t) π

(IIT-JEE 2009)

Integer Answer Type

1. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations

$$(y+z)\cos 3\theta = (xyz)\sin 3\theta$$

$$x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$

$$(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$$

have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$ is

(IIT-JEE 2010)

2. The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is

(IIT-JEE 2010)

3. The number of distinct solutions of the equation $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is **(JEE Advanced 2015)**

Fill in the Blanks Type

1. The solution set of the system of equations $x + y = 2\pi/3$, $\cos x + \cos y = 3/2$, where x and y are real, is ______. **(IIT-JEE 1986)**
2. The set of all x in the interval $[0, \pi]$ for which $2\sin^2 x - 3\sin x + 1 \geq 0$ is ______. **(IIT-JEE 1987)**
3. The general value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is ______. **(IIT-JEE 1996)**
4. The real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ are ______, ______, and ______. **(IIT-JEE 1997)**

True/False Type

1. There exists a value of θ between 0 and 2π that satisfies the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$. **(IIT-JEE 1984)**

Subjective Type

1. Find the coordinates of the points of intersection of the curves $y = \cos x$, $y = \sin 3x$ if $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. **(IIT-JEE 1982)**
2. Find all the solution of $4\cos^2 x \sin x - 2\sin^2 x = 3\sin x$. **(IIT-JEE 1983)**
3. Find the values of $x \in (-\pi, \pi)$ which satisfy the equation $8^{(1+|\cos x|+|\cos^2 x|+|\cos^3 x|+\ldots)} = 4^3$. **(IIT-JEE 1984)**
4. Find all values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying the equation

$$(1-\tan\theta)(1+\tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0.$$

(IIT-JEE 1996)

Answer Key

JEE Advanced

Single Correct Answer Type

1. a. 2. c. 3. b. 4. d. 5. c.
6. d. 7. c. 8. b. 9. d. 10. a.
11. c. 12. a. 13. d.

Multiple Correct Answers Type

1. d. 2. a., c. 3. c. 4. d. 5. c., b.
6. a., c., d.

Linked Comprehension Type

1. c.

Matching Column Type

1. (d) – (p, r) 2. (a) – (q, s)

Integer Answer Type

1. 3 2. 3 3. 8

Fill in the Blanks Type

1. ϕ 2. $\left[0, \frac{\pi}{6}\right] \cup \left\{\frac{\pi}{2}\right\} \cup \left[\frac{5\pi}{6}, \pi\right]$

3. $n\pi, n\pi \pm \frac{\pi}{3}\ n \in z$ 4. $-\frac{\pi}{2}, \frac{\pi}{2}, 0$

True/False Type

1. False

Subjective Type

1. $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right), \left(\frac{\pi}{8}, \cos\frac{\pi}{8}\right), \left(-\frac{3\pi}{8}, \cos\frac{3\pi}{8}\right)$

2. $x = n\pi,\ m\pi \pm (-1)^m \sin^{-1}\left(\frac{-1\pm\sqrt{5}}{4}\right)$; $m, n \in Z$

3. $\pm\frac{\pi}{3}, \pm\frac{2\pi}{3}$

4. $\pm\frac{\pi}{3}$

Hints and Solutions

JEE Advanced

Single Correct Answer Type

1. a. The given equation is

$$2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + \frac{1}{x^2}$$

where $0 < x \le \frac{\pi}{2}$

$$\text{L.H.S.} = 2\cos^2\left(\frac{x}{2}\right)\sin^2 x = (1+\cos x)\sin^2 x$$

$\because$ $1 + \cos x < 2$ and $\sin^2 x \le 1$ for $0 < x < \frac{\pi}{2}$

$\therefore$ $(1 + \cos x)\sin^2 x < 2$

and R.H.S. $= x^2 + \frac{1}{x^2} \ge 2$

Therefore, for $0 < x \le \frac{\pi}{2}$, the given equation is not possible for any real value of x.

2. c. $\sin x + \cos x = 1$

or $\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$

or $\sin x \cos\left(\frac{\pi}{4}\right) + \cos x \sin\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4}$

or $\sin\left(x + \frac{\pi}{4}\right) = \sin\frac{\pi}{4}$

$\Rightarrow$ $x + \left(\frac{\pi}{4}\right) = n\pi + (-1)^n\frac{\pi}{4}, n \in Z$

$\Rightarrow$ $x = n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{4}$

where $n = 0, \pm 1, \pm 2, \ldots$

3. b. The given equation is

$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

or $2\sin 2x\cos x - 3\sin 2x = 2\cos 2x\cos x - 3\cos 2x$

or $\sin 2x(2\cos x - 3) = \cos 2x(2\cos x - 3)$

or $\sin 2x = \cos 2x$ $\quad\left(\text{as } \cos x \ne \frac{3}{2}\right)$

or $\tan 2x = 1 \Rightarrow 2x = n\pi + \frac{\pi}{4}$

$\Rightarrow$ $x = \frac{n\pi}{2} + \frac{\pi}{8}, n \in Z$

4. d. The given equation is

$(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$

For this equation to have real roots, $D \ge 0$. Thus,

$\cos^2 p - 4\sin p(\cos p - 1) \ge 0$

or $\cos^2 p - 4\sin p\cos p + 4\sin^2 p + 4\sin p - 4\sin^2 p \ge 0$

or $(\cos p - 2\sin p)^2 + 4\sin p(1 - \sin p) \ge 0$

For every real value of p, we have

$(\cos p - 2\sin p)^2 \ge 0$ and $\sin p(1 - \sin p) \ge 0$

$\therefore$ $D \ge 0, \forall \sin p \in (0, \pi)$

5. c. The given equation is

$\tan x + \sec x = 2\cos x$

or $\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$

or $\sin x + 1 = 2\cos^2 x = 2 - 2\sin^2 x$

or $2\sin^2 x + \sin x - 1 = 0$

or $(2\sin x - 1)(\sin x + 1) = 0$

or $\sin x = \frac{1}{2}, -1$

$\Rightarrow$ $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} \in [0, 2\pi]$

But for $x = 3\pi/2$, $\tan x$ and $\sec x$ are not defined.

Therefore, there are only two solutions.

6. d. The given equation is

$2\sin^2\theta - 3\sin\theta - 2 = 0$

or $(2\sin\theta + 1)(\sin\theta - 2) = 0$

or $\sin = -\frac{1}{2}$ $\quad[\because \sin\theta - 2 = 0 \text{ is not possible}]$

or $\sin\theta = \sin(-\pi/6) = \sin(7\pi/6)$

$\Rightarrow$ $\theta = n\pi + (-1)^n(-\pi/6)$ or $\theta = n\pi + [(-1)^n 7\pi/6]$

Thus, $\theta = n\pi + (-1)^n 7\pi/6, n \in Z$

7. c. To simplify the determinant, let $\sin x = a$; $\cos x = b$. Then the equation becomes

$$\begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix} = 0$$

Operating $C_2 \to C_2 - C_1$; $C_3 \to C_3 - C_2$, we get

$$\begin{vmatrix} a & b-a & 0 \\ b & a-b & b-a \\ b & 0 & a-b \end{vmatrix} = 0$$

or $\quad a(a-b)^2 - (b-a)[b(a-b) - b(b-a)] = 0$

or $\quad a(a-b)^2 - 2b(b-a)(a-b) = 0$

or $\quad (a-b)^2(a-2b) = 0$

or $\quad a = b$ or $a = 2b$

or $\quad \dfrac{a}{b} = 1$ or $\dfrac{a}{b} = 2$

$\Rightarrow \quad \tan x = 1$ or $\tan x = 2$

But we have $-\dfrac{\pi}{4} \le x \le \dfrac{\pi}{4}$

$\Rightarrow \quad \tan\left(\dfrac{\pi}{4}\right) \le \tan x \le \tan\left(\dfrac{\pi}{4}\right)$

$\Rightarrow \quad -1 \le \tan x \le 1$

$\therefore \quad \tan x = 1 \quad \Rightarrow \quad x = \pi/4$

Therefore, there is only one real root.

8. b. We know that

$$-\sqrt{a^2+b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2+b^2}$$

$\Rightarrow \quad -\sqrt{74} \le 7\cos x + 5\sin x \le \sqrt{74}$

$\Rightarrow \quad -\sqrt{74} \le 2k+1 \le \sqrt{74}$

$\Rightarrow \quad -8 \le 2k+1 \le 8 \quad \Rightarrow \quad -4.5 \le k \le 3.5$

Considering only integral values, which means k can take eight integral values.

9. d. Given that $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$ where $\alpha, \beta \in [-\pi, \pi]$

Now $\cos(\alpha - \beta) = 1 \quad \Rightarrow \quad \alpha - \beta = 0 \quad$ or $\quad \alpha = \beta$

$\therefore \quad \cos(\alpha + \beta) = 1/e \quad \Rightarrow \quad \cos 2\alpha = 1/e$

$\because \quad 0 < 1/e < 1$ and $2\alpha \in [-2\pi, 2\pi]$

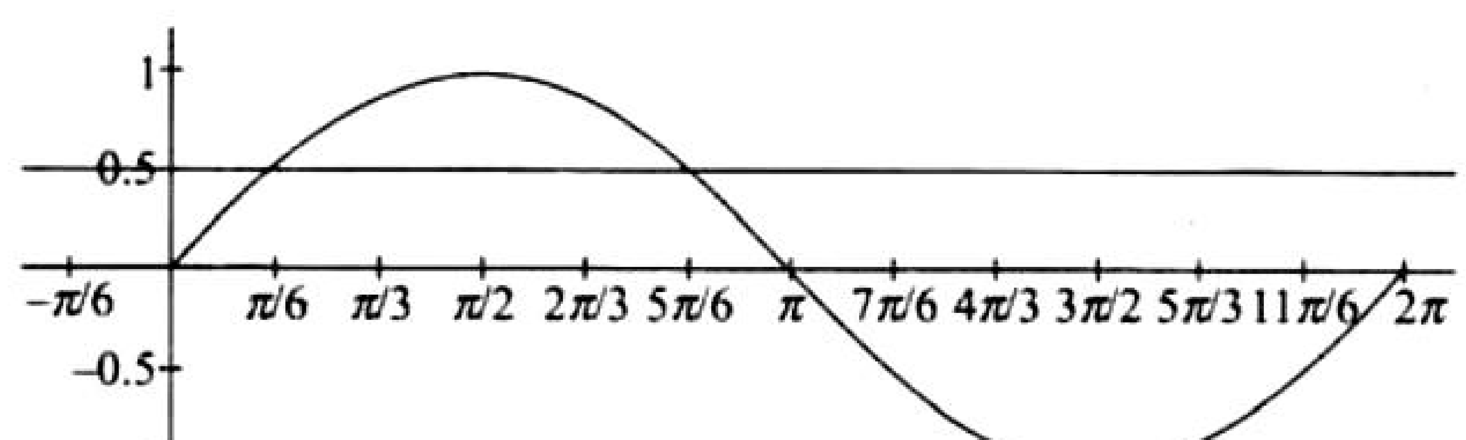
There will be two values of 2α satisfying $\cos 2\alpha = 1/e$ in $[0, 2\pi]$ and two in $[-2\pi, 0]$.

Therefore, there will be four values of α in $[-2\pi, 2\pi]$ and correspondingly four values of β. Hence, there are four sets of (α, β).

10. a. $2\sin^2\theta - 5\sin\theta + 2 > 0$

or $\quad (\sin\theta - 2)(2\sin\theta - 1) > 0$

or $\quad \sin\theta < 1/2 \qquad [\because -1 \le \sin\theta \le 1]$



From the graph $x \in (0, \pi/6) \cup (5\pi/6, 2\pi)$

11. c. $2\sin^2\theta - \cos 2\theta = 0$

or $\quad 1 - 2\cos 2\theta = 0$

or $\quad \cos 2\theta = \dfrac{1}{2}$

$\Rightarrow \quad 2\theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}, \dfrac{7\pi}{3}, \dfrac{11\pi}{3}$

or $\quad \theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6} \qquad$ (i)

where $\theta \in [0, 2\pi]$.

Also $\quad 2\cos^2\theta - 3\sin\theta = 0$

or $\quad 2\sin^2\theta + 3\sin\theta - 2 = 0$

or $\quad (2\sin\theta - 1)(\sin\theta + 2) = 0$

or $\quad \sin\theta = 1/2 \qquad [\because \sin\theta \ne -2]$

$\Rightarrow \quad \theta = \pi/6, 5\pi/6$, where $\theta \in [0, 2\pi] \qquad$ (ii)

Combining Eqs. (i) and (ii), we get $\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

Therefore, there are two solutions.

12. a. $(fogogof)(x) = \sin^2(\sin x^2)$

$(gogof)(x) = \sin(\sin x^2)$

$\therefore \quad \sin^2(\sin x^2) = \sin(\sin x^2)$

$\Rightarrow \quad \sin(\sin x^2)[\sin(\sin x^2) - 1] = 0$

$\Rightarrow \quad \sin(\sin x^2) = 0$ or 1

$\Rightarrow \quad \sin x^2 = n\pi$ or $2m\pi + \pi/2$, where $m, n \in I$

$\Rightarrow \quad \sin x^2 = 0$

$\Rightarrow \quad x^2 = n\pi \Rightarrow x = \pm\sqrt{n\pi}$, $n \in \{0, 1, 2, \ldots\}$.

13. d. $\sin x + 2\sin 2x - \sin 3x = 3$

$\Rightarrow \quad \sin x + 4\sin x\cos x - 3\sin x + 4\sin^3 x = 3$

$\Rightarrow \quad \sin x[-2 + 4\cos x + 4(1 - \cos^2 x)] = 3$

$\Rightarrow \quad \sin x[2 - (4\cos^2 x - 4\cos x + 1) + 1] = 3$

$\Rightarrow \quad 3 - (2\cos x - 1)^2 = 3\operatorname{cosec} x$

Now R.H.S. ≥ 3

But L.H.S. < 3

Hence, no solution.

Multiple Correct Answers Type

1. d. Since $a_1 + a_2\cos 2x + a_3\sin^2 x = 0$ for all x

Putting $x = 0$ and $x = \pi/2$, we get

$$a_1 + a_2 = 0 \text{ and } a_1 - a_2 + a_3 = 0$$

$\Rightarrow \quad a_2 = -a_1$ and $a_3 = -2a_1$

Therefore, the given equation becomes

$$a_1 - a_1\cos 2x - 2a_1\sin^2 x = 0, \ \forall x$$

or $\quad a_1(1 - \cos 2x - 2\sin^2 x) = 0, \ \forall x$

or $\quad a_1(2\sin^2 x - 2\sin^2 x) = 0, \ \forall x$

The above is satisfied for all values of a_1.

Hence, the infinite number of triplets $(a_1, -a_1, -2a_1)$ is possible.

Alternative Method:

$a_1 + a_2\cos 2x + a_3\sin^2 x = 0$ for real x

$\therefore \quad a_1 + a_2(1 - 2\sin^2 x) + a_3\sin^2 x = 0$ for real x

$\therefore \quad (a_1 + a_2) + (a_3 - 2a_2)\sin^2 x = 0$ for real x

$\therefore \quad a_1 + a_2 = 0$ and $a_3 - 2a_2 = 0$

$\therefore \quad a_2 = -a_1$ and $a_3 = 2a_2 = -2a_1$

Hence, infinite number of triplets $(a_1, -a_1, -2a_1)$ exist.

2. a., c. We have

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Operating $C_1 \to C_1 + C_2$, we get

$$\begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 2 & 1+\cos^2\theta & 4\sin 4\theta \\ 1 & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Operating $R_1 \to R_1 - R_2$; $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Expanding along R_1, we get $1 + 4\sin 4\theta + 1 = 0$

or $2(1 + 2\sin 4\theta) = 0$

or $\sin 4\theta = -1/2 \Rightarrow 4\theta = \pi + \pi/6$

or $2\pi - \pi/6$

$\Rightarrow 4\theta = 7\pi/6$ or $11\pi/6$

$\Rightarrow \theta = 7\pi/24$ or $11\pi/24$

3. c. $3\sin^2 x - 7\sin x + 2 = 0$

or $(\sin x - 2)(3\sin x - 1) = 0$

$\Rightarrow \sin x = 1/3 = \sin\alpha$, say ($\sin x = 2$, not possible)

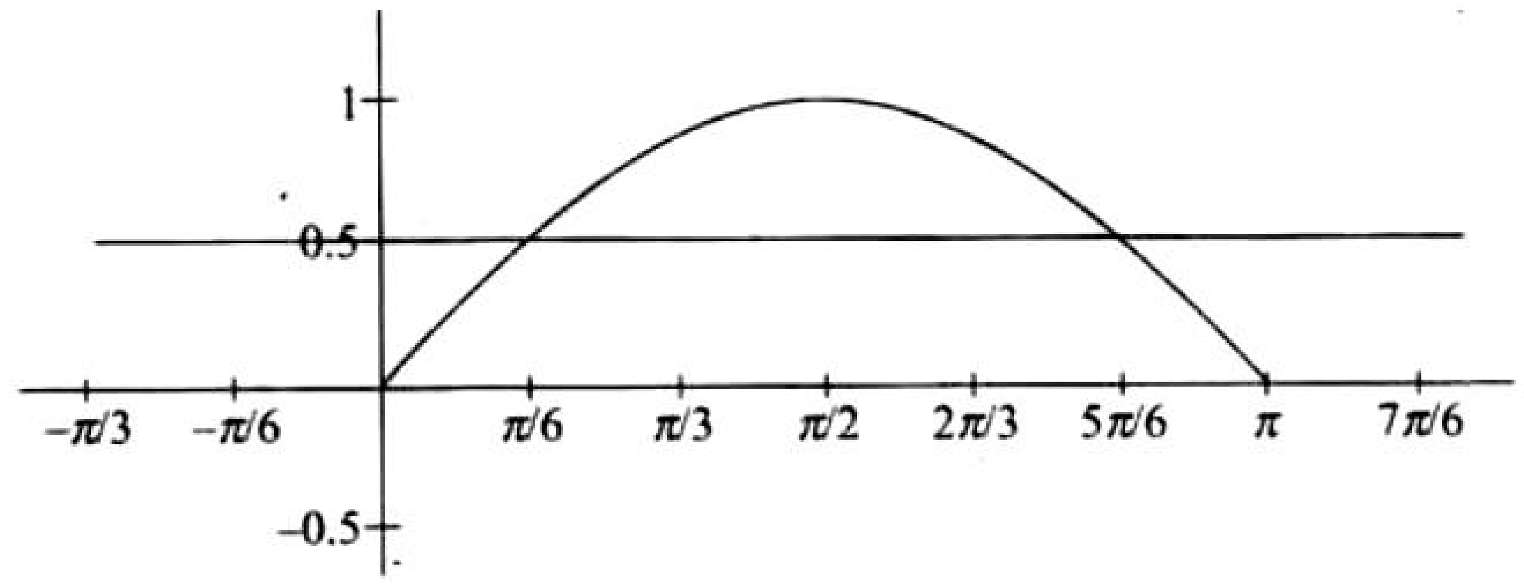
There exists two values in $(0, \pi)$, $(2\pi, 3\pi)$ and $(4\pi, 5\pi)$.

Hence there are six solutions.

4. d. $2\sin^2 x + 3\sin x - 2 > 0$

$(2\sin x - 1)(\sin x + 2) > 0$

$\Rightarrow 2\sin x - 1 > 0$ $\quad [\because -1 \le \sin x \le 1]$



$\Rightarrow \sin x > 1/2$

$\Rightarrow x \in (\pi/6, 5\pi/6)$ (i)

Also $x^2 - x - 2 < 0$

$\Rightarrow (x - 2)(x + 1) < 0 \Rightarrow -1 < x < 2$ (ii)

Combining Eqs. (i) and (ii), we get

$x \in (\pi/6, 2)$

5. c., d. $\displaystyle\sum_{m=1}^{6} \text{cosec}\left(\theta + \frac{(m-1)}{4}\pi\right)\text{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$

$$\Rightarrow \frac{1}{\sin(\pi/4)}\left[\frac{\sin(\theta + \pi/4 - \theta)}{\sin\theta \cdot \sin(\theta + \pi/4)} + \frac{\sin((\theta + \pi/2) - (\theta + \pi/4))}{\sin(\theta + \pi/4)\cdot\sin(\theta + \pi/2)} + \cdots + \frac{\sin((\theta + 3\pi/2) - (\theta + 5\pi/4))}{\sin(\theta + 3\pi/2)\cdot\sin(\theta + 5\pi/4)}\right] = 4\sqrt{2}$$

$$\Rightarrow \frac{1}{\sin(\pi/4)}\left[\frac{\sin(\theta + \pi/4)\cos\theta - \cos(\theta + \pi/4)\sin\theta}{\sin\theta\cdot\sin(\theta + \pi/4)} + \frac{\sin(\theta + \pi/2)\cos(\theta + \pi/4) - \cos(\theta + \pi/2)\sin(\theta + \pi/4)}{\sin(\theta + \pi/4)\cdot\sin(\theta + \pi/2)} + \ldots \frac{\sin(\theta + 3\pi/2)\cos(\theta + 5\pi/4) - \cos(\theta + 3\pi/2)\sin(\theta + 5\pi/4)}{\sin(\theta + 3\pi/2)\cdot\sin(\theta + 5\pi/4)}\right] = 4\sqrt{2}$$

$\Rightarrow \sqrt{2}\,[\cot\theta - \cot(\theta + \pi/4) + \cot(\theta + \pi/4) - \cot(\theta + \pi/2) + \ldots + \cot(\theta + 5\pi/4) - \cot(\theta + 3\pi/2)] = 4\sqrt{2}$

$\Rightarrow \tan\theta + \cot\theta = 4$

$\Rightarrow \tan\theta = 2 \pm \sqrt{3}$

$\Rightarrow \theta = \dfrac{\pi}{12}$ or $\dfrac{5\pi}{12}$

6. a., c., d.

$2\cos\theta(1 - \sin\varphi) = \dfrac{2\sin^2\theta}{\sin\theta}\cos\varphi - 1$

$= 2\sin\theta\cos\varphi - 1$

$\therefore 2\cos\theta - 2\cos\theta\sin\varphi = 2\sin\theta\cos\varphi - 1$

$\therefore 2\cos\theta + 1 = 2\sin(\theta + \varphi)$

$\tan(2\pi - \theta) > 0 \quad \Rightarrow \quad \tan\theta < 0$

and $-1 < \sin\theta < -\dfrac{\sqrt{3}}{2}$

$\Rightarrow \theta \in \left(\dfrac{3\pi}{2}, \dfrac{5\pi}{3}\right)$

$\Rightarrow 0 < \cos\theta < \dfrac{1}{2}$

$\dfrac{1}{2} < \sin(\theta + \varphi) < 1$

$\Rightarrow \dfrac{\pi}{6} + 2\pi < \sin(\theta + \varphi) < \dfrac{5\pi}{6} + 2\pi$

$2\pi + \dfrac{\pi}{6} - \theta_{max} < \varphi < 2\pi + \dfrac{5\pi}{6} - \theta_{min}$

$\Rightarrow \dfrac{\pi}{2} < \varphi < \dfrac{4\pi}{3}$

Linked Comprehension Type

1. c. $f(x) = (1 - x)^2\sin^2 x + x^2 \quad \forall x \in R$

For statement P:

$f(x) + 2x = 2(1 + x^2)$ (i)

$\Rightarrow (1 - x)^2\sin^2 x + x^2 + 2x = 2 + 2x^2$

$\Rightarrow (1 - x)^2\sin^2 x = x^2 - 2x + 2 = (x - 1)^2 + 1$

$\Rightarrow (1 - x)^2(\sin^2 x - 1) = 1$

$\Rightarrow -(1 - x)^2\cos^2 x = 1$

$\Rightarrow (1 - x)^2\cos^2 x = -1$

So equation (i) will not have real solution.

So, P is wrong.

For statement Q:

$2(1 - x)^2\sin^2 x + 2x^2 + 1 = 2x + 2x^2$ (ii)

$2(1 - x)^2\sin^2 x = 2x - 1$

$2\sin^2 x = \dfrac{2x - 1}{(1 - x)^2}$. Let $h(x) = \dfrac{2x - 1}{(1 - x)^2} - 2\sin^2 x$

Clearly, $h(0) = -1$, $\lim\limits_{x \to 1^-} h(x) = +\infty$

So by IVT, equation (ii) will have solution.

So, Q is correct.

Matching Column Type

1. **(d) – (p), (r)**

$\sin\theta = \cos\phi \Rightarrow \cos\left(\frac{\pi}{2} - \theta\right) = \cos\phi$

$\frac{\pi}{2} - \theta = 2n\pi \pm \phi, n \in Z$

$\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right) = -2n, n \in Z$

$\Rightarrow$ 0 and 2 are possible

Note: Solutions of the remaining parts are given in their respective chapters.

2. **(a) – (q), (s)**

We have $2\sin^2\theta + \sin^2 2\theta = 2$

$\Rightarrow 2\sin^2\theta + 4\sin^2\theta\cos^2\theta = 2$

$\Rightarrow \sin^2\theta + 2\sin^2\theta(1 - \sin^2\theta) = 1$

$\Rightarrow 3\sin^2\theta - 2\sin^4\theta - 1 = 0$

$\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{2}}, \pm 1$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{\pi}{2}.$

Note: Solutions of the remaining parts are given in their respective chapters.

Integer Answer Type

1. (1) Let $xyz = t$

$t\sin 3\theta - y\cos 3\theta - z\cos 3\theta = 0$ (1)

$t\sin 3\theta - 2y\sin 3\theta - 2z\cos 3\theta = 0$ (2)

$t\sin 3\theta - y(\cos 3\theta + \sin 3\theta) - 2z\cos 3\theta = 0$ (3)

$y_0 \cdot z_0 \neq 0$ hence homogeneous equation has non-trivial solution.

$$\therefore \quad D = \begin{vmatrix} \sin 3\theta & -\cos 3\theta & -\cos 3\theta \\ \sin 3\theta & -2\sin 3\theta & -2\cos 3\theta \\ \sin 3\theta & -(\cos 3\theta + \sin 3\theta) & -2\cos 3\theta \end{vmatrix} = 0$$

$\Rightarrow \sin 3\theta\cos 3\theta(\sin 3\theta - \cos 3\theta) = 0$

If $\sin 3\theta = 0$, then from equation (2)

$z = 0$, which is not possible

If $\cos 3\theta = 0$ and $\sin 3\theta \neq 0$, then

$t \cdot \sin 3\theta = 0$

$\Rightarrow t = 0$

$\Rightarrow x = 0$

From equation (2), $y = 0$ which is not possible

If $\sin 3\theta - \cos 3\theta = 0$, then

$\tan 3\theta = 1$

$\Rightarrow 3\theta = n\pi + \frac{\pi}{4}, n \in I$

$\Rightarrow x \cdot y \cdot z\sin 3\theta = 0$

$\Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{12}, n \in I$

$\Rightarrow x = 0, \sin 3\theta \neq 0$

$\Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}$

Hence, three solutions.

2. (3) $\tan\theta = \cot 5\theta$

$\Rightarrow \cos 6\theta = 0$

$\Rightarrow 4\cos^3 2\theta - 3\cos 2\theta = 0$

$\Rightarrow \cos 2\theta = 0$ or $\pm\frac{\sqrt{3}}{2}$ (i)

$\sin 2\theta = \cos 4\theta$

$\Rightarrow 2\sin^2 2\theta + \sin 2\theta - 1 = 0$

$\Rightarrow (2\sin 2\theta - 1)(\sin 2\theta + 1) = 0$

$\Rightarrow \sin 2\theta = -1$ or $\sin 2\theta = \frac{1}{2}$

$\Rightarrow \cos 2\theta = 0$ and $\sin 2\theta = -1$

$\Rightarrow 2\theta = -\frac{\pi}{2} \Rightarrow \theta = -\frac{\pi}{4}$

or $\cos 2\theta = \pm\frac{\sqrt{3}}{2}, \sin 2\theta = \frac{1}{2}$

$\Rightarrow 2\theta = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}$

$\therefore \theta = -\frac{\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$

3. (8) $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$

$\Rightarrow \frac{5}{4}\cos^2 2x + (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$

$+ (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x\cos^2 x(\sin^2 x + \cos^2 x) = 2$

$\Rightarrow \frac{5}{4}\cos^2 2x + 1 - \frac{1}{2}\sin^2 2x + 1 - \frac{3}{4}\sin^2 2x = 2$

$\Rightarrow \cos^2 2x = \sin^2 2x$

$\Rightarrow \tan^2 2x = 1$

$\Rightarrow \tan 2x = \pm 1$

$\Rightarrow 2x = n\pi \pm \frac{\pi}{4}, n \in Z$

$\Rightarrow x = (4n \pm 1)\frac{\pi}{8}, n \in Z$

$\Rightarrow x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

So, number of solutions = 8.

Fill in the Blanks Type

1. We have $\cos x + \cos y = \frac{3}{2}$

or $2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$

or $2\cos\frac{\pi}{3}\cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$ [using: $x + y = 2\pi/3$]

or $\cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$, which is not possible.

Hence, the system of equations has no solution.

2. We have $2\sin^2 x - 3\sin x + 1 \geq 0$

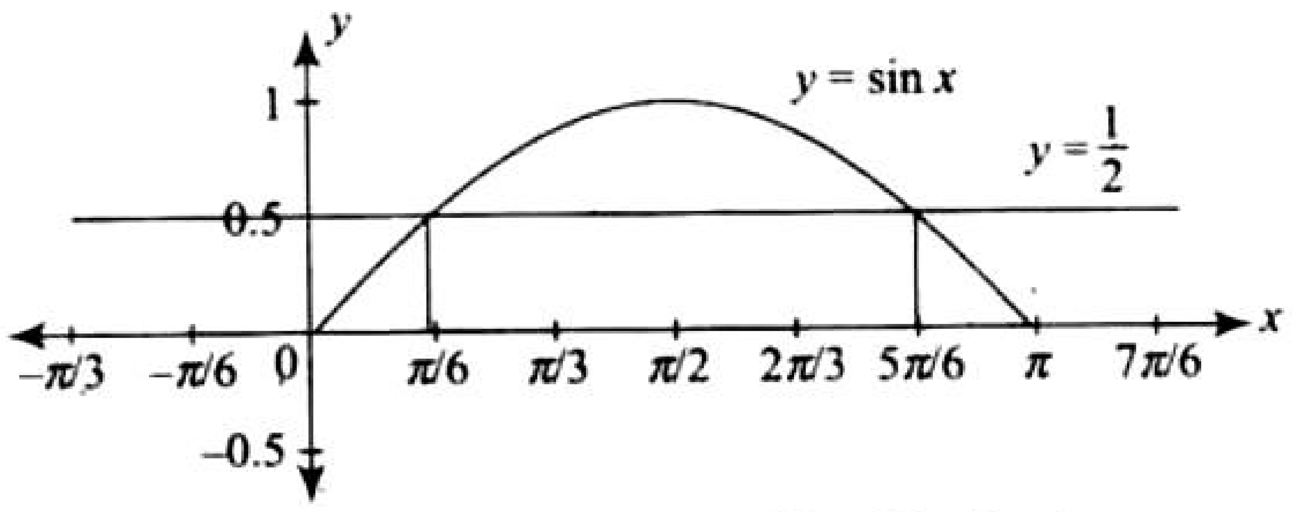
or $(2\sin x - 1)(\sin x - 1) \geq 0$

or $\left(\sin x - \frac{1}{2}\right)(\sin x - 1) \geq 0$

or $\sin x \leq \frac{1}{2}$ or $\sin x \geq 1$

But we know that $\sin x \leq 1$ and $\sin x \geq 0$ for $x \in [0, \pi]$.

Therefore, either $\sin x = 1$ or $0 \leq \sin x \leq \frac{1}{2}$



$\Rightarrow$ either $x = \pi/2$ or $x \in [0, \pi/6] \cup [5\pi/6, \pi]$

Combining, we get $x \in \left[0, \frac{\pi}{6}\right] \cup \left\{\frac{\pi}{2}\right\} \cup \left[\frac{5\pi}{6}, \pi\right]$.

3. $\tan^2\theta + \sec 2\theta = 1$

$$t^2 + \frac{1+t^2}{1-t^2} = 1, \text{ where } t = \tan\theta$$

or $\quad t^2(t^2 - 3) = 0$

or $\quad \tan\theta = 0, \pm\sqrt{3}$

$\Rightarrow \quad \theta = n\pi$ and $\theta = n\pi \pm \pi/3, n \in Z$.

4. $\cos^7 x = 1 - \sin^4 x$

$= (1 - \sin^2 x)(1 + \sin^2 x)$

$= \cos^2 x (1 + \sin^2 x)$

$\therefore \quad \cos x = 0$ or $x = \pi/2, -\pi/2,$

or $\quad \cos^5 x = 1 + \sin^2 x$

$\cos^5 x \le 1$ but $1 + \sin^2 x \ge 1$

$\Rightarrow \quad \cos^5 x = 1 + \sin^2 x = 1$

$\Rightarrow \quad \cos x = 1$ and $\sin x = 0$. [both these imply $x = 0$]

Hence, $x = -\frac{\pi}{2}, \frac{\pi}{2}$ and 0.

True/False Type

1. Given that equation is $\sin^4\theta - 2\sin^2\theta - 1 = 0$. Therefore,

$$\sin^2\theta = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$$

But $\sin^2\theta$ cannot be negative. Therefore,

$$\sin^2\theta = \sqrt{2} + 1$$

But as $-1 \le \sin\theta \le 1$, $\therefore \sin^2\theta \ne \sqrt{2} + 1$

Thus, there is no value of θ which satisfies the given equation. Therefore, statement is false.

Subjective Type

1. At the intersection point of $y = \cos x$ and $y = \sin 3x$, we have $\cos x = \sin 3x$

or $\quad \cos x = \cos\left(\frac{\pi}{2} - 3x\right)$

$\Rightarrow \quad x = 2n\pi \pm \left(\frac{\pi}{2} - 3x\right), n \in Z$

or $\quad x = \frac{\pi}{4}, \frac{\pi}{8}, -\frac{3\pi}{8}$ $\quad [\because -\pi/2 \le x \le \pi/2]$

Thus, the points are $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right), \left(\frac{\pi}{8}, \cos\frac{\pi}{8}\right)$

and $\left(-\frac{3\pi}{8}, \cos\frac{3\pi}{8}\right)$

2. The given equation is

$4\cos^2 x \sin x - 2\sin^2 x = 3\sin x$

or $\quad 4\cos^2 x \sin x - 2\sin^2 x - 3\sin x = 0$

or $\quad 4(1 - \sin^2 x)\sin x - 2\sin^2 x - 3\sin x = 0$

or $\quad \sin x[4\sin^2 x + 2\sin x - 1] = 0$

$\Rightarrow$ either $\sin x = 0$ or $4\sin^2 x + 2\sin x - 1 = 0$

If $\sin x = 0 \quad \Rightarrow \quad x = n\pi, n \in Z$

If $4\sin^2 x + 2\sin x - 1 = 0$

$\Rightarrow \quad \sin x = \frac{-1 \pm \sqrt{5}}{4}$

$\therefore \quad x = m\pi + (-1)^m \sin^{-1}\left(\frac{-1 \pm \sqrt{5}}{4}\right), m \in Z$

Thus, $x = n\pi, m\pi \pm (-1)^m \sin^{-1}\left(\frac{-1 \pm \sqrt{5}}{4}\right)$

where m and n are integers.

3. The given equation is

$$8^{(1 + |\cos x| + |\cos^2 x| + \cos^3 x| + \cdots)} = 4^3$$

or $\quad 2^{3(1 + |\cos x| + |\cos^2 x| + \cos^3 x| + \cdots\infty)} = 2^6$

or $\quad 3(1 + |\cos x| + |\cos^2 x| + \cos^3 x| + \cdots) = 6$

or $\quad 1 + |\cos x| + |\cos^2 x| + |\cos^3 x| + \cdots = 2$

or $\quad \frac{1}{1 - |\cos x|} = 2$

or $\quad |\cos x| = \frac{1}{2}$

$\Rightarrow \quad x = \pi/3, -\pi/3, 2\pi/3, -2\pi/3, \ldots$

The values of $x \in (-\pi, \pi)$ are $\pm\pi/3, \pm 2\pi/3$.

4. Given that $(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$

or $\quad (1 - \tan^2\theta)(1 + \tan^2\theta) + 2^{\tan^2\theta} = 0$

Let us put $\tan^2\theta = t$. Then

$(1 - t)(1 + t) + 2^t = 0 \quad$ or $\quad 1 - t^2 + 2^t = 0$

It is clearly satisfied by $t = 3$ as $-8 + 8 = 0$. We get

$\tan^2\theta = 3$

Therefore, $\theta = \pm\pi/3$ in the given interval.