3. Differentiation

EXERCISE 3.1

Q. 1 Find $\frac{dy}{dx}$ if,

1) $y = \sqrt{x + \frac{1}{x}}$

Solution:

Given : $y = \sqrt{x + \frac{1}{x}}$

Let $u = x + \frac{1}{x}$

Then $y = \sqrt{u}$

$$\therefore \frac{dy}{du} = \frac{d}{du}(u^{\frac{1}{2}}) = \frac{1}{2}u^{-\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{u}} = \frac{1}{2\sqrt{x + \frac{1}{x}}}$$

and $\frac{du}{dx} = \frac{d}{dx}\left(x + \frac{1}{x}\right)$

$$= \frac{d}{dx}(x) + \frac{d}{dx}(x^{-1})$$

$$= 1 + (-1)x^{-2} = 1 - \frac{1}{x^2}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2\sqrt{x + \frac{1}{x}}} \cdot \left(1 - \frac{1}{x^2}\right)$$

$$= \frac{1}{2}\left(x + \frac{1}{x}\right)^{-\frac{1}{2}}\left(1 - \frac{1}{x^2}\right).$$

2) $y = \sqrt[3]{a^2 + x^2}$

Solution:

Given : $y = \sqrt[3]{a^2 + x^2}$

Let $u = a^2 + x^2$

Then $y = \sqrt[3]{u}$

$$\therefore \frac{dy}{du} = \frac{d}{du}(u^{\frac{1}{3}}) = \frac{1}{3}u^{-\frac{2}{3}}$$

$$= \frac{1}{3}(a^2 + x^2)^{-\frac{2}{3}}$$

and $\frac{du}{dx} = \frac{d}{dx}(a^2 + x^2)$

$$= 0 + 2x = 2x$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{3}(a^2 + x^2)^{-\frac{2}{3}} \cdot 2x$$

$$= \frac{2x}{3}(a^2 + x^2)^{-\frac{2}{3}}.$$

3) $y = (5x^3 - 4x^2 - 8x)^9$

Solution:

Given : $y = (5x^3 - 4x^2 - 8x)^9$

Let $u = 5x^3 - 4x^2 - 8x$

Then $y = u^9$

$$\therefore \frac{dy}{du} = \frac{d}{du}(u^9) = 9u^8$$

$$= 9(5x^3 - 4x^2 - 8x)^8$$

and $\frac{du}{dx} = \frac{d}{dx}(5x^3 - 4x^2 - 8x)$

$$= 5\frac{d}{dx}(x^3) - 4\frac{d}{dx}(x^2) - 8\frac{d}{dx}(x)$$

$$= 5 \times 3x^2 - 4 \times 2x - 8 \times 1$$

$$= 15x^2 - 8x - 8$$

$$\therefore \frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$

$$=9(5x^3-4x^2-8x)^8(15x^2-8x-8).$$

Q. 2 Find $\frac{dy}{dx}$ *if*,

1) $y = \log(\log x)$

Solution:

Given : $y=\log(\log x)$

Let $u=\log x$

Then $y=\log u$

$$\therefore \frac{dy}{du}=\frac{d}{du}(\log u)$$

$$=\frac{1}{u}=\frac{1}{\log x}$$

and $\frac{du}{dx}=\frac{d}{dx}(\log x)=\frac{1}{x}$

$$\therefore \frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=\frac{1}{\log x}\times\frac{1}{x}$$

$$=\frac{1}{x\log x}.$$

2) $y = \log(10x^4+5x^3-3x^2+2)$

Solution:

Given : $y=\log(10x^4+5x^3-3x^2+2)$

Let $u=10x^4+5x^3-3x^2+2$

Then $y=\log u$

$$\therefore \frac{dy}{du}=\frac{d}{du}(\log u)=\frac{1}{u}$$

$$=\frac{1}{10x^4+5x^3-3x^2+2}$$

and $\frac{du}{dx}=\frac{d}{dx}(10x^4+5x^3-3x^2+2)$

$$=10\frac{d}{dx}(x^4)+5\frac{d}{dx}(x^3)-3\frac{d}{dx}(x^2)+\frac{d}{dx}(2)$$

$$=10\times4x^3+5\times3x^2-3\times2x+0$$

$$=40x^3+15x^2-6x$$

$$\therefore \frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$

$$=\frac{1}{10x^4+5x^3-3x^2+2}\times(40x^3+15x^2-6x)$$

$$=\frac{40x^3+15x^2-6x}{10x^4+5x^3-3x^2+2}.$$

3) $y = \log(ax^2+bx+c)$

Solution:

Given : $y=\log(ax^2+bx+c)$

Let $u=ax^2+bx+c$

Then $y=\log u$

$$\therefore \frac{dy}{du}=\frac{d}{du}(\log u)=\frac{1}{u}$$

$$=\frac{1}{ax^2+bx+c}$$

and $\frac{du}{dx}=\frac{d}{dx}(ax+bx+c)$

$$=a\frac{d}{dx}(x^2)+b\frac{d}{dx}(x)+\frac{d}{dx}(c)$$

$$=a\times2x+b\times1\times0=2ax+b$$

$$\therefore \frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=\frac{1}{ax^2+bx+c}\times(2ax+b)$$

$$=\frac{2ax+b}{ax^2+bx+c}.$$

Q. 2 Find $\frac{dy}{dx}$ *if*,

1) $y=e^{5x^2-2x+4}$

Solution:

Given : $y = e^{5x^2 - 2x + 4}$

Let $u = 5x^2 - 2x + 4$

Then $y = e^u$

$$\therefore \frac{dy}{du} = \frac{d}{du}(e^u) = e^u$$

$$= e^{5x^2 - 2x + 4}$$

and $\frac{du}{dx} = \frac{d}{dx}(5x^2 - 2x + 4)$

$$= 5\frac{d}{dx}(x^2) - 2\frac{d}{dx}(x) + \frac{d}{dx}(4)$$

$$= 5 \times 2x - 2 \times 1 + 0 = 10x - 2$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= e^{5x^2 - 2x + 4} \times (10x - 2)$$

$$= (10x - 2)e^{5x^2 - 2x + 4}.$$

2) $y = a^{(1+\log x)}$

Solution:

Given : $y = a^{(1 + \log x)}$

Let $u = 1 + \log x$

Then $y = a^u$

$$\therefore \frac{dy}{du} = \frac{d}{du}(a^u) = a^u \cdot \log a$$

$$= a^{(1 + \log x)} \cdot \log a$$

and $\frac{du}{dx} = \frac{d}{dx}(1 + \log x)$

$$= 0 + \frac{1}{x} = \frac{1}{x}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= a^{(1 + \log x)} \cdot \log a \cdot \frac{1}{x}.$$

3) $y = 5^{(x+\log x)}$

Solution:

Given : $y = 5^{(x + \log x)}$

Let $u = x + \log x$

Then $y = 5^u$

$$\therefore \frac{dy}{du} = \frac{d}{du}(5^u) = 5^u \cdot \log 5$$

$$= 5^{(x + \log x)} \cdot \log 5$$

and $\frac{du}{dx} = \frac{d}{dx}(x + \log x)$

$$= 1 + \frac{1}{x}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 5^{(x + \log x)} \cdot \log 5 \cdot \left(1 + \frac{1}{x}\right).$$

EXERCISE 3.2

Q.1 Find the rate of change of demand (x) of a commodity with respect to its price (y) if

1) y = 12 + 10x + 25x2

Solution:

Given : $y = 12 + 10x + 25x^2$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(12 + 10x + 25x^2)$$

$$= \frac{d}{dx}(12) + 10\frac{d}{dx}(x) + 25\frac{d}{dx}(x^2)$$

$$= 0 + 10 \times 1 + 25 \times 2x = 10 + 50x$$

By derivative of inverse function,

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} = \frac{1}{10 + 50x}$$

Hence, the rate of change of demand (x) with respect to price (y)

$$= \frac{dx}{dy} = \frac{1}{10 + 50x}.$$

2) $\mathbf{y = 18x + \log(x - 4)}$

Solution:

Given : $y = 18x + \log(x-4)$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}[18x + \log(x-4)]$$

$$= 18\frac{d}{dx}(x) + \frac{d}{dx}[\log(x-4)]$$

$$= 18 \times 1 + \frac{1}{x-4}\cdot\frac{d}{dx}(x-4)$$

$$= 18 + \frac{1}{x-4} \times (1-0)$$

$$= 18 + \frac{1}{x-4} = \frac{18x - 72 + 1}{x-4}$$

$$= \frac{18x - 71}{x-4}$$

By derivative of inverse function

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} = \frac{x-4}{18x-71}$$

Hence, the rate of change of demand (x) with respect to price (y)

$$= \frac{dx}{dy} = \frac{x-4}{18x-71}$$

3) $\mathbf{y = 25x + \log(1 + x2)}$

Solution:

Given : $y = 25x + \log(1+x^2)$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}[25x + \log(1+x^2)]$$

$$= 25\frac{d}{dx}(x) + \frac{d}{dx}[\log(1+x^2)]$$

$$= 25 \times 1 + \frac{1}{1+x^2}\cdot\frac{d}{dx}(1+x^2)$$

$$= 25 + \frac{1}{1+x^2} \times (0 + 2x)$$

$$= 25 + \frac{2x}{1+x^2} = \frac{25 + 25x^2 + 2x}{1+x^2}$$

$$= \frac{25x^2 + 2x + 25}{1+x^2}$$

By derivative of inverse function,

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} = \frac{1+x^2}{25x^2+2x+25}$$

Hence, the rate of change of demand (x) with respect to price (y)

$$= \frac{dx}{dy} = \frac{1+x^2}{25x^2+2x+25}.$$

Q.2 Find the marginal demand of a commodity where demand is x and price is y

1) $\mathbf{y = xe - x + 7}$

Solution:

Given : $y = xe^{-x} + 7$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(xe^{-x} + 7)$$

$$= \frac{d}{dx}(xe^{-x}) + \frac{d}{dx}(7)$$

$$= x\cdot\frac{d}{dx}(e^{-x}) + e^{-x}\cdot\frac{d}{dx}(x) + 0$$

$$= x \times e^{-x}\cdot\frac{d}{dx}(-x) + e^{-x} \times 1$$

$$= xe^{-x}(-1) + e^{-x}$$

$$= e^{-x}(-x+1) = \frac{1-x}{e^x}$$

By the derivative of inverse function,

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} = \frac{e^x}{1-x}$$

Hence, marginal demand $= \frac{dx}{dy} = \frac{e^x}{1-x}$.

2) $\mathbf{y = \frac{x+2}{x^2+1}}$

Solution:

) Given : $y = \frac{x+2}{x^2+1}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x+2}{x^2+1}\right)$$

$$= \frac{(x^2+1)\cdot\frac{d}{dx}(x+2) - (x+2)\cdot\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(1+0) - (x+2)(2x+0)}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2-4x}{(x^2+1)^2} = \frac{1-4x-x^2}{(x^2+1)^2}$$

By the derivative of inverse function,

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} = \frac{(x^2+1)^2}{1-4x-x^2}$$

Hence, marginal demand $= \frac{dx}{dy}$

$$= \frac{(x^2+1)^2}{1-4x-x^2}$$

3) $y = \frac{5x+9}{2x-10}$

Solution:

Given : $y = \frac{5x+9}{2x-10}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{5x+9}{2x-10}\right)$$

$$= \frac{(2x-10)\cdot\frac{d}{dx}(5x+9) - (5x+9)\cdot\frac{d}{dx}(2x-10)}{(2x-10)^2}$$

$$= \frac{(2x-10)(5\times 1+0) - (5x+9)(2\times 1-0)}{(2x-10)^2}$$

$$= \frac{10x-50-10x-18}{(2x-10)^2} = \frac{-68}{(2x-10)^2}$$

By the derivative of inverse function,

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} = -\frac{(2x-10)^2}{68}$$

Hence, marginal demand $= \frac{dx}{dy} = \frac{-(2x-10)^2}{68}$.

EXERCISE 3.3

Q. 1 Find $\frac{dy}{dx}$ if,

1) $y = x^{x^{2x}}$

Solution:

) $y = x^{x^{2x}}$

$\therefore \log y = \log x^{x^{2x}} = x^{2x}\cdot\log x$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{d}{dx}(x^{2x}\cdot\log x)$$

$$= x^{2x}\cdot\frac{d}{dx}(\log x) + (\log x)\cdot\frac{d}{dx}(x^{2x})$$

$$= x^{2x}\times\frac{1}{x} + (\log x)\cdot\frac{d}{dx}(x^{2x})$$

$$\therefore \frac{dy}{dx} = y\left[\frac{x^{2x}}{x} + (\log x)\cdot\frac{d}{dx}(x^{2x})\right]$$

$$= x^{x^{2x}}\left[\frac{x^{2x}}{x} + (\log x)\cdot\frac{d}{dx}(x^{2x})\right] \quad \ldots (1)$$

Let $u = x^{2x}$

Then $\log u = \log x^{2x} = 2x \log x$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{u}\cdot\frac{du}{dx} = 2\frac{d}{dx}(x \log x)$$

$$= 2\left[x\frac{d}{dx}(\log x) + (\log x)\cdot\frac{d}{dx}(x)\right]$$

$$= 2\left[x\times\frac{1}{x} + (\log x)\times 1\right]$$

$$\therefore \frac{du}{dx} = 2u(1+\log x)$$

$$\therefore \frac{d}{dx}(x^{2x}) = 2x^{2x}(1+\log x)$$

$\therefore$ from (1),

$$\frac{dy}{dx} = x^{x^{2x}}\left[\frac{x^{2x}}{x} + (\log x)\times 2x^{2x}(1+\log x)\right]$$

$$= x^{x^{2x}}\cdot x^{2x}\cdot\log x\left[2(1+\log x) + \frac{1}{x\log x}\right].$$

2) $y = x^{e^x}$

Solution:

$y = x^{e^x}$

$\therefore \ \log y = \log x^{e^x} = e^x \cdot \log x$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{d}{dx}(e^x \cdot \log x)$$

$$= e^x \cdot \frac{d}{dx}(\log x) + (\log x)\cdot\frac{d}{dx}(e^x)$$

$$= e^x \cdot \frac{1}{x} + (\log x)(e^x)$$

$$\therefore \ \frac{dy}{dx} = y\left[\frac{e^x}{x} + e^x \cdot \log x\right]$$

$$= x^{e^x}\cdot e^x\left[\frac{1}{x} + \log x\right].$$

3) $y = x^{x^x}$

Solution:

$y = e^{x^x}$

$\therefore \ \log y = \log e^{x^x} = x^x \log e$

$\therefore \ \log y = x^x$ ... $[\because \log e = 1]$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{d}{dx}(x^x)$$

$$\therefore \ \frac{dy}{dx} = y\cdot\frac{d}{dx}(x^x) = e^{x^x}\cdot\frac{d}{dx}(x^x) \quad \text{... (1)}$$

Let $u = x^x$

Then $\log u = \log x^x = x \log x$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{u}\cdot\frac{du}{dx} = \frac{d}{dx}(x \log x)$$

$$= x\cdot\frac{d}{dx}(\log x) + (\log x)\cdot\frac{d}{dx}(x)$$

$$= x \times \frac{1}{x} + (\log x) \times 1$$

$$\therefore \ \frac{du}{dx} = u(1 + \log x)$$

$$\therefore \ \frac{d}{dx}(x^x) = x^x(1 + \log x)$$

$\therefore$ from (1),

$$\frac{dy}{dx} = e^{x^x}\cdot x^x(1 + \log x).$$

Q. 2 Find $\frac{dy}{dx}$ if,

1) $y = \left[1 + \frac{1}{x}\right]^x$

Solution:

$$y = \left(1 + \frac{1}{x}\right)^x$$

$$\therefore \ \log y = \log\left(1 + \frac{1}{x}\right)^x = x \log\left(1 + \frac{1}{x}\right)$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{d}{dx}\left[x \log\left(1 + \frac{1}{x}\right)\right]$$

$$= x\frac{d}{dx}\left[\log\left(1 + \frac{1}{x}\right)\right] + \left[\log\left(1 + \frac{1}{x}\right)\right]\cdot\frac{d}{dx}(x)$$

$$= x \times \frac{1}{1 + \frac{1}{x}}\cdot\frac{d}{dx}\left(1 + \frac{1}{x}\right) + \left[\log\left(1 + \frac{1}{x}\right)\right] \times 1$$

$$= x \times \frac{x}{x+1} \times \left(0 - \frac{1}{x^2}\right) + \log\left(1 + \frac{1}{x}\right)$$

$$\therefore \ \frac{dy}{dx} = y\left[\frac{-1}{x+1} + \log\left(1 + \frac{1}{x}\right)\right]$$

$$= \left(1 + \frac{1}{x}\right)^x\left[\log\left(1 + \frac{1}{x}\right) - \frac{1}{1+x}\right].$$

2) $y = (2x + 5)^x$

Solution:

$y = (2x + 5)^x$

$\therefore \ \log y = \log (2x + 5)^x = x \log (2x + 5)$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{d}{dx}[x \log (2x + 5)]$$

$$= x\frac{d}{dx}[\log (2x + 5)] + [\log (2x + 5)]\cdot\frac{d}{dx}(x)$$

$$= x \times \frac{1}{2x + 5}\cdot\frac{d}{dx}(2x + 5) + [\log (2x + 5)] \times 1$$

$$= \frac{x}{2x + 5} \times (2 \times 1 + 0) + \log (2x + 5)$$

$$\therefore \ \frac{dy}{dx} = y\left[\frac{2x}{2x + 5} + \log (2x + 5)\right]$$

$$= (2x + 5)^x\left[\log (2x + 5) + \frac{2x}{2x + 5}\right].$$

3) $y = \sqrt[3]{\frac{(3x - 1)}{(2x + 3)(5 - x)^2}}$

Solution:

$$y = \sqrt[3]{\frac{3x-1}{(2x+3)(5-x)^2}}$$

$$\therefore \log y = \log\left[\frac{3x-1}{(2x+3)(5-x)^2}\right]^{\frac{1}{3}}$$

$$= \frac{1}{3}\log\left[\frac{3x-1}{(2x+3)(5-x)^2}\right]$$

$$= \frac{1}{3}[\log(3x-1) - \log(2x+3) - \log(5-x)^2]$$

$$= \frac{1}{3}\log(3x-1) - \frac{1}{3}\log(2x+3) - \frac{2}{3}\log(5-x)$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{3}\frac{d}{dx}[\log(3x-1)] - \frac{1}{3}\frac{d}{dx}[\log(2x+3)] - \frac{2}{3}\frac{d}{dx}[\log(5-x)]$$

$$= \frac{1}{3}\times\frac{1}{3x-1}\cdot\frac{d}{dx}(3x-1) - \frac{1}{3}\times\frac{1}{2x+3}\cdot\frac{d}{dx}(2x+3) - \frac{2}{3}\times\frac{1}{5-x}\cdot\frac{d}{dx}(5-x)$$

$$= \frac{1}{3(3x-1)}\times(3\times1-0) - \frac{1}{3(2x+3)}\times(2\times1+0) - \frac{2}{3(5-x)}\times(0-1)$$

$$\therefore \frac{dy}{dx} = y\left[\frac{3}{3(3x-1)} - \frac{2}{3(2x+3)} + \frac{2}{3(5-x)}\right]$$

$$= \frac{1}{3}\cdot\sqrt[3]{\frac{3x-1}{(2x+3)(5-x)^2}}\left[\frac{3}{3x-1} - \frac{2}{2x+3} + \frac{2}{5-x}\right].$$

Q. 3 Find $\frac{dy}{dx}$ *if*,

1) $y = (\log x)^x + x^{\log x}$

Solution:

$y = (\log x)^x + x^{\log x}$

Let $u = (\log x)^x$ and $v = x^{\log x}$

Then $y = u + v$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \ldots (1)$$

Take $u = (\log x)^x$

$\therefore \log u = \log(\log x)^x = x\log(\log x)$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{u}\cdot\frac{du}{dx} = \frac{d}{dx}[x\log(\log x)]$$

$$= x\cdot\frac{d}{dx}[\log(\log x)] + [\log(\log x)]\cdot\frac{d}{dx}(x)$$

$$= x\times\frac{1}{\log x}\cdot\frac{d}{dx}(\log x) + [\log(\log x)]\times 1$$

$$= x\times\frac{1}{\log x}\times\frac{1}{x} + \log(\log x)$$

$$\therefore \frac{du}{dx} = u\left[\frac{1}{\log x} + \log(\log x)\right]$$

$$= (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] \quad \ldots (2)$$

Also, $v = x^{\log x}$

$\therefore \log v = \log x^{\log x} = \log x\cdot\log x = (\log x)^2$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{v}\cdot\frac{dv}{dx} = \frac{d}{dx}(\log x)^2$$

$$= 2(\log x)\cdot\frac{d}{dx}(\log x)$$

$$= 2\log x\times\frac{1}{x}$$

$$\therefore \frac{dv}{dx} = v\left[\frac{2\log x}{x}\right]$$

$$= x^{\log x}\left[\frac{2\log x}{x}\right] \quad \ldots (3)$$

From (1), (2) and (3), we get

$$\frac{dy}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] + x^{\log x}\left[\frac{2\log x}{x}\right].$$

[**Note :** *Answer in the textbook is incorrect.*]

2) $y = (x)^x + (a)^x$

Solution:

Let $u = x^x$

Then $\log u = \log x^x = x\cdot\log x$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{u}\cdot\frac{du}{dx} = \frac{d}{dx}(x\cdot\log x)$$

$$= x\cdot\frac{d}{dx}(\log x) + (\log x)\cdot\frac{d}{dx}(x)$$

$$= x\times\frac{1}{x} + (\log x)\times 1$$

$$\therefore \frac{du}{dx} = u(1+\log x) = x^x(1+\log x) \quad \ldots (1)$$

Now, $y = u + a^x$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{d}{dx}(a^x)$$

$$= x^x(1+\log x) + a^x\cdot\log a \quad \ldots [\text{By (1)}]$$

3) $y = 10^{x^x} + 10^{x^{10}} + 10^{10^x}$

Solution:

$y = 10^{x^x} + 10^{x^{10}} + 10^{10^x}$

Let $u = x^x$

Then $\log u = \log x^x = x \log x$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{u} \cdot \frac{du}{dx} = \frac{d}{dx}(x \log x)$$

$$= x\frac{d}{dx}(\log x) + (\log x) \cdot \frac{d}{dx}(x)$$

$$= x \times \frac{1}{x} + (\log x) \times 1$$

$$\therefore \frac{du}{dx} = u(1 + \log x) = x^x(1 + \log x) \quad \ldots (1)$$

Now, $y = 10^u + 10^{x^{10}} + 10^{10^x}$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(10^u) + \frac{d}{dx}(10^{x^{10}}) + \frac{d}{dx}(10^{10^x})$$

$$= 10^u \cdot \log 10 \cdot \frac{du}{dx} + 10^{x^{10}} \cdot \log 10 \cdot \frac{d}{dx}(x^{10}) + 10^{10^x} \cdot \log 10 \cdot \frac{d}{dx}(10^x)$$

$$= 10^{x^x} \cdot \log 10 \cdot x^x(1 + \log x) + 10^{x^{10}} \cdot \log 10 \times 10x^9 + 10^{10^x} \cdot \log 10 \times 10^x \cdot \log 10 \quad \ldots \text{[By (1)]}$$

$$\therefore \frac{dy}{dx} = 10^{x^x} \cdot x^x \cdot (\log 10)(1 + \log x) + 10^{x^{10}}(10x^9) \log 10 + 10^{10^x} \cdot 10^x \cdot (\log 10)^2.$$

EXERCISE 3.4

Q. 1 Find $\frac{dy}{dx}$ *if*,

1) $\sqrt{x} + \sqrt{y} = \sqrt{a}$

Solution:

$\sqrt{x} + \sqrt{y} = \sqrt{a}$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$$

$$\therefore \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = -\frac{2\sqrt{y}}{2\sqrt{x}} = -\sqrt{\frac{y}{x}}.$$

2) $x^3 + y^3 + 4x^3y = 0$

Solution:

$x^3 + y^3 + 4x^3y = 0$

Differentiating both sides w.r.t. x, we get

$$3x^2 + 3y^2\frac{dy}{dx} + 4\left[x^3\frac{dy}{dx} + y\frac{d}{dx}(x^3)\right] = 0$$

$$\therefore 3x^2 + 3y^2\frac{dy}{dx} + 4x^3\frac{dy}{dx} + 4y \times 3x^2 = 0$$

$$\therefore 3y^2\frac{dy}{dx} + 4x^3\frac{dy}{dx} = -3x^2 - 12x^2y$$

$$\therefore (3y^2 + 4x^3)\frac{dy}{dx} = -3x^2(1 + 4y)$$

$$\therefore \frac{dy}{dx} = -\frac{3x^2(1 + 4y)}{3y^2 + 4x^3}.$$

3) $x^3 + x^2y + xy^2 + y^3 = 81$

Solution:

$x^3 + x^2y + xy^2 + y^3 = 81$

Differentiating both sides w.r.t. x, we get

$$3x^2 + \left[x^2\frac{dy}{dx} + y \cdot \frac{d}{dy}(x^2)\right] + \left[x \cdot \frac{d}{dx}(y^2) + y^2 \cdot \frac{d}{dx}(x)\right] + 3y^2\frac{dy}{dx} = 0$$

$$\therefore 3x^2 + x^2\frac{dy}{dx} + y \times 2x + x \times 2y \cdot \frac{dy}{dx} + y^2 \times 1 + 3y^2\frac{dy}{dx} = 0$$

$$\therefore x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -3x^2 - 2xy - y^2$$

$$\therefore (x^2 + 2xy + 3y^2)\frac{dy}{dx} = -(3x^2 + 2xy + y^2)$$

$$\therefore \frac{dy}{dx} = -\left(\frac{3x^2 + 2xy + y^2}{x^2 + 2xy + 3y^2}\right).$$

Q. 2 Find $\frac{dy}{dx}$ *if*,

1) $y.e^x + x.e^y = 1$

Solution:

$y \cdot e^x + x \cdot e^y = 1$

Differentiating both sides w.r.t. x, we get

$$\frac{d}{dx}(ye^x) + \frac{d}{dx}(xe^y) = 0$$

$$\therefore\ y \cdot \frac{d}{dx}(e^x) + e^x \cdot \frac{dy}{dx} + x \cdot \frac{d}{dx}(e^y) + e^y \cdot \frac{d}{dx}(x) = 0$$

$$\therefore\ y \cdot e^x + e^x \cdot \frac{dy}{dx} + x \cdot e^y \cdot \frac{dy}{dx} + e^y \times 1 = 0$$

$$\therefore\ (e^x + xe^y)\frac{dy}{dx} = -e^y - ye^x$$

$$\therefore\ \frac{dy}{dx} = -\left(\frac{e^y + ye^x}{e^x + xe^y}\right).$$

2) $x^y = e^{(x-y)}$

Solution:

$x^y = e^{(x-y)}$

$$\therefore\ \log x^y = \log e^{(x-y)}$$

$$\therefore\ y \log x = (x-y) \log e$$

$$\therefore\ y \log x = x - y \qquad \dots [\because \log e = 1]$$

$$\therefore\ y + y \log x = x$$

$$\therefore\ y(1 + \log x) = x$$

$$\therefore\ y = \frac{x}{1 + \log x}$$

$$\therefore\ \frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{1 + \log x}\right)$$

$$= \frac{(1 + \log x) \cdot \frac{d}{dx}(x) - x\frac{d}{dx}\ (1 + \log x)}{(1 + \log x)^2}$$

$$= \frac{(1 + \log x) \cdot 1 - x\left(0 + \frac{1}{x}\right)}{(1 + \log x)^2}$$

$$= \frac{1 + \log x - 1}{(1 + \log x)^2}$$

$$= \frac{\log x}{(1 + \log x)^2}.$$

3) $xy = \log(xy)$

Solution:

$xy = \log(xy)$

$$\therefore\ xy = \log x + \log y$$

Differentiating both sides w.r.t. x, we get

$$x \cdot \frac{dy}{dx} + y \cdot \frac{d}{dx}(x) = \frac{1}{x} + \frac{1}{y} \cdot \frac{dy}{dx}$$

$$\therefore\ x \cdot \frac{dy}{dx} + y \times 1 = \frac{1}{x} + \frac{1}{y} \cdot \frac{dy}{dx}$$

$$\therefore\ \left(x - \frac{1}{y}\right)\frac{dy}{dx} = \frac{1}{x} - y$$

$$\therefore\ \left(\frac{xy - 1}{y}\right)\frac{dy}{dx} = \frac{1 - xy}{x} = \frac{-(xy - 1)}{x}$$

$$\therefore\ \frac{1}{y}\frac{dy}{dx} = -\frac{1}{x}$$

$$\therefore\ \frac{dy}{dx} = -\frac{y}{x}.$$

Q.3 Solve the following.

1) *If* $x^5.y^7 = (x+y)^{12}$ *then show that,* $\frac{dy}{dx} = \frac{y}{x}$

Solution:

$$x^5 \cdot y^7 = (x+y)^{12}$$

$$\therefore\ \log(x^5 \cdot y^7) = \log(x+y)^{12}$$

$$\therefore\ \log x^5 + \log y^7 = \log(x+y)^{12}$$

$$\therefore\ 5 \log x + 7 \log y = 12 \log(x+y)$$

Differentiating both sides w.r.t. x, we get

$$5 \times \frac{1}{x} + 7 \times \frac{1}{y} \cdot \frac{dy}{dx} = 12 \times \frac{1}{x+y} \cdot \frac{d}{dx}(x+y)$$

$$\therefore\ \frac{5}{x} + \frac{7}{y} \cdot \frac{dy}{dx} = \frac{12}{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\therefore\ \frac{5}{x} + \frac{7}{y}\frac{dy}{dx} = \frac{12}{x+y} + \frac{12}{x+y} \cdot \frac{dy}{dx}$$

$$\therefore \left(\frac{7}{y}-\frac{12}{x+y}\right)\frac{dy}{dx}=\frac{12}{x+y}-\frac{5}{x}$$

$$\therefore \left[\frac{7x+7y-12y}{y(x+y)}\right]\frac{dy}{dx}=\frac{12x-5x-5y}{x(x+y)}$$

$$\therefore \left[\frac{7x-5y}{y(x+y)}\right]\frac{dy}{dx}=\frac{7x-5y}{x(x+y)}$$

$$\therefore \frac{1}{y}\cdot\frac{dy}{dx}=\frac{1}{x}$$

$$\therefore \frac{dy}{dx}=\frac{y}{x}.$$

2) If log(x + y) = log(xy) + a then show ***that,*** $\boldsymbol{\frac{dy}{dx}=\frac{-y^2}{x^2}}$

Solution:

$$\log(x+y)=\log(xy)+a$$

$$\therefore \log(x+y)=\log x+\log y+a$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{x+y}\cdot\frac{d}{dx}(x+y)=\frac{1}{x}+\frac{1}{y}\cdot\frac{dy}{dx}+0$$

$$\therefore \frac{1}{x+y}\cdot\left(1+\frac{dy}{dx}\right)=\frac{1}{x}+\frac{1}{y}\cdot\frac{dy}{dx}$$

$$\therefore \frac{1}{x+y}+\frac{1}{x+y}\cdot\frac{dy}{dx}=\frac{1}{x}+\frac{1}{y}\frac{dy}{dx}$$

$$\therefore \left(\frac{1}{x+y}-\frac{1}{y}\right)\frac{dy}{dx}=\frac{1}{x}-\frac{1}{x+y}$$

$$\therefore \left[\frac{y-x-y}{y(x+y)}\right]\frac{dy}{dx}=\frac{x+y-x}{x(x+y)}$$

$$\therefore \left[\frac{-x}{y(x+y)}\right]\frac{dy}{dx}=\frac{y}{x(x+y)}$$

$$\therefore -\frac{x}{y}\cdot\frac{dy}{dx}=\frac{y}{x}$$

$$\therefore \frac{dy}{dx}=-\frac{y^2}{x^2}.$$

3) ***If*** $\boldsymbol{e^x+e^y=e^{(x+y)}}$ ***then show that,*** $\boldsymbol{\frac{dy}{dx}=-e^{y-x}}$

Solution:

$$e^x+e^y=e^{(x+y)} \qquad \ldots (1)$$

Differentiating both sides w.r.t. x, we get

$$e^x+e^y\cdot\frac{dy}{dx}=e^{(x+y)}\cdot\frac{d}{dx}(x+y)$$

$$\therefore e^x+e^y\cdot\frac{dy}{dx}=e^{(x+y)}\cdot\left(1+\frac{dy}{dx}\right)$$

$$\therefore e^x+e^y\cdot\frac{dy}{dx}=e^{(x+y)}+e^{(x+y)}\cdot\frac{dy}{dx}$$

$$\therefore [e^y-e^{(x+y)}]\frac{dy}{dx}=e^{(x+y)}-e^x$$

$$\therefore (e^y-e^x-e^y)\frac{dy}{dx}=e^x+e^y-e^x \qquad \ldots \text{[By (1)]}$$

$$\therefore -e^x\cdot\frac{dy}{dx}=e^y$$

$$\therefore \frac{dy}{dx}=-\frac{e^y}{e^x}=-e^{y-x}.$$

EXERCISE 3.5

Q. 1 Find $\boldsymbol{\frac{dy}{dx}}$ ***if,***

1) $\boldsymbol{x=at^2, y=2at}$

Solution:

$x=at^2$, $y=2at$

Differentiating x and y w.r.t. t, we get

$$\frac{dx}{dt}=a\frac{d}{dt}(t^2)=a\times 2t=2at$$

$$\text{and } \frac{dy}{dt}=2a\frac{d}{dt}(t)=2a\times 1=2a$$

$$\therefore \frac{dy}{dx}=\frac{(dy/dt)}{(dx/dt)}=\frac{2a}{2at}=\frac{1}{t}.$$

2) $\boldsymbol{x=2at^2, y=at^4}$

Solution:

Refer to the solution of Q. 1 (1).

Ans. t^2.

3) $\boldsymbol{x=e^{3t}, y=e^{(4t+5)}}$

Solution:

$x = e^{3t}$, $y = e^{(4t+5)}$

Differentiating x and y w.r.t. t, we get

$$\frac{dx}{dt} = \frac{d}{dt}(e^{3t}) = e^{3t} \cdot \frac{d}{dt}(3t)$$

$$= e^{3t} \times 3 \times 1 = 3e^{3t}$$

and $$\frac{dy}{dt} = \frac{d}{dt}[e^{(4t+5)}] = e^{(4t+5)} \cdot \frac{d}{dt}(4t+5)$$

$$= e^{(4t+5)} \times (4 \times 1 + 0) = 4e^{(4t+5)}$$

$$\therefore \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{4e^{(4t+5)}}{3e^{3t}}$$

$$= \frac{4}{3} e^{4t+5-3t} = \frac{4}{3} e^{t+5}.$$

Q. 2 Find $\frac{dy}{dx}$ *if*,

1) $x = \left(u + \frac{1}{u}\right)^2, y = (2)^{\left(u + \frac{1}{u}\right)}$

Solution:

$$x = \left(u + \frac{1}{u}\right)^2, y = 2^{\left(u + \frac{1}{u}\right)} \quad \ldots (1)$$

Differentiating x and y w.r.t. u, we get,

$$\frac{dx}{du} = \frac{d}{du}\left(u + \frac{1}{u}\right)^2 = 2\left(u + \frac{1}{u}\right) \cdot \frac{d}{du}\left(u + \frac{1}{u}\right)$$

$$= 2\left(u + \frac{1}{u}\right)\left(1 - \frac{1}{u^2}\right)$$

and $$\frac{dy}{du} = \frac{d}{du}\left[2^{\left(u + \frac{1}{u}\right)}\right]$$

$$= 2^{\left(u + \frac{1}{u}\right)} \cdot \log 2 \cdot \frac{d}{du}\left(u + \frac{1}{u}\right)$$

$$= 2^{\left(u + \frac{1}{u}\right)} \cdot \log 2 \cdot \left(1 - \frac{1}{u^2}\right)$$

$$\therefore \frac{dy}{dx} = \frac{(dy/du)}{(dx/du)} = \frac{2^{\left(u + \frac{1}{u}\right)} \cdot \log 2 \cdot \left(1 - \frac{1}{u^2}\right)}{2\left(u + \frac{1}{u}\right)\left(1 - \frac{1}{u^2}\right)}$$

$$= \frac{2^{\left(u + \frac{1}{u}\right)} \cdot \log 2}{2\left(u + \frac{1}{u}\right)}$$

$$= \frac{y \log 2}{2\sqrt{x}} \quad \ldots \text{[By (1)]}$$

2) $x = \sqrt{1 + u^2}$, $\quad y = log(1 + u^2)$

Solution:

$x = \sqrt{1 + u^2}$, $y = \log(1 + u^2)$

Differentiating x and y w.r.t. u, we get

$$\frac{dx}{du} = \frac{d}{du}(\sqrt{1 + u^2}) = \frac{1}{2\sqrt{1 + u^2}} \frac{d}{du}(1 + u^2)$$

$$= \frac{1}{2\sqrt{1 + u^2}} \times (0 + 2u) = \frac{u}{\sqrt{1 + u^2}}$$

and $$\frac{dy}{du} = \frac{d}{du}[\log(1 + u^2)]$$

$$= \frac{1}{1 + u^2} \cdot \frac{d}{du}(1 + u^2)$$

$$= \frac{1}{1 + u^2} \times (0 + 2u) = \frac{2u}{1 + u^2}$$

$$\therefore \frac{dy}{dx} = \frac{(dy/du)}{(dx/du)} = \frac{\left(\frac{2u}{1 + u^2}\right)}{\left(\frac{u}{\sqrt{1 + u^2}}\right)}$$

$$= \frac{2u}{1 + u^2} \times \frac{\sqrt{1 + u^2}}{u} = \frac{2}{\sqrt{1 + u^2}}$$

3) *Differentiate* 5^x *with respect to log x*

Let $u = 5^x$ and $v = \log x$.

Then we want to find $\frac{du}{dv}$.

Differentiating u and v w.r.t. x, we get

$$\frac{du}{dx} = \frac{d}{dx}(5^x) = 5^x \cdot \log 5$$

and $$\frac{dv}{dx} = \frac{d}{dx}(\log x) = \frac{1}{x}$$

$$\therefore \frac{du}{dv} = \frac{(du/dx)}{(dv/dx)} = \frac{5^x \cdot \log 5}{\left(\frac{1}{x}\right)}$$

$$= x \cdot 5^x \cdot \log 5.$$

Q.3 Solve the following.

1) $\boldsymbol{If\ x = a\left(1-\frac{1}{t}\right), y = a\left(1+\frac{1}{t}\right)\ then,}$ $\boldsymbol{show\ that\ \frac{dy}{dx} = -1}$

Solution:

$$x = a\left(1-\frac{1}{t}\right),\ y = a\left(1+\frac{1}{t}\right)$$

Differentiating x and y w.r.t. t, we get

$$\frac{dx}{dt} = a\frac{d}{dt}\left(1-\frac{1}{t}\right) = a[0-(-1)t^{-2}] = \frac{a}{t^2}$$

and $\frac{dy}{dt} = a\frac{d}{dt}\left(1+\frac{1}{t}\right) = a[0+(-1)t^{-2}] = -\frac{a}{t^2}$

$$\therefore\ \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{\left(-\frac{a}{t^2}\right)}{\left(\frac{a}{t^2}\right)} = -1.$$

2) $\boldsymbol{If\ x = \frac{4t}{1+t^2}, y = 3\left(\frac{1-t^2}{1+t^2}\right)\ then,}$ $\boldsymbol{show\ that\ \frac{dy}{dx} = \frac{-9x}{4y}}$

Solution:

$$x = \frac{4t}{1+t^2},\ y = 3\left(\frac{1-t^2}{1+t^2}\right)$$

Differentiating x and y w.r.t. t, we get

$$\frac{dx}{dt} = \frac{d}{dt}\left(\frac{4t}{1+t^2}\right) = \frac{(1+t^2)\cdot\frac{d}{dt}(4t) - 4t\cdot\frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$= \frac{(1+t^2)(4) - 4t(0+2t)}{(1+t^2)^2}$$

$$= \frac{4+4t^2-8t^2}{(1+t^2)^2} = \frac{4-4t^2}{(1+t^2)^2}$$

$$= \frac{4(1-t^2)}{(1+t^2)^2}$$

and $\frac{dy}{dt} = 3\frac{d}{dt}\left(\frac{1-t^2}{1+t^2}\right)$

$$= 3\left[\frac{(1+t^2)\cdot\frac{d}{dt}(1-t^2) - (1-t^2)\cdot\frac{d}{dt}(1+t^2)}{(1+t^2)^2}\right]$$

$$= 3\left[\frac{(1+t^2)(0-2t) - (1-t^2)(0+2t)}{(1+t^2)^2}\right]$$

$$= 3\left[\frac{-2t-2t^3-2t+2t^3}{(1+t^2)^2}\right]$$

$$= \frac{-12t}{(1+t^2)^2}$$

$$\therefore\ \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{\left[\frac{-12t}{(1+t^2)^2}\right]}{\left[\frac{4(1-t^2)}{(1+t^2)^2}\right]}$$

$$\therefore\ \frac{dy}{dx} = \frac{-3t}{1-t^2} \quad \ldots (1)$$

$$\frac{-9x}{4y} = \frac{-9}{4}\cdot\frac{\left(\frac{4t}{1+t^2}\right)}{3\left(\frac{1-t^2}{1+t^2}\right)} = \frac{-3t}{1-t^2} \quad \ldots (2)$$

From (1) and (2)

$$\frac{dy}{dx} = -\frac{9x}{4y}.$$

3) If x = t. logt, y = t^t then, show that $\boldsymbol{\frac{dy}{dx} - y = 0}$

Solution:

$$x = t\cdot\log t$$

Differentiating w.r.t. t, we get

$$\frac{dx}{dt} = \frac{d}{dt}(t\cdot\log t)$$

$$= t\frac{d}{dt}(\log t) + (\log t)\cdot\frac{d}{dt}(t)$$

$$= t\times\frac{1}{t} + (\log t)\times 1 = 1 + \log t.$$

Also, $y = t^t$

$\therefore\ \log y = \log t^t = t\log t.$

Differentiating both sides w.r.t. t, we get

$$\frac{1}{y}\cdot\frac{dy}{dt} = \frac{d}{dt}(t\log t)$$

$$= t \cdot \frac{d}{dt}(\log t) + (\log t) \cdot \frac{d}{dt}(t)$$

$$= t \times \frac{1}{t} + (\log t) \times 1$$

$$\therefore \frac{dy}{dt} = y(1 + \log t)$$

$$\therefore \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{y(1+\log t)}{1+\log t} = y$$

$$\therefore \frac{dy}{dx} - y = 0.$$

EXERCISE 3.6

Q. 1 Find $\frac{d^2y}{dx^2}$ *if*,

1) $y = \sqrt{x}$

Solution:

$y = \sqrt{x}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{2\sqrt{x}}\right) = \frac{1}{2}\frac{d}{dx}(x^{-\frac{1}{2}})$$

$$= \frac{1}{2}\cdot\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1} = -\frac{1}{4}x^{-\frac{3}{2}}.$$

2) $y = x^5$

Solution:

$y = x^5$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x^5) = 5x^4$$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(5x^4) = 5\frac{d}{dx}(x^4)$$

$$= 5 \times 4x^3 = 20x^3.$$

3) $y = x^{-7}$

Solution:

$y = x^{-7}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x^{-7}) = -7x^{-8}$$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(-7x^{-8}) = -7\frac{d}{dx}(x^{-8})$$

$$= (-7)(-8)x^{-9} = 56x^{-9}.$$

Q. 2 Find $\frac{d^2y}{dx^2}$ *if*,

1) $y = e^x$

Solution:

$y = e^x$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(e^x) = e^x$$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(e^x) = e^x.$$

2) $y = e^{(2x+1)}$

Solution:

$y = e^{(2x+1)}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}[e^{(2x+1)}] = e^{(2x+1)} \cdot \frac{d}{dx}(2x+1)$$

$$= e^{(2x+1)} \times (2 \times 1 + 0) = 2e^{(2x+1)}$$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{d}{dx}[2e^{(2x+1)}] = 2\frac{d}{dx}[e^{(2x+1)}]$$

$$= 2e^{(2x+1)} \cdot \frac{d}{dx}(2x+1) = 2e^{(2x+1)} \times (2 \times 1 + 0)$$

$$= 4e^{(2x+1)}.$$

3) $y = e^{log x}$

Solution:

$y = e^{\log x} = x$ ... $[\because a^{\log_a x} = x]$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x) = 1$$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(1) = 0.$$