14. Linear Equations in one Variable (With Problems Based on Linear equations)

EXERCISE 14(A)

Solve the following equations:

Question 1. 20 = 6 + 2xSolution: 20 = 6 + 2x 20 - 6 = 2x 14 = 2x 7 = x x = 7Question 2. 15 + x = 5x + 3Solution: 15 - 3 = 5x - x12 = 4x

12 = 4x3 = xx = 3

Question 3.

 $\frac{3x+2}{x-6} = -7$

Solution: 3x + 2 = -7 (x - 6) (by cross multiplying) 3x + 2 = -7x + 42 3x + 7x = 42 - 2 10x = 40x = 4

Question 4.

3a - 4 = 2(4 - a)Solution: 3a - 4 = 8 - 2a3a + 2a = 8+45a = 12a = 2.4

Question 5.

3(b-4) = 2(4-b)

⇒	3b-12 = 8-2b
⇒	3b+2b = 8+12
⇒	5b = 20
⇒	$b = \frac{20}{5}$
⇒	b = 4

Question 6. $\frac{x+2}{9} = \frac{x+4}{11}$ Solution: 11(x+2) = 9(x+4)⇒ (by cross multiplying) 11x+22 = 9x+36⇒ 11x-9x = 36-22⇒ 2x = 14. ⇒ $x = \frac{14}{2}$ ⇒ x = 7⇒ Question 7. $\frac{x-8}{5} = \frac{x-12}{9}$ Solution: 9(x-8) = 5(x-12)⇒ (by cross multiplying) 9x-72 = 5x-60⇒ 9x-5x = -60+72⇒ 4x = 12⇒ $x = \frac{12}{4}$ ⇒

 \Rightarrow x = 3

Question 8.

5(8x + 3) = 9(4x + 7)

⇒	40x + 15 = 36x + 63
⇒	40x - 36x = 63 - 15
⇒`∖	4x = 48
⇒	$x = \frac{48}{4}$
⇒	x = 12

Question 9.

3(x+1) = 12 + 4(x-1)Solution: 3(x + 1) = 12 + 4(x - 1)3x + 3 = 12 + 4x - 43x - 4x = 12 - 4 - 3-x = 5 x = -5

Question 10. $\frac{3x}{4} - \frac{1}{4}(x - 20) = \frac{x}{4} + 32$ Solution:

⇒	$\frac{3x}{4} - \frac{x}{4} + 5 = \frac{x}{4} + 32$
⇒	$\frac{3x}{4} - \frac{x}{4} - \frac{x}{4} = 32 - 5$
⇒	$\frac{3x-x-x}{4} = 27$
⇒	$\frac{x}{4} = 27$
⇒	$x = 27 \times 4$
⇒	x = 108

Question 11. $3a - \frac{1}{5} = \frac{a}{5} + 5\frac{2}{5}$

⇒	$3a - \frac{a}{5} = 5\frac{2}{5} + \frac{1}{5}$
⇒	$3a - \frac{a}{5} = \frac{27}{5} + \frac{1}{5}$
⇒	$3a \times 5 - \frac{a}{5} \times 5 = \frac{27}{5} \times 5 + \frac{1}{5} \times 5$
	(Multiplying each term by 5)
⇒	15a-a = 27+1
⇒	14a = 28
⇒	$a = \frac{28}{14}$
⇒	a = 2

Question 12. $\frac{x}{3} - 2\frac{1}{2} = \frac{4x}{9} - \frac{2x}{3}$ Solution:

$$\Rightarrow \qquad \frac{x}{3} - \frac{5}{2} = \frac{4x}{9} - \frac{2x}{3}$$

Since, L.C.M. of denominators 3,2,9 and 3 = 18

$$\Rightarrow \frac{x}{3} \times 18 - \frac{5}{2} \times 18 = \frac{4x}{9} \times 18 - \frac{2x}{3} \times 18$$

[Multiplying each term by 18]
$$\Rightarrow 6x - 45 = 8x - 12x$$

$$\Rightarrow 6x + 12x - 8x = 45$$

$$\Rightarrow 18x - 8x = 45$$

$$\Rightarrow 10x = 45$$

$$\Rightarrow x = \frac{45}{10}$$

$$\Rightarrow x = 4.5$$

Question 13. $\frac{4(y+2)}{5} = 7 + \frac{5y}{13}$ Solution:

Jution.	
⇒	$\frac{4y+8}{5} = 7 + \frac{5y}{13}$
⇒	$\frac{4y+8}{5} = \frac{91+5y}{13}$
	(by cross multiplying)
⇒	13 (4y+8) = 5(91+5y)
⇒	52y + 104 = 455 + 25y
⇒	52y-25y = 455-104
⇒	27y = 351
⇒	$y = \frac{351}{27}$
⇒	y = 13

Question 14. $\frac{a+5}{6} - \frac{a+1}{9} = \frac{a+3}{4}$ Solution:

Since, L.C.M. of denominators 6,9 and 4 = 36

$$\therefore \quad \frac{a+5}{6} \times 36 - \frac{a+1}{9} \times 36 = \frac{a+3}{4} \times 36$$

(Multiplying each term by 36)

$$\Rightarrow 6(a+5) - 4(a+1) = 9(a+3)$$

$$\Rightarrow 6a+30-4a-4 = 9a+27$$

$$\Rightarrow 6a-4a-9a = 27-30+4$$

$$\Rightarrow 6a-13a = 1$$

$$\Rightarrow -7a = 1$$

$$\Rightarrow a = -\frac{1}{7}$$

Question 15. $\frac{2x-13}{5} - \frac{x-3}{11} = \frac{x-9}{5} + 1$

$$\Rightarrow \frac{2x-13}{5} - \frac{x-3}{11} = \frac{x-9}{5} + \frac{1}{1}$$

Since, L.C.M. of denominators 5,11,5 and 1 = 55

$$\therefore \frac{2x-13}{5} \times 55 - \frac{x-3}{11} \times 55 = \frac{x-9}{5} \times 55 + \frac{1}{1} \times 55$$

$$\Rightarrow 11(2x-13) - 5(x-3) = 11(x-9) + 55$$

$$\Rightarrow 22x - 143 - 5x + 15 = 11x - 99 + 55$$

$$\Rightarrow 22x - 5x - 11x = -99 + 55 + 143 - 15$$

$$\Rightarrow 6x = 198 - 114$$

$$\Rightarrow 6x = 84$$

$$\Rightarrow x = \frac{84}{6}$$

$$\Rightarrow x = 14$$

Question 16.

$$6(6x-5) - 5(7x-8) = 12(4-x) + 1$$

Solution:
 $6(6x-5) - 5(7x-8) = 12(4-x) + 1$
 $36x - 30 - 35x + 40 = 48 - 12x + 1$
 $\Rightarrow x + 12x = 49 - 10$
 $\Rightarrow 13x = 39 \Rightarrow x = \frac{19}{13} \Rightarrow x = 3$

Question 17.

$$(x-5)(x+3) = (x-7)(x+4)$$

Solution:
 $(x-5)(x+3) = (x-7)(x+4)$
 $\Rightarrow x^2 + 3x - 5x - 15 = x^2 + 4x - 7x - 28$
 $\Rightarrow -2x - 15 = -3x - 28$
 $\Rightarrow 3x - 2x = 15 - 28 \Rightarrow x = -13$

Question 18. (x-5)2 - (x+2)2 = -2Solution:

$$(x-5)^{2} - (x+2)^{2} = -2$$

$$\Rightarrow (x^{2} - 10x + 25) - (x^{2} + 4x + 4) = -2$$

$$\Rightarrow x^{2} - 10x + 25 - x^{2} - 4x - 4 = -2$$

$$\Rightarrow -10x - 4x + 25 - 4 = -2$$

$$\Rightarrow -14x = 4 - 2 - 25 = -23$$

$$\Rightarrow x = \frac{-23}{-14} = \frac{23}{14} = 1\frac{9}{14}$$

Question 19. (x - 1) (x + 6) - (x - 2) (x - 3) = 3Solution: (x - 1) (x + 6) - (x - 2) (x - 3) = 3 $x^2 - x + 6x - 6 - (x^2 - 3x - 2x + 6) = 3$ $\Rightarrow x^2 - x + 6x - 6 - x^2 + 3x + 2x - 6 = 3$ $\Rightarrow -x + 6x + 3x + 2x - 6 - 6 = 3$ -x + 11x - 6 - 6 = 3 10x = 15 $x = \frac{15}{10} = \frac{3}{2} = 1\frac{1}{2}$ Ans.

Question 20.

 $\frac{3x}{x+6} - \frac{x}{x+5} = 2$

Solution:

$$\frac{3x}{x+6} - \frac{x}{x+5} = 2$$

$$\Rightarrow \frac{3x(x+5) - x(x+6)}{(x+6)(x+5)} = 2$$

$$\Rightarrow \frac{3x^2 + 15x - x^2 - 6x}{x^2 + 5x + 6x + 30} = 2$$

$$\Rightarrow \frac{2x^2 + 9x}{x^2 + 11x + 30} = 2$$

$$\Rightarrow 2x^2 + 9x = 2 (x^2 + 11x + 30)$$

$$\Rightarrow 2x^2 + 9x = 2x^2 + 22x + 60$$

$$\Rightarrow 2x^2 - 2x^2 + 9x - 22x = 60 \Rightarrow -13x = 60$$

$$x = -\frac{60}{13} = -4\frac{8}{13}$$

Question 21.

$$\frac{1}{x-1} + \frac{2}{x-2} = \frac{3}{x-3}$$
Solution:

$$= \frac{1(x-2)+2(x-1)}{(x-1)(x-2)} = \frac{3}{x-3}$$

$$\Rightarrow \frac{x-2+2x-2}{x^2-2x-x+2} = \frac{3}{x-3}$$

$$\Rightarrow \frac{3x-4}{x^2-3x+2} = \frac{3}{x-3}$$

$$\Rightarrow (x-3) (3x-4) = 3 (x^2-3x+2)$$

$$\Rightarrow 3x^2 - 4x - 9x + 12 = 3x^2 - 9x + 6$$

$$\Rightarrow 3x^2 - 13x - 3x^2 + 9x = 6 - 12$$

$$\Rightarrow -4x = -6$$

$$x = \frac{-6}{-4} = \frac{3}{2} = 1\frac{1}{2}$$
Question 22.

 $\frac{x-1}{7x-14} = \frac{x-3}{7x-26}$

$$\frac{x-1}{7x-14} = \frac{x-3}{7x-26}$$

$$\Rightarrow (x-1) (7x-26) = (7x-14) (x-3)$$

$$\Rightarrow 7x^2 - 7x - 26x + 26 = 7x^2 - 14x - 21x + 42$$

$$\Rightarrow -33x + 26 = -35x + 42$$

$$\Rightarrow 35x - 33x = 42 - 26$$

$$\Rightarrow 2x = 16 \Rightarrow x = 8$$

Question 23. $\frac{1}{x-1} - \frac{1}{x} = \frac{1}{x+3} - \frac{1}{x+4}$ Solution: $\frac{1}{x-1} - \frac{1}{x} = \frac{1}{x+3} - \frac{1}{x+4}$ $\Rightarrow \frac{x - (x-1)}{(x-1)x} = \frac{(x+4) - (x+3)}{(x+3)(x+4)}$ $= \frac{1}{(x-1)x} = \frac{1}{(x+3)(x+4)}$ = (x+3)(x+4) = x(x-1) $\Rightarrow x^2 + 4x + 3x + 12 = x^2 - x$ $\Rightarrow x^2 + 7x - x^2 + x = -12$ 8x = -12 $x = -\frac{12}{8} = -\frac{3}{2} = -1\frac{1}{2}$

Question 24. Solve: $\frac{2x}{3} - \frac{x-1}{6} + \frac{7x-1}{4} = 2\frac{1}{6}$ Hence, find the value of 'a', if $\frac{1}{a}$ + 5x = 8. Solution:

 $\frac{2x}{x-1} + \frac{7x-1}{x-1}$

$$3 = 6 + 4$$

= $2\frac{1}{6}$
= $\frac{2x}{3} - \frac{x-1}{6} + \frac{7x-1}{4}$
= $\frac{13}{6}$
$$\frac{8x - 2x + 2 + 21x - 3 = 26}{12}$$

(L.C.M. of 3, 6, 4, 6 = 12)
 $\Rightarrow 27x - 1 = 26$
 $\Rightarrow 27x = 26 + 1$
 $\Rightarrow x = \frac{27}{27} = 1$

Now,
$$\frac{1}{a} + 5x = 8$$

 $\Rightarrow \frac{1}{a} + 5 \times 1 = 8$
 $\Rightarrow \frac{1}{a} + 5 = 8$
 $\Rightarrow \frac{1}{a} = 8 - 5 = 3$
 $\therefore 3a = 1$
 $\Rightarrow a = \frac{1}{3}$
 $\therefore x = 1 \text{ and } a = \frac{1}{3}$

Question 25. Solve: $\frac{4-3x}{5} + \frac{7-x}{3} + 4\frac{1}{3} = 0$ Hence find the value of 'p' if 2p - 2x + 1 = 0Solution:

$$\Rightarrow \frac{4-3x}{5} + \frac{7-x}{3} + 4\frac{1}{3} = 0$$

$$\Rightarrow \frac{4-3x}{5} + \frac{7-x}{3} + \frac{13}{3} = 0$$

$$\frac{12-9x+35-5x+65=0}{15}$$

(L.C.M. of 5, 3, 3 = 15)

$$-14x + 112 = 0$$

$$\Rightarrow -14x = -112$$

$$\Rightarrow x = \frac{-112}{-14}$$

$$= 8$$

Hence x = 8
Now, 3p - 2x + 1=0

$$\Rightarrow 3p - 16 + 1 = 0$$

$$\Rightarrow 3p - 15 = 0.$$

 \Rightarrow 3p=15 $\Rightarrow p = 5$

Question 26. Solve: $0.25 + \frac{1.95}{x} = 0.9$

Solution:

$$0.25 + \frac{1.95}{x} = 0.9$$
$$0.25x + 1.95 = 0.9x$$
$$\Rightarrow 0.9x - 0.25x = 1.95$$
$$\Rightarrow 0.65x = 1.95$$
$$\Rightarrow x = \frac{1.95}{0.65} = 3$$
Hence, $x = 3$

Question 27. Solve: $5x - (4x + \frac{5x-4}{7}) = \frac{4x-14}{3}$ Solution: (x - 5x - 4) = 4x - 14

$$5x - \left(\frac{4x + \frac{5x - 4}{7}}{7}\right) = \frac{4x - 14}{3}$$

$$5x - \left(\frac{28x + 5x - 4}{7}\right) = \frac{4x - 14}{3}$$

$$\frac{35x - 33x + 4}{7} = \frac{4x - 14}{3}$$

$$3 \times (2x + 4) = 7 \times (4x - 14)$$

$$6x + 12 = 28x - 98$$

$$22x = 98 + 12$$

$$x = \frac{110}{22} = 5$$

EXERCISE 14(B)

Question 1.

Fifteen less than 4 times a number is 9. Find the number. Solution: Let the required number be x 4 times the number = 4x

15 less than 4 times the number = 4x-15

According to the statement : 4x - 15 = 9 $\Rightarrow 4x = 9 + 15$

 $\Rightarrow 4x = 24$ $\Rightarrow x = 6$

Question 2.

If Megha's age is increased by three times her age, the result is 60 years. Find her age **Solution:**

Let Megha's age = x years Three times Megha's age = 3x years According to the statement : x + 3x = 60=> 4x = 60=> x = 15Megha's age = 15 years

Question 3.

28 is 12 less than 4 times a number. Find the number. **Solution:** Let the required number be x 4 times the number = 4x 12 less than 4 times the number = 4x - 12According to the statement 4x - 12 = 28 => 4x = 28 + 12 => 4x = 40 x = 10Required number = 10

Question 4.

Five less than 3 times a number is -20. Find the number. **Solution:** Let the required number = x 3 times the number = 3x5 less than 3 times the number = 3x - 5According to statement : 3x - 5 = -20 $\Rightarrow 3x = -20 + 5$ $\Rightarrow 3x = -15$ $\Rightarrow x = -5$ Required number = -5

Question 5.

Fifteen more than 3 times Neetu's age is the same as 4 times her age. How old is she?

Let Neetu's age = x years 3 times Neetu's age = 3x years Fifteen more than 3 times Neetu's age = (3x + 15) years 4 times Neetu's age = 4xAccording to the statement : 4x = 3x + 15 => 4x - 3x = 15 => x = 15Neetu's age = 15 years

Question 6.

A number decreased by 30 is the same as 14 decreased by 3 times the number; Find the number.

Solution:

Let the required number = x The number decreased by 30 = x - 3014 decreased by 3 times the number = 14 - 3xAccording to the statement : x - 30 = 14 - 3x => x + 3x = 14 + 30 => 4x = 44 x = 11Required number =11

Question 7.

A's salary is same as 4 times B's salary. If together they earn Rs.3,750 a month, find the salary of each. **Solution:** Let B's salary = Rs. x A's salary = Rs. 4x According to the statement : x + 4x = 3750=> 5x = 3750=> x = 750 $4x = 750 \times 4 = 3000$ A's salary = Rs. 3000 B's salary = Rs. 750

Question 8.

Separate 178 into two parts so that the first part is 8 less than twice the second part. **Solution:**

Let first part = x Second part = 178 - xAccording to the problem :

First Part = 8 less than twice the second part x = 2(178 - x) - 8=> x = 356 − 2x − 8 => x+2x = 356 - 8=> 3x = 348 => x = 116 First Part = 116 => Second Part = 178 - x = 178 - 116 = 62 First Part = 116 => Second Part = 62 Alternative Method : Let Second part = xFirst part = 2x - 8According to the problem : x + 2x - 8 = 178=> x + 2x = 178 + 8 => 3x = 186 => x = 62 First part = $2x - 8 = 2 \times 62 - 8 = 124 - 8 = 116$

Second part = 62

Question 9.

Six more than one-fourth of a number is two-fifth of the number. Find the number. **Solution:**

Let the required number = x

 \therefore One-fourth of the number = $\frac{x}{4}$

Two-fifth of the number =
$$\frac{2x}{5}$$

According to the statement :

	$\frac{2x}{5} = 6 + \frac{x}{4}$
⇒	$\frac{2x}{5} - \frac{x}{4} = \frac{6}{1}$
$\Rightarrow \frac{2x}{5}$	$\times 20 - \frac{x}{4} \times 20 = 6 \times 20$
	[Multiplying each term by 20 because L.C.M. of 5,4 and 1 = 20]
\Rightarrow	8x - 5x = 120
⇒	3x = 120
⇒	$x = \frac{120}{3}$

Required number = 40x = 40

Question 10.

The length of a rectangle is twice its width. If its perimeter is 54 cm; find its length. **Solution:**

Let width of the rectangle = x cm Length of the rectangle = 2x cm Perimeter of the rectangle = 2 [Length + Width] = 2 $[2x + x] = 2 \times 3x = 6x$ cm Given perimeter = 54 cm 6x = 54 => x = 9Length = $2x = 2 \times 9 = 18$ cm

Question 11.

A rectangle's length is 5 cm less than twice its width. If the length is decreased by 5 cm and width is increased by 2 cm; the perimeter of the resulting rectangle will be 74 cm. Find the length and the width of the origi¬nal rectangle.

Solution:

Let width of the original rectangle = x cm Length of the original rectangle = (2x - 5)cm Now, new length of the rectangle = 2x - 5 - 5 = (2x - 10) cm New width of the rectangle = (x + 2) cm New perimeter = 2[Length+Width] = 2[2x - 10 + x + 2] = 2[3x - 8] = (6x - 16) cm Given; new perimeter = 74 cm 6x - 16 = 74=> 6x = 74 + 16=> 6x = 90=>x = 15Length of the original rectangle = $2x - 5 = 2 \times 15 - 5 = 30 - 5 = 25$ cm Width of the original rectangle = x = 15 cm

Question 12.

The sum of three consecutive odd numbers is 57. Find the numbers. **Solution:** Let the three consecutive odd numbers be x, x+2, x+4. According to the statement : x + x + 2 + x + 4 = 57 => x + x + x = 57 - 2 - 4 => 3x = 51 => x = 17Three consecutive odd numbers are 17, 19, 21

Question 13.

A man's age is three times that of his son, and in twelve years he will be twice as old as his son would be. What are their present ages.

Solution:

Let present age of the son = x years present age of the man = 3x years In 12 years : Son's age will be = (x + 12) years The man's age will be = (3x + 12) years According to the statement : 3x + 12 = 2(x + 12)=> 3x + 12 = 2x + 24=> 3x - 2x = 24 - 12=> x = 12 $3x = 3 \times 12 = 36$ Hence, present age of the man = 36 years Present age of the son = 12 years.

Question 14.

A man is 42 years old and his son is 12 years old. In how many years will the age of the son be half the age of the man at that time?

Solution:

Man's age = 42 years Son's age = 12 years Let after x years the age of the son will be half the age of the man. Man's age after x years = (42 + x) years Son's age after x years = (12 + x) years According to the statement :

	$12+x = \frac{42+x}{2}$
\Rightarrow	2(12+x) = 42+x
	(by cross multiplying)
⇒	24+2x = 42+x
⇒	2x - x = 42 - 24
⇒	x = 18

Hence after 18 years, the age of the son will be half the age of the man

Question 15.

A man completed a trip of 136 km in 8 hours. Some part of the trip was covered at 15 km/hr and the remaining at 18 km/hr. Find the part of the trip covered at 18 km/hr. **Solution:**

Total distance of the trip = 136 km. Let part of the trip covered at 18 km/hr.

= x km.

 \therefore Distance of the trip covered at 15 km/hr = (136-x) km

Time taken by the man to cover x km

$$= \frac{\text{Distance}}{\text{Speed}} = \frac{x}{18} \text{ hours}$$

Time taken by the man to cover (136-x) km

$$=\frac{136-x}{15}$$
 hours

Total time taken by the man to cover a trip of 136 km = 8 hours



Question 16.

The difference of two numbers is 3 and the difference of their squares is 69. Find the numbers.

Solution:

Let one number = xSecond number = x + 3 [Difference of two numbers is 3] According to the statement :

	$(x+3)^2 - (x)^2 = 69$
⇒	$(x)^2 + (3)^2 + 2 \times x \times 3 - x^2 = 69$
⇒	$x^2 + 9 + 6x - x^2 = 69$
⇒	6x = 69-9
⇒	6x = 60
⇒	$x = \frac{60}{6}$
⇒	x = 10

One number = 10Second number = x + 3 = 10 + 3 = 13

Question 17.

Two consecutive natural numbers are such that one-fourth of the smaller exceeds onefifth of the greater by 1. Find the numbers.

Solution:

Let two consecutive natural numbers = x, x+1

 \therefore One-fourth of the smaller = $\frac{x}{4}$

One-fifth of the greater = $\frac{x+1}{5}$

According to the statement :

$$\frac{x}{4} = \frac{x+1}{5} + 1 \implies \frac{x}{4} - \frac{x+1}{5} = 1$$

$$\Rightarrow \frac{5x-4(x+1)}{20} = 1 \implies \frac{5x-4x-4}{20} = 1$$

$$\Rightarrow \frac{x-4}{20} = 1$$

$$\Rightarrow x - 4 = 20$$
(Cross-multiplying)
$$\Rightarrow x = 20 + 4 \implies x = 24$$

$$\therefore x + 1 = 24 + 1 = 25$$
Two consecutive numbers are 24 and 25

Question 18.

Three consecutive whole numbers are such that if they be divided by 5, 3 and 4 respectively; the sum of the quotients is 40. Find the numbers.

Solution:

Let the three consecutive whole numbers be x, x + 1 and x + 2According to the statement:

$$\frac{x}{5} + \frac{x+1}{3} + \frac{x+2}{4} = 40$$

$$\Rightarrow \frac{x}{5} \times 60 + \frac{x+1}{3} \times 60 + \frac{x+2}{4} \times 60 = 40 \times 60$$
[Multiplying each term by 60 because
L.C.M. of denominators = 60]
$$\Rightarrow 12x + 20(x+1) + 15(x+2) = 2400$$

$$\Rightarrow 12x + 20x + 15x + 30 = 2400$$

$$\Rightarrow 12x + 20x + 15x = 2400 - 20 - 30$$

$$\Rightarrow 47x = 2350$$

$$\Rightarrow 47x = 2350$$

$$\Rightarrow x = \frac{2350}{47}$$

$$x = 50$$

$$x + 1 = 50 + 1 = 51$$

$$x + 2 = 50 + 2 = 52$$
Three consecutive whole numbers are 50, 51 and 52

Question 19.

If the same number be added to the numbers 5, 11, 15 and 31, the resulting numbers are in proportion. Find the number.

Solution:

Let x be added to each number, then the numbers will be 5 + x, 11 + x, 15 + x and 31 + x

According to the condition

$$\frac{5+x}{11+x} = \frac{15+x}{31+x}$$

By cross multiplication,

(5+x)(31+x) = (15+x)(11+x)

- $\Rightarrow 155 + 5x + 31x + x^2 = 165 + 11x + 15x + x^2$
- $\Rightarrow 155 + 36x + x^2 = 165 + 26x + x^2$
- $\Rightarrow 36x + x^2 26x x^2 = 165 155$

$$\Rightarrow 10x = 10 \Rightarrow x = \frac{10}{10} = 1$$

Question 20.

The present age of a man is twice that of his son. Eight years hence, their ages will be in the ratio 7 : 4. Find their present ages.

Solution:

Let present age of son = x year Then age of his father = 2x 8 years hence, Age of son = (x + 8) years and age of father = (2x + 8) years According to the condition, $\frac{2x+8}{x+8} = \frac{7}{4}$ => 8x + 32 = 7x + 56 => 8x - 7x = 56 - 32 => x = 24 Present age of son = 24 years and age of father = 2x = 2 x 24 = 48 years Hence age of man = 48 years and age of his son = 24 years

EXERCISE 14(C)

Question 1.

Solve:
(i)
$$\frac{1}{3}x - 6 = \frac{5}{2}$$

(ii) $\frac{2x}{3} - \frac{3x}{8} = \frac{7}{12}$
(iii) $(x + 2)(x + 3) + (x - 3)(x - 2) - 2x(x + 1) = 0$
(iv) $\frac{1}{10} - \frac{7}{x} = 35$
(v) $13(x - 4) - 3(x - 9) - 5(x + 4) = 0$
(vi) $x + 7 - \frac{8x}{3} = \frac{17x}{6} - \frac{5x}{8}$
(vii) $\frac{3x - 2}{4} - \frac{2x + 3}{3} = \frac{2}{3} - x$
(viii) $\frac{x + 2}{4} - \frac{(11 - x)}{3} - \frac{1}{4} = \frac{3x - 4}{12}$
(ix) $\frac{2}{5x} - \frac{5}{3x} = \frac{1}{15}$
(x) $\frac{x + 2}{3} - \frac{x + 1}{5} = \frac{x - 3}{4} - 1$
(xi) $\frac{3x - 2}{3} + \frac{2x + 3}{2} = x + \frac{7}{6}$
(xii) $x - \frac{x - 1}{2} = 1 - \frac{x - 2}{3}$
(xiii) $\frac{9x + 7}{2} - (x - \frac{x - 2}{7}) = 36$
(xiv) $\frac{6x + 1}{2} + 1 = \frac{7x - 3}{3}$
Solution:
 $x = (i) \frac{1}{3}x - 6 = \frac{5}{2}$
 $\Rightarrow \frac{1}{3}x = \frac{5}{2} + \frac{6}{1}$

$$\Rightarrow \frac{1}{3}x = \frac{5 \times 1}{2 \times 1} + \frac{6 \times 2}{1 \times 2}$$

$$\Rightarrow \frac{1}{3}x = \frac{5}{2} + \frac{12}{2}$$

$$\Rightarrow \frac{1}{3}x = \frac{5 + 12}{2}$$

$$= \frac{1}{3}x = \frac{17}{2}$$

$$= x = \frac{17 \times 3}{2 \times 1} = \frac{51}{2} = 25\frac{1}{2}$$

(ii) $\frac{2x}{3} - \frac{3x}{8} = \frac{7}{12}$
L.C.M. of 3 and $8 = 2 \times 2 \times 2 \times 3 = 24$

$$\therefore \frac{2x \times 8}{3 \times 8} - \frac{3x \times 3}{8 \times 3} = \frac{7}{12}$$
$$= \frac{16x}{24} - \frac{9x}{24} = \frac{7}{12}$$

$$= \frac{16x}{24} - \frac{9x}{24} = \frac{7}{12}$$

$$= \frac{16x - 9x}{24} = \frac{7}{12}$$

$$= \frac{7x}{24} = \frac{7}{12}$$

$$= x = \frac{7 \times 24}{12 \times 7} = 2$$

$$\therefore x = 2$$
(*iii*) $(x + 2)(x + 3) + (x - 3)(x - 2) - 2x(x + 1) = 0$
Sol. $(x + 2)(x + 3) + (x - 3)(x - 2) - 2x(x + 1) = 0$

$$\Rightarrow [x^2 + (2 + 3)x + 2 \times 3] + [x^2 + (-3 - 2)x + (-3)(-2)] - 2x^2 - 2x = 0$$

$$\Rightarrow x^2 + 5x + 6 + x^2 - 5x + 6 - 2x^2 - 2x = 0$$

$$\Rightarrow x^2 + 5x + 6 + x^2 - 5x - 6 - 2x^2 - 2x = 0$$

$$\Rightarrow x^2 + x^2 - 2x^2 + 5x - 5x - 2x + 6 + 6 = 0$$

$$= -2x + 12 = 0$$
Subtracting 12 from both sides,
 $-2x + 12 - 12 = 0 - 12 \Rightarrow -2x = -12$
Dividing by -2

$$\frac{-2x}{-2} = \frac{-12}{-2} \Rightarrow x = 6$$

$$\therefore x = 6$$
Verification
L.H.S. = $(x + 2)(x + 3) + (x - 3)(x - 2) - 2x(x + 1)$

$$= (6 + 2)(6 + 3) + (6 - 3)(6 - 2) - 2 \times 6(6 + 1)$$

$$= 8 \times 9 + 3 \times 4 - 12 \times 7$$

$$= 72 + 12 - 84 = 84 - 84 = 0 = R.H.S.$$

$$(iv) \frac{1}{10} - \frac{7}{x} = 35$$

$$\Rightarrow \frac{-7}{x} = 35 - \frac{1}{10}$$

$$\Rightarrow \frac{-7}{x} = \frac{35 \times 10}{1 \times 10} - \frac{1 \times 1}{10 \times 1}$$

$$\Rightarrow \frac{-7}{x} = \frac{350 - 1}{10}$$

$$\Rightarrow \frac{1}{x} = \frac{350 - 1}{10 \times (-7)}$$

$$\Rightarrow x = \frac{349}{(-70)} = \frac{-70}{349}$$

$$(v) 13(x - 4) - 3(x - 9) - 5(x + 4) = 0$$

$$\Rightarrow 13(x - 4) - 3(x - 9) - 5(x + 4) = 0$$

$$\Rightarrow 13x - 52 - 3x + 27 - 5x - 20 = 0$$

$$\Rightarrow 13x - 3x - 5x - 52 + 27 - 20 = 0$$

$$\Rightarrow 13x - 8x - 72 + 27 = 0$$

$$\Rightarrow 5x - 45 = 0$$

Dividing by 5,

$$5x - 45$$

$$\frac{5x}{5} - \frac{45}{5} = 0 \Rightarrow x - 9 = 0 \Rightarrow x = 9$$

Verification,
L.H.S. =
$$13(x - 4) - 3(x - 9) - 5(x + 4)$$

= $13(9 - 4) - 3(9 - 9) - 5(9 + 4)$
= $13 \times 5 - 3 \times 0 - 5 \times 13$
= $65 - 0 - 65 = 0 = R.H.S.$
(vi) $x + 7 - \frac{8x}{3} = \frac{17x}{6} - \frac{5x}{8}$
 $\Rightarrow \frac{3(x + 7) - 8x}{3} = \frac{17x \times 4 - 5x \times 3}{24}$
 $\Rightarrow \frac{3x + 21 - 8x}{3} = \frac{68x - 15x}{24}$
 $\Rightarrow \frac{-5x + 21}{3} = \frac{53x}{24}$
 $\Rightarrow 3 \times 53x = 24(-5x + 21)$
 $\Rightarrow 159x = -120x + 504$
 $\Rightarrow 159x + 120x = 504$
 $\Rightarrow 279x = 504$
 $\Rightarrow x = \frac{504}{279} = \frac{168}{93} = \frac{56}{61}$
 $\therefore x = 1\frac{25}{31}$
(vii) $\frac{3x - 2}{4} - \frac{2x + 3}{3} = \frac{2}{3} - x$
 $= \frac{3(x - 2) - 4(2x + 3)}{12} = \frac{2 \times 1}{3 \times 1} - \frac{x \times 3}{1 \times 3}$
 $= \frac{9x - 6 - 8x - 12}{12} = \frac{2 - 3x}{3}$
 $= 3(x - 18) = 12(2 - 3x)$
 $= 3x - 54 = 24 - 36x$
 $= 3x + 36x = 24 + 54$

$$= 39x = 78$$

 $x = \frac{78}{39} = 2$
 $\therefore x = 2$
(viii) $\frac{x+2}{6} - \left(\frac{11-x}{3} - \frac{1}{4}\right) = \frac{3x-4}{12}$
 $\Rightarrow \frac{x+2}{6} - \left(\frac{4(11-x)-1\times3}{12}\right) = \frac{3x-4}{12}$
 $\Rightarrow \frac{x+2}{6} - \frac{44+4x+3}{12} = \frac{3x-4}{12}$
 $\Rightarrow \frac{2(x+2)-41+4x}{12} = \frac{3x-4}{12}$
 $\Rightarrow \frac{2x+4-41+4x}{12} = \frac{3x-4}{12}$
 $\Rightarrow \frac{6x-37}{12} = \frac{3x-4}{12}$
 $\Rightarrow \frac{6x-37}{12} = \frac{3x-4}{12}$
 $\Rightarrow 12(6x - 37) = 12(3x - 4)$
 $\Rightarrow 72x - 444 = 36x - 48$
 $\Rightarrow 72x - 36x = -48 + 444$
 $\Rightarrow 36x = 396$
 $\Rightarrow x = \frac{396}{36} = 11$
 $\therefore x = 11$
(ix) $\frac{2}{5x} - \frac{5}{3x} = \frac{1}{15}$
 $\Rightarrow \frac{2\times3}{5x\times3} - \frac{5\times5}{3x\times5} = \frac{1}{15}$
 $\Rightarrow \frac{-19}{15x} = \frac{1}{15}$

$$\Rightarrow -19 = x$$

$$\therefore x = -19$$
(x) $\frac{x+2}{3} - \frac{x+1}{5} = \frac{x-3}{4} - 1$
(L.C.M. of 3 and 5 = 15)

$$\Rightarrow \frac{5(x+2)-3(x+1)}{15} = \frac{x-3-4}{4}$$

$$\Rightarrow \frac{5x+10-3x-3}{15} = \frac{x-7}{4}$$

$$\Rightarrow \frac{2x+7}{15} = \frac{x-7}{4}$$

$$\Rightarrow 4(2x+7) = 15(x-7)$$

$$\Rightarrow 8x + 28 = 15x - 105$$

$$\Rightarrow 8x - 15x = -105 - 28$$

$$\Rightarrow -7x = -133$$

$$x = \frac{-133}{-7}$$

$$\therefore x = 19$$
(xi) $\frac{3x-2}{3} + \frac{2x+3}{2} = x + \frac{7}{6}$

$$\Rightarrow \frac{2(3x-2)+3(2x+3)}{6} = x + \frac{7}{6}$$

$$\Rightarrow \frac{6x-4+6x+9}{6} = \frac{6x+7}{6}$$

$$\Rightarrow \frac{12x+5}{6} = \frac{6x+7}{6}$$

$$\Rightarrow \frac{12x+5}{6} = \frac{6x+7}{6}$$

$$\Rightarrow 6(12x+5) = 6(6x+7)$$

$$\Rightarrow 72x+30 = 36x - 42$$

$$\Rightarrow 72x - 36x = 42 - 30$$

$$\Rightarrow 36x = 12$$

$$x = \frac{12}{36}$$

$$\therefore x = \frac{1}{3}$$

$$(xii) \ x - \frac{x-1}{2} = 1 - \frac{x-2}{3}$$

$$\Rightarrow \frac{2(x)-1(x-1)}{2} = \frac{3(1)-1(x-2)}{3}$$

$$\Rightarrow \frac{2x-x+1}{2} = \frac{3-x+2}{3}$$

$$\Rightarrow \frac{1x+1}{2} = \frac{5-x}{3}$$

$$\Rightarrow 3(x+1) = 2(5-x)$$

$$\Rightarrow 3x+3 = 10-2x$$

$$\Rightarrow 3x+2x = 10-3$$

$$\Rightarrow 5x = 7$$

$$\therefore x = \frac{7}{5}$$

$$(xiii) \ \frac{9x+7}{2} - \left(x - \frac{x-2}{7}\right) = 36$$

$$\Rightarrow \frac{9x+7}{2} - \left(\frac{7\times x - 1(x-2)}{7}\right) = 36$$

$$\Rightarrow \frac{9x+7}{2} - \left(\frac{6x-2}{7}\right) = 36$$

$$\Rightarrow \frac{9x+7}{2} - \left(\frac{6x-2}{7}\right) = 36$$

$$\Rightarrow \frac{7(9x+7) + 2(-6x+2)}{14} = 36$$

$$\Rightarrow \frac{63x+49-12x+4}{14} = 36$$

$$\Rightarrow \frac{51x + 53}{14} = 36$$

$$\Rightarrow 51x + 53 = 14 \times 36$$

$$\Rightarrow 51x = 504 - 53$$

$$\Rightarrow 51x = 459$$

$$\Rightarrow x = \frac{459}{51}$$

$$\therefore x = 9$$

$$(xiv) \frac{6x+1}{2} + 1 = \frac{7x-3}{3}$$

$$\Rightarrow \frac{(6x+1)+1\times 2}{2} = \frac{7x-3}{3}$$

$$\Rightarrow \frac{6x+1+2}{2} = \frac{7x-3}{3}$$

$$\Rightarrow \frac{6x+3}{2} = \frac{7x-3}{3}$$

$$\Rightarrow 3(6x+3) = 2(7x-3)$$

$$\Rightarrow 18x+9 = 14x-6$$

$$\Rightarrow 18x-14x = -6-9$$

$$\Rightarrow 4x = -15$$

$$\therefore x = \frac{-15}{4}$$

Question 2.

After 12 years, I shall be 3 times as old as 1 was 4 years ago. Find my present age. **Solution:** Let present age = x years According to question, (x + 12) = 3(x - 4)x + 12 = 3x - 12

2x = 24=> x = 12 years Present age = 12 years

Question 3.

A man sold an article for 7396 and gained 10% on it. Find the cost price of the article **Solution:** S.P. of article = ₹ 396 Gain = 10% Let cost price = ₹ x

$$\therefore \text{ S.P.} = \frac{x \times (100 + 10)}{100} = \frac{110}{100}x$$
$$\therefore \frac{110}{100}x = 396$$
$$\Rightarrow x = \frac{396 \times 100}{110} = 360$$

Cost price of an article = ₹ 360

Question 4.

The sum of two numbers is 4500. If 10% of one number is 12.5% of the other, find the numbers.

Solution:

Let the first number = x and the second number = y According to question, $x + y = 4500 \dots(i)$ and 10% x = 12.5% y i.e. 10x = 12.5y $x = \frac{12.5}{10}y$...(*ii*)

Substitute the value of x in equation (i),

$$\frac{12.5}{10}y + y = 45,000$$

12.5y + 10y = 45,000
22.5y = 45,000
45,000

$$y = \frac{1}{22.5} = 2000$$

Now, put the value of y in equation (ii)

$$x = \frac{12.5}{10} \times 2000$$

x = 2500Hence, the numbers are 2500 and 2000

Question 5. The sum of two numbers is 405 and their ratio is 8 : 7. Find the numbers. Solution:

Let the first number = x and the second number = 7 According to the question, x + y = 405(i) and the numbers are in the ratio 8 : 7

$$i.e. \ \frac{8x}{7y} = 1$$
$$\Rightarrow 8x = 7y$$
$$\Rightarrow x = \frac{7}{8}y$$

Now, substitute the value of x in equation (i)

$$\frac{7}{8}y + y = 405$$

7y + 8y = 405 × 8
15y = 3240
 $y = \frac{3240}{2}$

$$y = -15$$

$$y = 216$$

Now, put the value of y in equation (ii)

$$x=\frac{7}{8}\times 216$$

x = 189Hence, the numbers are 189 and 216

Question 6.

The ages of A and B are in the ratio 7 : 5. Ten years hence, the ratio of their ages will be 9 : 7. Find their present ages.

Solution:

Ratio in the present ages of A and B = 7 : 5 Let age of A = 7x years Let age of B = 5x years 10 years hence, Then age of A = 7x + 10 years and age of B = 5x + 10 years According to the condition, $\frac{7x+10}{5x+10} = \frac{9}{7}$ By crossing multiplication 7(7x + 10) = 9(5x + 10) => 49x + 70 = 45x + 90 => 49x - 45x = 90 - 70=> 4x = 20=> x = 5Present age of A = $7x = 7 \times 5 = 35$ years and present age of B = $5x = 5 \times 5 = 25$ years

Question 7.

Find the number whose double is 45 greater than its half.

Solution:

Let the required number = xDouble of it = 2x

and half of it = $\frac{x}{2}$

According to the condition,

$$2x - \frac{x}{2} = 45$$
$$\Rightarrow \frac{4x - x}{2} = 45 \Rightarrow \frac{3}{2}x = 45$$

$$\Rightarrow x = \frac{45 \times 2}{3} = 30$$

Required number = 30

Question 8.

The difference between the squares of two consecutive numbers is 31. Find the numbers.

Solution:

Let first number = xand The second number = x + 1According to the condition,

$$\therefore (x + 1)^2 - (x)^2 = 31$$

$$\Rightarrow x^2 + 2x + 1 - x^2 = 31$$

$$\Rightarrow 2x = 31 - 1 = 30$$

$$\Rightarrow x = \frac{30}{2} = 15$$

First number = 15and second number = 15 + 1 = 16Hence, the numbers are 15, 16

Question 9.

Find a number such that when 5 is subtracted from 5 times the number, the result is 4 more than twice the number.

Solution:

Let the required number = x 5 times of it = 5x Twice of it = 2x According to the condition, 5x - 5 = 2x + 4=> 5x - 2x = 4 + 5=> 3x = 9=> x = 3Required number = 3

Question 10.

The numerator of a fraction is 5 less than its denominator. If 3 is added to the numerator, and denominator both, the fraction becomes $\frac{2}{3}$. Find the original fraction. **Solution:**

Let denominator of the original fraction = x

Then numerator = x - 5

and fraction = $\frac{x-5}{x}$

According to the condition,

$$\frac{x-5+3}{x+3} = \frac{4}{5}$$

$$\Rightarrow \frac{x-2}{x+3} = \frac{4}{5}$$

$$\Rightarrow 5(x-2) = 4x + 12$$
(By cross multiplication)
$$\Rightarrow 5x - 10 = 4x + 12$$

$$\Rightarrow x = 22$$

$$\therefore \text{ Original fraction} = \frac{x-5}{x}$$

$$= \frac{22-5}{22} = \frac{17}{22}$$