Chapter 5

Simple Interest and Compound Interest

CHAPTER HIGHLIGHTS

- *Interest*
- *Simple Interest*
- *Compound Interest*
- *Compounding More Than Once a Year*
- *Present Value*
- *Repayment in Equal Instalments*

Interest

Interest is money paid to the lender by the borrower for using his money for a specified period of time. Various terms and their general representation are as follows:

1. INTEREST
   Money paid by borrower for using the lender's money. Denoted by I.
2. PRINCIPAL
   The original sum borrowed. Denoted by P.
3. TIME
   Time for which money is borrowed. Denoted by n. (n is expressed in number of periods, which is normally one year.)
4. RATE OF INTEREST
   Rate at which interest is calculated on the original sum. Denoted by r and is expressed as a percentage or decimal fraction.
5. AMOUNT
   Sum of principal and interest. Denoted by A.

Simple Interest

When interest is calculated every year (or every time period) on the original principal, i.e. the sum at the beginning of first year, such interest is called simple interest.

Here, year after year, even though the interest gets accumulated and is due to the lender, this accumulated interest is not taken into account for the purpose of calculating interest for latter years.

$$\text{Simple Interest} = \frac{Pnr}{100}$$

where P, n, r are as explained above.

$$\text{Total Amount } A = P + \frac{Pnr}{100} = P\left(1+\frac{nr}{100}\right)$$

Compound Interest

Under compound interest, the interest is added to the principal at the end of each period to arrive at the new principal for the next period.

In other words, the amount at the end of first year (or period) will become the principal for the second year (or period); the amount at the end of second year (or period) becomes the principal for the third year (or period); and so on.

If P denotes the principal at the beginning of Period 1, then, principal at the beginning of Period 2

$$= P\left(1+\frac{r}{100}\right)$$

$$= PR = \text{Amount at the end of Period 1,}$$

where $R = \left\{1+\left(\frac{r}{100}\right)\right\}$

P at the beginning of Period 3 = $P\left(1+\frac{r}{100}\right)^2$

$= PR^2$ = Amount at the end of Period 2

P at the beginning of Period $(n + 1)$

$$= P\left(1+\frac{r}{100}\right)^n = PR^n$$

= Amount at the end of Period n

Hence, the amount after n years (periods) = $PR^n = A$

$$\text{Interest} = I = A - P = P\,[R^n - 1]$$

The following table gives an example of how simple interest and compound interest operate, i.e. how the principal is for various years under simple interest and compound interest. A principal at the beginning of 1st year, of ₹100 and a rate of 10% p.a. are considered. The details are worked out for three years and shown below.

(All figures pertaining to principal, interest, and amount are in rupees)

	Under Simple Interest				Under Compound Interest			
Year	Principal at the beginn. of the year	Interest for the year	Interest till the end of the year	Amount at the end of the year	Principal at the beginn. of the year	Interest for the year	Interest till the end of the year	Amount at the end of the year
1	100	10	10	110	100	10	10	110
2	100	10	20	120	110	11	21	121
3	100	10	30	130	121	12.1	33.1	133.1

As can be seen from the table,

In case of simple interest,

1. The principal remains the same every year.
2. The interest for any year is the same as that for any other year.

In case of compound interest,

1. The amount at the end of an year is the principal for the next year.
2. The interest for different years is not the same.

The compound interest for the first year (where compounding is done every year) is the same as the simple interest for one year.

Compounding More Than Once a Year

We just looked at calculating the amount and interest when the compounding is done once a year. But, compounding can also be done more frequently than once a year. For example, the interest can be added to the principal every six months or every four months and so on.

If the interest is added to the principal every six months, we say that compounding is done twice a year. If the interest is added to the principal every four months, we say that compounding is done thrice a year. If the interest is added to the principal every three months, we say that compounding is done four times a year.

The formula that we discussed above for calculating the amount will essentially be the same, i.e.

$$\text{Amount} = P\left(1+\frac{r}{100}\right)^n$$

where r = rate % *per annum* and n = number of *years*, but the rate will *not* be for ONE YEAR but for the time period over which compounding is done and the power to which the term inside the bracket is raised (n in the above case) will *not* be the number of years but the number of years multiplied by the number of times compounding is done per year (this product is referred to as the total number of time periods).

For example, if a sum of ₹10000 is lent at the rate of 10% per annum and the compounding is done for every four months (thrice a year), then the amount will be equal to

$$10000\left(1+\frac{10}{3}\times\frac{1}{100}\right)^{2\times 3}$$

Here, the dividing factor of 3 in the rate and the multiplying factor of 3 in the power (multiplying the number of years)—both shown by arrow marks—are nothing but the NUMBER OF TIMES compounding is done in a year.

If compounding is done k times a year (i.e. once every 12/k months), at the rate of r% p.a. then in n years, the principal of P will amount to = $P\left(1+\frac{r}{k\cdot 100}\right)^{kn}$

When compounding is done more than once a year, the rate of interest given in the problem is called NOMINAL RATE OF INTEREST.

We can also calculate a rate of interest which will yield simple interest in one year equal to the interest obtained under the compound interest at the given nominal rate of interest. The rate of interest so calculated is called EFFECTIVE RATE OF INTEREST.

If the number of times compounding is done in a year is increased to infinity, we say that the compounding is done EVERY MOMENT and then the amount is given by $P \cdot e^{nr/100}$, where r is the rate % p.a. and n is the number of years.

The following points should also be noted, which are helpful in solving problems.

The difference between the compound interest and simple interest on a certain sum for two years is equal to the interest calculated for one year on one year's simple interest.

In mathematical terms, the difference between compound interest and simple interest for two years will be equal to $P(r/100)^2$, which can be written as $P(r/100)\,(r/100)$. In this, $Pr/100$ is the simple interest for one year, and when this is multiplied by $r/100$ again, it gives interest for one year on $Pr/100$, i.e. interest for one year on one year's simple interest.

The difference between the compound interest for k years and the compound interest for (k + 1) years is the interest for one year on the amount at the end of k^{th} year.

This can also be expressed in terms of the amount as follows:

The difference between the amount for k years and the amount for (k + 1) years under compound interest is the interest for one year on the amount at the end of the k^{th} year.

The difference between the compound interest for the k^{th} year and the compound interest for the $(k + 1)^{th}$ year is equal to the interest for one year on the compound interest for the k^{th} year.

PRESENT VALUE

Consider a given sum P and a rate of interest r.

We have seen that interest is cost of using the money over a period of time. That means a sum at the beginning of a period is always higher than the same amount after a period greater than or equal to 1.

Let the sum P that is being considered at a rate of interest r% p.a., becomes Y at the end of Year 1 and Z at the end of Year 2 (i.e. Y and Z are the amounts at the end of first and second years, respectively, on a principal of P).

Then, we can say that what is P today is equal to Y at the end of one year and equal to Z at the end of the second year. In other words, if an amount of Y were to come at the end of one year from now, its value today is equal to P. Similarly, if an amount of Z were to come at the end of two years from now, its value today is equal to P.

So, P is the PRESENT VALUE of Y coming at the end of one year and P is the PRESENT VALUE of Z coming at the end of two years.

Similarly, if we consider n years (or n periods in general), and X is the amount that P will become in n periods, then we say that P is the PRESENT VALUE of X coming at the end of n periods.

If we consider a series of payments Y_1 at the end of first year, Y_2 at the end of second year, and so on, the present value of the series of payments will then be equal to the sum of the present values of each of the payments calculated separately. If Z_1 is the present value of Y_1, Z_2 is the present value of Y_2, and so on, then the present value of the series of payments $Y_1, Y_2, \ldots$ is equal to $Z_1 + Z_2 + \ldots$

Present value can be looked at both under simple interest and compound interest.

If an amount of Y whose present value is P_1 comes at the end of Year 1 and an amount of Z whose present value is P_2 comes at the end of Year 2, then the present value of both the amounts together will be equal to $(P_1 + P_2)$, i.e. the present value of the stream of payments that come at different points of time is equal to the sum of the present values of the individual amounts coming in at various points of time.

Present Value under Simple Interest: The principal P is amounting to X in n periods. From this, we know that

$$X = P\left(1 + \frac{nr}{100}\right)$$

$$\Rightarrow \qquad P = \frac{X}{\left(1 + \frac{nr}{100}\right)}$$

Hence, in general, the present value P of an amount X coming (or due) after n periods is given by

$$\boxed{P = \frac{X}{\left(1 + \frac{nr}{100}\right)}}$$

where r is the rate percent per time period.

Present Value under Compound Interest: The principal P is amounting to X in n periods. From this, we know that

$$X = P\left(1 + \frac{r}{100}\right)^n$$

$$\Rightarrow \qquad P = \frac{X}{\left(1 + \frac{r}{100}\right)^n}$$

Hence, in general, the present value P of an amount X coming (or due) after n periods is given by

$$\boxed{P = \frac{X}{\left(1 + \frac{r}{100}\right)^n}}$$

where r is the rate percent per time period.

Repayment in Equal Instalments—Compound Interest

If a sum P borrowed is repaid in n equal instalments compound interest being calculated at $r\%$ per period of instalment, we can find out the value of each instalment. Let us consider the case of n equal ANNUAL instalments. (Even if the instalments are not annual, but monthly, the approach will remain the same except that the rate of interest taken should then be the rate per month and not rate per annum.)

Let each instalment (i.e. the amount paid at the end of each year) be X.

Instalment X paid after Year 1 gives a present value of $\dfrac{X}{\left(1+\dfrac{r}{100}\right)}$.

Instalment X, paid at the end of Year 2 gives a present value of $\dfrac{X}{\left(1+\dfrac{r}{100}\right)^2}$.

Similarly, instalment X paid for nth period (at the end of year n) gives a present value of $\dfrac{X}{\left(1+\dfrac{r}{100}\right)^n}$.

The sum of all these present values would be equal to the loan amount P (because only if the amount borrowed is equal to the amount repaid can we say that the loan is repaid).

$$\frac{X}{\left(1+\frac{r}{100}\right)} + \frac{X}{\left(1+\frac{r}{100}\right)^2} + \ldots + \frac{X}{\left(1+\frac{r}{100}\right)^n} = P$$

Call $\dfrac{1}{\left(1+\dfrac{r}{100}\right)} = k$

$$\Rightarrow \quad k = \frac{100}{100 + r}$$

The above equation can then be rewritten as

$$X\{k + k^2 + \ldots + k^n\} = P$$

The terms within the brackets form a G.P. with first term k and common ratio k.

The sum of this G.P. $= \dfrac{k\ (k^n - 1)}{(k-1)}$;

Thus $\dfrac{X \cdot k\ (k^n - 1)}{(k-1)} = P$

$$\Rightarrow \quad X = \frac{P\,(k-1)}{k\,(k^n - 1)}$$

$$= \frac{\left[P\left\{\frac{100}{100+r}\right\}-1\right]}{\left[\left\{\frac{100}{100+r}\right\}\right]\left[\left\{\frac{100}{100+r}\right\}^n - 1\right]} = \frac{P \cdot r}{100\left[1-\left\{\frac{100}{100+r}\right\}^n\right]}$$

$$\text{Each instalment} = \frac{P \cdot r}{100\left[1-\left\{\frac{100}{100+r}\right\}^n\right]}$$

Solved Examples

Example 1

Find the simple interest on a sum of ₹1000 at 10% p.a. for 4 years.

Solution

$$\text{Simple interest} = \frac{Pnr}{100}$$

$$\text{Interest} = \frac{(1000)(4)(10)}{100} = ₹\,400.$$

Example 2

A some of ₹4000 becomes ₹4500 in 2 years under simple interest. In how many years will ₹5000 become ₹5625 under simple interest at the same rate of interest?

Solution

Let the rate of interest be $R\%$ p.a.

Interest on ₹4000 = ₹500

$$500 = (4000)\left(\frac{R}{100}\right)(2)$$

$$R = 6.25\%$$

Interest on ₹5000 = ₹625

Let the required time be T years.

$$625 = (5000)\left(\frac{6 \cdot 25}{100}\right)T$$

$$\Rightarrow \quad T = 2.$$

Example 3

Find the value that ₹1000 would amount to under compound interest at 20% p.a., interest being compounded annually in 3 years.

Solution

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^N = 1000\left(1+\frac{20}{100}\right)^3 = ₹1728$$

Example 4

Find the sum that would amount to ₹6600 under simple interest in 4 years at 8% p.a.

Solution

Let the sum be ₹P.

Given that $P\left(1+4\left(\dfrac{8}{100}\right)\right) = 6600$

$$P = 5000.$$

Example 5

If a sum triples in 4 years under simple interest, find the time that it would take to become 5 times itself at the same rate of interest.

Solution

If the sum triples, the interest obtained will be twice the sum. This takes 4 years. If the sum becomes 5 times, the interest must be four times the sum.

∴ This takes a total of 8 years.

Example 6

A sum triples in 4 years under compound interest at a certain rate of interest, interest being compounded annually. Find the time it would take to become 9 times itself.

Solution

The sum triples in 4 years. If it becomes 9 times itself, it has tripled twice.

∴ This takes 8 years.

Let the sum of ₹P, triple in 4 years at R% p.a.

$$\Rightarrow \quad P\left(1+\frac{R}{100}\right)^4 = 3P$$

$$\Rightarrow \quad \left(1+\frac{R}{100}\right)^4 = 3 \qquad (1)$$

Let it take K years to become 9 times.

$$P\left(1+\frac{R}{100}\right)^K = 9P$$

$$\Rightarrow \quad \left(1+\frac{R}{100}\right)^K = 9$$

$$\Rightarrow \quad \left[\left(1+\frac{R}{100}\right)^4\right]^{\frac{K}{4}} = 3^2 \text{ from } \qquad (2),$$

$$3^{\frac{K}{4}} = 3^2$$

$$\Rightarrow \quad \frac{K}{4} = 2$$

$$\therefore \quad K = 8.$$

Example 7

If ₹4000 is lent at 10% p.a, interest being compounded annually, find the interest for the fourth year.

Solution

Interest for the fourth year = Amount at the end of the first 4 years – Amount at the end of the first 3 years

$$= 4000\left(1+\frac{10}{100}\right)^4 - 4000\left(1+\frac{10}{100}\right)^3$$

$$= 4000\,(1.4641 - 1.3310)$$

$$= 4000\,(0.1331) \quad \text{i.e. ₹ } 532.40.$$

Example 8

The interest on a sum is compounded every 3 months. If the rate of interest is 40% p.a., find the effective rate of interest per annum.

Solution

Let the sum be ₹100

Amount at the end of a year

$$= 100\left(1+\frac{40}{4(100)}\right)^4 = ₹146.41$$

∴ effective rate of interest = 46.41%.

Example 9

The compound interest and the simple interest on a sum at certain rate of interest for 2 years are ₹2760 and ₹2400, respectively. Find the sum and the rate of interest.

Solution

Let the sum be ₹P and let the rate of interest be R% p.a.

Difference between the compound interest and the simple interest = ₹360

$$\therefore \quad P\left(\frac{R}{100}\right)^2 = 360 \qquad (5)$$

$$P(2)\left(\frac{R}{100}\right) = 2400$$

$$\Rightarrow \quad \frac{PR}{100} = 1200 \qquad (6)$$

$$\therefore \quad \frac{PR}{100}\left(\frac{R}{100}\right) = 1200\left(\frac{R}{100}\right) = 360$$

$$R = 30$$

Substituting R in (5) or (6),

$$P = 4000.$$

Exercises

Direction for questions 1 to 20: Select the correct alternative from the given choices.

1. Find the amount obtained by investing ₹24,000 at 18% p.a. simple interest for five years
   (A) ₹21,600 (B) ₹44,000
   (C) ₹45,600 (D) ₹48,000

2. The simple interest for the second year on a certain sum at a certain rate of interest is ₹1000. Find the sum of the interest accrued on it for the 6^{th}, 7^{th}, and 8^{th} years.
   (A) ₹3200 (B) ₹3000
   (C) ₹3300 (D) ₹3630

3. In how many years will a sum of money become sixteen times itself at 30% p.a. simple interest?
(A) 25 (B) 40 (C) 30 (D) 50

4. A sum of money becomes ten times itself at simple interest. If the time period (in years) is numerically equal to the rate of interest, find the annual rate of interest.
(A) 25% (B) 20% (C) 30% (D) 90%

5. An amount of ₹2400 is due after six years under simple interest at 10% p.a. Find its present value (in ₹).
(A) 2000 (B) 1600 (C) 1800 (D) 1500

6. If ₹3000 amounts to ₹3630 in two years under compound interest, interest being compounded annually, what is the annual rate of interest?
(A) 10% (B) 21% (C) 11% (D) 10.5%

7. ₹5000 is invested for two years under compound interest at 10% p.a., interest being compounded annually. Find the interest earned (in ₹).
(A) 500 (B) 1000 (C) 2100 (D) 1050

8. A sum under compound interest, interest being compounded annually amounts to ₹6000 in two years and ₹7200 in three years. Find the rate of interest.
(A) 10% p.a. (B) 20% p.a.
(C) 18% p.a (D) 15% p.a

9. The compound interest on a sum for the third year is ₹2420, interest being compounded annually. The interest on it for the fourth year is ₹2662. Find the rate of interest.
(A) 10% p.a. (B) 11% p.a.
(C) 12% p.a (C) 13% p.a

10. A sum of money becomes four times itself in eight years at compound interest. In how many years will the same sum become sixteen times itself?
(A) 64 (B) 32 (C) 44 (D) 16

11. A sum becomes 2.197 times of itself in three years at compound interest. Find the rate of interest.
(A) 30% (B) 13% (C) 39.9% (D) 235

12. Find the interest (in ₹) earned in the first year on ₹200 at 20% p.a. compound interest, interest compounded every six months.
(A) 40 (B) 42 (C) 44 (D) 48

13. Find the effective rate of interest if the rate of interest is 40% p.a., and the interest is compounded quarterly?
(A) 42% p.a (B) 40% p.a.
(C) 44% p.a (D) 46.41% p.a.

14. Ashok borrowed a total of ₹84000 from two banks at compound interest, interest being compounded annually. One of the banks charged interest at 10% p.a. while the other charged interest at 20% p.a. If Ashok paid ₹13200 as the total interest after a year, find the difference of the sums he borrowed (in ₹).
(A) 24000 (B) 48000
(C) 54000 (D) 12000

15. If the annual rate of simple interest at which a sum is lent for two years increases by 10 percentage points, the interest realized would be ₹4000 more. Find the sum (in ₹).
(A) 20000 (B) 10000
(C) 8000 (D) 16000

16. If a sum was ₹10000 more it would fetch ₹4000 extra as simple interest, if it was lent at a certain rate of interest for two years. Find the annual rate of interest.
(A) 5% (B) 10% (C) 20% (D) 25%

17. A sum was invested under compound interest, interest being compounded annually. It fetches ₹14400 as interest in the second year and ₹17280 as interest in the third year. Find the annual rate of interest.
(A) 10% (B) 15%
(C) 20% (D) 25%

18. A sum takes T_1 years to double at R_1% p.a. simple interest. If it is lent at R_2% p.a. compound interest, interest being compounded annually, it would take the same time to double. Which of the following is always true if $T_1 > 1$?
(A) $R_1 > R_2$ (B) $0.5R_2 < R_1 < R_2$
(C) $R_1 = R_2$ (D) $R_2/3 < R_1 < R_2$

19. A sum takes two years to become 40% more under simple interest at a certain rate of interest. If it was lent at the same interest rate for the same time under compound interest, interest being compounded annually, it would amount to x% more than itself. Find x.
(A) 36 (B) 48
(C) 40 (D) 44

20. A sum was divided into two equal parts. One part was lent at 20% p.a. simple interest. The other part was lent at 20% p.a. compound interest, interest being compounded annually. The difference in the interests fetched by the parts in the second year is ₹400. Find the difference in the interests fetched by the parts in the fourth year (in ₹).
(A) 1414 (B) 1442
(C) 1456 (D) 1484

Answer Keys

1. C	**2.** B	**3.** D	**4.** C	**5.** D	**6.** A	**7.** D	**8.** B	**9.** A	**10.** D
11. A	**12.** B	**13.** D	**14.** D	**15.** A	**16.** C	**17.** C	**18.** A	**19.** D	**20.** C