

c) 2000 Pa

d) 5000 Pa

5. A body of weight 72 N moves from the surface of earth at a height half of the radius of earth, then gravitational force exerted on it will be: [1]

a) 36 N

b) 144 N

c) 50 N

d) 32 N

6. Two waves each of amplitude a and have a phase difference $\frac{\pi}{2}$. The amplitude and frequency of a resultant wave due to their superposition will be [1]

a) $2a, \frac{f}{2}$

b) $\sqrt{2}a, f$

c) $\frac{a}{\sqrt{2}}, \frac{f}{2}$

d) $\frac{a}{\sqrt{2}}, f$

7. What will be the ratio of the distances moved by a freely falling body from rest in 4th and 5th seconds of journey? [1]

a) 1 : 1

b) 16 : 25

c) 7 : 9

d) 4 : 5

8. The phase velocity (v_p) of a travelling wave is [1]

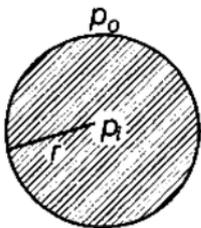
a) $v_p = \frac{\omega}{k}$

b) $v_p = \frac{c}{v_g}$

c) $v_p = \frac{d\omega}{dk}$

d) $v_p = c$

9. In figure, pressure inside a spherical drop is more than pressure outside. (S = surface tension and r = radius of bubble) [1]



The extra surface energy if radius of bubble is increased by Δr is

a) $2\pi r \Delta r S$

b) $4\pi r \Delta r S$

c) $10\pi r \Delta r S$

d) $8\pi r \Delta r S$

10. The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (u_p) whose radius and mean density are twice as that of earth is: [1]

a) $1 : 2\sqrt{2}$

b) $1 : 4$

c) $1 : \sqrt{2}$

d) $1 : 2$

11. A mass is revolving in a circle, which is in the plane of the paper. The direction of angular acceleration if any, is: [1]

a) Upward from the plane of the paper

b) Tangential

c) At right angles to the plane of the paper

d) Towards the radius

12. A chef, on finding his stove out of order, decides to boil the water for his wife's coffee by shaking it in a thermos flask. Suppose that he uses tap water at 15°C and that the water falls 30 cm each shake, the chef makes 30 shakes each minute. Neglecting any loss of thermal energy by the flask, how long must he shake the flask until the water reaches 100°C ? [1]

- c) 2:1
d) 1:2
- (b) A light body and a heavy body have the same kinetic energy. which one has greater linear momentum?
a) light body
b) both heavy and light body
c) Low body
d) heavy body
- (c) A spring is cut into two equal halves. How is the spring constant of each half affected?
a) becomes double
b) becomes triple
c) becomes 1/4th
d) becomes half

OR

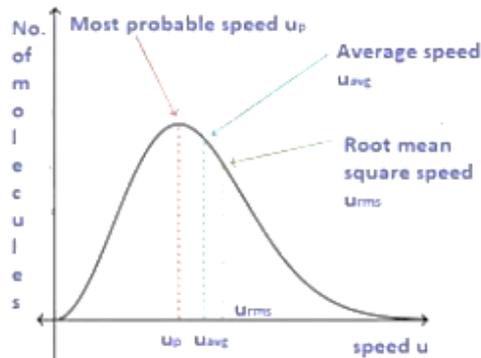
When spring is compressed, its potential energy:

- a) fall
b) decrease
c) first increase then decrease
d) increase
- (d) What type of energy is stored in the spring of a watch?
a) potential energy
b) Electrical energy
c) mechanical energy
d) kinetic energy

30. **Read the text carefully and answer the questions:**

[4]

Root mean square velocity (RMS value) is the square root of the mean of squares of the velocity of individual gas molecules and the Average velocity is the arithmetic mean of the velocities of different molecules of a gas at a given temperature.



- (a) Moon has no atmosphere because:
a) the escape velocity of the moon's surface is more than the r.m.s velocity of all molecules
b) it is far away from the surface of the earth
c) the r.m.s. velocity of all the gas molecules is more than the escape velocity of the moon's surface
d) its surface temperature is 10°C
- (b) For an ideal gas, $\frac{C_p}{C_v}$ is
a) ≤ 1
b) none of these
c) > 1
d) < 1
- (c) The root means square velocity of hydrogen is $\sqrt{5}$ times that of nitrogen. If T is the temperature of the gas then:

a) $T(\text{H}_2) = T(\text{N}_2)$

b) $T(\text{H}_2) < T(\text{N}_2)$

c) $T(\text{H}_2) \neq T(\text{N}_2)$

d) $T(\text{H}_2) > T(\text{N}_2)$

- (d) Suppose the temperature of the gas is tripled and N_2 molecules dissociate into an atom. Then what will be the rms speed of atom:

a) $v_0\sqrt{2}$

b) $v_0\sqrt{6}$

c) $v_0\sqrt{3}$

d) v_0

OR

The velocities of the molecules are $v, 2v, 3v, 4v$ & $5v$. The RMS speed will be:

a) $11v$

b) $v(12)^{11}$

c) v

d) $v(11)^{12}$

Section E

31. A person normally weighing 50 kg stands on a mass less platform which oscillates up and down harmonically at a frequency of 2.0 s^{-1} and an amplitude 5.0 cm. A weighing machine on the platform gives the persons weight against time. [5]
- Will there be any change in weight of the body, during the oscillation? Figure In extensible string.
 - If answer to part (a) is yes, what will be the maximum and minimum reading in the machine and at which position?

OR

Show that simple harmonic motion may be regarded as the projection of uniform circular motion along the diameter of the circle. Hence derive an expression for the displacement of a particle in S.H.M.

32. State triangle law of vector addition. Give analytical treatment to find the magnitude and direction of a resultant vector by using this law. [5]

OR

A quarterback, standing on his opponents 35-yard line, throws a football directly down field, releasing the ball at a height of 2.00 m above the ground with an initial velocity of 20.0 m/s, directed 30.0° above the horizontal.

- How long does it take for the ball to cross the goal line, 32.0 m from the point of release?
 - The ball is thrown too hard and so passes over the head of the intended receiver at the goal line. What is the ball's height above the ground as it crosses the goal line?
33. Two cylindrical hollow drums of radii R and $2R$, and of a common height h , are rotating with angular velocities ω_1 (anti-clockwise) and ω_2 (clockwise), respectively. Their axes, fixed are parallel and in a horizontal plane separated by $(3R + \delta)$. They are now brought in contact ($\delta \rightarrow 0$). [5]
- Show the frictional forces just after contact.
 - Identify forces and torques external to the system just after contact.
 - What would be the ratio of final angular velocities when friction ceases?

OR

Find the components along the x, y, z axes of the angular momentum l of a particle, whose position vector is r with components x, y, z and momentum is p with components p_x, p_y and p_z . Show that if the particle moves only in the x - y plane the angular momentum has only a z -component.

Solution

Section A

1. (a) $[M L^2 T^{-2} \text{ mol}^{-1} K^{-1}]$

Explanation: According to ideal gas equation for universal gas constant.

i.e., $pV = nRT$, where n is the number of moles of gases.

$$R = \frac{(p)(V)}{(n)(T)} = \frac{[ML^{-1} T^{-2}][L^3]}{[\text{mol}][K]}$$
$$= [ML^2 T^{-2} \text{ mol}^{-1} K^{-1}]$$

2.

(d) 13 dB

Explanation: $\Delta\beta = \beta_2 - \beta_1$

$$= 10 \log \frac{20I}{I_0} - 10 \log \frac{I}{I_0}$$

$$= 10 \times 1.3010 \approx 13 \text{ dB}$$

3.

(d) $\frac{ML\omega^2}{2}$

Explanation: The mass of the liquid acts at the centre of the tube.

Therefore, $r = \frac{L}{2}$

Force exerted by the liquid at the other end

= Centrifugal force

$$= Mr\omega^2 = M \left(\frac{L}{2}\right) \omega^2 = \frac{ML\omega^2}{2} .$$

4. (a) 6000 Pa

Explanation: The pressure at the bottom of the jar is due to the weight of a column of water of height $h = 50 \text{ cm} = 0.5 \text{ m}$ and the weight of a load of $m = 1 \text{ kg}$.

The total force acting on the base = $h\rho gA + mg$

$$= 0.5 \times 1000 \times 10 \times 0.01 + 1 \times 10 = 60 \text{ N}$$

$$\therefore \text{Pressure} = \frac{\text{force}}{\text{area}} = \frac{60}{0.01} = 6000 \text{ Nm}^{-2} \text{ or } 6000 \text{ Pa}$$

5.

(d) 32 N

Explanation: $F_{\text{surface}} = G \frac{Mm}{R_e^2}$

$$F_{\frac{R_e}{2}} = G \frac{Mm}{\left(\frac{R_e + R_e}{2}\right)^2} = \frac{4}{9} \times F_{\text{surface}} = \frac{4}{9} \times 72 = 32 \text{ N}$$

6.

(b) $\sqrt{2}a$, f

Explanation: $y_1 = a \sin 2\pi ft$

$$y_2 = a \sin\left(2\pi ft + \frac{\pi}{2}\right)$$

$$\therefore y = y_1 + y_2 = 2a \sin\left(2\pi ft + \frac{\pi}{4}\right) \cos \frac{\pi}{4}$$

$$= \frac{2a}{\sqrt{2}} \sin\left(2\pi ft + \frac{\pi}{4}\right)$$

$$= \sqrt{2}a \sin\left(2\pi ft + \frac{\pi}{4}\right)$$

Hence the resultant wave has amplitude $\sqrt{2}a$ and frequency f .

7.

(c) 7 : 9

Explanation: Distance covered in n^{th} second is given by

$$s_n = u + \frac{a}{2}(2n - 1)$$

Given: $u = 0$, $a = g$

$$\therefore s_4 = \frac{g}{2}(2 \times 4 - 1) = \frac{7g}{2}$$

$$s_5 = \frac{g}{2}(2 \times 5 - 1) = \frac{9g}{2}$$

$$\therefore \frac{s_4}{s_5} = \frac{7}{9}$$

8. (a) $v_p = \frac{\omega}{k}$

Explanation: $v_p = \frac{\omega}{k}$

9.

(d) $8\pi r \Delta r S$

Explanation: Suppose a spherical drop of radius r is in equilibrium. If its radius increases by Δr . The extra surface energy is $|4\pi(r + \Delta r)^2 - 4\pi r^2| S = 8\pi r \Delta r S$

10. (a) $1 : 2\sqrt{2}$

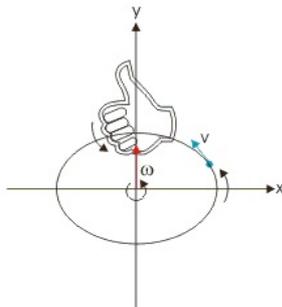
Explanation: $v_e = \sqrt{2gR} = R\sqrt{\frac{8}{3}\pi G\rho}$

$$\begin{aligned} \therefore \frac{v_s}{v_p} &= \frac{R\sqrt{\rho}}{R_p\sqrt{P_p}} \\ &= \frac{R\sqrt{\rho}}{2R \times \sqrt{2\rho}} = \frac{1}{2\sqrt{2}} \\ &= 1 : 2\sqrt{2} \end{aligned}$$

11.

(c) At right angles to the plane of the paper

Explanation: Angular acceleration is an axial vector. It is always directed along the axis of rotation according to the right-hand screw rule. Hence the direction of the angular acceleration vector is perpendicular to the plane in which the rotation takes place.



12. (a) 4.00×10^3 min

Explanation: Heat required to melt 50 g ice

$$= m l = 50 \times 80 = 4000 \text{ cal}$$

Heat given out by water in cooling from 80°C to 0°C

$$= mc\Delta T = 50 \times 1 \times 80 = 4000 \text{ cal}$$

13. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Explanation: If it is a completely inelastic collision then:

$$m_1 v_1 + m_2 v_2 = m_1 v + m_2 v$$

$$\text{or } v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$\text{KE} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$$

As \vec{P}_1 and \vec{P}_2 both simultaneously cannot be zero, therefore total KE cannot be lost.

14.

(b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

Explanation: Assertion and reason both are correct statements but reason is not correct explanation for assertion.

15.

(b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

Explanation: Assertion and reason both are correct statements but reason is not correct explanation for assertion.

16.

(c) A is true but R is false.

Explanation: Upto ordinary heights, the change in the distance of a projectile from the centre of earth is negligible compared to the radius of earth. Hence the projectile moves under a nearly uniform gravitational force and the path is parabolic. But for

the projectiles moving to a large height the gravitational force decreases quite rapidly (as $F \propto \frac{1}{r^2}$). Under such a rapidly decreasing variable force, the path of the projectile becomes elliptical.

Section B

17. Ratio of amplitudes(given) = $\frac{A_1}{A_2} = \frac{3}{5} \Rightarrow \sqrt{\frac{I_1}{I_2}} = \frac{3}{5}$ [as $A \propto \sqrt{I}$]

Now, $\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} \right)^2 = \left(\frac{\sqrt{I_1/I_2} + 1}{\sqrt{I_1/I_2} - 1} \right)^2$
 $= \left(\frac{3/5 + 1}{3/5 - 1} \right)^2$
 $= \frac{64}{4} = \frac{16}{1} = 16:1$

18. Let the orbital velocity of satellite be given by the relation

$$v = km^a r^b g^c$$

where, k is a dimensionless constant and a, b, c are unknown powers.

Writing dimensions on two sides of equation, we have

$$[M^0 L^1 T^{-1}] = [M]^a [L]^b [L T^{-2}]^c = [M^a L^{b+c} T^{-2c}]$$

By equating the powers on both sides, we have

$$a = 0, b + c = 1, -2c = -1$$

On solving these equations, we get

$$a = 0, b = +\frac{1}{2} \text{ and } c = +\frac{1}{2}$$

$$v = kr^{1/2} g^{1/2}$$

$\Rightarrow v = k\sqrt{rg}$, which is the required expression.

19. Let $x = 2.5 \times 10^{-6} = 0.0000025$ (2 significant figures)

$$y = 4.0 \times 10^{-4} = 0.00040 \text{ (2 significant figures)}$$

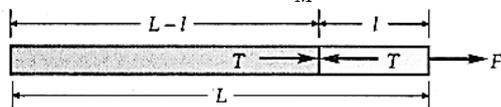
$$\therefore y - x = 0.00040 - 0.0000025 = 0.0003975$$

$$= 3.975 \times 10^{-4} = 4.0 \times 10^{-4} \text{ [Rounded off upto 2 significant figures]}$$

20. Let M be the mass of uniform rope of length L. Then

$$\text{Mass per unit length of rope} = \frac{M}{L}$$

$$\text{Acceleration in the rope} = \frac{F}{M}$$



Let T be the tension in the rope at a distance l from the end where the force F is applied.

Mass of length (L - l) of the rope is

$$M' = \frac{M}{L}(L - l)$$

As tension T is the only force on the length (L - l) of the rope, so

$$T = M' \times \frac{F}{M} = \frac{M}{L}(L - l) \times \frac{F}{M} = \left(1 - \frac{l}{L}\right) F$$

21. $g = \frac{GM}{R^2}$ or $M = \frac{gR^2}{G}$

This relation is true not only to the earth but for any heavenly body which is assumed to be spherical.

$$\text{Now, } g = 1.67 \text{ m s}^{-2}, R = 1.74 \times 10^6 \text{ m}$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^{-2} \text{ kg}^{-2}$$

$$\therefore \text{Mass of the moon, } M = \frac{1.67 \times (1.74 \times 10^6)^2}{6.67 \times 10^{-11}} \text{ kg}$$

$$= 7.58 \times 10^{22} \text{ kg}$$

OR

$$\text{Gravitational PE of mass m in orbit of radius } R = U = -\frac{GMm}{R}$$

$$\therefore U_i = \frac{GMm}{2R}$$

$$U_f = -\frac{GMm}{3R}$$

Energy required = Potential energy of the Earth(mass system when mass is at distance 3R) – Potential energy of the Earth (mass system when mass is at distance 2R)

$$\Delta U = U_f - U_i = GMm \left[\frac{1}{3} - \frac{1}{2} \right]$$

$$= \frac{GMm}{6R}$$

Section C

22. The velocity attained by the sphere after falling freely from height h is

$$v = \sqrt{2gh} \dots (i)$$

After entering water, the velocity of the sphere does not change. So v is also the terminal velocity of the sphere. Hence

$$v = \frac{2}{9} \frac{r^2}{\eta} (\rho - \rho') g$$

But $\rho = 10^4 \text{ kgm}^{-3}$, $\rho' = 10^3 \text{ kgm}^{-3}$, $r = 10^{-3} \text{ m}$, $g = 10 \text{ ms}^{-2}$, $\eta = 10^{-3} \text{ Nsm}^{-2}$

$$\therefore v = \frac{2}{9} \times \frac{(10^{-3})^2 \times (10^4 - 10^3) \times 10}{10^{-3}} = 20 \text{ ms}^{-1}$$

$$\text{From (i), } h = \frac{v^2}{2g} = \frac{20 \times 20}{2 \times 10} = 20 \text{ m}$$

23. Power of the drilling machine, $P = 10 \text{ kW} = 10 \times 10^3 \text{ W} = 10^4 \text{ W}$

Mass of the aluminum block, $m = 8.0 \text{ kg} = 8000 \text{ g}$

Time, $t = 2.5 \text{ min} = 2.5 \times 60 = 150 \text{ s}$

Specific heat of aluminium, $c = 0.91 \text{ J g}^{-1} \text{K}^{-1}$

Let rise in the temperature of the block after drilling = δT

Total energy of the drilling machine = $P \times T$

$$= 10 \times 10^3 \times 150 = 1.5 \times 10^6 \text{ J}$$

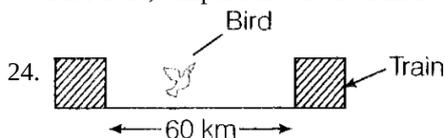
As only 50% of the energy is useful as per the question

$$\text{so useful energy, } \Delta Q = \frac{50}{100} \times 1.5 \times 10^6 = 7.5 \times 10^5 \text{ J}$$

But $\Delta Q = mc\Delta T$

$$\therefore \Delta T = \frac{\Delta Q}{mc} \\ = \frac{7.5 \times 10^5}{8 \times 10^3 \times 0.91} = 103^\circ \text{C}$$

Therefore, temperature of block increases by 103°C in drilling for 2.5 minutes.



Here, it is given that, $v_1 = 30 \text{ km/h}$, $v_2 = 60 \text{ km/h}$ and $d = 60 \text{ km}$

$$\text{Therefore, } v_1 = 30 \text{ km/h} = 30 \times \frac{5}{18} = 8.33 \text{ m/s}$$

$$\text{and } v_2 = 60 \text{ km/h} = 60 \times \frac{5}{18} = 16.67 \text{ m/s}$$

Also, $d = 60 \text{ km} = 60000 \text{ m}$ (because $1 \text{ km} = 1000 \text{ m}$)

Since the trains will collide in the middle, we have

$$\Delta x = 30 \text{ km}$$

$$\text{When trains collide, } v = \frac{\Delta x}{\Delta t} \Rightarrow \Delta t = \frac{\Delta x}{v}$$

$$\therefore \Delta t = \frac{30000}{8.33} = 3601$$

$$\text{Now, } v = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = v \Delta t$$

$$\therefore \Delta x = (16.67 \text{ m/s})(3601)$$

$$\approx 60028.67 \text{ m} \approx 60 \text{ km}$$

25. Here $m = 50 \text{ g} = 0.05 \text{ kg}$

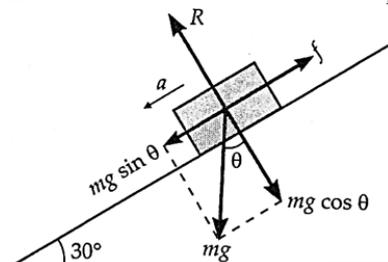
Angle of repose,

$$\alpha = 15^\circ$$

$$\therefore \mu = \tan \mu = \tan 15^\circ = 0.2679$$

New angle of inclination = $15 + 15 = 30^\circ$

Let a be the downward acceleration produced in the block.



Net downward force on the block is

$$F = mg \sin \theta - f$$

$$ma = mg \sin \theta - \mu mg \cos \theta \quad [\because f = \mu R = \mu mg \cos \theta]$$

$$\therefore a = g(\sin \theta - \mu \cos \theta)$$

$$= 9.8 (\sin 30^\circ - 0.2679 \cos 30^\circ)$$

$$= 9.8 (0.5 - 0.2679 \times 0.866)$$

$$= 9.8 \times 0.2680 = 2.6 \text{ ms}^{-2}$$

26. Specific heat of argon at constant pressure, $C_p = 0.125 \text{ cal / g / K}$

$$C_p = 0.125 \times 4.2 \times 1000 \text{ J / Kg / K (using 1 cal = 4.2 J)}$$

$$C_p = 525 \text{ J / Kg / K ... (i)}$$

Specific heat of argon at constant volume, $C_v = 0.075 \text{ cal / g / K}$

$$C_v = 0.075 \times 4.2 \times 1000 = 315 \text{ J / Kg / K}$$

The gas constant, r for 1 kg of gas is given by:-

$$r = C_p - C_v = 525 - 315 = 210 \text{ J / Kg / K}$$

$$\text{Normal pressure} = P = h P g = 0.76 \times 13600 \times 9.8 = 101292.8 \text{ N / m}^2$$

$$\text{Normal Temperature} = T = 273 \text{ K.}$$

Suppose $V =$ Volume of argon gas at N. T. P.

$$PV = nrT$$

for $n = 1$ mole

$$\frac{PV}{T} = r$$

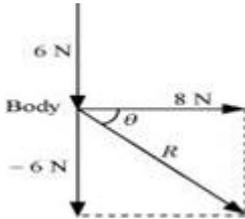
$$V = \frac{rT}{P} = \frac{210 \times 273}{101292.8} = 0.566 \text{ m}^3$$

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{1}{0.566} = 1.8 \text{ Kg / m}^3$$

Hence the density of argon is 1.8 Kg/m^3 .

27. Mass of the body, $m = 5 \text{ kg}$

The given situation can be represented as follows:



The resultant of two forces is given as:

$$R = \sqrt{(8)^2 + (-6)^2} = \sqrt{64 + 36} = 10 \text{ N}$$

θ is the angle made by R with the force of 8 N

$$\therefore \theta = \tan^{-1} \left(\frac{-6}{8} \right) = -36.87^\circ$$

The negative sign indicates clockwise direction, and the force of the magnitude is 8 N ,

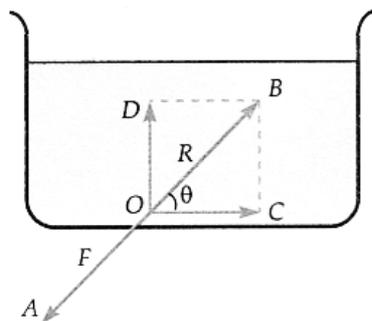
Using Newton's second law of motion, the acceleration (a) of the body is given as:

$$F = ma$$

$$\therefore a = \frac{F}{m} = \frac{10}{5} = 2 \text{ m/s}^2$$

This will be an acceleration in the positive direction, because the applied force acts as a push.

28. Consider a liquid contained in a vessel in the equilibrium state of rest. As shown in Fig., suppose the liquid exerts a force F on the bottom surface in an inclined direction OA . The surface exerts an equal reaction R to water along OB .



The reaction R along OB has two rectangular components:

i. Tangential component, $OC = R \cos \theta$

ii. Normal component, $OD = R \sin \theta$

Since a liquid cannot resist any tangential force, the liquid near O should begin to flow along OC. But the liquid is at rest, the force along OC must be zero.

$$\therefore R \cos \theta = 0$$

As $R \neq 0$, so $\cos \theta = 0$ or $\theta = 90^\circ$

Hence a liquid always exerts a force perpendicular to the surface of the container at every point.

OR

Suppose a sphere of radius r and density ρ falls in a fluid of density ρ' and viscosity η . When the sphere just enters the fluid, the net downward force on it is

$$F = \text{Weight of the sphere} - \text{Weight of the fluid displaced}$$

$$= \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho' g = \frac{4}{3}\pi r^3 (\rho - \rho') g$$

It is Given that, average acceleration as half of the initial acceleration.

\therefore Initial acceleration,

$$a = \frac{F}{m} = \frac{\frac{4}{3}\pi r^3 (\rho - \rho') g}{\frac{4}{3}\pi r^3 \rho} = \left(\frac{\rho - \rho'}{\rho}\right) g$$

When the sphere attains terminal velocity, its acceleration becomes zero.

$$\therefore \text{Average acceleration} = \frac{a+0}{2} = \left(\frac{\rho - \rho'}{2\rho}\right) g$$

Let the sphere take time t to attain the terminal velocity,

$$v = \frac{2}{9} \frac{r^2}{\eta} (\rho - \rho') g$$

Initial velocity, $u = 0$

Hence by using first equation of motion

$$v = u + at$$

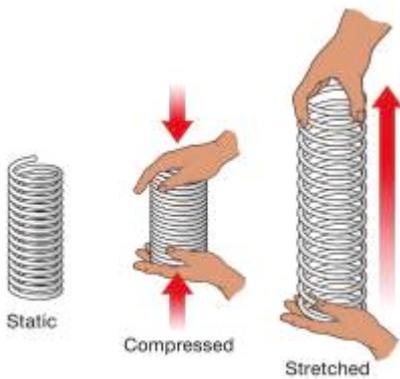
$$\frac{2}{9} \frac{r^2}{\eta} (\rho - \rho') g = 0 + \left(\frac{\rho - \rho'}{2\rho}\right) gt$$

$$\text{or } t = \frac{4}{9} \cdot \frac{r^2 \rho}{\eta}$$

Section D

29. Read the text carefully and answer the questions:

Elastic potential energy is Potential energy stored as a result of the deformation of an elastic object, such as the stretching of a spring. It is equal to the work done to stretch the spring, which depends upon the spring constant k as well as the distance stretched



(i) (c) 2:1

Explanation: 2:1

(ii) (d) heavy body

Explanation: heavy body

(iii) (a) becomes double

Explanation: becomes double

OR

(d) increase

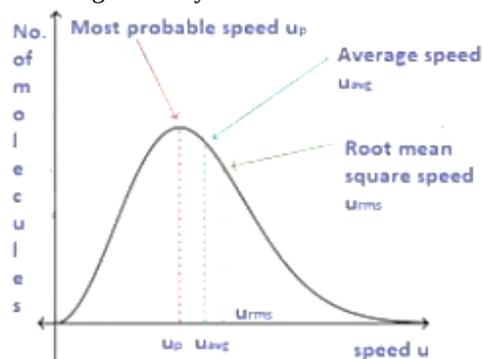
Explanation: increase

(iv) (a) potential energy

Explanation: potential energy

30. Read the text carefully and answer the questions:

Root mean square velocity (RMS value) is the square root of the mean of squares of the velocity of individual gas molecules and the Average velocity is the arithmetic mean of the velocities of different molecules of a gas at a given temperature.



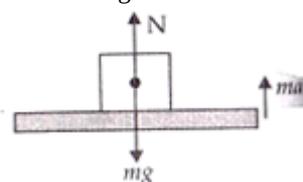
- (i) (c) the r.m.s. velocity of all the gas molecules is more than the escape velocity of the moon's surface
Explanation: The r.m.s. velocity of all the gas molecules is more than the escape velocity of the moon's surface.
- (ii) (c) > 1
Explanation: > 1
- (iii) (b) $T(H_2) < T(N_2)$
Explanation: $T(H_2) < T(N_2)$
- (iv) (b) $v_0 \sqrt{6}$
Explanation: $v_0 \sqrt{6}$

OR

- (d) $v(11)^{12}$
Explanation: $v(11)^{12}$

Section E

31. a. Weight in weight machine will be due to the normal reaction (N) by platform. Consider the top position of platform, two forces acting on it are due to weight of person and oscillator. They both act downward.



(mg = weight of the person with the oscillator is acting downwards, ma = force due to oscillation is acting upwards, N = normal reaction force acting upwards)

Now for the downward motion of the system with an acceleration a ,

$$ma = mg - N \dots(i)$$

When platform lifts from its lowest position to upward

$$ma = N - mg \dots(ii)$$

$a = \omega^2 A$ is value of acceleration of oscillator

\therefore From equation (i) we get,

$$N = mg - m\omega^2 A$$

Where A is amplitude, ω angular frequency and m mass of oscillator.

$$\omega = 2\pi\nu$$

$$\therefore \omega = 2\pi \times 2 = 4\pi \text{ rad/sec}$$

Again using $A = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$ we get

$$N = 50 \times 9.8 - 50 \times 4\pi \times 4\pi \times 5 \times 10^{-2}$$

$$= 50 [9.8 - 16\pi^2 \times 5 \times 10^{-2}] \text{ N}$$

$$= 50 [9.8 - 80 \times 3.14 \times 3.14 \times 10^{-2}] \text{ N}$$

$$\Rightarrow N = 50[9.8 - 7.89] = 50 \times 1.91 = 95.50 \text{ N}$$

So minimum weight is 95.50 N (for downward motion of the platform)

From equation (ii), $N - mg = ma$

For upward motion from the lowest to the highest point of oscillator,

$$\begin{aligned}
 N &= mg + ma \\
 &= m [9.81 + \omega^2 A] \quad \because a = \omega^2 A \\
 &= 50 [9.81 + 16\pi^2 \times 5 \times 10^{-2}] \\
 &= 50[9.81 + 7.89] = 50 \times 17.70 \text{ N} = 885 \text{ N}
 \end{aligned}$$

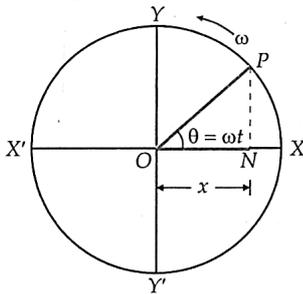
Hence, there is a change in weight of the body during oscillation.

b. The maximum weight is 885 N, when platform moves from lowest to upward direction.

And the minimum weight is 95.5 N, when platform moves from the highest point to downward direction.

OR

Relation between S.H.M. and uniform circular motion. As shown in figure, consider a particle P moving along a circle of radius A with uniform angular velocity ω . Let N be the foot of the perpendicular drawn from the point P to the diameter XX'. Then N is called the projection of P on the diameter XX'. As P moves along the circle from X to Y, Y to X', X' to Y' and Y' to X; N moves from X to O, O to X', X' to O and O to X. Thus, as P revolves along the circumference of the circle, N moves to and fro about the point O along the diameter XX'. The motion of N about O is said to be simple harmonic. Hence **simple harmonic motion** may be defined as the projection of uniform circular motion upon a diameter of a circle. The particle P is called the reference particle or generating particle and the circle along which the particle P revolves is called the circle of reference.



Displacement in simple harmonic motion. As shown in Figure, consider a particle moving in the anticlockwise direction with uniform angular velocity ω along a circle of radius A and centre O. Suppose at time $t = 0$, the reference particle is at point A such that $\angle XOA = \phi_0$. At any time t, suppose the particle reaches the point P such that $\angle AOP = \omega t$. Draw $PN \perp XX'$.

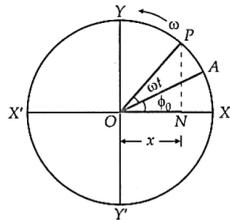


Fig. Displacement in S.H.M., epoch ($+\phi_0$)

Clearly, displacement of projection N from centre O at any instant t is $x = ON$.

In right-angled $\triangle ONP$,

$$\angle PON = \omega t + \phi_0$$

$$\therefore \frac{ON}{OP} = \cos(\omega t + \phi_0)$$

$$\text{or } \frac{x}{A} = \cos(\omega t + \phi_0)$$

$$\text{or } x = A \cos(\omega t + \phi_0)$$

This equation gives the displacement of a particle in S.H.M. at any instant t. The quantity $\omega t + \phi_0$ is called the phase of the particle and ϕ_0 is called the initial phase or phase constant or epoch of the particle. The quantity A is called the amplitude of the motion. It is a positive constant whose value depends on how the motion is initially started. Thus

$$\begin{array}{ccccccc}
 & & & \text{Phase} & & & \\
 x & = & A & \cos(\omega t & + & \phi_0) \\
 \uparrow & & \uparrow & \uparrow & & \uparrow & \\
 \text{Displacement} & & \text{Amplitude} & \text{Angular} & & \text{Initial} & \\
 & & & \text{frequency} & & \text{phase} &
 \end{array}$$

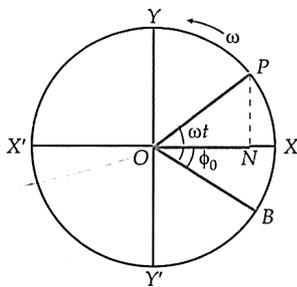


Fig. Epoch ($-\phi_0$)

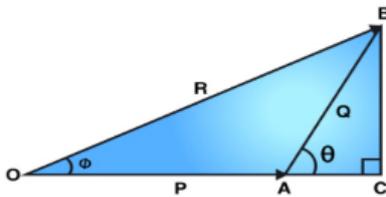
As shown in Figure, if the reference particle starts motion from the point P such that $\angle BOX = \phi_0$ and $\angle BOP = \omega t$, then

$$\angle PON = \omega t - \phi_0$$

$$\therefore x = A \cos(\omega t - \phi_0)$$

Here $-\phi_0$ is the initial phase of the S.H.M.

32. Triangle law of vector addition states that when two vectors are represented as two sides of the triangle taken in the same order, then the closing side of the triangle taken in the opposite order represents the magnitude and direction of the resultant vector. Consider two vectors, P and Q, respectively, represented by the sides OA and AB. Let vector R be the resultant of vectors P and Q.



From triangle OCB,

$$OB^2 = OC^2 + BC^2$$

In triangle ACB with θ as the angle between AC and AB

In $\triangle ABC$,

$$\frac{BC}{AB} = \sin \theta$$

$$\text{so } BC = AB \sin \theta = Q \sin \theta$$

$$\frac{AC}{AB} = \cos \theta$$

$$AC = AB \cos \theta = Q \cos \theta$$

$$\text{In } \triangle OBC, OB^2 = OC^2 + CB^2$$

$$OB^2 = (OA + AC)^2 + CB^2$$

$$R^2 = (P + Q \cos \theta)^2 + (Q \sin \theta)^2 \quad R^2 = P^2 + Q^2 + 2PQ \cos \theta$$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

The direction of result tan t vector can be found by following

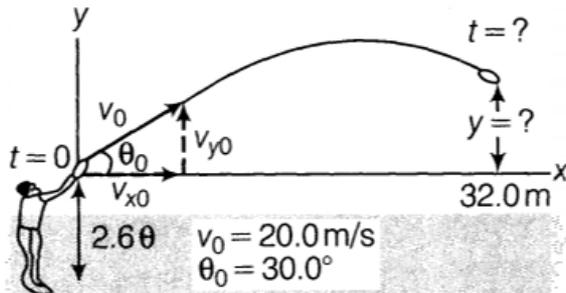
$$\tan \phi = \frac{BC}{OC} = \frac{Q \sin \theta}{P + Q \cos \theta}$$

Resultant act in the direction making an angle

$$\therefore \alpha = \tan^{-1} \left(\frac{\vec{Q} \sin \theta}{\vec{P} + \vec{Q} \cos \theta} \right) \text{ with direction of vector P.}$$

OR

To better visualise the solution described here, we first sketch the trajectory as shown in figure.



i. The problem here is to find t when $x = 32.0$ m. We can use $(x = v_{x0} t)$, if we first find v_{x0} . From figure, we see that $v_{x0} = v_0$

$$\cos \theta_0 = (20.0 \text{ m/s}) (\cos 30.0^\circ)$$

$$= 17.3 \text{ m/s}$$

Using the relation and solve for t .

$$x = v_{x0} t$$

$$t = \frac{x}{v_{x0}} = \frac{32.0 \text{ m}}{17.3 \text{ m/s}} = 1.85 \text{ s}$$

ii. We want to find y when $x = 32.0$ m, or since we have already found the time in part (a), we can state this, find y when $t = 1.85$ s. Using the relation,

$$y = v_{y0} t - \frac{1}{2} g t^2$$

$$\text{where } v_{y0} = v_0 \sin \theta_0 = (20.0 \text{ m/s}) (\sin 30.0^\circ)$$

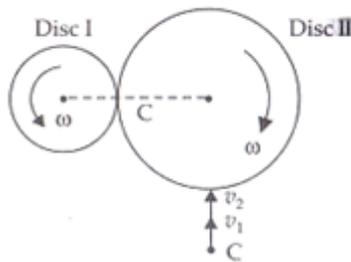
$$= 10.0 \text{ m/s}$$

$$\text{Thus, } y = (10.0 \text{ m/s})(1.85 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(1.85 \text{ s})^2 = 1.73 \text{ m}$$

Since, $y = 0$ is 2.00 m above the ground, this means the ball is 3.73 m above the ground as it crosses the goal line too much high to be caught at that point.

33. i. $\therefore v_1 = \omega R$

$$v_2 = \omega \cdot 2R = 2\omega R$$



The direction of v_1 and v_2 at point of contact C are tangentially upward. Frictional force (f) acts due to difference in velocities of disc 1 and 2. f on 1 due to 2 is $f_{12} =$ upward and $f_{21} =$ downward it will be equal and opposite by Newton's Third Law $f_{12} = -f_{21}$

ii. External forces acting on system are f_{12} and f_{21} which are equal and opposite so net force acting on system $f_{12} = -f_{21}$ or $f_{12} + f_{21} = 0$

$$|f_{12}| = |-f_{21}| = F$$

\therefore External torque $= F \times 3R$ (anti-clockwise)

As velocity of drum 2 is double i.e., $v_2 = 2v_1$ as in part (a).

iii. Let ω_1 (anti clockwise) and ω_2 (clockwise) are angular velocities of drum 1 and 2 respectively. Finally when their velocities become equal no force of friction will act due to no slipping at this stage $v_1 = v_2$ or $\omega_1 R = 2\omega_2 R$ or $\frac{\omega_1}{\omega_2} = \frac{2}{1}$

OR

$$l_x = y p_z - z p_y$$

$$l_y = z p_x - x p_z$$

$$l_z = x p_y - y p_x$$

The linear momentum of the particle in cartesian coordinate, $\vec{p} = p_x \hat{i} + p_y \hat{j} + p_z \hat{k}$

Position vector of the particle in cartesian coordinates, $\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$

As we know the angular momentum of a moving particle about a point is given as, $\vec{l} = \vec{r} \times \vec{p}$ where p and r are linear momentum and position vector respectively,

$$= (x \hat{i} + y \hat{j} + z \hat{k}) \times (p_x \hat{i} + p_y \hat{j} + p_z \hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$l_x \hat{i} + l_y \hat{j} + l_z \hat{k} = \hat{i} (y p_z - z p_y) - \hat{j} (x p_z - z p_x) + \hat{k} (x p_y - y p_x)$$

$$= \hat{i} (y p_z - z p_y) + \hat{j} (-x p_z + z p_x) + \hat{k} (x p_y - y p_x)$$

Comparing the coefficients of \hat{i} , \hat{j} , and \hat{k} we get the components of angular momentum as :

$$l_x = y p_z - z p_y$$

$$l_y = xp_z - zp_x \dots(i)$$

$$l_z = xp_y - yp_x$$

b) If the particle moves in the x-y plane only. Hence, the z-component of the position vector and z component of linear momentum vector become zero, i.e.,

$$z = p_z = 0$$

Thus, equation (i) reduces to:

$$l_x = 0$$

$$l_y = 0$$

$$l_z = xp_y - yp_x$$

Therefore, when the particle is confined to move in the x-y plane, the x and y components of linear momentum are zero and hence the direction of angular momentum is along the z-direction.