MATHEMATICS

(For 6th class)

H2Q4P8

ਸਮੱਗਰ ਸਿੱਖਿਆ ਅਭਿਆਨ

ਪੜ੍ਹੋ ਸਾਰੇ ਵਧੋ ਸਾਰੇ

ਸਿੱਖਿਆ ਅਤੇ ਭਲਾਈ ਵਿਭਾਗ, ਪੰਜਾਬ ਦਾ ਸਾਂਝਾ ਉਪਰਾਲਾ

Punjab School Education Board

Sahibzada Ajit Singh Nagar

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FOREWORD

The Punjab School Education Board has been continuously engaged in developing syllabi, producing and renewing text books according to the changing educational needs at the state and national level.

This book has been developed in accordance to the guidelines of National Curriculum Framework (NCF) 2005 and PCF 2013, after careful deliberations in workshops involving experienced teachers and experts from the board and field as well. All efforts have been made to make this book interesting with the help of activities and coloured figures. This book has been prepared with the joint efforts of subject experts of Board, SCERT and experienced teachers/experts of mathematics. Board is thankful to all of them.

The authors have tried their best to ensure that the treatment, presentation and style of the book in hand are in accordance with the mental level of the students of class VI. The topics, contents and examples in the book have been framed in accordance with the situations existing in the young learner's environment. A number of activities have been suggested in every lesson. These may be modified, keeping in view the availability of local resources and real life situations of the learners.

I hope the students will find this book very useful and interesting. The Board will be grateful for suggestions from the field for further improvement of the book.

Chairman

Punjab School Education Board

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Contents

SYMBOLS AND THEIR MEANING

$\therefore$	=	So/Therefore
$\because$	=	Because
$\Rightarrow$	=	Implies
$>$	=	Greater than
$<$	=	Less than
$\parallel$	=	Parallel
$\perp$	=	Perpendicular
Δ	=	Triangle
$\angle$	=	Angle
:	=	Ratio
: :	=	Proportion
i.e.	=	id est (that is)
e.g.	=	exempli gratia (for example)
etc.	=	Et cetera (the rest of same type)

REVISION OF FUNDAMENTAL OPERATIONS (+, –, ×, ÷)

Before building a strong and beautiful structure it is always good to test its foundation, on which that structure is supposed to stand. With same motive, in this chapter, Let us strengthen our previous knowledge and remove deficiency if any.

Exercise (Addition)

Addition (+)

1. Solve the following:

(a) 5999 + 1233 = ______

(b) 5219 + 3899 = ______

(c) 2009 + 7788 = ______

(d) 112 + 2709 = ______

(e) 3486 + 4306 = ______

(f) 506 + 909 = ______

2. Fill boxes operating as directed:

(a) 305 + 289 = ☐

(b) 2186 + 476 = ☐

(c) 332 + 4097 + 81 = ☐

(d) 77777 + 7777 + 777 + 77 + 7 = ☐

3. Fill empty boxes:

(a) 4 9 3 + 3 0 9 = 8 ☐ 2

(b) 2 6 3 6 + 5 9 9 = ☐ 2 3 ☐

(c) 9 7 3 9 + 6 5 2 8 = ☐ ☐ 2 6 ☐

4. Fill empty boxes operating as directed:

(a) 2017 + 928 + 74 = 3 ☐ ☐ 9

(b) 5077 + 537 + 98 = ☐ 7 1 ☐

(c) 3344 + 403 + 37 = ☐ ☐ 8 4

5. In a cricket match Virat scored 129 runs and Dhawan scored 97 runs. How many runs are made by both of them together.

Exercise (Subtraction)

1. Solve the following:

(a) 532 –289	(b) 643 –478	(c) 912 –289
(d) 604 –467	(e) 7800 –471	(f) 10000 –9999

2. Solve the following:

(a) 795 – 199 = ...

(b) 996 – 848 = ...

(c) 776 – 499 = ...

3. Fill empty boxes :

(a) 4 □ 6 – 2 7 □ □ 2 7	(b) 3 □ 6 8 – □ 7 4 5 1 6 □ 3	(c) 3 □ 6 0 – □ 8 9 4 0 2 □ 6

4. 196 litre of water has been used out of tank having 807 litre water. How much water has been left in tank?

5. Solve the following:

(a) 2048 + 3088 – 4017 = ...

(b) 48 + 37 – 23 + 49 – 63 = ...

(c) –103 + 63 + 36 – 37 + 269 = ...

Exercise (Multiplication)

1. Fill in the blanks :

(a) 7 × 0 = ... (b) 6 × 1 = ...

(c) 9 × 1 = ... (d) 71547×1 = ...

(e) 963 × 0 = ... (f) 23 × 47 × 0 × 32 = ...

(g) 1× 1 = ... (h) 0 × 0 = ...

2. Fill in the blanks:

(a) 20 × 30 = ... (b) 40 × 300 = ...

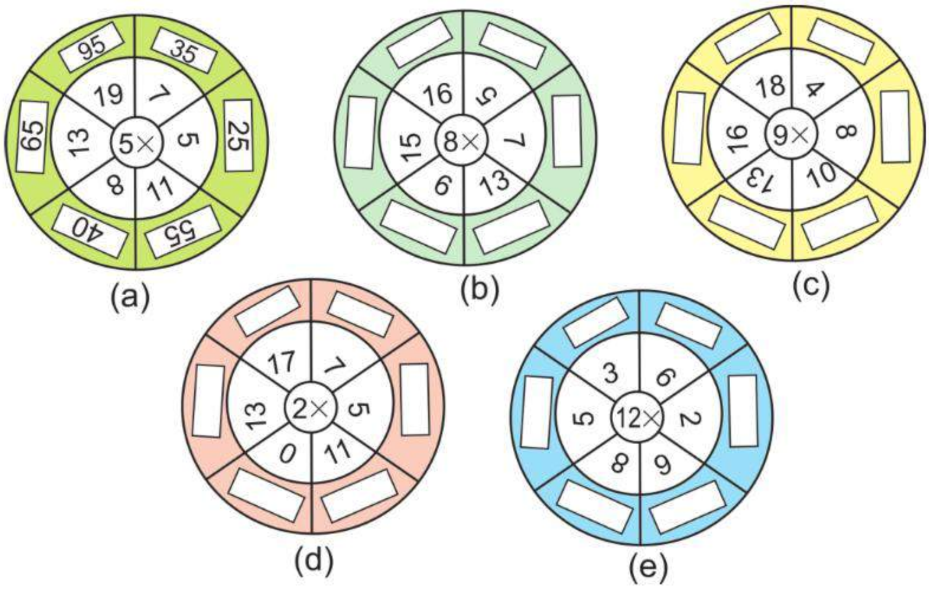
(c) 40 × 2000= .... (d) 90 × 9000 = ...

(e) 800 × 700 = ... (f) 12 × 200 = ...

(g) 8 × 11000 = ... (h) 7 × 1200 = ...

3. (a) 309 × 42 (b) 567 × 56 (c) 407 × 43 (d) 165 × 14

4. Fill the empty boxes:



5. Isha saves ₹ 48290 every month. How much money will she have after 2 years ?

6. Surinder bought 15346 chairs for auditorium. If each chair costs ₹398. How much money did Surinder paid?

7. A milk booth sells 448 litres of milk daily. How many litres of milk will it sell in 4 weeks?

8. Multiply :

(a) 3125 × 533 (b) 2391 × 236

(c) 4332 × 805 (d) 9219 × 78

(e) 473 × 999 (f) 234 × 11

Exercise (Division)

1. Fill in the blanks :

(a) 725 ÷ 1 = ... (b) 725 ÷ 725 = ...

(c) 0 ÷ 725 = ... (d) 823 ÷ 1 = ...

(e) 823 ÷ 823 = ... (f) 0 ÷ 823 = ...

(g) 0 ÷ 99999 = ... (h) .... ÷ 35 = 0

(i) 87450 ÷ .... = 1 (j) 8129 ÷ .... = 8129

2. Find quotient and remainder for the following :

(a) 1652 ÷ 7 (b) 5893 ÷ 6 (c) 7406 ÷ 6

(d) 11982 ÷ 5 (e) 28359 ÷ 12 (f) 12321 ÷ 11

3. Solve the following (Find Quotient and Reminder)

(a) 714 ÷ 7 (b) 618 ÷ 6 (c) 2416 ÷ 8

(d) 142114 ÷ 7 (e) 1384 ÷ 6 (f) 17126 ÷ 8

(g) 2107 ÷ 9 (h) 3046 ÷ 13 (i) 27661 ÷ 12

4. Fill empty boxes: (Remember : Dividend = Divisor × Quotient + Remainder)

	Dividend	**Divisor**	**Quotient**	**Reminder**
Example	138	11	12	6
(a)	158	13	12	☐
(b)	2168	☐	135	8
(c)	1689	14	☐	9
(d)	1414	14	☐	0
(e)	90	☐	12	6

5. Divide the largest 3-digit number by the largest 2 digit number and find the quotient and remainder.

6. A factory manufactured 936243 clips in 21 days. How many clips are manufactured in one day?

ANSWER KEY

Exercise (Addition)

1. (a) 7232 (b) 9118 (c) 9797 (d) 2821 (e) 7792 (f) 1415

2. (a) 594 (b) 2662 (c) 4510 (d) 86415

3. (a) 0 (b) 3, 5 (c) 1, 6, 7

4. (a) 0, 1 (b) 5, 2 (c) 3, 7

5. 226

Exercise (Subtraction)

1. (a) 243 (b) 165 (c) 623 (d) 137 (e) 7329 (f) 1

2. (a) 596 (b) 148 (c) 277

3.

(a)

	4	0	6
−	2	7	9
	1	2	7

(b)

	3	3	6	8
−	1	7	4	5
	1	6	2	3

(c)

	3	1	6	0
−	2	8	9	4
	0	2	6	6

4. 611 litre

5. (a) 1119 (b) 48 (c) 434

Exercise (Multiplication)

1. (a) 0 (b) 6 (c) 9 (d) 71547

(e) 0 (f) 0 (g) 1 (h) 0

2. (a) 600 (b) 12000 (c) 80000 (d) 810000

(e) 560000 (f) 2400 (g) 88000 (h) 8400

3. (a) 12978 (b) 31752 (c) 17501 (d) 2310

4. (b)

$8 \times 5 = 40$
$8 \times 7 = 56$
$8 \times 13 = 104$
$8 \times 6 = 48$
$8 \times 15 = 120$
$8 \times 16 = 128$

(c)

$9 \times 4 = 36$
$9 \times 8 = 72$
$9 \times 10 = 90$
$9 \times 13 = 117$
$9 \times 16 = 144$
$9 \times 18 = 162$

(d) $2 \times 7 = 14$

(e) $12 \times 6 = 72$

$2 \times 5 = 10$	$12 \times 2 = 24$
$2 \times 11 = 22$	$12 \times 9 = 108$
$2 \times 0 = 0$	$12 \times 8 = 96$
$2 \times 13 = 26$	$12 \times 5 = 60$
$2 \times 17 = 34$	$12 \times 3 = 36$

5. 1158960

6. 6107708

7. 12544

8. (a) 1665625 (b) 564276 (c) 3487260

(d) 719082 (e) 472527 (f) 2574

Exercise (Division)

1. (a) 725 (b) 1 (c) 0 (d) 823

(e) 1 (f) 0 (g) 0 (h) 0

(i) 87450 (j) 1

2. (a) Q = 236 ; R = 0 (b) Q = 982 ; R = 1

(c) Q = 1234; R = 2 (d) Q = 2396 ; R = 2

(e) Q = 2363 ; R = 3 (f) Q = 1120 ; R = 1

3. (a) Q = 102; R = 0 (b) Q = 103; R = 0 (c) Q = 302; R = 0

(d) Q = 20302; R = 0 (e) Q = 230; R = 4 (f) Q = 2140; R= 6

(g) Q = 234; R =1 (h) Q = 234; R = 4 (i) Q = 2305; R = 1

4. (a) 2 (b) 16 (c) 120

(d) 101 (e) 7

5. Q = 10 ; R = 9

6. 44583

Y7A3J4

1

KNOWING OUR NUMBERS

Objectives

In this chapter you will learn

(i) To read and write numbers in Indian as well as International system of Numeration.

(ii) To solve day to day life practical mathematical problems.

(iii) To perform basic operations on numbers and adopt them in routine life.

(iv) To make comparison between numbers.

(v) To use Brackets while solving mathematical problems.

(vi) To use Roman numeration system along with Hindu Arabic system of Numeration.

1.1 Introduction

We have enjoyed working with numbers in our previous classes. We have added, subtracted, multiplied and divided them. We also looked for patterns in number sequences and done many other interesting things with numbers. In this chapter we shall move forward on such interesting things with a bit of review and revision as well.

1.2 Few Basic terms

* **Natural Numbers :** Since our childhood, we are using numbers 1, 2, 3, 4, 5, ... etc. to count and calculate. These counting numbers are called natural numbers. Whatever, there is in nature, we count with these numbers, hence they are called both **Counting Numbers** as well as **Natural Numbers**.

- Smallest or first Natural Number is '1'.
- By adding 1 to any natural number we can get next natural number.
- There is no largest natural number.
- Set of natural numbers is represented by 'N'.

* **Digits :** To represent any number, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. These ten symbols are called digits.

1.3 Comparing Numbers

While comparing two numbers, we have to remember the following steps :

Step 1 : If the number of digits in the given numbers are not same, then the number with less number of digits will be smaller.

Step 2 : If the number of digits in both the numbers are same then,

(a) First compare the digits at the first place from the left. The number with the greater digit is greater than the other number.

(b) If the numbers have same digits at the first place, then compare the digits at the second place from left. The number with the greater digit is the greater one.

(c) Continue the process till you get unequal digit at the corresponding place.

(d) Apply the same process while comparing more than two numbers.

Example 1 : **Compare the following**

(a) 235 and 1023

(b) 47321 and 39874

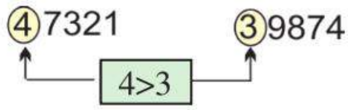
(c) 56398 and 56412

Solutions :

(a) 235 is the three digit number and 1023 is the four digit number So, number with more digits is greater.

Therefore 1023 is greater than 235.

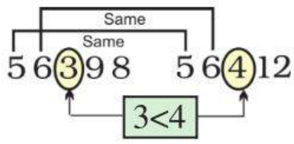
(b) <u>4</u>7321 is a five digit number.



<u>3</u>9874 is also a five digit number.

Compare the first digit from left side of both numbers. We observe these are 4 and 3 and $4 > 3$.

Therefore 47321 is greater than 39874.



(c) 56398 is a five digit number

56412 is also a five digit number.

We observe that first two digits from left side of both numbers are same. Both number start with 56.

Third digit from left side is different in both numbers and that are 3 and 4. and $3 < 4$.

Therefore 56412 is greater than 56398.

Example 2 : **Find the greatest and smallest among following numbers:**
1903, 9301, 1930, 9031, 9310

Solution : All the given numbers:

1903, 9301, 1930, 9031, 9310 are four digit numbers.

Let us examine the digit on extreme left side of each number.

First digit on the left side of two numbers is 1 and the first digit on the left side of other number is 9.

Smaller number will be observed between 1903 and 1930. And the greater number will be observed among 9301, 9031 and 9310. Then by observing second and third digit from left side we conclude that 1903 is the smallest and the 9310 is the greatest number.

1.3.1 Making Different numbers by shifting digits

Think about it, if the digits are shifted from one place to other in a number. Let us have an example 327. We can frame six different numbers (including 327) by just shifting of digits.

For Example : 327, 372, 237, 273, 732, 723

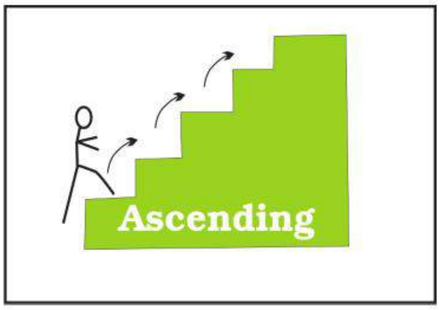
* Among these six numbers can you identify the largest and smallest number?

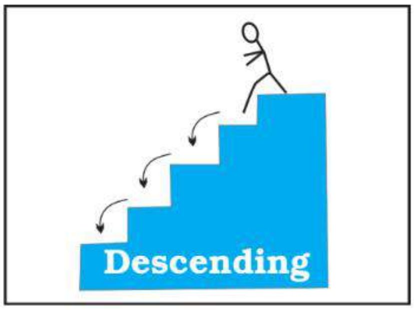
* Try to write all possible three digit numbers using digits 2, 3 and 5.

* Try to write all possible four digit numbers by using digits 3, 5, 7, 9 also, without repeating any digit. Write the greatest and smallest number among them.

1.3.2 Ascending (Increasing) and Descending (Decreasing) order.

Ascending order means the arrangement of numbers from the smallest to the greatest.





Descending order means the arrangement of numbers from the greatest to the smallest.

Example 3. **Arrange the numbers in ascending order 653, 1135, 47629, 2546, 7320**

Solution : The 3-digit number is the smallest number and the 5-digit number is the greatest number in the given numbers, number next to the smallest number is 1135, the next 4-digit number is 2546 followed by 7320.

Ascending order of given numbers is 653, 1135, 2546, 7320, 47629.

Example 4. **Use the given digits without repetition and make the greatest and smallest 4 digit number.**

(a) 2, 3, 1, 7 **(b) 4, 9, 0, 2**

Solution : (a) Digits to be used are 2, 3, 1, 7

Let us first arrange these digits

in ascending or descending order as per your choice.

Ascending order : 1, 2, 3 7

Now greatest 4 digit number with these digits = 7321

and the smallest 4 digit number with these digits = 1237

(b) Given digits are 4, 9, 0, 2

Let us arrange in ascending order = 0, 2, 4, 9

Now Greatest 4 digit number with these given digits = 9420

Now smallest 4 digits number with these given digits = 2049

Be Careful students 0249 is not the 4 digit number, because zero is not significant when it occupies the extreme left position. Here 0249 is the 3 digit number.

Example 5. **Using any one digit twice make the greatest and smallest 4 digit number.**

(a) 5, 2, 8 **(b) 7, 0, 2**

Solution : (a) Digits given are 5, 2, 8

Let us arrange them in ascending order as 2, 5, 8

Greatest 4 digit number repeating one digit = 8852

(We shall repeat the digit with highest face value i.e. 8)

Smallest 4 digit number repeating one digit = 2258

(We shall repeat the digit with lowest face value)

(b) Digits given are 7,. 0, 2

Let us arrange them in ascending order as 0, 2, 7

Greatest 4 digit number repeating one digit = 7720

(We shall repeat the digit with highest face value i.e. 7)

Smallest 4 digit number repeating one digit = 2007

(We shall repeat the digit with lowest face value i.e 0, but we can't place zero at extreme left place)

1.3.3 Place value and face value

The place value of a digit depends on its position, whereas the face value does not depend on its position. For example in the number 9678, the face value of 8 is 8. Similarly the face value of 7, 6 and 9 are also 7, 6 and 9 respectively.

However when we are concerned with place value.

The digit 8 has the place value	=	$8 \times 1 = 8$	(8 lies at units place)
The digit 7 has the place value	=	$7 \times 10 = 70$	(7 lies at tens place)
The digit 6 has the place value	=	$6 \times 100 = 600$	(6 lies at hundreds place)

The digit 9 has the place value = $9 \times 1000 = 9000$ (9 lies at thousands place)

In expanded form 9678 will be written as

$9678 = 9 \times 1000 + 6 \times 100 + 7 \times 10 + 8 \times 1$

It is evident from this that a number is the sum of the place values of all its digits.

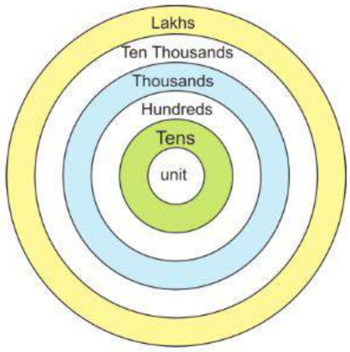
Place value of a digit = Face value × Position value

It is to be noted that position values of units, tens, hundreds, thousands, ten thousands, lakhs and so on..... are respectively 1, 10, 100, 1000, 10000, 100000, ... and so on.

Place value of 0 is 0 itself, where ever it may be.

Students! Let us play with numbers.

Draw concentric circles on a card board (as shown) or on ground. With marker write unit, tens, hundreds, thousands, ten thousands and so on....



Have some marbles and be ready to play. Throw these marbles gently on the card board with number circles.

Let us suppose that the marbles settle themselves as shown in the figure. Now can you identify the number.

Place value of Marbles in Ten thousands Circle = 1×10000

= 10000

Place value of Marbles in Thousands Circle = $2 \times 1000 = 2000$

Place value of Marbles in Hundreds Circle = $4 \times 100 = 400$

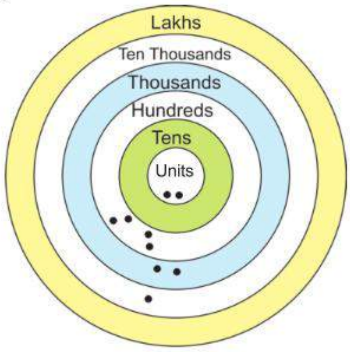
Place value of Marbles in Tens Circle = $0 \times 10 = 0$

Place value of Marbles in Units Circle = $2 \times 1 = 2$

Number = Sum of place values of all digits

= 10,000 + 2,000 + 400 + 0 + 2

= 12402



Now Let us prepare the place value chart

F.V. = Face Value

Roll No.	Name of Student	Lakhs	Ten Thousands	Thousands	Hundreds	Tens	Units	Number
		F.V. × 100000	F.V. × 10000	F.V. × 1000	F.V. × 100	F.V. × 10	F.V. × 1	Add all place values
1.	--	--	10000	2000	400	0	2	10,000+2000+ 400+0+2 =12402
2.								

Example 6. **Read and expand the numbers in the following table.**

Number	Number Name	Expansion
27000	Twenty Seven Thousand	$2 \times 10,000 + 7 \times 1000$
37600	Thirty Seven Thousand Six hundred	$3 \times 10000 + 7 \times 1000 + 6 \times 100$
56740		
69563		
42639		
29308		
20005		
19075		

1.4. Reading and Writing Numbers in Indian system of Numeration

According the 2011 census population of Punjab was approximately 2 Crore 77 lakh 43 thousand. In our daily routine life we need to speak numbers like thousands, Lakhs, Crores etc. It is the Indian system of Numeration.

In order to read numbers in the Indian system of Numeration, we make groups (periods) of place values like - 'Ones' , Thousands, Lakhs, Crores etc, separated by commas.

- The First three digits from the right of a number make unit period (or unit group)
- The next two digits from the right make thousands period.
- The next two digits from right make Lakhs period.
- The next two digits from right make crores period and so on.....

The digits in the same group or period are read together and the name of the period (except units) is read along with them.

Thus the number 8,76,54,321 is read as 'Eight Crores Seventy Six Lakhs Fifty Four Thousand Three hundred twenty one'.

Commas after each period are put to have a clear instant look.

Example 7. **Read the following numbers**

(a) 534632 **(b) 90763021**

Solution : (a) Given number is 534632.

Firstly place commas from the right side making group of 3, 2, 2 and so on ......

So given number is 5,34,632

It is clearly Five lakh Thirty Four thousand Six hundred thirty two.

(b) Given number is 90763021.

Firstly place commas from the right side making group of 3, 2, 2, 2 and so on....

So given number is 9,07,63,021

It is clearly Nine Crore Seven Lakh Sixty three thousand twenty one.

Example 8. **Write the number names as numerals.**

(a) Four lakh thirty two thousand six hundred seventy three.

(b) Six crore fifty three lakh twenty one thousand nine hundred seventy two.

Solution : (a) Four lakh thirty two thousand six hundred seventy three.

Crores	Lakhs	Thousands	Ones
0	04	32	673

= 4, 32, 673

(b) Six crore fifty three lakh twenty one thousand nine hundred seventy two.

= 6, 53, 21, 972

Example 9. **Find the difference of the place value and the face value of the digit 7 in 9745623.**

Solution : Given number is 9745623

Place value of 7 is 7,00,000

Face value of 7 is 7

Required difference is = 700000 – 7

= 699993

Example 10. **How many four digit numbers are there in all.**

Solution : Largest four digit number is 9999

Largest three digit number is 999

Total number of four digit numbers

= Largest four digit numbers – Largest three digit numbers

= 9999 – 999

= 9000

Example 11. **Write each of the numbers arranged in the following place value chart in words:**

	Crores		Lakhs		Thousands		Ones		
	Ten Crores 10,00,00,000	Crores 1,00,00,000	Ten Lakhs (10,00,000)	Lakhs (1,00,000)	Ten Thousands (10,000)	Thousands (1000)	Hundreds (100)	Tens (10)	Ones (1)
(i)		4	7	5	0	0	2	9	8
(ii)		7	8	0	5	1	0	2	4

Solution : (i) Given number is 4,75,00,298

Four crores seventy five lakh two hundred ninety eight

(ii) Given number is 7,80,51,024

Seven crore eighty lakh fifty one thousand twenty four.

Example 12. **Read the following numbers using place value chart.**

(a) 593268 **(b) 32067308**

Solution :

Crores		Lakhs		Thousands		Ones			Numbers
Ten Crores	Crores	Ten Lakhs	Lakhs	Ten Thousands	Thousands	Hundreds	Tens	Ones	
			5	9	3	2	6	8	Five lakh ninety three thousand two hundred sixty eight
	3	2	0	6	7	3	0	8	Three crore twenty lakh sixty seven thousand three hundred eight

1.5 International system of Numeration

In last section we have discussed about Indian system of Numeration. Now we shall learn about the International system of Numeration, which is followed by the most of countries of the world. In this system also, a number is split up into groups or periods. Starting from extreme right the number is split up into groups of three each. These groups are called Ones, Thousands, Millions and Billions. These groups are further sub-divided as follow:

Billions			Millions			Thousands			Ones		
Hundred Billions	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units

Example 13. **Insert Commas suitably and write the names according to:**

(a) Indian System of Numeration

(b) International System of Numeration 79530257

Solution : Indian system of Numeration

7,95,30,257

Seven crore ninety five lakh thirty thousand two hundred fifty seven.

International system of numeration

79,530,257

Seventy nine million five hundred thirty thousand two hundred fifty seven.

Example 14. **Write each of the numbers arranged in the following place value chart in words.**

Billions			Millions			Thousands			Ones		
Hundred Billions	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thou-sands	Ten Thou-sands	Thou-sands	Hundreds	Tens	Units
		2	3	4	1	2	9	8	3	4	6
				3	9	1	7	0	3	1	7

Solution : (i) Given number is 2, 341, 298, 346

Two billion three hundred forty one million two hundred ninety eight thousand three hundred forty six

(ii) Given number is 39, 170, 317

Thirty nine million one hundred seventy thousand three hundred seventeen.

• Relation Between Indian and International System of Numeration

Number	Indian System	International System
1	One	One
10	Ten	Ten
100	One Hundred	One Hundred
1000	One Thousand	One Thousand
10000	Ten Thousand	Ten Thousand
100000	One Lakh	One Hundred thousand
1000000	Ten Lakh	One Million
10000000	One Crore	Ten Million
100000000	Ten Crore	One Hundred Million
1000000000	One Arab	One Billion

Exercise 1.1

1. Write the smallest and the greatest number:
   (a) 30900, 30594, 30945, 30495
   (b) 10092, 10029, 10209, 10920
2. Arrange the numbers in ascending order:
   (a) 6089, 6098, 5231, 3953
   (b) 49905, 6073, 58904, 7392
   (c) 9801, 25751, 36501, 38802
3. Arrange the numbers in descending order:
   (a) 75003, 20051, 7600, 60632
   (b) 2934, 2834, 667, 3289
   (c) 1971, 45321, 88715, 92547
4. Use the given digits without repetition and make the greatest and smallest 4 digit number:
   (a) 6, 4, 3, 2 (b) 9, 7, 0, 3
   (c) 5, 4, 0, 3 (d) 3, 2, 7, 1
5. Using any one digit twice make the greatest and the smallest 4 digit number:
   (a) 2, 3, 7 (b) 5, 0, 3 (c) 2, 3, 0
   (d) 1, 3, 4 (e) 2, 5, 8 (f) 1, 2, 3
6. Read the following numbers using place value chart:
   (a) 638975 (b) 84321 (c) 29061058 (d) 60003608
7. Insert commas suitably and write the names according to Indian system of Numeration:
   (a) 98606873 (b) 7635172 (c) 89700057
   (d) 89322602 (e) 4503217 (f) 90032045
8. Insert commas suitably and write the names according to International system of Numeration:
   (a) 89832081 (b) 6543374 (c) 88976306
   (d) 9860001 (e) 90032045 (f) 4503217
9. Write the number names as numerals:
   (a) Seven lakh fifty four thousand
   (b) Nine crore fifty three lakh seventy four thousand five hundred twenty three
   (c) Six hundred forty seven thousand five hundred twenty five
   (d) Seventy two million three hundred thirty two thousand one hundred twelve.
   (e) Fifty eight million four hundred twenty three thousand two hundred two.
   (f) Twenty three lakh thirty thousand ten.
10. How many eight digit numbers are there in all.

11. Fill in the blanks:

(a) 1 Lakh = ... ten thousand

(b) 1 Million = ... hundred thousand

(c) 1 Crore = ... ten lakh

(d) 1 Crore = ... million

(e) 1 Million = ... lakh

1.6 Numbers in Length, Weight and Capacity

(Use Numbers in Practical Life):-

In previous classes we have learnt about units of measurement like length, weight and capacity. In this section we will learn to operate upon them in context of day to day practical problems of life.

- **Length :** We express the length of a pencil or paper in centimetres. Whereas to express the thickness of a paper clip, we find centimetres too big. We use smaller units i.e. millimetre. On the other hand to measure height of a building. We need bigger units that is metre. And we need bigger units to express distance between two cities, which is kilometre.

For example the distance between Bathinda and Chandigarh is about 230 kilometres.

		10 millimetres	=	1 centimetre
1 metre	=	100 centimetre	=	1000 millimetre
1 kilometre	=	1000 metres	=	1000000 millimetres

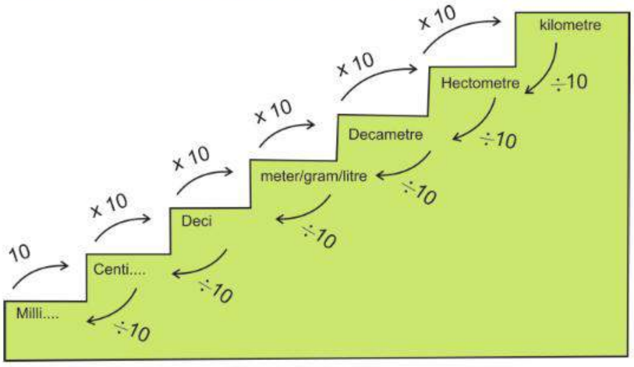
- **Weight :** We buy things like turmeric, ginger and garlic in grams whereas for buying things like flour, potatoes and rice we use bigger units like Kilograms. But medicine we consume is in milligrams.

1 kilogram	=	1000 gram
1 gram	=	1000 miligram

- **Capacity :** Usually when we drink water in a glass, it is approximately 250 to 300 mℓ. When we fill a bucket with water to have bath, it is nearly 20 to 25 litre.

1 litre	=	1000 millilitre

To Interchange the Units of Measurement



Fill all blanks with metre or gram or litre as follow:

milli .... : milimetre
miligram
millilitre

To Remember

King	Harry	Died,	Mother	Did not	Cry	Much
↓	↓	↓	↓	↓	↓	↓
Kilo	Hecto	Deca	Metre/ Gram/ Litre	Deci	Centi	Milli

Example 15. **In an election, the successful candidate registered 5765 votes and his nearest rival secured 3427 votes. By what margin did the successful candidate won the election?**

Solution : Number of votes secured by winning candidates = 5765

Number of votes secured by nearest rival = 3427

Winning Margin = 5765 – 3427

= 2338 Votes

Example 16. **A town newspaper is published everyday. One copy of that newspaper contain 13 pages and 11980 copies are printed daily. How many total pages are printed daily?**

Solution : Each copy of newspaper has 13 pages.

Hence 11980 copies will have 11980 × 13 pages

Now

```
   11980
     ×13
  ------
   35940
+ 119800
  ------
  155740
```

Hence everday 1,55,740 pages are printed.

Example 17. **A Shopkeeper has 48000 sheets of paper. He wants to bundle them in the form of reams. One ream of paper contain 480 sheets. How many reams will be made?**

Solution : Total number of sheets = 48000

Number of sheets in 1 Ream = 480

Number of Reams = Total Number of Sheets ÷ Sheets in one Ream

= 48000 ÷ 480

= 100 Ream

```
        100
480 ) 48000 (
     -480
     ----
       00
      - 0
     ----
        00
       - 0
      ----
         0
```

Example 18. **A vessel has 4 litre and 650mℓ of curd. In how many glasses, each of 25mℓ capacity, can it be distributed ?**

Solution : Volume of vessel having curd = 4 litre 650 mℓ

$= 4 \times 1000m\ell + 650m\ell$

$= 4000m\ell + 650m\ell$

$= 4650m\ell$

Capacity of One Glass = 25mℓ

Number of Glasses = Volume of Total Curd ÷ Capacity of One Glass

$= 4650\ m\ell \div 25m\ell$

$= 186$

$$\begin{array}{r|l} 25 & 4650\ (186 \\ & -25 \\ & \overline{215} \\ & -200 \\ & \overline{150} \\ & -150 \\ & \overline{0} \end{array}$$

4 litre 650 mℓ curd will be distributed in 186 glasses each of capacity 25mℓ.

Exercise 1.2

1. Convert the following measurements as directed:
   (a) 5 km into metre
   (b) 35 kilometre into metre
   (c) 2000 milligram into gram
   (d) 500 decigram into gram
   (e) 2000 millilitre into litre
   (f) 12 kilolitre into litre
2. In an election, the successful candidate registered 6317 votes whereas his nearest rival could attain only 3761 votes. By what margin did the successful candidates defeat his rival?
3. A monthly magazine having 37 pages is published on 20th day of each month. This month 23791 copies were printed. Tell us how many pages were printed in all?
4. A shopkeeper has 37 reams. One ream contain 480 pages and he wants to make quires of all these sheets to sell in retail. One quire of sheets contain 24 sheets. How many quires will be made?
5. Veerpal serves milk to the guests in glasses of capacity 250 mℓ each Suppose that the glasses are filled to capacity and there was 5 litre milk that got consumed. How many guests were served with milk?
6. A box of medicine contain 2,00,000 tablets each weighing 20mg. What is the total weight of tablets in box?
7. A bookstore sold books worth Rupees Two lakh eighty five thousand eight hundred ninety one in the first week of June. They sold books worth Rupees Four lakh seven hundred sixty eight in the second week of June. How much was the total sale for two weeks together?

8. A famous cricket player has so far scored 6978 runs in test matches. He wishes to complete 10,000 runs. How many more runs he need?

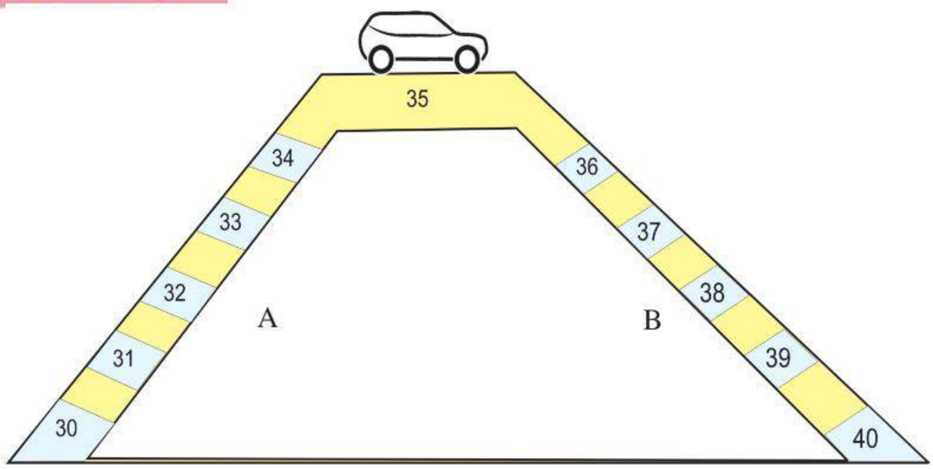
9. Surinder has ₹ 78592 with him. He placed an order for purchasing 39 radio sets at ₹1234 each. How much money will remain with him after the purchase?

10. A vessel has 3 litre 650 mℓ of curd. In how many glasses each of 25 mℓ capacity can it be distributed?

1.7 Estimation and Approximation

Numbers are very commonly used in our daily life. We need to answer many questions like 'How many'? But we need not to answer the exact number. We instantly answer a rough estimation. For example you tell one of your friends that you attended a marriage party yesterday and there was a big gathering of 600 people. 600 is your rough estimation. You have not counted them. There are many situations where it is sufficient to tell the estimated figure instead of telling the exact one. So we must know how to estimate (or round off) a number.

ACTIVITY



Let us try to understand strongly the concept of rounding off numbers with an activity. Prepare a two way inclined plane as shown in picture above.

A car on inclined at any point in portion A slips to lowest level at number 30 in above example. It means 31,32, 33, 34 when rounded off to nearest tens, they are rounded to 30.

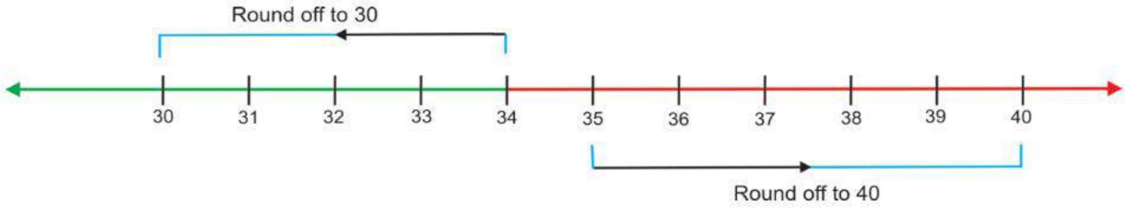
But when car is at any point in Portion B, slips to 40. It means 35, 36, 37, 38, 39 are rounded off to nearest tens are rounded to 40.

Rule I: Estimating or Rounding off numbers to Nearest Tens :

Follow the following rules to round off to nearest tens.

(a) If ones place digit is less than 5, replace ones digit by 0 and the other digits remain as they are.

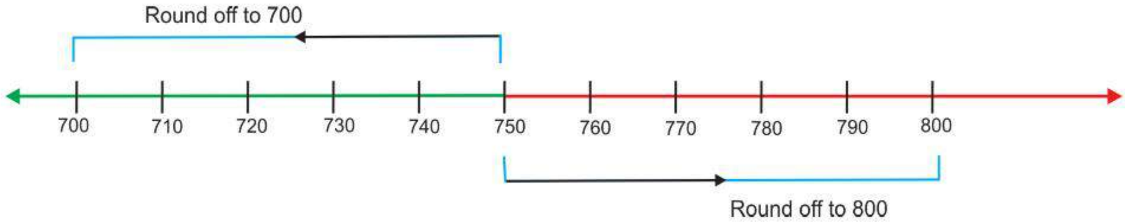
(b) If ones digits is more than or equal to 5, increase the tens digit by 1 and replace, ones digit by 0.



Rule II. Estimating or Rounding off numbers to Nearest Hundreds.

Follow the following rules to round off to nearest hundreds.

(a) If tens digit is less than 5, replace tens and ones digit by 0 and other digits remaining same.



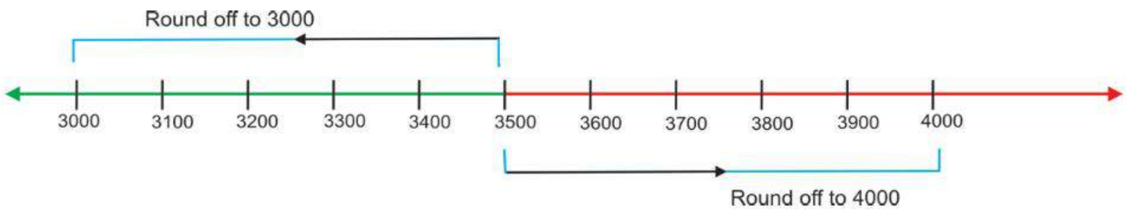
(b) If tens digit is more than or equal to 5, then increase the hundreds digit by 1 and replace each digit on its right by 0.

Rule III. Rounding off numbers to Nearest Thousands

Follow the following rules to round off to nearest thousands

(a) If hundreds digit is less than 5, replace hundreds, tens and ones digit by 0 and other digits remaining same.

(b) It hundreds digit is more than or equal to 5, then increase the thousands digit by 1 and replace each digit on its right by 0.



Example 19. Round off 36182 and 36827 to the nearest tens, nearest hundreds and nearest thousands.

Solution : Given numbers are 36182 and 36827

(i) Rounded off to nearest tens = 36180

Rounded off to nearest tens = 36830

(ii) Rounded off to nearest hundreds = 36200

Rounded off to nearest hundreds = 36800

(iii) Rounded off to nearest Thousands = 36000

Rounded off to nearest Thousands = 37000

1.7.1 To estimate sum, difference, product and quotient

There are many situations where we have to estimate the sum or difference or product or quotient of numbers. Upto which digit they should be rounded, it depends upon the necessity and requirement of data. Sometimes high degree of accuracy is required, sometimes we need to get result very quickly, although degree of accuracy is low. So all these things matter. Let us explain with examples.

Example 20. Estimate the sum 5290 + 17986 by rounding off to

(i) Hundreds Place (ii) Thousands Place

Solution : (i) While rounding off to hundreds place

5290 + 17986 = 5300 + 18000

= 23300

(ii) While rounding off to thousands place

5290 + 17986 = 5000 + 18000

= 23000

You can verify by performing actual addition that 23300 is more close, hence more reasonable.

Example 21. Estimate 5673 – 436

Solution : To begin with we round off to thousands

5673 rounds off to 6000

436 rounds off to – 0

Estimated difference = 6000

This is not a reasonable estimate. As the difference is greater than both numbers.

To get a closer estimate, Let us try rounding each number to hundreds place.

5673 rounds off to 5700

436 rounds off to – 400

Estimated Difference 5300

This is better and more meaningful estimate.

Example 22. Estimate the Products

(a) 87 × 313 **(b) 898 × 785**

(c) 63 × 182 **(d) 81 × 479**

By General Rule

General rule is rounding off each factor to its greater place, then multiplying the rounded off numbers.

Solution : (a) 87 × 313

87 rounds off to tens place = 90

313 rounds off to hundreds place = 300

Estimated product = 90×300

= 27000

(b) 898 × 785

898 rounds off to hundreds place = 900

785 rounds off to hundred place = 800

Estimated product = 900 × 800

= 720000

(c) 63 × 182

63 rounds off to tens place = 60

182 rounds off to hundreds place = 200

Estimated product = 60 × 200

= 12000

(d) 81 × 479

81 rounds off to tens place = 80

479 rounds off to hundreds place = 500

Estimted product = 80 × 500

= 40000

Example 23. **Find estimated quotient 2437 ÷ 125 by general rule**

Solution : 2437 ÷ 125

2437 rounded off to thousands place = 2000

125 rounded off to hundreds place = 100

Esttimated Quotient = 2000 ÷ 100

= 20

Exercise 1.3

1. Estimate each of the following using general rule:
   (a) 837 + 987 (b) 783 – 427 (c) 1391 + 2783
   (d) 28292 – 21496
2. Estimate the product using general rule:
   (a) 898 × 785 (b) 9 × 795 (c) 87 × 317
   (d) 9250 × 29
3. Estimate by rounding off to nearest hundred:
   (a) 439 + 334 + 4317 (b) 108734 – 47599
4. Estimate by rounding off to nearest tens:
   (a) 439 +334 + 4317 (b) 108734 – 47599

1.8. Use of Brackets

Brackets are symbols used in pairs to group things together and write statements in explicit form   Most commonly used brackets are

( )   : **Parentheses or common brackets**

{ }   : **Curly Brackets**

[ ]   : **Square Brackets or Box Brackets**

In this section, we shall learn about the use of parentheses only.

Consider the statement '4 is multiplied by the sum of 3 and 7'.

Using Parentheses this statement is written as $4 \times (3 + 7)$

Following Examples illutrates the use of brackets.

Example 24. **Write expression for each of the following statements using brackets and then simplify.**

(a) **Seven is multiplid by the sum of three and four.**

(b) **Sum of nine and four is multiplied by six.**

(c) **Divide the difference of eighteen and six by four.**

Solution : (a) $7 \times (3 + 4) = 7 \times 7 = 49$

(b) $(9 + 4) \times 6 = 13 \times 6 = 78$

(c) $(18 - 6) \div 4 = 12 \div 4 = 3$

1.8.1 Expanding Brackets

To expand brackets, we need to use distributive law. Distributive law states that a number outside a bracket performs the same operation with each term inside the bracket as follow:

$a(b+c) = ab + ac$

$a(b-c) = ab - ac$

Example 25. **Solve by expanding brackets**

(a) $7 \times (3 + 4)$ **(b)** $(9 + 4) \times 6$ **(c)** $(20 - 8) \div 4$

Solution : (a)
$$7 \times (3 + 4) = 7 \times 3 + 7 \times 4 = 21 + 28 = 49$$

(b)
$$(9 + 4) \times 6 = 9 \times 6 + 4 \times 6 = 54 + 24 = 78$$

(c)
$$(20 - 8) \div 4 = (20 \div 4) - (8 \div 4) = 5 - 2 = 3$$

Example 26. **Simplify:**

(a) 8×107 **(b)** 14×108

Solution : (a) $8 \times 107 = 8\times (100 + 7)$

$= 8 \times 100 + 8 \times 7$

$= 800 + 56$

$= 856$

(b) $14 \times 108 = (10 + 4) \times 108$

$= 10 \times 108 + 4 \times 108$

$= 10 \times (100 + 8) + 4 \times (100 + 8)$

$= 10 \times 100 + 10 \times 8 + 4 \times 100 + 4 \times 8$

$= 1000 + 80 + 400 + 32$

$= 1000 + 400 + 80 + 32$

$= 1512$

Exercise 1.4

1. Simplify each of following:

(a) 13×104 (b) 102×105 (c) 6×107

(d) 16×106 (e) 201×205 (f) 22×102

(g) $6 \times (4 + 3)$ (h) $(17 - 9) \times 3$ (i) $(20 + 4) \div 2$

1.9 Roman Numerals

We have already learnt the Indian system of numeration as well as International system of Numeration. There is another numeration system which was developed by Romans and widely used between 900 BC and 300BC. It was originated in ancient Rome. In Roman Numeration system only symbols were used to express numbers. There are 7 basic symbols in Roman system which are used to represent different numbers.

Roman Symbols	I	V	X	L	C	D	M
Numeral corresponding in Hindu Arebic System	1	5	10	50	100	500	1000

- K is also used to represent 1000.
- A common belief is that the symbols of Roman system were taken from the pictures of Hands and Fingers.

This system of Numeration is still used in many places like : In front of your class rooms (Class VI etc.), numbers on clock faces, parts of books, to denote historical events such as : world war I, world war II, etc.

Remember Corresponding to '0' of Hindu Arabic system, there is no symbol for 0 (Zero) in Roman Numeration system.

Using these 7 symbols, we can write number by following certain rules which are given below:

Rule 1 : If a symbol is repeated its value, it is added as many times as it occours.

For Example : II = 1 + 1 = 2

XXX = 10 + 10 + 10 = 30

CC = 100 + 100 = 200

It must be noted that symbol I, X, C, M never repeated more than three times, and V, L, D are never repeated.

Rule 2 : Any smaller Roman numeral that comes after a larger numeral is added to it.

For Example: VI = 5 + 1 = 6

VII = 5 +1 + 1 = 7

XIII = 10 + 1 + 1 + 1 = 13

XVII = 10 + 5 + 1 + 1 = 17

LXXV = 50 + 10 + 10 + 5 = 75

Rule 3 : Any smaller Roman numeral that comes before a larger numeral is subtracted from it.

For Example : IV = 5 – 1 = 4

IX = 10 – 1 = 9

XL = 50 – 10 = 40

XC = 100 – 10 = 90

Rule 4 : The symbols V, L and D are never written to the left of a larger value symbol.

i.e. V, L, D are never subtracted.

I V X L C D M

- I, X, C comes before larger value numerals i.e. I, X, C can be subtracted from larger value numerals.
- I, X, C can be subtracted from next two respective numerals.

  i.e. I can be subtracted from V and X only

  X can be subtracted from L and C only

  C can be subtracted from D and M only
- V, L, D are never subtracted.

Rule 5: If a smaller Roman numeral comes between two larger numerals then the smaller numeral is subtracted from the larger numeral following it:

For Example : XIX = X + IX = 10 + (10 – 1) = 19

LXIV = L + X + IV = 50 + 10 + (5–1) = 64

Rule 6 : If a bar is placed over a numeral then it is multiplied by 1000.

For Example: $\overline{\text{V}}$ = 5000, $\overline{\text{X}}$ = 10000

Let us write few numbers using these rules:

1	=	I	
2	=	II	
3	=	III	
4	= 5–1 =	IV	(∵ I cannot be repeated more than 3 times)
5	=	V	
6	=	VI	
7	=	VII	
8	=	VIII	
9	= 10–1=	IX	(∵ I cannot be repeated more than 3 times)
10	=	X	
20	=	10 + 10 =XX	
30	=	10 + 10 + 10 = XXX	
40	=	50 – 10 = XL	(∵ X cannot be repeated more than 3 times)
45	=	40 + 5 = XLV	
49	=	40 + 9 = XLIX	(It can't be written as 50 – 1= IL as I cannot be subtrated from L, I can be subtracted from V and X)
50	=	L	
60	=	50 + 10 = LX	
70	=	50 + 10 + 10 = LXX	
80	=	50 + 10 + 10 + 10 = LXXX	
90	=	100 – 10 = XC	
100	=	C	
400	=	500 – 100 = CD	
500	=	D	
900	=	1000 – 100 = CM	
1000	=	M	

Example 27. **Which of the following are meaningless.**

(a) XXXX (b) LXIX (c) VL (d) LIV (e) IL

Solution : (a) XXXX

Since X cannot be repeated more than 3 times so XXXX is meaningless.

(b) LXIX = LX + IX = 60 + 9 = 69

So LXIX is meaningful.

(c) VL

Since V can never be subtracted.

So VL is meaningless.

(d) LIV = L + IV = 50 + 4 = 54

So LIV is meaningful.

(e) IL

Since I can be subtracted only from V and X not from L.

So IL is meaningless.

Example 28. **Write the following in Hindu Arabic Numerals.**

(a) LXXI (b) CXLV (c) CCXLI

(d) CLXVII (e) MCCXLI

Solution : (a) LXXI = L+X+X+I = 50 + 10 + 10 + 1 = 71

(b) CXLV = C+XL+V = 100 + 40 + 5 = 145

(c) CCXLII = C + C + XL + II = 100 + 100 + 40 + 2 = 242

(d) CLXVII = C + L + X + VII = 100 + 50 + 10 + 7 = 167

(e) MCCXLI = M + CC + XL + I = 1000 + 200 + 40 + 1 = 1241

Example 29. **Express each of the following as a Roman Numeral.**

(a) 49 (b) 82 (c) 198

(d) 541 (e) 826

Solution : (a) 49 = 40 + 9 = XLIX

(b) 82 = 50 + 30 +2 = LXXXII

(c) 198 = 100 + 90 + 8= CXCVIII

(d) 541 = 500 + 40 + 1 = DXLI

(e) 826 = 500 + 300 + 20 + 6 = DCCCXXVI

Exercise 1.5

1. Which of the following are meaningless:

(a) IC (b) VD (c) XCVII

(d) IVC (e) XM

2. Write the following in Hindu Arabic Numerals:

(a) XXV (b) XLV (c) LXXIX (d) XCIX

(e) CLXIV (f) DCLXII (g) DLXIX (h) DCCLXVI

(i) CDXXXVIII (j) MCCXLVI

3. Express each of the following as Roman numerals:

(a) 29 (b) 63 (c) 94 (d) 99

(e) 156 (f) 293 (g) 472 (h) 638

(i) 1458 (j) 948 (k) 199 (l) 499

(m) 699 (n) 299 (o) 999 (p) 1000

Multiple Choice Questions

1. The number of digits are ....

(a) 9 (b) 10 (c) 8 (d) Infinite

2. The greatest 4 digit number using 1, 5, 2, 9 once is

(a) 9215 (b) 9512 (c) 5912 (d) 9521

3. The smallest 4 digit number using 2, 0, 3, 7 once is

(a) 0237 (b) 2037 (c) 7320 (d) 7023

4. Which of the following are in ascending order?

(a) 217, 271, 127, 721 (b) 217, 127, 721, 271

(c) 127, 217, 271, 721 (d) 721, 271, 217, 127

5. The face value of digit 4 in 23468 is :

(a) 4 (b) 400 (c) 40 (d) 468

6. The place value of digit 2 in 4123 is

(a) 23 (b) 2 (c) 20 (d) 200

7. The difference between place value and face value of 5 in 76542 is:

(a) 537 (b) 45 (c) 0 (d) 495

8. $5 \times 10000 + 3 \times 100 + 2 \times 10 + 2 =$ .... = ...

(a) 5322 (b) 53022 (c) 50322 (d) 53202

9. Four lakh two thousand three hundred fifty one = ...

(a) 42351 (b) 402351 (c) 420351 (d) 4002351

10. How many four digit numbers are there?

(a) 9999 (b) 9900 (c) 9000 (d) 9990

11. Seventeen million twenty four thousand fifty four = ...

(a) 172454 (b) 170024054

(c) 170240054 (d) 17024054

12. 1 Crore = ... million

(a) 1 (b) 10 (c) 100 (d) 1000

13. Rounded off 7213 to nearest thousands.

(a) 7200 (b) 7000 (c) 7210 (d) 7213

14. Rounded off 45553 to nearest hundreds.

(a) 45500 (b) 45550 (c) 45600 (d) 45650

15. Solve; $(9–4) \times 6 =$ ....

(a) 30 (b) 54 (c) 78 (d) 64

16. Which of the following number does not have symbol in Roman numerals?

(a) 0 (b) 1 (c) 10 (d) 1000

17. How many symbols are used in Roman Numerals?

(a) 5 (b) 8 (c) 9 (d) 7

18. Which of the following are meaningless?

(a) LXIX (b) XC (c) IL (d) LI

19. CLXVI = ...

(a) 164 (b) 144 (c) 176 (d) 166

20. XCIX + XLVI =

(a) CVL (b) CLV (c) CXLV (d) CXLIV

Learning Outcomes

After completion of this chapter students are now able to

(i) Read and the write numbers in Indian as well as International sytem of Numeration.

(ii) Solve day to day life practical mathematical problems.

(iii) Perform basic operations on numbers and adopt them in routine life.

(iv) Make comparison between numbers.

(v) Use brackets while solving mathematical problems.

(vi) Use Roman Numeration system along with Hindu Arabic system of Numeration.

ANSWER KEY

Exercise 1.1

1. (a) Smallest = 30495 Greatest = 30945

(b) Smallest = 10029 Greatest = 10920

2. (a) 3953, 5231, 6089, 6098 (b) 6073, 7392, 49905, 58904

(c) 9801, 25751, 36501, 38802

3. (a) 75003, 60632, 20051, 7600 (b) 3289, 2934, 2834, 667

(c) 92547, 88715, 45321, 1971

4. (a) 6432, 2346 (b) 9730, 3079

(c) 5430, 3045 (d) 7321, 1237

5. (a) 7732, 2237 (b) 5530, 3005

(c) 3320, 2003 (d) 4431, 1134

(e) 8852, 2258 (f) 3321, 1123

6. (a) Six Lakh thirty eight thousand nine hundred seventy five
   (b) Eighty four thousand three hundred twenty one
   (c) Two crore ninety lakh sixty one thousand fifty eight
   (d) Six crore three thousand six hundred eight

7. (a) 9,86,06,873
   Nine crore eighty six lakh six thousand eight hundred seventy three
   (b) 76,35,172
   Seventy six lakh thirty five thousand one hundred seventy two
   (c) 8,97,00,057
   Eight crore ninety seven lakh fifty seven.
   (d) 8,93,22,602
   Eight crore ninety three lakh twenty two thousand six hundred two.
   (e) 45,03,217
   Forty five lakh three thousand two hundred seventeen.
   (f) 9,00,32,045
   Nine crore thirty two thousand forty five.

8. (a) 89,832,081
   Eighty nine million eight hundred thirty two thousand eighty one
   (b) 6,543,374
   Six million five hundred fourty three thousand three hundred seventy four.
   (c) 88,976,306
   Eighty eight million nine hundred seventy six thousand three hundred six.
   (d) 9,860,001
   Nine million eight hundred sixty thousand one
   (e) 90,032,045
   Ninety million thirty two thousand forty five.
   (f) 4,503,217
   Four million five hundred three thousand two hundred seventeen.

9. (a) 7,54,000 (b) 9,53,74,523 (c) 647,525
   (d) 72,332, 112 (e) 58,423,202 (f) 23,30,010

10. 90000000

11. (a) Ten (b) Ten (c) Ten
   (d) Ten (e) Ten

Exercise 1.2

1. (a) 5000 (b) 35000 (c) 2
   (d) 50 (e) 2 (f) 12000

2. 2556 **3.** 880267 **4.** 740 **5.** 20

6. 4 kg **7.** 686659 **8.** 3022 **9.** 30466

10. 146

Exercise 1.3

1. (a) 1800 (b) 400 (c) 4000 (d) 10000

2. (a) 720000 (b) 8000 (c) 27000 (d) 270000

3. (a) 5000 (b) 61100

4. (a) 5090 (b) 61130

Exercise 1.4

1. (a) 1352 (b) 10710 (c) 642 (d) 1696
(e) 41205 (f) 2244 (g) 42 (h) 24
(i) 12

Exercise 1.5

1. a, b, d, e

2. (a) 25 (b) 45 (c) 79 (d) 99
(e) 164 (f) 662 (g) 569 (h) 766
(i) 438 (j) 1246

3. (a) XXIX (b) LXIII (c) XCIV (d) XCIX
(e) CLVI (f) CCXCIII (g) CDLXXII (h) DCXXXVIII
(i) MCDLVIII (j) CMXLVIII (k) CXCIX (l) CDXCIX
(m) DCXCIX (b) CCXCIX (c) CMXCIX (p) M

Multiple Choice Questions

(1) b	(2) d	(3) b	(4) c	(5) a
(6) c	(7) d	(8) c	(9) b	(10) c
(11) d	(12) b	(13) b	(14) c	(15) a
(16) a	(17) d	(18) c	(19) d	(20) c

E2F5X1

2

WHOLE NUMBERS

Objectives

In this chapter you will learn

(i) To understand extended number system from natural numbers to whole numbers.

(ii) To represent whole numbers on number line and operate on number line.

(iii) To understand properties of whole numbers.

(iv) To understand goemetrical pattern from whole numbers.

2.1 Introduction

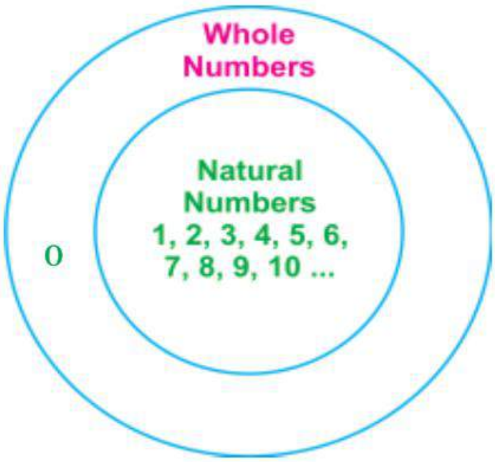
We have already learnt about the natural numbers i.e 1, 2, 3, 4, 5, ... Thus counting numbers are called Natural numbers. If forty students are present in class 6th and all of these students went to play ground to play games, then how many students are left behind in the classroom? Your answer will be no student is present in classroom. In this situation we say that zero ('0') student is present in class.

The counting numbers or natural numbers i.e. 1, 2, 3, 4, 5, ... together with the number '0' are called whole numbers.

Whole numbers are represented by W

Thus W = {0, 1, 2, 3, 4, 5, 6, 7, 8 .... }

2.2. Relation between Natural Numbers and Whole Numbers



N = {1, 2, 3, 4, 5,6 ,7, 8, 9, 10, 11 .... }

W = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ... }

All natural numbers are contained in collection of whole numbers.

- All natural numbers are whole numbers.
- All whole numbers are not natural numbers. (because '0' is a whole number but not natural

number)

- Smallest Natural number is 1.
- Smallest Whole number is 0.
- Greatest natural number cannot be written because by adding 1 to any natural number, we get larger natural number.
- Greatest whole number cannot be obtained because by adding 1 to any whole number, we get larger whole number.

2.3. Successor and Predecessor of a Whole Number

Successor : The successor of a whole number is the number obtained by adding 1 to it. Clearly the successor of 0 is 1; sucessor of 1 is 2, successor of 2 is 3 and so on......

Every whole number has successor.

Predecessor : The predecessor of a whole number is one less than the given number. Clearly the predecessor of 2 is 1, Predecessor of 1 is 0. But 0 does not have any predcessor in whole numbers. Every whole number other than zero has predecessor.

Example 1. **Write the successor of**

(a) 40099 **(b) 1000**

Solution : (a) Successor of 40099 = 40099 + 1

= 40100

(b) Sucessor of 1000 = 1000 + 1

= 1001

Example 2. **Write the predecessor of**

(a) 10000 **(b) 20099**

Solution : (a) Predecessor of 10000 = 10000 – 1

= 9999

(b) Predecessor of 20099 = 20099– 1

= 20098

2.4 Representation of Whole numbers on Number Line

Draw a line. Mark a point on it. Label it '0'. Mark a second point to the right of 0. Label it 1. The distance between these points labelled as 0 and 1 is called **unit distance**. On this line, mark a point to the right of 1 and at unit distance from 1 and label it 2. In this way go on labeling points at unit distance as 3, 4, 5, ... on the line.

You can go upto any whole number on the right in this manner. This is the number line for whole numbers.

Looking at above number line of whole numbers we observe that

(a) No whole number is on the left of zero ('0') and every whole number on right of '0' is greater than 0.

(b) A whole number on the right of given whole number is greater than the given whole number. e.g : 5>3, 7>6, and so on.

(c) A whole number on the left of given whole number (except 0) is less than the given whole number, e.g : 2<3, 5<7 and so on.

Number Line can be drawn in vertical form also.

2.4.1 Addition of Whole Numbers on number line

In order to add two whole numbers on the number line, we follow the following steps:

1. Draw a number line and mark whole numbers on it.
2. Mark the first given number on the number line.
3. Move as many units as the second number to the right of the first number.
4. The number obtained in step 3 represents the sum of two whole numbers.
5. Similarly, sum of three, four and five whole numbers can be found out.

Example 3. **Represent 3+5 on the number line.**

Solution : We draw a number line and move 3 steps from 0 to the right and mark this point as A. Now starting from A, we move 5 steps towards right and arrive at B.

OA = 3, AB = 5, OB = 8

Hence, OB = 3 + 5 = 8

2.4.2. Subtraction of Whole Numbers on number line

In order to subtract one whole number from another whole number on number line,following steps are followed:

1. Draw a number line and mark whole number on it.
2. Mark the first given number on the number line.
3. Starting from first number move as many units as the second number to the left of first number.
4. The number obtained in step 3 represents the required difference of the given whole numbers on the number line.

Example 4. **Represent 6-4 on the number line.**

Solution : We draw a number line.

Starting from point 0 (i.e. zero), we move 6 steps to the right and arrive at A. Now starting from A we move 4 units left of A and arrive at B.

OA = 6. AB = 4

Clearly OB = 6 – 4 = 2

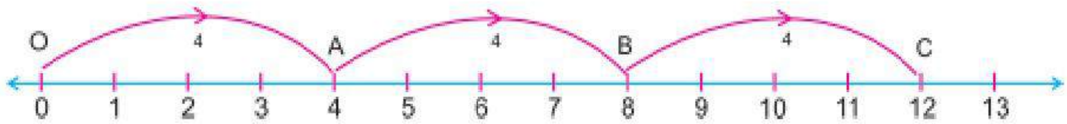
2.4.3 Product of Whole Numbers on number line

For multiplying the two whole numbers on number line, following steps are followed:

1. Draw a number line and mark whole numbers on it.
2. Starting from 0, we move to the right of 0 and count the units same as second number and it is considered as one jump.
3. Similar jumps are made equal to first number to reach at final point.
4. The final number represent the product of two whole numbers.

Example 5. **Find 3 × 4 using number line.**

Solution : We draw a number line.



Starting from 0 we move 4 units to the right of 0 to arrive at A. We make two more such same moves starting from A (total 3 moves of 4 unit each) to reach finally at C which represents 12.

Hence 4 × 3 = 12

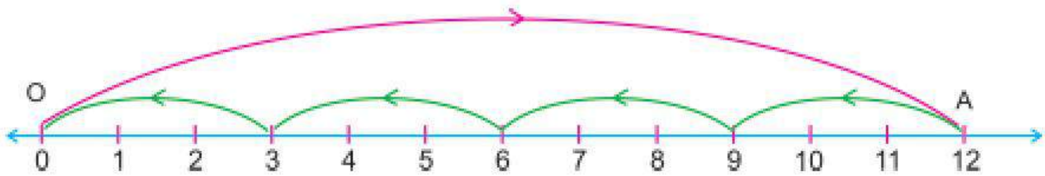
2.4.4 Division on number line

For division of two whole numbers, following steps are followed :

1. Draw a number line and mark whole numbers on it.
2. Starting from 0 we move to the right of 0 and reached at first number.
3. From First number we jumps towards zero taking one jump value equal to second number.
4. The number of jumps taken to reach at zero is quotient.

Example 6. **Find 12 ÷ 3 by number line.**

Solution : Draw a number line.



Starting from 0, we move 12 units to the right of 0 to arrive at A. Now from A take moves of 3 units to the left of A till we reach at '0'. We observe that there are 4 moves.

So 12 ÷ 3 = 4

Exercise 2.1

1. Answer the following questions.
   (a) Write the smallest whole number.
   (b) Write the smallest natural number.
   (c) Write the successor of 0 in whole numbers.
   (d) Write the predecessor of 0 in whole numbers.
   (e) Largest whole number.
2. Which of the following statements are True (T) and which are False (F)?
   (a) Zero is the smallest natural number.
   (b) Zero is the smallest whole number.
   (c) Every whole number is a natural number.
   (d) Every natural number is a whole number.
   (e) 1 is the smallest whole number.
   (f) The natural number 1 has no predecessor in natural numbers.
   (g) The whole number 1 has no predecessor in whole numbers.
   (h) Successor of the largest two digit number is smallest three digit number.
   (i) The successor of a two digit number is always a two digit number.
   (j) 300 is the predecessor of 299.
   (k) 500 is the successor of 499.
   (l) The predecessor of a two digit number is never a single digit number.
3. Write the successor of each of following:
   (a) 100909 (b) 4630999
   (c) 830001 (d) 99999
4. Write the predecessor of each of following:
   (a) 1000 (b) 208090
   (c) 7654321 (d) 12576
5. Represent the following numbers on the number line.
   2, 0, 3, 5, 7, 11, 15
6. How many whole numbers are there between 22 and 43?
7. Draw a number line to represent each of following on it.
   (a) 3 + 2 (b) 4 + 5 (c) 6 + 2
   (d) 8 – 3 (e) 7 – 4 (f) 7 – 2

(g) 3×3 (h) 2×5 (i) 3×5

(j) $9 \div 3$ (k) $12 \div 4$ (l) $10 \div 2$

8. Fill in the blanks with the appropriate symbol < or >:

(a) 25 .... 205 (b) 170 107

(c) 415 ....514 (d) 10001 9999

(e) 2300014 .... 2300041 (f) 99999 888888

2.5. Properties of Whole numbers

We have already learnt about four fundamental operations addition, subtraction, multiplication and division on numbers. Now let us study the properties of these operations on whole numbers.

⇒ Closure Property

- Closure property holds under addition of whole numbers. As we know that sum of two whole numbers is also a whole number.

  e.g: $7 + 5 = 12$ is a whole number

  $5 + 6 = 11$ is a whole number

  $0 + 4 = 4$ is a whole number

  Hence **Whole number + Whole number = Whole number**

- Closure Property holds under multiplication of whole numbers also. As we know that product of two whole numbers is also a whole number.

  e.g.: $7 \times 3 = 21$ is a whole number

  $4 \times 6 = 24$ is a whole number

  $0 \times 3 = 0$ is a whole number

  Hence **Whole number × Whole number = Whole number**

- Closure Property does not hold under subtraction of whole numbers. As difference of two whole numbers is not always a whole number.

  e.g. $7 - 9 = ?$ is not a whole number.

  $3 - 7 = ?$ is not a whole number.

  $7 - 4 = 3$ is a whole number.

- Closure property does not hold under division of whole numbers. As quotient of two whole numbers is not always a whole number.

  e.g. $8 \div 4 = 2$ is a whole number.

  but $5 \div 7 = \frac{5}{7}$ is a not a whole number

  $6 \div 5 = \frac{6}{5}$ is not a whole number

Division by Zero

Division by a number means subtracting that number repeatedly,

Let us find $8 \div 2$

8	6	4	2
–2	–2	–2	–2
6	4	2	0

Subtracting 2 again and again from 8

We reached '0' after 4 steps

$\therefore \quad 8 \div 2 = 4$

Hence $8 \div 2 = 4$

Let us try $2 \div 0$ now

2	2	2	2
–0	–0	–0	–0
2	2	2	2

In every move we get 2 again!

Will this ever stop?

No.

We say $2 \div 0$ is not defined

Hence $2 \div 0$ is not defined

Note :- *Divison by zero is not defined*

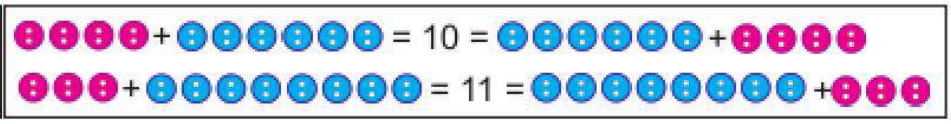
⇒ Commutative Property

- Commutativity holds under addition of whole numbers. You can add two whole numbers in any order. i.e. $a + b = b + a$

  e.g. $4 + 6 = 10 = 6 + 4$

  $3 + 8 = 11 = 8 + 3$

Button Activity



- Commutativity holds under multiplication of whole numbers also. You can multiply two whole numbers in any order. i.e. $a \times b = b \times a$

  e.g. $4 \times 5 = 20 = 5 \times 4$

  $3 \times 6 = 18 = 6 \times 3$

- But Subtraction is not commutative for whole numbers. In case of subtraction, If we change the order of whole numbers, result will not be same. $a - b \neq b - a$ (a and b are whole numbers)

  e.g. $10 - 3 = 7$

  $3 - 10 \neq 7$

- Similarly Division is not commutative for whole numbers.

  e.g. $12 \div 4 = 3$

  but $4 \div 12 \neq 3$

  $a \div b \neq b \div a$ (a and b are two different numbers, $a \neq 0$, $b \neq 0$)

⇒ Associativity

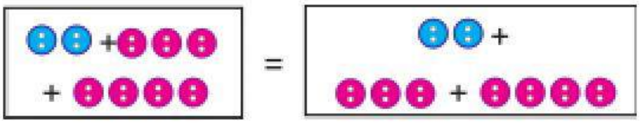
- **Addition** is associative for whole numbers if a, b, c are three whole numbers then $(a + b) + c = a + (b + c)$

  e.g.: $(2 + 3) + 4 = 5 + 4 = 9$

  & $2 + (3+4) = 2 + 7 = 9$

  ⇒ $(2 + 3) + 4 = 2 + (3 + 4)$

Button Activity



- **Multiplication** is also associative for whole numbers If a, b, c, are three whole numbers then $(a \times b) \times c = a \times (b \times c)$

  e.g. : $(2\times3) \times 4 = 6 \times 4 = 24$

  $2 \times (3 \times 4) = 2 \times 12 = 24$

  Hence $(2\times3) \times 4 = 2\times (3 \times 4)$

- **Subtraction and Division** are not associative for whole numbers. If a, b, c, are three whole numbers then

  $(a - b) - c \neq a - (b - c)$

  and $(a \div b) \div c \neq a \div (b \div c)$

⇒ Existence of Identity

- **Additive Identity:**

  If a is any whole number then

  $a + 0 = a = 0 + a$

  In other words, the sum of any whole number and zero is the number itself. The whole number 0 (zero) is called the **additive identity**.

- **Multiplicative Identity**

  $a \times 1 = a = 1 \times a$

  In other words, the product of any whole number and 1 is the number itself. The whole number 1 (one) is called the **multiplicative identity**.

- Identity element does not exist under subtraction and division of whole numbers as subtracting and division are not commutative.

Distributive of Multiplication over Addition

If a, b, c are any three whole numbers,

then (i) $a \times (b+c) = a \times b + a \times c$ (ii) $(b + c) \times a = b \times a + c \times a$

In other words, the multiplication of whole numbers distributes over their addition.

Verification : In order to verify this property, we take any three whole numbers a, b, c and find the values of the expression a× (b+c) and a ×b + a× c as shown below.

Whole Numbers a, b, c	Expression a× (b+c)	Expression a×b + a×c	Is a× (b+c) = a×b+a×c?
2, 3, 5	2 × (3+5) = 2×8 = 16	2×3+2×5 = 6 +10 = 16	Yes
3, 7, 15	3 × (7+15) = 3×22 = 66	3 × 7 + 3× 15 = 21 + 45 = 66	Yes
0, 4, 9	0×(4+9) = 0× 13 = 0	0 × 4+ 0 × 9 = 0 + 0 = 0	Yes

We see that the expression a × (b+c) and a×b + a× c are equal in each case.

Distributivity of Multiplication over Subtraction

Multiplication of whole numbers is also distributive over their subtraction. In other words, if a, b, c, are whole numbers such that b>c, then

(i) a × (b – c) = a × b – a × c

(ii) (b – c) × a = b ×a – c × a

Example 7: **Using suitable arrangement of terms find the product of**

(a) 25 × 4 × 384 **(b) 25 × 9 × 40 × 22637**

Solution : (a) 25 × 4 × 384 = (25 × 4) × 384 = 100 × 384 = 38400

(b) 25 × 9 × 40 × 25637 = (25 × 40) × (9 × 25637)

= 1000 × 230733

= 230733000

Example 8: **Find the product using properties of whole numbers:**

(a) 187 × 107 **(b) 42 × 96**

Solution : (a) 187 × 107 = 187 × (100+7)

= 187 × 100 + 187 × 7

= 18700 + 1309

= 20009

Distributive property of multiplication over addition

(b) 42 × 96 = 42 × (100–4)

= 42 × 100 – 42 × 4

= 4200 – 168

= 4032

Distributive property of multiplication over subtraction

Example 9: **Simplify using distributive property of multiplication.**

(a) $15 \times 32 + 15 \times 68$ (b) $125 \times 215 - 125 \times 15$

Solution : (a) $15 \times 32 + 15 \times 68 = 15 \times (32 + 68)$

$= 15 \times 100$

$= 1500$

Using Distributive property of multiplication over addition

(b) $125 \times 215 - 125 \times 15$

$= 125 \times (215 - 15)$

$= 125 \times 200$

$= 25000$

Using Distributive property of multiplication over subtraction

Example 10: **Divide 4567 by 2354 by actual division and check the result by division algorithm.**

Solution :

```
234 ) 4567 ( 19
     - 234↓
     ------
       2227
     - 2106
     ------
        121
     ------
```

Here Dividend = 4567, Divisor = 234

Quotient = 19, Remainder = 121

Check/ Verification : By Division Algorithm

Dividend = (Divisor×Quotient) + Remainder

$4567 = (234 \times 19) + 121$

or $4567 = 4446 + 121$

or $4567 = 4567$

Which is true

So, result is verified.

Example 11: **What is the largest 4 digit number divisible by 13?**

Solution : Largest 4 digit number = 9999

Let us divide it by 13.

```
13 ) 9999 ( 769
    -91 ↓
    ----
      89
     -78
    ----
     119
    -117
    ----
       2
```

On dividing 9999 by 13 we get remainder 2. We subtract 2 from 9999 to get number exactly divisible by 13.

So, 9999 – 2 = 9997 is the largest 4 digit number which is divisible by 13.

Exercise 2.2

1. Find the sum by suitable arrangement of terms:

(a) 837 + 208 + 363 (b) 1962 + 453 + 1538 + 647

2. Find the product by suitable arrangement of terms:

(a) $2 \times 1497 \times 50$ (b) $4 \times 263 \times 25$

(c) $8 \times 163 \times 125$ (d) $963 \times 16 \times 25$

(e) $5 \times 171 \times 60$ (f) $125 \times 40 \times 8 \times 25$

(g) $30921 \times 25 \times 40 \times 2$ (h) $4 \times 2 \times 1932 \times 125$

(i) $5462 \times 25 \times 4 \times 2$

3. Find the value of each of the following using distributive property:

(a) $(649\times8) + (649\times2)$ (b) $(6524 \times 69) + (6524 \times 31)$

(c) $(2986\times35)+(2986\times65)$ (d) $(6001 \times 172) - (6001 \times72)$

4. Find the value of the following :

(a) $493 \times 8 + 493 \times 2$ (b) $24579 \times 93 + 7 \times 24579$

(c) $3845 \times 5 \times 782 + 769 \times 25\times 218$

(d) $3297 \times 999 + 3297$

5. Find the product using suitable properties:

(a) 738×103 (b) 854×102 (c) 258×1008

(d) 736×93 (e) 816×745 (f) 2032×613

6. A taxi driver filled his car petrol tank with 40 litres of petrol on monday. The next day, he filled the tank with 50 litres of petrol. If the petrol costs ₹ 78 per litre, how much he spend in all on petrol?

7. A vendor supplies 32 litres of milk to a hotel in morning and 68 litres of milk in the evening. If the milk costs ₹ 35 per litre, how much money is due to the vendor per day?

8. We know that $0\times0=0$. Is there any other whole number which when multiplied by itself gives the product equal to the number itself? Find out the number.

9. Fill in the blanks:

(a) 15×0 = ... (b) $15 + 0$ = ...

(c) $15 - 0$ = ... (d) $15 \div 0$ = ...

(e) 0×15 = ... (f) $0 + 15$ = ...

(g) $0 \div 15$ = ... (h) 15×1 = ...

(i) $15 \div 1$ = ... (j) $1 \div 1$ = ...

10. The product of two Whole numbers is zero. What do you conclude. Explain with example.

11. Match the following:

(i)	$537 \times 106 = 537 \times 100 + 537 \times 6$	(a) Commutativity under multiplication
(ii)	$4 \times 47 \times 25 = 4 \times 25 \times 47$	(b) Commutativity under addition
(iii)	$70 + 1923 + 30 = 70 + 30 + 1923$	(c) Distributivity of multiplication over addition.

2.6 Patterns in Whole numbers

In this section, we shall try to arrange numbers in elementary shapes made up of dots. The shapes we take are (1) a line (2) a rectangle (3) a square and (4) a triangle. Every number should be arranged in one of these shapes. No other shape is allowed.

2.6.1 Representing whole numbers by line segments

If '•' represents 1, then 2, 3, 4, 5, ... can be represented by line segments as follow:

The number 2 is shown as 

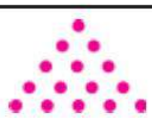
The number 3 is shown as 

The number 4 is shown as 

and so on.....

2.6.2 Triangular Numbers

Since whole numbers can be represented by triangles. Such numbers are called triangular numbers. 1, 3, 6, 10, 15 are some triangular numbers Let '•'represent 1. Following table shows the representation of whole numbers by triangles.

Triangular Number	Representation	Pattern
1		First triangular number = $\frac{1\times 2}{2} = 1$
3		Second triangular number = $\frac{2\times 3}{2} = 3$
6		Third triangular number = $\frac{3\times 4}{2} = 6$
10		Fourth triangular number = $\frac{4\times 5}{2} = 10$
15		Fifth triangular number = $\frac{5\times 6}{2} = 15$

1 is both triangular and square number.

By observing the above pattern possessed by triangular numbers, we can formulate the following rule:

nth triangular number = $\frac{n \times (n+1)}{2}$

2.6.3 Represently Whole Numbers by squares and Rectangles

Some whole numbers can be represented by squares and some by rectangles as shown below.

Square Number	Representation	Rectangular number	Representation
1		6	
4		8	
9		10	

Now complete the table

Number	Line	Rectangle	Square	Triangle
2	Yes	No	No	No
3	Yes	No	No	Yes
4	Yes	Yes	Yes	No
5	Yes	No	No	No
6				
7				
8				
9				
10				
11				
12				
13				

2.7. Patterns observations

Observation of patterns can guide you in simplifying process. Study the following:

(a) 237 + 9 = 237 + 10 –1 = 247 – 1 = 246

(b) 237 – 9 = 237 – 10 +1 = 227 + 1 = 228

(c) 237 + 99 = 237 + 100 –1= 337 –1 = 336

(d) 237 – 99 = 237 – 100 +1 = 137 + 1 = 138

Does this pattern help you to add or subtract numbers of the form 9, 99, 999 .....?

Here is one more pattern

(a) 84 × 9 = 84 × (10–1) = ? (b) 84 × 99 = 84 × (100 –1) = ?

(c) $84 \times 999 = 84 \times (1000-1) = ?$

Do you find a shortcut to multiply a number by numbers of the form 9, 99, 999, ...

Such shotcuts enable you to do sums verbally.

The following pattern suggests a way of multiplying by 5 or 25 or 125.

(a) $96 \times 5 = 96 \times \frac{10}{2} = \frac{960}{2} = 480$

(b) $96 \times 25 = 96 \times \frac{100}{4} = \frac{9600}{4} = 2400$

(c) $96 \times 125 = 96 \times \frac{1000}{8} = \frac{96000}{8} = 12000$

Exercise 2.3

1. If the product of two whole numbers is zero. Can we say that one or both of them will be zero? Justify through examples.

2. If the product of two whole numbers is 1. Can we say that one or both of them will be 1? Justify through examples.

3. Observe the pattern in the following and fill in the blanks:

$1 \times 1 = 1$

$11 \times 11 = 121$

$111 \times 111 = 12321$

$1111 \times 1111 =$ ....

$11111 \times 11111 =$ ....

4. Observe the pattern and fill in the blanks:

$1 \times 9 + 1 = 10$

$12 \times 9 + 2 = 110$

$123 \times 9 + 3 = 1110$

$1234 \times 9 + 4 = 11110$

$12345 \times 9 + 5 =$ ....

$123456 \times 9 + 6 =$ ....

5. Represent numbers from 24 to 30 according to rectangular, square or triangular pattern.

6. Study the following pattern:

$1 = 1 \times 1 = 1$

$1+3 = 2 \times 2 = 4$

$1+3+5 = 3 \times 3 = 9$

$1+3+5+7 = 4 \times 4 = 16$

Hence find the sum of

(a) First 12 odd numbers (b) First 50 odd numbers.

Multiple Choice Questions

1. The smallest whole number is

   (a) 0 (b) 1 (c) 2 (d) 3

2. The smallest natural number is

   (a) 0 (b) 1 (c) 2 (d) 3

3. The successor of 38899 is

   (a) 39000 (b) 38900 (c) 39900 (d) 38800

4. The predecessor of 24100 is

   (a) 24999 (b) 24009 (c) 24199 (d) 24099

5. The statement $4 + 3 = 3 + 4$ represents

   (a) Closure (b) Associative
   (c) Commutative property (d) Identity

6. Which of the following is the additive identity?

   (a) 0 (b) 1 (c) 2 (d) 3

7. The multiplicative identity is ....

   (a) 0 (b) 1 (c) 2 (d) 3

8. $15 \times 32 + 15 \times 68 =$

   (a) 1400 (b) 1600 (c) 1700 (d) 1500

9. The largest 4 digit number divisible by 13 is

   (a) 9997 (b) 9999 (c) 9995 (d) 9991

10. The successor of 3 digit largest number is

    (a) 100 (b) 998 (c) 1001 (d) 1000

Learning Outcomes

After completion of this chapter, the students are now able to

(i) Represent whole numbers on number line.

(ii) Operate upon whole numbers with the help of number line as well as arithmetically.

(iii) Use various properties of whole numbers.

(iv) Make different geometrical pattern for given whole number.

ANSWER KEY

Exercise 2.1

1. (a) 0 (b) 1 (c) 1 (d) Not Possible (e) Not Possible

2. (a) F (b) T (c) F (d) T (e) F (f) T
(g) F (h) T (i) F (j) F (k) T (l) F

3. (a) 100910 (b) 4631000 (c) 830002 (d) 100000

4. (a) 999 (b) 208089 (c) 7654320 (d) 12575

6. 20

8. (a) < (b) > (c) < (d) > (e) < (f) >

Exercise 2.2

1. (a) 1408 (b) 4600

2. (a) 149700 (b) 26300 (c) 163000 (d) 385200 (e) 51300
(f) 1000000 (g) 61842000 (h) 1932000 (i) 1092400

3. (a) 6490 (b) 652400 (c) 298600 (d) 600100

4. (a) 4930 (b) 2457900 (c) 19225000 (d) 3297000

5. (a) 76014 (b) 87108 (c) 260064 (d) 68448 (e) 607920 (f) 1245616

6. ₹ 7020 **7.** ₹ 3500

9. (a) 0 (b) 15 (c) 15 (d) Not Defined (e) 0
(f) 15 (g) 0 (h) 15 (i) 15 (j) 1

11. (i) $\longrightarrow$ c (ii) $\longrightarrow$ a (iii) $\longrightarrow$ b

Exercise 2.3

3. $1111 \times 1111 = 1234321$
$11111 \times 11111 = 123454321$

4. $12345 \times 9 + 5 = 11110$
$123456 \times 9 + 6 = 111111\ 0$

6. (a) $12 \times 12 = 144$ (b) $50 \times 50 = 2500$

Multiple Choice Questions

(1) a **(2)** b **(3)** b **(4)** d **(5)** c
(6) a **(7)** b **(8)** d **(9)** a **(10)** d

3

PLAYING WITH NUMBERS

Objectives

In this chapter you will learn

- To understand about factors and multiples.
- To provide information of prime-composite numbers, even-odd numbers etc.
- To provide information of divisiblility by different numbers.
- To acquire knowledge of HCF and LCM and their practical uses in life

3.1 Introduction

In previous classes, we have studied about factors, multiples, prime and composite numbers. In this chapter, we shall review these concepts and extend our study to include some new properties with suitable examples.

3.2 Factors

Vidhita arranges 12 balls in such a way that there are equal number of balls in each row.

→ 1 row with 12 balls

Total number of balls = $1 \times 12 = 12$

→ 2 rows with 6 balls each

Total number of balls = $2 \times 6 = 12$

→ 3 rows with 4 balls each

Total number of balls = 3 × 4 = 12

→ 4 rows with 3 balls each

Total number of balls = 4 × 3 = 12

→ 5 rows with equal number of balls in each row and having total 12 balls is not possible

→ 6 rows with 2 balls each

Total number of balls = 6 × 2 = 12

→ 7, 8, 9, 10 or 11 rows with equal number of balls in each row having total 12 balls is not possible.

→ 12 rows with 1 ball each

Total number of balls = 12 × 1 = 12

Here, we observe that 12 can be written as the product of two numbers in different ways.

12 = 1 × 12 ,   12 = 2 × 6,   12 = 3 × 4,

12 = 12 × 1,   12 = 6 × 2,   12 = 4 × 3

Thus 1, 2, 3, 4, 6 and 12 exactly divide 12. So the numbers 1, 2, 3, 4, 6 and 12 are the factors of 12.

* If a = b×c then b and c are factors of a and a is multiple of b and c.

Factors of a number exactly divide that number without leaving any remainder.

A factor of a number is an exact divisor of that number.

Number	Factors
2	**1, 2**
6	**1, 2, 3, 6**
10	**1, 2, 5, 10**
20	**1, 2, 4, 5, 10, 20**
24	**1, 2, 3, 4, 6, 8, 12, 24**

We conclude from the table:

- 1 is a factor of every number.
- Every number is factor of itself.
- Every number (other than 1) has atleast two factors, 1 and itself.
- Every factor of a number is always less than or equal to the number.
- A number has always finite number of factors.

Example 1: **Find all the factors of 15.**

Solution : $15 = 1 \times 15$, $15 = 15 \times 1$

$15 = 3 \times 5$, $15 = 5 \times 3$

So, 1, 3, 5 and 15 are factors of 15.

Example 2: **Find all the factors of 36.**

Solution :

$36 = 1 \times 36$ $\quad$ $36 = 9 \times 4$

$36 = 2 \times 18$ $\quad$ $36 = 12 \times 3$

$36 = 3 \times 12$ $\quad$ $36 = 18 \times 2$

$36 = 4 \times 9$ $\quad$ $36 = 36 \times 1$

$36 = 6 \times 6$

Note:- *There is no need of taking pairs 9×4, 12×3, 18×2, and 36×1. As they are repeating them selves in reverse order*

So, 1, 2, 3, 4, 6, 9, 12, 18, and 36 are factors of 36.

3.3 Multiples

In class 5th, we have studied about multiples that "Multiples of a number are obtained by multiplying it by any natural number".

Number	Multiples
1	1, 2, 3, 4, 5, ...
2	2, 4, 6, 8,10, ...
5	5, 10, 15, 20, 25, ...
8	8, 16, 24, 32, 40, ...
15	15, 30, 45, 60, 75, ...

We conclude from the table that:

- Every number is a multiple of itself.
- Every multiple of a number is greater than or equal to the number.
- The smallest multiple of a natural number is the number itself.
- There are infinite multiples of a number. So the largest multiple can not be defined.

Example 3: **Find the first six multiples of 4.**

Solution : First 6 multiples of 4 are

$4 \times 1 = 4, \quad 4 \times 2 = 8 \quad 4 \times 3 = 12, \quad 4 \times 4 = 16, \quad 4 \times 5 = 20, \quad 4 \times 6 = 24$

Example 4 : **Find the first five multiples of 13.**

Solution : First 5 multiples of 13 are 13, 26, 39, 52, 65.

3.3.1 Perfect Number

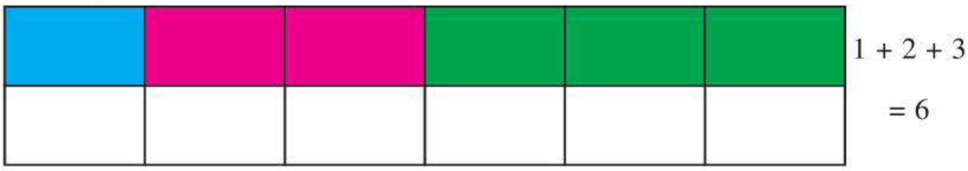
If the sum of all the factors of a number is two times the number then the number is called a perfect number.

The factors of 6 are 1, 2, 3 and 6

Also, $1 + 2 + 3 + 6 = 12 = 2 \times 6$

i.e sum of all factors of 6 = $2 \times$ Number

So, 6 is the perfect number.



* Other Perfect number are 28, 496 and 8128.

Note:- *Factors of 6 except the number and the number along itself make a rectangle as shown.So, it is the perfect number.*

3.3.2 Even numbers

All numbers which are multiples of 2 are called even numbers.

Or

Those numbers which are divisible by 2 are called even numbers e.g. 2, 4, 6, 8, 10,.....

* A number is an even number if 2 is a factor of it.

* All the even numbers end in 0, 2, 4, 6 or 8.

* If 2 is added to any even number then we get next consecutive even number.

3.3.3 Odd numbers

All numbers which are not multiples of 2 are called odd numbers. e.g. 1, 3, 5, 7, 9, 11, ...

* All the odd numbers end in 1, 3, 5, 7, 9.

* If 2 is added to any odd number then we get next consecutive odd number.

Note:- *A number is either even or odd. A number cannot be both odd as well as even.*

3.4 Prime and Composite Numbers

In the previous section we have learnt about factors and multiples of a number.

Let us consider the following table before discussing prime and composite numbers.

Numbers	Factors	Numbers of Factors
1	1	1
2	1, 2	2
3	1, 3	2
4	1, 2, 4	3
5	1, 5	2
6	1, 2, 3, 6	4
7	1, 7	2
8	1, 2 4, 8	4
9	1, 3, 9	3
10	1, 2, 5, 10	4
11	1, 11	2
12	1, 2, 3, 4, 6, 12	6

From the above table, we can divide the natural numbers in the following three categories:

(i) The numbers which have only **one factor**.

(ii) The numbers which have **exactly two factors (1 and itself)**

(iii) The numbers which have **more than two factors**.

We conclude that

(a) The natural number 1 is the only number which has exactly one factor, that number itself.

(b) The natural numbers 2, 3, 5, 7, 11, ... etc. have exactly two factors, 1 and the number itself. Such numbers are called **Prime Numbers**.

(c) The natural numbers 4, 6, 8, 9, 10, ... etc. have more than two factors. Such numbers are called **Composite Numbers.**

* 1 is the only number which is neither Prime nor Composite Number.
* 2 is the smallest prime number.
* 2 is the only even number which is prime. All other even numbers are composite numbers
* All prime numbers are odd except 2.
* All odd numbers are not prime numbers.

SIEVE Method
To find Prime and Composite Numbers

The Greek Mathematician Eratosthenes, found a very simple method for finding the prime and composite numbers in 3rd century B.C. He designed a table popularly known as **Sieve of Eratosthenes**.

In this table, He used natural numbers from 1 to 100.

The following steps are used to find prime and composite numbers form 1 to 100.

SIEVE OF ERATOSTHENES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Step 1:- As we know 1 is neither a prime nor a composite number. Thus put it in a square box.

Step 2:- Encircle 2 and cross out every multiple of 2 like 4, 6, 8, 10,12, ... etc.

Step 3:- Encircle next number 3 and cross out every multiple of 3 like 6, 9, 12, 15, 18, ... etc. Number already crossed need not be crossed again.

Step 4:- Encircle the next number 5 and cross out every multiple of 5 like 10, 15, 20, 25, ... etc.

Step 5:- Encircle the next number 7 and cross out every multiple of 7 like 14, 21, 28, 35, ... etc. continue this process till every number is either encircled or crossed out.

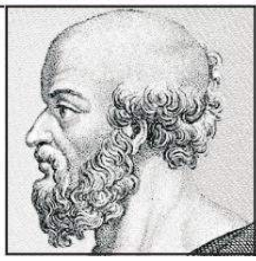
All the number that are encircled are the **prime numbers** and the number that are crossed out are the **composite numbers**.

Rule to check whether a number between 100 and 200 is prime or not: If the given number is divisible by any prime number less than 15 i.e. 2, 3, 5, 7, 11, 13 then it is composite other wise it is prime number.

→**Between 200 and 400:** If a number is divisible by any prime number less than 20, then it is composite number otherwise it is prime number.

Greek Mathematician Eratosthenes

He is best known for being the first person to calculate the **circumference of the Earth** and **tilt of the Earth's axis** with remarkable accurary. He was the founder of scientific chronology. He introduced the Sieve of Eratosthenes, an efficient method of finding prime numbers.



Eratosthenes (276 BC - 194BC)

3.4.1 Twin Primes

The pair of prime numbers having a difference of two are known as **twin primes**. The twin primes have only one composite number between them.

The twin primes from 1 to 100 are (3,5), (5,7), (11,13), (17,19),(29,31), (41,43), (59,61) and (71,73)

Numbers between any twin primes (above 5) is a multiple of 6.

3.4.2.Prime Triplet

A set of three consecutive prime numbers that differ by 2 is called a prime triplet. The only prime triplet is (3, 5, 7).

Gold Bach's Conjecture:- In 1742, a famous mathematician Goldbach gave a rule for which he could not provide a proof. So far no proof has been provided by anybody to contradict it by finding even one example.

"Every even number greater than 4 can be expressed as the sum of two odd prime numbers."

For example : 6 = 3 + 3, 10 = 3 + 7 or 5 + 5, 18 = 7 + 11, 24 = 11 + 13, 36 = 17 + 19 etc.

Example 5:- **Which of the following are prime numbers?**

(i) 37 (ii) 117 (iii) 191 (iv) 221

Solution : (i) Given number = 37

It is divisible by 1 and itself.

So it has exactly two factors

∴ 37 is a prime number

(ii) Given number = 117

We find that 117 is divisible by 3

∴ It has more than two factors.

∴ So it is not a prime number.

(iii) Given nuumber = 191

We find that 191 is not divisible by any of the numbers 2, 3, 5, 7, 11 and 13. So it is a prime number.

(iv) Given number = 221

We find that 221 is divisible by 13.

∴ It has more than two factors.

So it is not a prime number

Example 6: **Express each of the following numbers as a sum of two odd primes:**

(i) 20 (ii) 32 (iii) 48

Solution : (i) 20 = 3 + 17

= 7 + 13

(ii) 32 = 3 + 29

= 13 + 19

(iii) 48 = 5 + 43

= 7 + 41

= 11 + 37

= 17 + 31

= 19 + 29

Exercise 3.1

1. Write down all the factors of each of the following:-

(i) 18 (ii) 24 (iii) 45 (iv) 60 (v) 65

2. Write down the first six multiples of each of the following:-

(i) 6 (ii) 9 (iii) 11 (iv) 15 (v) 24

3. List all the numbers less than 100 that are multiples of

(i) 17 (ii) 12 (iii) 21

4. Which of the following are prime numbers ?

(i) 39 (ii) 129 (iii) 177 (iv) 203 (v) 237 (vi) 361

5. Express each of the following as sum of two odd prime numbers:-

(i) 16 (ii) 28 (iii) 40

6. Write all the prime numbers between the given numbers:-

(i) 1 to 25 (ii) 85 to 105 (iii) 120 to 140

7. Is 36 a perfect number?

8. Find the missing factors:-

(i) $5 \times ..... = 30$ (ii) $..... \times 6 = 48$ (iii) $7 \times ..... = 63$

(iv) $..... \times 8 = 104$ (iv) $..... \times 7 = 105$

9. List all 2-digit prime numbers, in which both the digits are prime numbers.

3.5 Common Factors and Multiples

In the previous section, we have learnt about the factors and the multiples of a number. In this section, we shall discuss the common factors or common multiples of two or more numbers.

Let's consider some examples:-

Example 7: **Find the common factors of 12 and 18.**

Solution : The factors of 12 = 1, 2, 3, 4, 6 and 12.

The factors of 18 = 1, 2, 3, 6, 9 and 18.

$\therefore$ Common factors of 12 and 18 are 1, 2, 3 and 6.

Example 8: **Find the common factors of 15, 24 and 30**

Solution : The factors of 15 = 1, 3, 5, 15

The factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

The factors of 30 = 1, 2, 3, 5, 6, 10, 15, 30

$\therefore$ Common factors of 15, 24 and 30 are 1 and 3.

Example 9: **Find the first four common multiples of 4 and 6.**

Solution : The multiples of 4 = 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52.

The multiples of 6 = 6, 12, 18, 24, 30, 36, 42, 48, 54

$\therefore$ The first four common multiples of 4 and 6 are 12, 24, 36 and 48.

Example 10: **Find the common factors of 16 and 25.**

Solution : The factors of 16 = 1, 2, 4, 8, 16

The factors of 25 = 1, 5, 25

$\therefore$ Common factors of 16 and 25 = 1

3.5.1 Co-Prime Numbers

Two numbers are said to be co-prime if they do not have a common factor other than 1.

In above example, common factors of 16 and 25 is 1. So these are called co-prime numbers. Other examples are (5, 7) ; (8, 9), (12, 13) etc.

* Two co-prime numbers need not to be both prime numbers.
* Two prime numbers are always co-prime.

3.6 Tests of Divisibility

To find whether a number is divisible by another number, we perform actual division and check whether the remainder is zero or not. But this is very time-consuming process. There are certain divisibility tests of numbers 2, 3, 4, 5, 6, 8, 9, 10 and 11 to check the divisibility whether a number is divisible by any of these numbers or not. In this section, we shall learn about these tests:

Divisibility by 2:- A number is divisible by 2, if its **units digit is even** i.e. 0, 2, 4, 6 or 8.

For example:-

(a) 2164, 12562, 83490 etc are divisible by 2.

(b) 6193, 82937, 14051 etc are not divisible by 2.

Divisibility by 4:- A number is divisible by 4, if the number **formed by its last two digits** is divisible by 4 or if the number ends in two zeros.

For example:-

(a) 61<u>24</u>, 256<u>32</u>, 843<u>00</u>, 124<u>96</u> etc. all are divisible by 4. As number formed by their last two digits are divisible by 4.

(b) 127<u>31</u> , 51<u>67</u> , 423<u>42</u> etc are not divisible by 4 because their last two digits are not divisible by 4.

Divisibility by 8:- A number is divisible by 8, if its **last three digits** is divisible by 8 or if the number ends in three zeros.

For example:-

(a) 214832 , 51616 , 2400 etc are divisible by 8.

(b) 613513 , 52642 , 1678093 etc are not divisible by 8 because last three digits are not divisible by 8.

Divisibility by 3:- A number is divisible by 3, if the **sum of its digits is divisible by 3**.

For example:-

(a) 258 is divisible by 3.

As Sum of digits = 2 + 5 + 8 = 15 is divisible by 3.

(b) 51062 is not divisible by 3.

As Sum of digits = 5 + 1 + 0 + 6 + 2 = 14, is not divisible by 3.

Divisibility by 9:- A number is divisible by 9, if the **sum of its digits is divisible by 9**.

For example:-

(a) 62154 is divisibly by 9.

As Sum of its digits = 6 + 2 + 1 + 5 + 4 = 18, is divisible by 9.

(b) 23509 is not divisibly by 9.

As Sum of its digits = 2 + 3 + 5 + 0 + 9 = 19, is not divisibly by 9.

Divisibility by 5:- A number is divisible by 5, if its **last digit is 0 or 5**.

For example:-

(a) 51680 , 235045 , 91435 etc. are divisible by 5, as their last digit is 0 or 5.

(b) 216803 , 52361 etc are not divisible by 5, as their last digit is not 0 or 5.

Divisibility by 10:- A number is divisible by 10, if its **last digit is 0**.

For example:-

(a) 62560 , 315680 , 25600 etc. are divisible by 10, As their last digit is 0.

(b) 2153 , 68024 , 519831 etc are not divisible by 10.

Divisibility by 6:- A number is divisible by 6, if the number is divisible **by 2 and 3**, both

Or

An **even number which is divisible by 3, will be divisible** by 6.

For example:- 25824 is divisible by 6

As it is an even number and sum of its digits = 2 + 5 + 8 + 2 + 4 = 21, is divisible by 3.

Divisibility by 11:- A number is divisible by 11, if the difference of the sum of its digits in odd places and sum of its digits in even places is either 0 or a multiple of 11.

For example:-

Odd place digits

4 3 5 2 0 4

Even place digits

(a) 435204 is divisible by 11.

Since sum of digits in odd places = 4 + 5 + 0 = 9 and sum of digits in even places = 3 + 2 + 4 = 9 their difference = 9 – 9 = 0

(b) 6574312 is not divisible by 11

Since sum of digits in odd places = 6 + 7 + 3 + 2 = 18 and sum of digits in even places = 5 + 4 + 1 = 10 their differences = 18 – 10 = 8, which is not divisible by 11.

Odd place digits
6 5 7 4 3 1 2
Even place digits

3.7 Some General Properties of Divisibility

Property 1:- Let a, b, c be three numbers. If a is divisible by b and b is divisible by c then a is divisible by c.

Or

If a number is divisible by another number, then it is divisible by each of the factors of that number.

For example:- 48 is divisible by 12 and 12 is divisible by 2, 3 and 6. So 48 is also divisible by 2, 3 and 6.

Consequences:-

* Since 4 is divisible by 2, So every number which is divisible by 4 is also divisible by 2.
* Since 6 is divisible by 2 and 3 both, So every number which is divisible by 6 is also divisible by 2 and 3 also.
* Since 9 is divisible by 3, So every number divisible by 9 is also divisible by 3.

Property 2:- If a and b are two co-prime numbers such that a number c is divisible by both a and b then c is also divisible by a × b.

Or

If a number is divisible by each of the two or more co-prime numbers then it is divisible by their product

For example:- Let us take two co-prime numbers 3 and 4.

3 is a factor of 72 and 4 is also a factor of 72. So 3 × 4 = 12 is, also a factor of 72.

Property 3:- If two numbers b and c are divisible by a then (b + c) is also divisible by a.

Or

If a number is a factor of each of two given numbers then it is a factor of their sum.

For example:- 45 and 70 both are divisible by 5. The sum of these two numbers is 45 + 70 = 115.

⇒ 115 is also divisible by 5.

Property 4:- If two numbers b and c are divisible by a then (b – c) or (c – b) is also divisible by a

Or

If a number is a factor of each of the two given numbers then it is a factor of their difference.

For example:- 84 and 45 both are divisible by 3.

The difference of these two numbers is 84 – 45 = 39

⇒ 39 is also divisible by 3.

Exercise 3.2

1. Find the common factors of the followings:-

(i) 16 and 24 (ii) 25 and 40 (iii) 24 and 36

(iv) 14, 35 and 42 (v) 15, 24 and 35

2. Find first three common multiples of the followings:-

(i) 3 and 5 (ii) 6 and 8 (iii) 2, 3 and 4

3. Which of the following numbers are divisible by 2 or 4?

(i) 52314 (ii) 678913 (iii) 4056784 (iv) 21536 (v) 412318

4. Which of the following numbers are divisible by 3 or 9?

(i) 654312 (ii) 516735 (iii) 423152 (iv) 704355 (v) 215478

5. Which of the following numbers are divisible by 5 or 10?

(i) 456803 (ii) 654130 (iii) 256785 (iv) 412508 (v) 872565

6. Which of the following numbers are divisible by 8?

(i) 457432 (ii) 5134214 (iii) 7232000 (iv) 5124328 (v) 642516

7. Which of the following numbers are divisible by 6?

(i) 425424 (ii) 617415 (iii) 3415026 (iv) 4065842 (v) 725436

8. Which of the following numbers are divisible by 11?

(i) 4281970 (ii) 8049536 (iii) 1234321 (iv) 6450828 (v) 5648346

9. State True or False:-

(i) If a number is divisible by 24, then it is also divisible by 3 and 8.

(ii) 60 and 90 both are divisible by 10 then their sum is not divisible by 10.

(iii) If a number is divisible by 8 then it is also divisible by 16.

(iv) If a number is divisible by 15 then it is also divisible by 3.

(v) 144 and 72 are divisible by 12 then their difference is also divisible by 12.

10. If a number is divisible by 5 and 9 then by which other number will that number be always divisible?

11. Which of the following pairs are co-prime?

(i) 25, 35 (ii) 16, 21 (iii) 24, 41 (iv) 48, 33 (v) 20, 57

3.8 Prime Factorisation:- (Canonical Form)

In the previous sections, we have learnt about factors of a number, prime numbers and composite numbers. If a number is composite then it can be written as the product of two of its factors, the factors may be both prime or both composite or either prime or composite.

If composite, the factors can be split again, this process will be continued when we get all prime factors.

Thus "Prime Factorisation is the process by which a composite number is rewritten as the product of prime factors."

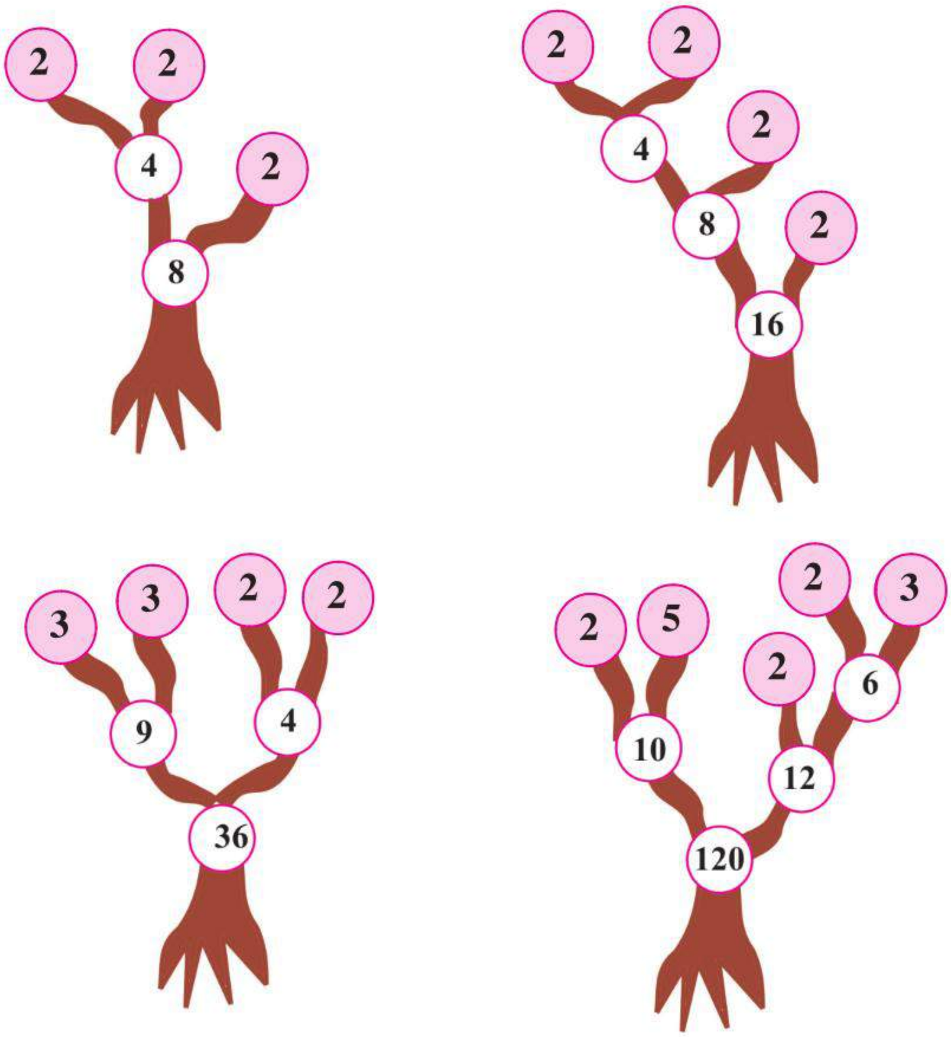
> * **Fundamental Theorem of Arithmetic:-** Every Composite number can be factorised into prime factors in one and only one way apart from the order of the factors.

Prime factorisation can be done by two methods:-

- Factor Tree Method
- Division Method

3.8.1 Factor Tree Method

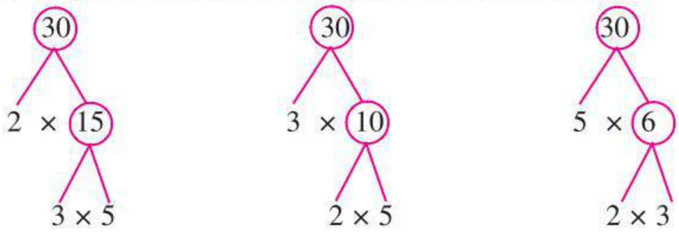
In each step of the factor tree, we write the given composite number as the product of its smallest prime factors and another factor untill we get all the prime factors.



2 × 5 2 ×

3

Let us find the prime factors of 30 using the factor tree method.



So In each case, the prime factorisation of 30 is 2 × 3 × 5

3.8.2.Division Method

Let us find the prime factors of 360 using the division method.

2	360
2	180
2	90
3	45
3	15
	5

Step I:- Divide the number by any prime number which will exactly divide it.

Let us find the prime factors of 30 using the factor tree method.

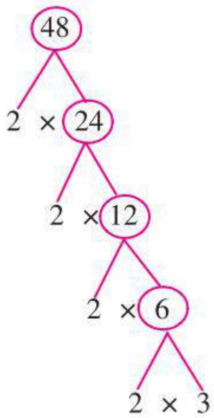
Step II:- Continue dividing the quotient by any prime number till we get the quotient itself as a prime number.

So, The prime factorisation of 360 is 2 × 2 × 2 × 3 × 3 × 5

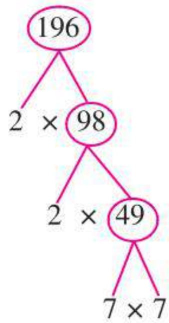
Example 11:- Find the prime factors of the following by factor tree method:-

(i) 48 (ii) 196 (iii) 150

Solution : (i)



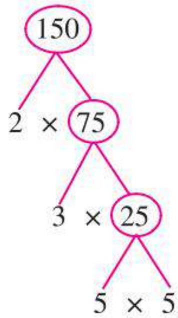
(ii)



∴ Prime factorisation of 48 = 2 × 2 × 2 × 2 × 3

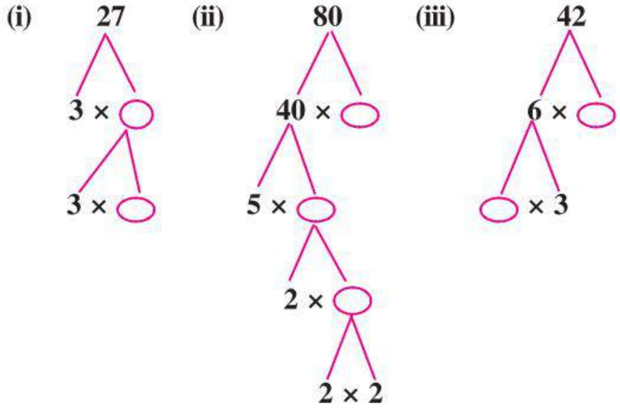
∴ Prime factorisation of 196 = 2 × 2 × 7 × 7

(iii)

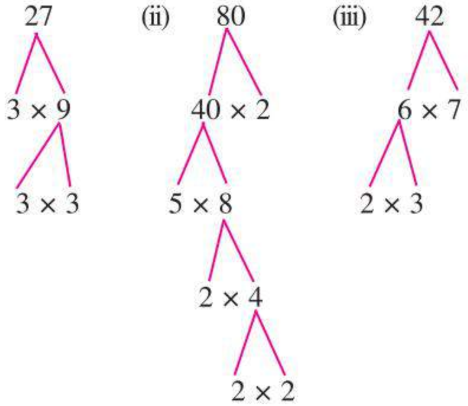


∴ Prime factorisation of 150 = 2 × 3 × 5 × 5

Example 12:- **Complete each factor tree**



Solution : (i) (ii) (iii)



Example 13:- **Find prime factors of the following numbers:-**

(i) 216 (ii) 375 (iii) 920

Solution : (i)

2	216
2	108
2	54
3	27
3	9
	3

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

(ii)

3	375
5	125
5	25
	5

$375 = 3 \times 5 \times 5 \times 5$

(iii)

2	920
2	460
2	230
5	115
	23

$920 = 2 \times 2 \times 2 \times 5 \times 23$

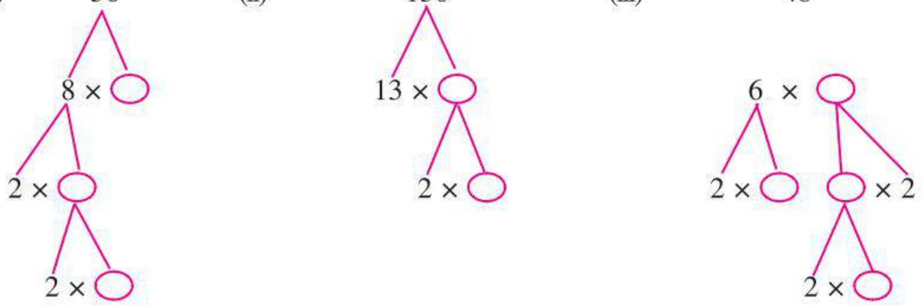
Exercise 3.3

1. Find prime factors of the following numbers by factor tree method:-

   (i) 96 (ii) 120 (iii) 180

2. Complete each factor tree:-

   (i) 56 (ii) 130 (iii) 48



3. Find the prime factors of the following numbers by division method:-

   (i) 420 (ii) 980 (iii) 225 (iv) 150 (v) 324

3.9. Highest common Factor (H.C.F.) Or Greatest Common Divisor (G.C.D)

In the previous sections, we have learnt about factors and multiples, common factors. In this section, we shall learn about the highest common factor among the common factors of the given numbers. which is also known as H.C.F. or G.C.D. of the numbers.

"The highest common factor (H.C.F.) of two or more numbers is the greatest or the largest among common factors".

In other words, H.C.F. of two or more numbers is the **largest number that divides all the numbers completely.**

For example:- Consider the numbers 24 and 42.

Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Factors of 42 = 1, 2, 3, 6, 7, 14, 21, 42

Common factors of 24 and 42 = 1, 2, 3, 6

out of these common factors, we find that 6 is the highest or greatest common factor.

So HCF of 24 and 42 is 6.

* HCF of two or more numbers can never be zero because 1 as a factor will be common to all numbers.
* HCF of two co-prime numbers is always 1.
* HCF is always smaller than or equal to the smallest of the given numbers.

There are two common methods to find H.C.F. of two or more numbers.

* Prime Factorisation Method

* Continued division mehtod

Here, we shall learnt about these two methods.

3.9.1. Prime Factorisation Method:- To find H.C.F., we follow the following steps:

Step 1:- Make the prime factors of each of the given number.

Step 2:- Find the common prime factors of the given numbers.

Step 3:- The product of all common factors (of step 2) is the H.C.F. of given numbers.

Example 14: **Find HCF of the following numbes:-**

(i) 36 and 48 (ii) 30 and 75 (iii) 108 and 144

(iv) 42, 63 and 210 (v) 125, 175 and 250

Solution : (i) First we write the prime factorisation of each of the given numbers.

2	36	2	48
2	18	2	24
3	9	2	12
	3	2	6
			3

$\therefore$ $36 = 2 \times 2 \times 3 \times 3$

and $48 = 2 \times 2 \times 2 \times 2 \times 3$

We find that 2 occurs two times and 3 occurs once as common factors.

$\therefore$ HCF of 36 and 48 $= 2 \times 2 \times 3 = 12$

(ii) First we write the prime factorisation of each of given numbers

2	30	3	75
3	15	5	25
	5		5

$\therefore$ $30 = 2 \times 3 \times 5$

and $75 = 3 \times 5 \times 5$

We find that 3 occurs once and 5 occurs once as common factors.

$\therefore$ HCF of 30 and 75 $= 3 \times 5 = 15$

(iii) First we write the prime factorisation of each of the given numbers.

2	108	2	144
2	54	2	72
3	27	2	36
3	9	2	18
	3	3	9
			3

$\therefore$ 108 = 2 × 2 × 3 × 3 × 3

and 144 = 2 × 2 × 2 × 2 × 3 × 3

We find that 2 occurs twice and 3 occurs twice as common factors.

$\therefore$ HCF of 108 and 144 = 2 × 2 × 3 × 3 = 36

(iv) First we write the prime factorisation of each of given numbers.

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$\therefore$ 42 = 2 × 3 × 7

63 = 3 × 3 × 7

and 210 = 2 × 3 × 5 × 7

We find that 3 occurs once and 7 occurs once as common factors.

$\therefore$ HCF of 42, 63 and 210 = 3 × 7 = 21

(v) First we write the prime factorisation of each of given numbes

$$\begin{array}{r|l} 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 5 & 175 \\ \hline 5 & 35 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 250 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

$\therefore$ 125 = 5 × 5 × 5

175 = 5 × 5 × 7

and 250 = 2 × 5 × 5 × 5

We find that 5 occurs twice as common factors.

$\therefore$ HCF of 125, 175 adn 250 = 5 × 5 = 25

3.9.2 Continued Division Method (Euclid's Algorithm)

Euclid, a Greek mathematician derived an interesting method to find HCF of two or more numbers. This method is known as **Euclid's algorithm** or **Long division method**.

Euclid's Algorithm (step for finding HCF)

Step 1:- From the given numbers, Identity the greater number.

Step 2:- Take the greater number as dividend and the smallest number as divisor.

Step 3:- Find the quotient and remainder.

Step 4:- If the remainder is zero then the divisor is the required HCF.

Step 5:- If the remainder is non-zero then take the remainder as new divisor and the last divisor as the new dividend.

Step 6:- Repeat the steps till the remainder obtained is zero.

Step 7:- The last divisor for which the remainder is zero is the required H.C.F.

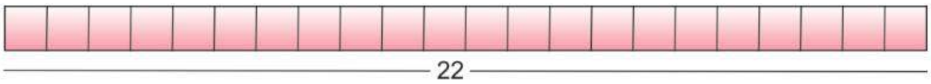
Let us perform an activity based on this method.

To Find HCF by cutting and pasting of paper.

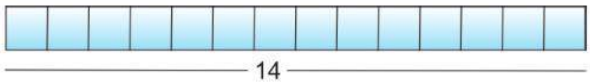
Material used:- A meaning scale, a pencil, chart paper, coloured pencils or sketch pens, eraser etc.

Procedure:- To find HCF of 14 and 22.

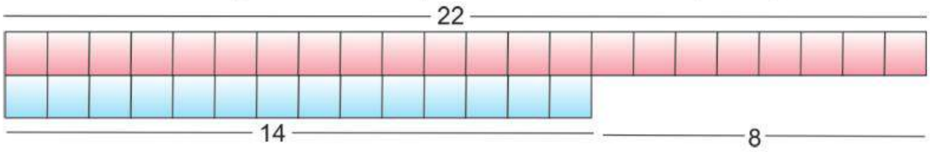
1. Take white coloured chart and cut a strip and divide it into 22 square boxes with pencil and scale and fill red colour in it.



2. Take another strip and divide it into 14 square boxes and fill blue colour in it.

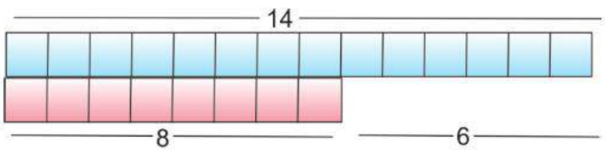


3. Divide the larger number 22 by smaller number 14 as (shown)



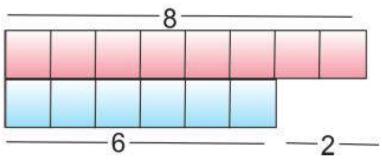
14) 22 (1
– 14
8

4. Divide the smaller number 14 by remainder 8.



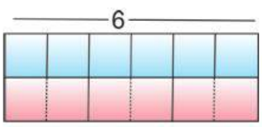
8) 14 (1
– 8
6

5. Divide 8 by 6.



6) 8 (1
– 6
2

6. Divide 6 by 2.



2) 6 (3
– 6
0

Observation:- The required HCF of 14 and 22 is 2 which is the last divisor in the above process as it leaves remainder 0.

Example 15: **Find HCF of the following by division method:-**

(i) 144, 252 (ii) 58, 70 (iii) 25, 44

Solution : (i) Given numbers are 144 and 252

```
144) 252 ( 1
   - 144
     108)144 ( 1
        - 108
           36 )108 (3
              - 108
                  0
```

Hence, 36 is the HCF of 144 and 252.

(ii) Given numbers are 58 and 70.

```
58) 70 ( 1
  - 58
    12) 58 (4
      - 48
        10) 12 (1
          - 10
             2 ) 10 (5
               -10
                 0
```

Hence, 2 is H.C.F. of 58 and 70

(iii) Given numbers are 25 and 44

```
25) 44 ( 1
  - 25
    19) 25 (1
      - 19
         6 ) 19 (3
           - 18
              1 ) 6 ( 6
                  6
                  0
```

Hence 1, is H.C.F. of 25 and 44.

H.C.F. of More than two numbers:-

To find H.C.F. of three numbers, we proceed as follows:-

Step 1:- Find H.C.F. of any two of them.

Step 2:- Find H.C.F. of remaining number and HCF obtained in step 1.

Step 3:- HCF of step 2 is the required HCF of three numbers.

Example 16: **Find H.C.F. of 50, 125 and 195.**

Solution : Given numbers are 50, 125 and 195.

Consider any two numbers, say 50 and 125.

```
50 ) 125 ( 2
   – 100
     25 ) 50 ( 2
        – 50
           0
```

∴ HCF of 50 and 125 is 25.

Now, we find HCF of 25 and 195

```
25 ) 195 ( 7
   – 175
     20 ) 25 ( 1
        – 20
           5 ) 20 ( 4
             – 20
                0
```

∴ HCF of 25 and 195 is 5.

⇒ HCF of 50, 125 and 195 is 5.

3.9.3. Applications of H.C.F. (Word Problems)

In this section, we shall discuss some applications of HCF in solving some practical daily life problems. Let's illustrate these problems with following examples:-

Example 17:- Find the greatest number which divides 250 and 188 leaving the remainder 2 in each case.

Solution : Given that, required number when divides 250 and 188, the remainder is 2 in each case.

⇒ $250 - 2 = 248$ and $188 - 2 = 186$ are completely divisible by the required number.

⇒ Required number is the highest common factor of 248 and 186.

Since it is given that required number is the largest number.

∴ Required number is the HCF of 248 and 186.

```
186 ) 248 ( 1
    – 186
       62 ) 186 ( 3
          – 186
              0
```

∴ Required number (HCF) is 62

Example 18: **Find the greatest number which divides 645 and 792 leaving a remainder 7 and 9 respectively.**

Solution : Required greatest number = HCF of $(645 - 7)$ and $(792 - 9)$

= HCF of 638 and 783.

638) 783 (1
− 638
145) 638 (4
− 580
58) 145 (2
− 116
29) 58 (2
− 58
0

∴ Required greatest number = 29

Example 19 : **Find the greatest number that divides 135, 245 and 385 leaving a remainder 5 in each case.**

Solution: Required Number= HCF of (135 − 5), (245 − 5) and (385 − 5)

= HCF of 130, 240 and 380

Now,

2	130
5	65
	13

2	240
2	120
2	60
2	30
3	15
	5

2	380
2	190
5	95
	19

∴ $130 = 2 \times 5 \times 13$

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

and $380 = 2 \times 2 \times 5 \times 19$

∴ HCF $= 2 \times 5 = 10$

Hence, Required number = 10

Example 20:- **Two tankers contain 434 litres and 465 litres of diesel respectively. Find maximum capacity of a container that can measure the diesel of both containers exact number of times.**

Solution : We have to find, maximum capacity of a container which measure both containers.

⇒ We required the maximum number which divides 434 and 465 completely.

⇒ Required Number = HCF of 434, 465

2	434
7	217
	31

5	465
3	93
	31

$434 = 2 \times 7 \times 31$

and $465 = 5 \times 3 \times 31$

∴ HCF of 434 and 465 = 31

So Required capacity of container = 31 litres

Example 21: The length, breadth and height of a room are 8m 25cm, 6m 75cm and 4m 50cm respectively. Find the longest tape which can measure the three dimensions of the room exactly.

Solution : We have to find the longest tape which measure the given dimensions of room.

So We required the maximum number which divides 8m 25cm, 6m 75cm and 4m 50cm

$\therefore$ Required length of tape = HCF of 8m 25cm, 6m 75cm and 4m 50cm

= HCF of 825 cm, 675cm and 450cm [$\because$ 1m = 100 cm]

Now Take any two numbers, say 825 and 675

```
675) 825 ( 1
    - 675
     150) 675 (4
         - 600
           75) 150 (2
              - 150
                  0
```

Here, HCF of 825 and 675 is 75.

Now to find HCFof 75 and 450

```
75) 450 (6
   - 450
       0
```

$\therefore$ HCF of 835, 675 and 450 = 75

$\therefore$ Hence, length of longest tape = 75cm

Example 22: A floor of a room is 9m × 4.75m. It is to be paved with square tiles of marble of the same size. Find the greatest measurement of each tile.

Solution: We have to find square tile of greatest measurement which paved the floor marble exactly

$\therefore$ Required size of tile = HCF of 9m and 4.75m

= HCF of 900cm and 475cm [$\because$ 1 m = 100cm]

```
475) 900 ( 1
    - 475
     425) 475 ( 1
         - 425
           50 ) 425 ( 8
               - 400
                 25) 50 ( 2
                    - 50
                       0
```

$\therefore$ HCF of 900 cm and 475cm = 25cm

Hence, Side of each square tile is 25cm.

Example 23 : **Reduce $\frac{312}{507}$ to the lowest term (Simplest form).**

Solution : In order to reduce a given fraction to the lowest terms, we divide the numerator and denominator by their HCF.

Now, we find HCF of 312 and 507.

```
312) 507 ( 1
    -312
     195) 312 (1
         -195
          117 ) 195 (1
               - 117
                  78)117 (1
                     - 78
                      39 )78( 2
                         - 78
                           0
```

Clearly HCF of 312 and 507 is 39

Now $\frac{312}{507} = \frac{312 \div 39}{507 \div 39} = \frac{8}{13}$

[Divide numerator and denomiantor by 39].

Exercise 3.4

1. Find H.C.F. of the following numbers by prime factorisation:-

   (i) 30, 42 (ii) 135, 225 (iii) 180, 192 (iv) 49, 91, 175 (v) 144, 252, 630

2. Find H.C.F. of the following numbers using division method:-

   (i) 170, 238 (ii) 54, 144 (iii) 72, 88 (iv) 96, 240, 336 (v) 120, 156, 192

3. What is the HC.F. of two prime numbers?
4. What is the H.C.F. of two consecutive even numbers?
5. What is the H.C.F of two consecutive natural numbers?
6. What is the H.C.F. of two consecutive odd numbers?
7. Find the greatest number which divides 245 and 1029, leaving a remainder 5 in each case.
8. Find the greatest number that can divide 782 and 460 leaving remainder 2 and 5 respectively.
9. Find the greatest number that will divide 398, 437 and 540 leaving remainders 7, 12 and 13 respectively.
10. Two different containers contain 529 litres and 667 litres of milk respectively. Find the maximum capacity of container which can measure the milk of both containers in exact number of times.
11. There are 136 apples, 170 mangoes and 255 oranges. These are to be packed in boxes containing the same number of fruits. Find the greatest number of fruits possible in each box.

12. Three pieces of timber 54m, 36m and 24m long, have to be divided into planks of the same length. What is the greatest possible length of each plank?

13. A room measures 4.8m and 5.04m. Find the size of the largest square tile that can be used to tile the floor without cutting any tile

14. Reduce each of the following fractions to lowest forms:-

(i) $\frac{85}{102}$ (ii) $\frac{52}{130}$ (iii) $\frac{289}{391}$

3.10 Lowest Common Multiple

In previous sections, we have learnt about the highest common factors of two or more numbers and we have learnt common multiples of two or more numbers. In this section, we shall learn about the lowest of the common multiples of given numbers which is known as L.C.M. of the numbers.

"Lowest common Multiple (L.C.M.) of two or more numbers is the smallest number which is a multiple of each of the numbers."

Or

LCM is the smallest number which is divisible by all the given numbers.

For example:- Consider number 6 and 8.

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, ...

Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, ...

Common multiples are 24, 48, ...

Clearly, 24 is the smallest among common multiples.

$\therefore$ LCM of 6 and 8 is 24.

* LCM of the numbers is exactly divisible by each number.
* LCM of two numbers is the greater number of them, if one of the numbers is multiple of other.
* LCM of given numbers is not less than any of the given numbers.

There ae two methods to find LCM of two or more numbers:-

* Prime Factorisation Method
* Common Division Method

Now, we shall learn about these two methods:-

3.10.1. Prime Factorisation Method

To find LCM, we follow the following steps:-

Step 1:- Make the prime factors of each of the given number.

Step 2:- Find the product of all different prime factors with maximum number of times each factor appear.

Step 3:- The product of those factors is the required LCM.

Example 24: Find LCM of the following numbers:-

(i) 20, 30 (ii) 36, 120 (iii) 72, 84 (iv) 40, 75, 126 (v) 108, 135, 162

Solution : (i)

$$\begin{array}{r|l} 2 & 20 \\ \hline 2 & 10 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$\therefore$ $20 = 2 \times 2 \times 5$

$30 = 2 \times 3 \times 5$

We find that in these prime factorisation 2 occurs maximum two times, 3 and 5 occurs maximum once.

$\therefore$ LCM of 20 and 30 $= 2 \times 2 \times 3 \times 5 = 60$

(ii)

$$\begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$\therefore$ $36 = 2 \times 2 \times 3 \times 3$

$120 = 2 \times 2 \times 2 \times 3 \times 5$

In these prime factorisation, 2 occurs maximum 3 times, 3 occurs maximum 2 times and 5 occurs maximum once.

$\therefore$ LCM of 36 and 120 $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

(iii)

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

$\therefore$ $72 = 2 \times 2 \times 2 \times 3 \times 3$

$84 = 2 \times 2 \times 3 \times 7$

In these prime factorisation, 2 occurs maximum 3 times, 3 occurs maximum 2 times and 7 occurs maximum once.

$\therefore$ LCM of 72 and 84

$= 2 \times 2 \times 2 \times 3 \times 3 \times 7 = 504$

(iv)

$\therefore$ $40 = 2 \times 2 \times 2 \times 5$

$75 = 3 \times 5 \times 5$

$126 = 2 \times 3 \times 3 \times 7$

$$\begin{array}{r|l} 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 3 & 75 \\ \hline 5 & 25 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

In these prime factorisation, 2 occurs maximum 3 times, 3 and 5 occurs maximum twice and 7 occurs maximum once

$\therefore$ LCM of 40, 75 and 126 $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 = 12600$

(v) $108 = 2 \times 2 \times 3 \times 3 \times 3$

$135 = 3 \times 3 \times 3 \times 5$

$162 = 2 \times 3 \times 3 \times 3 \times 3$

$$\begin{array}{r|l} 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 162 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

In these prime factorisation 2 occurs maximum 2 times, 3 occurs maximum 4 times and 5 occurs maximum once.

$\therefore$ LCM of 108, 135 and 162 $= 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 = 1620$

3.10.2. Common Division Method

To find L.C.M. of two or more numbers, we follow the following steps:-

Step 1:- Arrange the given numbers in a row separated by commas.

Step 2:- Obtain a number which divides exactly atleast two of the given numbers.

Step 3:- Write the quotients just below them which are divisible by the chosen number and carry forward the numbers which are not divisible by that number.

Step 4:- Repeat the process till no two of the given numbers divisible by the same number.

Step 5:- The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers.

Example 25: **Find the L.C.M. of the following numbers:-**

(i) 45, 60 **(ii) 12, 18 and 20** **(iii) 30, 40 and 75**

(iv) 84, 90 and 120 **(v) 56, 72 and 144**

Solution : (i)

$$\begin{array}{r|l} 3 & 45,\ 60 \\ \hline 5 & 15,\ 20 \\ \hline & 3,\ 4 \end{array}$$

$\therefore$ LCM of 45 and 60 $= 3 \times 5 \times 3 \times 4 = 180$

(ii)

$$\begin{array}{r|l} 2 & 12,\ 18,\ 20 \\ \hline 2 & 6,\ 9,\ 10 \\ \hline 3 & 3,\ 9,\ 5 \\ \hline & 1,\ 3,\ 5 \end{array}$$

$\therefore$ LCM of 12, 18 and 20 $= 2 \times 2 \times 3 \times 3 \times 5 = 180$

(iii)

2	30, 40, 75
5	15, 20, 75
3	3, 4, 15
	1, 4, 5

$\therefore$ LCM of 30, 40 and 75 = $2 \times 5 \times 3 \times 4 \times 5 = 600$

(iv)

2	84, 90, 120
2	42, 45, 60
3	21, 45, 30
5	7, 15, 10
	7, 3, 2

$\therefore$ LCM of 84, 90 and 120 = $2 \times 2 \times 3 \times 5 \times 7 \times 3 \times 2 = 2520$

(v)

2	56, 72, 144
2	28, 36, 72
2	14, 18, 36
3	7, 9, 18
3	7, 3, 6
	7, 1, 2

$\therefore$ LCM of 56, 72 and 144 = $2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 2 = 1008$

3.10.3 Applications of LCM (Word Problems)

In this section, we shall discuss some applications of LCM in solving some practical daily life problems. Let's illustrate these problems with following examples:-

Example 26: Find the smallest number which is divisible by 12, 15 and 24.

Solution : We know that the smallest number divisible by 12, 15 and 24 is their LCM.

So, We calculate LCM of 12, 15 and 24

2	12, 15, 24
2	6, 15, 12
3	3, 15, 6
	1, 5, 2

$\therefore$ LCM = $2 \times 2 \times 3 \times 5 \times 2 = 120$

Hence Required number = 120

Example 27: Find the least number when divided by 6, 15 and 21 leaves remainder 4 in each case.

Solution : We know that the least number divisible by 6, 15 and 21 is their LCM.

So, the required number must be 4 more than their LCM.

We calculate LCM of 6, 15 and 21

3	6, 15, 21
	2, 5, 7

$\therefore$ LCM = $3 \times 2 \times 5 \times 7 = 210$

Hence, Required number = $210 + 4 = 214$

Example 28: Find the greatest 3-digit number exactly divisible by 18, 24 and 36

Solution : First, find LCM of 18, 24 and 36

$\therefore$ LCM = $2 \times 2 \times 3 \times 3 \times 2 = 72$

2	18, 24, 36
2	9, 12, 18
3	9, 6, 9
3	3, 2, 3
	1, 2, 1

Now the greatest 3-digit number is 999

We find that when 999 is divided by 72, the remainder is 63.

Hence, greatest number of 3 digits which is exactly divisible by 18, 24 and 36 is = $999 - 63 = 936$

```
72 ) 999 ( 13
    - 72
    ----
     279
    - 216
    -----
      63
```

Example 29: Find the 4-digit smallest number which is exactly divisible by 15, 20 and 24.

Solution : First, find LCM of 15, 20 and 24

2	15, 20, 24
2	15, 10, 12
3	15, 5, 6
5	5, 5, 2
	1, 1, 2

$\therefore$ LCM = $2 \times 2 \times 3 \times 5 \times 2 = 120$

Now, 4 digit smallest number is 1000.

We find that when 1000 is divided by 120, the remainder is 40.

```
120 ) 1000 ( 8
     - 960
     -----
        40
```

$\therefore$ Smallest 4-digit number, which is exactly divisible by 15, 20 and 24 = $1000 + (120 - 40) = 1080$

Hence, required number = 1080

Example 30: In a morning walk, three persons step off together. Their steps measure 70cm, 80cm and 75cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps?

Solution : The distance covered by each one of them has to be same as well as minimum.So, the required minimum distance each should walk would be L.C.M. of the measure of their steps.

$\therefore$ LCM = $2 \times 5 \times 7 \times 8 \times 15 = 8400$ cm

2	70, 80, 75
5	35, 40, 75
	7, 8, 15

Hence required distance = 8400 cm or 84m

Example 31 : Four bells toll at intervals of 2, 3, 4 and 5 seconds. The bells toll together at 8 a.m. when will they again toll together?

Solution : The bells will toll together at a time which is a multiple of four intervals 2, 3, 4 and 5 seconds.

So, first we find LCM of 2, 3, 4 and 5

2	2, 3, 4, 5
	1, 3, 2, 5

$\therefore$ LCM = $2 \times 3 \times 2 \times 5 = 60$

Thus, the bells will toll together after 60 seconds or 1 minute.

First they toll together at 8a.m. then they will toll together after 1 minute i.e. 8:01a.m.

3.11 Relation Between H.C.F. and LCM:-

* HCF of given numbers is always a factor of LCM **or** LCM is a multiple of H.C.F.
* The product of HCF and LCM of two numbers is equal to product of both given numbers. If a and b are two numbers then $a \times b$ = HCF $\times$ LCM

For example:- Consider two numbers 12 and 18

$12 = 2 \times 2 \times 3$

and $18 = 2 \times 3 \times 3$

HCF = $2 \times 3 = 6$

and LCM = $2 \times 2 \times 3 \times 3 = 36$

Now Product of given numbers = $12 \times 18 = 216$

Product of their HCF and LCM = $6 \times 36 = 216$

Hence, **Product of two numbers = Product of their HCF and LCM**

Note:- *This result is true only for two numbers*

Example 32: Can two numbers have 18 as their HCF and 42 as their LCM. Give reasons in support of your answer.

Solution : We know that HCF of given numbers is a factor of their LCM.

But 18 is not a factor of 42.

So, there cannot be two numbers with HCF 18 and LCM 42.

Example 33: The HCF and LCM of two numbers are 15 and 75 respectively. If one number is 25 find other number.

Solution : I^{st} number × 2^{nd} number = HCF × LCM

$$2^{nd} \text{ number} = \frac{\text{HCF} \times \text{LCM}}{I^{st} \text{ number}} = \frac{15 \times 75}{25} = 45$$

Hence other number is 45.

Exercise 3.5

1. Find LCM of following numbers by prime factorisation method:-
   - (i) 45, 60 (ii) 52, 56 (iii) 96, 360
   - (iv) 36, 96, 180 (v) 18, 42, 72
2. Find LCM of the following by common division method:-
   - (i) 24, 64 (ii) 42, 63 (iii) 108, 135, 162
   - (iv) 16, 18, 48 (v) 48, 72, 108
3. Find the smallest number which is divisible by 6, 8 and 10.
4. Find the least number when divided by 10, 12 and 15 leaves remainder 7 in each case.
5. Find the greatest 4-digit number exactly divisible by 12, 18 and 30.
6. Find the smallest 4-digit number exactly divisible by 15, 24 and 36.
7. Four bells toll at intervals of 4, 7, 12 and 14 seconds.The bells toll together at 5a.m. When will they again toll together?
8. Three boys step off together from the same spot their steps measures 56cm, 70cm and 63cm respectively. At what distance from the starting point will they again step together?
9. Can two numbers have 15 as their HCF and 65 as their LCM. Give reasons in support of your answer.
10. Can two numbers have 12 as their HCF and 72 as their LCM. Give reasons in support of your answer.
11. The HCF and LCM of two numbers are 13 and 182 respectively. If one of the numbers is 26. Find other number.
12. The LCM of two co-prime numbers is 195. If one number is 15 then find the other number.
13. The HCF of two numbers is 6 and product of two numbers is 216. Find their LCM.

Multiple Choice Questions

1. Which number is a factor of every number?

(a) 0 (b) 1 (c) 2 (d) 3

2. How many even numbers are prime?

(a) 1 (b) 2 (c) 3 (d) 4

3. The smallest composite number is ....

(a) 1 (b) 2 (c) 3 (d) 4

4. Which of the following number is a perfect number?

(a) 8 (b) 6 (c) 12 (d) 18

5. Which of the following is not a multiple of 7?

(a) 35 (b) 48 (c) 56 (d) 91

6. Which of the following is not a factor of 36?

(a) 12 (b) 6 (c) 9 (d) 8

7. The number of prime numbers upto 25 are

(a) 9 (b) 10 (c) 8 (d) 12

8. Which mathematician gave the method to find prime and composite numbers?

(a) Aryabhatta (b) Ramayan (c) Eratosthenes (d) Goldbach

9. The statement "Every even number greater than 4 can be expressed as the sum of two odd prime numbers" is given by

(a) Goldbach (b) Eratosthenes (c) Aryabhatta (d) Ramanujan

10. Which of the following is a prime number?

(a) 221 (b) 195 (c) 97 (d) 111

11. Which of the following number is divisble by 4?

(a) 52369 (b) 25746 (c) 21564 (d) 83426

12. Which of the following is not true?

(a) If a number is factor of two numbers then it is also factor of their sum

(b) If a number is factor of two numbers then it is also factor of their difference.

(c) 15 and 24 are co-prime to each other.

(d) 1 is neither prime nor composite.

13. Which of the following pair is co-prime?

(a) (12, 25) (b) (18, 27) (c) (25, 35) (d) (21, 56)

14. Which of the following number is divisible by 8?

(a) 123568 (b) 412580 (c) 258124 (d) 453230

15. Prime factorisation of 84

(a) $2 \times 2 \times 3 \times 2 \times 7$ (b) $7 \times 2 \times 3 \times 3$

(c) $2 \times 3 \times 7 \times 2$ (d) $3 \times 2 \times 3 \times 2 \times 7$

16. HCF of 25 and 45 is

(a) 15 (b) 5 (c) 225 (d) 135

17. If LCM of two numbers is 36 then which of the following can not be their HCF?

(a) 9 (b) 12 (c) 8 (d) 18

18. The LCM of two co-prime numbers is 143. If one number is 11 then find other number.

(a) 132 (b) 154 (c) 18 (d) 13

19. Find the greatest number which divides 145 and 235 leaving the remainder 1 in each case.

(a) 24 (b) 18 (c) 19 (d) 17

20. The greatest 4 digit number which is divisible by 12, 15 and 20

(a) 9990 (b) 9000 (c) 9960 (d) 9999

Learning Outcomes

After completion of this chapter, the students are now able to :

1. Understand about factors and multiples.
2. Give information about different types of numbers.
3. Check the divisibility of number without actual division.
4. Apply knowledge of HCF and LCM and can use them in daily life.

ANSWER KEY

Exercise 3.1

1. (i) 1, 2, 3, 6, 9, 18 (ii) 1, 2, 3, 4, 6, 8, 12, 24

(iii) 1, 3, 5, 9, 15, 45 (iv) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

(v) 1, 5, 13, 65

2. (i) 6, 12, 18, 24, 30, 36 (ii) 9, 18, 27, 36, 45, 54

(iii) 11, 22, 33, 44, 55, 66 (iv) 15, 30, 45, 60, 75, 90

(v) 24, 48, 72, 96, 120, 144

3. (i) 17, 34, 51, 68, 85 (ii) 12, 24, 36, 48, 60, 72, 84, 96

(iii) 21, 42, 63, 84

4. (ii), (iv)

5. (i) $16 = 3 + 13 = 5 + 11$ (ii) $28 = 11 + 17$

(iii) $40 = 3 + 37 = 11 + 29 = 17 + 23$

6. (i) 2, 3, 5, 7, 11, 13, 17, 19, 23 (ii) 89, 97, 101, 103

(iii) 127, 129, 131, 137, 139

7. No

8. (i) 6 (ii) 8 (iii) 9 (iv) 13 (v) 15

9. 23, 37, 53, 73

Exercise 3.2

1. (i) 1, 2, 4, 8 (ii) 1, 5 (iii) 1, 2, 3, 4, 6, 12 (iv) 1, 7 (v) 1

2. (i) 15, 30, 45 (ii) 24, 48, 72 (iii) 12, 24, 36

3. Divisible by 2:- (i), (iii), (iv), (v)

Divisible by 4:- (iii), (iv)

4. Divisible by 3:- (i), (ii), (iv), (v)

Divisible by 9:- (ii), (v)

5. Divisible by 5:- (ii), (iii), (v)

Divisible by 10:- (ii)

6. (i), (iii), (iv) **7.** (i), (iii), (v) **8.** (i), (ii), (iii), (v)

9. (i) True (ii) False (iii) False (iv) True (v) True

10. 45 **11.** (ii), (iii), (v)

Exercise 3.3

1. (i) $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$ (ii) $120 = 2 \times 2 \times 2 \times 3 \times 5$

(iii) $180 = 2 \times 2 \times 3 \times 3 \times 5$

2. (i) 7, 4, 2 (ii) 10, 5 (iii) 8, 3, 4, 2

3. (i) $420 = 2 \times 2 \times 3 \times 5 \times 7$ (ii) $980 = 2 \times 2 \times 5 \times 7 \times 7$

(iii) $225 = 3 \times 3 \times 5 \times 5$ (iv) $150 = 2 \times 3 \times 5 \times 5$

(v) $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$

Exercise 3.4

1. (i) 6 (ii) 45 (iii) 12 (iv) 7 (v) 18

2. (i) 34 (ii) 18 (iii) 8 (iv) 48 (v) 12

3. 1 **4.** 2 **5.** 1 **6.** 1 **7.** 16

8. 65 **9.** 17 **10.** 23 litres **11.** 17 **12.** 6m

13. 24cm **14.** (i) $\frac{5}{6}$ (ii) $\frac{2}{5}$ (iii) $\frac{17}{23}$

Exercise 3.5

1. (i) 180 (ii) 728 (iii) 1440 (iv) 1440 (v) 504

2. (i) 192 (ii) 126 (iii) 1620 (iv) 144 (v) 432

3. 120 **4.** 67 **5.** 9900 **6.** 1080 **7.** 5 : 01 : 24 cm

8. 2520cm **9.** No **10.** Yes **11.** 91 **12.** 13

13 36

Multiple Choice Questions

1. b **2.** a **3.** d **4.** b **5.** b **6.** d **7.** a **8.** c

9. a **10.** c **11.** c **12.** c **13.** a **14.** a **15.** c **16.** b

17. c **18.** d **19.** b **20.** c

V4M6C3

4

INTEGERS

Objectives

In this chapter you will learn

(i) To understand about the extended number system from natural numbers to integers.
(ii) To represent integers on number line and operations on number line.
(iii) To identify greater or smaller integer out of given set of integers .
(iv) To solve problems involving addition and subtraction of integers.

4.1 Introduction

We have already learnt about the natural numbers, i.e. 1, 2, 3,4, 5, ... which we also called counting numbers. We also learnt about whole numbers i.e. 0, 1, 2, 3, 4, 5, ... which is the extension of natural numbers. We have studied earlier in whole number system that sum of two whole numbers is always a whole number, but difference of two whole numbers is not always a whole number (Do you remember 7 – 5 = 2) But what is 5 – 7 =?. To answer this problem we need to extend our number system from whole numbers to integers. Let us look at few more real life examples:

- Sachin goes to a hill sation, the temperature of that hill station is 0ºC. Further 2 degrees fall in temperature causes the temperature to be 2ºC below 0ºC. Can you tell the present temperature?
- Ramesh and Arjun went to a shop to buy a pen. The price of pen is ₹25. But Ramesh had only ₹20 in his pocket. He borrowed ₹5 from Arjun and bought the pen. Now Ramesh is left with no money or ₹0 in his pocket. But he has to remember the amount borrowed (should he or not?) He writes ₹5 in his note book. How will he express the money borrowed in numbers?

  In the above examples, we feel the need of introducing special numbers to deal with the situations of borrowing and going below 0.

Infact we need to extend our number system beyond whole numbers.

For Example : Earning and Spending, East and West, Deposit and Withdrawls, Above sea level and below sea level, above freezing point and below freezing point, etc. In fact for this we need negative numbers to express spending (Contrary to positive earning) and to express other similar cases.

4.2. Negative Numbers

Negative number is a real number less than zero. Negative numbers represent opposite to the positive numbers. If positive represents a movement to the right, then negative represents a movement to the left. If positive represents above sea level, then negative represents below sea level. If positive represents a deposit, negative represents a withdrawl. They are often used to represent the magnitude of a loss or deficiency. A debt owned by someone, may be considered as his negative asset. Negative numbers are used to describe values on a scale that goes below zero, such as celsius and Fahrenheit scales of Temparature.

Negative numbers are usually written with a minus sign in front of number like –1 (pronounced as : minus one or negative one). Numbers less than zero are negative and greater than zero are positive. **Zero itself is neither positive nor negative**. Zero is non-negative non positive number.

As in common sense oppsoite of opposite is the original thing, like wise negative of a negative is positive.

For Example : – $(-1) = 1$

In this way we got new range of numbers that we called negative numbers (negative integers) and these are:

–1, –2, –3, –4, –5, ...

4.3 Integers

The first number to be discoverd were Natural Numbers (counting numbers) ie. 1, 2, 3, 4 .... Then we included zero (0) to the set (collection) of natural numbers, we got new set of numbers known as whole numbers i.e 0, 1, 2, 3, 4.......... .Now we found that there are negative numbers too. If we include negative numbers to the set of whole numbers we get a new set of numbers called Integers as 0, ±1, ±2, ±3, ±4, ±5, ...

(........... –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5..........)

1, 2, 3, 4, 5..............

are called positive integers.

–1, –2, –3, –4, –5 ....

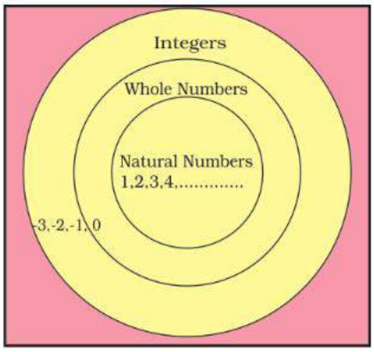
are called Negative integers.

0 (zero) is neither positive nor negative

N : {1, 2, 3, 4.........}

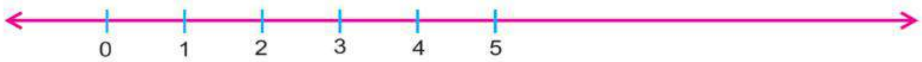
W : {0, 1, 2, 3, 4..........}

Z or I : {.........–4, –3, –2, –1, 0, 1, 2, 3, 4............}

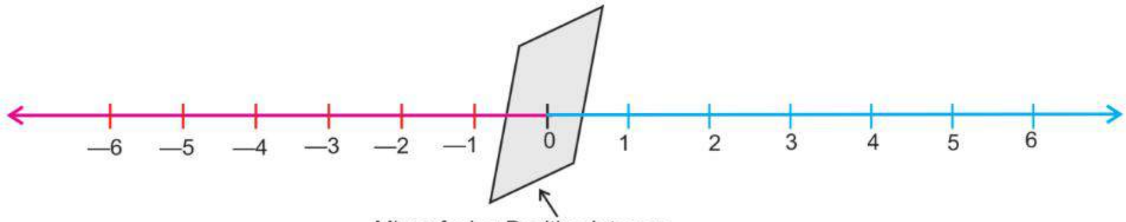


4.4 Representation of Integers on Number Line

Consider the part of a number line with whole numbers 0, 1, 2........ marked on it (we have already done it in previous chapter)



If we place mirror at 0 (Zero) facing towards the numbers 1, 2, 3, ... we get an image of the part of the line extending in opposite direction in the mirror. This part of the line are points which are images of 1, 2, 3 ...., Draw these images and mark the images of these points 1, 2, 3 .... as –1, –2, –3 .... respectively.



We got a line extending indefinitely in both directions where zero (0) lies in the centre. While facing towards zero, the numbers on the right of zero are positive integers 1, 2, 3, 4 .... and to the left of zero are negative integers (which are all images of positive integers) –1, –2, –3, –4........ . Here –1 is the image of 1, –2 is the image of 2 and so on.

4.5 Ordering of Integers

Any number on the number line is greater than any other number appearing on its left, and any number on the number line is less than any other number appearing on its right.

Some Important Observations:

(a) Every integer has its successor as well as predecessor.

(b) Every positive integer is greater than 0 and every negative integer is less than 0.

(c) The greater integer between the two given integers is the lesser integer between the negative of these integers

e..g $15 > 13$ but $-15 < -13$

(d) A number farther from 0 on the right has larger value.

(e) A number farther from 0 on the left side has smaller value.

(f) Smallest positive Integer is 1.

But largest positive integer (or Just Integer) is not possible to write in.

(g) Largest Negative integer is '–1'

but smallest Negative integer (or just integer) is not possible to write in.

(h) 0 is neither positive integer nor negative integer.

(i) Every positive integer is greater than every negative integer.

(j) 0 is greater than all negative integers.

Example 1: Write the opposite of the following

(a) 300 feet above sea level.

(b) Withdrawal of ₹500 from Bank Account.

Solution : (a) 300 feet below sea level.

(b) Deposit of ₹500 in Bank Account.

Example 2 : Represent the following situations in integers.

(a) Height of Mount Everest is 8848 m above sea level.

(b) A submarine is at a depth of 600 m below sea level.

(c) A loss of ₹200

(d) Share market gained 200 points today.

Solution : (a) +8848 (Here '+' represents above the sea level in metres)

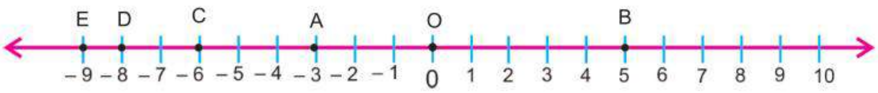
(b) –600 (Here '–' represents below the sea level in metres)

(c) –200 (Here '–' is loss in Rupees)

(d) +200 ('+' is points gained)

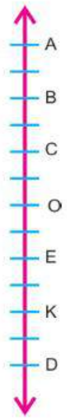
Example 3: Represent the following numbers on number line : –3, +5, –6, 0, –8, –9

Solution :



Point O represents zero, Point A represents –3, Point B represents +5, Point C reprsents –6, Point D represents –8, Point E represents –9.

Example 4: Given figure is vertical number line.



representing integers in which O represent zero. Answer the following.

(a) If point D is –6, then which point is +6.

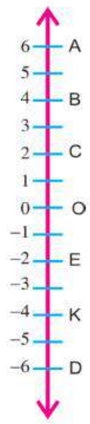
(b) Is A negative or a positive integer

(c) Write integers from B to E

(d) Write point on the number line having least value.

(e) Which number is represented by C.

Solution : Let us write integers on this vertical number line taking O as origin.



We shall write positive integers above '0' (zero) and negative integers below '0' (zero) in sequence.

(a) Given that D is –6 and hence A is +6.

(b) A is positive integer.

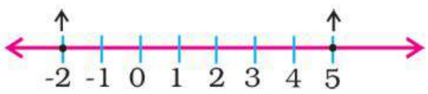
(c) Integers from B to E are :4, 3, 2, 1, 0, –1,–2

(d) Point on the number line having least value is D.

(e) C represents +2.

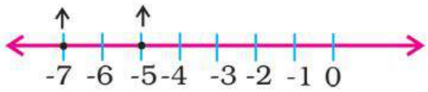
Example 5. In each of the following pairs, which number is to the right of the other on the number line.

(a) –2, 5 **(b) –7, –5**



Solution : (a) 5 lies to the right of – 2.

(b) –5 lies to the right of –7



Example 6: **Which of the following lies to the left of the other on number line ?**

(a) –10, –20 (b) 7, –6

Solution : (a) –20 lies to the left of –10

(b) –6 lies to the left of 7

Example 7: **Write all the integer between the given pairs.**

(a) –30 and –20 (b) –8 and –15

Between mean excluded "end points".

Solution: (a) The integers lying between –30 and –20 are –29, –28, –27, –26, –25, –24, –23, –22, –21

(b) The integers lying between –8 and –15 are –14, –13, –12, –11, –10, –9

Example 8: **Write four negative integers greater than –9.**

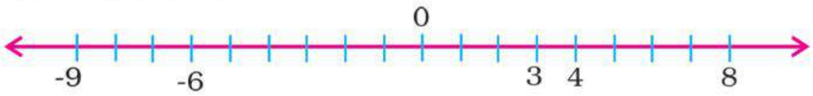
Solution : Greater integers lies on the right side on number line.

Integers on the right of (–9) on number line are : –8, –7, –6,–5.

Example 9 : **Arrange the following integers in ascending order.**

(a) –9, 3, 4, –6, 8

Solution :



Given integers in ascending order are :;

–9, –6, 3, 4, 8

Exercise 4.1

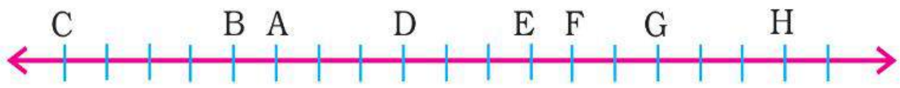
1. Write two examples from day to day life in which we can use positive and negative integers.
2. Write the opposite of the following:
   (a) A profit of ₹ 500 (b) A withdrawal of ₹70 Rs from bank account.
   (c) A deposit of ₹1000 (d) 326 B.C
   (e) 500m below Sea level (f) 25° above 0°C
3. Represent the situations mentioned in Q2 in integers.
4. Represent the following situations in Integers.
   (a) A deposit of ₹ 500.
   (b) An Aeroplane is flying at a height two thousand metre above the sea level.
   (c) A withdrawal of ₹700 from Bank Account.
   (d) A diver dives to a depth of 6 feet below ground level.

5. Represent the following numbers on number line.

(a) –5 (b) +6 (c) 0 (d) +1

(e) –9 (f) –4 (g) +8 (h) +3

6. Integers are represented on a horizontal number line as shown where A represents –2. With reference to the number line, answer the following questions :



(a) Which point represent –3?

(b) Locate the point which represents the opposite of B and name it P.

(c) Write integers for the points C and E.

(d) Which point marked on the number line has the least value?

7. In each of the following pairs, which number is to the right of other on the number line?

(a) 2, 9 (b) –3, –8 (c) 0, –5

(d) –11, 10 (e) –9, 9 (f) 2, –200

8. Write all the integers between the given pairs (write them in increasing order)

(a) 0 and –6 (b) –6 and +6 (c) –9 and –17

(d) –19 and –5

9. (a) Write five negative integers greater than '–15'.

(b) Write five integers smaller than '–20'.

(c) Write five integers greater than 0.

(d) Write five integers smaller than 0.

10. Encircle the greater integer in each given pair.

(a) –5, –7 (b) 0, –3 (c) 5, 7

(d) –9, 0 (e) –9, –11 (f) –4, 4

(g) –10, –100 (h) 10, 100

11. Arrange the following integers in ascending order:

(a) 0, –7, –9, 5, –3, 2, –4

(b) 8, –3, 7, 0, –9, –6

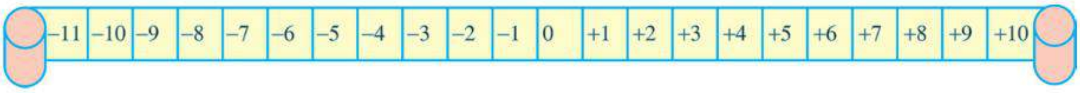
12. Arrange the following integers in descending order:

(a) –9, 3, 4, –6, 8, –3

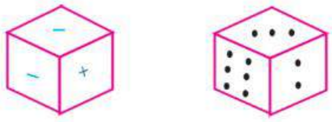
(b) 4, 8, –3, –2, 5, 0

4.6 Understanding Integers with a game

Make a number strip marked with integers –30 to +30.



Take two dice, one marked 1 to 6 and other marked with three '+' signs and three '–' signs.



Two players can play the game at a time. Players will keep different coloured buttons at the zero position on number strip.

Let player A starts the game. He throws both dice simultaneously. If on one dice there appear '+' sign and on another dice there appears 3 then it means the outcome is +3. Player A picks his button and places it on '+3'.

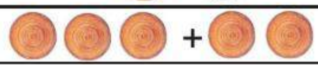
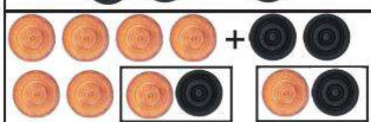
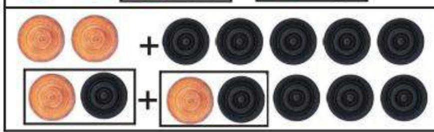
Now it is turn of B. He throws both dice simultaneously. On one die he gets '–' sign and on other dice he gets '4'. It means he has got '–4'. Player B picks his button and places it at –4.

Now it is turn of A. He throws both dice and gets '–5'. He has to move 5 steps to the left of his present position '+3'. Thus he reaches '–2' and places his button there (+3 – 5 = –2). The game continues this way. And player who reaches –30 will be considered out. And the player who reaches +30 first wins the game.

4.7 Addition of Integers (Understanding with an Activity)

Take carrom coins (Black and White) to perform this activity.

Let us assume that each white carrom coin represent +1 and each black carrom coin represent –1.

Carrom coins	**Integer Represented**
	= 2
	= –2
	= 0
	= (+3) + (+2) = +5
	= (–2) + (–1) = –3
	(+4) + (–2) = (+2) + 0 + = +2
	= (+2) + (–5) = 0 + 0 + (–3) = –3

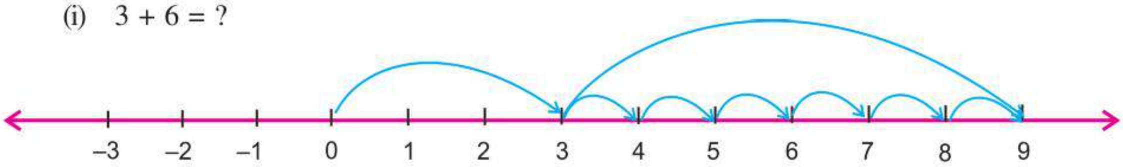
Observation:

1. We do addition when we have two positive integers like (+3) + (+2) = +5.
2. We do additon when we have two negative integers but the answer takes the negative sign (–) [minus sign] like (–2) + (–1) = –3.
3. When we have one positive and one negative integer, we must subtract, but answer will take the sign of largest integer (Ignoring the sign of integer, decide which is bigger) like (+2) +(–5) = –3.

4.7.1 Addition of Integers using Number line

It is not always easy to add integers using carrom coins. Let us try to perform these operations on number line of integers.

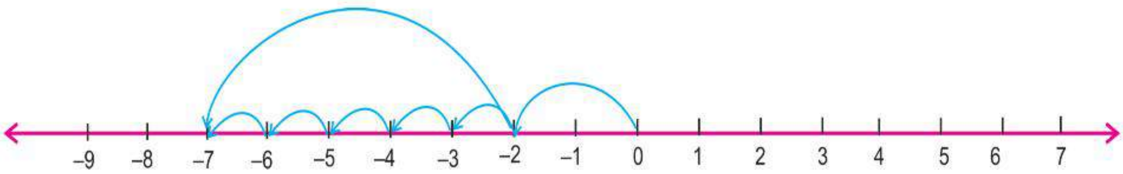
(i) 3 + 6 = ?



On number line we first move 3 steps to the right of zero (suggested by '+' sign) them moved ahead 6 steps to the right of 3. We finally reaches at 9. Thus '+9' is the final answer.

⇒ 3 + 6 = 9

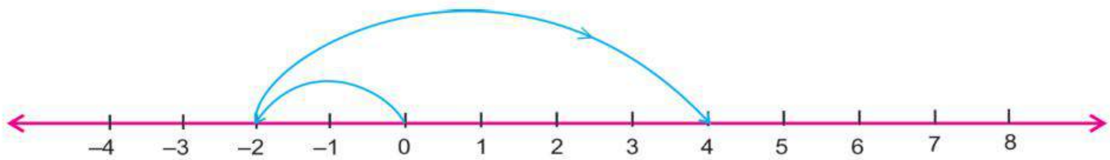
(ii) (–2) + (–5) = ?



On number line we first move 2 steps to the left of zero (left direction is suggested by minus sign then we moved ahead 5 steps to the left of '–2' We finally reaches at –7. Thus '–7' is the answer.

⇒ (–2) + (–5) = –7

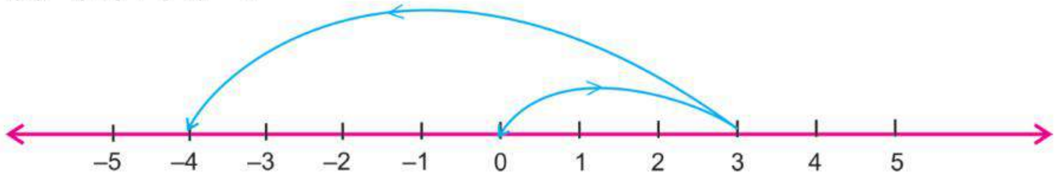
(iii) (–2) + (6) = ?



On number line we first move 2 steps to the left of zero (suggested by minus sign), then we moved six steps to the right of '–2' (Direction suggested by plus sign). We finally reaches at +4. Thus '+4' is the answer.

⇒ (–2) + (6) = +4

(iv) (+3) + (–7) = ?



On number line we first move 3 steps to the right of zero (Right direction suggested by plus sign) and reaches at +3. Then we move 7 steps to the left of '+3' (left direction suggested by minus sign) and reaches at –4. Thus answer is –4.

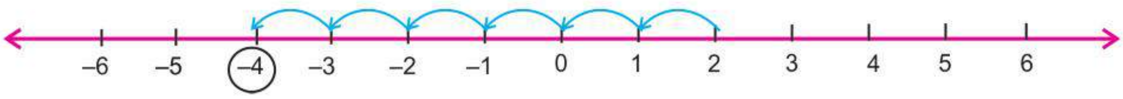
⇒ (+3) + (–7) = –4

Example 10. **Using number line write the integer which is**

(a) 6 less than 2 **(b) 3 less than –2**

Solution : (a) 6 less than 2 = ?

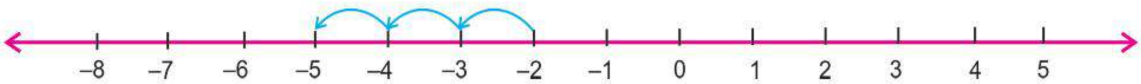
We need to find the integer which is 6 less than 2. So we shall start with '+2' and procced 6 steps to the left of '+2' as shown below.



Therefore 6 less than 2 is '–4'

(b) 3 less than –2 = ?

We need to find the integer which is 3 less than –2. So we shall start with –2 and proceed 3 steps to the left of –2 as shown below.



Therefore 3 less than –2 is '–5'.

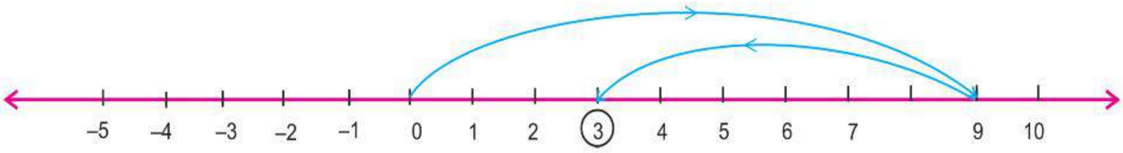
Example 11. **Using Number line add the following integer.**

(a) 9 + (–6) **(b) (–5) + 10**

(c) (–2) + 5 + (–3)

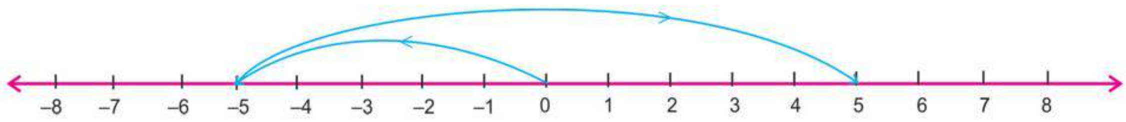
Solution : (a) 9 + (–6)

On number line we shall start from 0 and move 9 steps to the right of zero. Then we shall move six steps to the left of '+9' . We finally reach at +3. Thus +3 is the answer.



Hence 9 + (–6) = +3

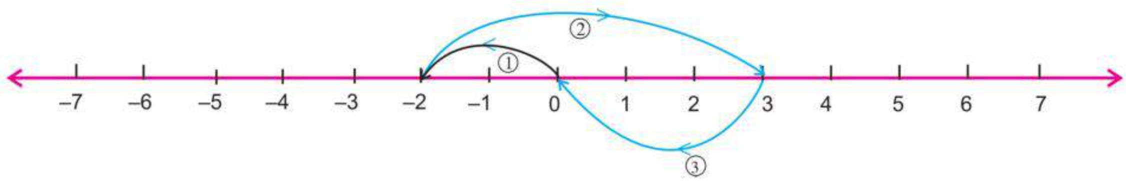
(b) (–5) + 10 = ?



On number line we shall move 5 steps to the left of zero (left side suggested by minus sign). Then we shall move 10 steps to the right of '–5' and finally reach at '+5'. Thus '+5' is the answer.

Hence (–5) + 10 = +5

(c) (–2) + 5 + (–3)



Step I : On number line we shall move 2 steps to the left of zero suggested by minus sign of '–2'.

Step II: The we shall move 5 steps to the right of '–2' suggested by plus sign of '+5' and we reach of '+3'.

Sign III : Then we shall move 3 steps to the left of '+3' as suggested by minus sign of '–3' and finally we reach at zero.

Hence answer is zero. Hence (–2) + 5 + (–3) = 0

Example 12. **Add without using number line.**

(a) 19 + (–13) **(b) 19 + 13**

(c) (–19) + (–13) **(d) (–19) + 13**

(e) 21 + (–13) + 8 + 7 + (–19) + (–11) + 2

Solution : (a) 19 + (–13)

= + (19 –13)

= +6

When we have one positive and one negative integer. We subtract them, but the answer takes the sign of bigger integer (ignoring the sign of integer, decide which is bigger)

(b) 19 + 13

= 32

We simply add when we have two positive integers.

(c) (–19) + (–13)

= – (19+13)

= –32

We add when we have two negative integers. But the answer takes the minus sign.

(d) (–19) + 13

= – (19 – 13)

= –6

When we have one positive and one negative integer, we subtract them, But the answer takes the sign of bigger integer (Ignoring the sign of integer, decide which is bigger)

(e) 21 + (–13) + 8 + 7 + (–19) + (–11) + 2

We arrange the numbers so that, the positive and negative integers are grouped together. We have

21 + 8 + 7 + 2 + (–13) + (–19) + (–11)

= 38 + (–43)

= 38 – 43

= –5

	+	–
	21	13
	8	19
	7	11
	2	
Add	+38	–43

Pick the sign. of larger number.

Example 13. **Write the successor and predecessor of the following:**

(a) –69 **(b) 59**

Solution : (a) Successor of –69 = –69 + 1 = –68

Predecessor of –69 = –69 – 1 = –70

(b) Successor of 59 = 59 + 1 = 60

Predecessor of 59 = 59 – 1 = 58

Exercise 4.2

1. Using number line write the integer which is

(a) 5 less than –1 (b) 5 more than –5

(c) 2 less than 5 (d) 3 less than –2

2. Using number line, add the following integers:

(a) 9 + (–3) (b) 5 + (–11)

(b) (–1) + (–4) (d) (–5) + 12

(e) (–1) + (–2) + (–4) (f) (–2) + 4 + (–5)

(g) (–3) + (5) + (–4)

3. Add without using number line:

(a) 18 + 13 (b) 18 + (–13)

(c) (–18) + 13 (d) (–18) + (–13)

(e) 180 + (–200) (f) 777 + (–67)

(g) 1262 + (–366) + (–962) (h) 30 + (–27) + 21 + (–19) + (–3) + (11) + (–9)

(i) (–7) + (–9) + 4 + 16 (j) 37 + (–2) + (–65) + (–8)

4. Write the successor and predecessor of the following:

(a) –15 (b) 27 (c) –79 (d) 0

(e) 29 (f) –18 (g) –21 (h) 99

(i) –1 (j) –13

5. Complete the following addition table:

+	–3	–4	–2	+1	+2	+3
–2						
–3						
0						
+1						
+2						

4.8. Additive Inverse

Two integers which when added to each other give the sum zero, are called additive inverse of each other.

e.g.: (–3) + (3) = 0

Here (–3) is additive inverse of 3

and (3) is additive inverse of –3

4.9 Subtraction of Integers:

Subtraction is an operation which is just the reverse of addition. We use the following rule for subtraction:

If a and b are two integers, to subtract b from a, we change the sign of b and add it in a .

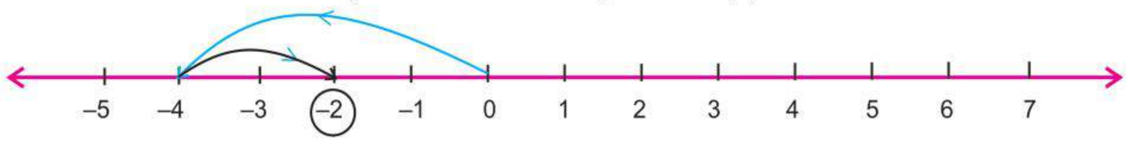
e.g. : a – b = a + (–b)

Example 14. **Using number line find the value of**

(a) –4 – (–2) **(b) 7 – (–2)**

Solution : (a) –4– (–2)

= –4 + (additive inverse of –2) = –4 + (2)

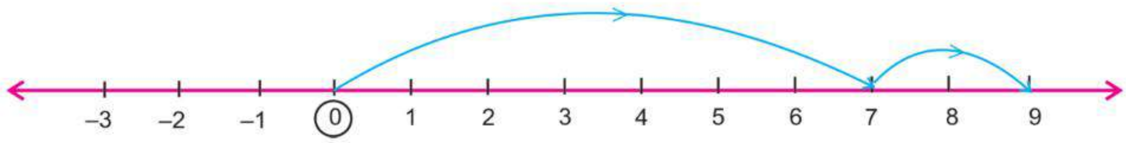


Hence –4 – (–2) = –2

(b) 7 – (–2)

= 7 + (additive inverse of –2)

= 7 + (2)



Hence 7 – (–2) = 9

Example 15. **Subtract the following:**

(a) 27 from 42 **(b) –13 from 91**

(c) 16 from –84 **(d) –61 from –41**

Solution : (a) 42 – (+27) = 42 + (–27)
= 42 – 27
= 15

(b) 91 – (–13) = 91 +(13)
= 91 + 13
= 104

(c) –84 – (16) = –84 + (–16)
= –84 – 16
= –100

(d) –41 – (–61) = –41 + (61)
= –41 + 61
= +20

Example 16. **Solve.**

(a) (–13) + 32 –8 –1

(b) 19 – (–45) – (–3)

Solution : (a) (–13) + 32 – 8 – 1
= – 13 + 32 – 8 – 1
= 32 – 13 – 8 – 1
= 32 – 22
= 10

(b) 19 – (–45) – (–3)
= 19 + (45) + (3)
= 19 + 45 + 3
= 67

Exercise 4.3

1. Fill the suitable integer in box :

(a) 2 + ☐ = 0 (b) ☐ + 11 = 0

(c) –5 + ☐ = 0 (d) ☐ + (–9) = 0

(e) 3 + ☐ = 0 (f) ☐ + 0 = 0

2. Subtract using number line:

(a) 5 from –7 (b) –3 from –6

(c) –2 from 8 (d) 3 from 9

3. Subtract without using number line:

(a) –6 from 16 (b) –51 from 55

(c) 75 from –10 (d) –31 from –47

4. Find:

(a) 35 – (20) (b) (–20) – (13)

(c) (–15) – (–18) (d) 72 – (90)

(e) 23 – (–12) (f) (–32) – (–40)

5. Simplify:

(a) 2 – 4 + 6 – 8 – 10 (b) 4 – 2 + 2 – 4 – 2 + 2

(c) 4 – (–9) + 7 – (–3) (d) (–7) + (–19) + (–7)

Multiple Choice Questions

1. How many integers are between –3 to 3?

(a) 5 (b) 6 (c) 4 (d) 3

2. Which of the following integer is greater than –3?

(a) –5 (b) –4 (c) 0 (d) –10

3. Which of the following integers are in ascending order?

(a) –5, –9, –7, –8 (b) –9, –8, –7, –5

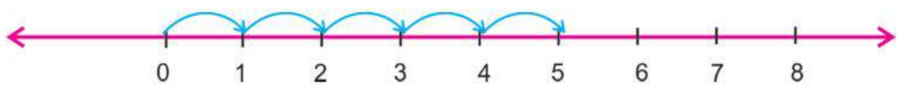
(c) –5, –7, –8, –8, –9 (d) –8, –5, –9, –7

4. Which of the following integers are in descending order ?

(a) 3, 0 , –2, –5 (b) –5, –2, 0, 3

(c) –5, 3, –2, 0 (d) –2, 0, –5, 3

5. The given number line represents:



(a) 5+1 (b) 1+5 (c) 1+1+1+1+1 (d) 5+5+5+5+5

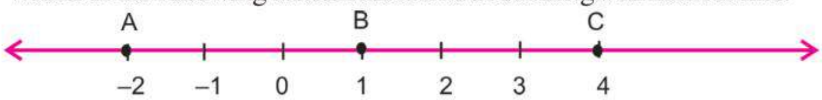
6. 3 less than –2 =

(a) –5 (b) –6 (c) 5 (d) 6

7. (–2) + 8 =

(a) –6 (b) –10 (c) 10 (d) 6

8. Which of the following statements is true about the given number line.



(a) Value of A is greater than value of B.

(b) Value of A is greater than value of C.

(c) Value of B is less than value of C.

(d) Value of C is less than value of B.

9. (–7) + (–12) + 11 =

(a) –19 (b) 30 (c) –23 (d) –8

10. 15 – (–12) + (–27) =

(a) 0 (b) –54 (c) –24 (d) 54

Learning Outcomes

After completion of this chapter, the students are now able to

- Understand the extended number system from natural number to integers
- Represent integers on the number line and operate on number line
- Identify greater or smaller integers out of a given set of integers.
- Solve problems involving addition and subtraction of integers

ANSWER KEY

Exercise - 4.1

2. (a) A loss of ₹500 (b) deposit of ₹70 in bank account (c) withdrawal of ₹1000

(d) 326 AD (e) 500m above sea level (f) 25° below 0°C

3. (a) +500 (b) –70 (c) +1000

(d) –326 (e) –500m (f) +25

4. (a) +500 (b) +2000 (c) –700 (d) –6

6. (a) B (b) +3 (c) C = –7, E = +4 (d) C

7. (a) 9 (b) –3 (c) 0

(d) 10 (e) 9 (f) 2

8. (a) –5, –4, –3, –2, –1

(b) –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5

(c) –16, –15, –14, –13, –12, –11, –10

(d) –18, –17, –16, –15, –14, –13, –12, –11, –10, –9, –8, –7, –6

10. (a) –5 (b) 0 (c) 7 (d) 0
(e) –9 (f) 4 (g) –10 (h) 100

11. (a) –9, –7, –4, –3, 0, 2,5 (b) –9, –6, –3, 0, 7, 8

12. (a) 8, 4, 3, –3, –6, –9 (b) 8, 5, 4, 0, –2,–3

Exercise - 4.2

1. (a) –6 (b) 0 (c) 3 (d) –5

2. (a) 6 (b) –6 (c) –5 (d) 7 (e) –7 (f)–3 (g) –2

3. (a) 31 (b) 5 (c) –5 (d) –31 (e) –20 (f) 710
(g) –66 (h) 4 (i) 4 (j) –38

4. (a) –14 and –16 (b) 28 and 26 (c) –78 and –80
(d) 1 and –1 (e) 30 and 28 (f) –17 and –19
(g) –20 and –22 (h) 100 and 98 (i) 0 and –2
(j) –12 and –14

5.

+	–3	–4	–2	+1	+2	+3
–2	–5	–6	–4	–1	0	+1
–3	–6	–7	–5	–2	–1	0
0	–3	–4	–2	+1	+2	+3
+1	–2	–3	–1	+2	+3	+4
+2	–1	–2	0	+3	+4	+5

Exercise - 4.3

1. (a) –2 (b) –11 (c) 5 (d) 9 (e) –3 (f) 0

2. (a) –12 (b) –3 (c) 10 (d) 6

3. (a) 22 (b) 106 (c) –85 (d) –16

4. (a) 15 (b) –33 (c) 3 (d) –18 (e) 35 (f) 8

5. (a) –14 (b) 0 (c) 23 (d) –33

Multiple Choice Questions

(1) a (2) c (3) b (4) a (5) c
(6) a (7) d (8) c (9) d (10) a

5

FRACTIONS

Objectives

In this chapter you will learn

- To understand about different types of fractions.
- To use fraction in practical life.
- To use fraction in different units i.e. money, length and temparature

5.1 Introduction

In previous classes, we have studied that if an object is divided in equal parts then one or more parts of the object is called fraction. In our daily life, we perform many activities of a fraction like a mother is preparing breakfast for her kids and the first kid demands for half chapatti and the second kid demands for one third chapati. Their demand of half chapati and one third chapatti represents fraction of whole chapati. A fraction means a part of whole or a group.

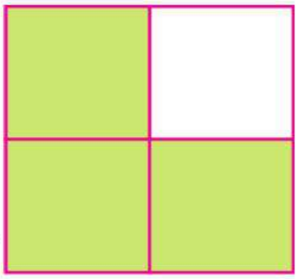
$$\text{Fraction} = \frac{\text{Parts of an object}}{\text{Total part of an object}}$$

The upper part of a fraction is called **numerator** and the lower part is called **denominator.**

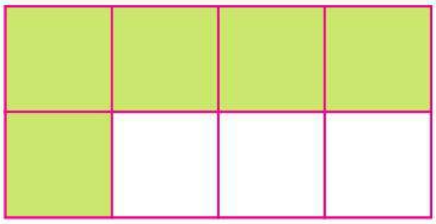
Look at the following figures:

(i) A square has been divided into four equal parts. 3 parts out of 4 have been shaded i.e. three-fourth has been shaded.

Mathematically, we say $\frac{3}{4}$ portion has been shaded. i.e. $\frac{3}{4} = \frac{\text{Shaded Parts}}{\text{Total number of Parts}} = \frac{\text{Numerator}}{\text{Denominator}}$



(ii) A rectangle has been divided into eight equal parts. 5 parts out of 8 have been shaded i.e. $\frac{5}{8}$ portion of rectangular sheet has been shaded and $\frac{3}{8}$ remains unshaded.

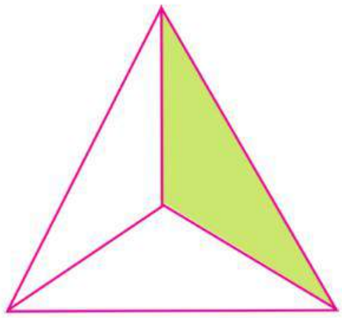


$$\frac{5}{8} = \frac{\text{Shaded Parts}}{\text{Total number of Parts}} = \frac{\text{Numerator}}{\text{Denominator}}$$

and $\frac{3}{8} = \frac{\text{Unshaded Parts}}{\text{Total number of Parts}} = \frac{\text{Numerator}}{\text{Denominator}}$

(iii) A triangle has been divided into three equal parts.

one part out of 3 has been shaded i.e. $\frac{1}{3}$ part has been shaded.



i.e. $\frac{1}{3} = \frac{\text{Shaded Parts}}{\text{Total number of Parts}} = \frac{\text{Numerator}}{\text{Denominator}}$

With the above discussion, we arrive at the definition of fractions.

"A fraction is a ratio representing part(s) of the whole".

* A fraction is of the form $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$, In $\frac{a}{b}$, a is called the numerator and b is called the denominator."

Consider the fraction $\frac{3}{5}$. This fraction is read as **"three - fifth"** which means that 3 parts out of 5 equal parts. In the fraction $\frac{3}{5}$, 3 is called the **numerator** and 5 is called the **denominator**.

Following are some more fractions:

Fraction	Meaning of the fraction	Numerator	Denominator
$\frac{2}{7}$ or Two-Seventh	Two equal parts out of seven equal parts	2	7
$\frac{3}{4}$ or Three-Fourth	Three equal parts out of four equal parts	3	4
$\frac{5}{11}$ or Five-Eleventh	Five equal parts out of eleven equal parts	5	11

Example 1: **Write the fraction for each of the following:-**

(i) Half (ii) Two-Fifth (iii) Five-Seventh

Solution : (i) Half = one out of two = $\frac{1}{2}$

(ii) Two-Fifth = Two out of five = $\frac{2}{5}$

(iii) Five-Seventh = Five out of Seven = $\frac{5}{7}$

Example 2 : **Write the numerator and the denominator for the followings:-**

(i) $\frac{7}{10}$ (ii) $\frac{3}{5}$ (iii) $\frac{9}{13}$

Solution : (i) Given fraction is $\frac{7}{10} = \frac{\text{Numerator}}{\text{Denominator}}$

$\therefore$ Numerator = 7 and Denominator = 10

(ii) Given fraction is $\frac{3}{5} = \frac{\text{Numerator}}{\text{Denominator}}$

$\therefore$ Numerator = 3 and Denominator = 5

(iii) Given fraction is $\frac{9}{13} = \frac{\text{Numerator}}{\text{Denominator}}$

$\therefore$ Numerator = 9 and Denominator = 13

Example 3: **(i) What fraction of a year is 4 months?**

(ii) What fraction of a day is 10 hours?

(iii) What fraction of a week is 2 days?

Solution : (i) We know 1 year = 12 months

$\therefore$ Required fraction = $\frac{4}{12}$

(ii) We know 1 day = 24 hours

$\therefore$ Required fraction = $\frac{10}{24}$

(iii) We know 1 week = 7 days

$\therefore$ Required fraction = $\frac{2}{7}$

Example 4: **Write the natural numbers from 1 to 15. What fraction of them are prime numbers?**

Solution : Natural numbers from 1 to 15 are:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 i.e. 15 in number

Prime numbers out of these numbers are:

2, 3, 5, 7, 11, 13, i.e. 6 in number.

$\therefore$ Required fraction = $\frac{6}{15}$

Example 5 : A bag contains 8 balls out of which 3 are blue and 5 are white. What fraction of ball represents blue and white?

Solution : Here, 3 out of 8 balls are blue

Fraction which represents blue balls = $\frac{3}{8}$

Now, 5 out of 8 balls are white

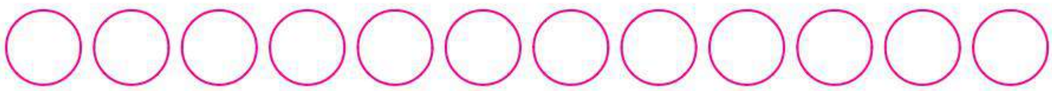
Fraction which represents while balls = $\frac{5}{8}$

5.2 Fraction and Division (Fraction as a part of collection)

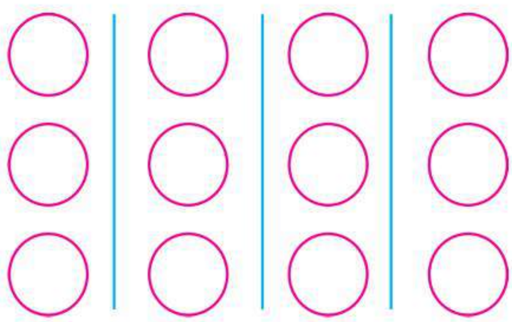
A fraction represents parts of a collection, the numerator being the number of parts we have and the denominator being the total number of parts in the collection.

For Example :

Let us take a collection of 12 balls and we want to get $\frac{1}{4}$ of the collection.

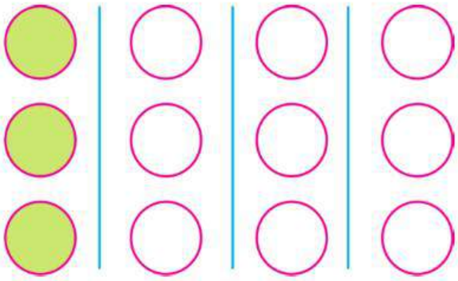


Step 1. In order to find $\frac{1}{4}$ out of the 12 balls, we divide the 12 balls into four equal groups/parts.



Each group/part contains 3 balls

Step 2. Now, we shade 1 group/part out of 4 parts.



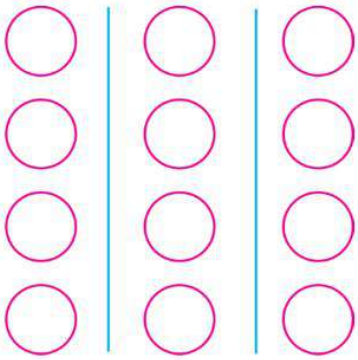
On counting, we find that the total number of shaded balls is 3.

In other words, $\frac{1}{4}$ of 12 balls = 3 balls

i.e. $\frac{1}{4} \times 12 = \frac{1 \times 12}{4} = \frac{12}{4} = 12 \div 4 = 3$ balls

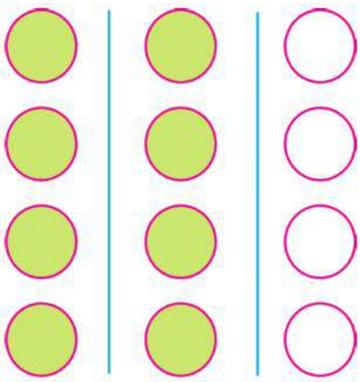
For Example : Find $\frac{2}{3}$ of 12 balls

Step 1. In order to find $\frac{2}{3}$ out of 12 balls, we divide the 12 balls into 3 equal groups/parts.



Each group/part contains 4 balls

Step 2. Now, we shade 2 groups/parts out of 3 parts.



On counting, we find that the total number of shaded balls are 8.

In other words, $\frac{2}{3}$ of 12 balls = 8 balls

i. e. $\frac{2}{3} \times 12 = \frac{2 \times 12}{3} = \frac{24}{3} = 24 \div 3 = 8$ balls

Example 6:- (i) What is $\frac{1}{4}$ of 16? (ii) What is $\frac{2}{5}$ of 20?

(iii) What is $\frac{3}{4}$ of 24?

Solution : (i) $\frac{1}{4}$ of $16 = \frac{1}{4} \times 16 = \frac{1 \times 16}{4} = \frac{16}{4} = 16 \div 4 = 4$

(ii) $\frac{2}{5}$ of $20 = \frac{2}{5} \times 20 = \frac{2 \times 20}{5} = \frac{40}{5} = 40 \div 5 = 8$

(iii) $\frac{3}{4}$ of $24 = \frac{3}{4} \times 24 = \frac{3 \times 24}{4} = \frac{72}{4} = 72 \div 4 = 18$

Example 7: Jayant has 24 oranges. he ate $\frac{1}{6}$ of them, then

(i) How many oranges he ate?

(ii) How many does he have left?

Solution : Total oranges = 24

(i) Oranges he ate = $\frac{1}{6}$ of $24 = \frac{1}{6} \times 24 = \frac{1 \times 24}{6} = \frac{24}{6} = 24 \div 6 = 4$ oranges

(ii) Number of oranges left out = 24 – 4 = 20 oranges

Example 8: Jasmine has a packet of 20 biscuits. She gave $\frac{1}{4}$ of them to Harneet and $\frac{3}{5}$ of them to Sophia. The rest she keeps.

(i) How many biscuits does Harneet have?

(ii) How many biscuits does Sophia have?

(iii) How many biscuits does Jasmine keep?

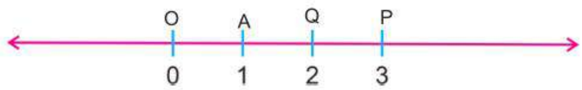
Solution : (i) Harneet have biscuits = $\frac{1}{4}$ of $20 = \frac{1}{4} \times 20 = \frac{1 \times 20}{4} = \frac{20}{4} = 20 \div 4 = 5$ biscuits

(ii) Sophia have biscuits = $\frac{3}{5}$ of $20 = \frac{3}{5} \times 20 = \frac{3 \times 20}{5} = \frac{60}{5} = 60 \div 5 = 12$ biscuits

(iii) Jasmine keep biscuits = 20 – 5 – 12 = 3 biscuits

5.3 Fractions on Number Line

In previous chapter, we have learnt about the representation of whole numbers on a number line. To represent whole numbers on a number line, we draw a straight line and mark point 0 on it. Now starting from 0, mark points, A,Q,P etc, on the line to the right of 0 at equal distance. These points represent 1, 2, 3 etc and O represents 0.

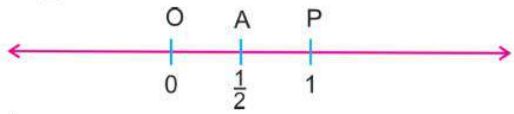


We can represent fractions on number line.

- **In order to represent $\frac{1}{2}$ on the number line:-** Draw the number line and mark a point P to represent 1.

  Now divide the gap between O and P into two equal parts. Let A be the point of division.

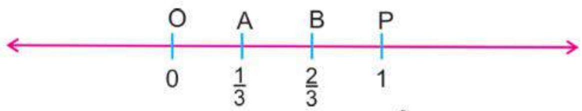
  then A represents $\frac{1}{2}$.



- **To represent $\frac{1}{3}$ on the number line:-** Draw the number line and mark a point P to represent 1.

  Now divide the gap between O and P into three equal parts. Let A and B be the points of division.

  Then A represents $\frac{1}{3}$ and B represents $\frac{2}{3}$.



* By using the same logic, point O represents $\frac{0}{3}$ and point P represents $\frac{3}{3}$

Thus we have $\frac{0}{3} = 0$ and $\frac{3}{3} = 1$

There are infinite fractions between 0 and 1.

Example 9: (i) **Represent $\frac{4}{5}$ on a number line.**

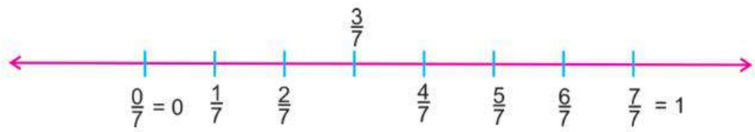
(ii) **Represent $\frac{3}{7}$ on a number line.**

Solution : (i) In order to represent $\frac{4}{5}$ on a number line. We divide the gap between 0 and 1 into 5 equal parts, which are $\frac{0}{5}=0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$ and $\frac{5}{5}=1$ (as shown)

Then we mark fourth point from 0 to the right (as shown)

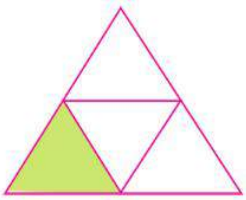
(ii) In order to represent $\frac{3}{7}$ on a number line we divide the gap between 0 and 1 into 7 equal parts, which are $\frac{0}{7}=0, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{7}{7}$ (as shown)

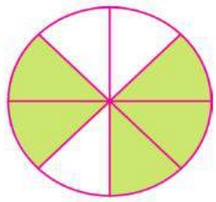
Then, we mark third point from 0 to the right (as shown)

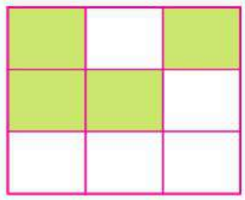


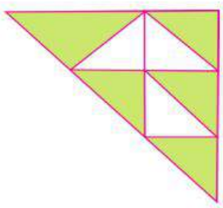
Exercise 5.1

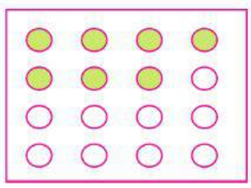
1. Write the fraction representing the shaded portion:-

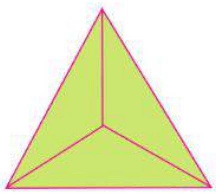
(i)


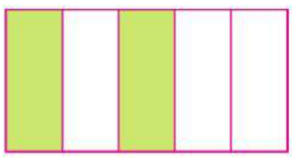
(ii)


(iii)


(iv)


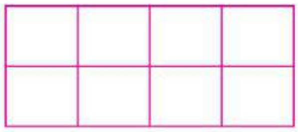
(v)


(vi)


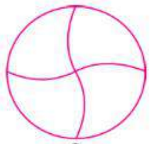
(vii)


(viii)

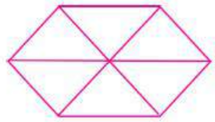

2. Colour the part according to the given fraction:-

(i) 

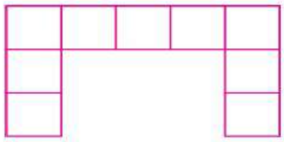
$\frac{5}{8}$

(ii) 

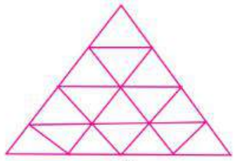
$\frac{3}{4}$

(iii) 

$\frac{2}{6}$

(iv) 

$\frac{7}{9}$

(v) 

$\frac{11}{16}$

3. Write the fraction for each of the folloiwng:-

(i) Three-Fourth (ii) Seven-Tenth (iii) A Quarter

(iv) Five-Eighth (v) Three-Twelvth

4. Write the fraction for the followings:-

(i) numerator = 5, denominator = 9 (ii) numerator = 2, denominator = 11

(iii) numerator = 6, denominator = 7

5. Write the numerator and the denominator for the followings:-

(i) $\frac{2}{3}$ (ii) $\frac{1}{4}$ (iii) $\frac{5}{11}$ (iv) $\frac{9}{13}$ (v) $\frac{17}{16}$

6. Express:-

(i) 1 day as a fraction of 1 week.

(ii) 40 seconds as a fraction of 1 minute.

(iii) 15 hours as a fraction of 1 day.

(iv) 2 months as a fraction of 1 year.

(v) 45cm as a fraction of 1 metre.

7. Write the numbers from 1 to 25.

(i) What fraction of them are even numbers?

(ii) What fraction of them are prime numbers?

(iii) What fraction of them are multiples of 3?

8. In class 6^{th}, there are 24 boys and 18 girls. What fraction of total students represent boys and girls.

9. A bag contians 6 red balls and 7 blue balls. What fraction of balls represent red and blue colour?

10. Sidharth has a cake. He cuts it into 10 equal parts. He gave 2 parts to Naman, 3 parts to Nidhi, 1 parts to Seema and the remaining four parts he kept for himself. Find

(i) What fraction of cake, he gave to Naman?

(ii) What fraction of cake, he gave to Nidhi?

(iii) What fraction of cake, he kept for himself?

(iv) Who has more cake than others?

11. In a box, there are 12 apples, 7 oranges and 5 guavas. What fraction of fruits in box represents each?

12. Dishmeet has 20 pens. He gives one-fourth to Balkirat. How many pens Dishmeet and Balkirat have?

13. Represent the following fraction on the number line?

(i) $\frac{2}{5}$ (ii) $\frac{5}{7}$ (iii) $\frac{3}{10}, \frac{5}{10}, \frac{1}{10}$ (iv) $\frac{3}{8}, \frac{5}{8}, \frac{7}{8}$

14. Find:-

(i) $\frac{3}{5}$ of 20 books (ii) $\frac{5}{8}$ of 32 pens (iii) $\frac{1}{6}$ of 36 Copies

(iv) $\frac{4}{7}$ of 21 apples (v) $\frac{3}{4}$ of 28 pencils

15. Balkirat had a box of 36 erasers. He gave $\frac{1}{2}$ of them to Rani, $\frac{2}{9}$ of them to Yuvraj and keeps the rest.

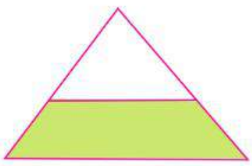
(i) How many erasers does Rani get?

(ii) How many erasers does Yuvraj get?

(iii) How many erasers does Harnik keep?

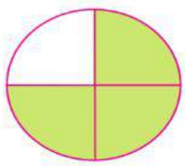
16. State True/False.

(i)



$\frac{1}{2}$

(ii)

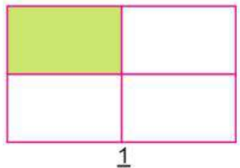


$\frac{3}{4}$

(iii)

$\frac{2}{3}$

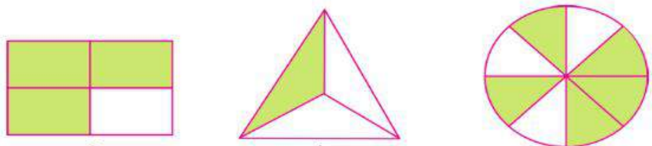
(iv)



$\frac{1}{4}$

5.4 Types of Fractions

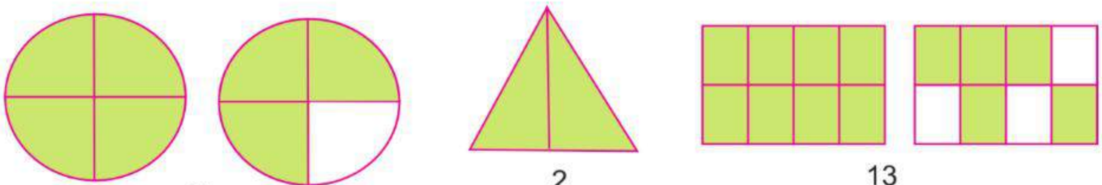
Proper Fraction:- Fractions whose numerator is less than the denominator are called the proper fractions.



$\frac{3}{4}$ $\frac{1}{3}$ $\frac{5}{8}$

These are proper fractions.

Improper Fraction:- Fractions whose numerator is either equal or greater than the denominator are called improper fractions.

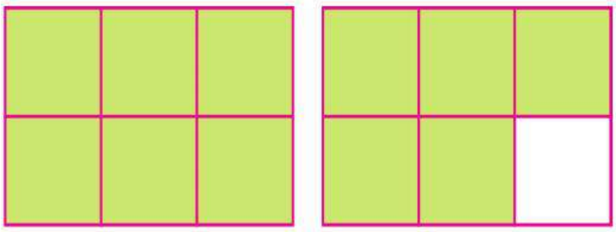


$\frac{7}{8}$ $\frac{2}{2}$ $\frac{13}{16}$

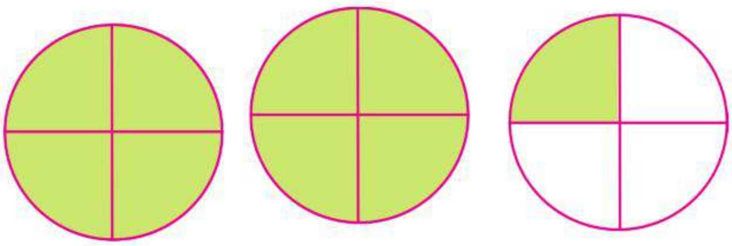
* **Improper fractions are converted to a mixed fraction.**

Mixed Fraction:- A fraction which is a combination of a whole number and a proper fraction is called a mixed fraction.

Following are mixed fractions:



This is $\frac{11}{6} = 1 + \frac{5}{6}$ is written as $1\frac{5}{6}$



This is $\frac{9}{4} = 2 + \frac{1}{4}$ and is written as $2\frac{1}{4}$

5.4.1 Conversion of Improper fractions into Mixed fractions

Step 1. Obtain the Improper fraction

Step 2. Divide the numerator by the denominator and obtain the quotient and the remainder

Step 3. Write the mixed fraction as:

$$\text{Quotient}\ \frac{\text{Remainder}}{\text{Denominator}} \qquad \text{Or} \qquad \text{Quotient}\ \frac{\text{Remainder}}{\text{Divisor}}$$

5.4.2 Conversion of Mixed fractions into Improper fractions

Step1. Obtain Mixed fraction which is in form of

$$\text{Quotient}\ \frac{\text{Remainder}}{\text{Divisor}}\ \text{Or}\ (\text{Quotient}) + \frac{\text{Remainder}}{\text{Divisor}}$$

Step 2. Write the fraction having numerator equal to the (Quotient × Divisor + Remainder) and denominator same as divisor in step 1.

$$\text{Thus, Improper fraction} = \frac{\text{Quotient} \times \text{Denominator} + \text{Remainder}}{\text{Denominator}}$$

Example 10:- Classify the following as proper and Improper fractions:

$$\frac{2}{5}, \frac{7}{8}, \frac{11}{5}, \frac{6}{11}, \frac{9}{4}, \frac{5}{13}, \frac{6}{6}$$

Solution : Proper fractions : $\frac{2}{5}, \frac{7}{8}, \frac{6}{11}, \frac{5}{13}$

Improper fractions : $\frac{11}{5}, \frac{9}{4}, \frac{6}{6}$

Example 11:- Express each of the following as mixed fractions:-

(i) $\frac{16}{5}$ **(ii)** $\frac{19}{4}$ **(iii)** $\frac{28}{3}$

Solution : (i) $\frac{16}{5}$

Divisor 5) 16 (3 Quotient
– 15
1 Remainder

$$\therefore \text{Mixed fraction} = \text{Quotient}\ \frac{\text{Remainder}}{\text{Divisor}}$$

$$= 3\frac{1}{5}$$

Aliter :- $\frac{16}{5} = \frac{5+5+5+1}{5} = \frac{5}{5}+\frac{5}{5}+\frac{5}{5}+\frac{1}{5}$

$= 1 + 1 + 1 + \frac{1}{5} = 3\frac{1}{5}$

(ii) $\frac{19}{4}$ Divisor 4) 19 (4 Quotient

-16

3 Remainder

$\therefore$ Mixed fraction = Quotient $\frac{\text{Remainder}}{\text{Divisor}} = 4\frac{3}{4}$

(iii) $\frac{28}{3}$ Divisor 3) 28 (9 Quotient

-27

1 Remainder

$\therefore$ Mixed fraction = Quotient $\frac{\text{Remainder}}{\text{Divisor}} = 9\frac{1}{3}$

Example 12: Express each of the following mixed fractions as improper fractions:-

(i) $5\frac{3}{4}$ **(ii)** $7\frac{2}{5}$ **(iii)** $8\frac{2}{7}$

Solution : (i) $5\frac{3}{4} = \frac{5\times4+3}{4} = \frac{20+3}{4} = \frac{23}{4}$

(ii) $7\frac{2}{5} = \frac{7\times5+2}{5} = \frac{35+2}{5} = \frac{37}{5}$

(iii) $8\frac{2}{7} = \frac{8\times7+2}{7} = \frac{56+2}{7} = \frac{58}{7}$

Exercise 5.2

1. Classify the following as proper and improper fractions:-

$\frac{5}{4}, \frac{9}{13}, \frac{6}{11}, \frac{3}{2}, \frac{5}{2}, \frac{6}{6}, \frac{7}{9}, \frac{2}{15}, \frac{4}{17}, \frac{7}{8}$

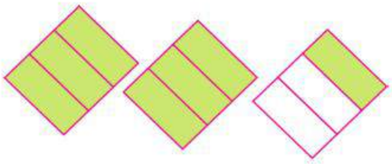
2. Express each of the following as mixed fractions, Also represent with diagrams.

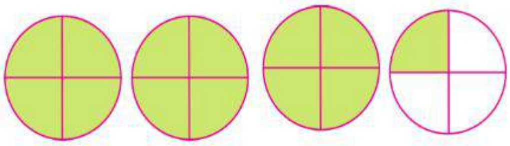
(i) $\frac{27}{5}$ (ii) $\frac{13}{4}$ (iii) $\frac{43}{8}$ (iv) $\frac{51}{7}$ (v) $\frac{20}{3}$

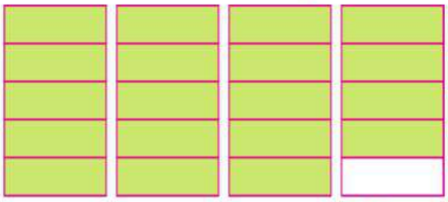
3. Express each of the following mixed fractions as improper fractions:-

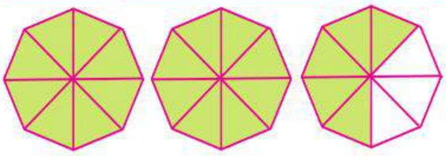
(i) $2\frac{1}{3}$ (ii) $5\frac{2}{7}$ (iii) $4\frac{3}{5}$ (iv) $3\frac{3}{4}$ (v) $9\frac{5}{8}$

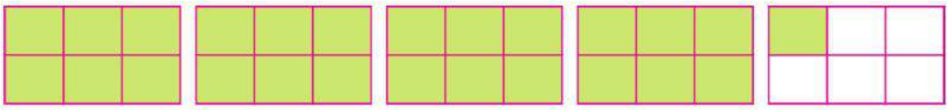
4. Express the shaded portion as Improper fraction and Mixed fraction:-

(i) 

(ii) 

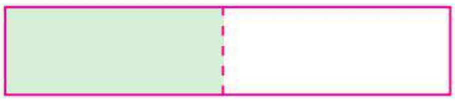
(iii) 

(iv) 

(v) 

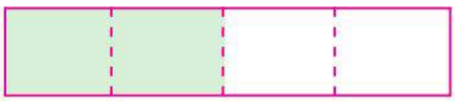
5.5 Equivalent Fractions

- Fold the piece of paper into two equal parts. Unfold the paper Colour one of the parts.



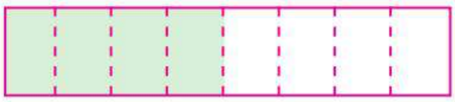
  Each part represents $\frac{1}{2}$ of the paper.

- Now, fold it again and then unfold.



  The coloured part $\frac{1}{2}$ now represents $\frac{2}{4}$ of the paper.

- Now, fold it again and then unfold.



  The coloured part now represents $\frac{4}{8}$ of the paper.

We can observed from above that $\frac{1}{2}, \frac{2}{4}$ and $\frac{4}{8}$ all represent the same part of the paper.

Such fractions are called Equivalent fractions.

- **Equivalent fractions are the fractions which represent the same value.**

Look at the above activity, we have equivalent fractions, $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}$

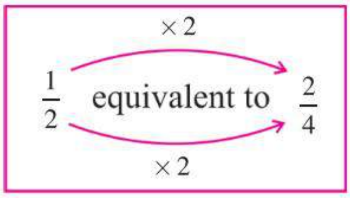
Numerator and denominator of $\frac{2}{4}$ are twice the numerator and denominator of $\frac{1}{2}$

i.e. $\frac{2}{4} = \frac{1 \times 2}{2 \times 2}$

Similarly numerator and denominator of $\frac{4}{8}$ are four times the numerator and denominator of $\frac{1}{2}$.

i.e. $\frac{4}{8} = \frac{1 \times 4}{2 \times 4}$

We observe that $\frac{2}{4}, \frac{4}{8}$ are obtained by multiplying the numerator and denominator of $\frac{1}{2}$ by 2 and 4 respectively.



Thus an equivalent fraction of a given fraction can be obtained by multiplying its numerator and denominator by the same number (other than zero),

Further $\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$

$\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$

We observe that if we divide the numerator and denominators of $\frac{2}{4}, \frac{4}{8}$ each by their common factors i.e. 2 and 4 respectively, we get an equivalent fraction $\frac{1}{2}$.

Thus an equivalent fraction of a given fraction can be obtained by dividing its numerator and denominator by their common factors (other than 1)

So in above discussion, we can find equivalent fractions of any given fraction by

* Mulitiplying its numerator and denominator by same number (other than 0)
* Dividing its numerator and denominator by their common factor (other than 1).

Example 13:- Find three equivalent fractions of the followings:-

(i) $\frac{2}{5}$ (ii) $\frac{3}{4}$ (iii) $\frac{7}{9}$

Solution : (i) Equivalent fractions of $\frac{2}{5}$ are

$$\frac{2\times 2}{5\times 2}=\frac{4}{10}\ ;\ \frac{2\times 3}{5\times 3}=\frac{6}{15}\ ;\ \frac{2\times 4}{5\times 4}=\frac{8}{20}$$

(ii) Equivalent fractions of $\frac{3}{4}$ are

$$\frac{3\times 2}{4\times 2}=\frac{6}{8}\ ;\ \frac{3\times 3}{4\times 3}=\frac{9}{12}\ ;\ \frac{3\times 4}{4\times 4}=\frac{12}{16}$$

(iii) Equivalent fractions of $\frac{7}{9}$ are

$$\frac{7\times 2}{9\times 2}=\frac{14}{18}\ ;\ \frac{7\times 3}{9\times 3}=\frac{21}{27}\ ;\ \frac{7\times 4}{9\times 4}=\frac{28}{36}$$

Example 14: Write the lowest equivalent fractions (Simplest form) of:-

(i) $\frac{9}{15}$ (ii) $\frac{24}{32}$ (iii) $\frac{60}{75}$ (iv) $\frac{20}{36}$ (v) $\frac{56}{84}$

Solution : (i) $\frac{9}{15}=\frac{9\div 3}{15\div 3}=\frac{3}{5}$

[Divide the numerator and denominator by HCF of 9 and 15 i.e. 3]

(ii) $\frac{24}{32}=\frac{24\div 8}{32\div 8}=\frac{3}{4}$

[Divide the numerator and denominator by HCF of 24 and 32 i.e. 8]

(iii) $\frac{60}{75}=\frac{60\div 15}{75\div 15}=\frac{4}{5}$

[Divide the numerator and denominator by HCF of 60 and 75 i.e. 15]

(iv) $\frac{20}{36}=\frac{20\div 4}{36\div 4}=\frac{5}{9}$

[Divide the numerator and denominator by HCF of 20 and 36 i.e. 4]

(v) $\frac{56}{84}=\frac{56\div 28}{84\div 28}=\frac{2}{3}$

[Divide the numerator and denominator by HCF of 56 and 84]

> * A fraction is said to be in lowest terms or simplest form if there is no common factor (other than 1) between its numerator and denominator.

5.4.1 Checking for Equivalent Fractions (by Cross product)

If the product (multiplication) of the numerator of the first and the denominator of the second is equal to the product of the denominator of the first and the numerator of the Second than the given fractions are said to be equivalent condsider an example.

$$\frac{2}{7} \times \frac{6}{21}$$

$2 \times 21 = 42$ and $6 \times 7 = 42$

Both the cross products are same.

Thus two fractions are equivalent if their cross products are equal

Example 15: Are the following fractions equivalent?

(i) $\frac{3}{4}$ **and** $\frac{12}{16}$ **(ii)** $\frac{5}{6}$ **and** $\frac{25}{30}$ **(iii)** $\frac{4}{7}$ **and** $\frac{24}{27}$

Solution : (i) We have $\frac{3}{4} \times \frac{12}{16}$

By cross product, $3 \times 16 = 48$ and $12 \times 4 = 48$

Since two products are same.

So, the given fractions are equivalent.

(ii) We have $\frac{5}{6} \times \frac{25}{30}$

By cross product, $5 \times 30 = 150$ and $25 \times 6 = 150$

Since two products are same.

So, the given fractions are equivalent.

(iii) We have $\frac{4}{7} \times \frac{24}{27}$

By cross product, $4 \times 27 = 108$ and $24 \times 7 = 168$

Since two products are not same

So, the given fractions are not equivalent

Example 16: Replace □ in each of the following by the correct numbers:-

(i) $\frac{3}{4} = \frac{15}{\square}$ **(ii)** $\frac{2}{5} = \frac{\square}{30}$ **(iii)** $\frac{20}{28} = \frac{5}{\square}$

Solution : (i) Observe the numerators, we have $15 \div 3 = 5$

So we multiply both numerator and denominator of $\frac{3}{4}$ by 5.

i.e $\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$

Aliter :- We can solve it by cross product also.

$$\frac{3}{4} = \frac{15}{\square}$$

Let x be the correct number in $\square$.

$\Rightarrow \quad \frac{3}{4} \times \frac{15}{x} \quad \Rightarrow 3 \times x = 15 \times 4$

$$\Rightarrow x = \frac{15 \times 4}{3} = 20$$

(ii) Observe the denominators, we have $30 \div 5 = 6$

So, we multiply the numerator and denominator of $\frac{2}{5}$ by 6

i.e. $\frac{2}{5} = \frac{2 \times 6}{5 \times 6} = \frac{12}{30}$

(iii) Observe the numerators, we have $20 \div 5 = 4$

So, we divide the numerator and denominator of $\frac{20}{28}$ by 4

i.e. $\frac{20}{28} = \frac{20 \div 4}{28 \div 4} = \frac{5}{7}$

Example 17: **(i)** **Write a fraction equivalent to $\frac{2}{3}$ with numerator 16.**

(ii) **Write a fraction equivalent to $\frac{5}{7}$ with denominator 28.**

(iii) **Write a fraction equivalent to $\frac{30}{45}$ with numerator 6.**

Solution : (i) We have to find the equivalent fraction to $\frac{2}{3}$ with numerator 16.

or $\frac{2}{3} = \frac{16}{\square}$

Observe the numerators, we have $16 \div 2 = 8$

So we multiply the numerator and denominator of $\frac{2}{3}$ by 8

i.e $\frac{2}{3} = \frac{2 \times 8}{3 \times 8} = \frac{16}{24}$

Hence, equivalent fraction is $\frac{16}{24}$.

(ii) We have to find the equivalent fraction to $\frac{5}{7}$ with denominator 28.

or $\frac{5}{7} = \frac{\square}{28}$

Observe the denominators, we have $28 \div 7 = 4$

So, we multiply the numerator and denomiator of $\frac{5}{7}$ by 4.

i.e $\frac{5}{7} = \frac{5 \times 4}{7 \times 4} = \frac{20}{28}$

Hence equivalent fraction is $\frac{20}{28}$.

(iii) We have to find the equivalent fraction to $\frac{30}{45}$ with numerator 6.

or $\frac{30}{45} = \frac{6}{\square}$

Observe the numerators, we have $30 \div 6 = 5$

So we divide the numerator and denominator by $\frac{30}{45}$ by 5.

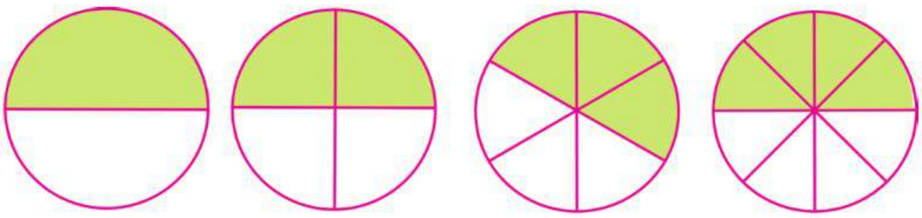
i.e $\frac{30}{45} = \frac{30 \div 5}{45 \div 5} = \frac{6}{9}$

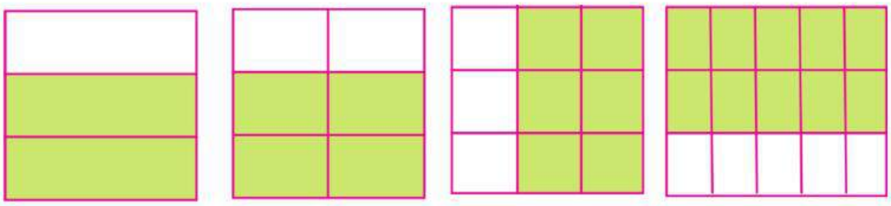
Hence, Equivalent fraction in $\frac{6}{9}$.

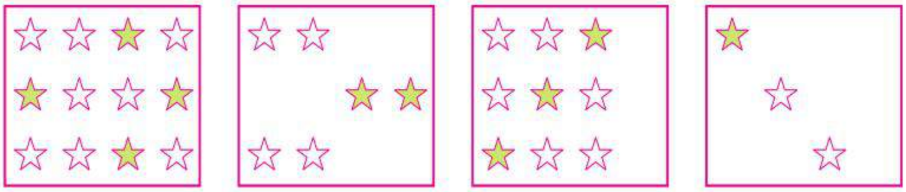
Exercise 5.3

1. Write the fraction for the shaded part and check whether these fractions are equivalent or not?

(i)



(ii) 

(iii) 

2. Find four equivalent fractions of the followings:-

(i) $\frac{1}{4}$ (ii) $\frac{3}{5}$ (iii) $\frac{7}{9}$ (iv) $\frac{5}{11}$ (v) $\frac{2}{3}$

3. Write the lowest equivalent fraction (simplest form) of:-

(i) $\frac{10}{25}$ (ii) $\frac{27}{54}$ (iii) $\frac{48}{72}$ (iv) $\frac{150}{60}$ (v) $\frac{162}{90}$

4. Are the following fractions equivalent or not?

(i) $\frac{5}{12}, \frac{25}{60}$ (ii) $\frac{6}{7}, \frac{36}{42}$ (iii) $\frac{7}{9}, \frac{56}{72}$

5. Replace $\square$ in each of the following by the correct number.

(i) $\frac{2}{7} = \frac{12}{\square}$ (ii) $\frac{5}{8} = \frac{35}{\square}$ (iii) $\frac{24}{36} = \frac{6}{\square}$ (iv) $\frac{30}{48} = \frac{\square}{8}$ (v) $\frac{7}{4} = \frac{42}{\square}$

6. Find the equivalent fraction of $\frac{3}{5}$, having

(i) numerator 18 (ii) denominator 20 (iii) numerator 24

7. Find the equivalent fraction of $\frac{24}{40}$, having

(i) numerator 6 (ii) numerator 48 (iii) denominator 20

5.6 Like, Unlike Fractions and Unit Fractions

Like Fraction:- Fractions with the same denominators are called like fractions

e.g. $\frac{5}{7}, \frac{1}{7}, \frac{3}{7}$ are like fractions.

Unlike Fractions:- Fractions having different denominators are called unlike fractions

e.g. $\frac{2}{5}, \frac{3}{4}, \frac{7}{8}$ etc. are unlike fractions.

* **Unit Fractions:-** A fraction having 1 as its numerator is called unit fraction

e.g. $\frac{1}{3}, \frac{1}{2}, \frac{1}{7}, \frac{1}{11}, \frac{1}{8}$ etc.

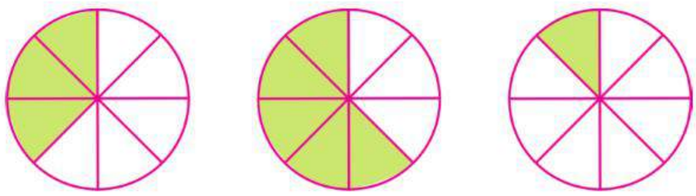
5.7 Comparing and Ordering of Fractions

We have already learnt how to compare natural numbers, whole number and integers. Here we will learn to compare the fractions, which are divided into three categories:-

5.7.1 Fractions with the Same Denominator :

Let us consider some fractions with the same denominator $\frac{3}{8}, \frac{5}{8}, \frac{1}{8}$

The pictorial representation of these fractions are given below:



$\frac{3}{8}$ $\frac{5}{8}$ $\frac{1}{8}$

A B C

By observing pictures,

we can conclude that shaded part of B > shaded part of A > shaded part of C

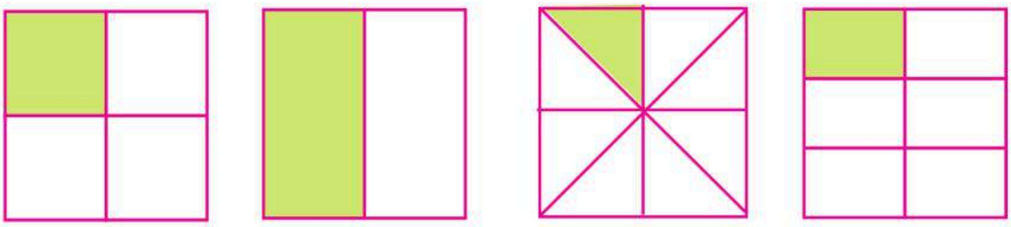
i.e $\frac{5}{8} > \frac{3}{8} > \frac{1}{8}$

Thus, If two or more fractions having the same denominator then the fraction with greater numerator is greater fraction.

5.7.2 Fraction with the same Numerator

Let us consider some fractions with the same numerator $\frac{1}{4}, \frac{1}{2}, \frac{1}{8}, \frac{1}{6}$

The pictorial representation of these fractions are given below:



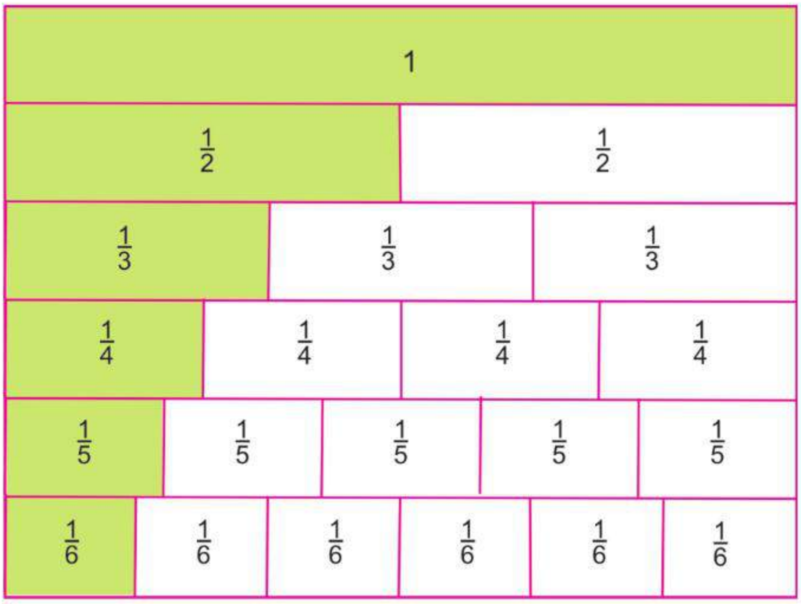
$\frac{1}{4}$ (A) $\frac{1}{2}$ (B) $\frac{1}{8}$ (C) $\frac{1}{6}$ (D)

By observing pictures, we can conclude that shaded part of B> Shaded part of A> shaded part of D> shaded part of C

i.e $\frac{1}{2} > \frac{1}{4} > \frac{1}{6} > \frac{1}{8}$

Thus, If two or more fractions having the same numerator then the fraction with a small denominator is greater.

Representation of equivalent fractions.



In this way we can conclude that If two or more fractions have same numerator then the fractions with smaller denominator is greater.

i.e $1 > \frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \frac{1}{5} > \frac{1}{6}$

This activity can be performed by using chart and colours.

5.7.3 Fraction with different numerators and denominators

To compare the fractions with different numerators and denominators. First we convert that fraction into like fractions using the following steps:

Step 1: Find the LCM of the denominators of the fractions.

Step 2: Convert each fraction into its equivalent fraction with denominator equal to the LCM obtained in Step 1.

Step 3: Compare the numerators of the equivalent fractions whose denominator are same.

Example 18: Which is larger $\frac{2}{3}$ or $\frac{5}{6}$?

Solution : First find the LCM of 3 and 6,

Now we convert the given fraction into a fraction with denominator 6,

We have $\frac{2}{3}=\frac{2\times2}{3\times2}=\frac{4}{6}$ and

second fraction is $\frac{5}{6}$ clearly $\frac{4}{6}<\frac{5}{6} \Rightarrow \frac{2}{3}<\frac{5}{6}$

3	3, 6
2	1, 2
	1, 1

$\therefore$ LCM of 3 and 6 is $3 \times 2 = 6$

Aliter :-

If two fractions are there then we can compare the fractions with cross-product also

$\frac{2}{3}$ $\frac{5}{6}$

$2 \times 6 = 12$ and $5 \times 3 = 15$

So $12 < 15 \Rightarrow \frac{2}{3}<\frac{5}{6}$

Example 19: Arrange the fractions in $\frac{3}{4},\frac{7}{8},\frac{15}{12}$ descending order.

Solution : First find the LCM of denominator 4, 8 and 12.

Now, we convert the given fractions into a fraction with denominator 24, we have

$$\frac{3}{4}=\frac{3\times6}{4\times6}=\frac{18}{24}$$

$$\frac{7}{8}=\frac{7\times3}{8\times3}=\frac{21}{24}$$

and $$\frac{5}{12}=\frac{5\times2}{12\times2}=\frac{10}{24}$$

2	4, 8, 12
2	2, 4, 6
2	1, 2, 3
3	1, 1, 3
	1, 1, 1

LCM = $2 \times 2 \times 2 \times 3 = 24$

$\Rightarrow \quad \frac{21}{24} > \frac{18}{24} > \frac{10}{24} \qquad \Rightarrow \quad \frac{7}{8} > \frac{3}{4} > \frac{5}{12}$

Example 20: Arrange the following fractions in ascending order $\frac{2}{3}, \frac{5}{9}, \frac{3}{5}, \frac{7}{15}$

Solution : First find LCM of denominators 3, 9, 5 and 15

Now, we convert the given fractions into fractions with denominators 45, we have

3	3, 9, 5, 15
3	1, 3, 5, 5
5	1, 1, 5, 5
	1, 1, 1, 1

LCM = 3 × 3 × 5 = 45

$\frac{2}{3} = \frac{2 \times 15}{3 \times 15} = \frac{30}{45}$

$\frac{5}{9} = \frac{5 \times 5}{9 \times 5} = \frac{25}{45}$

$\frac{3}{5} = \frac{3 \times 9}{5 \times 9} = \frac{27}{45}$

$\frac{7}{15} = \frac{7 \times 3}{15 \times 3} = \frac{21}{45}$

∴ Ascending order is

$\Rightarrow \quad \frac{21}{45} < \frac{25}{45} < \frac{27}{45} < \frac{30}{45}$

$\Rightarrow \quad \frac{7}{15} < \frac{5}{9} < \frac{3}{5} < \frac{2}{3}$

Example 21: A boy reads $\frac{2}{5}$ of a book on the first day and $\frac{1}{4}$ of the same book on the second day on which day did he read the major part of the book?

Solution : Here, we have to compare both fractions $\frac{2}{5}$ and $\frac{1}{4}$ and find which is larger

By Cross Product $\frac{2}{5} \times \frac{1}{4}$

$2 \times 4 = 8$ and $1 \times 5 = 5$

So $8 > 5 \quad \Rightarrow \quad \frac{2}{5} > \frac{1}{4}$

Hence, he read the major part of the book on the first day.

Example 22: Arun exercised for $\frac{3}{4}$ of an hour while Jaspreet exercised for $\frac{3}{10}$ of an hour. Who exercised for longer time?

Solution : Arun exercised = $\frac{3}{4}$ of an hour

Jaspreet exercised = $\frac{3}{10}$ of an hour

Clearly, In both fractions, numerators are same.

So fraction with smaller denominator is larger.

i.e $\frac{3}{4} > \frac{3}{10}$

Hence, Arun exercised for a longer time.

Aliter :-

Arun exercised $= \frac{3}{4}$ of an hour $= \frac{3}{4} \times 60$ minutes [$\because$ 1 hours = 60 minutes]

$=$ 45 minutes

Jaspreet exercised $= \frac{3}{10}$ of an hour $= \frac{3}{10} \times 60$ minutes

$=$ 18 minutes

$\Rightarrow$ 45 > 18

Hence Arun exercised for a longer time.

Exercise 5.4

1. Find the different set of like fractions:

$\frac{3}{7}, \frac{5}{11}, \frac{2}{7}, \frac{6}{13}, \frac{3}{11}, \frac{1}{11}, \frac{2}{13}, \frac{5}{13}, \frac{6}{7}, \frac{10}{13}$

2. Write any three like fractions of:-

(i) $\frac{2}{5}$ (ii) $\frac{1}{4}$ (iii) $\frac{11}{6}$

3. Encircle unit fractions:-

$\frac{6}{11}, \frac{2}{3}, \frac{1}{8}, \frac{15}{7}, \frac{1}{9}, \frac{1}{7}, \frac{3}{3}$

4. Fill in the boxes with >, < or =

(i) $\frac{4}{7} \square \frac{6}{7}$ (ii) $\frac{4}{5} \square \frac{3}{5}$ (iii) $\frac{7}{8} \square \frac{0}{8}$ (iv) $\frac{2}{3} \square \frac{5}{3}$ (v) $\frac{5}{13} \square \frac{7}{13}$

5. Compare using >, < or =

(i) $\frac{5}{7} \square \frac{5}{9}$ (ii) $\frac{1}{3} \square \frac{1}{2}$ (iii) $\frac{6}{11} \square \frac{6}{13}$ (iv) $\frac{11}{12} \square \frac{11}{17}$ (v) $\frac{7}{13} \square \frac{7}{10}$

6. Compare using > , < or =

(i) $\frac{5}{6} \square \frac{2}{5}$ (ii) $\frac{3}{4} \square \frac{1}{3}$ (iii) $\frac{3}{7} \square \frac{5}{9}$ (iv) $\frac{7}{10} \square \frac{4}{5}$ (v) $\frac{7}{7} \square 1$

7. Arrange the following fractions in ascending order:-

(i) $\frac{7}{10}, \frac{3}{10}, \frac{5}{10}$ (ii) $\frac{6}{7}, \frac{1}{7}, \frac{4}{7}$ (iii) $\frac{5}{8}, \frac{7}{8}, \frac{1}{8}, \frac{3}{8}$ (iv) $\frac{5}{7}, \frac{5}{9}, \frac{5}{3}$

(v) $\frac{3}{11}, \frac{3}{7}, \frac{3}{13}$ (vi) $\frac{1}{4}, \frac{1}{6}, \frac{5}{12}$ (vii) $\frac{2}{7}, \frac{11}{35}, \frac{9}{14}, \frac{13}{28}$ (viii) $\frac{1}{3}, \frac{4}{9}, \frac{5}{12}, \frac{4}{15}$

8. Arrange the following fractions in descending order:-

(i) $\frac{5}{9}, \frac{7}{9}, \frac{1}{9}$ (ii) $\frac{3}{11}, \frac{5}{11}, \frac{2}{11}, \frac{7}{11}$ (iii) $\frac{2}{7}, \frac{2}{13}, \frac{2}{9}$

(iv) $\frac{1}{5}, \frac{1}{3}, \frac{1}{8}, \frac{1}{2}$ (v) $\frac{1}{6}, \frac{5}{12}, \frac{5}{18}, \frac{2}{3}$ (vi) $\frac{3}{4}, \frac{9}{20}, \frac{11}{15}, \frac{17}{30}$

9. Kasvi covered $\frac{1}{3}$ of her journey by car, $\frac{1}{5}$ by rickshaw and $\frac{2}{15}$ on foot. Find by which means, she covered the major part of her journey.

10. Father distributed his property among his three sons. The eldest one got $\frac{3}{10}$, the middle got $\frac{1}{6}$ and the youngest got $\frac{1}{5}$ part of the property. State how the property was distribued in ascending order.

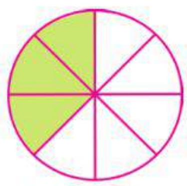
5.8 Operations on Fractions

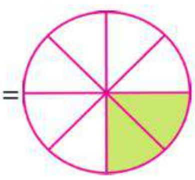
In the previous section, we have learnt the like, unlike fractions and their comparison. In this section, we shall learn about addition and subtraction of the fractions. We follow certain methods for doing these operations:

5.8.1 Addition and Subtraction of like fractions

- **Addition:-** Look at the adjoining figure, there are 8 parts in a circle Let us represent

$\frac{3}{8} + \frac{2}{8}$

$\frac{3}{8} =$ 

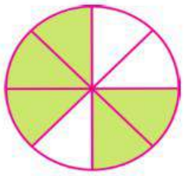
$\frac{2}{8} =$ 

Shade 1:- Out of 8, 3 parts are shaded green.

Shade 2:- Out of 8, 2 parts are shaded green.

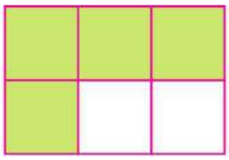
Count the total number of shaded parts there are 5 out of 8 i.e. $\frac{5}{8}$

i.e $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

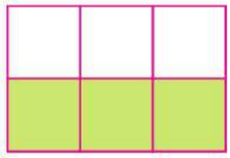


- Now Let us represent $\frac{4}{6} + \frac{3}{6}$

$\frac{4}{6} =$



, $\frac{3}{6} =$

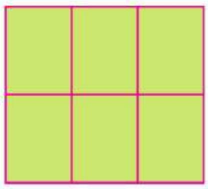


Shade 1:- Out of 6, 4 boxes are shaded with green colour.

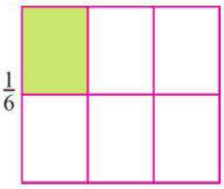
Shade 2:- Here, we are to shade 3 parts out of 6, but we have 2 boxes left and we cannot shade more than 2. So we make one more same size figure with 6 parts and shade the remaining1 in those parts.

ie. $\frac{4}{6} + \frac{3}{6} = \frac{7}{6} = 1\frac{1}{6}$

$\frac{6}{6} = 1$



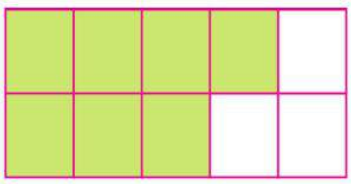
$\frac{1}{6}$



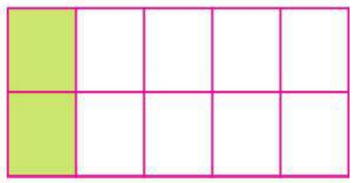
- **Subtraction of Like fractions:-** Let us subtract $\frac{2}{10}$ from $\frac{7}{10}$

i.e $\frac{7}{10} - \frac{2}{10}$

$\frac{7}{10} =$

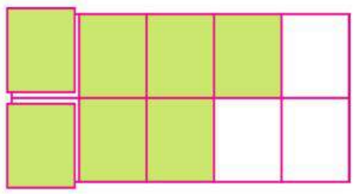


$\frac{2}{10} =$



- Shade 7 parts out of 10 and 2 out of 10 as shown
- Take away 2 shaded parts out of 10.

Now, the remainder shaded parts are 5 thus $\frac{7}{10} - \frac{2}{10} = \frac{5}{10}$



Thus the sum and difference of two or more like fractions can be obtained as follows:-

Step 1:- Add/Subtract the numerators of all fractions.

Step 2:- Retain the common denominator of all fractions.

Step 3:- Write the fraction as $\frac{\text{Sum / difference of numerators}}{\text{Common denominator}}$

Example 23: Simplity:-

(i) $\frac{3}{10}+\frac{4}{10}$ (ii) $\frac{5}{11}+\frac{2}{11}+\frac{1}{11}$ (iii) $\frac{5}{14}+\frac{8}{14}+\frac{2}{14}$

(iv) $\frac{5}{8}-\frac{3}{8}$ (v) $\frac{4}{7}+\frac{5}{7}-\frac{6}{7}$ (vi) $\frac{8}{9}-\frac{2}{9}-\frac{3}{9}$

Solution : (i) $\frac{3}{10}+\frac{4}{10}=\frac{3+4}{10}=\frac{7}{10}$

(ii) $\frac{5}{11}+\frac{2}{11}+\frac{1}{11}=\frac{5+2+1}{11}=\frac{8}{11}$

(iii) $\frac{5}{14}+\frac{8}{14}+\frac{2}{14}=\frac{5+8+2}{14}=\frac{15}{14}$

(iv) $\frac{5}{8}-\frac{3}{8}=\frac{5-3}{8}=\frac{2}{8}=\frac{1}{4}$ $\left[\frac{2}{8}=\frac{2\div 2}{8\div 2}=\frac{1}{4}\right]$

(v) $\frac{4}{7}+\frac{5}{7}-\frac{6}{7}=\frac{4+5-6}{7}=\frac{3}{7}$

(vi) $\frac{8}{9}-\frac{2}{9}-\frac{3}{9}=\frac{8-2-3}{9}=\frac{3}{9}$

5.8.2 Addition / Subtraction of Unlike Fractions :

In last section, we have learnt about addition and subtraction of like decimals. In this section we shall learn about addition and subtraction of unlike fraction. First we convert unlike fractions to equivalent like fractions and then we add or subtract, We follow these steps:-

Step 1:- Find the L.C.M of the denominators.

Step 2:- Convert each fraction into equivalent fraction with denominator equal to the L.C.M. obtained in step 1.

Step 3:- Add or subtract like fractions as required.

Let us illustrate this with following examples:-

Example 24: **Add $\frac{2}{3}$ and $\frac{3}{10}$**

Solution : To add $\frac{2}{3}$ and $\frac{3}{10}$, we take LCM of denominators 3 and 10 = 30

New, we convert the given fractions into equivalent fractions with denominator 30:

$$\frac{2}{3}=\frac{2\times10}{3\times10}=\frac{20}{30} \text{ and } \frac{3}{10}=\frac{3\times3}{10\times3}=\frac{9}{30}$$

Thus, $\frac{2}{3}+\frac{3}{10}=\frac{20}{30}+\frac{9}{30}=\frac{20+9}{30}=\frac{29}{30}$

Aliter :- $\frac{(\text{Ist fraction})\times \text{LCM}+(\text{2nd fraction})\times \text{LCM}}{\text{LCM of denominators}}$

i.e $\frac{\frac{2}{3}\times30+\frac{3}{10}\times30}{30}=\frac{2\times10+3\times3}{30}=\frac{20+9}{30}=\frac{29}{30}$

Aliter :- Cross Product Method.

$$\frac{a}{b}\times\frac{c}{d}=\frac{a\times d+b\times c}{b\times d}$$

Thus $\frac{2}{3}+\frac{3}{10}=\frac{2\times10+3\times3}{3\times10}=\frac{20+9}{30}=\frac{29}{30}$

This is applicable when denominators has no common factor

Example 25: Add $\frac{5}{6}+\frac{1}{4}$

Solution : To add $\frac{5}{6}$ and $\frac{1}{4}$, we take LCM of denominators 6 and 4 = 12

Now, convert the given fractions into equivalent fractions with denominator 12.

$$\frac{5}{6}=\frac{5\times2}{6\times2}=\frac{10}{12} \text{ and } \frac{1}{4}=\frac{1\times3}{4\times3}=\frac{3}{12}$$

2	6, 4
	3, 2

LCM = 2 × 3 × 2 = 12

Thus $\frac{5}{6}+\frac{1}{4}=\frac{10}{12}+\frac{3}{12}=\frac{10+3}{12}=\frac{13}{12}$

Aliter :-

$$\frac{(\text{Ist fraction})\times \text{LCM}+(\text{2nd fraction})\times \text{LCM}}{\text{LCM of denominators}}$$

i.e $\frac{\left[\frac{5}{6}\times12\right]+\left[\frac{1}{4}\times12\right]}{12}=\frac{10+3}{12}=\frac{13}{12}$

Example 26:- **Add $3\frac{1}{4}+2\frac{4}{5}$**

Solution : Convert mixed fractions to Improper fraction

[∴ LCM of 4 and 5 is 20, So convert each fraction into an equivalent fraction with denominator 20]

ie. $3\frac{1}{4}+2\frac{4}{5} = \frac{13}{4}+\frac{14}{5}$

$= \frac{13\times 5}{4\times 5}+\frac{14\times 4}{5\times 4}=\frac{65}{20}+\frac{56}{20}$

$= \frac{65+56}{20}=\frac{121}{20}=6\frac{1}{20}$

∵ 20) 121 (6
  −120
  1

Aliter :-

$3\frac{1}{4}+2\frac{4}{5}$

$= 3+\frac{1}{4}+2+\frac{4}{5} = 5+\left[\frac{1}{4}+\frac{4}{5}\right]$

$= 5 + \left(\frac{1\times 5}{4\times 5}+\frac{4\times 4}{5\times 4}\right)$

Convert into equivalent fractions with denominator 20

$= 5 + \left(\frac{5}{20}+\frac{16}{20}\right)$

$= 5 + \left(\frac{21}{20}\right) = 5 + \left(1\frac{1}{20}\right)$

∵ 20) 21 (1
  − 20
  1

$= 5 + 1 + \frac{1}{20} = 6 + \frac{1}{20} = 6\frac{1}{20}$

Exmaple 27:- **Simplify:-**

(i) $\frac{3}{4}-\frac{5}{8}$ **(ii)** $\frac{2}{3}-\frac{1}{4}$

Solution : (i) $\frac{3}{4}-\frac{5}{8}$

To subtract $\frac{3}{4}$ and $\frac{5}{8}$, we take LCM of denominators 4 and 8 = 8

2	4, 8
2	2, 4
	1, 2

LCM = 2 × 2 × 2 = 8

Now Equivalent fractions with denominators 8.

$\frac{3}{4}=\frac{3\times 2}{4\times 2}=\frac{6}{8}$ and $\frac{5}{8}$

Thus, $\frac{3}{4}-\frac{5}{8}=\frac{6}{8}-\frac{5}{8}=\frac{6-5}{8}=\frac{1}{8}$

(ii) $\frac{2}{3}-\frac{1}{4}$

To subtract $\frac{2}{3}$ and $\frac{1}{2}$, we take LCM of denominator 3 and 4 = 12

So $\frac{2}{3}-\frac{1}{4}=\frac{2\times4}{3\times4}-\frac{1\times3}{4\times3}$

$=\frac{8}{12}-\frac{3}{12}=\frac{8-3}{12}=\frac{5}{12}$

Example 28: Subtract $1\frac{1}{2}$ from $4\frac{3}{5}$

Solution : (i) $4\frac{3}{5}-1\frac{1}{2}$

First convert mixed fraction into improper fraction.

i.e $4\frac{3}{5}-1\frac{1}{2}=\frac{23}{5}-\frac{3}{2}$

[∵ LCM of 5 and 2 is 10, So convert each fraction into an equivalent fraction of denominator 10]

$=\frac{23\times2}{5\times2}-\frac{3\times5}{2\times5}$

$=\frac{46}{10}-\frac{15}{10}=\frac{46-15}{10}$

$=\frac{31}{10}=3\frac{1}{10}$

∵ 10) 31 (3
–30
1

Aliter :-

$4\frac{3}{5}-1\frac{1}{2}=(4-1)+\left(\frac{3}{5}-\frac{1}{2}\right)$

$=3+\left(\frac{3\times2}{5\times2}-\frac{1\times5}{2\times5}\right)=3+\left(\frac{6}{10}-\frac{5}{10}\right)$

$=3+\frac{1}{10}=3\frac{1}{10}$

Example 29: Simplify:-

(i) $\frac{2}{3}+\frac{3}{8}-\frac{5}{6}$ **(ii)** $\frac{3}{5}+\frac{7}{10}-\frac{1}{4}$ **(iii)** $\frac{5}{12}-\frac{1}{6}+\frac{4}{15}$

Solution : (i) $\frac{2}{3}+\frac{3}{8}-\frac{5}{6}$

First, take LCM of 3, 8 and 6 = 24 then Convert each fraction into equivalent fraction with denominator 24.

$$\therefore \frac{2}{3}+\frac{3}{8}-\frac{5}{6}=\frac{2\times 8}{3\times 8}+\frac{3\times 3}{8\times 3}-\frac{5\times 4}{6\times 4}$$

$$=\frac{16}{24}+\frac{9}{24}-\frac{20}{24}=\frac{16+9-20}{24}=\frac{5}{24}$$

3	3, 8, 6
2	1, 8, 2
2	1, 4, 1
	1, 2, 1

LCM = 3 × 2 × 2 × 2 = 24

(ii) $\frac{3}{5}+\frac{7}{10}-\frac{1}{4}$

First, take LCM of 5, 10 and 4 = 20 then convert each fraction into equivalent fraction with denominator 20.

$$\therefore \frac{3}{5}+\frac{7}{10}-\frac{1}{4}=\frac{3\times 4}{5\times 4}+\frac{7\times 2}{10\times 2}-\frac{1\times 5}{4\times 5}$$

$$=\frac{12}{20}+\frac{14}{20}-\frac{5}{20}=\frac{12+14-5}{20}=\frac{21}{20}$$

2	5, 10, 4
5	5, 5, 2
	1, 1, 2

LCM = 3 × 5 × 2 = 20

(iii) $\frac{5}{12}-\frac{1}{6}+\frac{4}{15}$

LCM of denominators = 60

$$=\frac{5\times 5}{12\times 5}-\frac{1\times 10}{6\times 10}+\frac{4\times 4}{15\times 4}$$

$$=\frac{25}{60}-\frac{10}{60}+\frac{16}{60}=\frac{25-10+16}{60}=\frac{31}{60}$$

2	12, 6, 15
3	6, 3, 15
	2, 1, 5

LCM = 2 × 3 × 2 × 5 = 60

5.8.3 Adding or Subtracting a fraction with a whole number

In this section, we shall learn to add or subtract a fraction with a whole number. We express the whole number in a fraction by writing 1 in the denominator.

Then we add or subtract as we have discussed in the previous section.

Example 30: Simplify

(i) $4+\frac{2}{3}$ **(ii)** $2-\frac{5}{6}$

Solution : (i) $4+\frac{2}{3}=\frac{4}{1}+\frac{2}{3}=\frac{4\times 3}{1\times 3}+\frac{2}{3}$

(∵ LCM of 1 and 3 is 3)

$$=\frac{4\times 3+2\times 1}{3}=\frac{12+2}{3}=\frac{14}{3}=4\frac{2}{3}$$

Aliter :-

In additon, we can write it direct also.

i.e. $4+\frac{2}{3}=4\frac{2}{3}$

(ii) $2-\frac{5}{6}=\frac{2}{1}-\frac{5}{6}$

(∵ LCM of 1 and 6 is 6)

$=\frac{2\times 6}{1\times 6}-\frac{5}{6}=\frac{12}{6}-\frac{5}{6}$

$=\frac{12-5}{6}=\frac{7}{6}=1\frac{1}{6}$

$$6\,\overline{)\,7\,(}\,1$$
$$\underline{-6}$$
$$\underline{1}$$

Example 31: **Sophia ran $\frac{3}{8}$ km in the morning and $2\frac{7}{10}$ km in the evening. How much did she run in that day?**

Solution : Total distance Sophia ran = $\frac{3}{8}+2\frac{7}{10}$

$=\frac{3}{8}+\frac{27}{10}$

$=\frac{3\times 5}{8\times 5}+\frac{27\times 4}{10\times 4}$

$=\frac{15}{40}+\frac{108}{40}=\frac{15+108}{40}=\frac{123}{40}=3\frac{3}{40}$

∴ Sophia ran $3\frac{3}{40}$ km on that day.

2	8, 10
	4, 5

LCM of 8 and 10 = $2\times 4\times 5=40$

∵ $40\,\overline{)\,123\,(}\,3$
$$\underline{-120}$$
$$\underline{3}$$

Example 32:- **A piece of wire $3\frac{3}{4}$ metres long broke into two pieces. One piece was $2\frac{5}{6}$ metre long. How long is the other piece?**

Solution : Length of Second piece = $3\frac{3}{4}-2\frac{5}{6}$

∴ LCM of 4 and 6 = 12

$=\frac{15}{4}-\frac{17}{6}$

$=\frac{15\times 3}{4\times 3}-\frac{17\times 2}{6\times 2}=\frac{45}{12}-\frac{34}{12}=\frac{45-34}{12}=\frac{11}{12}$

So, length of other piece is $\frac{11}{12}$ m.

Example 33: **Pankaj bought a notebook for ₹ $11\frac{1}{2}$, pencil for ₹ $2\frac{3}{4}$ and a colour box for ₹ $6\frac{2}{5}$. How much money did he spend?**

Solution : Total money he spend $= 11\frac{1}{2} + 2\frac{3}{4} + 6\frac{2}{5}$

$= \frac{23}{2} + \frac{11}{4} + \frac{32}{5}$ ($\because$ LCM of 2, 4, 5 = 20)

$= \frac{23\times10}{2\times10} + \frac{11\times5}{4\times5} + \frac{32\times4}{5\times4}$

$= \frac{230}{20} + \frac{55}{20} + \frac{128}{20} = \frac{230+55+128}{20}$

$= \frac{413}{20} = 20\frac{13}{20}$

$$\begin{array}{r} 20\,)\,413\,(\,20 \\ -400 \\ \hline 13 \end{array}$$

He spent ₹ $20\frac{13}{20}$ in all.

Exercise 5.5

1. Add the following:-

(i) $\frac{3}{7}+\frac{2}{7}$ (ii) $\frac{2}{11}+\frac{4}{11}$ (iii) $\frac{6}{13}+\frac{5}{13}$ (iv) $\frac{5}{14}+\frac{9}{14}+\frac{3}{14}$

(v) $\frac{1}{4}+\frac{2}{3}$ (vi) $\frac{1}{6}+\frac{5}{12}$ (vii) $\frac{3}{10}+\frac{4}{15}$ (viii) $\frac{3}{8}+\frac{1}{4}$

(ix) $\frac{5}{9}+4$ (x) $\frac{4}{7}+\frac{2}{3}+\frac{5}{21}$ (ix) $\frac{3}{4}+\frac{7}{12}+\frac{2}{3}$ (xii) $\frac{3}{5}+\frac{1}{3}$

2. Subtract the following:-

(i) $\frac{5}{9}-\frac{2}{9}$ (ii) $\frac{6}{17}-\frac{3}{17}$ (iii) $\frac{7}{10}-\frac{3}{10}$ (iv) $\frac{11}{13}-\frac{6}{13}-\frac{2}{13}$

(v) $\frac{5}{12}-\frac{1}{4}$ (vi) $\frac{3}{5}-\frac{2}{10}$ (vii) $\frac{6}{7}-\frac{2}{3}$ (viii) $\frac{5}{6}-\frac{1}{4}$

(ix) $\frac{8}{3}-\frac{5}{9}$ (x) $2-\frac{1}{7}$ (xi) $\frac{13}{7}-\frac{3}{4}-\frac{1}{14}$ (xii) $\frac{17}{24}-\frac{5}{16}-\frac{1}{3}$

3. Simplify the following:-

(i) $4\frac{2}{5}+2\frac{1}{5}$ (ii) $5\frac{3}{4}+2\frac{1}{6}$ (iii) $6\frac{1}{2}+2\frac{2}{3}$ (iv) $4\frac{3}{4}-1\frac{5}{6}$

(v) $2\frac{7}{10}-1\frac{2}{15}$ (vi) $5-3\frac{1}{2}$ (vii) $7+\frac{7}{4}+5\frac{1}{6}$ (viii) $2\frac{1}{8}+1\frac{1}{2}-\frac{7}{16}$

(ix) $5\frac{2}{3}+6-3\frac{1}{4}$ (x) $2-\frac{7}{16}$ (xi) $6+1\frac{1}{2}$ (xii) $2\frac{5}{6}-3\frac{5}{8}+2$

4. An iron pipe of length $6\frac{2}{3}$ metres long was cut into two pieces. One piece is $4\frac{3}{7}$ metre long. What is the length of other piece?

5. Ashok bought $\frac{7}{10}$ kg of mangoes and Tarun $\frac{11}{15}$ kg of apples. How much fruit did he buy in all?

6. Avi did $\frac{3}{5}$ of his homework on Saturday and $\frac{1}{10}$ of the same homework on Sunday. How much of the homework did he do over the weekend?

7. Charan spent $\frac{1}{4}$ of his pocket money on a movie and $\frac{3}{8}$ on a new pen and $\frac{1}{8}$ on a pencil. What fraction of his pocket money did he spend?

8. Simar lives at a distance of 4km from the school. Prabhjot lives at a distance of $\frac{2}{3}$ km less than simar's distance from the school. How far does prabhjot live from the school?

5.9 Multiplication of a fraction by a Whole Number

We have learnt the addition and subtraction of different type of fractions. Here we shall learn the multiplication of a fraction with a whole number in a brief view.

We know that in case of whole numbers, **multiplication means repeated addition.**

e.g. $4 \times 6 = 6 + 6 + 6 + 6 = 24$

Similarly, using the same rule, we multiply a fraction by a whole number.

Let us consider an example.

$$\frac{1}{3}\times 5 = 5\times\frac{1}{3} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3} \qquad = \frac{1+1+1+1+1}{3} = \frac{5}{3}$$

or, we can say that $\frac{1}{3}\times 5 = 5\times\frac{1}{3} = \frac{5\times 1}{3} = \frac{5}{3}$

Thus, To multiply a fraction by a whole number, we multiply the numerator of the fraction by the whole number and the denominator of the fraction remains same.

Example 34:- Multiply :-

(i) $\frac{1}{8}\times 3$ **(ii)** $\frac{5}{12}\times 4$ **(iii)** $6\times\frac{7}{10}$

Solution : (i) $\frac{1}{8}\times 3 = \frac{1}{8}\times\frac{3}{1} = \frac{1\times 3}{8} = \frac{3}{8}$

(ii) $\frac{5}{12}\times 4 = \frac{5}{12}\times\frac{4}{1} = \frac{5\times 4}{12} = \frac{20}{12} = \frac{20\div 4}{12\div 4} = \frac{5}{3}$

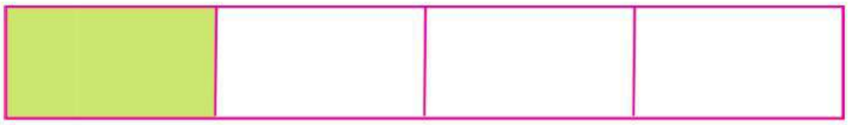
($\because$ HCF of 12 and 20 is 4. Convert in Simplest form)

(iii) $6\times\frac{7}{10}=\frac{6}{1}\times\frac{7}{10}=\frac{6\times7}{10}=\frac{42}{10}=\frac{42\div2}{10\div2}=\frac{21}{5}$

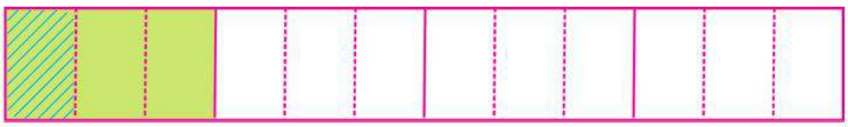
(∵ HCF of 42 and 10 is 2. Convert in Simplest form)

5.10 Division of a Fraction by a natural number

Let us divide $\frac{1}{4}$ by 3



Divide the whole figure in 4 equal parts. Now, we divide each part into 3 equal parts. Such that the whole figure is divided into 12 equal parts.



In figure, double shaded part represents $\frac{1}{12}$ of the whole figure.

i.e. $\frac{1}{4}\div3=\frac{1}{12}$

Also $\frac{1}{4}\div\frac{3}{1}=\frac{1}{4}\times\frac{1}{3}=\frac{1\times1}{4\times3}=\frac{1}{12}$

Thus, to divide a fraction by a natural number, we multiply the fraction by the reciprocal of the natural number.

Example 35: Divide:-

(i) $\frac{1}{6}$ by 2 (ii) $\frac{2}{3}$ by 5 (iii) $\frac{2}{5}$ by 4

Solution : (i) $\frac{1}{6}\div2=\frac{1}{6}\div\frac{2}{1}=\frac{1}{6}\times\frac{1}{2}=\frac{1\times1}{6\times2}=\frac{1}{12}$

(ii) $\frac{2}{3}\div5=\frac{2}{3}\div\frac{5}{1}=\frac{2}{3}\times\frac{1}{5}=\frac{2\times1}{3\times5}=\frac{2}{15}$

(iii) $\frac{2}{5}\div4=\frac{2}{5}\div\frac{4}{1}=\frac{2}{5}\times\frac{1}{4}=\frac{2\times1}{5\times4}=\frac{2}{20}=\frac{1}{10}$

Exercise 5.6

1. Multiply:-

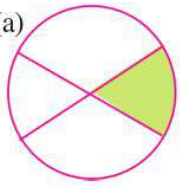
(i) $\frac{1}{5} \times 4$ (ii) $\frac{2}{7} \times 3$ (iii) $\frac{5}{8} \times 2$ (iv) $\frac{7}{12} \times 4$ (v) $10 \times \frac{4}{5}$

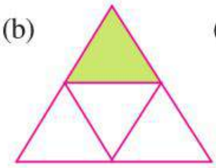
2. Divide:-

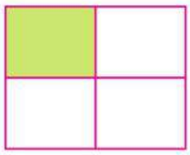
(i) $\frac{1}{4} \div 5$ (ii) $\frac{3}{5} \div 3$ (iii) $\frac{5}{8} \div 3$ (iv) $\frac{6}{7} \div 2$ (v) $\frac{12}{15} \div 6$

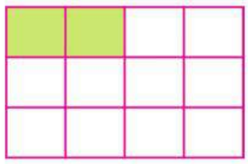
Multiple Choice Questions

1. Which of the following does not represent any fraction ?

(a)


(b)


(c)


(d)


2. Which of the following is a proper fraction?

(a) $\frac{5}{5}$ (b) $\frac{12}{11}$ (c) $\frac{7}{9}$ (d) 7

3. Which of the following is an improper fraction?.

(a) $\frac{5}{8}$ (b) $2\frac{3}{4}$ (c) $\frac{7}{11}$ (d) $\frac{15}{16}$

4. The fractions having 1 as numerator are called .... fractions.

(a) Like (b) Unlike (c) Unit (d) Proper

5. The fractions having same denominators are called .... fractions.

(a) Proper (b) Unit (c) Improper (d) Like

6. The fraction having different denominators are called .... fractions.

(a) Unlike (b) Like (c) Improper (d) Unit

7. Express 8 hours as a fraction of 1 day.

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{8}{1}$ (d) $\frac{1}{8}$

8. Find : $\frac{2}{5}$ of ₹ 20

(a) ₹ 8 (b) ₹ 10 (c) ₹ 12 (d) ₹ 40

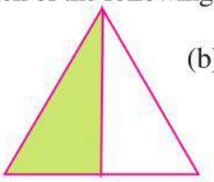
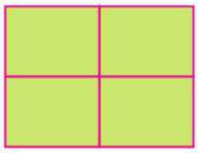
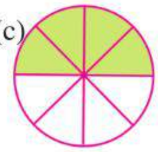
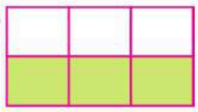
9. Write $\frac{19}{4}$ as mixed fraction

(a) $3\frac{4}{5}$ (b) $4\frac{4}{3}$ (c) $4\frac{3}{4}$ (d) $5\frac{1}{4}$

10. $7\frac{2}{3}$ = ...

(a) $\frac{17}{3}$ (b) $\frac{23}{3}$ (c) $\frac{13}{3}$ (d) $\frac{42}{3}$

11. Which of the following diagram represents an improper fraction?

(a)  (b)  (c)  (d) 

12. Which of the following fraction is an equivalent of $\frac{5}{7}$?

(a) $\frac{25}{49}$ (b) $\frac{20}{35}$ (c) $\frac{35}{49}$ (d) $\frac{35}{28}$

13. Replace $\square$ by the correct number in $\frac{5}{8} = \frac{20}{\square}$

(a) 32 (b) 24 (c) 40 (d) 16

14. Which of the following are in ascending order?

(a) $\frac{2}{3}, \frac{2}{7}, \frac{2}{5}$ (b) $\frac{2}{3}, \frac{2}{5}, \frac{2}{7}$ (c) $\frac{2}{7}, \frac{2}{3}, \frac{2}{5}$ (d) $\frac{2}{7}, \frac{2}{5}, \frac{2}{3}$

15. Which of the following are in descending order?

(a) $\frac{1}{8}, \frac{1}{3}, \frac{1}{9}$ (b) $\frac{1}{3}, \frac{1}{8}, \frac{1}{9}$ (c) $\frac{1}{8}, \frac{1}{9}, \frac{1}{3}$ (d) $\frac{1}{3}, \frac{1}{9}, \frac{1}{8}$

16. $\frac{4}{6} + \frac{3}{6}$ = ...

(a) $\frac{7}{12}$ (b) $\frac{7}{8}$ (c) $1\frac{1}{6}$ (d) $1\frac{1}{12}$

17. $\frac{4}{9} + \frac{5}{9} - \frac{2}{9}$ = ...

(a) $\frac{7}{9}$ (b) $\frac{7}{18}$ (c) $\frac{11}{9}$ (d) $\frac{5}{9}$

18. $\frac{2}{3}+\frac{1}{6}=$ ....

(a) $\frac{3}{9}$ (b) $\frac{5}{6}$ (c) $\frac{7}{6}$ (d) $\frac{5}{9}$

19. $4-\frac{1}{3}=$ ....

(a) $4\frac{1}{3}$ (b) $3\frac{1}{3}$ (c) $4\frac{2}{3}$ (d) $3\frac{2}{3}$

20. Divide $\frac{1}{6}$ by 2

(a) $\frac{1}{3}$ (b) $\frac{1}{12}$ (c) $\frac{1}{18}$ (d) 12

Learning Outcomes

After completion of this chapter the students are now able to

- Recognise different types of fractions and their diagrammical representation
- Use fractions in daily life.
- Use fractions in different units i.e. money, length and temperature.

ANSWER KEY

Exercise 5.1

1. (i) $\frac{1}{4}$ (ii) $\frac{5}{8}$ (iii) $\frac{4}{9}$ (iv) $\frac{5}{8}$ (v) $\frac{7}{16}$ (vi) $\frac{3}{3}$ (vii) $\frac{2}{5}$ (viii) $\frac{4}{7}$

3. (i) $\frac{3}{4}$ (ii) $\frac{7}{10}$ (iii) $\frac{1}{4}$ (iv) $\frac{5}{8}$ (v) $\frac{3}{12}$ 4. (i) $\frac{5}{9}$ (ii) $\frac{2}{11}$ (iii) $\frac{6}{7}$

5. (i) numerator = 2, denominator = 3 (ii) numerator = 1, denominator = 4
(iii) numerator = 5, denominator = 11 (iv) numerator = 9, denominator = 13
(v) numerator = 17, denominator = 16

6. (i) $\frac{1}{7}$ (ii) $\frac{40}{60}$ or $\frac{2}{3}$ (iii) $\frac{15}{24}$ or $\frac{5}{8}$ (iv) $\frac{2}{12}$ or $\frac{1}{6}$ (v) $\frac{45}{100}$ or $\frac{9}{20}$

7. (i) $\frac{12}{25}$ (ii) $\frac{9}{25}$ (iii) $\frac{8}{25}$ 8. $\frac{24}{42}$ or $\frac{4}{7}$, $\frac{18}{42}$ or $\frac{3}{7}$ 9. $\frac{6}{13}, \frac{7}{13}$

10. (i) $\frac{2}{10}$ or $\frac{1}{5}$ (ii) $\frac{3}{10}$ (iii) $\frac{4}{10}$ or $\frac{2}{5}$ (iv) Sidharth

11. Apples = $\frac{12}{24}$ or $\frac{1}{2}$, Oranges = $\frac{7}{24}$, Guava = $\frac{5}{24}$ **12.** Dishmeet = 15, Balkirat = 5

14. (i) 12 books (ii) 20 pens (iii) 6 copies (iv) 12 apples (v) 21 pencils

15. (i) 18 (ii) 8 (iii) 10

16. (i) False (ii) True (iii) True (iv) True

Exercise 5.2

1. Proper fractions :- $\frac{9}{13}, \frac{6}{11}, \frac{7}{9}, \frac{2}{15}, \frac{4}{17}, \frac{7}{8}$ Improper fractions:- $\frac{5}{4}, \frac{3}{2}, \frac{5}{2}, \frac{6}{6}$

2. (i) $5\frac{2}{5}$ (ii) $3\frac{1}{4}$ (iii) $5\frac{3}{8}$ (iv) $7\frac{2}{7}$ (v) $6\frac{2}{3}$

3. (i) $\frac{7}{3}$ (ii) $\frac{37}{7}$ (iii) $\frac{23}{5}$ (iv) $\frac{15}{4}$ (v) $\frac{77}{8}$

4. (i) $\frac{7}{3}, 2\frac{1}{3}$ (ii) $\frac{13}{4}, 3\frac{1}{4}$ (iii) $\frac{19}{5}, 3\frac{4}{5}$ (iv) $\frac{21}{8}, 2\frac{5}{8}$ (v) $\frac{25}{6}, 4\frac{1}{6}$

Exercise 5.3

1. (i) $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$; yes (ii) $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{10}{15}$; yes (iii) $\frac{4}{12}, \frac{2}{6}, \frac{3}{9}, \frac{1}{3}$; yes

2. (i) $\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20}$ (ii) $\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{15}{25}$

(ii) $\frac{7}{9} = \frac{14}{18} = \frac{21}{27} = \frac{28}{36} = \frac{35}{45}$ (iv) $\frac{5}{11} = \frac{10}{22} = \frac{15}{33} = \frac{20}{44} = \frac{25}{55}$

(v) $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$

3. (i) $\frac{2}{5}$ (ii) $\frac{1}{2}$ (iii) $\frac{2}{3}$ (iv) $\frac{3}{2}$ (v) $\frac{9}{5}$ **4.** (i) yes (ii) yes (iii) yes

5. (i) 42 (ii) 56 (iii) 9 (iv) 5 (v) 24

6. (i) $\frac{18}{30}$ (ii) $\frac{12}{20}$ (iii) $\frac{24}{40}$ **7.** (i) $\frac{6}{10}$ (ii) $\frac{48}{80}$ (iii) $\frac{12}{20}$

Exercise 5.4

1. $\frac{3}{7}, \frac{2}{7}, \frac{6}{7}$; $\frac{5}{11}, \frac{3}{11}, \frac{1}{11}$; $\frac{6}{13}, \frac{2}{13}, \frac{5}{13}, \frac{10}{13}$ **3.** $\frac{1}{8}, \frac{1}{9}, \frac{1}{7}$

4. (i) $<$ (ii) $>$ (iii) $>$ (iv) $<$ (v) $<$

5. (i) $>$ (ii) $<$ (iii) $>$ (iv) $>$ (v) $<$

6. (i) $>$ (ii) $>$ (iii) $<$ (iv) $<$ (v) $=$

7. (i) $\frac{3}{10}, \frac{5}{10}, \frac{7}{10}$ (ii) $\frac{1}{7}, \frac{4}{7}, \frac{6}{7}$ (iii) $\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}$ (iv) $\frac{5}{9}, \frac{5}{7}, \frac{5}{3}$

(v) $\frac{3}{13}, \frac{3}{11}, \frac{3}{7}$ (vi) $\frac{1}{6}, \frac{1}{4}, \frac{5}{12}$ (vii) $\frac{2}{7}, \frac{11}{35}, \frac{13}{28}, \frac{8}{14}$ (viii) $\frac{4}{15}, \frac{1}{3}, \frac{5}{12}, \frac{4}{9}$

(ix) $\frac{3}{16}, \frac{3}{8}, \frac{7}{12}, \frac{2}{3}$ (x) $\frac{2}{9}, \frac{7}{12}, \frac{11}{18}, \frac{5}{6}$

8. (i) $\frac{7}{9}, \frac{5}{9}, \frac{1}{9}$ (ii) $\frac{7}{11}, \frac{5}{11}, \frac{3}{11}, \frac{2}{11}$ (iii) $\frac{2}{7}, \frac{2}{9}, \frac{2}{13}$ (iv) $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{8}$

(v) $\frac{2}{3}, \frac{5}{12}, \frac{5}{18}, \frac{1}{6}$ (vi) $\frac{3}{4}, \frac{11}{15}, \frac{17}{30}, \frac{9}{20}$

9. Car **10.** $\frac{1}{6}, \frac{1}{5}, \frac{3}{10}$

Exercise 5.5

1. (i) $\frac{5}{7}$ (ii) $\frac{6}{11}$ (iii) $\frac{11}{13}$ (iv) $\frac{17}{14}$ (v) $\frac{11}{12}$ (vi) $\frac{7}{12}$ (vii) $\frac{17}{30}$

(viii) $\frac{5}{8}$ (ix) $\frac{41}{9}$ (x) $\frac{31}{21}$ (xi) 2 (xii) $\frac{14}{15}$

2. (i) $\frac{1}{3}$ (ii) $\frac{3}{17}$ (iii) $\frac{2}{5}$ (iv) $\frac{3}{13}$ (v) $\frac{1}{6}$ (vi) $\frac{2}{5}$ (vii) $\frac{4}{21}$

(viii) $\frac{7}{12}$ (ix) $\frac{19}{9}$ (x) $\frac{13}{7}$ (xi) $\frac{29}{28}$ (xii) $\frac{1}{16}$

3. (i) $6\frac{3}{5}$ (ii) $7\frac{11}{12}$ (iii) $9\frac{1}{6}$ (iv) $2\frac{11}{12}$ (v) $1\frac{17}{30}$ (vi) $1\frac{1}{2}$ (vii) $13\frac{11}{12}$

(viii) $3\frac{3}{16}$ (ix) $8\frac{5}{12}$ (x) $1\frac{9}{16}$ (xi) $7\frac{1}{2}$ (xii) $1\frac{3}{8}$

4. $2\frac{5}{21}$ m **5.** $1\frac{13}{30}$ kg **6.** $\frac{7}{10}$ **7.** $\frac{3}{4}$ **8.** $3\frac{1}{3}$

Exercise 5.6

1. (i) $\frac{4}{5}$ (ii) $\frac{6}{7}$ (iii) $\frac{5}{4}$ (iv) $\frac{7}{3}$ (v) 8 **2.** (i) $\frac{1}{20}$ (ii) $\frac{1}{5}$ (iii) $\frac{5}{24}$ (iv) $\frac{3}{7}$ (v) $\frac{2}{15}$

Multiple Choice Questions

1. a **2.** c **3.** b **4.** c **5.** d **6.** a **7.** b **8.** a **9.** c **10.** b
11. b **12.** c **13.** a **14.** d **15.** b **16.** c **17.** a **18.** b **19.** d **20.** b

6

DECIMALS

Objectives

In this chapter, you will learn

1. To understand about decimals places.
2. To provide knowledge of addition and subtraction of decimals.
3. To use decimals in length, capacity and weight etc.
4. To use decimals in daily life problems.

6.1 Introduction

The word decimal comes from Latin word **"Decem"** meaning 10. We have learnt about decimals in earlier classes. In this class, we shall study decimals as an extension of place value table as fractions and addition and subtraction of decimals.

6.2 Decimal Number

Consider the number 2145. the number 2145 in terms of place value can be written as

$2145 = 2000 + 100 + 40 + 5 = 2 \times 1000 + 1 \times 100 + 4 \times 10 + 5 \times 1$

It is observed that in the place value table each place value is 10 times of the next place value on its right. For example, place value of tens is 10 times the place value of ones. Place value of hundreds is 10 times of the place value of tens and place value of thousands is 10 times of the place value of hundreds and it continuous in the same way.

We can notice that as we move from left to right, the place value is divided by 10. If we extend this system to the right of units place digit 5 (as in above example), the place value of the digit to the right of 5 (unit digit) will be **tenths** i.e. $\frac{1}{10}$, the place value of next digit will be **hundredths** i.e. $\frac{1}{100}$ and next will be **thousandths** i.e. $\frac{1}{1000}$ and so on.

In such a situation, the place value table will take the following shape:

Thousands (1000)	Hundreds (100)	Tens (10)	Ones (1)	Tenths $\left(\frac{1}{10}\right)$	Hundredths $\left(\frac{1}{100}\right)$	Thousandths $\left(\frac{1}{1000}\right)$

Consider a number 5432.167

Thousands (1000)	Hundreds (100)	Tens (10)	Ones (1)	Tenths $\left(\frac{1}{10}\right)$	Hundredths $\left(\frac{1}{100}\right)$	Thousandths $\left(\frac{1}{1000}\right)$
5	4	3	2	1	6	7

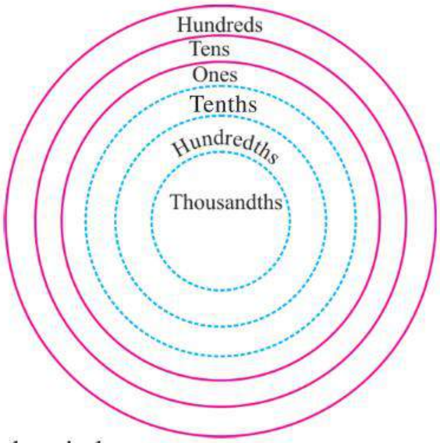
The number 5432.167 in terms of place value can be written as 5432 + . 167

$$\underbrace{5\times1000+4\times100+3\times10+2\times1}_{\text{whole number part}} + \underbrace{1\times\frac{1}{10}+6\times\frac{1}{100}+7\times\frac{1}{1000}}_{\text{fractional part}}$$

To separate whole number and fractional part of the number, we put small dot in between which is called **decimal**.

The number is read as *"Five thousand four hundred thirty two point one six seven."*

Or *Five thousand four hundred thirty two and one hundred sixty seven thousandths."*



Students, Let's play an activity with numbers. Draw concentric circles on a card board (as shown). Write ones, tens, hundreds and tenths, hundredths, thousandths so on.

Take some marbles and throw these marbles gently on the this cardboard. Let us suppose that the marbles settle themselves in the place value circles as shown in following figure.

Observe the number of marbles in different place value circles.

Number of marbles in hundreds circle = 6

So place value = 6 ×100 = 600

Number of marbles in tens circle $= 2$

Place value $= 2 \times 10 = 20$

Number of marbles in ones circle $= 4$

Place value $= 4 \times 1 = 4$

Number of marbles in tenths circle $= 1$

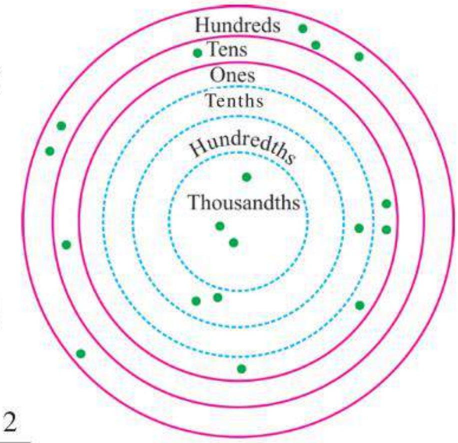
Place value $= 1 \times \frac{1}{10} = \frac{1}{10}$

Number of marbles in hundredths circle $= 2$

Place value $= 2 \times \frac{1}{100} = \frac{2}{100}$

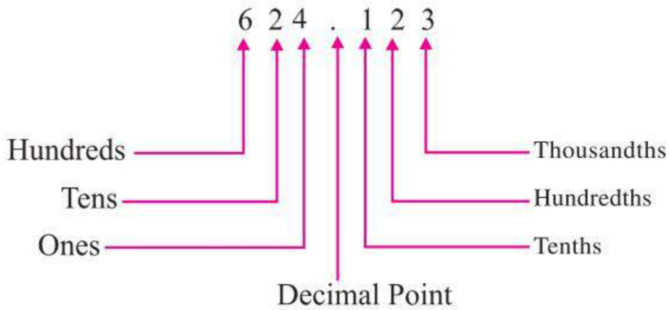
Number of marbles in thousandths circle $= 3$

Place value $= 3 \times \frac{1}{1000} = \frac{3}{1000}$



Hence Decimal number $= 600 + 20 + 4 + \frac{1}{10} + \frac{2}{100} + \frac{3}{1000}$

$= 624.123$



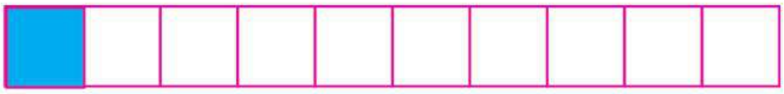
Now we shall discuss decimal fractions : tenths, hundredths, thousandths etc. in detail.

6.3 Decimal Fractions

In the previous section, we have learnt that decimals are an extension of our number system. In this section, we shall see that decimals are another name of fractions whose denominators are 10,100, 1000 etc. Let us first define tenths, hundredths, thousandths etc. as fractions.

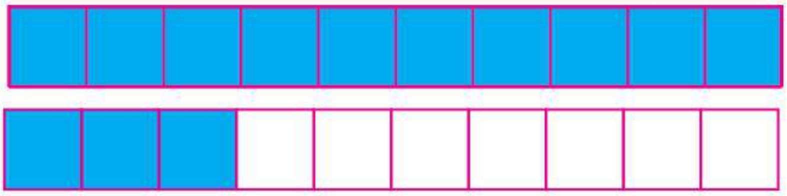
6.3.1 Tenths

⇒ Consider a rectangle divided into ten equal parts and shade one part. The shaded part represents one-tenths of the whole figure. It is written as $\frac{1}{10}$ or .1, which is read as 'one tenth' or 'point one' or 'decimal one'.



⇒ The following figure is divided into ten equal parts and seven parts are shaded. The shaded part represents seven-tenths of the whole figure. It is written as $\frac{7}{10}$ or .7 and is read as 'point seven' or 'decimal seven'.

⇒ Similarly, In the following figure, one whole rectangle is shaded and 3 parts of another same rectangle are shaded.



The shaded part is written as $1\frac{3}{10}$ = 1.3 and read as 'one point 3' or 'one and three tenths'.

So, $.1 = \frac{1}{10} = 1$ tenths

Let's illustrate some examples:

Example 1. **Write each of the following as decimals:**

(i) Seven and three tenths **(ii) Two tenths**

(iii) Twenty four and one tenth

Solution : (i) Seven and three tenths $= 7 + \frac{3}{10}$

$= 7\frac{3}{10} = 7.3$

(ii) Two tenths $= \frac{2}{10} = .2$

(iii) Twenty four and one tenth $= 24\frac{1}{10} = 24.1$

Example 2. **Write each of the following as decimals:**

(i) $\mathbf{10 + 3 + \frac{2}{10}}$ **(ii)** $\mathbf{200 + 7 + \frac{5}{10}}$ **(iii)** $\mathbf{\frac{9}{10}}$

Solution ; (i) $10 + 3 + \frac{2}{10}$

There are 1 tens, 3 ones and 2 tenths.

$\therefore \quad 10 + 3 + \frac{2}{10} = 13 + \frac{2}{10} = 13.2$

(ii) $200 + 7 + \frac{5}{10}$

There are 2 hundreds, 7 ones and 5 tenths

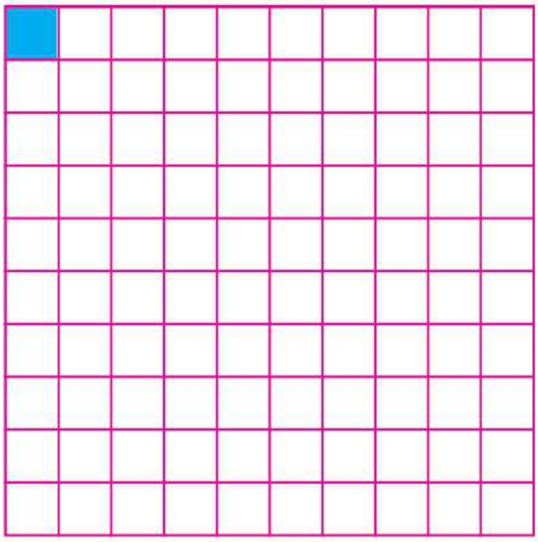
$\therefore \quad 200 + 7 + \frac{5}{10} = 207 + \frac{5}{10} = 207.5$

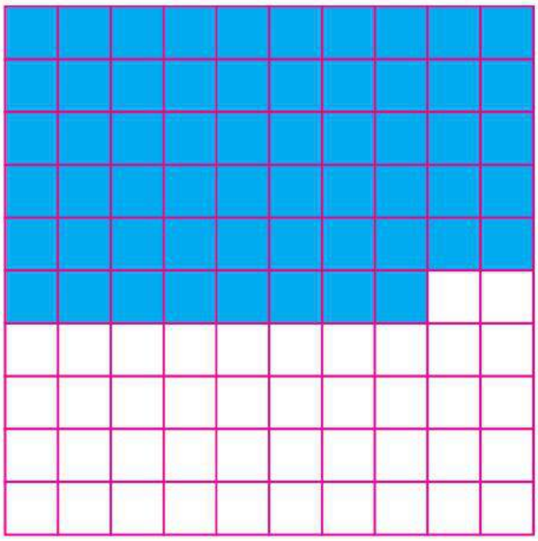
(iii) $\frac{9}{10}$, There are only 9 tenths

$\therefore \quad \frac{9}{10} = .9$

6.3.2 Hundredths

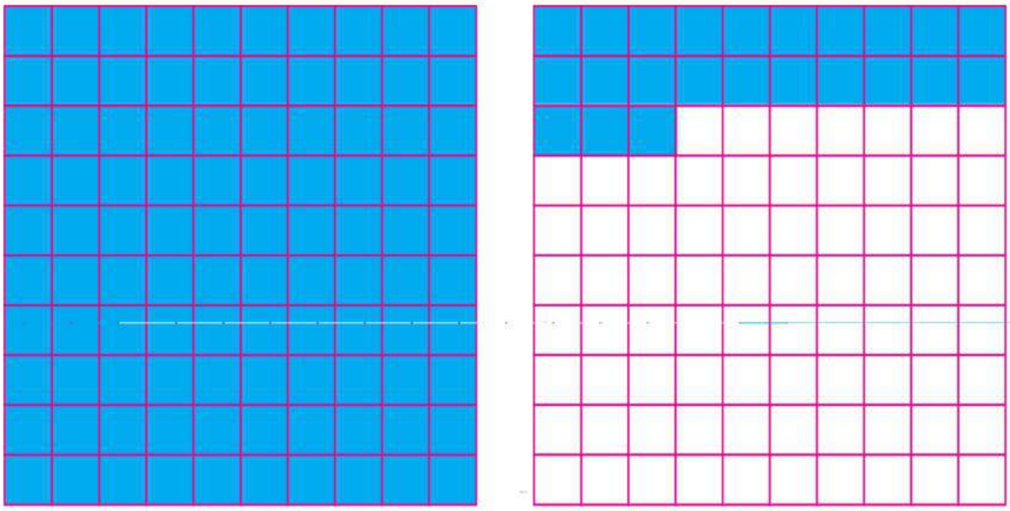
⇒ In the following figure, a square is divided into 100 equal parts and 1 part is shaded. Thus, the shaded part represents one-hundredths of the whole figure and is written as $\frac{1}{100}$ or .01 and read as 'one-hundredth' or 'point zero one' or 'decimal zero one'.





⇒ In the figure, a square is divided into 100 equal parts out of which 58 are shaded. Then the shaded part represents fifty-eight hundredths of the whole and is written as $\frac{58}{100}$ or .58 and read as 'point five eight' or 'decimal five eight'.

⇒ Similarly, In the figure, one whole square is shaded and 23 parts of other similar squares are shaded.



The shaded part is written as $1\frac{23}{100}$ or 1.23 and read as 'one point two three' or 'one and twenty three hundredths.

6.3.3 Thousandths

If an object is divided into 1000 equal parts then each part is one-thousandths of the whole. It is written as $\frac{1}{1000}$ or .001 and read as 'point zero zero one.'

- If we take 19 parts out of 1000 equal parts than 19 parts make $\frac{19}{1000}$ of the whole and written as .019 and read as 'point zero one nine'.

  Similarly we have,

$$\frac{102}{1000} = .102, \quad \frac{519}{1000} = .519, \quad \frac{1439}{1000} = 1.439$$

$$\frac{12508}{1000} = 12.508 \text{ etc.}$$

How to mark decimal ?

⇒ A fraction of the form $\frac{\text{Number}}{10}$ is written as a decimal obtained by putting decimal point by leaving **one right most digit**.

e.g. $\frac{34}{10} = 3.4$

⇒ A fraction of the form $\frac{\text{Number}}{100}$ is written as a decimal obtained by putting decimal point by leaving **two right most digits**. If the number is short of digits, insert zeroes at the left of the number.

e.g. (i) $\frac{1258}{100} = 12.58$ (ii) $\frac{6}{100} = .06$

⇒ A fraction of the form $\frac{\text{Number}}{1000}$ is written as a decimal obtaind by putting decimal point by leaving three right most digits. If the number is short of digits, insert zeros at the left of the number.

e.g. (i) $\frac{2345}{1000} = 2.345$ (ii) $\frac{16}{1000} = .016$

In decimals, we have some distinctions which are as follows:

(i) A decimal number may contain a whole number and a decimal part .4, .23, 6.25 etc.

(ii) If the decimal number consists only decimal part then zero can be written in the whole part

i.e. .3 = 0.3

.05 = 0.05 etc.

(iii) If the decimal numbers consists only whole part then zero can be written in the decimal part.

i..e 2 = 2.0

40 = 40.0 etc.

Let us consider some examples to understand the basic idea of decimals:

Example 3. **Write each of the following in numbers.**

(i) Twenty four point five

(ii) Sixty nine point three eight

(iii) One thousand fifty two point zero seven

(iv) Zero point six zero nine.

(v) Two point zero zero one.

Solution : (i) Twenty four point five = 24.5

(ii) Sixty nine point three eight= 69.38

(iii) One thousand fifty two point zero seven = 1052.07

(iv) Zero point six zero nine = 0.609

(v) Two point zero zero one = 2.001

Example 4. **Write each of the following in figures.**

(i) Five and seven tenths

(ii) Twenty nine and sixty one hundredths

(iii) Eighty two and one hundred fifty two thousandths

(iv) Four hundredths

(v) Seventy and two thousandths

Solution : (i) Five and seven tenths = $5+\frac{7}{10}=5.7$

(ii) Twenty nine and sixty one hundredths = $29+\frac{61}{100}=29.61$

(iii) Eighty two and one hundred fifty two thousandths = $82+\frac{152}{1000}=82.152$

(iv) Four hundredths = $\frac{4}{100}=0.04$

(v) Seventy and two thousandths = $70+\frac{2}{1000}=70.002$

Example 5. **Write the following decimals in the place value table:**

(i) 125.67 **(ii) 5.3** **(iii) 0.56**

(iv) 3.148 **(v) 10.007**

Solution :

Number	Thousands	Hundreds	Tens	ones	Tenths	Hundredths	Thousandths
125.67	–	1	2	5	6	7	–
5.3	–	–	–	5	3	–	–
0.56	–	–	–	0	5	6	–
3.148	–	–	–	3	1	4	8
10.007	–	–	1	0	0	0	7

Example 6. **Write the following decimal numbers in words:**

(i) 64.58 **(ii) 0.63** **(iii) 7.006** **(iv) 712.05**

(v) 0.725

Solution : (i) 64.58 = Sixty four point five eight

Or Sixty four and fifty eight hundredths

(ii) 0.63 = Zero point six three

Or Sixty three hundredths

(iii) 7.006 = Seven point zero zero six

Or Seven and six thousandths

(iv) 712.05 = Seven hundred twelve point zero five

Or Seven hundred twelve and five hundredths

(v) 0.725 = Zero point seven two five

Or Seven hundred twenty five thousandths

Example 7. **Write the decimals shown in the following place value table:**

	Thousands (1000)	Hundreds (100)	Tens (10)	Ones (1)	Tenth $\left(\frac{1}{10}\right)$	Hundredths $\left(\frac{1}{100}\right)$	Thousandth $\left(\frac{1}{1000}\right)$
(i)	–	5	6	0	3	4	–
(ii)	1	0	2	3	0	5	2
(iii)	2	1	5	0	0	0	6
(iv)	–	–	2	1	1	2	–
(v)	–	–	–	5	0	0	4

Solution : (i) We have $5 \times 100 + 6 \times 10 + 0 \times 1 + 3 \times \frac{1}{10} + 4 \times \frac{1}{100}$

$= 500 + 60 + 0 + \frac{3}{10} + \frac{4}{100} = 560.34$

(ii) We have $1 \times 1000 + 0 \times 100 + 2 \times 10 + 3\times 1 + 0 \times \frac{1}{10} + 5 \times \frac{1}{100} + 2\times \frac{1}{1000}$

$= 1000 + 0 + 20 + 3 + 0 + \frac{5}{100} + \frac{2}{1000}$

$= 1023.052$

(iii) We have $2\times 1000 + 1 \times 100 + 5 \times 10 + 0\times 1 + 0\times \frac{1}{10} + 0 \times \frac{1}{100} + 6 \times \frac{1}{1000}$

$= 2000 + 100 + 50 + 0 + 0 + 0 + \frac{6}{1000} = 2150.006$

(iv) We have $2\times 10 + 1 \times 1 + 1 \times \frac{1}{10} + 2 \times \frac{1}{100}$

$= 20 + 1 + \frac{1}{10} + \frac{2}{100} = 21.12$

(v) We have $5 \times 1 + 0 \times \frac{1}{10} + 0 \times \frac{1}{100} + 4\times \frac{1}{1000}$

$= 5 + 0 + 0 + \frac{4}{1000} = 5.004$

Example 8. **Write the following decimals in expanded form:**

(i) 5.6 (ii) 2.12 (iii) 14.89 (iv) 45.067 (v) 130.008

Solution : (i) 5.6 $= 5 + .6 = 5 + \frac{6}{10}$

(ii) 2.12 $= 2 + .12 = 2 + .1 + .02$

$= 2 + \frac{1}{10} + \frac{2}{100}$

(iii) 14.89 $= 14 + .89$

$= 10 + 4 + .8 + .09$

$= 10 + 4 + \frac{8}{10} + \frac{9}{100}$

(iv) 45.067 $= 45 + .067$

$= 40 + 5 + .06 + .007$

$= 40 + 5 + \frac{6}{100} + \frac{7}{1000}$

(v) 130.008 $= 130 + .008$

$= 100 + 30 + \frac{8}{1000}$

Exercise 6.1

1. Write each of the following in figures:

(i) Seventy two point one four.

(ii) Two hundred fifty seven point zero eight

(iii) Eight point two five six.

(iv) Forty five and twenty three hundredths.

(v) Six hundred twenty one and two hundred fifty three thousandths

(vi) Twelve and eight thousandths.

2. Write the following decimal numbers in words:

(i) 12.52 (ii) 7.148 (iii) 0.24 (iv) 5.018 (v) .009

3. Write the following decimals in the place value table:

(i) 21.569 (ii) 0.64 (iii) 3.51 (iv) 14.087 (v) 3.002

4. Write the following as decimals:

(i) $40 + \frac{2}{10}$ (ii) $700 + 5 + \frac{3}{10} + \frac{4}{100}$

(iii) $10 + \frac{5}{100} + \frac{3}{1000}$ (iv) $\frac{7}{10} + \frac{4}{1000}$ (v) $\frac{5}{1000}$

5. Write the decimals shown in the following place value table:

	Thousands	Hundreds	Tens	Ones	Tenth	Hundredths	Thousandths
(i)	–	5	2	4	1	2	–
(ii)	2	0	3	4	2	1	–
(iii)	–	–	6	1	0	2	3
(iv)	–	–	–	4	0	0	1
(v)	–	1	0	0	0	3	

6. Expand the following decimals.

(i) 2.5 (ii) 18.43 (iii) 4.05 (iv) 13.123 (v) 245.456 (vi) 20.057

6.4 Conversion of decimals and fractions

In last section, we have learnt about reading and writing of decimals and their expanded form. Now we shall learn the conversion of decimals into fractions and vice versa.

6.4.1 Decimals into fractions

Consider a decimal say 2.3 which can be written as:

$$2.3 = 2 + .3 = 2 + \frac{3}{10}$$

$$= \frac{20}{10} + \frac{3}{10} = \frac{20+3}{10} = \frac{23}{10}$$

i.e. $2.3 = \frac{2.3}{1} = \frac{23}{10}$

$$2.3 = \frac{23}{10} = \frac{\text{Number without decimal}}{\text{1 in place of decimal followed by as many zeroes as the number of digits after decimal}}$$

Example 9. **Convert the following decimals into fraction and reduce it to its lowest terms.**

(i) 2.5 (ii) 1.52 (iii) .006 (iv) 24.6 (v) 4.32

Solution : (i) 2.5

$$2.5 = \frac{25}{10} = \frac{25 \div 5}{10 \div 5} = \frac{5}{2} \quad \text{(HCF of 25 and 10 = 5)}$$

Here, numerator of the fraction is the given number without decimal, i..e 25. since the number of digits after decimals in 2.5 is 1, So the denominator of the fraction is 1 followed by one zero.

(ii) 1.52

$$1.52 = \frac{152}{100} = \frac{152 \div 4}{100 \div 4} \quad \text{(HCF of 152 and 100 is 4)}$$

$$= \frac{38}{25}$$

Here, numerator of the fraction is the given number without decimal i.e. 152 since the number of digits after decimals in 1.52 are 2, So the denominator of the fraction is 1 followed by two zeros.

(iii) $.006 = \frac{6}{1000} = \frac{6 \div 2}{1000 \div 2}$ (HCF of 6 and 1000 is 2)

$$= \frac{3}{500}$$

(iv) $24.6 = \frac{246}{10} = \frac{246 \div 2}{10 \div 2}$ (HCF of 246 and 10 is 2)

$$= \frac{123}{5}$$

(v) $4.32 = \frac{432}{100} = \frac{432 \div 4}{100 \div 4}$ (HCF of 432 and 100 is 4)

$$= \frac{108}{25}$$

6.4.2 Fractions into decimals

As we have learnt in last section that the fractions with denominators 10, 100 or 1000 can easily be converted into decimals.

e.g. $\frac{43}{10} = 4.3$; $\frac{125}{100} = 1.25$; $\frac{65}{1000} = 0.065$

$\frac{2143}{1000} = 2.143$; $\frac{619}{100} = 6.19$ etc

> To convert a fraction with denominator 10,100 or 1000 into decimal, place the decimal point (from right to left) in the numerator after as many digits as there are zeroes (after) in the denominator.

But some fractions have denominators other than 10,100 or 1000, those can be changed into fractions with denominators 10,100 or 1000 by finding their equivalent fractions or by division method.

In this class, we shall learn only that fractions whose denominators are multiples of 2 or 5 or both.

Equivalent Fractions Method

⇒ Consider an example, say $\frac{3}{5}$

Here denominator is 5. So we need to convert it into 10,100 or 1000. We know that after multiplying 5 by 2, we get 10.

$$\therefore \quad \frac{3}{5} = \frac{3\times 2}{5\times 2} = \frac{6}{10} = 0.6$$

Now consider $\frac{5}{4}$ Here denominatior is 4, so we need to convert it into 10, 100 or 1000. We know that after multiplying 4 by 25, we get 100

$$\therefore \quad \frac{5}{4} = \frac{5\times 25}{4\times 25} = \frac{125}{100} = 1.25$$

Division Method

As above method is quite difficult in many cases when we have denominator 8, 16 or 40 etc. So we have another method, Division method which is quite easy. Consider an example say, $\frac{14}{5}$

We know, Here dividend = 14

And divisor = 5

Step 1. Divide 14 by 5 we know quotient and remainder will be 2 and 4 respectively.

$$\begin{array}{r} 2 \\ 5\,\overline{)\,14\,(} \\ -\,10 \\ \hline 4 \\ \hline \end{array}$$

Step 2. In dividend, insert decimal point and put zero right to it i.e. 14 = 14.0

$$\begin{array}{r} 2 \\ 5\,\overline{)\,14.0\,(} \\ -\,10 \\ \hline 4 \\ \hline \end{array}$$

Step 3: Take 0 of 14.0 down to the remainder 4 which becomes 40.

Step 4 : In quotient, put decimal point after 2 i.e. 2.

Step 5. Now divide 40 by 5, we have quotient 8.

i.e. $\frac{14}{5} = 2.8$

$$\begin{array}{r} 2.8 \\ 5\,\overline{)\,14.0\,(} \\ -\,10\downarrow \\ \hline 4\,0 \\ -4\,0 \\ \hline 0 \\ \hline \end{array}$$

Let's consider some examples.

Example 10. Convert the following fractions into decimals by equivalent fraction method.

(i) $\frac{5}{10}$ **(ii)** $\frac{423}{100}$ **(iii)** $\frac{9}{1000}$ **(iv)** $\frac{15}{2}$ **(v)** $\frac{12}{25}$

(vi) $\frac{23}{20}$

Solution : (i) $\frac{5}{10}$ = .5 or 0.5 (Here denominator is 10)

(ii) $\frac{423}{100}$ = 4.23 (Here denominator is 100)

(iii) $\frac{9}{1000}$ = .009 or 0.009 (Here denominator is 1000)

(iv) $\frac{15}{2}$

Here denominator is 2, convert into equivalent fraction with denominator 10 by multiplying it by 5.

$$\therefore \quad \frac{15}{2} = \frac{15 \times 5}{2 \times 5} = \frac{75}{10} = 7.5$$

(v) $\frac{12}{25}$

Here denominator is 25.

Convert into equivalent fraction with denominator 100 by multiplying it by 4.

$$\therefore \quad \frac{12}{25} = \frac{12 \times 4}{25 \times 4} = \frac{48}{100} = .48 \text{ or } 0.48$$

(vi) $\frac{23}{20}$

Here denominator is 20.

Convert into equivalent fraction with denominator 100 by multiplyig it by 5.

$$\therefore \quad \frac{23}{20} = \frac{23 \times 5}{20 \times 5} = \frac{115}{100} = 1.15$$

Example 11. Convert the following fraction into decimal by division method:

(i) $\mathbf{\frac{13}{2}}$ **(ii)** $\mathbf{\frac{34}{5}}$ **(iii)** $\mathbf{\frac{47}{4}}$ **(iv)** $\mathbf{\frac{21}{8}}$ **(v)** $\mathbf{\frac{18}{25}}$

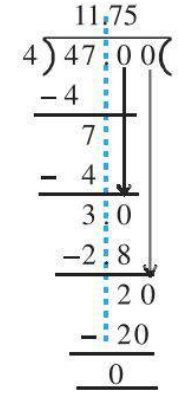
Solution : (i) $\frac{13}{2} = 6.5$

$$\begin{array}{r|l} & 6.5 \\ 2 & 13.0 \\ & -12\downarrow \\ \hline & 1\,0 \\ & -1\,0 \\ \hline & 0 \end{array}$$

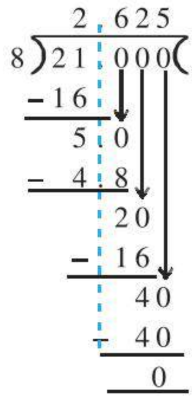
(ii) $\frac{34}{5} = 6.8$

$$\begin{array}{r|l} & 6.8 \\ 5 & 34.0 \\ & -30\downarrow \\ \hline & 4\,0 \\ & -4\,0 \\ \hline & 0 \end{array}$$

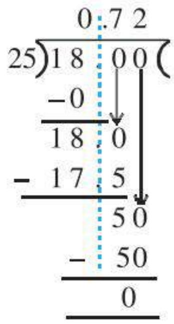
(iii) $\frac{47}{4} = 11.75$



(iv) $\frac{21}{8} = 2.625$



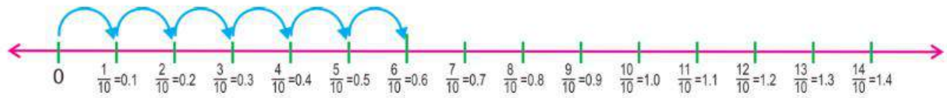
(v) $\frac{18}{25} = 0.72$



6.5 Representation of Decimals on Number line

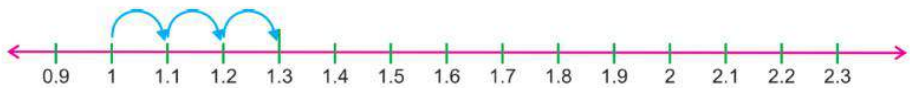
We have learnt to represent natural numbers, whole numbers, integers and fractions on number line. Here we shall represent decimals on a number line. Representation of decimal is similar like of fractions.

⇒ Let's represent .6 or $\frac{6}{10}$ on number line. The fraction $\frac{6}{10}$ is smaller than 1 but greater than 0. So we divide the distance from 0 to 1 on the number line into 10 equal parts and count 6 steps starting from 0 towards the right.



Since $\frac{6}{10} = 0.6$, 0.6 represents the same point on the number line as $\frac{6}{10}$.

⇒ Now we represent 1.3 on the number line. We know that 1.3 = 1 + .3 i.e. 1 + 3 tenths is greather than 1 but smaller than 2. So we start from 1 and count 3 steps towards the right.

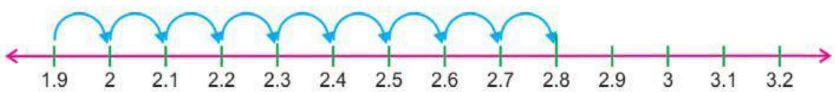


Example 12. **Represent the following decimals on the number line:**

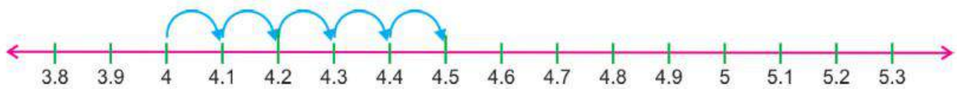
(i) 0.4 **(ii) 2.8** **(iii) 4.5**

Solution : (i) 0.4 lies between 0 and 1.

(ii) 2.8 lies between 2 and 3.



(iii) 4.5 lies between 4 and 5.



6.6 Like and Unlike decimals

As we know the number of digits contained in the decimal part of a decimal number gives the number of decimal places.

e.g. ⇒ 5.34 has two decimal places.

⇒ 4.156 has three decimal places.

⇒ 42.01 has two decimal places.

Like Decimals : The decimals with the same number of decimal places are called like decimals. e.g. 2.56, 42.01, 1.68,2.30 are like decimals, each having two places of decimals.

Unlike Decimals: The decimals having different number of decimal places are called unlike decimals e.g. 2.1, 3.14, 42.356 are unlike decimal as they contain one, two, three decimal places respectively.

Now, convert all unlike decimals into like decimals by putting zeroes at the end of the decimal number so that all of them have same number of decimal places.

i.e. 2.1 = 2.100 ; 3.14 = 3.140; 42.356 are all like decimals.

Adding extra zeroes to the right of a decimal does not change its value

i.e 2.5 = 2.50 = 2.500

6.7 Comparing Decimals

To compare decimals following steps are followed:

Step 1 : Convert unlike decimals to like decimals.

Step 2 : Compare the whole number part. Number with greater whole number part will be the greater decimal number.

e.g. 6.8 and 5.9

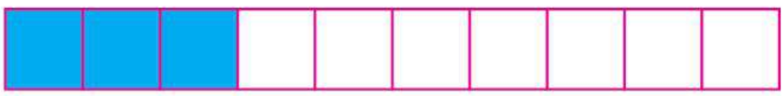
6>5

$\therefore$ 6.8 > 5.9

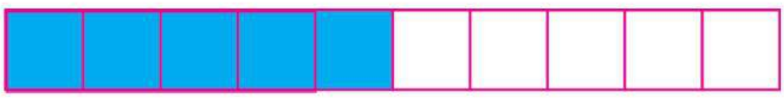
Step 3: If the whole number part is equal then compare the digits in the tenths place, the decimal number having greater number at the tenths place will be greater.

e.g. (i) Compare 0.3 and 0.5

$0.3 = \frac{3}{10}$ i.e. 3 out of 10 parts are shaded



and $0.5 = \frac{5}{10}$ i.e. 5 out of 10 parts are shaded.



$\therefore$ 0.5 > 0.3

As

same

0.3 0.5

3<5

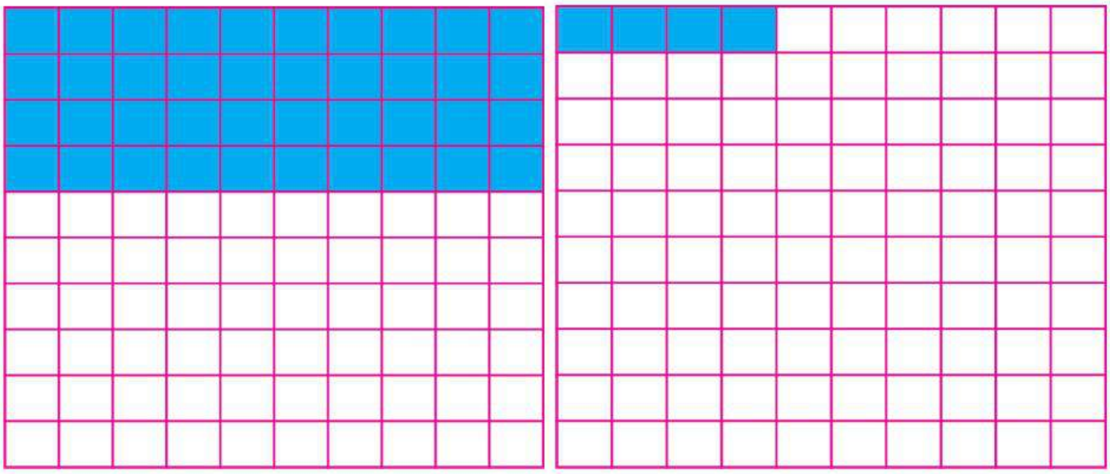
$\therefore$ 0.3 < 0.5 or 0.5 > 0.3

(ii) Now compare 0.4 and 0.04

$0.4 = \frac{4}{10} = \frac{40}{100}$ i.e. 40 out of 100 parts are shaded.

and $0.04 = \frac{4}{100}$ i.e. 4 out of 100 parts are shaded.

Now 40 > 4



Thus $\frac{40}{100} > \frac{4}{100}$ or .4 > .04

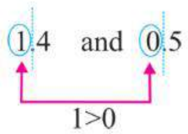
Step 4 : If the digits in tenths place are also equal then compare the digits in the hundredths place and so on.

Example 13. **Which is greater?**

(i) 1.4 and 0.5 **(ii) 3.18 and 13.28**

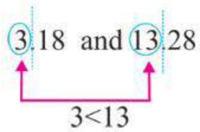
(iii) 4.3 and 4.03 **(iv) 5.168 and 5.169** **(v) 24.3 and 24.31**

Solution : (i)



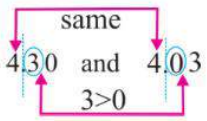
Since 1>0, so 1.4 > 0.5

(ii)



Since 3<13, so 3.18 < 13.28

(iii)

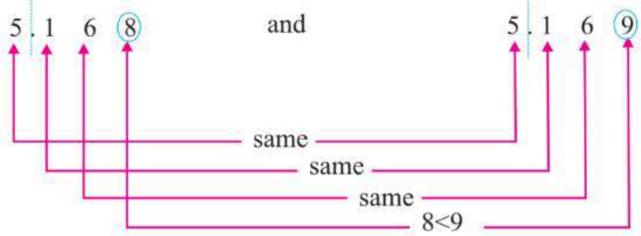


(Convert 4.3 into 4.30)

Since whole part is same then comparing tenths digit, 3 > 0

$\Rightarrow$ 4.3 > 4.03

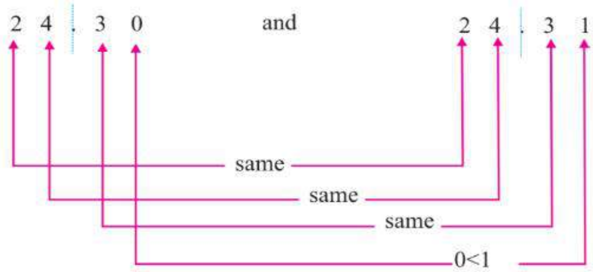
(iv)



Comparing the thousandths digit 8<9.

$\Rightarrow$ 5.169>5.168

(v)



Comparing hundredths digit, 0<1

$\Rightarrow$ 24.31> 24.3

Exercise 6.2

1. Convert the following decimal numbers into fractions and reduce it to lowest form.

(i) 1.4 (ii) 2.25 (iii) 18.6 (iv) 4.04 (v) 21.6

2. Convert the following fractions into decimal numbers :

(i) $\frac{7}{100}$ (ii) $\frac{12}{10}$ (iii) $\frac{215}{100}$ (iv) $\frac{18}{1000}$ (v) $\frac{245}{10}$

3. Convert the following fractions into decimal numbers by equivalent fraction method:

(i) $\frac{5}{2}$ (ii) $\frac{3}{4}$ (iii) $\frac{28}{5}$ (iv) $\frac{135}{20}$ (v) $\frac{17}{4}$

4. Convert the following fractions into decimals by long division method:

(i) $\frac{17}{2}$ (ii) $\frac{33}{4}$ (iii) $\frac{76}{5}$ (iv) $\frac{24}{25}$ (v) $\frac{5}{8}$

5. Represent the following decimals on number line:

(i) 0.7 (ii) 1.6 (iii) 3.7 (iv) 6.3 (v) 5.4

6. Write three decimal numbers between:

(i) 1.2 and 1.6 (ii) 2.8 and 3.2 (iii) 5 and 5.5

7. Which number is greater:

(i) 0.4 or 0.7 (ii) 2.6 or 2.5 (iii) 1.23 or 1.32

(iv) 12.3 or 12.4 (v) 18.35 or 18.3 (vi) 12 or 1.2

(vii) 5.06 or 5.061 (viii) 2.34 or 23.3 (ix) 13.08 or 13.078

(x) 2.3 or 2.03

8. Arrange the decimal numbers in ascending order:

(i) 2.5, 2, 1.8, 1.9 (ii) 3.4, 4.3, 3.1, 1.3

(iii) 1.24, 1.2, 1.42, 1.8

9. Arrange the deecimal numbers in descending order:

(i) 4.1, 4.01, 4.12, 4.2 (ii) 1.3, 1.03, 1.003, 13

(iii) 8.02, 8.2, 8.1, 8.002

6.8 Use of decimals in daily life

Decimals are very useful in our daily life in form of money, weight, capacity etc. In this section, we shall learn about use of decimals in different fields of our life.

6.8.1 CURRENCY (Money)

Conversion of paise into rupees :

In India, money is expressed in rupees and paise.

i.e. 100 paise = ₹1

So 1 paisa is one hundredth of a rupee.

i.e. 1 paisa = ₹$\frac{1}{100}$ = ₹ 0.01

Similarly 2 paise = ₹$\frac{2}{100}$ = ₹0.02

5 paise = ₹$\frac{5}{100}$ = ₹0.05

45 paise = ₹$\frac{45}{100}$ = ₹0.45

Let's consider some examples:

Example 14: **Write the following money in rupees using decimals:**

(i) 60 paise **(ii) 125 paise** **(iii) 5 rupees 50 paise**

(iv) 18 rupees 99 paise **(v) 25 rupees 5 paise**

Solution : (i) 60 paise $= ₹\frac{60}{100} = ₹0.60$ $(\because 1 \text{ paisa} = ₹\frac{1}{100})$

(ii) 125 paise $= ₹\frac{125}{100} = ₹1.25$ $(\because 1 \text{ paisa} = ₹\frac{1}{100})$

(iii) 5 rupees 50 paise

= (5 rupees) + (50 paise)

$= ₹5 + ₹\frac{50}{100} = ₹\ 5 + ₹\ 0.50 = ₹\ 5.50$ $(\because 1 \text{ paisa} = ₹\frac{1}{100})$

(iv) 18 rupees 99 paise

= (18 rupees) + (99 paise)

$= ₹18 + ₹\frac{99}{100}$ $(\because 1 \text{ paisa} = ₹\frac{1}{100})$

$= ₹18 + ₹0.99 = ₹18.99$

(v) 25 rupees 5 paise

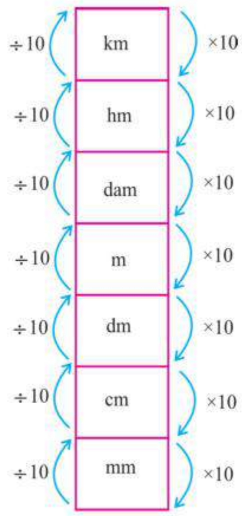
= (25 rupees) + (5 paise)

$= ₹25 + ₹\frac{5}{100}$ $(\because 1 \text{ paisa} = ₹\frac{1}{100})$

$= ₹25 + ₹0.05 = ₹25.05$

6.8.2 Length or Distance

Conversion of centimetre into metre :



We know that 100cm = 1 metre

$1\text{cm} = \frac{1}{100}\text{m} = 0.01\text{m}$

Similarly $2\text{cm} = \frac{2}{100}\text{m} = 0.02\text{m}$

$7\text{cm} = \frac{7}{100}\text{m} = 0.07\text{m}$

$35\text{cm} = \frac{35}{100}\text{m} = 0.35\text{m}$

Example 15. **Express as metre using decimal.**

(i) 4cm **(ii) 185cm** **(iii) 3m 32cm**

Solution : (i) $4\text{cm} = \frac{4}{100}\text{m} = 0.04\text{m}$ $(\because 1\text{cm} = \frac{1}{100}\text{m})$

(ii) $185\text{cm} = \frac{185}{100}\text{m} = 1.85\text{m}$ $\quad (\because \ 1\text{cm} = \frac{1}{100}\text{m})$

(iii) $3\text{m } 32\text{cm} = 3\text{m} + 32\text{cm}$

$= 3\text{m} + \frac{32}{100}\text{m}$ $\quad (\because \ 1\text{cm} = \frac{1}{100}\text{m})$

$= 3\text{m} + 0.32\text{m} = 3.32\text{m}$

Conversion of millimere into centimetre

We know that 10mm = 1 cm

$\Rightarrow \quad 1\text{mm} = \frac{1}{10}\text{cm} = 0.1 \text{ cm}$

Similarly $\quad 3\text{mm} = \frac{3}{10}\text{cm} = 0.3\text{cm}$

$8\text{mm} = \frac{8}{10}\text{ cm} = 0.8\text{cm}$

Example 16. **Express as centimetre using decimals:**

(i) 5mm **(ii) 28mm** **(iii) 5cm 3mm**

Solution : (i) $5\text{mm} = \frac{5}{10}\text{ cm} = 0.5\text{cm}$ $\quad (1\text{mm} = \frac{1}{10}\text{cm})$

(ii) $28\text{mm} = \frac{28}{10}\text{cm} = 2.8\text{cm}$ $\quad (1\text{mm} = \frac{1}{10}\text{cm})$

(iii) $5\text{cm } 3\text{mm} = 5\text{cm} + 3\text{mm}$

$= 5\text{cm} + \frac{3}{10}\text{ cm} = 5\text{ cm} + 0.3\text{ cm}$

$= 5.3\text{cm}$

Conversion of metre into kilometre

- We know that 1000m = 1km

$\Rightarrow \quad 1\text{m} = \frac{1}{1000}\text{km} = 0.001\text{km}$

Similarly $\quad 42\text{m} = \frac{42}{1000}\text{km} = 0.042\text{km}$

$180\text{m} = \frac{180}{1000}\text{km} = 0.180\text{km}$

Example 17. **Express as kilometre using decimals:**

(i) 35m **(ii) 1250m** **(iii) 5km 45m**

Solution : (i) $35\text{m} = \frac{35}{1000}\text{km} = 0.035\text{km}$ $\quad (1\text{m} = \frac{1}{1000}\text{km})$

(ii) $1250\text{m} = \frac{1250}{1000}\text{km} = 1.250\text{km}$ $\quad (1\text{m} = \frac{1}{1000}\text{km})$

(iii) $5\text{km } 45\text{m} = 5\text{km} + 45\text{m}$

$= 5\text{km} + \frac{45}{1000}\text{km}$

$= 5\text{km} + 0.045\text{ km} = 5.045\text{km}$

6.8.3 Weight

Conversion of grams into kilograms :

We know that $1000\text{g} = 1\text{kg}$

$1\text{g} = \frac{1}{1000}\text{kg} = 0.001\text{kg}$

Similarly $8\text{g} = \frac{8}{1000}\text{kg} = 0.008\text{ kg}$

$72\text{g} = \frac{72}{1000}\text{kg} = 0.072\text{kg}$

$430\text{g} = \frac{430}{1000}\text{kg} = 0.430\text{kg}$

Example 18. **Express as kilogram using decimals:**
(i) 3g (ii) 765g (iii) 4kg 80g

Solution : (i) $3\text{g} = \frac{3}{1000}\text{kg} = 0.003\text{kg}$ $\quad (\because \; 1\text{g} = \frac{1}{1000}\text{kg})$

(ii) $765\text{g} = \frac{765}{1000}\text{kg} = 0.765\text{kg}$ $\quad (\because \; 1\text{g} = \frac{1}{1000}\text{kg})$

(iii) $4\text{kg } 80\text{g} = 4\text{kg} + 80\text{g}$

$= 4\text{kg} + \frac{80}{1000}\text{kg} = 4\text{kg} + 0.080\text{ kg}$

$= 4.080\text{kg}$

6.8.4 Capacity

Conversion of millilitre into litre

We know that $1000\text{ m}\ell = 1\text{litre}$

$1\text{ m}\ell = \frac{1}{1000}\ell = 0.001\ell$

Similarly $9\text{ m}\ell = \frac{9}{1000}\ell = 0.009\ell$

$65\text{ m}\ell = \frac{65}{1000}\ell = 0.065\ell$

$325\text{ m}\ell = \frac{325}{1000}\ell = 0.325\ell$

Example 19. **Express as litre using decimals :**

(i) 50 mℓ **(ii) 665mℓ** **(iii) 2ℓ25mℓ**

Solution: (i) $50m\ell = \frac{50}{1000}\ell = 0.050\ell$ $\quad (1m\ell = \frac{1}{1000}\ell)$

(ii) $665m\ell = \frac{665}{1000}\ell = 0.665\ell$ $\quad (1m\ell = \frac{1}{1000}\ell)$

(iii) $2\ell\ 25m\ell = 2\ell + 25m\ell$

$= 2\ell + \frac{25}{1000}\ell = 2\ell + 0.025\ell$

$= 2.025\ell$

Exercise 6.3

1. Express as rupee using decimals:

(i) 35 paise (ii) 4 paise (iii) 240 paise

(iv) 12 rupees 25 paise (v) 24 rupees 5 paise

2. Express as metre using decimals:

(i) 5cm (ii) 62cm (iii) 135cm (iv) 5m 20cm (v) 12m 8cm

3. Express as centimetre using decimals:

(i) 2mm (ii) 28mm (iii) 8cm 4mm

4. Express as kilometre using decimals:

(i) 7m (ii) 50m (iii) 425m (iv) 2475m (v) 3km 225m

5. Express as kilogram using decimals:

(i) 5g (ii) 75g (iii) 423g (iv) 1265g (v) 5kg 418g

6. Express as litre using decimals:

(i) 2mℓ (ii) 80mℓ (iii) 725mℓ (iv) 3ℓ 423mℓ (v) 8ℓ 20mℓ

6.9 Addition of Decimals

Addition of decimals is same as addition of whole numbers. The only difference is that we ensure that all decimal points will be in same column before addition. We use the following steps to add the decimals.

Step 1. Draw a dotted line to represents decimal point.

Step 2. Write the decimals in column so that tenths place digit comes under tenths place digits, hundredths comes under hundredths and so on.

Step 3. Convert the given decimals into like decimals.

Step 4. Add as we add whole numbers.

The following examples will make this concept more clear.

Example 20. **Add the followings**

(i) 4.23 + 5.69 **(ii) 3.15 + 4.234** **(iii) 1.2 + 18.67**

(iv) 2.4 + 1.35 + 24.567 **(v) 13.25 + 2.4 + 18**

Solution : (i) We have 4.23 + 5.69

- Draw a dotted line for representation of decimal point.
- Ensure that all decimal points of given decimals must be on this line below each other as shown.
- Add the numbers.

Hence the required answer is 9.92

$$\begin{array}{r} 4.23 \\ +\ 5.69 \\ \hline 9.92 \\ \hline \end{array}$$

(ii) We have 3.15 + 4.234

- Draw a dotted line for representation of decimal point.
- Convert the given decimals into like decimals and add numbers.

Hence, the required answer is 7.384

$$\begin{array}{r} 3.150 \\ +\ 4.234 \\ \hline 7.384 \\ \hline \end{array}$$

(iii) We have 1.2 + 18.67

- Draw a dotted line for representation of decimal point.
- Convert the given decimals into like decimals and add.

Hence, the required answer is 19.87

$$\begin{array}{r} 01.20 \\ +\ 18.67 \\ \hline 19.87 \\ \hline \end{array}$$

> **Common Error:-** This is common error that students can add in this way
>
> - It is not correct as decimals points are not in a column.
>
> So to avoid this mistake ensure that decimal points lie on a vertical line.
>
> $$\begin{array}{r} .12 \\ +\ 18.67 \\ \hline 18.79 \\ \hline \end{array}$$

(iii) We have 2.4 + 1.35 + 24.567

- Draw a dotted line for representation of decimal point.
- Convert into like decimals and add.

Hence, the required answer is 28.317

$$\begin{array}{r} 02.400 \\ 01.350 \\ +\ 24.567 \\ \hline 28.317 \\ \hline \end{array}$$

(iv) We have 13.25 + 2.4 + 18

- Draw a dotted line for representation of decimal point.
- Convert into like decimals as 18 can be written as 18.00 and then add

Hence, the required answer is 33.65

$$\begin{array}{r} 13.25 \\ 02.40 \\ +\ 18.00 \\ \hline 33.65 \\ \hline \end{array}$$

6.10 Subtraction of Decimals

Subtraction of decimals is same as subtraction of whole numbers. The only difference is to ensure that all decimal points will be in same column before subtraction. We use the following steps for subtraction of decimals.

Step 1. Draw a dotted line to represent decimal point.

Step 2. Write the decimals in column so that tenths place digit comes under tenths place digit, hundredths comes under hundredths and so on.

Step 3. Convert the given decimals into like decimals.

Step 4. Subtract as we subtract in whole numbers.

Let's consider some examples:

Example 21. **Subtract the decimals:**

(i) 14.82 – 5.97 **(ii) 25.18 – 18.07** **(iii) 42.3 – 15.78**

(iv) 47.39 – 13.412 **(v) 40 – 4.156**

Solution : (i) We have 14.82 – 5.97

- Draw a dotted line for representation of decimal point.
- Ensure that all decimal points of given decimals must be on this line.
- Subtract the numbers.

	1 4	.	8	2
–	0 5	.	9	7
	8	.	8	5

Hence, the required answer is 8.85

(ii) We have 25.18 – 18.07

- Draw a dotted line for representation of decimal point.
- Ensure that all decimal points of given decimals must be on this line.
- Subtract the numbers.

	2	5	.	1	8
–	1	8	.	0	7
		7	.	1	1

Hence, the required answer is 7.11

(iii) We have 42.3 – 15.78

- Draw a dotted line for representation of decimal point.
- Convert the given decimals into like decimals and then subtract

	4	2	.	3	0
–	1	5	.	7	8
	2	6	.	5	2

Hence, the required answer is 26.52

(iv) We have 47.39 – 13.412

- Draw a dotted line for representation of decimal point.
- Convert the given decimals into like decimals and then subtract

	4	7	.	3	9	0
–	1	3	.	4	1	2
	3	3	.	9	7	8

Hence, the required answer is 33.978

(v) We have 40 – 4.156

- Draw a dotted line for representation of decimals point.
- Convert the given decimals into like decimals and then subtract

	4	0	.	0	0	0
–	0	4	.	1	5	6
	3	5	.	8	4	4

Hence, the required answer is 35.844

Example 22 : (i) Subtract 12.83 from 19.672.

(ii) Subtract 24.67 from 32.

Solution:

(i)

	19.672
–	12.830
	6.842

(ii)

	32.00
–	24.67
	7.33

6.11 Word Problems

In this section, we shall deal with daily life problems of decimals in addition and subtraction.

Example 23. **Three bags contain 45kg, 38.16kg and 47.258kg of rice respectively. What is the total weight of the rice in the bags?**

Solution : Total weight of the rice in the bags = Sum of weight of all three bags

$$\begin{array}{r} 45.000 \\ 38.160 \\ +\ 47.258 \\ \hline 130.418 \\ \hline \end{array}$$

Hence, total weight of rice is 130.418 kg.

Example 24. **Mandeep buys books worth ₹ 86.75, pencils for ₹ 28.2 and geometry box for ₹ 54.25. How much she has to pay?**

Solution : Price of books = ₹ 86.75

Price of pencils = ₹ 28.20

Price of geometry box = ₹ 54.25

Total amount she has to pay = ₹ 86.75 + ₹28.20 + ₹54.50

= ₹169.20

Example 25. **The height of Raman and Aashish are 1.64 m and 0.98 m respectively. How much Aashish is shorter than Raman?**

Solution : To solve sum, we subtract heights of boths

Height of Raman = 1.64 m

Height of Aashish = 0.98 m

∴ Differnce of heights = 1.64 m – 0.98 m = 0.66 m

Hence, Aashish is 0.66m shorter than Raman.

Example 26. **From a ribbon of length 25m, two pieces of 8.2m and 5.65 m were cut. Find the length of the remaining part.**

Solution : Given, Total length of the ribbon = 25 m

Length of first piece = 8.2 m

and length of 2nd piece = 5.65 m

Now length of both pieces = 8.2 m + 5.65 m = 13.85 m

∴ Length of Remaining part = (Total length) – (Sum of length of both pieces)

25.00 m – 13.85m = 11.15 m

Hence the required length is 11.15m.

Exercise 6.4

1. Solve the following:

(i) 12.15 + 4.87 (ii) 23.5 + 13.47 (iii) 12.56 + 6.234

(iv) 24.25 –13.12 (v) 18.8 – 4.26 (vi) 42.34 – 5.256

(vii) 45.4 + 13.25 + 28.68 (viii) 52.9 + 26.893 + 13.62

(ix) 42 – 27.563 (x) 64.26 – 43.589 + 13.42

(xi) 18.3 + 2.56 – 11.643 (xii) 66.5 – 13.49 – 29.712

2. (i) Subtract 21.92 from 32.683

(ii) Subtract 14.812 from 23.

3. What should be added to 3.412 to get 7?

4. Khan spent ₹63.25 for Maths book and ₹48.99 for English book. Find the total amount spent by Khan.

5. Samar walked 3km 450m in morning and 2km 585m in evening. How much distance did he walk in all?

6. Sheetal has ₹190.50 in her pocket. She buys a school bag for ₹123.99. How much money is left with her now?

7. A piece of 18.56m long ribbon is cut into three pieces. If the length of two pieces are 8.75m and 3.125m respectively. Find the length of the third piece.

8. Veerpal bought vegetables weighing 20kg. Out of this 6kg 750kg are onions, 5kg 25g are potatoes and rest are tomatoes. What is the weight of the tomatoes?

9. Ashish's school is 28km far from his house. He covers 14km 250m by bus, 12km 650m by car and the remaining distance by foot. How much distance does he cover on foot?

Multiple Choice Questions

1. $3+\frac{2}{10}$ = ...

(a) 302 (b) 3.2 (c) 3.02 (d) 30.2

2. $200 + 4 + \frac{5}{10}$ = ...

(a) 24.5 (b) 204.05 (c) 204.5 (d) 24.05

3. $\frac{7}{100}$ = ...

(a) .07 (b) 700 (c) .007 (d) 7

4. $50+\frac{3}{1000}$ = ...

(a) 50.3 (b) 503000 (c) 50.0003 (d) 50.003

5. Seventy and four thousandths = ...

(a) 74000 (b) 70.004 (c) .00074 (d) .074