SAMPLE QUESTION PAPER (STANDARD) - 03

Class 10 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. $\triangle$PQR $\sim$ $\triangle$XYZ and the perimeters of $\triangle$PQR and $\triangle$XYZ are 30 cm and 18 cm respectively. If QR = 9 cm, then, YZ is equal to **[1]**

   a) 4.5 cm. b) 5.4 cm.

   c) 12.5 cm. d) 9.5 cm.

2. The zeros of the polynomial $x^2 + \frac{1}{6}x - 2$ are **[1]**

   a) $\frac{-4}{3}, \frac{3}{2}$ b) -3, 4

   c) $\frac{-3}{2}, \frac{4}{3}$ d) None of these

3. For what value of k, do the equations kx - 2y = 3 and 3x + y = 5 represent two lines intersecting at a unique point? **[1]**

   a) all real values except -6 b) k = 3

   c) k = 6 d) k = -3

4. The graphic representation of the equations x + 2y = 3 and 2x + 4y + 7 = 0 gives a pair of **[1]**

   a) parallel lines b) none of these

   c) coincident lines d) intersecting lines

5. In a $\triangle$ABC, AD is the bisector of $\angle$BAC. If AB = 8cm, BD = 6cm and DC = 3 cm. Find AC **[1]**

   a) 6 cm b) 4 cm

c) 3 cm | d) 8 cm

6. A girl has a cube one letter written on each face, as shown below: **[1]**

M, N, P, M, N, M

The cube is thrown once. The probability of getting M is

a) $\frac{1}{3}$ | b) $\frac{1}{5}$
c) $\frac{1}{2}$ | d) $\frac{1}{4}$

7. A contractor planned to install a slide for the children to play in a park. If he prefers to have a slide whose top is at a height of 1.5 m and is inclined at an angle of 30° to the ground, then the length of the slide would be **[1]**

a) $\sqrt{3}$ m | b) 3 m
c) 1.5 m | d) $2\sqrt{3}$ m

8. Consider the following frequency distribution of the heights of 60 students of a class : **[1]**

Height (in cm)	Number of students
150-155	15
155-160	13
160-165	10
165-170	8
170-175	9
175-180	5

The sum of the lower limit of the modal class and upper limit of the median class is

a) 310 | b) 330
c) 320 | d) 315

9. A vertical stick 1.8 m long casts a shadow 45 cm long on the ground. At the same time, what is the length of the shadow of a pole 6 m high? **[1]**

a) 13.5 m | b) 1.35 m
c) 1.5 m | d) 2.4 m

10. The LCM of two numbers is 1200. Which of the following cannot be their HCF? **[1]**

a) 500 | b) 200
c) 600 | d) 400

11. The number of quadratic equations having real roots and which do not change by squaring their roots is **[1]**

a) 3 | b) 1
c) 4 | d) 2

12. The perimeter of the triangle formed by the points (0, 0), (1, 0) and (0, 1) is **[1]**

a) $2 + \sqrt{2}$ | b) 3
c) $\sqrt{2} + 1$ | d) $1 \pm \sqrt{2}$

13. The wickets taken by a bowler in 10 cricket matches are 2, 6, 4, 5, 0, 3, 1, 3, 2, 3. The mode of the data is **[1]**

a) 1

b) 2

c) 4

d) 3

14. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ is equal to **[1]**

a) $tan\,\theta\ -\ sec\,\theta$

b) $-sec\,\theta\ -\ tan\,\theta$

c) $sec\,\theta\ +\ tan\,\theta$

d) $sec\,\theta$ - $tan\,\theta$

15. If two trees of height ‘x’ and ‘y’ standing on the two ends of a road subtend angles of 30° and 60° respectively at the midpoint of the road, then the ratio of x : y is **[1]**

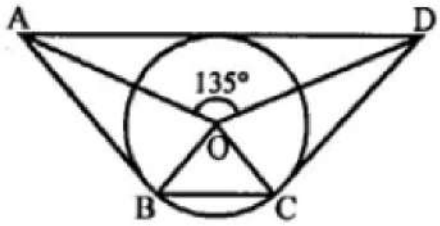
a) 1 : 3

b) 1 : 2

c) 3 : 1

d) 1 : 1

16. In the given figure, If ∠AOD = 135° then ∠BOC is equal to **[1]**



a) 45°

b) 25°

c) 52.5°

d) 62.5°

17. Out of the given statements **[1]**

A. The areas of two similar triangles are in the ratio of the corresponding altitudes.

B. If the areas of two similar triangles are equal, then the triangles are congruent.

C. The ratio of areas of two similar triangles is equal to the ratio of their corresponding medians.

D. The ratio of the areas of two similar triangles is equal to the ratio of their corresponding sides.

The correct statement is

a) (C)

b) (B)

c) (A)

d) (D)

18. A quadratic equation $ax^2 + bx + c = 0$ has non-real roots, if **[1]**

a) $b^2 - 4ac > 0$

b) $b^2 - 4ac = 0$

c) $b^2 - 4ac < 0$

d) $b^2 - ac = 0$

19. **Assertion (A):** $x^2 + 7x + 12$ has no real zeros **[1]**

Reason (R): A quadratic polynomial can have at the most two zeroes.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 350 cm^2. **[1]**

Reason (R): Total surface area of a cuboid is 2(lb + bh + hl)

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/hr less than that of the fast train, find the speeds of the two trains. **[2]**

22. In what ratio does the point C(4, 5) divide the join of A(2, 3) and B(7, 8)? **[2]**

23. Find the HCF and LCM of 6, 72 and 120 using fundamental theorem of arithmetic. **[2]**

24. If $\cot\theta = \frac{15}{8}$, then evaluate: $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$. **[2]**

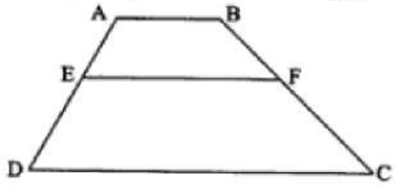
OR

If cos (A-B) = $\frac{\sqrt{3}}{2}$ and sin (A + B) = $\frac{\sqrt{3}}{2}$, find A and B, where (A + B) and {A- B) are acute angles.

25. In a $\triangle ABC$, AD is the bisector of $\angle A$, meeting side BC at D. If AC = 4.2 cm, DC = 6 cm and BC = 10 cm, find AB. **[2]**

OR

If EF $\|$DC $\|$ AB, prove that $\frac{AE}{ED} = \frac{BF}{FC}$



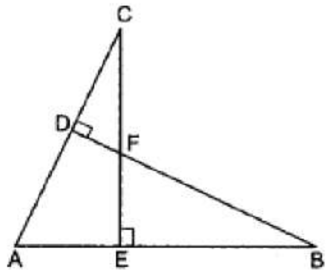
Section C

26. A two-digit number is such that the product of its digits is 15. If 18 is added to the number, the digits interchange their places. Find the number. **[3]**

27. In Fig. if $BD \perp AC$ and $CE \perp AB$, prove that **[3]**

i. ΔAEC ~ ΔADB

ii. $\frac{CA}{AB} = \frac{CE}{DB}$



28. Find the distance between the pair of points $(a\sin\alpha, -b\cos\alpha)$ and $(-a\cos\alpha, b\sin\alpha)$. **[3]**

OR

Find the co-ordinates of the points which divide the line segment joining the points (5, 7) and (8, 10) in 3 equal parts.

29. Prove that $\sqrt{5}+\sqrt{3}$ is irrational. **[3]**

30. From a balloon vertically above a straight road, the angles of depression of two cars at an instant are found to be 45° and 60°. If the cars are 100 m apart, find the height of the balloon. **[3]**

OR

The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of15 seconds, the angle of elevation changes to 30°. If the aeroplane is flying at a constant height of $1500\sqrt{3}$m, find the speed of the plane in km/hr.

31. Find the mean of the following data, using step-deviation method: **[3]**

Class	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75
Frequency	6	10	16	15	24	8	7

Section D

32. Solve graphically system of linear equations. Also, find the coordinates of the points where the lines meet the axis of x in each system: **[5]**

2x + 3y = 8

x - 2y = -3

OR

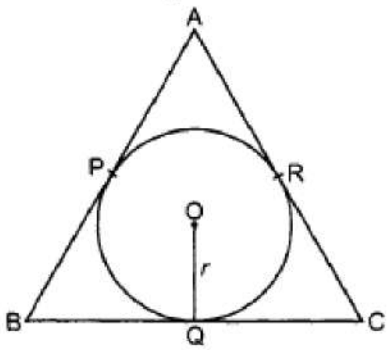
Use a single graph paper and draw the graph of the following equations:

2y - x = 8; 5y - x = 14, y - 2x = 1

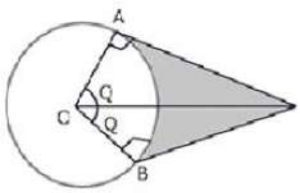
Obtain the vertices of the triangle so obtained.

33. In figure, the sides AB, BC and CA of triangle ABC touch a circle with centre O and radius r at P, Q and R respectively. Prove that **[5]**

i. AB + CQ = AC + BQ
ii. Area (ABC) = $\frac{1}{2}$ (perimeter of $\triangle$ABC) x r



34. An elastic belt is placed around therein of a pulley of radius 5cm. One point on the belt is pulled directly away from the center O of the pulley until it is at P, 10cm from O. Find the length of the best that is in contact with the rim of the pulley. Also, find the shaded area. **[5]**



OR

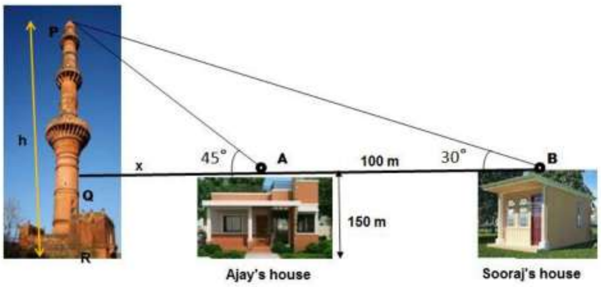
Find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres (Use π = 22/7).

35. The king, queen and jack of clubs are removed from a deck of 52 cards. The remaining cards are mixed together and then a card is drawn at random from it. Find the probability of getting (i) a face card, (ii) a card of heart, (iii) a card of clubs (iv) a queen of diamond. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

The houses of Ajay and Sooraj are at 100 m distance and the height of their houses is the same as approx 150 m. One big tower was situated near their house. Once both friends decided to measure the height of the tower. They measure the angle of elevation of the top of the tower from the roof of their houses. The angle of elevation of ajay's house to the tower and sooraj's house to the tower are 45^o and 30^o respectively as shown in the figure.



(i) Find the height of the tower.

(ii) What is the distance between the tower and the house of Sooraj?

(iii) Find the distance between top of the tower and top of Sooraj's house?

OR

Find the distance between top of tower and top of Ajay's house?

37. **Read the text carefully and answer the questions:** **[4]**

The students of a school decided to beautify the school on an annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 metre. The flags are stored at the position of the middlemost flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time.

(i) How much distance did she cover in pacing 6 flags on either side of center point?

(ii) Represent above information in Arithmetic progression

OR

What is the maximum distance she travelled carrying a flag?

(iii) How much distance did she cover in completing this job and returning to collect her books?

38. **Read the text carefully and answer the questions:** **[4]**

Rohan makes a project on coronavirus in science for an exhibition in his school. In this Project, he picks a sphere which has volume 38808 cm^3 and 11 cylindrical shapes each of Volume 1540 cm^3 with 10 cm length.

(i) Find the area covered by cylindrical shapes on the surface of a sphere.

(ii) Find the diameter of the sphere.

OR

Find the curved surface area of the cylindrical shape.

(iii) Find the total volume of the shape.

Solution

SAMPLE QUESTION PAPER (STANDARD) - 03

Class 10 - Mathematics

Section A

1. **(b)** 5.4 cm.

   Explanation: Given: $\triangle PQR \sim \triangle XYZ$

   $\therefore \frac{\text{Perimeter of } \Delta PQR}{\text{Perimeter of } \Delta XYZ} = \frac{QR}{YZ}$

   $\Rightarrow \frac{30}{18} = \frac{9}{YZ}$

   $\Rightarrow$YZ = 5.4 cm

2. **(c)** $\frac{-3}{2}, \frac{4}{3}$

   Explanation: $x^2 + \frac{1}{6}x - 2 = \frac{6x^2+x-12}{6}$

   $6x^2 + x - 12 = 6x^2 + 9x - 8x - 12 = 3x(2x + 3) - 4(2x + 3)$

   = (2x + 3)(3x - 4)

   $\therefore$ the zeros are $\frac{-3}{2}$ and $\frac{4}{3}$

3. **(a)** all real values except -6

   Explanation: For a unique intersecting point, we have $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

   $\therefore \frac{k}{3} \neq \frac{-2}{1} \Rightarrow k \neq -6$

4. **(a)** parallel lines

   Explanation: Given: Two equations, x + 2y = 3

   ⇒ x + 2y – 3 = 0 .... (i)

   2x + 4y + 7 = 0 .... (ii)

   We know that the general form for a pair of linear equations in 2 variables x and y is $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

   Comparing with above equations,

   we have $a_1 = 1, b_1 = 2, c_1 = -3; a_2 = 2, b_2 = 4, c_2 = 7$

   $\frac{a_1}{a_2} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}; \frac{c_1}{c_2} = \frac{-3}{7}$

   Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

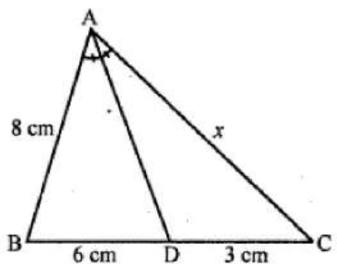
   $\therefore$Both lines are parallel to each other.

5. **(b)** 4 cm

   Explanation:

   In $\triangle$ABC, AD is the bisector of $\angle$BAC

   AB = 8 cm, BD = 6 cm and DC = 3 cm

   Let AC = x

   $\because$ In $\triangle$ ABC, AD is the bisector of $\angle$A

   $\therefore \frac{AB}{AC} = \frac{BD}{DC} \Rightarrow \frac{8}{x} = \frac{6}{3}$

   $\Rightarrow x = \frac{8 \times 3}{6} = 4$

   $\therefore \text{AC} = 4\text{cm}$

6. **(c)** $\frac{1}{2}$

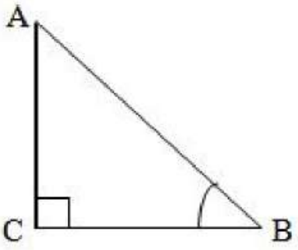
   Explanation: Number of possible outcomes (getting M) = 3

Number of total outcomes = 6

$\therefore$ Required Probability = $\frac{3}{6} = \frac{1}{2}$

7. **(b)** 3 m

Explanation:



Here, Height of the slide = AC = 1.5 m,

Angle of elevation = $\theta = 30^\circ$ To find: Length of slide = AB

$\therefore \sin 30^\circ = \frac{AC}{AB}$

$\Rightarrow \frac{1}{2} = \frac{1.5}{AB}$

$\Rightarrow AB = 3$ m

8. **(d)** 315

Explanation: 150-155 is the modal class

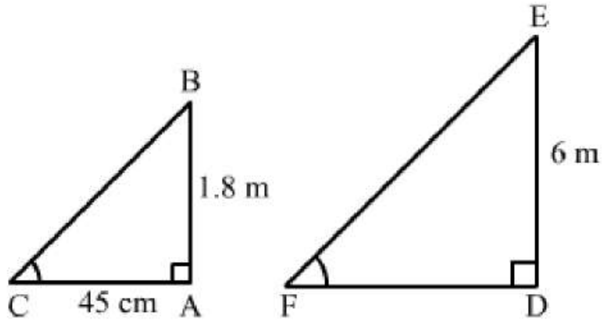
Height in cm	Number of students (f)	Cumulative Frequency (CF)
150-155	15	15
155-160	13	28
160-165	10	38
165-170	8	46
170-175	9	55
175-180	5	60
Total	60	

Here, $\frac{N}{2}$ = 30, the commulative frequency just above 30 is 38 and the corresponding class is 160-165 which is the median class.

hence the required sum = 115 + 165 = 315

9. **(c)** 1.5 m

Explanation:



Let AB and AC be the vertical stick and its shadow, respectively.

According to the question:

AB = 1.8 m

AC = 45 cm = 0.45 m

Again, let DE and DF be the pole and its shadow, respectively.

According to the question:

DE = 6 m

DF = ?

Now, in right-angled triangles ABC and DEF, we have:

$\angle$BAC = $\angle$EDF = 90°

$\angle$ACB = $\angle$DFE (Angular elevation of the Sun at the same time)

Therefore, by AA similarity theorem,

we get:$\triangle$ABC$\sim\triangle$DEF

$\Rightarrow \frac{AB}{AC} = \frac{DE}{DF} \Rightarrow \frac{1.8}{0.45} = \frac{6}{DF} \quad \Rightarrow DF = \frac{6\times0.45}{1.8} = 1.5\text{m}$

10. **(a)** 500

Explanation: It is given that the LCM of two numbers is 1200 .

We know that the HCF of two numbers is always the factor of LCM.

500 is not the factor of 1200.

So this cannot be the HCF.

11. **(a)** 3

Explanation: We are given that quadratic equations have real roots and the quadratic equation does not change by squaring their roots. We have to find the number of quadratic equations.

The possible roots (1,1),(1,0),(0,0)

The general formula of quadratic equation is;

x^2- (sum of roots)x + product of roots

So, we have;

Case-I: When roots are 1 and 1

$x^2 - (1+1)x + 1 = 0$

$x^2 - 2x + 1 = 0$

Case-II: When roots are 1 and 0

$x^2 - x = 0$

Case-III: When roots are 0 and 0

Then, $x^2 = 0$

Therefore, 3 possible quadratic equation.

12. **(a)** $2+\sqrt{2}$

Explanation: Let the vertices of $\triangle\text{ABC}$ be A(0, 0), B(1, 0) and C(0, 1)

Now length of AB $= \sqrt{(1-0)^2 + (0-0)^2}$

$= \sqrt{(1)^2 + 0^2} = \sqrt{1^2} = 1$

Length of AC $= \sqrt{(0-0)^2 + (1-0)^2} = \sqrt{0^2 + (1)^2}$

$= \sqrt{1^2} = 1$

and length of BC $= \sqrt{(0-1)^2 + (1-0)^2}$

$= \sqrt{(1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$

Perimeter of $\triangle\text{ABC}$ = Sum of sides

$= 1 + 1 + \sqrt{2} = 2 + \sqrt{2}$

13. **(d)** 3

Explanation: In the given data, the frequency of 3 is more than those other wickets taken by a bowler.

Therefore, Mode of given data is 3.

14. **(c)** $sec\,\theta\ +\ tan\,\theta$

Explanation: Given: $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$

$= \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} \times \sqrt{\frac{1+\sin\theta}{1+\sin\theta}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$

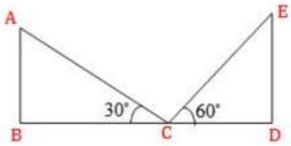
$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

$= \frac{1+\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$= \sec\theta + \tan\theta$

15. **(a)** 1 : 3

Explanation: 

Here two trees AB and ED are of height x x and y y respectively. And BC = CD

$\therefore \tan 30^\circ = \frac{x}{\mathrm{BC}}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{\mathrm{BC}}$

$\Rightarrow x = \frac{\mathrm{BC}}{\sqrt{3}}$ And $\tan 60^\circ = \frac{y}{\mathrm{CD}}$

$\Rightarrow \sqrt{3} = \frac{y}{\mathrm{CD}}$

$\Rightarrow y = \mathrm{CD}\sqrt{3} = \mathrm{BC}\sqrt{3}$ [BC = CD]

Now, $\frac{x}{y} = \frac{\mathrm{BC}}{\sqrt{3} \times \mathrm{BC}\sqrt{3}}$

= $\frac{1}{3}$

$\Rightarrow x : y = 1 : 3$

16. **(a)** 45°

Explanation: In the given figure, ∠AOD = 135°

We know that if a circle is inscribed in a quadrilateral, the opposite sides subtend supplementary angles.

∠AOD + ∠BOC = 180°

135° + ∠BOC = 180°

⇒∠BOC = 180° – 135° = 45°

17. **(b)** (B)

Explanation: If the areas of two similar triangles are equal, then the triangles are congruent

Option (i) is wrong since "The areas of two similar triangles are in the ratio of the square of the corresponding altitudes.

Like those options (iii) and (iv) are also wrong.

18. **(c)** $b^2 - 4ac < 0$

Explanation: The roots of the quadratic equation $ax^2 + bx + c = 0$, In this formula the term $b^2 - 4ac$ is called the discriminant. If $b^2 - 4ac = 0$, so the equation has a single repeated root. If $b^2 - 4ac > 0$, the equation has two real roots. If $b^2 - 4ac < 0$, the equation has two complex roots.

19. **(d)** A is false but R is true.

Explanation: $x^2 + 7x + 12 = 0$

$\Rightarrow x^2 + 4x + 3x + 12 = 0$

$\Rightarrow x(x + 4) + 3(x + 4) = 0$

$\Rightarrow (x + 4)(x + 3) = 0$

$\Rightarrow (x + 4) = 0$ or $(x + 3) = 0$

$\Rightarrow x = -4$ or $x = -3$

Therefore, $x^2 + 7x + 12$ has two real zeroes.

20. **(d)** A is false but R is true.

Explanation: A is false but R is true.

Section B

21. Let the speed of the slow train be x km/hr

Then, the speed of the fast train = (x+10) km/hr

As we know that Time = $\frac{Distance}{Speed}$

Time taken by the fast train to cover 600 km = $\frac{600}{x+10}$ hrs

Time taken by the slow train to cover 600 km = $\frac{600}{x}$ hrs

$\therefore \frac{600}{x} - \frac{600}{x+10} = 3$

$\Rightarrow \frac{600(x+10) - 600x}{x(x+10)} = 3$

$\Rightarrow \frac{6000}{x^2 + 10x} = 3$

$\Rightarrow 3x^2 + 30x - 6000 = 0$

$\Rightarrow 3(x^2 + 10x - 2000) = 0 \, or \, x^2 + 10x - 2000 = 0$

$\Rightarrow x^2 + 50x - 40x - 2000 = 0$

$\Rightarrow x(x + 50) - 40(x + 50) = 0$

$\Rightarrow (x + 50)(x - 40) = 0$

Either x = -50 or x = 40

But the speed of the train cannot be negative. So, x= 40

Hence, the speed of the two trains are 40km/hr and 50km/hr respectively.

22. Let the point C(4, 5) divides the join of A(2, 3) and B(7, 8) in the ratio k:1

The point C is $\left(\frac{7k+2}{k+1}, \frac{8k+3}{k+1}\right)$

But C is (4, 5)

$\Rightarrow \frac{7k+2}{k+1} = 4$

or 7k + 2 = 4k + 4

or 3k = 2

$\therefore k = \frac{2}{3}$

Thus, C divides AB in the ratio 2:3

23. 6 = 2 × 3

$72 = 8 \times 9 = 2^3 \times 3^2$

$120 = 8 \times 15 = 2^3 \times 3 \times 5$

HCF(6,72,120) =2×3 = 6

LCM (6,12,120)= $2^3 \times 3^2 \times 5$= 360

24. Given $\cot\theta = \frac{15}{8}$

To evaluate: $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$

$= \frac{2(1+\sin\theta)(1-\sin\theta)}{2(1+\cos\theta)(1-\cos\theta)}$

$= \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$

$= (\cot\theta)^2 = \left(\frac{15}{8}\right)^2 = \frac{225}{64}$

Hence, the value of the given expression is $\frac{225}{64}$.

OR

We have,

cos (A -B) = $\frac{\sqrt{3}}{2}$

⇒cos(A-B) = cos30°

A - B =30° .....(i)

Again, sin (A + B) = $\frac{\sqrt{3}}{2}$

⇒sin(A+B)= sin 60°

A + B = 60°.........(ii)

Adding, (i) and (ii),

2A =90°

∴A =45°

Put A=45^o in (ii),

B = 60° - A = 60° - 45° = 15°

Therefore, A=45^o and B=15^o

25. It is given that AC = 4.2 cm, DC = 6 cm and BC = 10 cm

In $\triangle ABC$, AD is the bisector of $\angle A$, meeting side BC at D

We have to find AB

Since AD is $\angle A$ bisector

So $\frac{AC}{AB} = \frac{DC}{BD}$

Then, $\frac{4.2}{AB} = \frac{6}{4}$

⇒ 6 AB = 4.2 × 4

$\Rightarrow AB = \frac{4.2\times4}{6}$

$= \frac{16.8}{6}$

Hence, AB = 2.8 cm

OR

Construction: Join AC and name the point as P where AC cuts EF

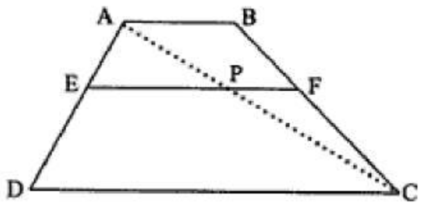
Proof: In $\triangle$ ADC, since EP $\parallel$ DC

$\therefore$ By the basic proportional theorem, we get

$\frac{AE}{ED} = \frac{AP}{PC}$ .....(1)

In $\triangle$ABC, since PF $\parallel$ AB

$\therefore$ By the basic proportional theorem, we get



$\frac{AP}{PC} = \frac{BF}{FC}$ ......(ii)

Comparing (i) and (ii),

$\frac{AE}{ED} = \frac{BF}{FC}$ Hence proved

Section C

26. Assume digit at ten's place = x and digit at unit's place = y

Therefore number = 10x + y

Also xy = 15 ⇒ x = $\frac{15}{y}$ ...(i)

According to given situation we have,

10x + y + 18 = 10y + x

⇒9x - 9y + 18 = 0

⇒x - y + 2 = 0

⇒$\frac{15}{y}$ - y + 2 = 0 (From (i))

⇒15 - y^2 + 2y = 0

⇒y^2 - 2y - 15 = 0

On factorizing the above quadratic equation we get

(y - 5) (y + 3) = 0

⇒y = 5, y = -3 [y = -3 is rejected]

Put the value of y = 5 in equation (i), we obtain

x = $\frac{15}{5}$ = 3

$\therefore$ Number = 3 × 10 + 5 = 35.

27. i. Resorting to the given figure we observe that In Δ's AEC and ADB,

$\angle$AEC = $\angle$ADB = 90° [$\because CE \perp AB$ and $BD \perp AC$]

and, $\angle$EAC = $\angle$DAB [Each equal to $\angle$ A]

Therefore, by AA-criterion of similarity, we obtain

ΔAEC~ ΔADB

ii. We have,

ΔAEC ~ ΔADB [As proved above]

$\Rightarrow \frac{CA}{BA} = \frac{EC}{DB}$ {For similar triangles corresponding sides are proportional}

$\Rightarrow \frac{CA}{AB} = \frac{CE}{DB}$

28. Required distance $= \sqrt{(asin\alpha + acos\alpha)^2 + (-bcos\alpha - b\sin\alpha)^2}$

$= \sqrt{a^2sin^2\alpha + a^2cos^2\alpha + 2a^2sin\alpha cos\alpha + b^2cos^2\alpha + b^2sin\alpha + 2b^2sin\alpha cos\alpha}$

$= \sqrt{a^2(sin^2\alpha + cos^2\alpha) + b^2(sin^2\alpha + cos^2\alpha) + (2a^2 + 2b^2)sin\alpha cos\alpha}$

$= \sqrt{a^2 + b^2 + 2(a^2 + b^2)sin\alpha cos\alpha}$

$= \sqrt{(a^2 + b^2)[1 + 2sin\alpha cos\alpha]}$

$= \sqrt{(a^2 + b^2)[sin^2\alpha + cos^2\alpha + 2sin\alpha cos\alpha]}$

$= \sqrt{(a^2 + b^2)[sin\alpha + cos\alpha]^2}$

$= (sin\alpha + cos\alpha)\sqrt{a^2 + b^2}$

OR

Let P(x, y) and Q(x_1, y_1) trisect AB.

P divides AB in the ratio 1 : 2

$\therefore \quad x = \frac{1(8)+2(5)}{3} = 6$

$y = \frac{1(10)+2(7)}{3} = 8$

Hence, P(6, 8)

And Q is the mid point of PB.

$x_1 = \frac{6+8}{2} = 7$

$y_1 = \frac{8+10}{2} = 9$

Hence, Q(7, 9)

29. Let assume that $\sqrt{5} + \sqrt{3}$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write $\sqrt{5} = \frac{p}{q} - \sqrt{3}$

$(\sqrt{5})^2 = \left(\frac{p}{q} - \sqrt{3}\right)^2$

$5 = \frac{p2}{q2} - \frac{2p\sqrt{3}}{q} + 3$

5 - 3 = $\frac{p^2}{q^2} - \frac{2p\sqrt{3}}{q}$

$\frac{p^2}{q^2} - 2 = \frac{2p\sqrt{3}}{q}$

$\frac{p2-2q2}{qp}$ = √3

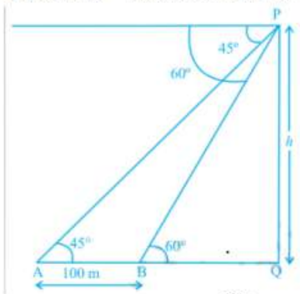
$\frac{p2-2q2}{qp}$ is a rational number as p and q are integers. This contradicts the fact that √3 is irrational, so our assumption is incorrect.

Therefore $\sqrt{5} + \sqrt{3}$ is irrational.

30. Let the height of the balloon at P be h meters (see Fig).

Let A and B be the two cars.

Thus AB = 100 m. From ΔPAQ, AQ = PQ = h



Now from ΔPBQ, $\frac{\text{PQ}}{\text{BQ}} = \tan 60^\circ$

or $\frac{h}{h-100} = \sqrt{3}$

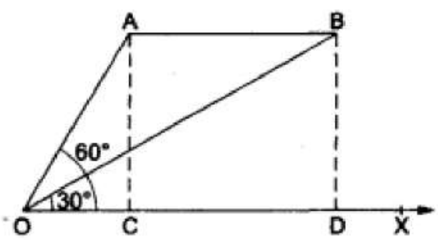
or $h = \sqrt{3}(h - 100)$

Therefore, $h = \frac{100\sqrt{3}}{\sqrt{3}-1} = 50(3 + \sqrt{3})$

i.e, Height of the balloon is $50(3 + \sqrt{3})m$

OR

Let A and B be the two positions of the aeroplane.



Let $AC \perp OX$ and $BD \perp OX$. Then,

$\angle COA = 60^\circ, \angle DOB = 30^\circ$

and AC = BD = $1500\sqrt{3}$m.

From right ΔOCA, we have

$\frac{OC}{AC} = \cot 60^\circ = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{OC}{1500\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow OC = 1500\text{m}$

From right ΔODB,we have

$\frac{OD}{BD} = \cot 30^\circ = \sqrt{3} \Rightarrow \frac{OD}{1500\sqrt{3}\text{m}} = \sqrt{3}$

$\Rightarrow OD = (1500 \times 3)\text{m} = 4500\text{m}.$

$\therefore CD = (OD - OC) = (4500 - 1500)m = 3000m.$

Thus, the aeroplane covers 300m in 15 seconds.

$\therefore$ speed of the aeroplance $= \left(\frac{3000}{15} \times \frac{60\times60}{1000}\right)$ km/hr

= 720 km /hr.

31.

Class Interval	Frequency(f_i)	Mid value x_i	$u_i = \frac{x_i - A}{h} = \frac{x_i - 40}{10}$	$(f_i \times u_i)$
5 - 15	6	10	-3	-18
15 - 25	10	20	-2	-20
25 - 35	16	30	-1	-16
35 - 45	15	40 = A	0	0
45 - 55	24	50	1	24
55 - 65	8	60	2	16
65 - 75	7	70	3	21
	$\Sigma f_i = 86$			$\Sigma(f_i \times u_i)$= 7

Thus, A = 40, h = 10, $\sum f_i = 86$ and $\sum f_i u_i = 7$

Mean = $A + \left\{h \times \frac{\sum f_i u_i}{\sum f_i}\right\}$

$= 40 + \left\{10 \times \frac{7}{86}\right\}$

= 40 + 0.81

= 40.81

Section D

32. The given system of equations is

$2x + 3y = 8$

$x - 2y = -3$

Now,

$2x + 3y = 8$

$\Rightarrow 2x = 8 - 3y$

$\Rightarrow x = \frac{8-3y}{2}$

When $y = 2$, we have

$x = \frac{8-3\times2}{2} = 1$

When y = 4, we have

$x = \frac{8-3\times4}{2} = -2$

x	1	-2
y	2	4

We have,

$x - 2y = -3$

$\Rightarrow x = 2y - 3$

When y = 0, we have

x = 2 × 0 - 3 = -3

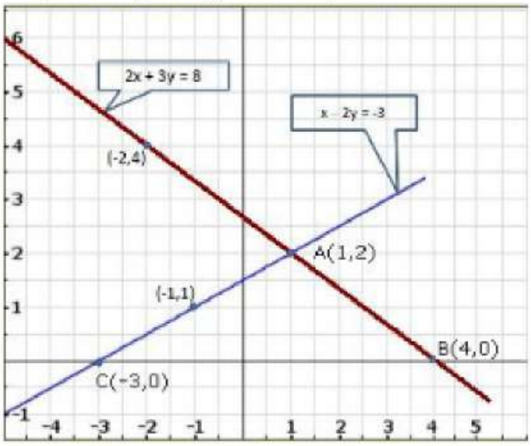
When y = 1, we have

x = 2 × 1 - 3 = -1

Thus, we have the following table;

x	-3	-1
y	0	1

Graph of the given system of equations:



Clearly, the lines intersect at A(1, 2).

Hence, x = 1, y= 2 is the solution of the given system of equations.

We also observe that the lines represented by the equations 2x + 3y = 8 and x - 2y = -3 meet x-axis at B(4, 0) and C(-3, 0) respectively.

OR

We have to use a single graph paper and draw the graph of the following equations:

2y - x = 8; 5y - x = 14, y - 2x = 1

Also, we have to obtain the vertices of the triangle so obtained.

Graph of $2y - x = 8$:

We have, $2y - x = 8 \Rightarrow x = 2y - 8$

When y = 2, we have

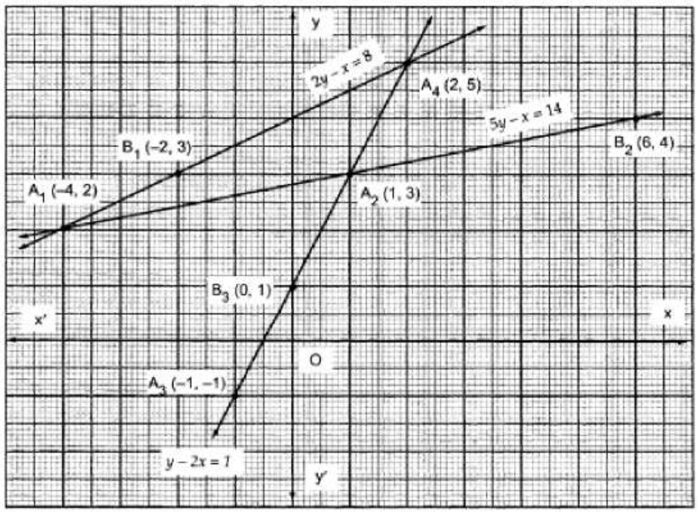
x = 2 × 2 - 8 = - 4

When y = 3, we have

x = 2 ×3 - 8 = 6 - 8 = -2

x	-4	-2
y	2	3

Plot the points $A_1(-4, 2)\ and\ B_1(-2, 3)$ on the graph paper. Join A_1 and B_1 and extend it on both sides to obtain the graph of $2y - x = 8$ as shown in Fig.



Graph of $5y - x = 14$:

We have, $5y - x = 14 \Rightarrow x = 5y - 14$

When y = 3, we have x = 5 × 3 - 14 = 15 - 14 = 1

When y = 4, we have x = 5 × 4 - 14 = 20 - 14 = 6

Thus, we have the following table:

x	1	6
y	3	4

Plot the points $A_2(1, 3)$ and $B_2(6, 4)$ on a graph paper. Join A_2 and B_2 and extend it on both sides to obtain the graph of 5y - x = 14 as shown in Fig.

Graph of $y - 2x = 1$:

We have, $y - 2x = 1 \Rightarrow y = 2x + 1$

When x = -1, we have y = 2 × -1 + 1 = -2 + 1 = -1

When x = 0, we have y = 2 × 0 + 1 = 1

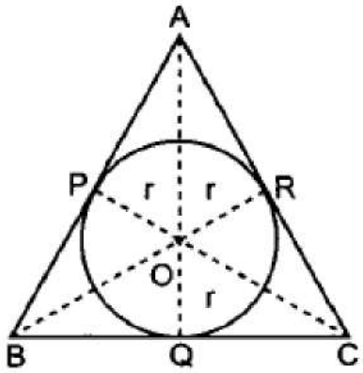
x	-1	0
y	-1	1

Plot the points $A_3(-1 - 1)$ and B_3 (0,1) on the same graph paper. Join A_3 and B_3 and extend it on both sides to obtain the graph of y - 2x = 1 as shown in Fig.

From the graph of the three equations, we find that the three lines taken in pairs intersect each other at points $A_1(-4,2)$, $A_2(1, 3)$ and $A_4(2, 5)$.

Hence, the vertices of the required triangle are (-4, 2), (1, 3) and (2,5).

33. Given, the sides AB, BC and CA of triangle ABC touch a circle with centre O and radius r at P, Q and R respectively.



(i) AP = AR [Tangents from A] ...(i)

Similarly, BP = BQ ...(ii)

CR = CQ ...(iii)

Now,

∵AP = AR

⇒ (AB-BP) = (AC - CR)

⇒ AB + CR = AC + BP

⇒ $AB + CQ = AC + BQ$ [Using eq. (ii) and (iii)]

(ii) Let AB = x, BC = y, AC = z

∴Perimeter of △ABC =x + y + z ...(iv)

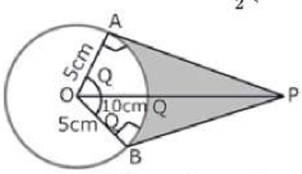
Area of △ABC = $\frac{1}{2}$ [area of △AOB + area of △BOC + area of △AOC]

⇒ Area of △ABC = $\frac{1}{2}$ × AB × OP + $\frac{1}{2}$ × BC × OQ + $\frac{1}{2}$ × AC × OR

⇒ Area of △ABC = $\frac{1}{2} \times x \times r + \frac{1}{2} \times y \times r + \frac{1}{2} \times z \times r$

⇒ Area of △ABC = $\frac{1}{2}(x + y + z) \times$ r

⇒ Area of △ABC = $\frac{1}{2}$(Perimeter of △ABC) × r

34. 

$\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2}$

⇒ $\theta = 60^o$

⇒ ∠AOB = 2 × θ = 120^o

$\therefore$ ARC AB = $\frac{120\times 2\times\pi\times5}{360}cm = \frac{10\pi}{3}cm$ $\left[\because l = \frac{\theta}{360}\times 2\pi r\right]$

Length of the belt that is in contact with the rim of the pulley

= Circumference of the rim - length of arc AB

= $2\pi \times 5$ cm - $\frac{10\pi}{3}$cm

= $\frac{20\pi}{3}$cm

Now, the area of sector OAQB = $\frac{120\times\pi\times5\times5}{360}cm^2 = \frac{25\pi}{3}cm^2$ $\left[\because Area = \frac{\theta}{360}\times\pi r^2\right]$

Area of quadrilateral OAPB = 2(Area of $\triangle$OAP) = $25\sqrt{3}$ cm^2

$[\because AP = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}\ cm]$

Hence, shaded area = $25\sqrt{3} - \frac{25\pi}{3} = \frac{25}{3}[3\sqrt{3} - \pi]\ cm^2$

OR

We have to find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres.

For the first triangle, we have a = 35, b = 53 and c = 66.

$\therefore \quad s = \frac{a+b+c}{2} = \frac{35+53+66}{2} = 77\text{cm}$

Let Δ_1 be the area of the first triangle. Then,

$\Delta_1 = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \Delta_1 = \sqrt{77(77-35)(77-53)(77-66)} = \sqrt{77\times42\times24\times11}$

$\Rightarrow \quad \Delta_1 = \sqrt{7\times11\times7\times6\times6\times4\times11} = \sqrt{7^2\times11^2\times6^2\times2^2} = 7\times11\times6\times2 = 924\text{cm}^2$ ...(i)

For the second triangle, we have a = 33,b = 56,c = 65

$\therefore \quad s = \frac{a+b+c}{2} = \frac{33+56+65}{2} = 77\text{cm}$

Let Δ_2 be the area of the second triangle. Then,

$\Delta_2 = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \Delta_2 = \sqrt{77(77-33)(77-56)(77-65)}$

$\Rightarrow \quad \Delta_2 = \sqrt{77\times44\times21\times12} = \sqrt{7\times11\times4\times11\times3\times7\times3\times4} = \sqrt{7^2\times11^2\times4^2\times3^2}$

$\Rightarrow \quad \Delta_2 = 7\times11\times4\times3 = 924\text{cm}^2$

Let r be the radius of the circle. Then,

Area of the circle = Sum of the areas of two triangles

$\Rightarrow \quad \pi r^2 = \Delta_1 + \Delta_2$

$\Rightarrow \quad \pi r^2 = 924 + 924$

$\Rightarrow \quad \frac{22}{7}\times r^2 = 1848$

$\Rightarrow \quad r^2 = 1848\times\frac{7}{22} = 3\times4\times7\times7 \Rightarrow r = \sqrt{3\times2^2\times7^2} = 2\times7\times\sqrt{3} = 14\sqrt{3}\text{cm}$

35. No of cards removed=3

No. of all possible outcomes n = 52 - 3 = 49

i. No, of face cards left = 12 - 3 = 9 so m=9

$so\ P(E) = \frac{m}{n} = \frac{9}{49}$

ii. No. of cards of heart in the deck = 13 so m=13

$so\ P(E) = \frac{m}{n} = \frac{13}{49}$

iii. No. of cards of clubs = 13 - 3 = 10

$so\ P(E) = \frac{m}{n} = \frac{10}{49}$

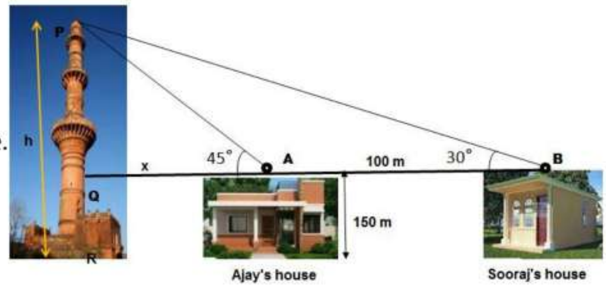
iv. There is only one queen of diamond so m=1

$so\ P(E) = \frac{m}{n} = \frac{1}{49}$

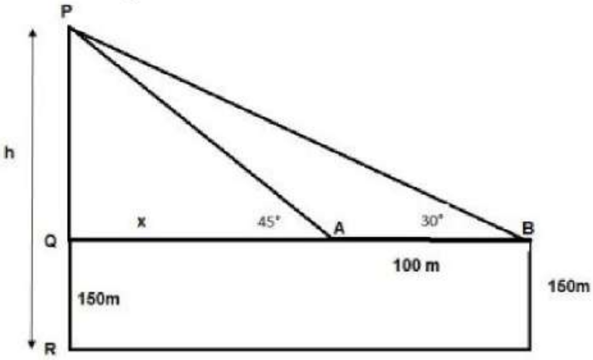
Section E

36. **Read the text carefully and answer the questions:**

The houses of Ajay and Sooraj are at 100 m distance and the height of their houses is the same as approx 150 m. One big tower was situated near their house. Once both friends decided to measure the height of the tower. They measure the angle of elevation of the top of the tower from the roof of their houses. The angle of elevation of ajay's house to the tower and sooraj's house to the tower are 45^o and 30^o respectively as shown in the figure.



(i) The above figure can be redrawn as shown below:



Let PQ = y

In △PQA,

tan 45 = $\frac{PQ}{QA} = \frac{y}{x}$

$1 = \frac{y}{x}$

x = y ...(i)

In △PQB,

tan 30 = $\frac{PQ}{QB} = \frac{PQ}{x+100} = \frac{y}{x+100} = \frac{x}{x+100}$

$\frac{1}{\sqrt{3}} = \frac{x}{x+100}$

$x\sqrt{3} = x + 100$

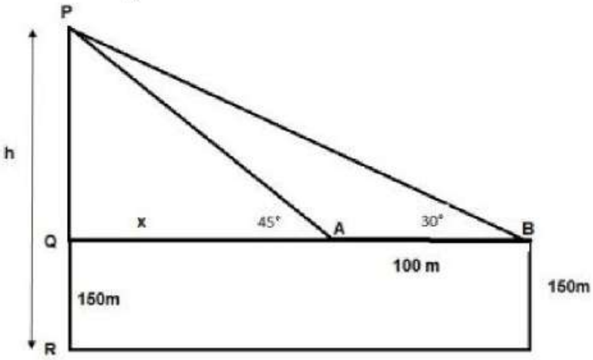
$x = \frac{100}{\sqrt{3}-1} = 136.61$ m

From the figure, height of tower h = PQ + QR

= x + 150 = 136.61 + 150

h = 286.61 m

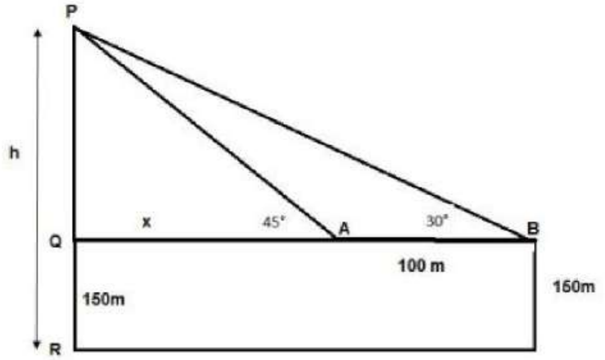
(ii) The above figure can be redrawn as shown below:



Distance of Sooraj's house from tower = QA + AB

= x + 100 = 136.61 + 100 = 236.61 m

(iii) The above figure can be redrawn as shown below:



Distance between top of tower and Top of Sooraj's house is PB

In △PQB

sin $30^o = \frac{PQ}{PB}$

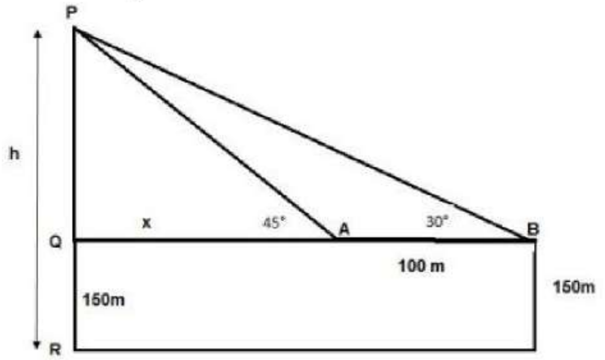
$\Rightarrow PB = \frac{PQ}{\sin 30^o}$

$\Rightarrow$ PB = $\frac{y}{\frac{1}{2}} = 2 \times 136.61$

$\Rightarrow$ PB = 273.20 m

OR

The above figure can be redrawn as shown below:



Distance between top of the tower and top of Ajay's house is PA

In △PQA

sin $45^o = \frac{PQ}{PA}$

$\Rightarrow PA = \frac{PQ}{\sin 45^o}$

$\Rightarrow$ PA = $\frac{y}{\frac{1}{\sqrt{2}}} = \sqrt{2} \times 136.61$

$\Rightarrow$ PA = 193.20 m

37. **Read the text carefully and answer the questions:**

The students of a school decided to beautify the school on an annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 metre. The flags are stored at the position of the middlemost flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time.

(i) Distance covered in placing 6 flags on either side of center point is 84 + 84 = 168 m

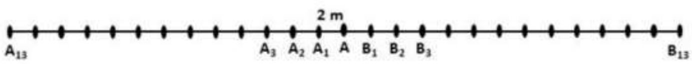
$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_6 = \frac{6}{2}[2 \times 4 + (6 - 1) \times 4]$

$\Rightarrow S_6 = 3[8 + 20]$

$\Rightarrow S_6 = 84$

(ii)



Let A be the position of the middle-most flag.

Now, there are 13 flags ($A_1, A_2 ... A_{12}$) to the left of A and 13 flags ($B_1, B_2, B_3 ... B_{13}$) to the right of A.

Distance covered in fixing flag to A_1 = 2 + 2 = 4 m

Distance covered in fixing flag to A_2 = 4 + 4 = 8 m

Distance covered in fixing flag to A_3 = 6 + 6 = 12 m

...

Distance covered in fixing flag to A_{13} = 26 + 26 = 52 m

This forms an A.P. with,

First term, a = 4

Common difference, d = 4

and n = 13

OR

Maximum distance travelled by Ruchi in carrying a flag

= Distance from A_{13} to A or B_{13} to A = 26 m

(iii) $\because$ Distance covered in fixing 13 flags to the left of A = S_{13}

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{13} = \frac{13}{2}[2 \times 4 + 12 \times 4]$

$= \frac{13}{2} \times [8 + 48]$

$= \frac{13}{2} \times 56$

= 364

Similarly, distance covered in fixing 13 flags to the right of A = 364

Total distance covered by Ruchi in completing the task

= 364 + 364 = 728 m

38. **Read the text carefully and answer the questions:**

Rohan makes a project on coronavirus in science for an exhibition in his school. In this Project, he picks a sphere which has volume 38808 cm^3 and 11 cylindrical shapes each of Volume 1540 cm^3 with 10 cm length.

(i) Given Volume of cylinder = $1540cm^3$.

Surface covered by cylindrical shapes on sphere is area of circular base of cylinder

Volume of cylinder = $\pi r^2 h$ = 1540

$\Rightarrow 1540 = \frac{22}{7} \times r^2 \times 10$

$\Rightarrow r^2 = \frac{1540 \times 7}{22 \times 10} = 49$

$\Rightarrow$ r = 7 cm

Surface area covered by cylindrical shapes = $11\pi r^2$

$\Rightarrow S = 11 \times \frac{22}{7} \times 7 \times 7$

$\Rightarrow$ S = 1694 cm^2

Surface covered by cylindrical shapes on sphere = $1694cm^2$

(ii) Volume of sphere = $38808cm^3$

Volume of sphere $= \frac{4}{3} \times \pi \times r^3$

$\Rightarrow 38808 = \frac{4}{3} \times \frac{22}{7} \times r^3$

$\Rightarrow r^3 = \frac{38808 \times 3 \times 7}{22 \times 4} = 21^3$

$\Rightarrow$ r = 21 cm

$\Rightarrow$ Diameter = 42 cm

OR

For cylinder height = h = 10cm and radius = r = 7cm

Curved surface area of cylinder = $2\pi rh$

$\Rightarrow CSA = 2 \times \frac{22}{7} \times 7 \times 10$

$\Rightarrow$ CSA = 440 cm^2

(iii) Given Volume of Sphere = 38808 cm^3 and Volume of each cylinder = 1540cm^3

Total volume of shape = volume of sphere + 11 $\times$ volume of cylinder

= 38808 + 11 $\times$ 1540

= 38808 +16940

= 55748 cm^3