# ☆ State Space Analysis :

⇒ State ⇒ future behaviour

PI (present i/p) Past history

⇒ The state gives the future behaviour of the sys. based on the present I/p & past history of the system.

⇒ The past history (Initial condition) of the sys. described by the state variable.

⇒ The resistive ckt not having any state variable, because the o/p does not depends on the past history of the system.

⇒ The resistive ckt o/p depends on only i/p.

⇒ The resistive ckt cannot store any energy i.e. No past history, No state variables.

⇒ The resistive ckt is called memoryless system.

## * No. of State Variable's:

⇒ If the RLC ckt given then the

$$\boxed{\text{no. of State Variable = Sum of Inductors \& Capacitors.}}$$

⇒ If the differential eqⁿ is given,

$$\boxed{\text{No. of State Variable = Order of diff}^n \text{ eq}^n.}$$

## * Standard form of State model:-

$$\text{Diff}^n \text{ State Vector} \longrightarrow \dot{X} = AX + BU \longrightarrow \text{State eq}^n \,|\, \text{Dynamic eq}^n$$

$$Y = CX + DU \longrightarrow \text{O/P eq}^n$$

$Y$ → O/P vector, $X$ → State vector, $U$ → I/P vector

⇒ $A \longrightarrow$ State matrix.

$B \longrightarrow$ I/P matrix.

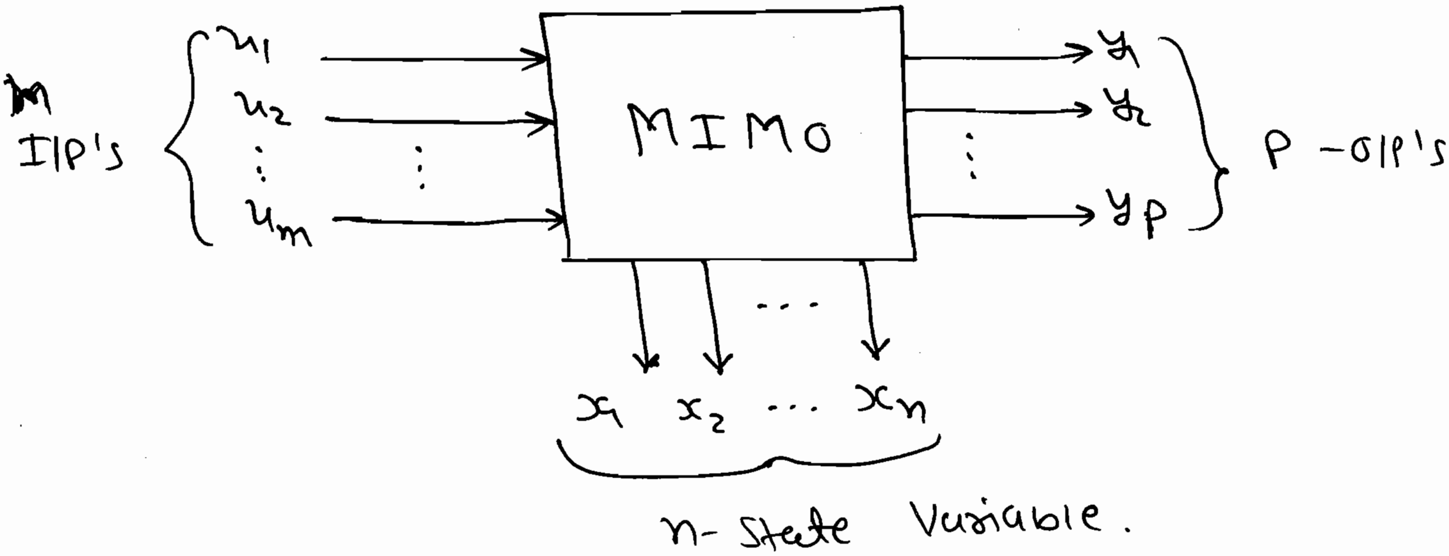
$C \longrightarrow$ O/P matrix.

$D \longrightarrow$ Transmission matrix.

## * Order of Matrices:-

⇒ Consider the multi-I/P, multi O/P system as shown in fig.

$\Rightarrow$




$\Rightarrow$ I/P Vector $(U) = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}_{m\times 1}$.

$\Rightarrow$ O/P Vector $(Y) = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix}$ $p\times 1$.

$\Rightarrow$ State Vector $(X) = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$

$\Rightarrow$ $$\underset{n\times 1}{\dot{X}} = \underset{n\times n}{A}\,\underset{n\times 1}{X} + \underset{n\times m}{B}\,\underset{m\times 1}{U}$$

⇒ The order of differential state vector must be equal to order of the state vector.

$$\underbrace{Y}_{p\times 1} = \underbrace{C}_{p\times n}\underbrace{X}_{n\times 1} + \underbrace{D}_{p\times m}\underbrace{U}_{m\times 1}$$

## * State Model to Differential eqn:-

(a) write the state model to following systems:

① $\dddot{y} + 3\ddot{y} + 5\dot{y} + 7y = 10u.$

S.C. with opposite sign of co-efficient.

Soln: The No. of State Variable required is 3, $\boxed{n=3}$

Let, $y = x_1$ — ①

$\dot{x}_1 = \dot{y} = x_2$ — ②

$\dot{x}_2 = \ddot{y} = x_3$ — ③

$\dot{x}_3 = \dddot{y} =$ ——— ④.

⇒ To get the $\dot{x}_3$ interms of state variable substitute all eqn in the given sys.

$\therefore \dot{x}_3 + 3x_3 + 5x_2 + 7x_1 = 10u.$

$\Rightarrow \dot{x}_3 = -7x_1 - 5x_2 - 3x_3 + 10u$ — ⑤

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} [U].$$

$$[y] = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

**S.C.**

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ \multicolumn{5}{l}{\text{coefficient from last to first with opposite sign}} \end{bmatrix}$$

Last Row

⇒ The above state model is Controllable Cannonical form. $\boxed{\text{CCF}}$

⇒ The state models are not unique, there are four types of state model.

1. Controllable Canonical form.
2. Observable Cannonical form.
3. Diagonalization (or) Normal form.
4. Jordon Cannoical form.

## ✱ Observable Canonical form:-

$$\Rightarrow A_{OCF} = (A_{CCF})^T = \begin{bmatrix} 0 & 0 & -7 \\ 1 & 0 & -5 \\ 0 & 1 & -3 \end{bmatrix}$$

$$\Rightarrow \quad B_{CCF} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} \begin{matrix} \text{end} \\ \uparrow \\ \text{start} \end{matrix} \Rightarrow \quad B_{OCF} = \begin{bmatrix} b_3 \\ b_2 \\ b_1 \\ b_0 \end{bmatrix} \begin{matrix} \text{start} \\ \downarrow \\ \text{end} \end{matrix}$$

$$\Rightarrow \quad \mathcal{C}_{CCF} \Rightarrow \quad B_{OCF} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}.$$

$$\Rightarrow \quad C_{CCF} = \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \end{bmatrix},$$

start $\longrightarrow$ End

$$C_{OCF} = \begin{bmatrix} c_3 & c_2 & c_1 & c_0 \end{bmatrix}$$

end $\longleftarrow$ start.

$$C_{OCF} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

$\boxed{Q}$ $\ddddot{y} + 2\dddot{y} + 4\ddot{y} + 6\dot{y} + 8y = 5U.$

Soln:

$y = x_1$

$\dot{x}_1 = \dot{y} = x_2$

$\dot{x}_2 = \ddot{y} = x_3$

$\dot{x}_3 = \dddot{y} = x_4.$

$\dot{x}_4 = \ddddot{y}$

$\dot{x}_4 + 2x_4 + 4x_3 + 6x_2 + 8x_1 = 5U.$

$\therefore \dot{x}_4 = -8x_1 - 6x_2 - 4x_3 - 2x_4 + 5U.$

$$\therefore A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -8 & -6 & -4 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}.$$

## State model to the Transfer Function:

**Q** Write the State model to the given TF.

$$\frac{Y(s)}{U(s)} = \frac{2s+3}{s^2+5s+6}.$$

Soln:

$$\frac{Y(s)}{U(s)} = \frac{2s+3}{s^2+5s+6}.$$

$\dot{x}_1 = x_2$   $x_1$   $x_1$

$\dot{x}_2$   $\dot{x}_1 = x_2$ — ①

$$\dddot{y} = \dot{x}_3 = s^3$$

so, $\boxed{s^n = \dot{x}_n}$

$$\Rightarrow \quad U = \dot{x}_2 + 5x_2 + 6x_1$$

$$\therefore \boxed{\dot{x}_2 = -6x_1 - 5x_2 + U.} \quad — ②.$$

$$\Rightarrow \quad Y = 2\dot{x}_1 + 3x_1$$

$$\boxed{Y = 2x_2 + 3x_1}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [U].$$

$$[Y] = \begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Short-cuts:

$$\frac{Y(s)}{KU(s)} = \frac{K\,(2s+3)}{s^2+5s+6}.$$

with same sign of co-eff. $\leftarrow$ C matrix

$\leftarrow$ A matrix with opposite sign of coeffs.

$\boxed{Q}$

$$\frac{Y(s)}{U(s)} = \frac{2s^3+4s^2+6}{s^5+3s^4+5s^3+7s^2+9s+10}.$$

Soln:

$$\frac{Y(s)}{2\,U(s)} = \frac{2\,(s^3+2s^2+3)}{s^5+3s^4+5s^3+7s^2+9s+10}.$$

$$\underset{\dot{x}_5}{s^5} + \underset{\dot{x}_4}{3s^4} + \underset{\dot{x}_3}{5s^3} + \underset{\dot{x}_2}{7s^2} + \underset{\dot{x}_1}{9s} + \underset{x_1}{10}$$

$\therefore$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -10 & -9 & -7 & -5 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}.$$

$$C = \begin{bmatrix} 3 & 0 & 2 & 1 & 0 \end{bmatrix}.$$

## * Diagonalization form:-

$$\Rightarrow \quad \frac{Y(s)}{U(s)} = \frac{1}{(s+1)(s+2)(s+3)}.$$

$$= \frac{1/2}{s+1} - \frac{1}{(s+2)} + \frac{1/2}{(s+3)}.$$

$$\therefore \quad y = \frac{\frac{1}{2}U}{(S+1)} + \frac{-1U}{(S+2)} + \frac{\frac{1}{2}U}{(S+3)}.$$

$$y = x_1 + x_2 + x_3.$$

let, $\quad x_1 = \dfrac{\frac{1}{2}U}{(S+1)}, \quad x_2 = \dfrac{-1U}{(S+1)}, \quad x_3 = \dfrac{\frac{1}{2}U}{(S+3)}.$

$$\Rightarrow \quad Sx_1 + x_1 = \frac{1}{2}U$$

$$\therefore \quad \dot{x}_1 + x_1 = \frac{1}{2}U$$

$$\boxed{\dot{x}_1 = -x_1 + \frac{1}{2}U} \quad \text{— (1)}$$

$$\Rightarrow \quad Sx_2 + x_2 = -U.$$

$$\therefore \quad \dot{x}_2 + x_2 = -U.$$

$$\Rightarrow \quad \boxed{\dot{x}_2 = -x_2 - U.} \quad \text{— (2)}.$$

$$\Rightarrow \quad Sx_3 + 3x_3 = \frac{1}{2}U.$$

$$\therefore \quad \dot{x}_3 + 3x_3 = \frac{1}{2}U$$

$$\therefore \quad \boxed{\dot{x}_3 = -3x_3 + \frac{1}{2}U.} \quad \text{— (3)}.$$

Partial fraction

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ -1 \\ \frac{1}{2} \end{bmatrix} [U].$$

Poles (or) Eigen Values.

$$\Rightarrow [y] = \underbrace{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}}_{\text{Always } 111\ldots} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\Rightarrow$ In the Diagonalization form B & C matrix are interchange.

## * Jordan Canonical form:-

$$\Rightarrow \frac{Y(s)}{U(s)} = \frac{1}{(s+2)^2 (s+3)}$$

$$= \frac{1}{(s+2)^2} + \frac{-1}{(s+2)} + \frac{1}{s+3}$$

$$\Rightarrow y = \frac{1U}{(s+2)^2} + \frac{-1U}{(s+2)} + \frac{1U}{(s+3)}$$

$$\Rightarrow y = x_1 - x_2 + x_3.$$

where, $x_1 = \dfrac{U}{(s+2)^2} = \dfrac{U}{(s+2)} \cdot \dfrac{1}{(s+2)}$

$$\therefore x_1 = \frac{x_2}{(s+2)}.$$

$$\therefore sx_1 + 2x_1 = x_2.$$

$$\therefore \boxed{\dot{x}_1 = -2x_1 + x_2} \quad \text{— (1)}$$

$$\Rightarrow x_2 = \frac{U}{(s+2)} \Rightarrow sx_2 + 2x_2 = U$$

$$\Rightarrow \boxed{\dot{x}_2 = -2x_2 + U.} \quad \text{— (2)}$$

$$\Rightarrow \quad x_3 = \frac{U}{S+3}.$$

$$\therefore \quad Sx_3 + 3x_3 = U.$$

$$\therefore \quad \boxed{\dot{x}_3 = -3x_3 + U} \quad \text{---} \quad (3).$$

Repeated Roots. i.e. Jordan block

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \left[\begin{array}{cc|c} -2 & 1 & 0 \\ 0 & -2 & 0 \\ \hline 0 & 0 & -3 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \left[\begin{array}{c} 0 \\ 1 \\ \hline 1 \end{array}\right] [U].$$

zero's

one's

$$[Y] = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Partial fraction

$\boxed{Q}$ $$\frac{Y}{U} = \frac{1}{(S+5)^3 (S+10)}.$$

Sol^n:

$$A = \begin{bmatrix} -5 & 1 & 0 & 0 \\ 0 & -5 & 1 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -10 \end{bmatrix}$$

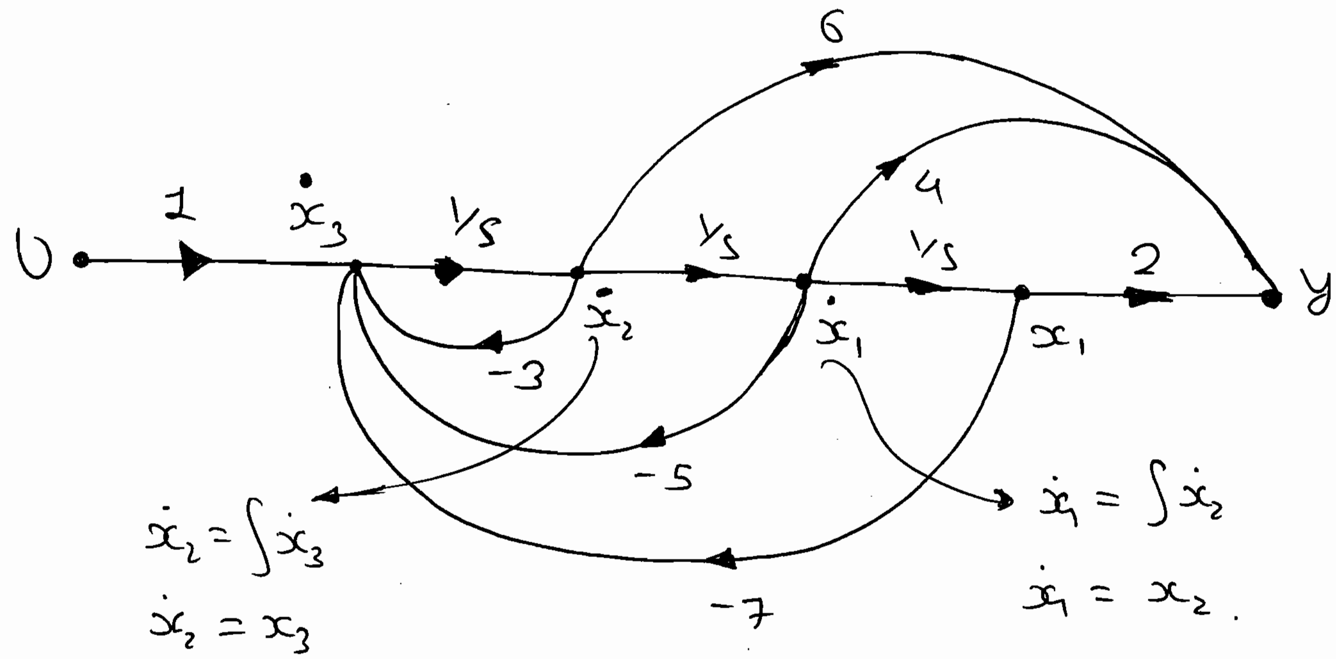
$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix}.$$

$$k_1 \downarrow (S+5)^3 \quad k_2 \downarrow (S+5)^2 \quad k_3 \downarrow (S+5) \quad k_4 \downarrow (S+10)$$

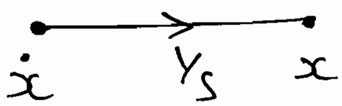
## * State Model to the signal flow graph:

**Q** Write the state model to the following signal flow graph.

**Soln:**




$\Rightarrow$ To select the node as a state variable, the incoming branch to that particular node must be a integrator, like




$\Rightarrow \quad \dot{x}_3 = 1 \cdot U - 3\dot{x}_2 - 5\dot{x}_1 - 7x_1.$

$$\boxed{\dot{x}_3 = U - 3x_3 - 5x_2 - 7x_1}$$

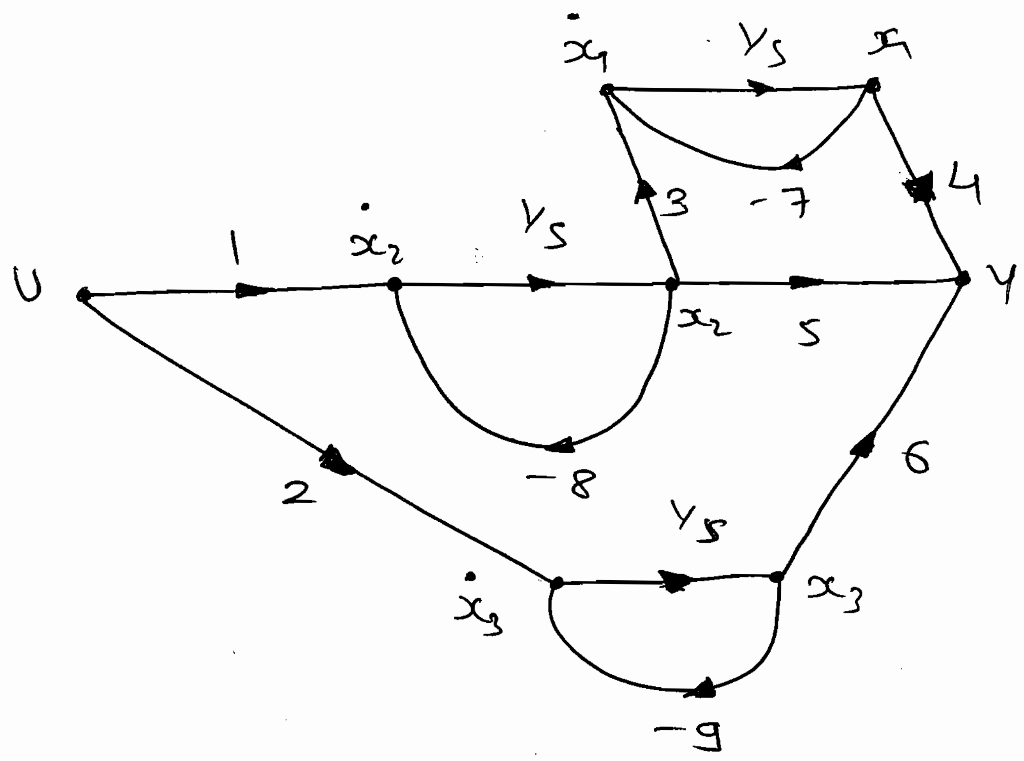
$\therefore \quad y = 2x_1 + 4\dot{x}_1 + 6\dot{x}_2$

$$\therefore \boxed{y = 2x_1 + 4x_2 + 6x_3.}$$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [u].$$

$$[y] = \begin{bmatrix} 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

$\boxed{a}$




Sol$^n$:

$$\dot{x}_1 = -7x_1 + 3x_2.$$

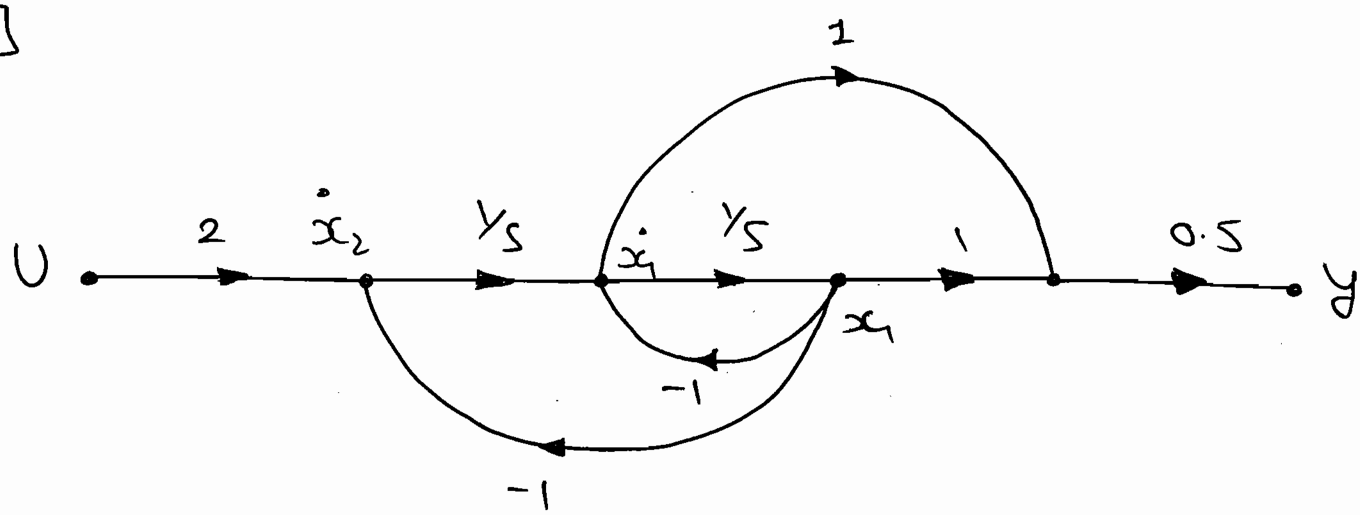
$$\dot{x}_2 = -8x_2 + 1.u. \qquad y = 4x_1 + 5x_2 + 6x_3$$

$$\dot{x}_3 = -9x_3 + 2.u.$$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -7 & 3 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} [u].$$

$$\therefore [y] = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Q




Soln:

$$\dot{x}_1 = \int \dot{x}_2 - x_1 = x_2 - x_1$$

$$\dot{x}_2 = 2U - x_1.$$

$$\therefore y = (\dot{x}_1 + x_1)\,0.5$$

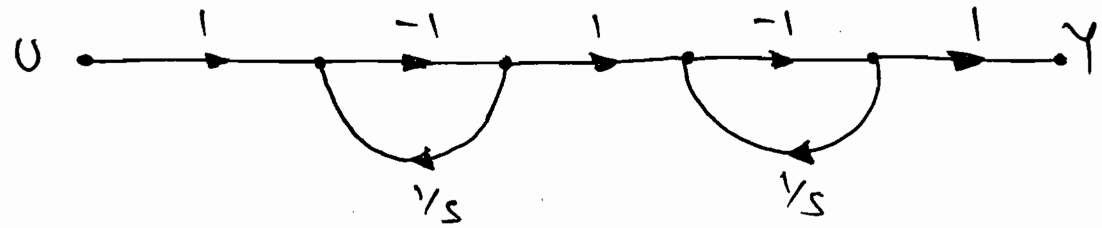
$$\therefore y = (x_2 - \cancel{x_1} + \cancel{x_1})\,0.5$$

$$\therefore \boxed{y = 0.5x_2}$$

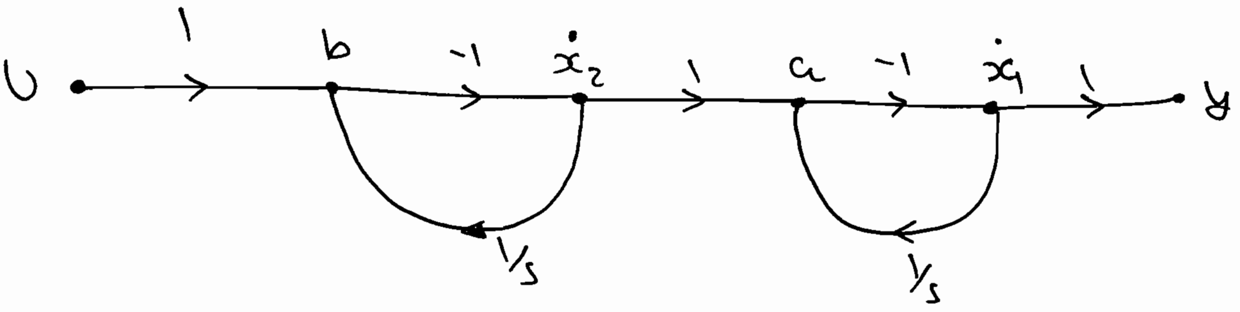
$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} [U].$$

$$\therefore [y] = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Q



Sol<sup></sup>n:




$\Rightarrow \quad a = \dot{x}_2 + \frac{1}{s}\cdot \dot{x}_1$

$a = \dot{x}_2 + x_1$

$\Rightarrow \quad \cancel{x_2} \; \cancel{=} \; b = 1\cdot U + \dot{x}_2/s.$

$\therefore \; b = U + x_2$

$\dot{x}_2 = -b$

$\therefore \; \boxed{\dot{x}_2 = -U - x_2.}$

$\dot{x}_1 = -a.$

$\therefore \; \dot{x}_1 = -x_1 - \dot{x}_2$

$\boxed{\dot{x}_1 = -x_1 + x_2 + U.}$

$\therefore \; y = \dot{x}_1$

$\therefore \; \boxed{y = -x_1 + x_2 + U.}$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} [U].$$

$$\therefore \; [y] = \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [1][U].$$

(*) State model to the Electrical N/w:

$\Rightarrow$ Select the state variables as Voltage across capacitors, and current through the ~~conductor~~. Inductor.

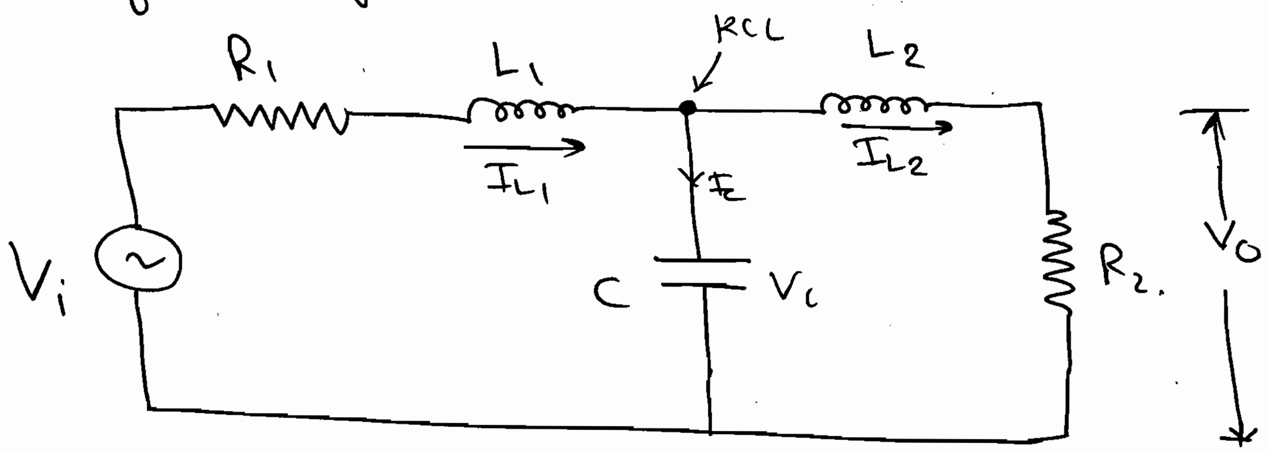
⇒ The no. of state variables = sum of Inductors & Capacitors.

⇒ Write the independent KCL & KVL eqn.

⇒ At capacitor jn apply KCL & apply KVL through the Inductor.

⇒ The resultant eqn should consist, state variables, differential state variables, I/P variables & o/p variables.

**Q** Write the state model to the following system:




Soln:

SV (State variable) = $\begin{bmatrix} V_c \\ I_{L_1} \\ I_{L_2} \end{bmatrix}$.

⇒ KCL at cap. jn

$$\therefore \quad -I_{L_1} + I_{L_2} + C\frac{dV_c}{dt} = 0.$$

$$\therefore \quad C\frac{dV_c}{dt} = I_{L_1} - I_{L_2}.$$

$$\therefore \boxed{\dot{V}_c = \frac{I_{L_1}}{C} - \frac{I_{L_2}}{C}} \quad \text{— ①.}$$

→ $\underline{\underline{KVL_1}}$:

$$V_i - I_{L_1} \cdot R_1 - L_1 \frac{dI_{L_1}}{dt} - V_c = 0.$$

$$\therefore L_1 \frac{dI_{L_1}}{dt} = -I_{L_1}R_1 - V_c + V_i.$$

$$\therefore \boxed{\dot{I}_{L_1} = \frac{-R_1}{L_1} \cdot I_{L_1} - \frac{V_c}{L_1} + \frac{V_i}{L_1}.} \quad \text{— ②.}$$

→ $\underline{\underline{KVL_2}}$:

$$V_c - L_2 \frac{dI_{L_2}}{dt} - I_{L_2} \cdot R_2 = 0.$$

$$\therefore L_2 \frac{dI_{L_2}}{dt} = -I_{L_2} \cdot R_2 + V_c.$$

$$\therefore \boxed{\frac{dI_{L_2}}{dt} = -\frac{R_2}{L_2} \cdot I_{L_2} + \frac{V_c}{L_2}.} \quad \text{— ③.}$$

$$\Rightarrow \begin{bmatrix} \dot{V}_c \\ \dot{I}_{L_1} \\ \dot{I}_{L_2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} & -\frac{1}{C} \\ -\frac{1}{L_1} & -\frac{R_1}{L_1} & 0 \\ \frac{1}{L_2} & 0 & -\frac{R_2}{L_2} \end{bmatrix} \begin{bmatrix} V_c \\ I_{L_1} \\ I_{L_2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_1} \\ 0 \end{bmatrix} [V_i]$$

$$V_o = I_{L_2} \cdot R_2.$$

$$\therefore [V_o] = [0 \quad 0 \quad R_2] \begin{bmatrix} V_c \\ I_{L_1} \\ I_{L_2} \end{bmatrix}.$$

## * Transfer function from the State model:-

=>

$$T.F. = C[SI - A]^{-1} \cdot B + D.$$

$$T.F. = C \cdot \frac{adj[SI-A]}{|SI-A|} \cdot B + D.$$

=> The det of $SI-A$ i.e. $|SI-A| = 0$ gives the Char. eqn.

=> The roots of the CE is called Poles which are called eigen values.

Q) Find the T.F. to the given State model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} [U].$$

$$[y] = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Soln: add $s$ diagonally & change the sign of coefficient for to get $|SI-A|^{-1}$.

$$SI - A = \begin{bmatrix} s+2 & 3 \\ -4 & s-2 \end{bmatrix}.$$

$$\therefore (SI-A)^{-1} = \frac{adj(SI-A)}{|SI-A|} = \frac{\begin{bmatrix} s-2 & -3 \\ 4 & s+2 \end{bmatrix}}{s^2-4+12}.$$

$$\therefore \; T.F = C\,(SI-A)^{-1}.\,B. + D.$$

$$= \frac{\begin{bmatrix} 1 & 1 \end{bmatrix}_{1\times 2} \begin{bmatrix} s-2 & -3 \\ +4 & s+2 \end{bmatrix}_{2\times 2} \begin{bmatrix} 3 \\ 5 \end{bmatrix}}{s^2+8}$$

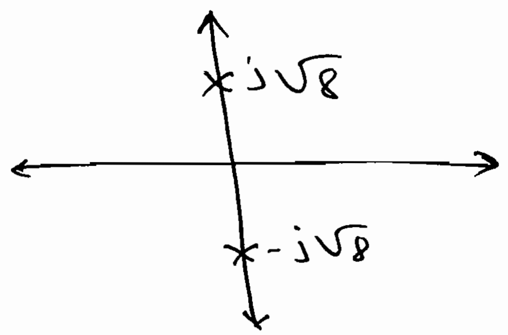
$$= \frac{\begin{bmatrix} s+2 & s-1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}}{s^2+8}$$

$$= \frac{3s+6+5s-5}{s^2+8}$$

$$\therefore \; \boxed{TF = \frac{8s+1}{s^2+8}}$$

CE $\rightarrow$ $s^2+8=0 \Rightarrow s = \pm j\sqrt{8}$.

Marginally stable (or) Undamped sys.




$\omega_n = \sqrt{8}$ rad/sec

M.S

Q $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} [u].$$

$$[y] = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Sol$^n$:

$$sI - A = \begin{bmatrix} s & -3 \\ 2 & s+5 \end{bmatrix}.$$

$$(sI-A)^{-1} = \frac{\text{adj}(sI-A)}{|sI-A|}.$$

$$= \frac{\begin{bmatrix} s+5 & 3 \\ -2 & s \end{bmatrix}}{s^2+5s+6}.$$

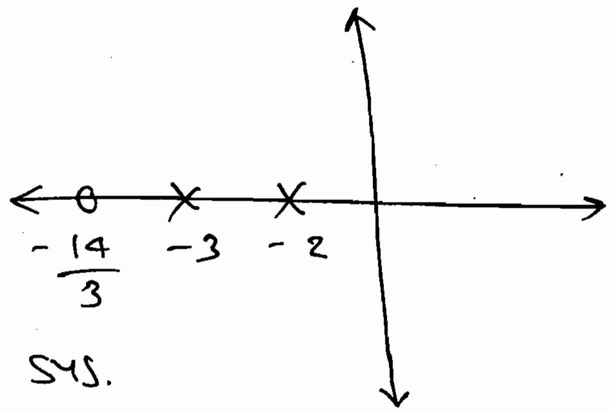
$$\therefore \text{T.F.} = C[sI-A]^{-1}B + D.$$

$$= \frac{\begin{bmatrix} 2 & 1 \end{bmatrix}\begin{bmatrix} s+5 & 3 \\ -2 & s \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{s^2+5s+6}$$

$$= \frac{\begin{bmatrix} 2s+8 & 6+s \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{s^2+5s+6}$$

$$= \frac{2s+8+6+s}{s^2+5s+6}.$$

$$\boxed{\text{T.F.} = \frac{3s+14}{s^2+5s+6}}.$$




Stable / over-damped sys.

## * Solution to the State eqn :-

$\Rightarrow$ $\dot{X} = AX + BU$ $\rightarrow$ Non-Homogenous State eqn.

### $\boxed{\text{M-I}}$ : Laplace Transform method.

$$\Rightarrow \quad sX(s) - X(0) = AX(s) + BU(s).$$

$$\therefore \quad sX(s) - AX(s) = X(0) + BU(s).$$

$$\therefore \quad (sI - A)\, X(s) = X(0) + BU(s).$$

$$\therefore \quad X(s) = (sI - A)^{-1} X(0) + [sI - A]^{-1} \cdot BU(s).$$

$\Rightarrow$ Apply I.L.T.

$$\therefore \quad \boxed{x(t) = \underbrace{L^{-1}\left\{(sI-A)^{-1} X(0)\right\}}_{\text{Zero I/P Response due I.C.}} + \underbrace{L^{-1}\left\{(sI-A)^{-1} BU(s)\right\}}_{\text{Zero State Response due to I/P.}}}$$

$-I$

$\Rightarrow$ The zero I/P resp. (ZIR) is due to Initial condition.

$\Rightarrow$ The zero State resp. (ZSR) is due to I/P.

### $\boxed{\text{M-II}}$ : Classical Method.

$$\boxed{x(t) = \underbrace{e^{At} x(0)}_{ZIR} + \underbrace{\int_0^t \overbrace{e^{A(t-\tau)}}^{\phi(t-\tau)} \cdot B \cdot u(\tau)\, d\tau}_{ZSR}} \quad \text{—(II)}$$

$\Rightarrow$ Compare ZIR terms, ***

$$\boxed{\phi(t) = e^{At} = L^{-1}[SI - A]^{-1}}$$

STM : State Transmission matrix.

$\Rightarrow$ $[SI-A]^{-1} = L[\phi(t)] = \phi(s)$.

$\therefore$ $\boxed{\phi(s) = [SI - A]^{-1}}$

$\Rightarrow$ Compare ZSR term:-

$$\boxed{\int_0^t \phi(t-\tau) B u(\tau)\, d\tau = L^{-1}[\phi(s) \cdot B \cdot U(s)]}$$

$\Rightarrow$ $\boxed{x(t) = e^{At} \cdot x(0) + L^{-1}[\phi(s) \cdot B \cdot U(s)]}$ ***

* **Properties of STM:-**

$\Rightarrow$ STM : $\phi(t) = e^{At}$.

① $\phi(0) = e^0 = I$ (Identity matrix).

② $\phi^K(t) = (e^{At})^K = e^{A(Kt)} = \phi(Kt)$.

e.g. $\phi^{-1}(t) = \phi(-t)$.

(3) $\phi(t_1 + t_2) = \phi(t_1) \cdot \phi(t_2)$.

(4) $\phi(t_2 - t_1) \cdot \phi(t_1 - t_0) = \phi(t_2 - t_0)$.

$\boxed{Q}$ Obtain the Complete sys. response of the system given below:

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad X(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad y = [1 \ -1] X.$$

Soln:

Homogenous State eqn:

$\dot{X} = AX \rightarrow$ is called as homogenous State eqn. $(U = 0)$.

$\rightarrow x(t) = \phi(t) \cdot X(0) = Z.I.R. = e^{At} \cdot X(0)$.

$$\boxed{\begin{aligned} x(t) &= \phi(t) \cdot X(0) \\ x(t) &= L^{-1}[(SI - A)^{-1} \cancel{X}(0)] \\ x(t) &= L^{-1}[(SI - A)^{-1} \cdot X(0)] \end{aligned}}$$

$\Rightarrow$

$\Rightarrow$ The given state model is homogeneous

Hence the soln is

$x(t) = Z.I.R. = e^{At} \cdot X(0) = \phi(t) \cdot X(0)$.

$\Rightarrow \xrightarrow{STM} \phi(t) = e^{At} = L^{-1}[SI - A]^{-1}$.

$$\phi(t) = L^{-1}\left[(SI - A)^{-1}\right].$$

$$\Rightarrow (SI - A) = \begin{bmatrix} s & -1 \\ 2 & +s \end{bmatrix}$$

$$\Rightarrow (SI - A)^{-1} = \frac{\operatorname{adj}(SI - A)}{|SI - A|}.$$

$$(SI - A)^{-1} = \frac{\begin{bmatrix} s & +1 \\ -2 & s \end{bmatrix}}{s^2 + 2}.$$

$$\therefore \phi(t) = L^{-1}\begin{bmatrix} \frac{s}{s^2+2} & \frac{1}{s^2+2} \\ \frac{-2}{s^2+2} & \frac{s}{s^2+2} \end{bmatrix}.$$

$$\therefore \phi(t) = \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}.$$

$$\Rightarrow x(t) = ZIR = \phi(t).\, X(0).$$

$$= \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}_{2\times 2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2\times 1}$$

$$x(t) = \begin{bmatrix} \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ \sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix} \quad \cancel{-\sqrt{2}\sin\sqrt{2}t} \; + \cancel{\cos\sqrt{2}t}]$$

$\Rightarrow$ The Complete time Response is called $y(t)$.

→ Substitute $x$ in $y$.

$$\therefore \; y(t) = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix}.$$

$$\therefore \; y(t) = \cancel{\cos\sqrt{2}t} + \frac{1}{\sqrt{2}}\sin\sqrt{2}t + \sqrt{2}\sin\sqrt{2}t - \cancel{\cos\sqrt{2}t}.$$

$$\therefore \; \boxed{y(t) = \frac{3}{\sqrt{2}}\sin\sqrt{2}t.}$$

$\boxed{Q}$ Obtain the time response for unit - step I/P for a sys. given by.

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 5 \end{bmatrix} [0]$$

$$X[0] = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad y = \begin{bmatrix} 0 & 1 \end{bmatrix} X.$$

Sol$^n$:

~~$\phi(t)$~~ $x(t) = \phi(t)\,X(0) + L^{-1}\left[\phi(s) \cdot B\,U(s)\right].$

$\Rightarrow \; \phi(t) = L^{-1}\left[(sI-A)^{-1}\right].$

⇒ The given state model is non-homogeneous. Hence, sol$^n$ is

$$x(t) = Z.I.R. + Z.S.R.$$

$\Rightarrow \xrightarrow{Z.I.R.} e^{At} \cdot X(0) \Rightarrow \phi(t) \cdot X(0).$

$$\Rightarrow \quad \phi(t) = L^{-1}\left[(SI-A)^{-1}\right].$$

$$\therefore \quad SI - A = \begin{bmatrix} S & -1 \\ +2 & S+3 \end{bmatrix}.$$

$$(SI-A)^{-1} = \frac{adj\,(SI-A)}{|SI-A|}.$$

$$= \frac{\begin{bmatrix} S+3 & +1 \\ -2 & S \end{bmatrix}}{S^2+3S+2}.$$

$$\therefore \quad = \frac{\begin{bmatrix} S+3 & +1 \\ -2 & S \end{bmatrix}}{(S+2)(S+1)}.$$

$$\therefore \quad \phi(t) = L^{-1}\begin{bmatrix} \dfrac{(S+3)}{(S+2)(S+1)} & \dfrac{1}{(S+2)(S+1)} \\ \dfrac{-2}{(S+2)(S+1)} & \dfrac{S}{(S+2)(S+1)} \end{bmatrix}$$

$\therefore$ ~~$\phi(t) =$~~

$$\phi(t) = L^{-1}\begin{bmatrix} \dfrac{2}{S+1} - \dfrac{1}{S+2} & \dfrac{1}{S+1} - \dfrac{1}{S+2} \\ \dfrac{-2}{(S+1)} + \dfrac{2}{S+2} & \dfrac{-1}{S+1} + \dfrac{2}{S+2} \end{bmatrix}$$

$$\Rightarrow \quad \phi(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$\Rightarrow \text{Z.I.R.} = \phi(t) \cdot x(0).$$

$$= \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

$$\therefore \text{Z.I.R.} = \phi(t) \cdot x(0).$$

$$= \begin{bmatrix} 2e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} \end{bmatrix}.$$

$$\Rightarrow \text{ZSR} = L^{-1}\left[\phi(s) \cdot B\, U(s)\right].$$

$$= L^{-1}\left[\begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} \left[\frac{1}{s}\right]\right]$$

$$= L^{-1}\left[\begin{bmatrix} \frac{s+3}{(s+1)(s+2} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix} \begin{bmatrix} 0 \\ 5/s \end{bmatrix}\right]$$

$$= L^{-1}\begin{bmatrix} \frac{5}{s(s+1)(s+2)} \\ \frac{5}{(s+1)(s+2)} \end{bmatrix}$$

$$= L^{-1}\begin{bmatrix} \frac{5}{2S} - \frac{5}{S+1} + \frac{5}{2(S+2)} \\ \frac{5}{(S+1)} - \frac{5}{S+2} \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \frac{5}{2} - 5e^{-t} + \frac{5}{2}.e^{-2t} \\ 5e^{-t} - 5e^{-2t} \end{bmatrix}}_{ZSR}.$$

$\Rightarrow \quad x(t) = ZIR + ZSR.$

$$\Rightarrow \quad x(t) = \begin{bmatrix} \frac{5}{2} - 3e^{-t} + \frac{3}{2}e^{-2t} \\ 3e^{-t} - 3e^{-2t} \end{bmatrix}.$$

$\Rightarrow y = [0 \quad 1]\, x(t).$

$$y[t] = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{5}{2} - 3e^{-t} + \frac{3}{2}.e^{-2t} \\ 3e^{-t} - 3e^{-2t} \end{bmatrix}.$$

$$\therefore \quad \boxed{y(t) = 3e^{-t} - 3e^{-2t}}$$

# * Controllability & Observability :-

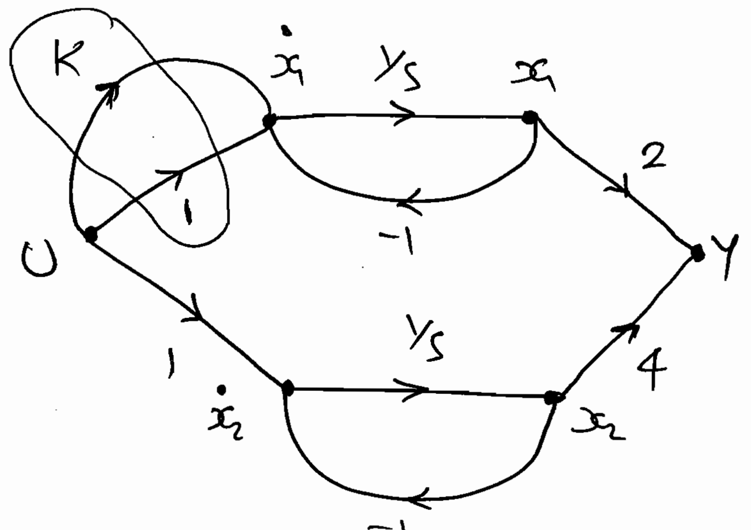
## ① Controllability :-

⇒ A sys. is ~~xxxxxxx~~ said to be controllable if it is possible to transfer the initial states to the desired ~~sate~~ state in a finite time interval by the controlled I/P.

⇒ If the SFG is given to check the controllability observe the continuous path from I/P to each & every state variable.

⇒ If the path is exist then it is called controllable.

$\boxed{Q}$ Find the K value to become the system uncontrollable.



$\underline{\underline{\text{Sol}^n}}$: To become the system uncontrollable no path exist bet$^n$ the $u$ to $\dot{x}_i$

$$\longrightarrow K+1 = 0 \Rightarrow \boxed{K=-1}$$

* Kalaman's test for Controllability ($Q_c$):-

$$\Rightarrow \boxed{Q_c = [\, B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B \,].}$$

Controllable

$$\Rightarrow \boxed{\begin{array}{c}\text{Rank of } Q_c = \text{Rank of } A \\ |Q_c| \neq 0.\end{array}}$$

$\boxed{Q}$ Check the Controllability to the given system.

$$\frac{Y(s)}{U(s)} = \frac{1}{s^3 + 2s^2 + 3s + 4}.$$

$\underline{\underline{\text{Sol}^n}}$:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

$$\Rightarrow \quad Q_c = \begin{array}{c} \begin{matrix} B & AB & A^2B \end{matrix} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -2 & +1 \end{bmatrix} \end{array} \Rightarrow |Q_c| = 1(-1) \neq 0$$

So, Controllable.

## ② Observability:

⇒ A sys. is said to be observable if it is possible to determine the initial states of the sys. by observing the o/p in a finite time interval.

### * Kalaman's test for observability:-

⇒ $Q_o = [\, C^T \quad A^T C^T \quad (A^T)^2 . C^T \; \cdots \; (A^T)^{n-1} . C^T \,]$.

$$Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

⇒ Observability

$$\boxed{\begin{array}{c} \text{Rank of } Q_o = \text{Rank of } A. \\ |Q_o| \neq 0 \end{array}}$$

Q Check the controllability & observability for the following system.

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u.$$

$y = [1 \quad 1] X$

Soln:

$Q_c = [B \quad AB]$

$\Rightarrow Q_c = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$

$|Q_c| = -2 + 2 = 0 \Rightarrow$ Not Controllable.

$\Rightarrow Q_o = \begin{bmatrix} C \\ CA \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\therefore |Q_o| = 1 - 1 = 0 \Rightarrow$ Not observable.

$\boxed{Q}$ $\dot{x}_1 = -2x_1 + x_2 + U.$

$\dot{x}_2 = -x_2 + U.$

$y = x_1 + x_2.$

Soln:

$A = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$

$C = [1 \quad 1].$

$\rightarrow Q_c = \begin{array}{cc} B & AB \\ \end{array} \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \Rightarrow |Q_c| = 0$

$\Rightarrow$ Not Controllable.

$$\Rightarrow \quad Q_o = \begin{bmatrix} C \\ CA \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix}$$

$\therefore |Q_o| = 2 \Rightarrow$ Observable.

$\Rightarrow$ The Pole-Zero Cancellation makes the system un-Controllable & un-observable (or) Controllable & unobservalble (or) uncontrollable & Observable.

## * Gribbson test for Controllability & observability.

$\Rightarrow$ The Gribbson test is valid for only diagonalization form & Jordan Canonical form.

e.g.

Non-zero, Controllable

$$\dot{x}(t) = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix} u(t).$$

non-zero controllable $\rightarrow$ Controllable.

$$y(t) = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} x(t)$$

Non zero — Observable

Non-zero — observable $\rightarrow$ Observable.

⇒ So, the given system is both observable and controllable.

Jordan block.

e.g.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} U.$$

non-zero

Controllable

$$[y] = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Non-zero ∴ observable

Non-zero observable.

Observable.

⇒ So, Sys. is controllable & observable.