Exercise: 7.4

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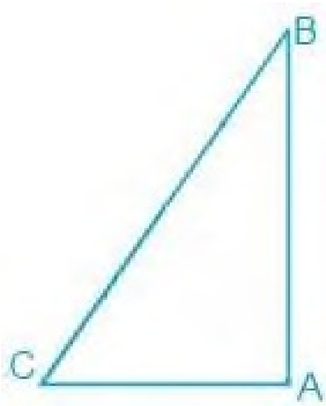
1. Show that in a right-angled triangle, the hypotenuse is the longest side.



Solution:

It is known that ABC is a triangle right angled at B.

We know that,

A + B + C = 180°

Now, if B + C = 90° then A has to be 90°.

Since A is the largest angle of the triangle, the side opposite to it must be the largest.

So, AB is the hypotenuse which will be the largest side of the above right-angled triangle i.e. ΔABC.

2. In Fig. 7.48, sides AB and AC of ΔABC are extended to points P and Q respectively. Also, PBC < QCB. Show that AC > AB.

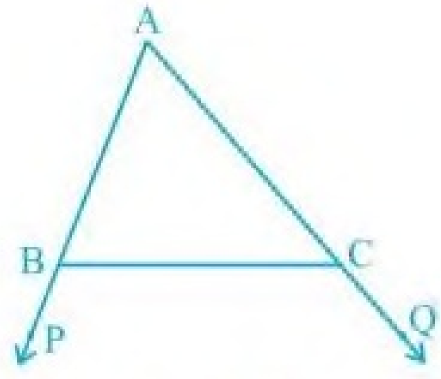


Fig. 7.48

Solution:

It is given that PBC < QCB

We know that ABC + PBC = 180°

So, ABC = 180°-PBC

Also,

ACB + QCB = 180°

Therefore, ACB = 180° - QCB

Now, since PBC < QCB,

∴ ABC > ACB

Hence, AC > AB as sides opposite to the larger angle is always larger.

3. In Fig. 7.49, B < A and C < D. Show that AD < BC.

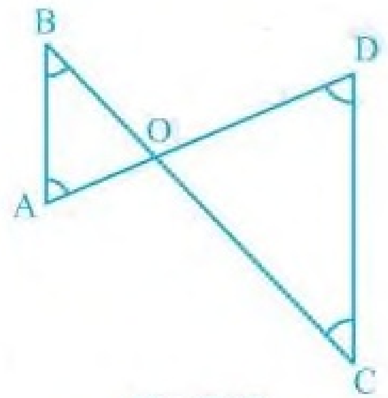


Fig. 7.49

Solution:

In the question, it is mentioned that angles B and angle C is smaller than angles A and D respectively i.e. B < A and C < D.

Now,

Since the side opposite to the smaller angle is always smaller

AO < BO … (i)

And OD < OC … (ii)

By adding equation (i) and equation (ii) we get

AO+OD < BO + OC

So, AD < BC

4. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see Fig. 7.50).

Show that A > C and B > D.

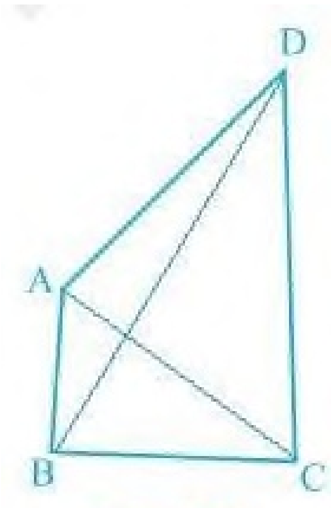


Fig. 7.50

Solution:

In ΔABD, we see that

AB < AD < BD

So, ADB < ABD … (i)

(Since angle opposite to longer side is always larger)

Now, in ΔBCD,

BC < DC < BD

Hence, it can be concluded that

BDC < CBD … (ii)

Now, by adding equation (i) and equation (ii) we get,

ADB + BDC < ABD + CBD

ADC < ABC

B > D

Similarly, In triangle ABC,

ACB < BAC … (iii)

(Since the angle opposite to the longer side is always larger)

Now, In ΔADC,

DCA < DAC … (iv)

By adding equation (iii) and equation (iv) we get,

ACB + DCA < BAC+DAC

⇒ BCD < BAD

∴ A > C

5. In Fig 7.51, PR > PQ and PS bisect QPR. Prove that PSR > PSQ.

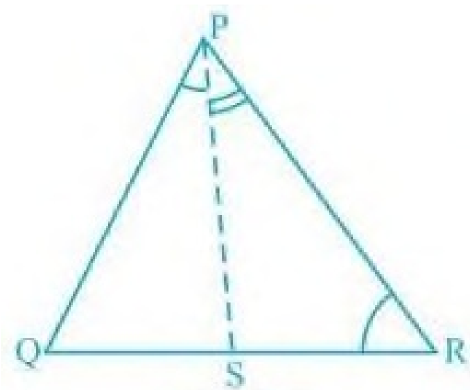


Fig. 7.51

Solution:

It is given that PR > PQ and PS bisects QPR

Now we will have to prove that angle PSR is smaller than PSQ i.e. PSR > PSQ

Proof:

QPS = RPS … (ii) (As PS bisects ∠QPR)

PQR > PRQ … (i)

(Since PR > PQ as angle opposite to the larger side is always larger)

PSR = PQR + QPS … (iii)

(Since the exterior angle of a triangle equals to the sum of opposite interior angles)

PSQ = PRQ + RPS … (iv)

(As the exterior angle of a triangle equals to the sum of opposite interior angles)

By adding (i) and (ii)

PQR +QPS > PRQ +RPS

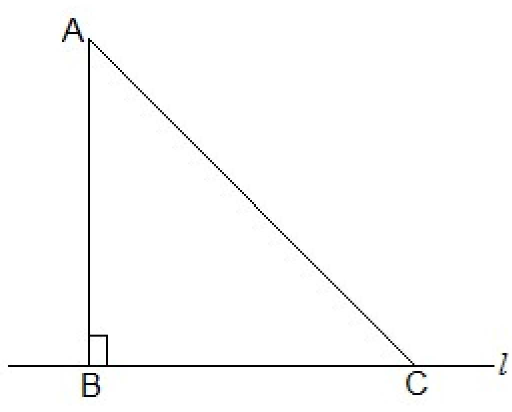
Thus, from (i), (ii), (iii) and (iv), we get

PSR > PSQ

6. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Solution:

First, let "*l*" be a line segment and "B" be a point lying on it. A line AB perpendicular to *l* is now drawn. Also, let C be any other point on *l*. The diagram will be as follows:



To prove:

AB < AC

Proof:

In ΔABC, $B = 90°$

Now, we know that

$A+B+C = 180°$

$\therefore A + C = 90°$

Hence, C must be an acute angle which implies $C < B$

So, $AB < AC$ (As the side opposite to the larger angle is always larger)