Engineering Mathematics Test 5

Number of Questions: 25 **Time: 60 min.**

Numerical Methods

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. In the process of finding an approximate root of $f(x) = 0$ in $[a, b]$ (where $f(a)$ and $f(b)$ are of opposite signs) by Regula – Falsi method, we assume that the curve $f(x) = 0$ in between $x = a$ and $x = b$ can be approximated to ________.
   (A) a parabola
   (B) a straight line
   (C) a hyperbola
   (D) a rectangular hyperbola

2. The iterative formula to find a root of the equation $f(x) = x^3 - 5x + 7 = 0$ by Newton Raphson method is ________.
   (A) $x_{k+1} = \dfrac{x_k^3 + 5x - 7}{3x_k^2 + 5}$ (B) $x_{k+1} = \dfrac{2x_k^3 + 5x}{3x_k^2 + 7}$
   (C) $x_{k+1} = \dfrac{2x_k^3 - 7}{3x_k^2 - 5}$ (D) $x_{k+1} = \dfrac{x_k^3 - 5x}{3x_k^2 + 7}$

3. With $x_0 = 0.5$ as the initial approximation, the value of the root of $f(x) = x + \sin x - 1 = 0$, after first iteration by Newton Raphson method is ________.
   (A) 0.7456 (B) 0.5110
   (C) 0.4998 (D) 0.2644

4. Applying the secant method, the first approximation to the root of $f(x) = xe^x - 2 = 0$, starting with function value at $x = 0.5$ and $x = 1$ is ________.
   (A) 1.1756 (B) 0.4035
   (C) 0.8104 (D) 0.5473

5. The extreme (minimum or maximum) point of a function $f(x)$ is to be determined by solving $\dfrac{df(x)}{dx} = 0$ using the Newton Raphson method. Let $f(x) = x^3 - 4x^2 + 5$ and $x_0 = 3$ be the initial guess of x. The value of x after first iteration (x_1) is _______.
   (A) 2.70 (B) 4.33
   (C) 3.30 (D) 1.77

6. In the process of evaluating $\int_0^{\frac{\pi}{2}} \left(x^3 + \sin 2x + 5\right) dx$ using Simpson's Rule with $h = \dfrac{\pi}{8}$, the absolute value of the error does not exceed _______.
   (A) 0.12351×10^{-4} (B) 1.03503×10^{-4}
   (C) 3.01243×10^{-4} (D) 6.2475×10^{-4}

7. The following table gives the velocity ν of a particle at time t.

T(in seconds)	0	2	4	6	8	10	12
n(in m/sec)	6	10	16	20	22	30	40

   The distance moved by the particle in 12 seconds, when calculated by the Trapeziodal rule with $h = 2$ is ________.
   (A) 200 meters (B) 210 meters
   (C) 242 meters (D) 262 meters

8. A curve is drawn to pass through the points given by the following table

X	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Y	1.0	1.7	2.5	3.4	4.1	3.7	2.9

   The area bounded by the curve, the x – axis and the lines $x = 1$ and $x = 4$, when calculated by the Simpson's $\dfrac{3}{8}th$ Rule is _____ square units.
   (A) 8.7562 (B) 5.7435
   (C) 6.7134 (D) 8.4296

9. The absolute error (correct up to 4 decimal places) in calculating the value $\log_e 2$ by trapezoidal rule, with 4 intervals using the formulae $\log_e 2 = \int_1^2 \dfrac{dx}{x}$ is _______.
   (A) 0.1314 (B) 0.0039
   (C) 0.0000 (D) 0.0004

10. With reference to finding solution of a differential equation by numerical methods, which of the following methods is NOT a predictor correct method?
   (A) Picard's method
   (B) Modified Euler's method
   (C) Adams – Bash forth method
   (D) Milne's method

11. The differential equation $\dfrac{dy}{dx} - x^2 = y$; $y(0) = 1$ is to be solved by the modified Euler's method. With $h = 0.1$, the value of y_1 correct to four decimal places is _______.
   (A) 1.2046 (B) 1.1058
   (C) 0.9954 (D) 0.8764

12. Using Taylor's series method, the solution of the differential equation $\dfrac{dy}{dx} - xy = 1$ with $y(0) = 3$ at $x = 0.1$ with $h = 0.1$ is correct upto three decimal places is ________.
   (A) 3.1153 (B) 2.9847
   (C) 4.1572 (D) 3.7893

13. The solution of the differential equation $\frac{dy}{dx} = x + y$; $y(0) = 0$ at $x = 0.2$ by Runge Kutta method of fourth order with $h = 0.2$ is ______.

(A) 1.0034 (B) 0.0456
(C) 0.9984 (D) 0.0214

14. Consider an equation $f(x) = 0$ for which $x = 4.50$ is an exact root. In the process of finding a root of $f(x) = 0$ by a numerical method, the approximations obtained in four successive iterations are 4.45, 4.54, 4.47 and 4.52 respectively. Then these approximate values of the root of $f(x) = 0$ are ______.

(A) precise but not accurate
(B) not precise but accurate
(C) both precise and accurate
(D) neither precise nor accurate

15. For an equation $f(x) = 0$, if x_e is the exact root and x_a is the approximate root, then the percentage error is ______.

(A) $(x_e - x_a) \times 100$ (B) $|x_e - x_a| \times 100$
(C) $\frac{(x_e - x_a)}{x_e} \times 100$ (D) $\frac{|x_e - x_a|}{|x_e|} \times 100$

16. Consider the following two statements

P: Truncation error in numerical analysis arise when approximations are used to estimate some quantity.

Q: Round off error in numerical analysis occurs because of the computing devices inability to deal with certain number.

Then

(A) Both *P* and *Q* are true
(B) *P* is true but *Q* is false
(C) *P* is false but *Q* is true
(D) Both *P* and *Q* are false

17. In the process of fitting a quadratic equation of the form $y = a + bx + cx^2$ to a set of n points (x_1, y_1), (x_2, y_2), ….., (x_n, y_n) by the method of least squares, which of the following is not a normal equation?

(A) $\sum y_i = na + b\sum x_i + c\sum x_i^2$
(B) $\sum x_i y_i = a\sum x_i + b\sum x_i^2 + c\sum x_i^3$
(C) $\sum x_i y_i^2 = a\sum x_i^2 + b\sum x_i^3 + c\sum x_i^4$
(D) $\sum x_i^2 y_i = a\sum x_i^2 + b\sum x_i^3 + c\sum x_i^4$

18. If $y = 3x + 7$ is the best fit for 6 pairs of values of x and y by the method of least squares and $\sum y = 150$, then $\sum x$ is ______.

(A) 144 (B) 102
(C) 46 (D) 36

19. In the process of fitting a curve $y = \frac{x^2}{ax + b}$ to a given set of n pairs of values of x and y by converting it into a linear form $Y = a + bX$, X and Y respectively stand for ______.

(A) $\frac{1}{x^2}$ and $\frac{x^2}{y}$ (B) $\frac{1}{x}$ and $\frac{x}{y}$
(C) x^2 and $\frac{y}{x^2}$ (D) x and $\frac{y}{x}$

20. In the process of fitting a curve $\exp(y) = ab^x$ to a given set of n pairs of values of x and y by converting it into a linear form $y = A + Bx$, A and B respectively stand for ______

(A) ln a and ln b (B) ln a and $\log_{10} b$
(C) $\log_{10} a$ and ln b (D) $\log_{10} a$ and $\log_{10} b$

21. If Δ denotes the forward difference operator then the value of $\Delta^{18}\left[(1+2x^3)(1-3x^4)(1+4x^5)(1-5x^6)\right]$ is ______.

(A) $5! \times 18!$
(B) $6! \times 18!$
(C) $5! \times 17!$
(D) $6! \times 17!$

22. The central difference operator δ is defined as

$y_r - y_{r-1} = \delta y_{r-\frac{1}{2}}$

Then which of the following is an identity? (Note that Δ and ∇ denote the forward and the backward difference operators respectively)

(A) $\Delta y_5 = \nabla y_4 = \delta y_{\frac{7}{2}}$
(B) $\Delta y_5 = \nabla y_4 = \delta y_4$
(C) $\Delta y_4 = \nabla y_5 = \delta y_{\frac{9}{2}}$
(D) $\Delta y_4 = \nabla y_5 = \delta y_4$

23. Match the following

	Group – I		Group – II
P.	To extrapolate the values of y to the left of y_0 when x values are equally spaced	1.	Newton's divided difference formula
Q.	To interpolate the values of y near the end value y_n when x values are equally spaced	2.	Lagrange's interpolation formula
R.	To split the given function into partial fractions	3.	Newton's forward interpolation formula
S.	To interpolate the values of y when x values are unequally spaced.	4.	Newton's backward interpolation formula

(A) $P-(1), Q-(2), R-(3), S-(4)$
(B) $P-(3), Q-(2), R-(4), S-(1)$
(C) $P-(3), Q-(4), R-(2), S-(1)$
(D) $P-(2), Q-(1), R-(4), S-(3)$

24. The 9th divided difference of a polynomial of degree 8 is ______.
(A) zero
(B) a non – zero constant
(C) a linear polynomial
(D) a quadratic polynomial

25. If $f(0) = -12$, $f(3) = 6$ and $f(4) = 12$, then the value of $f(6)$ obtained by the Lagrange's interpolation formula is ______.
(A) 18 (B) 24
(C) 20 (D) 26

Answer Keys

1. B	**2.** C	**3.** B	**4.** C	**5.** A	**6.** B	**7.** C	**8.** A	**9.** B	**10.** A
11. B	**12.** A	**13.** D	**14.** B	**15.** D	**16.** A	**17.** C	**18.** D	**19.** B	**20.** A
21. A	**22.** C	**23.** C	**24.** A	**25.** B					

Hints and Explanations

1. Standard Result. Choice (B)

2. Given, $f(x) = x^3 - 5x + 7 = 0$
$\Rightarrow f^1(x) = 3x^2 - 5$.
By Newton Raphson's method, the interative formulae to find a root is

$$x_{k+1} = x_k - \frac{f(x_k)}{f^1(x_k)} = x_k - \frac{(x_k^3 - 5x_k + 7)}{(3x_k^2 - 5)}$$

$$\therefore \quad x_{k+1} = \frac{2x_k^3 - 7}{3x_k^2 - 5}.$$ Choice (C)

3. Given, $f(x) = x + \sin x - 1 = 0$
$\Rightarrow f^1(x) = 1 + \cos x$ and $x_0 = 0.5$
By Newton Raphson's method

$$x_1 = x_0 - \frac{f(x_0)}{f^1(x_0)}$$

$$= (0.5) - \frac{(0.5 + \sin(0.5) - 1)}{(1 + \cos(0.5))}$$

$= 0.5110.$ Choice (B)

4. Here, $f(x) = xe^x - 2 = 0$
By the secant method, the approximate root of $f(x) = 0$ after first iteration is given by

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} \quad \text{---------- (1)}$$

Here, $x_0 = 0.5$ and $x_1 = 1$
$\therefore \quad f(x_0) = f(0.5) = -1.1756$ and $f(x_1) = f(1) = 0.7183$
Substituting these in (1) we have

$$x_2 = \frac{(0.5)(0.7183) - (1)(-1.1756)}{(0.7183) - (-1.1756)}$$

$\therefore \quad x_2 = 0.8104.$ Choice (C)

5. Given, $f(x) = x^3 - 4x^2 + 5$

$$\frac{df(x)}{dx} = 0 \Rightarrow 3x^2 - 8x = 0$$

Let, $g(x) = 3x^2 - 8x = 0$.

$\therefore$ We have to find the approximate root of $g(x) = 0$ after first interaction by the Newton Raphson method with $x_0 = 3$.
$\therefore \quad g^1(x) = 6x - 8$
By Newton Raphson method

$$x_1 = x_0 - \frac{g(x_0)}{g^1(x_0)}$$

$$= 3 - \frac{(3(3)^2 - 8(3))}{6(3) - 8} = 2.7.$$ Choice (A)

6. We have $\int_0^{\frac{\pi}{2}} (x^3 + \sin 2x + 5)dx$.

Let $y = f(x) = x^3 + \sin 2x + 5$.

The absolute value of the maximum error in Simpson's Rule is

$$|E|_{\max} = \frac{(b-a)h^4}{180} m \quad \text{----------- (1)}$$

Where $m = \max - \{y_0^{(iv)}, y_2^{(iv)}, y_4^{(iv)}\}$

Here, $h = \frac{\pi}{8}$ and $y^{(iv)} = 16 \sin 2x$, $a = 0$, $b = \frac{\pi}{2}$

$$\therefore \quad m = \max \left[16\sin 0, 16\sin\left(2\times\frac{\pi}{4}\right), 16\sin\left(2\times\frac{\pi}{2}\right)\right] = 16$$

From (1)

$$|E_{\max}| = \frac{\left(\frac{\pi}{2}\right)\left(\frac{\pi}{8}\right)}{180} \times 16 = 1.0350 \times 10^{-4}$$

The absolute value of the maximum error cannot exceed 1.03503×10^{-4}.

Choice (B)

7. Given velocity of the particle at various times is

T	0	2	4	6	8	10	12
n	6	10	16	20	22	30	40

Distance traveled in 12 seconds = $\int_0^{12} \nu dt$.

By trapezoidal rule

$$\int_0^{12} \nu dt = \frac{h}{2}[(\nu_0 + \nu_6) + 2(\nu_1 + \nu_2 + \nu_3 + \nu_4 + \nu_5)]$$

$$= \frac{2}{2}[(6 + 40) + 2(10 + 16 + 20 + 22 + 30)]$$

= 242 meters. Choice (C)

8. Let $y = f(x)$ be the curve, that pass through the points

X	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Y	1.0	1.7	2.5	3.4	4.1	3.7	2.9

∴ The area bounded by the curve $y = f(x)$, x – axis andthe lines $x = 1$ and $x = 4$ is $\int_1^4 f(x)dx$.

By Simpson's $\frac{3}{8}th$ Rule, we have

$$\int_1^4 f(x)dx = \frac{3h}{8}[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2y_3]$$

$$= \frac{3\times(0.5)}{8}[(1.0 + 2.9) + 3(1.7 + 2.5 + 4.1 + 3.7) + 2 \times 3.4] = 8.7562.$$

Choice (A)

9. Let $y = f(x) = \frac{1}{x}$

Here, $a = 1$, $b = 2$ and $n = 4$

$$\therefore \quad h = \frac{b-a}{n} = 0.25$$

X	1	1.25	1.5	1.75	2
F(x)	1	0.8	0.667	0.5714	0.5

We have $\log_e 2 = \int_1^2 \frac{dx}{x}$

By the trapezoidal rule, we have

$$\int_1^2 \frac{dx}{x} = \int_1^2 f(x)dx = \frac{h}{2}[(y_0 + y_4) + 2(y_1 + y_2 + y_3)]$$

$$\frac{0.25}{2}[(1+ 0.5) + 2(0.8 + 0.6667 + 0.5714)]$$

$$\therefore \quad \log_e 2 = \int_1^2 \frac{dx}{x} = 0.6970 \qquad \text{---------- (1)}$$

The exact value of $\log_e 2 = 0.6931$ -------- (2)

The absolute error in calculating $\log_e 2$ by the trapezoidal rule = 0.6970 – 0.6931 = 0.0039. Choice (B)

10. A predictor corrector method is one in which we predict the solution first and then we improve it for accuracy Picard's method is not a predictor corrector method and all other methods are predictor corrector methods. Choice (A)

11. Given different equation is $\frac{dy}{dx} - x^2 = y$ and $y(0) = 1$

$$\Rightarrow \frac{dy}{dx} = x^2 + y$$

∴ $f(x, y) = x^2 + y$, $x_0 = 0$, $y_0 = y(x_0) = 1$ and $h = 0.1$

By Euler's method

$$y_1^{(0)} = y_0 + h(f(x_0, y_0) = y_0 + h\left(x_0^2 + y_0\right)$$

$$= 1 + (0.1)(0 + 1)$$

$$y_1^{(0)} = 1.1$$

By modified Euler's method

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) f\left(x_1 y_1^{(0)}\right)\right]$$

$$= y_0 + \frac{h}{2}\left[\left(x_0^2 + y_0\right) + \left(x_1^2 + y_1^{(0)}\right)\right]$$

$$= 1 + \frac{(0.1)}{2}[(0 + 1) + (10.1)^2 + 1.1)] = 1.1055$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f\left(x_1, y_1^{(1)}\right)\right]$$

$$= y_0 + \frac{h}{2}\left[\left(x_0^2 + y_0\right) + \left(x_1^2 + y_1^{(1)}\right)\right]$$

$$= 1 + \frac{(0.1)}{2}[(0 + 1) + ((0.1)^2 + 1.1055)]$$

$$= 1.1058$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f\left(x_1, y_1^{(2)}\right)\right]$$

$$= y_0 + \frac{h}{2}\left[\left(x_0^2 + y_0\right) + \left(x_1^2 + y_1^{(2)}\right)\right]$$

$$= 1 + \frac{(0.1)}{2}[(0 + 1) + ((0.1)^2 + 1.1058)]$$

$$= 1.1058$$

The solution of a given different equation at $x_1 = 0.1$ is $y_1 = 1.1058$. Choice (B)

12. Given differential equation is $\frac{du}{dx} - xy = 1$ and $y(0) = 3$

$$\Rightarrow \frac{dy}{dx} = 1 + xy$$

Here $f(x, y) = 1 + xy$, $x_0 = 0$, $y_0 = y(x_0) = 3$ and $h = 0.1$

By Taylar's series we have

$$y_1 = y(x_1) = y_0 + h_0 y_0^1 + \frac{h^2}{2!} y_0^{11} + \frac{h^3}{3!} y_0^{111} + \ldots.. \infty \qquad \text{----(1)}$$

$$y_0^1 = \left(\frac{dy}{dx}\right)_{x=0} = f(x_0, y_0) = 1 + x_0 y_0 = 1 + (0)(3) = 1$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(1 + xy) = xy^1 + y$$

$$\therefore \quad y_0^{11} = \left(\frac{d^2y}{dx^2}\right)_{at x = x_0} = x_0 \; y_0^1 + y_0 = 0 \times 1 + 3 = 3$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d}{dx}(xy^1 + y) = xy^{11} + 2y^1$$

$$\therefore \quad y_0^{111} = \frac{d^3y}{dx^3}_{at x = x_0} = x_0 \; y_0^{11} + 2\, y_0^1 = 0 \times 3 + 2 \times 1 = 2$$

$$\frac{d^4y}{dx^4} = \frac{d}{dx}\left(\frac{d^3y}{dx^3}\right) = \frac{d}{dx}(xy^{11} + 2y^1) = xy^{111} + 3y^{11}$$

$$y_0^{(iv)} = \frac{d^4y}{dx^4}_{at x = x_0} = x_0 \; y_0^{111} + 3\, y_0^{11} = 0 \times 2 + 3 \times 3 = 9$$

Substituting these in (1), we have

$$y_1 = 3 + (0.1) \times 1 + \frac{(0.1)^2}{2!} \times 3 + \frac{(0.1)^3}{3!} \times 2 + \frac{(0.1)^4}{4!} \times 9 + \ldots = 3.1153.$$

Choice (A)

13. Given differential equation is $\frac{dy}{dx} = x + y, y(0) = 0$

Here $f(x, y) = x + y$, $x_0 = 0$; $y_0 = 0$ and $h = 0.2$

$\therefore \quad x_1 = x_0 + h = 0.2$

By R – K method of fourth order we have

$Y_{\text{at } x = 0.1} = y_1 = y_0 + \Delta y$ ---------- (1)

Where $\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$ ------- (2)

Here $k_1 = h\, f(x_0, y_0) = h(x_0 + y_0)$

$= (0.2)\,(0 + 0)$

$\therefore \quad k_1 = 0$

$$k_2 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= h\left(\left(x_0 + \frac{h}{2}\right) + \left(y_0 + \frac{k_1}{2}\right)\right)$$

$$= (0.2)\left[\left(0 + \frac{(0.2)}{2}\right) + \left(0 + \frac{0}{2}\right)\right]$$

$\therefore \quad k_2 = 0.02$

$K_3 = 0.022$ and $k_4 = h\, f(x_0 + h, y_0 + k_3)$

$= h\,[(x_0 + h) + (y_0 + k_3)]$

$= (0.2)\,[(0 + 0.2) + (0 + 0.022)]$

$\therefore \quad k_4 = 0.0444$

$\therefore$ From (2),

$$\Delta y = \frac{1}{6}[0 + 2 \times 0.02 + 2 \times 0.022 + 0.0444]$$

$\Delta y = 0.0214$

$\therefore$ From (1), $y_1 = y_0 + \Delta y = 0 + 0.0214 = 0.0214.$

Choice (D)

14. The four approximations given are not relatively close to each other. So, they are not precise. All the four approximations are close to the exact root $x = 4.50$. So, they are accurate. Choice (B)

15. By definition. Choice (D)

16. By definitions of the round off and the truncation errors. Choice (A)

17. Given set of n points are $(x_1, y_1,)$, $(x_2, y_2), \ldots, (x_n, y_n)$

We have to fit the quadratic equation

$y = a + bx + cx^2$ $\rightarrow$ (1)

to the given set of n points

Here a, b and c are constants to be determined such that

$$S = \sum\left[y_i - \left(a + bx_i + cx_i^2\right)\right]^2 \rightarrow (2)$$

is minimum

S is minimum for those values of a, b and c at which

$$\frac{\partial S}{\partial a} = 0, \frac{\partial S}{\partial b} = 0 \text{ and } \frac{\partial S}{\partial c} = 0.$$

i.e., $-2\left(\sum\left[y_i - \left(a + bx_i + cx_i^2\right)\right]\right) = 0$

$$-2x_i\left(\sum\left[y_i - \left(a + bx_i + cx_i^2\right)\right]\right) = 0$$

and $-2x_i^2\left(\sum\left[y_i - \left(a + bx_i + cx_i^2\right)\right]\right) = 0$

$\therefore$ the normal equations are

$$\Rightarrow \sum y_i = na + b\sum x_i + c\sum x_i^2$$

$$\sum x_i y_i = a\sum x_i + b\sum x_i^2 + c\sum x_i^3$$

And $\sum x_i^2 y_i = a\sum x_i^2 + b\sum x_i^3 + c\sum x_i^4$

$\therefore$ the equation given in option (C) is not a normal equation. Choice (C)

18. Given that $y = 3x + 7$ is the best fit for 6 pairs of values of x and y also given $\sum y = 150$

$\therefore$ We know that

$\sum y = 3\sum x + n7$

(Here n = number of points = 6)

$\Rightarrow$ $150 = 3\sum x + 6 \times 7$

$\Rightarrow$ $3\sum x = 108$

$\Rightarrow$ $\sum x = 36.$ Choice (D)

19. Given curve is $y = \frac{x^2}{ax + b}$

$$\Rightarrow \frac{1}{y} = \frac{ax + b}{x^2}$$

$$\Rightarrow \frac{}{y} = \frac{a}{x} + \frac{b}{x}$$

$$\Rightarrow \frac{x}{y} = a + \frac{b}{x}$$

$$\Rightarrow \frac{x}{y} = a + \frac{b}{x}$$

Which is of the linear from $Y = a + bX$

Where $X = \frac{1}{x}$ and $Y = \frac{x}{y}$ Choice (B)

20. Given curve is $\exp(y) = ab^x$

i.e., $e^y = ab^x$

Applying logarithm (ln) on both sides,

We have

$\ln(e^y) = \ln(ab^x)$

$\Rightarrow\ y = \ln a + \ln b^x$

$\Rightarrow\ y = \ln a + x\ln b$

Which is of the form

$y = A + Bx$

Where $A = \ln a$ and $B = \ln b$. Choice (A)

21. We have $\Delta^{18}\left[(1+2x^3)(1-3x^4)(1+4x^5)(1-5x^6)\right]$

$= \Delta^{18}\left[2x(-3)x4x(-5)x^{18} + k_1x^{17} + k_2x^{16} + \ldots + k_{15}x^3 + 1\right]$

$= 5!\ \Delta^{18}\ [x^{18}]$

$(\because \Delta^{18}[x^n] = 0$ for $n < 18)$

$= 5! \times 18!.$ Choice (A)

22. We know that $\Delta y_{r-1} = y_r - y_{r-1}$ → (1)

$\nabla y_r = y_r - y_{r-1}$ → (2)

and given that $\delta y_{r-\frac{1}{2}} = y_r - y_{r-1}$ → (3)

From (1), (2) and (3), we have

$\Delta y_{r-1} = \nabla y_r = \delta y_{r-\frac{1}{2}}$ → (4)

Among the options given, we can get option (C) by taking $r = 5$ in (4),

$\therefore\ \Delta y_4 = \nabla y_5 = \delta y_{\frac{9}{2}}$ Choice (C)

23. Standard result Choice (C)

24. We know that the n^{th} divided difference of any polynomial of degree less than n is always zero. Choice (A)

25. Given pairs of values of x and $f(x)$ are

X	0	3	4
f(x)	–12	6	12

By Lagrange's interpolation formula, we have

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0)$$

$$+ \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

$$= \frac{(x-3)(x-4)}{(0-3)(0-4)} \times (-12) + \frac{(x-0)(x-4)}{(3-0)(3-4)} \times 6$$

$$+ \frac{(x-0)(x-3)}{(4-0)(4-3)} \times 12$$

By taking $x = 6$ on both sides,

We have

$$f(6) = \frac{(6-3)(6-4)}{3\times 4} \times (-12) + \frac{(6-0)(6-4)}{3\times(-1)} \times 6$$

$$+ \frac{(6-0)(6-3)}{4\times 1} \times 12$$

$\therefore\ f(6) = 24$ Choice (B)