HEIGHTS AND DISTANCES

(A) OBJECTIVE TYPE QUESTIONS — 1 Mark Each

Stand Alone MCQs (1 Mark Each)

1. A pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then the Sun's elevation is:

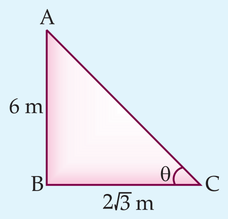
(A) 60° (B) 45°

(C) 30° (D) 90° $\boxed{A}$

Ans. Option (A) is correct.

Explanation: In ΔABC, ∠ B = 90°

$$\tan\theta = \frac{6}{2\sqrt{3}} = \sqrt{3} = \tan 60° \Rightarrow \theta = 60°$$



2. The angle of depression of a car parked on the road from the top of 150 m high tower is 30°. The distance of the car from the tower (in metres) is:

(A) $50\sqrt{3}$ (B) $150\sqrt{3}$

(C) $150\sqrt{2}$ (D) 75 $\boxed{A}$

Ans. Option (B) is correct.

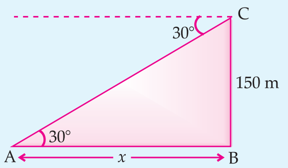
Explanation: In ΔABC, ∠B=90°

$$\tan\theta = \frac{CB}{AB}$$

$$\tan 30° = \frac{150}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{150}{x}$$

$$x = 150\sqrt{3} \text{ m}$$



3. The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle θ with the ground level such that $\tan\theta = \frac{15}{8}$, then the kite is at what height from the ground ?

(A) 75 m (B) 79.41 m

(C) 80 m (D) 72.5 m $\boxed{A}$

Ans. Option (A) is correct.

Explanation: $\tan\theta = \frac{15}{8}$

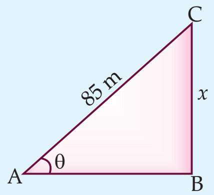
$\sin\theta = \frac{15}{17}$ ...(i)

Now, $\sin\theta = \frac{x}{85}$ ...(ii)

From, equation (i) and (ii),

$$\frac{15}{17} = \frac{x}{85}$$

$$x = 75 \text{ m}$$



4. If the height of a vertical pole is $\sqrt{3}$ times the length of its shadow on the ground, then the angle of elevation of the Sun at that time is:

(A) 30° (B) 60°

(C) 45° (D) 75° $\boxed{C}$ + $\boxed{A}$

Ans. Option (B) is correct.

Explanation: Let the length of shadow is x,

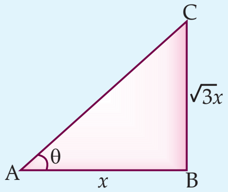
Then height of pole = $\sqrt{3}\,x$

Now, $\tan\theta = \frac{CB}{AB}$

$$\tan\theta = \frac{\sqrt{3}x}{x}$$

$$\tan\theta = \sqrt{3}$$

$$\tan\theta = \tan 60°$$
$$\theta = 60°$$



5. The angle of depression of a car, standing on the ground, from the top of a 75 m high tower, is 30°. The distance of the car from the base of the tower (in m.) is:

(A) $25\sqrt{3}$ (B) $50\sqrt{3}$

(C) $75\sqrt{3}$ (D) 150 C + A

Ans. Option (C) is correct.

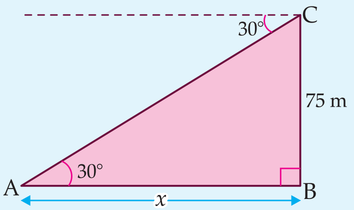
Explanation: In ΔABC, ∠B = 90°

$$\tan\theta = \frac{CB}{AB}$$

$$\tan 30° = \frac{75}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{75}{x}$$

$$x = 75\sqrt{3} \text{ m}$$

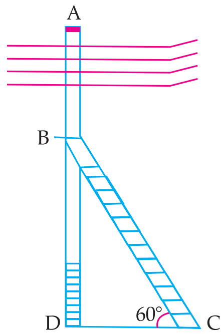


Case-based MCQs (1 Mark Each)

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

AI **I. Read the following text and answer the questions that follow, on the basis of the same.**

An electrician has to repaired an electric fault on the pole of height 5 m. He needs to reach a point 1.3 m below the top of the pole to undertake the repair work (see figure). **[CBSE QB, 2021]**



1. What is the length of BD ?

(A) 1.3 m (B) 5 m

(C) 3.7 m (D) None of these

Ans. Option (C) is correct.

Explanation: From figure, the electrician is required to reach at the point B on the pole AD.

So, $BD = AD - AB$

$= (5 - 1.3)$ m = 3.7 m

2. What should be the length of ladder, when inclined at an angle of 60° to the horizontal ?

(A) 4.28 m (B) $\frac{3.7}{\sqrt{3}}$ m

(C) 3.7 m (D) 7.4 m

Ans. Option (A) is correct.

Explanation: In ΔADC,

$$\sin 60° = \frac{BD}{BC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{3.7}{BC}$$

$$BC = \frac{3.7\times 2}{\sqrt{3}}$$

$$\Rightarrow \quad BC = 4.28 \text{ m (approx.)}$$

3. How far from the foot of pole should she place the foot of the ladder ?

(A) 3.7 (B) 2.14

(C) $\frac{1}{\sqrt{3}}$ (D) None of these

Ans. Option (B) is correct.

Explanation: In ΔBDC,

$$\therefore \quad \cot 60° = \frac{DC}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{DC}{3.7}$$

$$\Rightarrow \quad DC = \frac{3.7}{\sqrt{3}}$$

$$\Rightarrow \quad DC = 2.14 \text{ m (approx.)}$$

4. If the horizontal angle is changed to 30°, then what should be the length of the ladder ?

(A) 7.4 m (B) 3.7 m

(C) 1.3 m (D) 5 m

Ans. Option (A) is correct.

Explanation: In ΔBDC,

$$\therefore \quad \sin 60° = \frac{BD}{BC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{3.7}{BC}$$

$$\Rightarrow \quad BC = 3.7 \times 2 = 7.4 \text{ m}$$

5. What is the value of $\angle B$?

(A) $60°$ **(B)** $90°$

(C) $30°$ **(D)** $180°$

Ans. Option (C) is correct.

> *Explanation:* In ΔADC, angle D is $90°$.
> So, by angle sum property.
> $$\angle B + \angle D + \angle C = 180°$$
> or, $$\angle B = 180° - (90° + 60°) = 30°$$

II. Read the following text and answer the questions that follows, on the basis of the same.

A group of students of class X visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kings way), is about 138 feet (42 metres) in height. **[CBSE QB, 2021]**

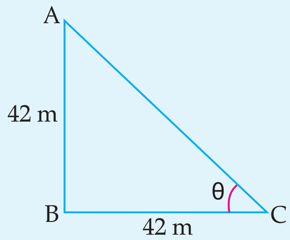
1. What is the angle of elevation if they are standing at a distance of 42 m away from the monument ?

(A) $30°$ **(B)** $45°$

(C) $60°$ **(D)** $0°$

Ans. Option (B) is correct.

> *Explanation:* Height of India gate = 42 m
> Distance between students and India Gate = 42 cm



Now,
$$\tan\theta = \frac{AB}{BC}$$
$$\tan\theta = \frac{42}{42}$$
$$\tan\theta = 1$$
$$\tan\theta = \tan 45°$$
$$\theta = 45°$$

Hence, angle of elevation = $45°$

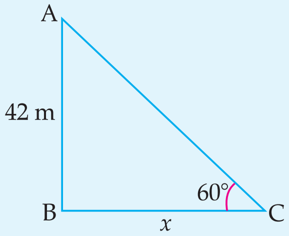
2. They want to see the tower at an angle of $60°$. So, they want to know the distance where they should stand and hence find the distance.

(A) 25.24 m **(B)** 20.12 m

(C) 42 m **(D)** 24.64 m

Ans. Option (A) is correct.

> *Explanation:* Height of India Gate = 42 cm
> Angle of elevation = $60°$
> Let the distance between students and India gate = x m.



Now,
$$\tan\theta = \frac{AB}{BC}$$
$$\tan 60° = \frac{42}{x}$$
$$\sqrt{3} = \frac{42}{x}$$
$$x = \frac{42}{\sqrt{3}}$$
$$x = \frac{42\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$$
$$= \frac{42\sqrt{3}}{3}$$
$$= 14\sqrt{3}\text{ m} = 25.24\text{ m}$$

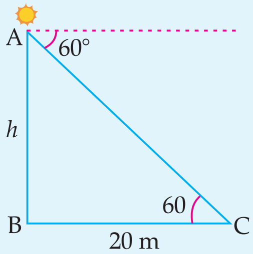
3. If the altitude of the Sun is at $60°$, then the height of the vertical tower that will cast a shadow of length 20 m is:

(A) $20\sqrt{3}$ m **(B)** $\frac{20}{\sqrt{3}}$ m

(C) $\frac{15}{\sqrt{3}}$ m **(D)** $15\sqrt{3}$ m

Ans. Option (A) is correct.

> *Explanation:*



Let, the height of tower = h

Now,
$$\tan\theta = \frac{AB}{BC}$$
$$\tan 60^\circ = \frac{h}{20}$$
$$\sqrt{3} = \frac{h}{20}$$
$$h = 20\sqrt{3}$$

4. The ratio of the length of a rod and its shadow is 1 : 1. The angle of elevation of the Sun is:

(A) 30° **(B)** 45°

(C) 60° **(D)** 90°

Ans. Option (B) is correct.

5. The angle formed by the line of sight with the horizontal when the object viewed is below the horizontal level is:

(A) corresponding angle

(B) angle of elevation

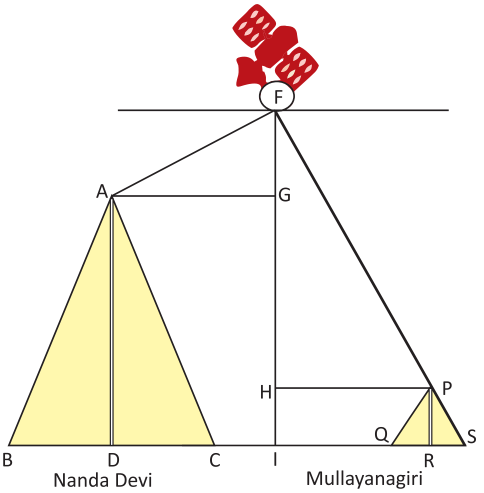
(C) angle of depression

(D) complete angle

Ans. Option (C) is correct.

III. Read the following text and answer the questions that follows, on the basis of the same: U

A satellite flying at height h is watching the top of the two tallest mountains in Uttarakhand and Karnataka, them being Nanda Devi (height 7,816 m) and Mullayanagiri (height 1,930 m). The angles of depression from the satellite, to the top of Nanda Devi and Mullayanagiri are 30° and 60° respectively. If the distance between the peaks of two mountains is 1937 km, and the satellite is vertically above the midpoint of the distance between the two mountains. **[CBSE QB, 2021]**



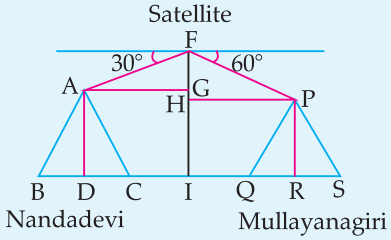
1. The distance of the satellite from the top of Nanda Devi is:

(A) 1118.36 km **(B)** 577.52 km

(C) 1937 km **(D)** 1025.36 km

Ans. Option (A) is correct.

Explanation:



Now,
$$AG = \frac{1937}{2}\text{ km}$$
$$\cos\theta = \frac{AG}{AF}$$
$$\cos 30^\circ = \frac{\frac{1937}{2}}{AF}$$
$$\frac{\sqrt{3}}{2} = \frac{1937}{2AF}$$
$$AF = \frac{1937}{\sqrt{3}}$$
$$AF = 1118.36\text{ km}$$

2. The distance of the satellite from the top of Mullayanagiri is:

(A) 1139.4 km **(B)** 577.52 km

(C) 1937 km **(D)** 1025.36 km

Ans. Option (C) is correct.

Explanation: In ΔFPH,
$$\cos\theta = \frac{PH}{FP}$$
$$\cos 60^\circ = \frac{1937}{2\text{FP}}$$
$$\frac{1}{2} = \frac{1937}{2FP}$$
$$FP = 1937\text{ km}$$

3. The distance of the satellite from the ground is:

(A) 1139.4 km **(B)** 567 km

(C) 1937 km **(D)** 1025.36 km

Ans. Option (B) is correct.

Explanation:

Distance of satellite from the ground = FI

$= FG + GI$

$$\left(FG = AG\tan 30^\circ = \frac{1937}{2}\times\frac{1}{\sqrt{3}}\right)$$
$$= \left(\frac{1937}{2\sqrt{3}} + 7.816\right)\text{km}$$
$$= 566.99 \simeq 567\text{ km}$$

4. What is the angle of elevation if a man is standing at a distance of 7816 m from Nanda Devi ?

(A) 30° **(B)** 45°

(C) 60° **(D)** 0°

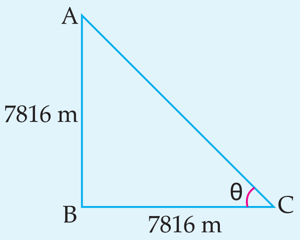
Ans. Option (B) is correct.

Explanation: Height of Nanda Devi Mountain

$= 7816$ m

Distance between man and mountain

$= 7816$ m.



$$\tan\theta = \frac{AB}{BC}$$

$$\tan\theta = \frac{7816}{7816}$$

$$\tan\theta = 1$$

$$\tan\theta = \tan 45°$$

$$\theta = 45°$$

5. If a mile stone very far away from, makes 45° to the top of Mullanyangiri mountain. So, find the distance of this mile stone from the mountain.

(A) 1118.327 km **(B)** 566.976 km

(C) 1930 km **(D)** 1025.36 km

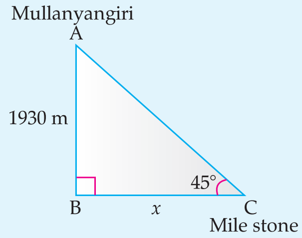
Ans. Option (C) is correct.

Explanation:

$$\tan 45° = \frac{AB}{x}$$

$$\Rightarrow \quad 1 = \frac{1930}{x}$$

$$\Rightarrow \quad x = 1930 \text{ m}$$



(B) SUBJECTIVE QUESTIONS

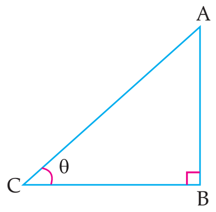
Very Short Answer Type Questions

(1 Mark Each)

AI **1. The ratio of the length of a vertical rod and the length of its shadow is $1 : \sqrt{3}$. Find the angle of elevation of the sun at that moment ?**

A **[CBSE Delhi Set-I, 2020]**

Sol. Let AB be a vertical rod and BC be its shadow.
From the figure, $\angle ACB = \theta$.
In ΔABC,



$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} \quad \left[\because \frac{AB}{BC} = \frac{1}{\sqrt{3}}(\text{Given})\right]$$

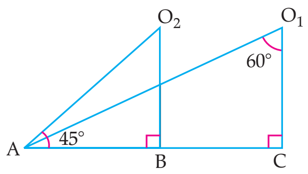
$$\Rightarrow \quad \tan\theta = \tan 30°$$

$$\Rightarrow \quad \theta = 30°$$

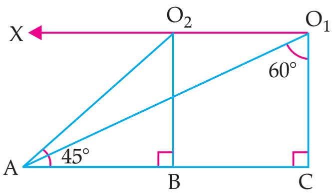
Hence, the angle of elevation of the sun is 30°.

AI **2. In the given figure, find the angles of depressions from the observing positions O_1 and O_2 respectively of the object A.**

A **[CBSE OD Set-I, 2020]**



Sol.



Draw $\quad O_1X \parallel AC$

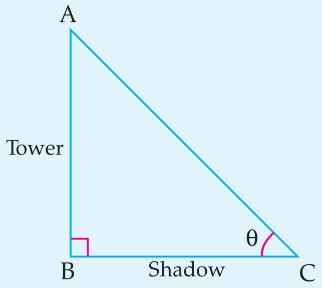
$\therefore \quad \angle AO_1X = 90° - 60° = 30°$

and $\quad \angle AO_2X = \angle BAO_2 = 45°$

3. The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3} : 1$. What is the angle of elevation of the Sun ?

U **[CBSE Delhi Set-I, II, 2017][CBSE Term-II, 2016]**

Sol.



Let the height of tower be AB and its shadow be BC.

$$\therefore \quad \frac{AB}{BC} = \tan\theta$$

$$= \frac{\sqrt{3}}{1}$$

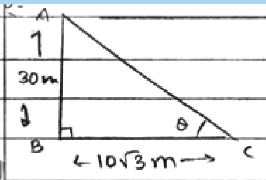
$$= \tan 60°$$

Hence, the angle of elevation of Sun = 60°. 1

[CBSE Marking Scheme, 2017]

4. **If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the Sun ?**
U [CBSE OD & Comptt. OD Set-I, II, III 2017] [CBSE Foreign Set-I, II, III, 2015]

Topper Answer, 2017

2. 

Tower AB is 30m and shadow BC is $10\sqrt{3}$m

In ΔABC which is right triangle,

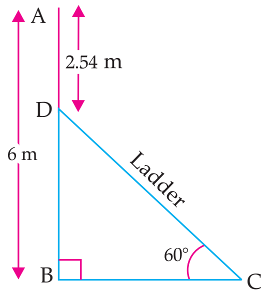
$\tan\theta = \dfrac{AB}{BC} = \dfrac{30^3}{10\sqrt{3}}$

$\tan\theta = \sqrt{3}$

but $\tan 60° = \sqrt{3}$. $\therefore \theta = 60°$.

so, angle of elevation of sun is 60°. 1

5. **In the given figure, AB is a 6 m high pole and DC is a ladder inclined at an angle of 60° to the horizontal and reaches up to point D of pole. If AD = 2.54 m, find the length of the ladder. (use $\sqrt{3}$ = 1.73)**



C + A [CBSE Delhi Set I, II, III, 2016]

Sol. Given, $AD = 2.54$ m

$\therefore$ $DB = 6 - 2.54 = 3.46$ m

In ΔBCD, $\angle B = 90°$

$$\sin 60° = \frac{BD}{DC}$$

$$\frac{\sqrt{3}}{2} = \frac{3.46}{DC}$$

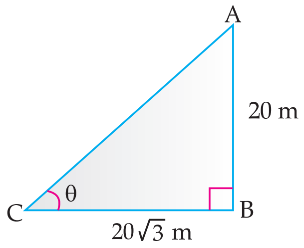
$$\therefore \quad DC = \frac{3.46 \times 2}{\sqrt{3}}$$

$$= \frac{3.46 \times 2}{1.73}$$

$$= 4 \text{ m}$$

$\therefore$ Length of ladder = 4 m.

AI 6. **In figure, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. Find the Sun's altitude.**



C + A [CBSE OD Set-I, II, III, 2015]

Sol. Let the $\angle ACB$ be θ, $\angle B = 90°$

$$\tan\theta = \frac{AB}{BC}$$

$$\tan\theta = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30° \quad \mathbf{1}$$

$\Rightarrow$ $\theta = 30°$

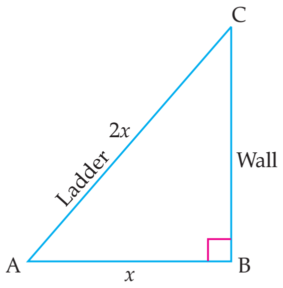
Thus, the Sun's altitude is 30°.

[CBSE Marking Scheme, 2015]

7. **If the length of the ladder placed against a wall is twice the distance between the foot of the ladder and the wall. Find the angle made by the ladder with the horizontal.** A [CBSE SA-II, 2015]

Sol. Let the distance between the foot of the ladder and the wall, AB be x.

Then AC, the length of the ladder = $2x$



In ΔABC, $\angle B = 90°$

$$\cos A = \frac{x}{2x}$$

$$\Rightarrow \quad \cos A = \frac{1}{2} = \cos 60° \Rightarrow \angle A = 60°$$

Short Answer Type Questions-I

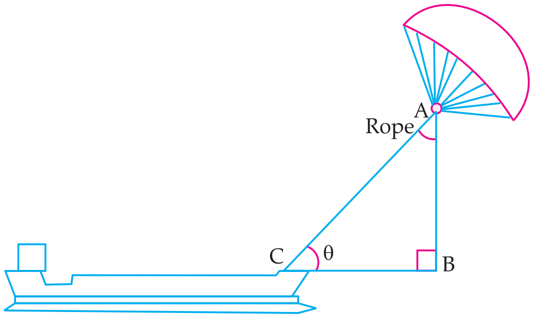
(2 Marks Each)

1. **'Sky sails' is that genre of engineering science that uses extensive utilization of wind energy to move a vessel in the sea water. The 'Sky sails' technology allows the towing kite to gain a height of anything between 100 metres – 300 metres. The sailing kite**

is made in such a way that it can be raised to its proper elevation and then brought back with the help of 'telescopic mast' that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing, answer the questions:

(i) In the given figure, if $\sin\theta = \cos(3\theta - 30°)$, where θ and $3\theta - 30°$ are acute angles, then find the value of θ.



(ii) What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 200 m ? C [CBSE SQP, 2020-21]

Sol. (i) $\cos(90° - \theta) = \cos(3\theta - 30°)$

$\Rightarrow \quad 90° - \theta = 3\theta - 30°$

$\Rightarrow \quad \theta = 30°$ **1**

(ii) $\dfrac{AB}{AC} = \sin 30°$

$\dfrac{200}{AC} = \dfrac{1}{2}$

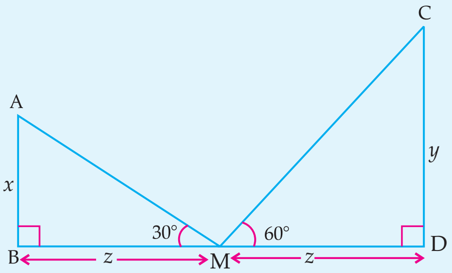
$\therefore$ Length of rope $= AC = 400$ m **1**

[CBSE SQP Marking Scheme, 2020]

2. The tops of two towers of height x and y, standing on the ground, subtend the angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x : y$.

A [CBSE Delhi Set-I, II, III, 2015]

Sol.



Let M be the centre of the line joining their feet.

Let $BM = MD = z$

$\therefore \quad \tan\theta = \dfrac{\text{perpendicular}}{\text{base}}$

In ΔABM, $\dfrac{x}{z} = \tan 30°$

$\Rightarrow \quad x = z \times \dfrac{1}{\sqrt{3}}$ ...(i) ½

In ΔCDM, $\dfrac{y}{z} = \tan 60°$

$y = z \times \sqrt{3}$ ...(ii) ½

From (i) and (ii), $\dfrac{x}{y} = \dfrac{z \times \frac{1}{\sqrt{3}}}{z \times \sqrt{3}}$

$\therefore \quad \dfrac{x}{y} = \dfrac{1}{3}$

$x : y = 1 : 3$ **1**

[CBSE Marking Scheme, 2015]

3. From the top of light house, 40 m above the water, the angle of depression of a small boat is 60°. Find how far the boat is from the base of the light house. U [CBSE Term-II, 2015]

Sol. Let AB be the light house and C be the position of the boat.

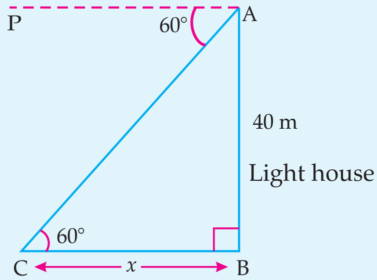
Since, $\angle PAC = 60° \quad \therefore \angle ACB = 60°$ **1**

Let BC be x m.

In ΔABC, $\dfrac{AB}{BC} = \tan 60°$

$\Rightarrow \quad \dfrac{40}{x} = \sqrt{3}$

$x = \dfrac{40}{\sqrt{3}}$



$\Rightarrow \quad x = \dfrac{40}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

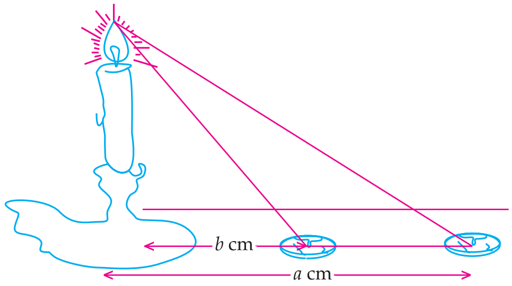
$= \dfrac{40\sqrt{3}}{3}$ m

Hence, the boat is $\dfrac{40\sqrt{3}}{3}$ m away from the foot of the light house. [CBSE Marking Scheme, 2015] **1**

Short Answer Type Questions-II

(3 Marks Each)

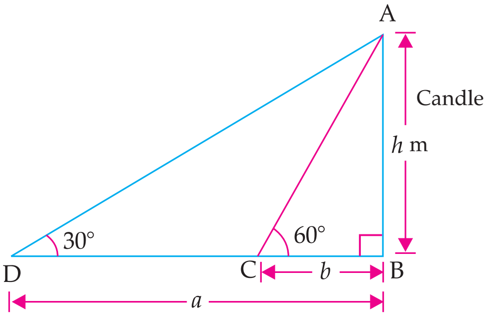
AI 1.



If the angles of elevation of the top of the candle from two coins distant 'a' cm and 'b' cm ($a > b$) from its base and in the same straight line from it are 30° and 60°, then find the height of the candle.

C + A [CBSE SQP, 2020-21]

Sol.



Let $\quad AB$ = candle

C and D are two coins.

In ΔACB, $\quad \tan 60° = \dfrac{AB}{BC} = \dfrac{h}{b}$

$$\sqrt{3} = \frac{h}{b}$$

$$h = b\sqrt{3} \quad \text{...(i)}$$

In ΔADB, $\quad \tan 30° = \dfrac{AB}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{h}{a}$$

$$h = \frac{a}{\sqrt{3}} \quad \text{...(ii)}$$

Multiplying (i) and (ii), we get

$$h^2 = b\sqrt{3} \times \frac{a}{\sqrt{3}}$$

$$h^2 = ba$$

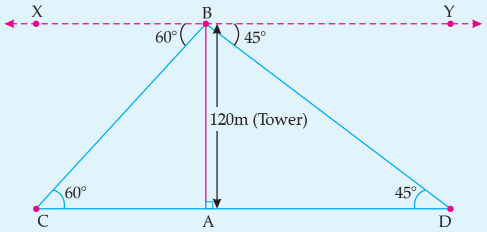
$$h = \sqrt{ab} \text{ m}$$

Hence, the height of the candle is $\sqrt{ab}$ m.

2. From the top of a 120 m high tower, a man observes two cars on the opposite sides of the tower and in straight line with the base of tower with angles of depression as 60° and 45°. Find the distance between two cars.

A [CBSE Comptt. Delhi/OD Set-I, II, III, 2017]

Sol.



½

In ΔABD, $\angle ADB = \angle DBY = 45°$ (alternate angles)

and in ΔABC, $\angle BCA = \angle XBC = 60°$

$$\frac{AB}{AD} = \tan 45°$$

$$\frac{120}{AD} = 1$$

$$\Rightarrow \quad AD = 120 \text{ m} \quad \text{...(i) } \mathbf{1}$$

Now, In ΔABC, $\dfrac{AB}{CA} = \tan 60°$

$$\frac{120}{CA} = \sqrt{3}$$

$$\Rightarrow \quad CA = \frac{120}{\sqrt{3}} = 40\sqrt{3} \text{ m} \quad ½$$

$$CD = AD + CA$$

$$= 120 + 40\sqrt{3}$$

$$= 120 + 40 \times 1{\cdot}732$$

$$= 120 + 69{\cdot}28 = 189{\cdot}28 \text{ m}$$

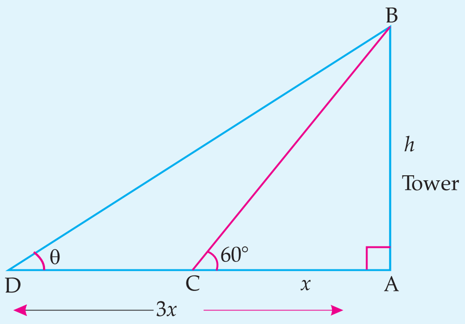
Hence the distance between two cars = 189·28 m. **1**

[CBSE Marking Scheme, 2017]

3. The shadow of a tower at a time is three times as long as its shadow when the angle of elevation of the Sun is 60°. Find the angle of elevation of the Sun at the time of the longer shadow.

A [CBSE Foreign Set-I, II, III, 2017]

Sol.



½

In ΔABC, $\quad \dfrac{AB}{AC} = \tan 60°$

$$\frac{h}{x} = \sqrt{3}$$

$$\Rightarrow \quad h = x\sqrt{3} \quad \mathbf{1}$$

In ΔABD, $\quad \dfrac{AB}{AD} = \tan\theta$

$$\frac{h}{3x} = \tan\theta$$

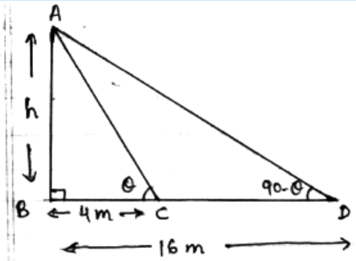
$$\Rightarrow \quad \frac{x\sqrt{3}}{3x} = \frac{1}{\sqrt{3}} = \tan 30° \quad \mathbf{1}$$

$$\therefore \quad \theta = 30° \quad ½$$

[CBSE Marking Scheme, 2017]

4. On a straight line passing through the foot of a tower, two points C and D are at distances of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower. **A [CBSE OD Set-I, II, III, 2017]**

Topper Answer, 2017



It is given that ∠ACB and ∠ADB are complementary.
Let them be θ and 90-θ respectively.
Now,
In right Δ ABC,

$\tan\theta = \frac{AB}{BC} = \frac{h}{4}$

$\tan\theta = \frac{h}{4}$ —(1)

In right ΔABD,

$\tan(90-\theta) = \frac{AB}{BD} = \frac{h}{16}$

$\cot\theta = \frac{h}{16}$

$\tan\theta = \frac{16}{h}$ —(2)

$\tan(90-\theta) = \cot\theta$

$\cot\theta = \frac{1}{\tan\theta}$

From (1) and (2),

$\tan\theta = \frac{h}{4} = \frac{16}{h}$

$h^2 = 4\times16$

$h = \sqrt{4\times16}$

$\therefore h = 2\times4$

$h = 8$ m (ignoring -ve value).

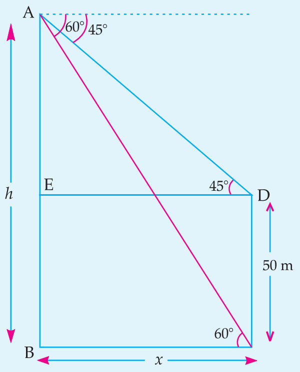
$\therefore$ height of tower is 8 m.

3

5. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building. (Use $\sqrt{3}$ = 1.73)

A **[CBSE Delhi Set I, II, III, 2016]**

Sol.



½

$$\tan 45° = \frac{h-50}{x}$$

$\Rightarrow$ $x = h - 50$ ½

$$\tan 60° = \frac{h}{x}$$ ½

$\Rightarrow$ $$x = \frac{h}{\sqrt{3}}$$

Hence $$h - 50 = \frac{h}{\sqrt{3}}$$ ½

$$\sqrt{3}h - 50\sqrt{3} = h$$
$$\sqrt{3}h - h = 50\sqrt{3}$$
$$h(\sqrt{3}-1) = 50\sqrt{3}$$
$$h = \frac{50\sqrt{3}}{\sqrt{3}-1}$$
$$h = \frac{50\sqrt{3}(\sqrt{3}+1)}{3-1}$$
$$h = \frac{50(3+\sqrt{3})}{2}$$

$\Rightarrow$ $h = 75 + 25\sqrt{3} = 75 + 43.25$

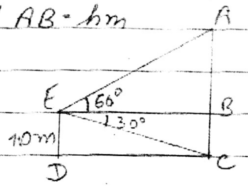
$= 118.25$ m 1

[CBSE Marking Scheme, 2016]

6. **A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30°. Find the distance of the hill from the ship and the height of the hill.** A **[CBSE OD Set-II, 2016]**

Topper Answer, 2016

Let AC = be height of hill and AB = h m



In Δ BCE,

where BC = 10 m & ∠BEC = 30°

$\tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{BC}{BE} = \frac{1}{\sqrt{3}} \Rightarrow \frac{10}{BE} = \frac{1}{\sqrt{3}}$

$\Rightarrow 10\sqrt{3}\text{ m} = BE = CD$

Distance of hill from ship = $10\sqrt{3}$ m = 10×1.732 m

= 17.32 m

In Δ ABE, where AB = h m, BE = $10\sqrt{3}$ m & ∠AEB = 60°

$\tan 60° = \sqrt{3}$

$\Rightarrow \frac{h}{10\sqrt{3}} = \sqrt{3} \Rightarrow h = 10\sqrt{3} \times \sqrt{3}$

h = 30 m

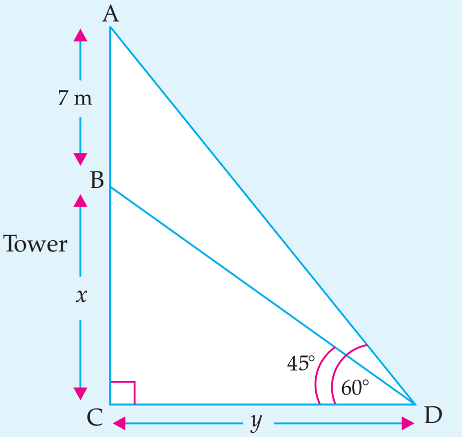
Height of hill = h + 10 m

$\boxed{40\text{ m}}$

3

7. **A 7 m long flagstaff is fixed on the top of a tower standing on the horizontal plane. From point on the ground, the angles of elevation of the top and bottom of the flagstaff are 60° and 45° respectively. Find the height of the tower correct upto one place of decimal. (Use $\sqrt{3}$ = 1.73)** A **[CBSE Foreign Set II, 2016]**

Sol.



½

In ΔBCD, $\frac{BC}{CD}$ = tan 45°

i.e., $\frac{x}{y}$ = tan 45° = 1

$\Rightarrow$ $x = y$ **1**

In ΔACD, $\frac{x+7}{y}$ = tan 60° = $\sqrt{3}$

$\Rightarrow$ $y\sqrt{3} = x + 7$

Putting $y = x$, then

$x\sqrt{3} = x + 7$

$\Rightarrow$ $7 = (\sqrt{3}-1)x$ **1+½**

$x = \frac{7(\sqrt{3}+1)}{2}$

$= \frac{7(2.73)}{2}$

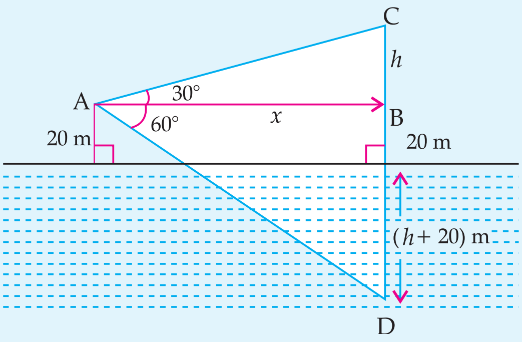
$= \frac{19.21}{2} = 9.60$

= 9.6 m

[CBSE Marking Scheme, 2016]

AI 8. **At a point A, 20 metre above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A ?** A **[CBSE OD Set-I, II, III, 2015]**

Sol.



In ΔABC,

$$\frac{h}{x} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = \sqrt{3}h \qquad \text{...(i) ½}$$

In ΔABD,

$$\frac{40+h}{x} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad x = \frac{40+h}{\sqrt{3}} \qquad \text{...(ii) ½}$$

From (i) and (ii),

$$\therefore \quad \sqrt{3}h = \frac{40+h}{\sqrt{3}}$$

$$\Rightarrow \quad 3h = 40 + h \text{ m}$$

$$\Rightarrow \quad h = 20 \text{ m} \qquad \mathbf{1}$$

$$\therefore \quad x = 20\sqrt{3} \text{ m}$$

$$\therefore \quad AC = \sqrt{(BC)^2 + (AB)^2}$$

$$= \sqrt{(20)^2 + (20\sqrt{3})^2}$$

$$= \sqrt{400 + 1200}$$

$$= 40 \text{ m} \qquad \mathbf{1}$$

Hence, the distance of the cloud = 40 m

[CBSE Marking Scheme, 2015]

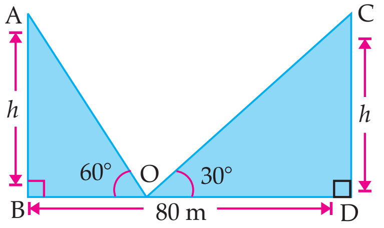
Long Answer Type Questions

(5 Marks Each)

AI **1. The two palm trees are of equal heights and are standing opposite to each other on either side of the river, which is 80 m wide. From a point O between them on the river the angles of elevation of the top of the trees are 60° and 30°, respectively. Find the height of the trees and the distances of the point O from the trees. (use $\sqrt{3}$ = 1.73)**

A **[CBSE SQP, 2020-21]**

Sol.



Let BD = width of river = 80 m

$AB = CD$

= height of both palm trees = h

$BO = x$ m

$OD = 80 - x$ m

In ΔABO,

$$\tan 60° = \frac{h}{x}$$

$$\sqrt{3} = \frac{h}{x} \qquad \text{...(i)}$$

$$h = \sqrt{3}x$$

In ΔCDO,

$$\tan 30° = \frac{h}{(80-x)}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{(80-x)} \qquad \text{...(ii)}$$

Solving (i) and (ii), we get

$$x = 20\ m$$

$$h = \sqrt{3}x \qquad \text{[From eqn. (i)]}$$

$$= 1.73 \times 20$$

$$= 34.6$$

The height of the trees = h = 34.6 m

$BO = x$ = 20 m

$DO = 80 - x$

= 80 – 20

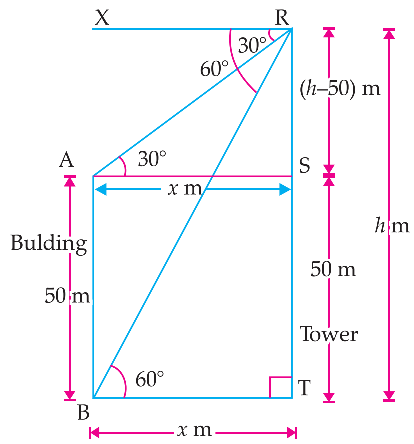
= 60 m

∴ The distances of the point O from the trees are 20 m and 60 m respectively.

AI **2. The angles of depression of the top and bottom of a building 50 meters high as observed from the top of a tower are 30° and 60° respectively. Find the height of the tower, and also the horizontal distance between the building and the tower.**

A **[CBSE SQP, 2020-21]**

Sol.



Let height of building, AB = 50 m

and height of tower, $RT = h$ m

$BT = AS = x$ m

$AB = ST$ = 50 m

$RS = TR - TS = (h - 50)$ m

In ΔARS, $\tan 30° = \frac{RS}{AS}$

$$\frac{1}{\sqrt{3}} = \frac{h-50}{x} \quad ...(i)$$

In ΔRBT, $\tan 60° = \frac{RT}{BT}$

$$\sqrt{3} = \frac{h}{x} \quad ...(ii)$$

Solving (i) and (ii), we get

$$h = 75$$

From (ii), $x = \frac{h}{\sqrt{3}} = \frac{75}{\sqrt{3}}$

$$= 25\sqrt{3}$$

Hence, height of the tower = 75 m

Distance between the building and the tower

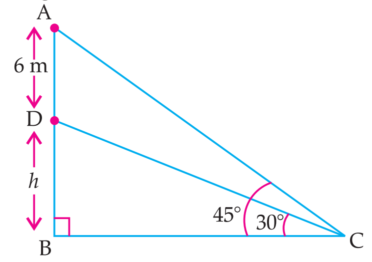
$$= 25\sqrt{3}$$
$$= 25 \times 1.732$$
$$= 43.30 \text{ m}$$

AI **3. A vertical tower stands on horizontal plane and is surmounted by a vertical flag-staff of height 6 m. The angles at a point on the bottom and top of the flag-staff with the ground are 30° and 45° respectively. Find the height of the tower.**

(Take $\sqrt{3}$ = 1.73) U **[CBSE Delhi Set-I, 2020]**

Sol. According to question,

AD is a flagstaff and BD is a tower.



In ΔABC,

$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow \quad 1 = \frac{h+6}{BC}$$

$$\Rightarrow \quad BC = h + 6 \quad ...(i)$$

In ΔDBC, $\tan 30° = \frac{DB}{BC}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{h+6} \quad \text{[from (i)]}$$

$$\Rightarrow \quad h\sqrt{3} = h + 6$$

$$\Rightarrow \quad h\sqrt{3} - h = 6$$

$$\Rightarrow \quad h(\sqrt{3}-1) = 6$$

$$\Rightarrow \quad h = \frac{6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\Rightarrow \quad h = \frac{6(\sqrt{3}+1)}{2}$$

$$\Rightarrow \quad h = 3(\sqrt{3}+1)$$

$$= 3(1.73 + 1)$$

$$\Rightarrow \quad h = 3 \times 2.73$$

$$\Rightarrow \quad h = 8.19 \text{ m}$$

$\therefore$ The height of the tower is 8.19 m

4. From a point on the ground, the angles of elevation of the bottom and the top of a tower are 45° and 60° respectively above of 20 m high building. Find the height of the tower. A **[CBSE OD Set-I, 2020]**

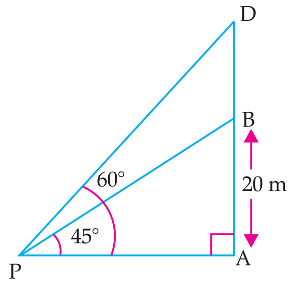
Sol. Let the height of the tower be BD

In ΔPAB

$$\tan 45° = \frac{AB}{AP}$$

$$\Rightarrow \quad 1 = \frac{20}{AP}$$

$$\Rightarrow \quad AP = 20 \text{ m}$$



In ΔPAD,

$$\tan 60° = \frac{AD}{AP} = \frac{20 + BD}{20}$$

$$\Rightarrow \quad \sqrt{3} = \frac{20+BD}{20}$$

$$\Rightarrow \quad 20 + BD = 20\sqrt{3}$$

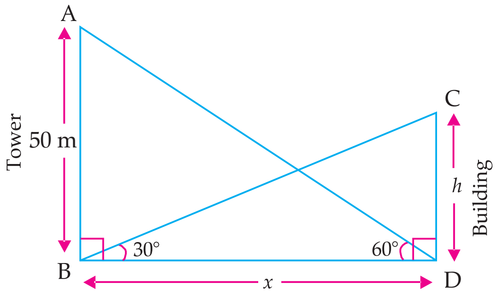
$$\Rightarrow \quad BD = 20\sqrt{3} - 20$$

$$= 20(\sqrt{3}-1)$$
$$= 20(1.732 - 1)$$
$$= 20 \times 0.732$$
$$= 14.64 \text{ m.}$$

Hence, the height of the tower is 14.64 m.

5. The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of a tower from the foot of the building is 60°. If the tower is 50 m high, then find the height of the building. A **[CBSE OD Set-II, 2020]**

Sol. According to question,



In ΔABD,

$$\tan 60° = \frac{AB}{BD}$$

$$\Rightarrow \qquad \sqrt{3} = \frac{50}{BD}$$

$$\Rightarrow \qquad BD = \frac{50}{\sqrt{3}}$$

Now in ΔBDC,

$$\tan 30° = \frac{CD}{BD}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{h}{\frac{50}{\sqrt{3}}} = \frac{h\sqrt{3}}{50}$$

$$\Rightarrow \qquad 3h = 50$$

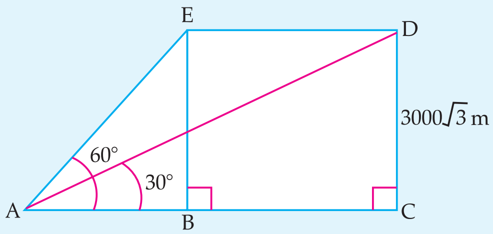
$$\Rightarrow \qquad h = \frac{50}{3} = 16.67$$

Hence, the height of the building is 16.67 m.

AI 6. The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30 seconds the angle of elevation becomes 30°. If the air plane is flying at a constant height of $3000\sqrt{3}$ m, find the speed of the aeroplane.

A **[CBSE SQP, 2020]**

Sol.



Correct figure. **1**

In ΔABE, $\frac{BE}{AB} = \tan 60°$ **1**

$\Rightarrow \qquad AB = 3000$ m

In ΔDAC, $\frac{DC}{AC} = \tan 30°$ **1**

$\Rightarrow \qquad AC = 9000$ m

$BC = AC - AB = 6000$ m **1**

$\therefore$ Speed of aeroplane $= \frac{6000}{30}$ m / sec

$= 200$ m/sec **1**

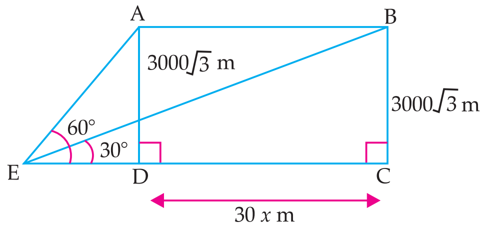
[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

$$\angle AED = 60° \text{ and } \angle BEC = 30°$$

$AD = BC = 3000\sqrt{3}$ m

Let the speed of the aeroplane $= x$ m/s



Then, $AB = DC = 30 \times x$

$= 30x$ m ...(i)

In ΔAED, $\angle D = 90°$

$$\tan 60° = \frac{AD}{DE}$$

$$\sqrt{3} = \frac{3000\sqrt{3}}{DE}$$

$DE = 3000$ m ...(ii)

In ΔBEC, $\angle C = 90°$

$$\tan 30° = \frac{BC}{EC}$$

$$\frac{1}{\sqrt{3}} = \frac{3000\sqrt{3}}{DE + CD}$$

$DE + CD = 3000 \times 3$

$3000 + 30x = 9000$ [from eqs. (i) and (ii)]

$30x = 6000$

$x = 200$ m/s

Hence, the speed of plane $= 200$ m/s.

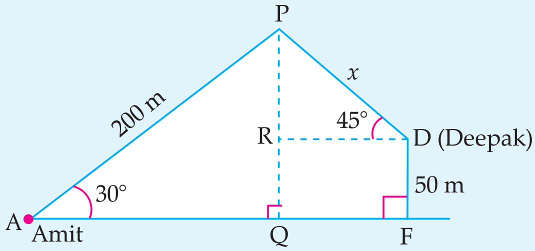
$$= 200 \times \frac{18}{5} = 720 \text{ km/h}$$

AI 7. Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point in between them on the road, the angles of elevation of the top of poles are 60° and 30° respectively. Find the height of the poles and the distances of the point P from the poles. A **[CBSE Delhi, Set-I, 2019]**

Sol. Try yourself similar to Q.No. 1 L.A.T.Q.

AI 8. Amit, standing on a horizontal plane, and a bird flying at a distance of 200 m from him at an elevation of 30°. Deepak standing on the roof of a 50 m high building, and the angle of elevation of the same bird to be 45°. Amit and Deepak are on opposite sides of the bird. Find the distance of the bird from Deepak. A **[CBSE OD Set-I, 2019]**

Sol.



1

In ΔAPQ, $\quad \dfrac{PQ}{AP} = \sin 30° = \dfrac{1}{2}$ **1**

$PQ = (200)\left(\dfrac{1}{2}\right) = 100$ m **1**

$PR = 100 - 50 = 50$ m **1**

In ΔPRD, $\quad \dfrac{PR}{PD} = \sin 45° = \dfrac{1}{\sqrt{2}}$

$PD = (PR)(\sqrt{2}) = 50\sqrt{2}$ m **1**

[CBSE Marking Scheme, 2019]

Detailed Solution:

Let P be the position of the bird, A be the position of Amit, D be the position of Deepak and FD be the building at which Deepak is standing at height 50 m.

Given, $AP = 200$ m and $FD = 50$ m

In ΔPQA, $\quad \angle Q = 90°$

$$\sin 30° = \frac{PQ}{PA}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{PQ}{200}$$

$$\Rightarrow \quad PQ = \frac{200}{2} = 100 \text{ m}$$

$$\therefore \quad PR = PQ - RQ$$
$$= PQ - FD$$
$$= (100 - 50) \text{ m}$$
$$= 50 \text{ m}$$

In ΔPRD, $\quad \angle R = 90°$

$$\sin 45° = \frac{PR}{PD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{50}{PD}$$

$$\Rightarrow \quad PD = 50\sqrt{2} \text{ m}$$

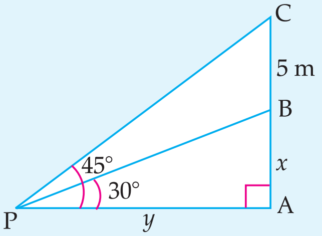
$$= 50 \times 1.414 \text{ m}$$
$$= 70.7 \text{ m}$$

Thus, the distance of the bird from Deepak is 70.7 m.

9. From a point P on the ground, the angle of elevation of the top of a tower is 30° and that of the top of the flag-staff fixed on the top of the tower is 45°. If the length of the flag-staff is 5 m, find the height of the tower. (Use $\sqrt{3} = 1.732$)

A **[CBSE OD Set-III, 2019]**

Sol. Let AB be a tower and BC be a flagstaff.



1

In ΔPAC, according to question,

$$\frac{AC}{AP} = \tan 45° = 1 \quad \textbf{1}$$

$$\Rightarrow \quad x + 5 = y \quad \text{...(i) } ½$$

In ΔPAB, $\quad \dfrac{x}{y} = \tan 30° = \dfrac{1}{\sqrt{3}}$

$$\frac{x}{x+5} = \frac{1}{\sqrt{3}} \quad \text{[from eq. (i)]}$$

$$\Rightarrow \quad x = \frac{5}{\sqrt{3}-1}$$

$$= \frac{5(\sqrt{3}+1)}{2}$$

$$= \frac{13.66}{2}$$

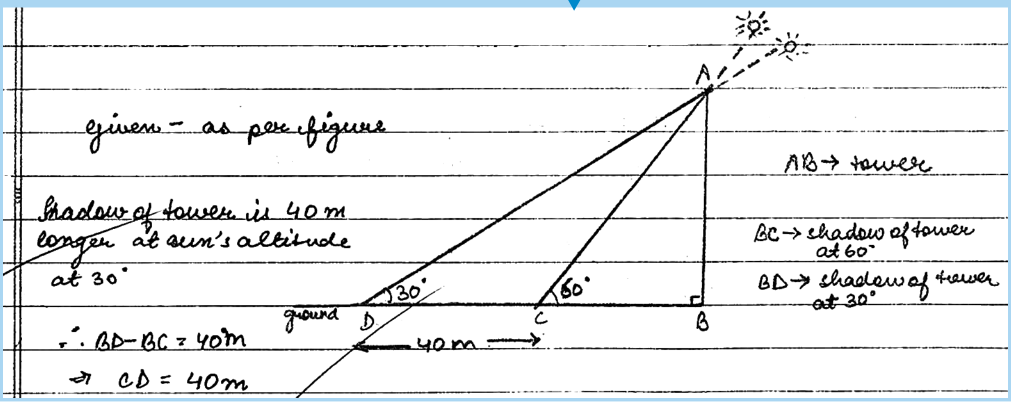
$$= 6.83 \quad 1½$$

$\therefore$ Height of tower $= 6.83$ m **1**

[CBSE Marking Scheme, 2019]

10. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60°. Find the height of the tower. (Given $\sqrt{3} = 1.732$) **[CBSE Delhi Term, 2019]**

Topper Answer, 2019



In ΔACB,

$$\tan 60^\circ = \frac{AB}{BC}$$

$\Rightarrow \sqrt{3} \times BC = AB \quad \Rightarrow BC = \frac{AB}{\sqrt{3}}$ —①

In ΔADB,

$\tan 30^\circ = \frac{AB}{BD} \quad \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{40+BC}$

$\Rightarrow 40 + BC = \sqrt{3} \times AB$

$\Rightarrow 40 + \frac{AB}{\sqrt{3}} = \sqrt{3} \times AB$ [Put $BC = \frac{AB}{\sqrt{3}}$ from ①].

$\Rightarrow AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 40$

$\Rightarrow AB\left(\frac{3-1}{\sqrt{3}}\right) = 40$

$\Rightarrow AB \times \frac{2}{\sqrt{3}} = 40 \quad \Rightarrow AB = \frac{40 \times \sqrt{3}}{2}$ m

$\Rightarrow AB = \underline{20\sqrt{3}\text{ m}}$

given, use $\sqrt{3} = 1{\cdot}732$ $\quad \therefore AB = 20 \times 1{\cdot}732$ m $= \underline{34{\cdot}64\text{ m}}$

$\therefore$ Height of tower $= 34{\cdot}64$m.

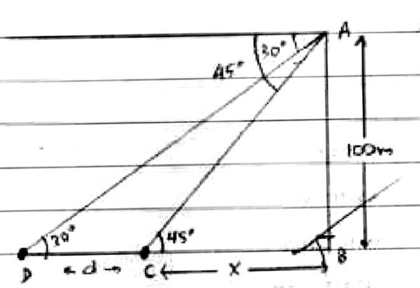
5

AI 11. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. [Use $\sqrt{3} = 1{\cdot}732$] C + A **[CBSE Delhi/OD 2018]**

Topper Answer, 2018

Diagram:

4



AB→ lighthouse = 100m high

C→ boat 1

D→ boat 2.

To find: $\overline{CD}$ or d.

(distance b/w ships)

We know,

$\tan\angle ACB = \frac{Opp.}{adj.} = \frac{AB}{BC}$. $\qquad \tan\angle ADB = \frac{opp.}{adj.} = \frac{AB}{BD}$.

$\rightarrow \tan 45^\circ = \frac{100}{x}$ $\qquad \rightarrow \tan 30^\circ = \frac{100}{x+d}$

$1 = \frac{100}{x}$ $\qquad \frac{1}{\sqrt{3}} = \frac{100}{d+100}$ [x=100]

$\Rightarrow x = 100$ m.

$100 + d = 100\sqrt{3}$.

$\rightarrow d = 100\sqrt{3} - 100. = 100(\sqrt{3}-1)$

Given, $\sqrt{3} = 1.732$,

$\Rightarrow d = 100(1.732 - 1)$

$= 100 \times 0.732 = 73.2m.$

→ The distance between the boats is 73.2 m. 5

COMMONLY MADE ERROR

- Most candidates are unable to draw the diagram as per the given data and lose their marks. Some candidates do calculation errors while putting the values of $\sqrt{3}$ = 1.73 instead of 1.732 and hence write inaccurate answer.

ANSWERING TIP

- Students should do rounding off at the end while calculating the final answer.

12. A man on the top of a vertical observation tower observes a car moving at uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how long will the car take to reach the observation tower from this point ? C + A **[CBSE SQP, 2018]**

Sol. Let the speed of car by x m/minute

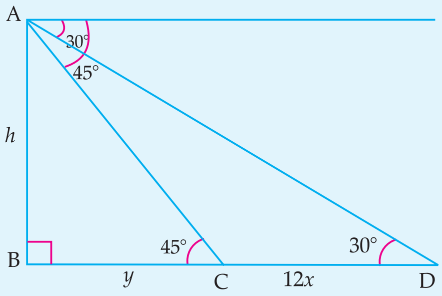
In ΔABC,

$$\frac{h}{y} = \tan 45° \qquad 1$$

$$\Rightarrow \quad h = y \qquad ½$$

In ΔABD, $$\frac{h}{y+12x} = \tan 30° \qquad 1$$

$$\Rightarrow \quad h\sqrt{3} = y + 12x \qquad ½$$



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$$y\sqrt{3} - y = 12x$$

$$y = \frac{12x}{\sqrt{3}-1} = \frac{12x(\sqrt{3}+1)}{2}$$

$$\Rightarrow \quad y = 6x(\sqrt{3}+1)$$

Hence, time taken from C to $B = 6(\sqrt{3}+1)$ minutes

[CBSE Marking Scheme, 2018] 1

Detailed Solution:

Let AB be the tower of height h and x be the distance between Point P to Point Q

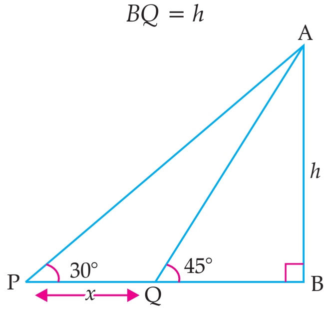
$$\angle AQB = 45°$$

Now, in ΔABQ,

$$\tan 45° = \frac{AB}{BQ}$$

$$\Rightarrow \quad 1 = \frac{h}{BQ}$$

$$\Rightarrow \quad BQ = h$$



In ΔAPB, $$\tan 30° = \frac{AB}{PB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+h}$$

$$\Rightarrow \quad x + h = h\sqrt{3}$$

i.e., $$x = h\left(\sqrt{3}-1\right)$$

Thus, $$\text{Speed} = \frac{h\left(\sqrt{3}-1\right)}{12} \text{ m/min}$$

Time for remaining distance,

$$= \frac{h}{\frac{h\left(\sqrt{3}-1\right)}{12}}$$

$$= \frac{12\left(\sqrt{3}+1\right)}{3-1}$$

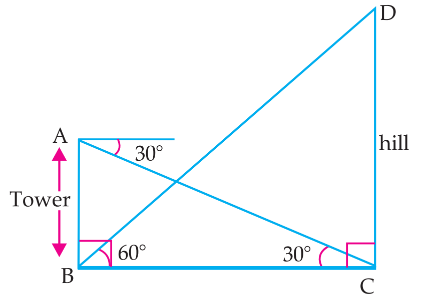
$$= \frac{12}{2}(\sqrt{3}+1)$$

$$= 6\left(\sqrt{3}+1\right) \text{ min}$$

13. The angle of elevation of the top of a hill from the foot of a tower is 60° and the angle of depression from the top of the tower of the foot of the hill is 30°. If tower is 50 meter high, find the height of the hill. A [CBSE Comptt. Set-I, II, III, 2018]

[CBSE Delhi Set-I, II, III, 2015]

Sol.



Let $AB = 50$ m be the height of the tower and CD be the height of hill.

Now, in ΔABC,

$$\angle ABC = 90°$$

$$\tan 30° = \frac{AB}{BC}$$

or, $$BC = \frac{50}{\tan 30°} = \frac{50\times\sqrt{3}}{1} \text{ m}$$

or, $$BC = 50\sqrt{3} \text{ m}$$

Again in ΔBCD, $\angle BCD = 90°$

$$\tan 60° = \frac{DC}{BC}$$

or, $$DC = BC \tan 60°$$
$$= 50\sqrt{3}\times\sqrt{3} \text{ m}$$

$\Rightarrow$ $$DC = 150 \text{ m}$$

∴ The height of hill is 150 m.

COMMONLY MADE ERROR

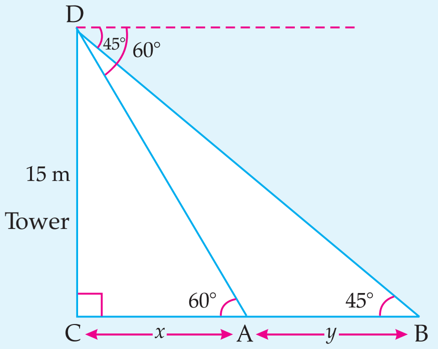
- The concept of angle of depression is not clear to many students. That's why they are not able to draw the diagram correctly.

ANSWERING TIP

- The concept of angle of depression and angle of elevation must be clear to the students.

14. Two points A and B are on the same side of a tower and in the same straight line with its base. The angle of depression of these points from the top of the tower are 60° and 45° respectively. If the height of the tower is 15 m, then find the distance between these points. C + A [CBSE Delhi Set-I, 2017]

Sol.



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In ΔDCA, $$\frac{DC}{CA} = \tan 60°$$

$\Rightarrow$ $$\frac{15}{x} = \sqrt{3}$$ ½

$\Rightarrow$ $$x = \frac{15}{\sqrt{3}}$$

$\Rightarrow$ $$x = 5\sqrt{3}$$ 1

In ΔDCB, $$\frac{DC}{CB} = \tan 45° = \frac{15}{x+y} = 1$$ ½

$\Rightarrow$ $$x + y = 15$$ 1

$\Rightarrow$ $$5\sqrt{3} + y = 15$$

$\Rightarrow$ $$y = 15 - 5\sqrt{3}$$
$$= 5\left(3-\sqrt{3}\right) \text{ m}$$

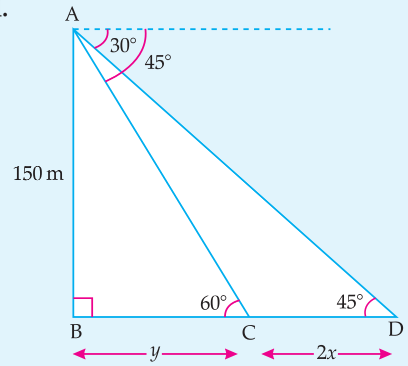
Hence, the distance between the points
$$= 5\left(3-\sqrt{3}\right) \text{ m.}$$

[CBSE Marking Scheme, 2017] 1

15. A moving boat observed from the top of a 150 m high cliff, moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat.

A [CBSE Delhi Set-I, 2017]

Sol.



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Let the speed of the boat be x m/min.

∴ Distance covered in 2 minutes $= 2x$

∴ $$CD = 2x$$

Let BC be y m.

In ΔABC, $$\frac{AB}{BC} = \tan 60°$$ ½

$\Rightarrow$ $$\frac{150}{y} = \sqrt{3}$$

$$\Rightarrow \qquad y = \frac{150}{\sqrt{3}}$$

$$\Rightarrow \qquad y = 50\sqrt{3} \text{ m} \qquad \text{...(i)}$$

In ΔABD, $\dfrac{AB}{BD} = \tan 45°$ ½

$$\Rightarrow \qquad \frac{150}{y+2x} = 1$$

$$\Rightarrow \qquad y + 2x = 150 \qquad \text{...(ii)} \; \mathbf{1}$$

Substituting the value of y from (i) in (ii),

$$50\sqrt{3} + 2x = 150$$

$$2x = 150 - 50\sqrt{3}$$

$$2x = 50\left(3-\sqrt{3}\right)$$

$$x = 25\left(3-\sqrt{3}\right) \text{ m} \qquad \mathbf{1}$$

$$\text{Speed of the boat} = 25\left(3-\sqrt{3}\right) \text{ m/min}$$

$$= \frac{25\left(3-\sqrt{3}\right)\times 60}{1000}$$

$$= \frac{3}{2}\left(3-\sqrt{3}\right) \text{ km/h}$$

$$= 1.902 \text{ km/h} \qquad \mathbf{1}$$

[CBSE Marking Scheme, 2017]

16. The angle of depression of two ships from an aeroplane flying at the height of 7500 m are 30° and 45°. If both the ships are in the same line that one ship is exactly behind the other, find the distance between the ships.

C + A **[CBSE Foreign Set-II 2017]**

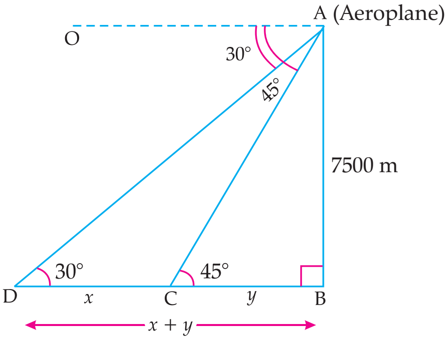
Sol. Let AB be the height of the aeroplane, then AB = 7500 m.

Also let D and C be the positions of two ships on the same line. From the point A of an aeroplane, the angles of depression of two ships D and C are $\angle OAD = 30°$ *i.e.,* $\angle BDA = 30°$ and $\angle OAC = 45°$ *i.e.,* $\angle BCA = 45°$

Let distance between two ships

$$DC = x \text{ m}$$

and $\qquad BC = y$ m.



In ΔABC, $\dfrac{AB}{BC} = \tan 45°$

$$\Rightarrow \qquad \frac{7500}{y} = 1$$

$$\Rightarrow \qquad y = 7500$$

In ΔABD, $\dfrac{AB}{BD} = \tan 30°$

$$\frac{7500}{x+y} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \qquad x + y = 7500\sqrt{3}$$

$$x + 7500 = 7500\sqrt{3}$$

$$x = 7500\sqrt{3} - 7500$$

$$= 7500\left(\sqrt{3}-1\right)$$

$$= 7500(1.732 - 1)$$

$$= 7500 \times 0.732$$

$$= 5490 \text{ m}$$

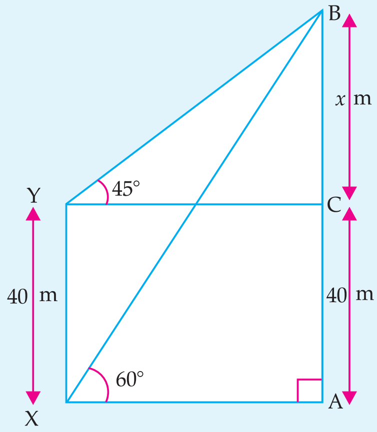
Hence, the distance between two ships

$$= 5490 \text{ m}$$

17. The angle of elevation of the top B of a tower AB from a point X on the ground is 60°. At a point Y, 40 m vertically above X, the angle of elevation of the top is 45°. Find the height of the tower AB and the distance XB. A **[CBSE Term-II, 2016]**

Sol. In ΔYCB, we have

$$\tan 45° = \frac{BC}{YC}$$



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$$1 = \frac{x}{YC}$$

$$YC = x \text{ m}$$

$$\Rightarrow \qquad XA = x \text{ m}$$

In ΔXAB,

$$\tan 60° = \frac{AB}{XA} \qquad ½$$

$$\sqrt{3} = \frac{x+40}{x} \qquad \tfrac{1}{2}$$

$$\sqrt{3}x = x + 40$$

$$x\sqrt{3} - x = 40$$

$$x = \frac{40}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \qquad \tfrac{1}{2}$$

$$= 20(\sqrt{3}+1)$$

$$= \left(20\sqrt{3}+20\right) \text{ m}$$

∴ Height of the tower,

$$AB = x + 40$$

$$= 20\sqrt{3} + 20 + 40 \qquad \tfrac{1}{2}$$

$$= 20\sqrt{3} + 60$$

$$= 20(\sqrt{3} + 3) \text{ m} \qquad \tfrac{1}{2}$$

$$20 \times 4.732 = 94.64 \text{ m}$$

In ΔXAB, $\sin 60° = \frac{AB}{BX}$ ½

$$\frac{\sqrt{3}}{2} = \frac{AB}{BX} \qquad \tfrac{1}{2}$$

$$BX = \frac{20(\sqrt{3}+3)2}{\sqrt{3}}$$

$$= 40\left(\sqrt{3}+1\right) \text{ m}$$

$$= 40 \times 2.732 \text{ m}$$

$$= 109.28 \text{ m} \qquad \tfrac{1}{2}$$

[CBSE Marking Scheme, 2016]

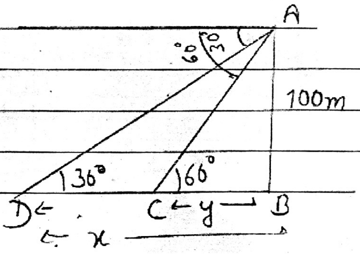
18. As observed from the top of a light house, 100 m high above sea level, the angles of depression of a ship, sailing directly towards it, changes from 30° to 60°. Find the distance travelled by the ship during the period of observation. (Use $\sqrt{3}$ = 1.73) C **[CBSE OD Set-II, 2016]**

Topper Answer, 2017

~~Q10 →~~

In Δ ABC, ∠ACB = 60°

$\tan 60° = \sqrt{3}$



$\Rightarrow \frac{100}{BC} = \sqrt{3}$

$\frac{100}{\sqrt{3}} = BC = y$

In Δ ABD, ∠ADB = 30°

$\tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \frac{AB}{BD} = \frac{1}{\sqrt{3}} \Rightarrow \frac{100}{BD} = \frac{1}{\sqrt{3}}$

$100\sqrt{3} = BD = \cancel{y}\ x$

Required distance travelled by ship = $-(y - x) = x - y$

$\Rightarrow 100\sqrt{3} - 100$ m

$x - y \Rightarrow 100\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right] = 100\left[\frac{3-1}{\sqrt{3}}\right] = \frac{100 \times 2}{\sqrt{3}}$

$CD = x - y \Rightarrow \frac{100 \times 2 \times \sqrt{3}}{\sqrt{3}\ \sqrt{3}} = \frac{200\sqrt{3}}{3}$ m

$CD = \frac{200 \times 1.73 \text{ m}}{3} = \frac{346}{3}$ m

$\Rightarrow \boxed{115.33 \text{ m}}$

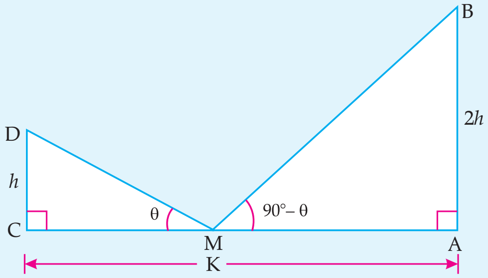
19. Two posts are k metre apart and the height of one is double that of the other. If from the mid-point of the line segment joining their feet, an observer finds the angles of elevation of their tops to be complementary, then find the height of the shorter post. A **[CBSE Term-II, 2015]**

Sol. Let AB and CD be the two posts such that

$$AB = 2CD.$$

Let M be the mid-point of CA.

Let $\angle CMD = \theta$ and $\angle AMB = 90° - \theta$



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Clearly, $\quad CM = MA = \frac{1}{2}k$

Let $CD = h$ m, then AB = $2h$ m

Now, $\quad \frac{AB}{AM} = \tan(90° - \theta) = \cot\theta$ **1**

$$\Rightarrow \quad \frac{2h}{\left(\frac{k}{2}\right)} = \cot\theta$$

$$\Rightarrow \quad \frac{2h}{\left(\frac{k}{2}\right)} = \cot\theta$$

$$\Rightarrow \quad \cot\theta = \frac{4h}{k} \quad \text{...(i)} \ \mathbf{1}$$

Also in ΔCMD, $\frac{CD}{CM} = \tan\theta$

$$\Rightarrow \quad \frac{h}{\frac{k}{2}} = \tan\theta$$

$$\Rightarrow \quad \tan\theta = \frac{2h}{k} \quad \text{...(ii)} \ \mathbf{1}$$

Multiplying (i) and (ii),

$$\frac{4h}{k} \times \frac{2h}{k} = 1$$

$$\therefore \quad h^2 = \frac{k^2}{8}$$

$$\Rightarrow \quad h = \frac{k}{2\sqrt{2}}$$

$$= \frac{k\sqrt{2}}{4} \text{ m} \quad \mathbf{1}$$

[CBSE Marking Scheme, 2015]