CBSE Class 11 Mathematics

Important Questions

Chapter 3

Trigonometric Functions

1 Marks Questions

1. Find the radian measure corresponding to 5° 37' 30''

Ans. $\left(\frac{\pi}{32}\right)^c$

2. Find the degree measure corresponding to $\left(\frac{11}{16}\right)^c$

Ans. 39°22'30''

3. Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°

Ans. $\frac{5\pi}{12}$ cm

4. Find the value of $\frac{19\pi}{3}$

Ans. $\sqrt{3}$

5. Find the value of sin(–1125°)

Ans. $\frac{-1}{\sqrt{2}}$

6. Find the value of tan 15°

Ans. $2-\sqrt{3}$

7. If sin A = $\frac{3}{5}$ and $\frac{\pi}{2}$ < A < , find cos A

Ans. $\frac{-4}{5}$

8. If tan A = $\frac{a}{a+1}$ and tan B = $\frac{1}{2a+1}$ then find the value of A + B.

Ans. $45°$

9. Express sin 12θ + sin 4θ as the product of sines and cosines.

Ans. 2 sin8θ cos4θ

10. Express 2 cos4x sin2x as an algebraic sum of sines or cosines.

Ans. sin 6x – sin2x

11. Write the range of cos θ

Ans. [–1,1]

12. What is domain of sec θ?

Ans. $R - \left\{(2n+1)\frac{\pi}{2};\ n \in z\right\}$

13.Find the principal solutions of cotx = 3

Ans. $\frac{5\pi}{6}, \frac{11\pi}{6}$

14. Write the general solution of cos θ = 0

Ans. $(2n+1)\frac{\pi}{2},\ n \in z$

15. If sinx = $\frac{\sqrt{5}}{3}$ **and 0 < x <** $\frac{\pi}{2}$ **find the value of cos 2x**

Ans. $-\frac{1}{9}$

16. If cosx = $\frac{-1}{3}$ **and x lies in quadrant III, find the value of sin** $\frac{x}{2}$

Ans. $\frac{\sqrt{6}}{3}$

17.Convert into radian measures. – 47^0 30′

Ans. - 47^0 30′ = - $\left(47+\frac{30}{60}\right)^0$

$= -\left(47\frac{1}{2}\right)^0$

$= -\left(\frac{95}{2}\times\frac{\pi}{180}\right)$ radians

$= -\ \frac{19\pi}{72}$ radians.

18.Evaluate tan 75^0.

Ans. tan 75 = tan (45 + 30)

$$= \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30}$$

$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

19.Prove that Sin (40 + θ). Cos (10 + θ) – Cos (40 + θ). Sin (10 + θ) = $\frac{1}{2}$

Ans. L. H. S = Sin (40 + θ). Cos (10 + θ) – Cos (40 + θ). Sin (10 +θ)

= Sin $\left[40 + \not{\theta} - 10 - \not{\theta}\right] = \text{Sin } 30 = \frac{1}{2}$

20.Find the principal solution of the eq. Sin x = $\frac{\sqrt{3}}{2}$

Ans. Sin x = $\frac{\sqrt{3}}{2}$

$$\text{Sin x} = \text{Sin } \frac{\pi}{3}$$

$$x = \frac{\pi}{3}$$

$$Sin \text{ x} = \text{Sin } \left(\pi - \frac{\pi}{3}\right)$$

$$x = \frac{2\pi}{3}$$

21.Prove that $\text{Cos}\left(\frac{\pi}{4}+x\right)+\text{Cos}\left(\frac{\pi}{4}-x\right)=\sqrt{2}\ \text{Cos x}$

Ans. L. H. S = $\text{Cos}\left(\frac{\pi}{4}+x\right)+\text{Cos}\left(\frac{\pi}{4}-x\right)$

$2\ \text{Cos}\ \frac{\pi}{4}.\ \text{Cos x}\ [\because\ \text{Cos (A + B) + Cos (A - B) = 2 Cos A. Cos B}$

$= 2\ \frac{1}{\sqrt{2}}.\ \text{Cos x}\qquad = \sqrt{2}\ \text{Cos x}$

22.Convert into radian measures. -37^0 30'

Ans. - 37^0 30' = - $\left(37+\frac{\not{3}\not{0}^{\,1}}{\not{6}\not{0}^{\,2}}\right)^0$

$=-\left(\frac{75}{2}\right)^0$

$=\ -\ \frac{75}{2}\times\frac{\pi}{180}\,radien$

$=-\frac{5\pi}{24}$

23.Prove Sin (n+1) x Sin (n+2) x + Cos (n+1) x. Cos (n+2) x = Cos x

Ans.L.H.S = Cos (n+1) x Cos (n+2) x + Sin (n+1) x Sin (n+2) x

=Cos $\left[(n+1)x-(n+2)x\right]$

=Cos x

24.Find the value of Sin $\frac{31\pi}{3}$

Ans. Sin $\frac{31\pi}{3}$ = Sin $\left(10\pi+\frac{\pi}{3}\right)$

= sin $\left(2\pi\times 5+\frac{\pi}{3}\right)$

= Sin $\frac{\pi}{3}$

= $\frac{\sqrt{3}}{2}$

25.Find the principal solution of the eq. tan x = $\frac{-1}{\sqrt{3}}$

Ans. tan x = $-\frac{1}{\sqrt{3}}$

$\tan x=\tan\left(\pi-\frac{\pi}{6}\right)$

$x=\frac{5\pi}{6}$

$\tan x=-\frac{1}{\sqrt{3}}$

$\tan x=\tan\left(2\pi-\frac{\pi}{6}\right)$

$x=\frac{11\pi}{6}$

26.Convert into radian measures. $5^{0}37'\ 30''$

Ans. $5^0\ 37^1\ 30^{11}$ = 5^0 + $\left(37+\frac{30}{60}\right)^1$

$$=5^0+\left(\frac{75}{2}\right)^1$$

$$=5^0+\left(\frac{75}{2}\times\frac{1}{60}\right)^0$$

$$=5^0+\left(\frac{5}{8}\right)^0$$

$$=\left(5\frac{5}{8}\right)^0$$

$$=\left(\frac{45}{8}\right)^0$$

$$=\frac{45}{8}\times\frac{\pi}{180}\ \text{radian}$$

$$=\frac{\pi}{32}\ \text{radian.}$$

27.Prove Cos 70^0. Cos 10^0 + **Sin 70^0. Sin 10^0** = $\frac{1}{2}$

Ans. L. H. S. = Cos (70 – 10) = Cos 60 = $\frac{1}{2}$

28.Evaluate 2 Sin $\frac{\pi}{12}$.

Ans.2 Sin $\frac{\pi}{12}=2\ \text{Sin}\left[\frac{\pi}{4}\ -\ \frac{\pi}{6}\right]$

$$= 2\left[\sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}\right]$$

$$= 2\left[\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}\right]$$

$$= \frac{\sqrt{3}-1}{\sqrt{2}}$$

29.Find the solution of Sin x = $-\frac{\sqrt{3}}{2}$

Ans. Sin x = $-\frac{\sqrt{3}}{2}$

$$\text{Sin x} = \text{Sin}\left(\pi + \frac{\pi}{3}\right)$$

$$\text{Sin x} = \text{Sin}\ \frac{4\pi}{3}$$

$$\text{if Sin}\ \theta = \text{Sin}\ \alpha$$

$$\theta = \text{n}\ \pi + (-1)^n.\ \alpha$$

$$x = \text{n}\,\pi + (-1)^n.\frac{4\pi}{3}$$

30.Prove that $\frac{\text{Cos } 9^0 - \text{Sin } 9^0}{\text{Cos } 9^0 + \text{Sin } 9^0} = \tan 36^0$

Ans.L. H. S = tan 36^0

$$= \tan(45^0 - 9^0)$$

$$= \frac{1 - \tan 9^0}{1 + \tan 9^0}$$

$$= \frac{\text{Cos } 9^0 - \text{Sin } 9^0}{\text{Cos } 9^0 + \text{Sin } 9^0}$$

31.Find the value of tan $\frac{19\pi}{3}$.

Ans. $\tan \frac{19\pi}{3} = \tan\left(6\pi + \frac{\pi}{3}\right)$

$= \tan\left(3 \times 2\pi + \frac{\pi}{3}\right)$

$= \tan \frac{\pi}{3} = \sqrt{3}$

32.Prove Cos 4x = 1 – 8 Sin^2 x. Cos^{2}x

Ans. L. H. S = Cos 4x

$= 1 - 2\,\text{Sin}^2\,2x \quad \left[\because \text{Cos } 2\theta = 1 - 2\text{Sin}^2\,\theta\right.$

$= 1 - 2\,(\text{Sin}2x)^2$

$= 1 - 2\,(2\,\text{Sin x. Cos x})^2$

$= 1 - 2\,(4\,\text{Sin}^2\,x\,.\,\text{Cos}^2\,x)$

$= 1 - 8\,\text{Sin}^2\,x.\,\text{Cos}^2\,x.$

33.Prove

$$\frac{\text{Cos}(\pi+x)\ .\ \text{Cos}(-x)}{\text{Sin}(\pi-x).\ \text{Cos}\left(\frac{\pi}{2}+x\right)} = \text{Cot}^2\ x$$

Ans.L. H. S = $\frac{-\text{Cos x. Cos x}}{\text{-Sin x. Sin x}} = \text{Cot}^2\ x.$

34.Prove that tan 56^0 = $\frac{\text{Cos }11^0 + \text{Sin }11^0}{\text{Cos }11^0 - \text{Sin }11^0}$

Ans.L. H. S = tan 56^0

= tan (45^0 + 11^0)

$$= \frac{\tan 45^0 + \tan 11^0}{1 - \tan 45^0\ .\ \tan 11^0}$$

$$= \frac{1 + \tan 11^0}{1 - \tan 11^0}$$

$$= \frac{\text{Cos }11^0 + \text{Sin }11^0}{\text{Cos }11^0 - \text{Sin }11^0} = \text{R H S}$$

35.Prove that Cos 105^0 + **Cos 15^0** = **Sin 75^0 – Sin 15^0**

Ans.L. H. S = Cos 105^0 + Cos 15^0

$= \text{Cos}(90^0 + 15^0) + \text{Cos}(90^0 - 75^0)$

$= -\ \text{Sin }15^0 + \text{Sin }75^0$

$= \text{Sin }75^0 - \text{Sin }15^0$

36.Find the value of Cos (- 1710^0).

Ans. Cos (-1710^0) = Cos (1800-90)[Cos (-θ) = Cos θ

$= \text{Cos} [5 \times 360 + 90]$

$= \text{Cos} \frac{\pi}{2} = 0$

37.A wheel makes 360 revolutions in 1 minute. Through how many radians does it turn in 1 second.

Ans.N. of revolutions made in 60 sec. = 360

N. of revolutions made in 1 sec = $\frac{360}{60} = 6$

Angle moved in 6 revolutions = $2\pi \times 6 = 12\pi$

38.Prove $\text{Sin}^2 6x - \text{Sin}^2 4x = \text{Sin}2x. \text{Sin} 10x$.

Ans.L. H. S = $\text{Sin}^2 6x - \text{Sin}^2 4x$

$= \text{Sin} (6x + 4x). \text{Sin} (6x - 4x)$

$= \text{Sin} 10x . \text{Sin} 2x$

39.Prove that $\frac{\tan 69 + \tan 66}{1 - \tan 69. \tan 66} = -1$

Ans.L. H. S = tan (69 + 66)

= tan (135)

= tan (90 + 45)

= - tan 45

= -1

40.Prove that $\frac{\text{Sin x}}{1 + \text{Cos x}} = \tan \frac{x}{2}$

Ans.L. H. S

$$\frac{2Sin\frac{x}{2}.Cos\frac{x}{2}}{2Cos^{x}\frac{x}{2}}$$

= tan $\frac{x}{2}$

CBSE Class 12 Mathematics

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Chapter 3

Trigonometric Functions

4 Marks Questions

Prove the following Identities

1.The minute hand of a watch is 1.5 cm long. How far does it tip move in 40 minute?

Ans. r = 1.5 cm

Angle made in 60 mint = 360^0

Angle made in 1 min = $\frac{360}{60}$ = 60^0

Angle made in 40 mint = 6×40

= 240^0

$\Theta = \frac{l}{r}$

$\cancel{240}^{\,4} \times \frac{\pi}{\cancel{180}\,3} = \frac{l}{1.5}$

$\frac{4 \times 3.14}{\cancel{3}} = \frac{l}{\frac{\cancel{15}\,\cancel{5}}{\cancel{10}\,2}}$

$2 \times 3.14 = l$

$6.28 = l$

$l = 6.28$ cm

2. Show that tan 3x. tan 2x. tan x = tan 3x – tan 2x – tan x

Ans. Let 3x = 2x + x

tan 3x = tan (2x + x)

$$\frac{\tan 3x}{1} = \frac{\tan 2x + \tan x}{1 - \tan 2x . \tan x}$$

$$\tan 3x\,(1 - \tan 2x . \tan x) = \tan 2x + \tan x$$

$$\tan 3x - \tan 3x . \tan 2x . \tan x = \tan 2x + \tan x$$

$$\tan 3x . \tan 2x . \tan x = \tan 3x - \tan 2x - \tan x$$

3.Find the value of tan $\frac{\pi}{8}$.

Ans. Let x = $\frac{\pi}{8}$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} t$$

$$\tan\left(2 \frac{\pi}{8 \; 4}\right) = \frac{2 \tan \pi/8}{1 - \tan^2 \pi/8}$$

$$1 = \frac{2 \tan \pi/8}{1 - \tan^2 \pi/8}$$

put tan $\pi/8 =$

$$\frac{1}{1} = \frac{2t}{1-t^2}$$

$$2t = 1 - t^2$$

$t^2 + 2t - 1 = 0$

$$t = \frac{-2 \pm 2\sqrt{2}}{2\times 1}$$

$= -1 \pm \sqrt{2}$

$= \pm\sqrt{2} - 1$

$= \sqrt{2} - 1$ or $-\sqrt{2} - 1$

$\tan \pi/8 = \sqrt{2} - 1$

4.Prove that $\dfrac{Sin(x+y)}{Sin(x-y)} = \dfrac{\tan x + \tan y}{\tan x - \tan y}$

Ans.L.H.S = $\dfrac{Sin(x+y)}{Sin(x-y)}$

$$= \frac{Sinx.Cosy + Cosx.Siny}{Sinx.Cosy - Cosx.Siny}$$

$DividingNandDbyCosx.Cosy$

$$= \frac{\tan x + \tan y}{\tan x - \tan y}$$

5.If in two circles, arcs of the same length subtend angles 60^0 and 75^0 at the centre find the ratio of their radii.

Ans. $\theta = \dfrac{l}{r_1}$

$$60 \times \frac{\pi}{18} = \frac{l}{r_1}$$

$$r_1 = \frac{3l}{\pi} \quad (1)$$

$$\theta = \frac{l}{r_2}$$

$$75 \times \frac{\pi}{18} = \frac{l}{r_2}$$

$$r_2 = \frac{12l}{5\pi} \quad (2)$$

(1) ÷ (2)

$$\frac{r_1}{r_2} = \frac{\frac{3l}{\pi}}{\frac{12l}{5\pi}}$$

$$= \frac{3l}{\pi} \times \frac{5\pi}{12l}$$

$$= 5:4$$

6.Prove that Cos 6x= 32 Cos^{6}x – 48 Cos^4 x + 18 Cos^2 x-1

Ans.L.H.S. = Cos 6x

$$= \cos 2(3x) = 2\cos^2 3x - 1 = \cos 2(3x)$$

$$= 2(4\cos^3 x - 3\cos x)^2 - 1$$

$$= 2\left[16\cos^6 x + 9\cos^2 x - 24\cos^4 x\right] - 1$$

$$= 32\cos^6 x + 18\cos^2 x - 48\cos^4 x - 1$$

$$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$$

7.Solve Sin2x-Sin4x+Sin6x=o

Ans. $Sin\ 6x+\sin 2x-\sin 4x=0$

$$2\sin\left(\frac{6x+2x}{2}\right).\cos\left(\frac{6x-2x}{2}\right)-\sin 4x=0$$

$$2\sin 4x.\cos 2x-\sin 4x=0$$

$$\sin 4x\left[2\cos 2x-1\right]=0$$

$$\sin 4x=0$$

$$4x=n\pi$$

$$x=\frac{n\pi}{4}$$

$$2\cos 2x-1=0$$

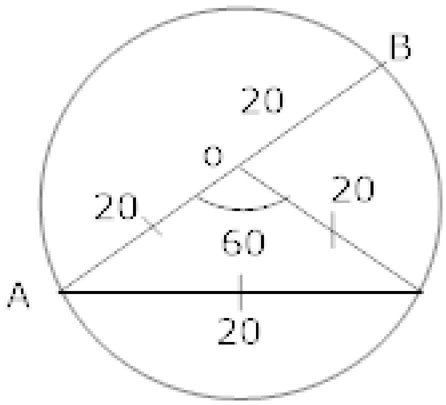
$$\cos 2x=\cos\frac{\pi}{3}$$

$$2x=2n\pi\pm\frac{\pi}{3}$$

$$x=n\pi\pm\frac{\pi}{6}$$

8.In a circle of diameter 40cm, the length of a chord is 20cm. Find the length of minor are of the chord.

Ans.



$\Theta = 60^0$

$$\theta = \frac{l}{r}$$

$$60 \times \frac{\pi}{180\ 3} = \frac{l}{20}$$

$$l = \frac{20\pi}{3} \text{ cm. s}$$

9.Prove that tan 4x = $\dfrac{4 \tan x\ (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$

Ans. L. H. S = tan 4x

$$= \frac{2 \tan 2x}{1 - \tan^2 2x}$$

$$= \frac{2 . \dfrac{2 \tan x}{1 - \tan^2 x}}{1 - \left(\dfrac{2 \tan x}{1 - \tan^2 x}\right)^2}$$

$$= \frac{\frac{4\tan x}{1-\tan^2 x}}{\frac{(1-\tan^2 x)^2 - 4\tan^2 x}{(1-\tan^2 x)^2}}$$

$$= \frac{4\tan x}{(1-\tan^2 x)} \times \frac{(1-\tan^2 x)}{1+\tan^4 x - 2\tan^2 x - 4\tan^2 x}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$$

10.Prove that (Cos x + Cos y)^2 + (Sin x – Sin y)^2 = 4 Cos^2 $\left(\frac{x+y}{2}\right)$

Ans. L. H. S = (Cos x + Cos y)^2 + (Sin x – Sin y)^2

$$= \left(2\ \mathrm{Cos}\ \frac{x+y}{2} . \mathrm{Cos}\ \frac{x-y}{2}\right)^2 + \left(2\ \mathrm{Cos}\left(\frac{x+y}{2}\right) . \mathrm{Sin}\left(\frac{x-y}{2}\right)\right)^2$$

$$= 4\ \mathrm{Cos}^2\ \frac{x+y}{2} . \mathrm{Cos}^2\left(\frac{x-y}{2}\right) + 4\ \mathrm{Cos}^2\ \frac{x+y}{2} . \mathrm{Sin}^2\ \frac{x-y}{2}$$

$$= 4\ \mathrm{Cos}^2\left(\frac{x+y}{2}\right)\left[\mathrm{Cos}^2\ \frac{x-y}{2} + Sin^2\ \frac{x-y}{2}\right]$$

$$= 4\ \mathrm{Cos}^2\left(\frac{x+y}{2}\right)$$

11.If Cot x = - $\frac{5}{12}$, **x lies in second quadrant find the values of other five trigonometric functions.**

Ans. Cot x = $-\frac{5}{12}$

$\tan x = -\frac{12}{5}$

$Sec^2 x = 1 + \tan^2 x$

$Sec\, x = \pm\frac{13}{5}$

$Sec\, x = -\frac{13}{5}$ [$\because$ x lies in IInd quad.

$Cos\, x = -\frac{5}{13}$

$Sin\, x = \tan x .\ Cos\, x$

$= \frac{-12}{5} \times \left(\frac{-5}{13}\right) \qquad = \frac{12}{13}$

$Cosec\, x = \frac{13}{12}$

12.Prove that $\frac{Sin\,5x - 2Sin\,3x + Sin\,x}{Cos\,5x - Cos\,x} = \tan x$

Ans.L. H. S = $\frac{Sin\,5x + Sinx - 2\,Sin\,3x}{Cos\,5x - Cos\,x}$

$= \frac{2Sin\,3x.\ Cos\,2x - 2\,Sin\,3x}{-2\,Sin\,3x.\ Sin\,2x}$

$= \frac{\cancel{2\,Sin\,3x}\,(Cos\,2x - 1)}{-\cancel{2\,Sin\,3x}.\ Sin\,2x}$

$= \frac{\cancel{-}(1 - Cos\,2x)}{\cancel{-}Sin\,2x}$

$$= \frac{2\ \text{Sin}^2\ x}{2\ \text{Sin}\ x.\ \text{Cos}\ x}$$

$$= \frac{\text{Sin}\ x}{\text{Cos}\ x} = \tan x$$

13.Prove that Sin x + Sin 3x + Sin 5x + Sin 7x = 4 Cos x. Cos 2x. Sin 4x

Ans.L. H. S. = Sin x + Sin 3x + Sin 5x + Sin 7x

= Sin x + Sin 7x + Sin 3x + Sin 5x

$$= 2\ \text{Sin}\ \left(\frac{x+7x}{2}\right).\ \text{Cos}\ \left(\frac{x-7x}{2}\right) + 2\ \text{Sin}\ \left(\frac{3x+5x}{2}\right)\ \text{Cos}\ \left(\frac{3x-5x}{2}\right)$$

$$= 2\ \text{Sin}\ 4x.\ \text{Cos}\ 3x + 2\ \text{Sin}\ 4x.\ \text{Cos}\ x$$

$$= 2\ \text{Sin}\ 4x\ [\text{Cos}\ 3x + \text{Cos}\ x]$$

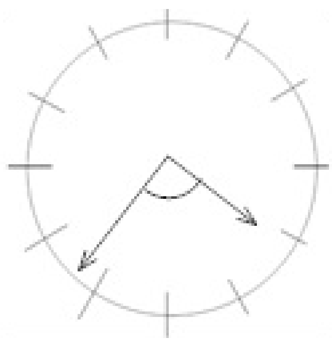
$$= 2\ \text{Sin}\ 4x\ \left[2\ \text{Cos}\ \left(\frac{3x+x}{2}\right).\ \text{Cos}\ \left(\frac{3x-x}{2}\right)\right]$$

$$= 2\ \text{Sin}\ 4x\ [\ 2\ \text{Cos}\ 2x.\ \text{Cos}\ x]$$

$$= 4\ \text{Cos}\ x.\ \text{Cos}\ 2x.\ \text{Sin}\ 4x$$

14.Find the angle between the minute hand and hour hand of a clock when the time is 7. 20.

Ans. Angle made by mint hand in 15 mint= 15 $\times$ 6 = 90^0



Angle made by hour hand in 1 hr = 30^0

in 60 minute = $\frac{30}{60} = \frac{1}{2}$

[$\because$ Angle Traled by hr hand in 12 hr = 360^0

in 20 minute = $\frac{1}{2} \times 20 = 10^0$

Angle made = 90 + 10 = 100^0

15.Show that $\sqrt{2+\sqrt{2+2\text{ Cos }4\theta}} = 2\text{ Cos }\theta$

Ans.L H. S = $\sqrt{2+\sqrt{2+2\text{Cos }4\theta}}$

$$= \sqrt{2+\sqrt{2(1+\text{Cos }4\theta)}}$$

$$= \sqrt{2+\sqrt{2.2\text{ Cos}^2\ 2\theta}}$$

$$= \sqrt{2+2\text{ Cos }2\theta}$$

$$= \sqrt{2(1+\text{Cos }2\theta)}$$

$$= \sqrt{2.2\text{ Cos}^2\theta}$$

$$= 2\text{ Cos }\theta$$

16.Prove that Cot 4x (Sin 5x + Sin 3x) = Cot x (Sin 5x – Sin 3x)

Ans.L. H. S = Cot 4x (Sin 5x + Sin 3x)

$$= \frac{\text{Cos }4x}{\text{Sin }4x}\left[2\text{ Sin }\frac{5x+3x}{2}.\text{ Cos }\frac{5x-3x}{2}\right]$$

$$= \frac{\text{Cos } 4x}{\cancel{\text{Sin } 4x}} \; 2 \cancel{\text{Sin } 4x}. \text{Cos } x$$

$$= 2 \text{ Cos } 4x. \text{ Cos } x$$

R. H. S = Cot x (Sin 5x – Sin 3x)

$$= \frac{\text{Cos x}}{\text{Sin x}}\left[2 \text{ Cos } \frac{5x+3x}{2}. \text{ Sin } \frac{5x-3x}{2}\right]$$

$$= \frac{\text{Cos x}}{\cancel{\text{Sin x}}}\left[2 \text{ Cos } 4x. \cancel{\text{Sin x}}\right]$$

$$= 2 \text{ Cos } 4x. \text{ Cos } x$$

L. H. S = R. H. S

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Important Questions

Chapter 3

Trigonometric Functions

6 Marks Questions

1. Find the general solution of sin2x + sin4x + sin6x = 0

Ans. $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}, n \in z$

2. Find the general solution of cos θ cos2 θ cos3θ = $\frac{1}{4}$

Ans. $(2n+1)\frac{\pi}{8}, n\pi \pm \frac{\pi}{3}, n \in z$

3.If Sin α + Sin β = a and Cos α + Cos β = b show that Cos (α + β) = $\frac{b^2-a^2}{b^2+a^2}$

Ans. $b^2 + a^2 = (\text{Cos } \alpha + \text{Cos } \beta)^2 + (\text{Sin } \alpha + \text{Sin } \beta)^2$

$= \text{Cos}^2 \alpha + \text{Cos}^2 \beta + 2 \text{ Cos } \alpha. \text{ Cos } \beta + \text{Sin}^2 \alpha + \text{Sin}^2 \beta + 2Sin\ \alpha. \text{ Sin } \beta$

$= 1 + 1 + 2 (\text{Cos } \alpha. \text{ Cos } \beta + \text{Sin } \alpha . \text{ Sin } \beta)$

$= 2 + 2 \text{ Cos } (\alpha - \beta) \quad (1)$

$b^2 - a^2 = (\text{Cos } \alpha + \text{Cos } \beta)^2 - (\text{Sin } \alpha + \text{Sin } \beta)^2$

$= (\text{Cos}^2 \alpha - \text{Sin}^2 \beta) + (\text{Cos}^2 \beta - \text{Sin}^2 \alpha) + 2 \text{ Cos } (\alpha + \beta)$

$= \text{Cos } (\alpha + \beta) \text{ Cos } (\alpha - \beta) + \text{Cos } (\beta + \alpha) \text{Cos } (\alpha - \beta) + 2 \text{ Cos } (\alpha + \beta)$

$= 2 \text{ Cos } (\alpha + \beta). \text{ Cos } (\alpha - \beta) + 2 \text{ Cos } (\alpha + \beta)$

$= Cos\ (\alpha + \beta)\ [2\ \text{Cos}\ (\alpha - \beta) + 2]$

$= \ \text{Cos}\ (\alpha + \beta).\ (b^2 + a^2)$ [from (1)

$$\frac{b^2 - a^2}{b^2 + a^2} = \text{Cos}\ (\alpha + \beta)$$

4. Prove that Cos α + Cos β + Cos γ + Cos (α + β + γ)

$$= 4\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\beta+\gamma}{2}\right).\ \text{Cos}\left(\frac{\gamma+\alpha}{2}\right)$$

Ans.L. H. S.

$$= \ \text{Cos}\ \alpha + \text{Cos}\ \beta + \text{Cos}\ \gamma + \text{Cos}\ (\alpha + \beta + \gamma)$$

$$= \text{Cos}\ \alpha + \text{Cos}\ \beta) + [\text{Cos}\ \gamma + \text{Cos}\ (\alpha + \beta + \gamma)]$$

$$= \ 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\alpha-\beta}{2}\right) + 2\ \text{Cos}\left(\frac{\alpha+\beta+\gamma+\gamma}{2}\right).\ \text{Cos}\left(\frac{\alpha+\beta+\gamma-\gamma}{2}\right)$$

$$= \ 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\alpha-\beta}{2}\right) + 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\alpha+\beta+2\gamma}{2}\right)$$

$$= 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[\text{Cos}\left(\frac{\alpha-\beta}{2}\right) + \ \text{Cos}\left(\frac{\alpha+\beta+2\gamma}{2}\right)\right]$$

$$= \ 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[2\ \text{Cos}\left(\frac{\frac{\alpha-\beta}{2}+\frac{\alpha+\beta+2\gamma}{2}}{2}\right).\ \text{Cos}\left(\frac{\frac{\alpha+\beta+2\gamma}{2}-\frac{\alpha-\beta}{2}}{2}\right)\right]$$

$$= \ 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[2\ \text{Cos}\left(\frac{\alpha+\gamma}{2}\right).\ \text{Cos}\left(\frac{\beta+\gamma}{2}\right)\right]$$

$$= 4\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\beta+\gamma}{2}\right).\ \text{Cos}\left(\frac{\gamma+\alpha}{2}\right)$$

5.Prove that Sin3x +Sin2x-Sin2x=4Sinx.Cos $\frac{x}{2}$. Cos $\frac{3x}{2}$

Ans.

$$(\sin 3x - \sin x) + \sin 2x$$

$$= 2\cos\left(\frac{3x+x}{2}\right).\sin\left(\frac{3x-x}{2}\right) + \sin 2x$$

$$= 2\cos 2x.\sin x + \sin 2x$$

$$= 2\cos 2x.\sin x + 2\sin x.\cos x$$

$$= 2\sin x[\cos 2x + \cos x]$$

$$= 2\sin x\left[2\cos x\frac{3x}{2}.\cos\frac{x}{2}\right]$$

$$= 4\sin x\cos\frac{3x}{2}\cos\frac{x}{2}$$

6.Prove that 2Cos $\frac{\pi}{13}$.Cos $\frac{9\pi}{13}$ + Cos $\frac{3\pi}{13}$ + Cos $\frac{5\pi}{13}$ =0

Ans.L.H.S.

$$= 2\cos\frac{\pi}{13}.\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= \cos\left(\frac{\pi}{13} + \frac{9\pi}{13}\right) + \cos\left(\frac{\pi}{13} - \frac{9\pi}{13}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= \cos\frac{10\pi}{13} + \cos\frac{8\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= \cos\left(\pi - \frac{3\pi}{13}\right) + \cos\left(\pi - \frac{5\pi}{13}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= -\cos\frac{3\pi}{13} - \cos\frac{5\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= 0$$

7. Find the value of tan (α + β) Given that

$$\text{Cot } \alpha = \frac{1}{2}, \alpha \in \left(\pi, \frac{3\pi}{2}\right) \text{ and Sec } \beta = -\frac{5}{3}, \beta \in \left(\frac{\pi}{2}, \pi\right)$$

Ans.

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta} \quad (1)$$

$$\text{Cot } \alpha = \frac{1}{2},$$

$$\Rightarrow \tan\alpha = 2$$

$$1 + \tan^2\beta = \text{Sec}^2\beta$$

$$1 + \tan^2\beta = \left(\frac{-5}{3}\right)^2 \quad \left[\because \text{Sec } \beta = \frac{-5}{3} 1\right)$$

$$\tan\beta = \pm\frac{4}{3}$$

$$\tan\beta = -\frac{4}{3} \quad \left[\because \beta \in \left(\frac{\pi}{2} / x\right)\right.$$

put tan α, and tan β in eq. (

$$\tan(\alpha+\beta) = \frac{2-\frac{4}{3}}{1-2\left(-\frac{4}{3}\right)}$$

$$= \frac{2}{11}$$

8.Prove that $\frac{\text{Sec } 8A - 1}{\text{Sec } 4A - 1} = \frac{\tan 8A}{\tan 2A}$

Ans.L. H. S=

$$\frac{\frac{1}{Cos\ 8A}-1}{\frac{1}{\text{Cos } 4A}-1}$$

$$= \frac{1 - \text{Cos } 8A}{1 - \text{Cos } 4A} \times \frac{\text{Cos } 4A}{\text{Cos } 8A}$$

$$= \frac{2\ \text{Sin}^2\ 4A}{2\ \text{Sin}^2\ 2A} . \frac{\text{Cos } 4A}{\text{Cos } 8A}$$

$$= \frac{(2\ \text{Sin } 4A.\ \text{Cos } 4A).\ \text{Sin } 4A}{2\ \text{Sin}^2\ 2A.\ \text{Cos } 8A}$$

$$= \frac{\text{Sin } 8A.\ (\cancel{2}\ \cancel{\text{Sin}}\ \cancel{2}A\ .\ \text{Cos } 2A)}{\cancel{2}\ \cancel{\text{Sin}}^{\cancel{2}}\ 2A.\ \text{Cos } 8A}$$

$$= \frac{\text{Sin } 8A\ \text{Cos } 2A}{\text{Sin } 2A.\ \text{Cos } 8A}$$

$$= \frac{\tan\ 8A}{\tan 2A}$$

9.Prove that $\text{Cos}^2\ x + \text{Cos}^2\left(x+\frac{\pi}{3}\right)+Cos^2\left(x-\frac{\pi}{3}\right)=\frac{3}{2}$

Ans.L. H. S = $\frac{1 + \text{Cos } 2x}{2} + \frac{1 + \text{Cos } \left(2x + \frac{2\pi}{3}\right)}{2} + \frac{1 + \text{Cos } \left(2x - \frac{2\pi}{3}\right)}{2}$

$= \frac{1}{2}\left[1+1+1+\text{Cos } 2x + \text{Cos } \left(2x + \frac{2\pi}{3}\right) + \text{ Cos } \left(2x - \frac{2\pi}{3}\right)\right]$

$= \frac{1}{2}\left[3 + Cos 2x + Cos\left(2x + \frac{2\pi}{3}\right) + Cos\left(2x - \frac{2\pi}{3}\right)\right]$

$= \frac{1}{2}\left[3 + Cos 2x + 2Cos\left(\frac{2x + \frac{\cancel{2}\pi}{\cancel{3}} + 2x - \frac{\cancel{2}\pi}{\cancel{3}}}{2}\right). \text{Cos } \left(\frac{\cancel{2}\cancel{x} + \frac{2\pi}{3} - \cancel{2}\cancel{x} + \frac{2\pi}{3}}{3}\right)\right]$

$= \frac{1}{2}\left[3 + \text{Cos } 2x + 2 \text{ Cos } 2x. \text{ Cos } \frac{{}^{2}\cancel{4}\pi}{\cancel{6}_{3}}\right]$

$= \frac{1}{2}\left[3 + \text{Cos } 2x + 2 \text{ Cos } 2x. \text{ Cos } \frac{2\pi}{3}\right]$

$= \frac{1}{2}\left[3 + \text{Cos } 2x + 2 \text{ Cos } 2x. \text{ Cos } \left(\pi - \frac{\pi}{3}\right)\right]$

$= \frac{1}{2}\left[3 + \text{Cos } 2x + \cancel{2} \text{ Cos } 2x. \left(\frac{-1}{\cancel{2}}\right)\right]$

$= \frac{3}{2}$.

10..Prove that Cos 2x. Cos $\frac{x}{2}$ - $\text{Cos } 3x. \text{ Cos } \frac{9x}{2} = \text{Sin } 5x \text{ Sin } \frac{5x}{2}$

Ans.L. H. S = $\frac{1}{2}\left[2 \text{ Cos } 2x \text{ Cos } \frac{x}{2} - 2\text{Cos } 3x. \text{ Cos } \frac{9x}{2}\right]$

$= \frac{1}{2}\left[\text{Cos } \left(2x + \frac{x}{2}\right) + Cos\left(2x - \frac{x}{2}\right) - Cos\left(\frac{9x}{2} + 3x\right) - Cos\left(\frac{9x}{2} - 3x\right)\right]$

$$= \frac{1}{2}\left[Cos\,\frac{5x}{2} + \cancel{Cos}\,\frac{\cancel{3}x}{\cancel{2}} - Cos\,\frac{15x}{2} - \cancel{Cos}\,\frac{\cancel{3}x}{\cancel{2}} \right]$$

$$= \frac{1}{2}\left[Cos\,\frac{5x}{2} - Cos\,\frac{15x}{2} \right]$$

$$= \frac{1}{2}\left[-2\,Sin\left(\frac{\frac{5x}{2} + \frac{15x}{2}}{2} \right).Sin\left(\frac{\frac{5x}{2} - \frac{15x}{2}}{2} \right) \right]$$

$$= -\,\text{Sin } 5x.\ \text{Sin}\left(\frac{-5x}{2} \right)$$

$$= \text{Sin } 5x.\ \text{Sin}\ \frac{5x}{2}.$$

11.Prove that Cos 20^0. Cos 40^0. Cos 60^0 Cos 80^0 = $\frac{1}{16}$

Ans.L. H. S = Cos 20^0. Cos 40^0. Cos 60^0. Cos 80^0.

= Cos 60. Cos 20^0. Cos 40^0. Cos 80.

$$= \frac{1}{2}.\frac{1}{2}\ \text{Cos}40^0\ (\ 2\ \text{Cos}20.\ \text{Cos}80)^0$$

$$= \frac{1}{4}\text{Cos}40^0\ [\text{Cos}\ (80 + 20) + \text{Cos}\ (80 - 20)]$$

$$= \frac{1}{4}\text{Cos}40^0\left[\text{Cos}\ 100^0 + \text{Cos}\ 60 \right]$$

$$= \frac{1}{4}\text{Cos}40^0\left[\text{Cos}\ 100 + \frac{1}{2} \right]$$

$= \frac{1}{8}\left(2 \text{ Cos } 100. \text{ Cos } 40^0\right) + \frac{1}{8} \text{ Cos } 40$

$= \frac{1}{8}\left[\text{Cos } (100 + 40^0) + \text{Cos } (100 - 40^0)\right] + \frac{1}{8} Cos\, 40^0$

$= \frac{1}{8}\left[\text{Cos } 140^0 + \text{Cos } 60\right] + \frac{1}{8} \text{ Cos } 40^0$

$= \frac{1}{8}\left[\text{Cos } 140 + \frac{1}{2}\right] + \frac{1}{8} \text{ Cos } 40^0$

$= \frac{1}{8} \text{ Cos } 140^0 + \frac{1}{16} + \frac{1}{8} \text{ Cos } 40^0$

$= \frac{1}{8} \text{ Cos } (180 - 40) + \frac{1}{16} + \frac{1}{8} \text{ Cos } 40^0$

$= -\frac{1}{8} \cancel{\text{Cos } 40^0} + \frac{1}{16} + \frac{1}{8} \cancel{\text{Cos } 40^0}$

$= \frac{1}{16}$

12.If tan x = $\frac{3}{4}$, $\pi <$ x $< \frac{3\pi}{2}$. Find the value of Sin $\frac{x}{2}$, Cos $\frac{x}{2}$ and tan $\frac{x}{2}$.

Ans.$\pi <$ x $< \frac{3\pi}{2}$ [Given

Cos x is – tive

$\Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

Sin $\frac{x}{2}$ is + tive and Cos $\frac{x}{2}$ is - tive.

$1 + \tan^2 x = \text{Sec}^2 x \;\; \frac{5}{4}$

$1 + \left(\frac{3}{4}\right)^2 = \text{Sec}^2 x$

Sec x = ±

Cos x = ± $\frac{5}{4}$

Cos x = − $\frac{5}{4}$ $\left[\because \ \pi < x < \frac{3\pi}{2}\right.$

$$\text{Sin } \frac{x}{2} = \sqrt{\frac{1 - Cos x}{2}}$$

$$= \sqrt{\frac{1 + \frac{4}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$$

$$\text{Cos } \frac{x}{2} = -\sqrt{\frac{1 + Cos x}{2}}$$

$$= -\sqrt{\frac{1 - \frac{4}{5}}{2}} = -\sqrt{\frac{1}{10}} = \frac{-1}{\sqrt{10}}$$

$$\tan \frac{x}{2} = \frac{\frac{3}{\sqrt{10}}}{\frac{-1}{\sqrt{10}}} = -3$$