Quadratic Equations

IIT Foundation Material

SECTION - I Straight Objective Type

- 1. $\frac{x-2}{x-1} = \frac{1-2}{x-2}$ $x-1 \neq 0 \text{ but } x = 1$ $\Rightarrow \text{ above equations has no roots}$ Hence (a) is the correct option.
- $|x^2| 3|x| + 2 = 0$ 2. x^2 is always positive $\Rightarrow |x^2| = x^2$ |x| = x for x > 0= x for x > 0If $x > 0 \Longrightarrow |x^2| - 3|x| + 2 = 0$ is $x^2 - 3x + 2 = 0$ \Rightarrow (x-1)(x-2) = 0 \Rightarrow x = 1 or 2 If $x < 0 \Longrightarrow |x^2| - 3|x| + 2 = 0$ is $x^2 - 3(-x) + 2 = 0$ $\Rightarrow x^2 + 3x + 2 = 0$ \Rightarrow (x+1)(x+2)=0 $\Rightarrow x = -1 \text{ or } -2$ $x = \{1, 2, -1, -2\}$ Hence (a) is the correct option.

3.
$$(x-a)(x-b) = abx^{2}$$
$$\Rightarrow x^{2} - (a+b)x + ab = abx^{2}$$
$$\Rightarrow (1-ab)x^{2} - (a+b)x + ab = 0$$

$$\Delta = b^2 - 4ac$$

= (a+b)² - 4(1-ab)(ab)
= a² + b² - 2ab - 4(ab - a²b²)
= a² + b² - 2ab - 4ab + 4ab + 4a²b²
= a² + b² + 4a²b² - 6ab \ge 0
 \Rightarrow The roots are real
 \Rightarrow Hence (a) is the correct option.

4.
$$(b-c)x^2 + (c-a)x + (a-b) = 0$$

 $b-c+c-a-b=0$
 \Rightarrow Discriminant of the above equation is zero.
 \Rightarrow The above equation has equal roots
Hence (a) is the correct option.

$$\frac{(x+2)(x-5)}{(x-3)(x+6)} = \frac{x-2}{x+4}$$

$$\frac{x^2 - 3x - 10}{x^2 + 3x - 18} = \frac{x-2}{x-4}$$

$$\Rightarrow (x-2)(x^2 + 3x - 18) - (x^2 3x - 10)(x-4) = 0$$
is a linear equation in the first degree

$$\Rightarrow \text{ The number of roots of the equation is 1.}$$
Hence (c) is the correct option.

6. $7(q-r)x^{2}+(r-p)x+(p-q)=0$ The sum of the coefficients of all terms is equal to zero Since q-r+r-p+p-g=0 $\Rightarrow 1$ is one of its root Let α is another root $1+\alpha = \frac{(r-p)}{q-r}or\alpha - \frac{p-q}{q-r}$ $\Rightarrow \frac{p-q}{q-r}$,1 are the roots of the above equation Hence (b) is the correct option.

7. The expression $ax^2 + bx + c, a > 0$ is positive for all real value of x only if $b^2 - 4ac < 0$ Hence (c) is the correct option.

8.
$$(x-b)(x-c) + (x-c)(x-a) + (x-a)(x-b) = 0$$

 $\Rightarrow 3x^2 - 2x(a+b+c) + (ab+bc+ac) = 0$
The weater of the scheme area

The roots of the above equation is always negative. Hence (b) is the correct option.

9. The sum of the roots of a Quadratic equation $ax^2 + bx + c = 0$ is $-\frac{b}{a}$ \Rightarrow For the equation $(M+1)x^2 + 2mx + 3 = 0$ Sum of the roots $= \frac{-2m}{m+1} = 1$ $\Rightarrow -2m = m+1$ -3m = 1 $m = -\frac{1}{3}$ Hence (d) is the correct option.

10. The roots of the equation $2^{2x} - 10.2^{x} + 16 = 0$ Let $2^{x} = a$

 $a^2 - 10a + 16 = 0$ (a-8)(a-2)=0A = 8 $2^{x} = 8$ $2^{x} = 2^{3}$ x = 3a = 2 $2^{x} = 2^{1}$ x = 1x = 1 or 3

The roots of the equation are 1, 3Hence (b) is the correct option.

 $x^2 = px+1$ is a factor of $ax^3 + bx + c$ then when $ax^3 + bx + c$ is 11. divided by $x^2 px + 1$, it leaves the remainder '0'

$$x^{2} + px + 1 \begin{vmatrix} ax^{3} + bx + c \\ ax^{3} + a p x^{2} + a x \end{vmatrix} a x$$

 $-a p x^{2} + (b-a) x + c = 0$ $a^2 - c^2 = -ab$. Hence (b) is the correct option.

 $\frac{a}{x-a} + \frac{b}{x-b} = 1$ 12. a(x-b) + b(x-a) = (x-a)(x-b) \Rightarrow а

 \Rightarrow

$$ax-ab+bx-ab$$
$$=x^{2}-(a+b)x+ab$$

$$\Rightarrow x^2 - 2(a+b)x + 3ab = 0$$

The roots are equal and in opposite sign Let α , $-\alpha$ are the roots

$$(\alpha) + (-\alpha) = \frac{+2(a+b)}{1}$$

$$\Rightarrow a+b=0$$

Hence (b) is the correct option.

13. If
$$a > b$$
 then the equation

$$x^{2} + (a+b)x + ab < 0$$

$$(x+a)(x+b) < 0$$

$$(x-(-a))(x-(-b)) < 0$$

$$\Rightarrow -b < x < -a \quad (\because a > b)$$

14.
$$x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots \dots \alpha}}}}$$
$$x = \sqrt{6 + x}$$
$$x^2 = 6 + x$$
$$\Rightarrow x^2 - x - 6 = 0$$
$$\Rightarrow (x - 3)(x + 2) = 0$$
$$\Rightarrow x = 3$$
Hence (c) is the correct option.

15. If
$$2.x^{\frac{1}{3}} + 2.x^{-\frac{1}{3}} = 5$$

Let $x^{-\frac{1}{3}} = a$
 $\Rightarrow 2a + \frac{2}{a} = 5$
 $\Rightarrow 2a^2 + 2 = 5a$
 $\Rightarrow 2a^2 - 5a + 2 = 0$
 $\Rightarrow (2a - 1)(a - 2) = 0$
 $a = \frac{1}{2}ora = 2$

$$x^{\frac{1}{3}} = 2 \implies x = 2^3 = 8$$
$$x^{\frac{1}{3}} = 2^{-1} \implies x = 2^{-3} = \frac{1}{8}$$

16. α, β are the roots of the equation $ax^{2} + bx + b = 0$ $\Rightarrow \alpha + \beta = -\frac{b}{a}$ $\alpha \beta = -\frac{b}{a}$ $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{b}{a}}$ $\frac{\alpha + \beta}{\sqrt{\alpha\beta}} + \sqrt{\frac{b}{a}}$ $\frac{-\frac{b}{a}}{\sqrt{\frac{b}{a}}} + \sqrt{\frac{b}{a}}$ $= -\sqrt{\frac{b}{a}} + \sqrt{\frac{b}{a}}$ = 0Hence (a) is the correct option.

17.
$$|x-1|+|x-2|+|x-3| \ge 6$$
 then
Let *x* < 1
⇒ $-(x-1)-(x-2)-(x-3) \ge 6$
 $-3x+6 \ge 6$
 $-3x \ge 0$
 $x \le 0$

$$x \le 0 \text{ is one of its solutions.}$$

Let $1 \le x < 2$
 $-(x-1)-(x-2)-(x-3) \ge 6$
 $x-1-x+2-x+3 \ge 6$
 $-x+4 \ge 6$
 $-x \ge 2$
 $x \le -2$
 $x \le -2 \ne 1 \le x < 2$
Let $2 \le x < 3$
 $-(x-1)+(x-2)-(x-3) \ge 6$
 $2x-4-x+3 \ge 6$
 $x-1\ge 6$
 $x\ge 7$
 $x\ge 7 \ne 2 \le x < 3$
Let $x\ge 3$
 $x-1+x-2+x-3 \ge 6$
 $3x-6\ge 6$
 $3x\ge 12$
 $x\ge 4$
 $x\ge 4$
 $x\ge 4 \le x\ge 3$
 $\Rightarrow x\ge 3$ is another solution.
Hence (c) is the correct option.

18.
$$x^{2}+6x-27>0$$
 and $x^{2}-3x-4<0$
 $(x+9)(x-3)>0$ and $(x-4)(x+1)<0$
 $[x-(-9)[x-3]>0]$ and $(x-4)(x-(-1))<0$
 $\Rightarrow x<-9, x>3 \text{ and } x>4 \text{ and } x\geq -1$
 $x<-9, x>3 \text{ and } x>4 \text{ and } x\geq -1$
 $x<-9, x>3 \text{ and } x>4 \text{ and } x\geq -1$
 $x<-3$
 $x>3$
 $x<4$
Hence (a) is the correct ention

19.
$$x^2 - 1 < 0$$
 and $x^2 - x - 2 \ge 0$
 $x^2 - 1 < 0 \implies -1 < x < 1$
 $x^2 - x - 2 \ge 0 \implies (x - 2)(x + 1) \ge 0$
 $\implies x \le -1$ and $x \ge 2$
 $x = \{-1\}$
Hence (d) is the correct option.

- **20.** The quadric equation $\equiv x^{2} + 19x + 60 = 0$ This the roots are - 15 and - 4 but correct equation $\equiv x^{2} + 16x + 60 = 0$ $\Rightarrow (x+10)(x+6) = 0 \Rightarrow -10, -6 \text{ are the roots}$ Hence (b) is the correct option.
- **21.** $x^2 ax + b = 0$ and $x^2 + ax b = 0$ have common root. Let *a* is the common root.

$$\Rightarrow \alpha^{2} - a\alpha + b = 0$$

and
$$\alpha^{2} + b\alpha - a = 0$$

$$\Rightarrow \alpha^{2} - a\alpha + b = \alpha^{2} + b\alpha - a$$

$$\Rightarrow a + b = (a + b)\alpha$$

$$\Rightarrow \alpha = 1 \text{ and } a + b = 0$$

Hence (b) is the correct option.

22. Let
$$\alpha$$
 is the common root for the Quadratic creation
 $ax^2 + 2cx + b = 0$ and $ax^2 - 2bx + c = 0$ ($b \neq c$)
 $a\alpha^2 + 2c\alpha + b = 0$ and $a\alpha^3 + 2b\alpha + c = 0$
 $\Rightarrow a\alpha^2 + 2ca + b = a\alpha^2 + 2b\alpha + c = 0$
 $\Rightarrow (2c - 2b)\alpha = c - b$

$$\Rightarrow 2(c-b)\alpha = c-b$$

$$\Rightarrow \alpha = \frac{1}{2}c = b$$

$$\alpha = \frac{1}{2} \Rightarrow \alpha = \left(\frac{1}{2}\right)^2 + 2c\left(\frac{1}{2}\right) + b = 0$$

$$\Rightarrow \frac{a}{4} + c + b = 0$$

$$\Rightarrow a + 4b + 4c = 0$$

Hence (c) is the correct option.

- **23.** If the roots of $ax^2 + bx + c = 0a > 0$ be greater than unity then a+b+c < 0Hence (c) is the correct option.
- 24. The Number of real roots of the equation $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$ is 3 Since $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$ and x-3=0i.e., x = 1, 2, 3Hence (d) is the correct option.
- **25.** The ratio of the roots of the equation $x^2 + bx + c = 0$ is the same as that of $x^2 + qx + r = 0$ then $b^2r = q^2c$ Hence (d) is the correct option.
- **26.** (x+1)(x+2)(x+3)(x+4) = 120 $\Rightarrow (x+1)(x+2)(x+3)(x+4)$ = (1+1)(1+2)(1+3)(1+4) = 120 $\Rightarrow x = 1$

27.
$$\left|3 + \frac{1}{x}\right| = 2$$

 $3 + \frac{1}{x} = 2 \text{ or } 3 + \frac{1}{x} = -2$
 $\frac{1}{x} = -1 \text{ or } \frac{1}{x} = -5$
 $x = -1 \text{ or } x = -\frac{1}{5}$
 $x = -1, -\frac{1}{5}$

Hence (c) is the correct option.

If $f(x) = 2x^3 + mx^2 - 13x + n$ 28. 2, is root of $f(\mathbf{x}) = \Rightarrow f(2) = 0$ $\Rightarrow 2(2)^3 + m(2)^2 - 13(2) + n = 0$ $\Rightarrow 16+4m-26+n=0$ $\Rightarrow 4m+n=10$ is a root of $f(\mathbf{x}) = \Rightarrow \mathbf{f}(3) = 0$ $2(3^3) + m(3^2) - 13(3) + n = 0$ \Rightarrow $\Rightarrow 54+9m-39+n=0$ $\Rightarrow 9m-15+n=0$ 9m + m = 159m + m = 154m + n = 105m = 5 $m = 1 \Longrightarrow n = 6$ Hence (c) is the correct option.

29.
$$x^2 - (2+m)x + (m^2 - 4m + 4)$$
 has coincident roots
 $\Rightarrow m = 0$ and $m = 2$
Since $\Rightarrow \frac{m^2 - 4m + 4}{1}$
 $= (m-2)^2$
Hence (b) is the correct option.

30. Let α , 3α are the roots of the equation (x-1)(7-x) = m $\Rightarrow x^2 - 8x + 7 = -m$ $\Rightarrow \alpha + 3\alpha = 8 \Rightarrow 4\alpha = 8$ $3\alpha^2 = \frac{7+n}{1} \Rightarrow 4\alpha = 8$ $\Rightarrow \alpha^2 = \frac{7+m}{1}$

$$\Rightarrow \ u = \frac{1}{3}$$
$$\Rightarrow 4 = \frac{7+m}{3} \Rightarrow m = 5$$

Hence (d) is the correct option.

Section - II

Assertion - Reason Questions

31.
$$4x^{2}-8x+3=0$$
$$4x^{2}-2x-6x+3=0$$
$$2x(2x-1)-3(2x-1)=0$$
$$\Rightarrow (2x-3)-3(2x-1)=0$$
$$\Rightarrow x=\frac{1}{2} \text{ or } \frac{3}{2}$$
$$\Rightarrow \text{ The roots of the equation are}$$
$$\frac{1}{2} \text{ or } \frac{3}{2}$$

Since The roots of an Quadratic equation

 $ax^2 + bx + c = 0$ is $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Hence (a) is the correct option.

32. The sum of the roots of are Quadratic equation $ax^2 + bx + c = 0$ is $\frac{-b}{a}$ and the product of the roots $= \frac{c}{a}$ \Rightarrow The product of the roots of $9x^2 + 4x - 11 = 0$ is $\frac{-11}{9}$ Hence (a) is the correct option.

33. If 2 is one of the root of $x^2 - 5x + k = 0$ $\Rightarrow 2^2 - 5(2) + k = 0 \Rightarrow 4 - 10 + k = 0$ $\Rightarrow k = 6$ Δ is called discriminant of $ax^2 + bx + c = 0$ where $\Delta = b^2 - 4ac$ $b^2 - 4ac = 0 \Rightarrow$ The roots are real and equal $b^2 - 4ac > 0 \Rightarrow$ The roots are real and unequal $b^2 - 4ac < 0 \Rightarrow$ The roots are imaginary Hence (b) is the correct option.

34.
$$px^2 + qxx + r = 0$$

Let $\alpha, 3\alpha$ are the roots of $px^2 + qx + r = 0$
 $\Rightarrow \qquad \alpha + 3\alpha = \frac{q}{p}, \alpha.3\alpha = \frac{r}{p}$
 $\alpha = \frac{q}{p}, 3\alpha^2 = \frac{r}{p}$

$$\alpha = \frac{q}{4p}, \alpha^{2} \frac{r}{3p}$$

$$\Rightarrow \qquad \frac{q^{2}}{16p^{2}} = \frac{r}{3p}$$

$$\Rightarrow \qquad 3q^{2} = 16pr$$
The Quadratic equation with roots
 α, β is $x^{2} - (\alpha + \beta)x + \alpha\beta$
Hence (b) is the correct option.

35.
$$x^2 + x + 1 = 0$$

 $\Delta = (1)^2 - 4(1)(1) = -3 < 0$
If the discriminant $\Delta < 0$
The equation $x^2 + x + 1 = 0$ has no real roots $ax^2 + bx + c = 0$ has two real roots then $\Delta > 0$
Hence (b) is the correct option.

36. If
$$3x^2 - 7x + 6 = a(x-2)^2 + b(x-2) + c$$

 $a+b+c=?$
 $3x^2 - 7x + 6$
 $= a(x^2 - 4x + 4)^2 + b(x-2) + c$
 $3x^2 - 7x + 6 = a(x^2)$
 $+x(-4a+b) + 4a - 2b + c$
 $\Rightarrow a = 3, -4a+b = -7$
 $-12+b=-7$
 $b=5$
 $4a-2b+c=6$
 $12-10+c=6$
 $c=4$
 $a+b+c=3+5+4=12$

The roots of $x^4 - kx^3 + kn^2 + \ln + m = 0$ are am vm cm d. The minimum value of $a^2 + b^2 + c^2 + d^2 = -1$ Hence (b) is the correct option.

37.
$$|x-2|+|x-3|=7$$

 $(x-2)+(x-3)=7$
 $2x-5=7$
 $x=6$
 $-(x-2)-(x-3)=7$
 $-2x=4$
 $x=-1$
 $x=-1,6$
 $x^{2}+ax+b=0$ and $x^{2}+bx+a=0$
 $(a \neq b)$ have a common root than $a+b=-1$
Hence (b) is the correct option.

38. $e^{\sin x} - e^{-\sin x} - 4 = 0$ has no real roots If the discriminant of $ax^2 + bx + c = 0$ is negative then it has no real roots Hence (b) is the correct option.

39. The roots of $x^2 + 5|x| + 4 = 0$ are not real $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ has no solutions Hence (b) is the correct option.

40.
$$x^2 - 7x + 2m = 0, x^2 - 11x + 4m = 0$$

have a common root
Let α is the common root
 $\alpha^2 - 7\alpha + 2m = 0$ and $\alpha^2 - 11\alpha + 4m = 0$
 $\alpha^2 - 7\alpha + 2m = \alpha^2 - 11\alpha + 4m$
 $\Rightarrow 4\alpha = 2m$

 $m = 2\alpha$ m = 6The Number of real solutions of the equations $(x+4)^3 + (x+3)^3 + (x+2)^3 + (x+1)^2 + (x-5)^3 + 180 = 0$ is one Hence (b) is the correct option.

SECTION - III Linked Comprehension Type

41. If one root of the equation $(l-m)x^2 + lx + 1 = 0$ is double other

i.e.
$$\alpha + 2\alpha = \frac{l}{l-m}$$
 and $\alpha \cdot 2\alpha = \frac{l}{l-m}$
 $3\alpha = \frac{l}{l-m}$
 $2\alpha^2 = \frac{l}{l-m}$
 $\alpha = \frac{l}{3(l-m)}$
 $\alpha^2 = \frac{l}{1-n}$
 $\frac{l^2}{9(1-m)^2} = \frac{1}{l-m}$
 $\frac{l^2}{9(1-m)^2} = 1 \Rightarrow l^2 - 9l + 9m = 0$
 \Rightarrow greatest value of $m = \frac{9}{8}$
Hence (b) is the correct option.

- **42.** $ax^2 + bx + c = 0$ and $x^2 + x + 1 = 0$ have a common root then a + b + c = 0. Hence (b) is the correct option.
- **43.** $x^2 = 11x + k = 0$ and $x^2 14x + 2k = 0$ have a common root say α $\alpha^2 - 11\alpha + k = \alpha^2 - 14\alpha + 2k$ $\Rightarrow 3\alpha = k$

$$\alpha^{2} - 11\alpha + 3\alpha = 0$$

$$\Rightarrow \qquad \alpha^{2} - 8\alpha = 0$$

$$\Rightarrow \qquad \alpha(\alpha - 8) = 0$$

$$\Rightarrow \qquad \alpha = 0$$

$$\Rightarrow \qquad k = 3 \times 8 = 24$$

Hence (c) is the correct option.

44.
$$x^2 + 2(k-1)x + k + 5 = 0$$
 have positive root
 $\Rightarrow 4(k-1)^2 - 4(1)(k+5) = 0$
 $\Rightarrow k \in (-\alpha, -1)$
Hence (a) is the correct option.

45.
$$(a-b)^2 x^2 + 2(a+b-2c)x+1=0$$

 $\Delta = 4(a+b-2c)-4(a-b)^2.1$
 < 0
 $\Delta < 0 \Rightarrow$ The roots are imaginary.
Hence (c) is the correct option.

46.
$$ax + by = 1$$

 $cx^2 + by^2 = 1$ have only one solution
 $\Rightarrow x = \frac{a}{c}$
Hence (a) is the correct option.

47.
$$(x-2)(x-5) > 0$$

 $\Rightarrow x < 2$ and $x > 5$
Hence (a) is the correct option.

48. (x-2)(x-5) > 0 $\Rightarrow 2 < x < 5$ Hence (b) is the correct option. **49.** (x+2)(x-5) > 0i.e. [x-(-2)][x-5] > 0 $\Rightarrow x < -2$ and x > 5Hence (c) is the correct option.

50. 2x+3y+5=0, 6x+9y+15=0 $\frac{2}{3}=\frac{6}{9}\neq\frac{5}{15}$ \Rightarrow Number of solutions is zero Hence (c) is the correct option.

51. 2r+3y+5=0 and 2x+3y-7=0 $\frac{2}{2}=\frac{3}{3}\neq\frac{5}{7}$ Hence (c) is the correct option.

52. 2x+3y+5=0 and 3x+2y+7=0 $\frac{2}{3} \neq \frac{3}{2} \neq \frac{5}{7}$ \Rightarrow Number of solution is 1 Hence (a) is the correct option.



53.



.



.

