Chapter 24. Solution of Right Triangles [Simple 2-D Problems Involving One Right-angled Triangle]

Exercise 24

Solution 1:

(i)

From the figure we have

$$\sin 60^0 = \frac{20}{x}$$
$$\frac{\sqrt{3}}{2} = \frac{20}{x}$$
$$x = \frac{40}{\sqrt{3}}$$

(ii)

From the figure we have

$$\tan 30^0 = \frac{20}{x}$$
$$\frac{1}{\sqrt{3}} = \frac{20}{x}$$
$$x = 20\sqrt{3}$$

(iii)

From the figure we have

$$\sin 45^0 = \frac{20}{x}$$
$$\frac{1}{\sqrt{2}} = \frac{20}{x}$$
$$x = 20\sqrt{2}$$

Solution 2:

(i)

From the figure we have

$$\cos A = \frac{10}{20}$$

$$\cos A = \frac{1}{2}$$

$$\cos A = \cos 60^0$$

$$A = 60^0$$

(ii)

From the figure we have

$$\sin A = \frac{\frac{10}{\sqrt{2}}}{10}$$

$$\sin A = \frac{1}{\sqrt{2}}$$

$$\sin A = \sin 45^0$$

$$A = 45^0$$

(iii)

From the figure we have

$$\tan A = \frac{10\sqrt{3}}{10}$$

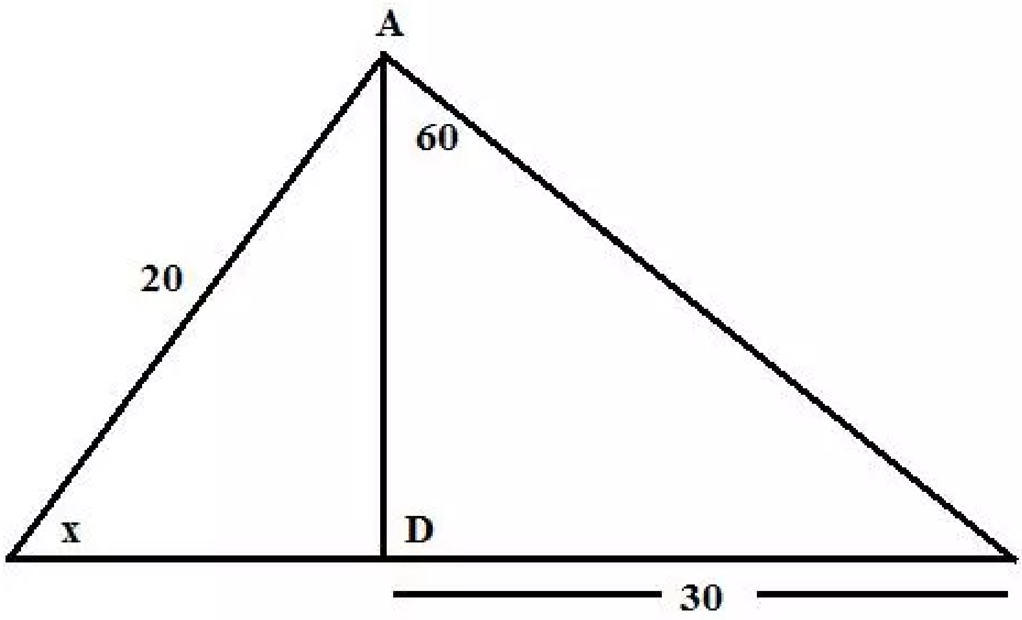
$$\tan A = \sqrt{3}$$

$$\tan A = \sin 60^0$$

$$A = 60^0$$

Solution 3:

The figure is drawn as follows:



The above figure we have

$$\tan 60^0 = \frac{30}{AD}$$

$$\sqrt{3} = \frac{30}{AD}$$

$$AD = \frac{30}{\sqrt{3}}$$

Again

$$\sin x = \frac{AD}{20}$$

$$AD = 20 \sin x$$

Now

$$20 \sin x = \frac{30}{\sqrt{3}}$$

$$\sin x = \frac{30}{20\sqrt{3}}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$\sin x = \sin 60^0$$

$$x = 60^0$$

Solution 4:

(i)

From the right triangle ABE

$$\tan 45^0 = \frac{AE}{BE}$$
$$1 = \frac{AE}{BE}$$
$$AE = BE$$

Therefore AE = BE = 50 m.

Now from the rectangle BCDE we have

DE = BC = 10 m.

Therefore the length of AD will be:

AD = AE + DE = 50 + 10 = 60 m.

(ii)

From the triangle ABD we have

$$\sin B = \frac{AD}{AB}$$
$$\sin 30 = \frac{AD}{100} \quad \left[\begin{array}{l}\text{Since } \angle ACD \text{ is the exterior} \\ \text{angle of the triangle ABC}\end{array}\right]$$
$$\frac{1}{2} = \frac{AD}{100}$$
$$AD = 50 \text{ m}$$

Solution 5:

From right triangle ABC,

$$\tan 60° = \frac{AC}{BC}$$
$$\Rightarrow \sqrt{3} = \frac{AC}{40}$$
$$\Rightarrow AC = 40\sqrt{3} \ cm$$

From right triangle BDC,

$$\tan 45° = \frac{DC}{BC}$$
$$\Rightarrow 1 = \frac{DC}{40}$$
$$\Rightarrow DC = 40 \ cm$$

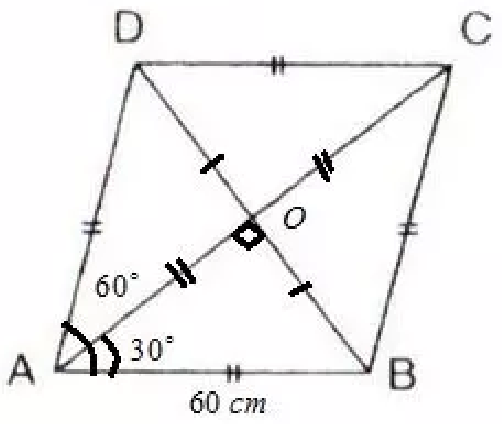
From the figure, it is clear that $AD = AC - DC$

$$\Rightarrow AD = 40\sqrt{3} - 40$$
$$\Rightarrow AD = 40(\sqrt{3} - 1)$$
$$\Rightarrow AD = 29.28 \ cm$$

Solution 6:

We know, diagonals of a rhombus bisect each other at right angles and also bisect the angle of vertex.

The figure is shown below:



Now

$$OA = OC = \frac{1}{2}AC, OB = OD = \frac{1}{2}BD; \angle AOB = 90^\circ$$

And $\angle OAB = \frac{60^\circ}{2} = 30^\circ$

Also given $AB = 60cm$

In right triangle AOB

$$\sin 30^\circ = \frac{OB}{AB}$$
$$\frac{1}{2} = \frac{OB}{60}$$
$$OB = 30\,cm$$

Also

$$\cos 30^\circ = \frac{OA}{AB}$$
$$\frac{\sqrt{3}}{2} = \frac{OA}{60}$$
$$OA = 51.96\,cm$$

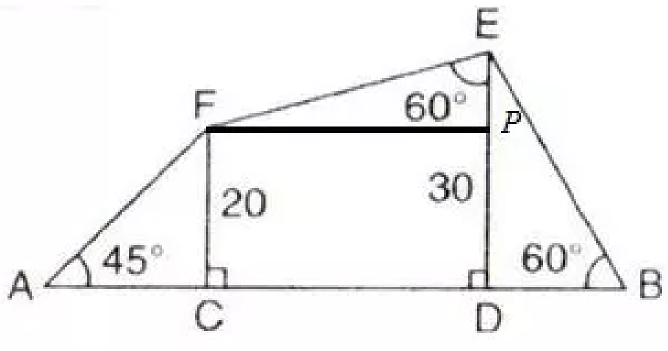
Therefore,

Length of diagonal $AC = 2 \times OA = 2 \times 51.96 = 103.92\,cm$.

Length of diagonal $BD = 2 \times OB = 2 \times 30 = 60\,cm$.

Solution 7:

Consider the figure



From right triangle ACF

$$\tan 45^\circ = \frac{20}{AC}$$
$$1 = \frac{20}{AC}$$
$$AC = 20\,cm$$

From triangle DEB

$$\tan 60^\circ = \frac{30}{BD}$$
$$\sqrt{3} = \frac{30}{BD}$$
$$BD = \frac{30}{\sqrt{3}}$$
$$= 17.32\,cm$$

Given $FC = 20, ED = 30$, So $EP = 10\,cm$

Therefore

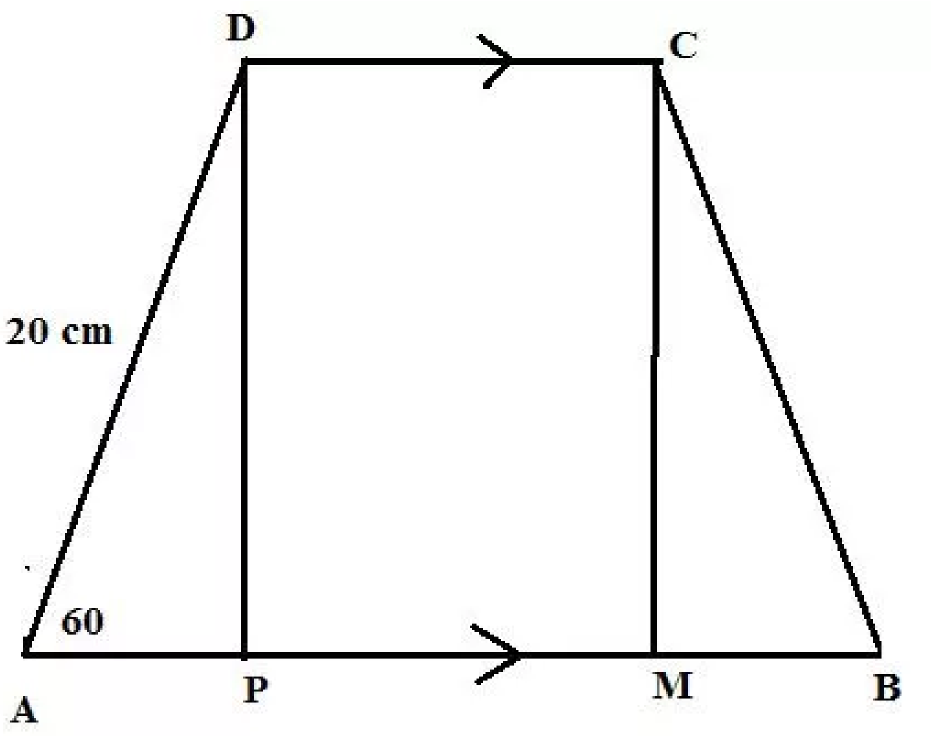
$$\tan 60^0 = \frac{FP}{EP}$$
$$\sqrt{3} = \frac{FP}{10}$$
$$FP = 10\sqrt{3}$$
$$= 17.32\text{ cm}$$

Thus AB = AC + CD + BD = 54.64 cm.

Solution 8:

First draw two perpendiculars to AB from the point D and C respectively. Since AB|| CD therefore PMCD will be a rectangle.

Consider the figure,



(i)

From right triangle ADP we have

$$\cos 60^0 = \frac{AP}{AD}$$
$$\frac{1}{2} = \frac{AP}{20}$$
$$AP = 10$$

Similarly from the right triangle BMC we have BM = 10 cm.

Now from the rectangle PMCD we have CD = PM = 20 cm.

Therefore

AB = AP + PM + MB = 10 + 20 + 10 = 40 cm.

(ii)

Again from the right triangle APD we have

$$\sin 60^0 = \frac{PD}{20}$$
$$\frac{\sqrt{3}}{2} = \frac{PD}{20}$$
$$PD = 10\sqrt{3}$$

Therefore the distance between AB and CD is $10\sqrt{3}$.

Solution 9:

From right triangle AQP

$$\tan 30^0 = \frac{AQ}{AP}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{AP}$$

$$AP = 10\sqrt{3}$$

Also from triangle PBR

$$\tan 45^0 = \frac{PB}{BR}$$

$$1 = \frac{PB}{8}$$

$$PB = 8$$

Therefore,

AB = AP + PB = $10\sqrt{3} + 8$.

Solution 10:

From right triangle ADE

$$\tan 45^0 = \frac{AE}{DE}$$

$$1 = \frac{AE}{30}$$

$$AE = 30 \text{ cm}$$

Also, from triangle DBE

$$\tan 60^0 = \frac{BE}{DE}$$

$$\sqrt{3} = \frac{BE}{30}$$

$$BE = 30\sqrt{3} \text{ cm}$$

Therefore AB = AE + BE = 30 + $30\sqrt{3} = 30\left(1+\sqrt{3}\right)$ cm

Solution 11:

(i)

From the triangle ADC we have

$$\tan 45^0 = \frac{AD}{DC}$$
$$1 = \frac{2}{DC}$$
$$DC = 2$$

Since AD || DC and $AD \perp EC$, ABCD is a parallelogram and hence opposite sides are equal.

Therefore AB = DC = 2 cm

(ii)

Again

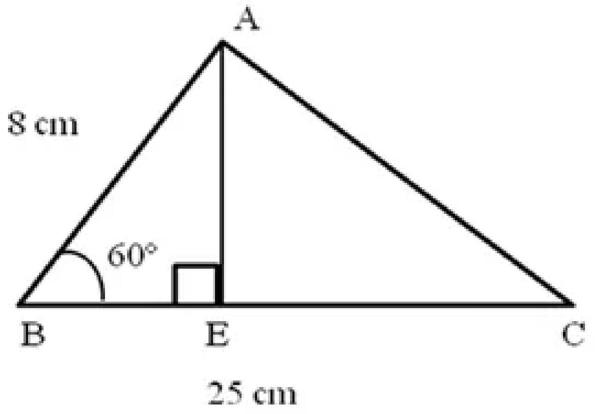
$$\sin 45^0 = \frac{AD}{AC}$$
$$\frac{1}{\sqrt{2}} = \frac{2}{AC}$$
$$AC = 2\sqrt{2}$$

(iii)

From the right triangle ADE we have

$$\sin 60^0 = \frac{AD}{AE}$$
$$\frac{\sqrt{3}}{2} = \frac{2}{AE}$$
$$AE = \frac{4}{\sqrt{3}}$$

Solution 12:



Let $BE = x$, and $EC = 25 - x$

In ΔABC,

$$\sin 60^\circ = \frac{AE}{AB}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AE}{8}$$

$$\Rightarrow AE = 8 \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow AE = 4\sqrt{3} \text{ cm}$$

(i) $BE^2 = AB^2 - AE^2$

$$\Rightarrow BE^2 = 8^2 - (4\sqrt{3})^2$$

$$\Rightarrow BE^2 = 64 - 48$$

$$\Rightarrow BE^2 = 16$$

$$\Rightarrow BE = 4 \text{ cm}$$

(ii) $EC = BC - BE$

$= 25 - 4$

$= 21$

In right ΔAEC,

$$AC^2 = AE^2 + EC^2$$

$$\Rightarrow AC^2 = (4\sqrt{3})^2 + 21^2$$

$$\Rightarrow AC^2 = 48 + 441$$

$$\Rightarrow AC^2 = 489$$

$$\Rightarrow AC = 22.11 \text{ cm}$$

Solution 13:

(i)

From right triangle ABC

$$\tan 30^0 = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{12}{BC}$$

$$BC = 12\sqrt{3} \text{ cm}$$

(ii)From the right triangle *ABD*

$$\cos A = \frac{AD}{AB}$$

$$\cos 60^0 = \frac{AD}{AB}$$

$$\frac{1}{2} = \frac{AD}{12}$$

$$AD = \frac{12}{2}$$

$$= 6 \text{ cm}$$

(iii)From right triangle ABC

$$\sin B = \frac{AB}{AC}$$

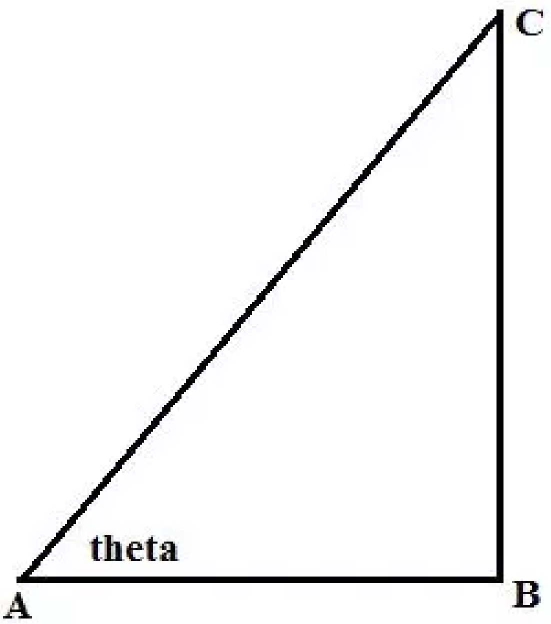
$$\sin 30^0 = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{12}{AC}$$

$$AC = 24 \text{ cm}$$

Solution 14:

Consider the figure



(i) Here AB is $\sqrt{3}$ times of BC means

$$\frac{AB}{BC} = \sqrt{3}$$
$$\cot\theta = \cot 30^0$$
$$\theta = 30^0$$

(ii)

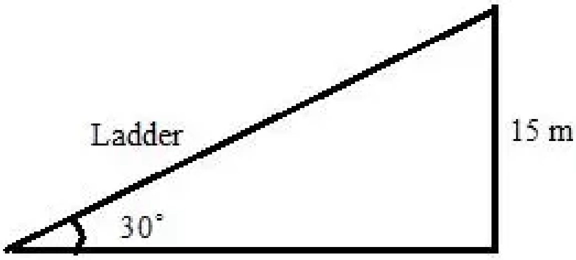
Again from the figure

$$\frac{BC}{AB} = \sqrt{3}$$
$$\tan\theta = \sqrt{3}$$
$$\tan\theta = \tan 60^0$$
$$\theta = 60^0$$

Therefore, magnitude of angle A is 30°

Solution 15:

Given that the ladder makes an angle of 30o with the ground and reaches upto a height of 15 m of the tower which is shown in the figure below:



Suppose the length of the ladder is x m

From the figure

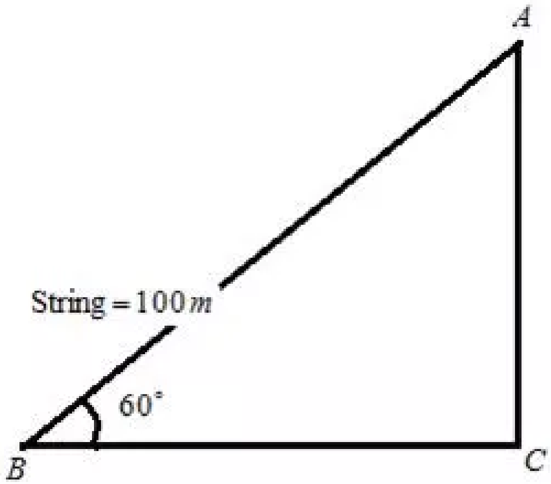
$$\frac{15}{x} = \sin 30^\circ \qquad \left[\because \frac{\text{Perp.}}{\text{Hypot.}} = \sin\right]$$

$$\frac{15}{x} = \frac{1}{2}$$

$$x = 30\,m$$

Therefore the length of the ladder is 30m.

Solution 16:



Suppose that the greatest height is x m.

From the figure

$$\frac{x}{100} = \sin 60^\circ \qquad \left[\because \frac{\text{Perp.}}{\text{Hypot.}} = \sin\right]$$

$$\frac{x}{100} = \frac{\sqrt{3}}{2}$$

$$x = 86.6\,m$$

Therefore the greatest height reached by the kite is $86.6m$.

Solution 17:

(i)Let $BC = x m$

$BD = BC + CD = (x+20) cm$

In $\triangle ABD$,

$\tan 30^\circ = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{AB}{x + 20}$

$x+20 = \sqrt{3}AB$ .....(1)

In $\triangle ABC$

$\tan 45^\circ = \frac{AB}{BC}$

$1 = \frac{AB}{x}$

$AB = x$ ... (2)

From (1)

$AB + 20 = \sqrt{3}AB$

$AB(\sqrt{3}-1) = 20$

$AB = \frac{20}{(\sqrt{3} - 1)}$

$= \frac{20}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1}{(\sqrt{3}+1)}$

$= \frac{20(\sqrt{3}+ 1)}{3-1} = 27.32 \ cm$

From (2)

$AB = x = 27.32cm$

Therefore $BC = x = AB = 27.32cm$

Therefore, $AB = 27.32cm$, $BC = 27.32cm$

(ii)

Let $BC = xm$

$BD = BC + CD = (x + 20)\ cm$

In $\triangle ABD$,

$\tan 30^{\circ} = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{AB}{x + 20}$

$x + 20 = \sqrt{3}\,AB$ ...(1)

In $\triangle ABC$

$\tan 60^{\circ} = \frac{AB}{BC}$

$\sqrt{3} = \frac{AB}{x}$

$x = \frac{AB}{\sqrt{3}}$ ...(2)

From (1)

$$\frac{AB}{\sqrt{3}} + 20 = \sqrt{3}AB$$
$$AB + 20\sqrt{3} = 3AB$$
$$2AB = 20\sqrt{3}$$
$$AB = \frac{20\sqrt{3}}{2}$$
$$= 10\sqrt{3} = 17.32cm$$

From (2)

$$x = \frac{AB}{\sqrt{3}} = \frac{17.32}{\sqrt{3}} = 10cm$$

Therefore $BC = x = 10cm$

Therefore,

$AB = 17.32cm, BC = 10cm$

(iii)

Let $BC = xm$

$BD = BC + CD = (x + 20)cm$

In $\triangle ABD$,

$$\tan 45° = \frac{AB}{BD}$$

$$1 = \frac{AB}{x + 20}$$

$x + 20 = AB$ ...(1)

In $\triangle ABC$

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{x}$$

$$x = \frac{AB}{\sqrt{3}} \quad \text{... (2)}$$

From (1)

$$\frac{AB}{\sqrt{3}} + 20 = AB$$

$$AB + 20\sqrt{3} = \sqrt{3}AB$$

$$AB\left(\sqrt{3} - 1\right) = 20\sqrt{3}$$

$$AB = \frac{20\sqrt{3}}{\left(\sqrt{3}-1\right)}$$

$$= \frac{20\sqrt{3}}{\left(\sqrt{3}-1\right)} \times \frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$$

$$= \frac{20\sqrt{3}\left(\sqrt{3}+1\right)}{3-1} = 47.32\,cm$$

From (2)

$$x = \frac{AB}{\sqrt{3}} = \frac{47.32}{\sqrt{3}} = 27.32\,cm$$

$$\therefore BC = x = 27.32\,cm$$

Therefore,

$$AB = 47.32\,cm, BC = 27.32\,cm$$

Solution 18:

(i) From ΔAPB

$$\tan 30^\circ = \frac{AB}{PB}$$

$$\frac{1}{\sqrt{3}} = \frac{150}{PB}$$

$$PB = 150\sqrt{3} = 259.80\,m$$

Also, from ΔABQ

$$\tan 45^\circ = \frac{AB}{BQ}$$

$$1 = \frac{150}{BQ}$$

$$BQ = 150\,m$$

Therefore,

$$PQ = PB + BQ$$

$$= 259.80 + 150$$

$$= 409.80\,m$$

(ii) From ΔAPB

$$\tan 30^\circ = \frac{AB}{PB}$$

$$\frac{1}{\sqrt{3}} = \frac{150}{PB}$$

$$PB = 150\sqrt{3}$$

$$= 259.80\,m$$

Also, from ΔABQ

$$\tan 45^\circ = \frac{AB}{BQ}$$

$$1 = \frac{150}{BQ}$$

$$BQ = 150\,m$$

Therefore,

$$PQ = PB - BQ$$

$$= 259.80 - 150$$

$$= 109.80\,m$$

Solution 19:

Given $\tan x^{o} = \frac{5}{12}$ $\tan t^{o} = \frac{3}{4}$ and AB = 48 m;

Let length of $BC = xm$

From $\triangle ADC$

$$\tan x^{\circ} = \frac{DC}{AC}$$
$$\frac{5}{12} = \frac{DC}{48+x}$$
$$240+5x = 12CD \qquad \dots(1)$$

Also, from $\triangle BDC$

$$\tan y^{\circ} = \frac{CD}{BC}$$
$$\frac{3}{4} = \frac{CD}{x}$$
$$x = \frac{4CD}{3} \qquad \dots(2)$$

From (1)

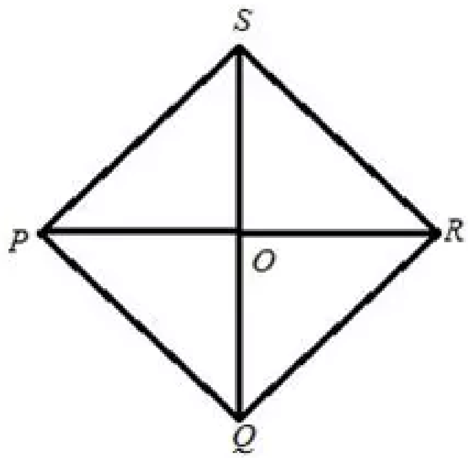
$$240+5\left(\frac{4CD}{3}\right) = 12CD$$
$$240+\frac{20CD}{3} = 12CD$$
$$720+20CD = 36CD$$
$$16CD = 720$$
$$CD = 45$$

Therefore, length of CD is 45 m.

Solution 20:

Since in a rhombus all sides are equal.

The diagram is shown below:



Therefore $PQ = \frac{96}{4} = 24\,cm$, Let $\angle PQR = 120^\circ$.

We also know that in rhombus diagonals bisect each other perpendicularly and diagonal bisect the angle at vertex.

Hence POR is a right angle triangle and

$$POR = \frac{1}{2}(PQR) = 60^{\circ}$$

$$\sin 60^{\circ} = \frac{\text{Perp.}}{\text{Hypot.}} = \frac{PO}{PQ} = \frac{PO}{24}$$

But

$$\sin 60^{\circ} = \frac{\sqrt{3}}{2}$$

$$\frac{PO}{24} = \frac{\sqrt{3}}{2}$$

$$PO = 12\sqrt{3} = 20.784$$

Therefore,

$$PR = 2PO = 2 \times 20.784 = 41.568cm$$

Also,

$$\cos 60^{\circ} = \frac{\text{Base}}{\text{Hypot.}} = \frac{OQ}{24}$$

But

$$\cos 60^{\circ} = \frac{1}{2}$$

$$\frac{OQ}{24} = \frac{1}{2}$$

$$OQ = 12$$

Therefore, $SQ = 2 \times OQ = 2 \times 12 = 24cm$

So, the length of the diagonal $PR = 41.568cm$ and $SQ = 24cm$.