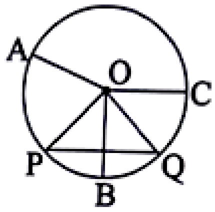
3. Theorems Related to Circle

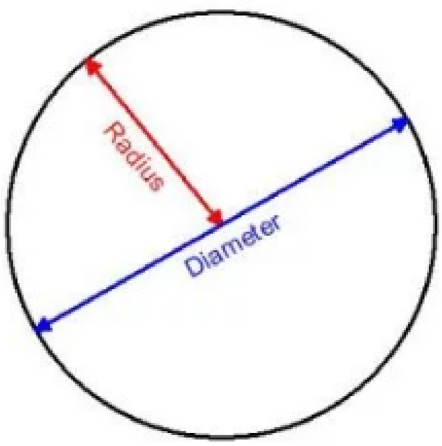
Let us Work Out 3.1

1. Question

Let us see the adjoining figure of the circle with centre O and write the radii which are situated in the segment PAQ.



Answer



Radius is the line segment that joins center and circumference of a circle.

So, OA, OC, OP, OQ and OB are the radii.

2. Question

Let us write in the following ☐ by understanding it.

i. In a circle, there are ☐ number of points.

ii. The greatest chord of the circle is ☐.

iii. The chord divides the circular region into two ☐.

iv. All diameters of the circle pass through ☐.

v. If two segments are equal, then their two arcs are ☐ in length.

vi. The sector of the circular region is the region enclosed by the arc and the two ☐.

vii. The length of the line segment joining the point outside the circle and the center is ☐ than the length of radius.

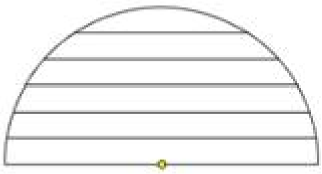
Answer

(i) Infinite

The circle is a collection of infinite number of points lying on a plane, each of the points is equidistant from a fixed point on that plane.

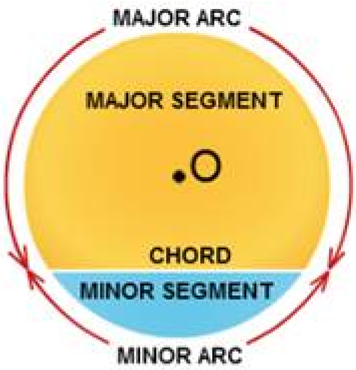
(ii) Diameter

A diameter is the greatest chord of any circle.

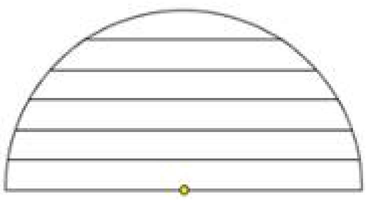


(iiii) Segment

Any chord of a circle divide the circle in two parts, the one with major area is known as major segment and the other part is known as minor segment.



(iv) Origin



As you can see in this image, Diameter has to pass through origin.

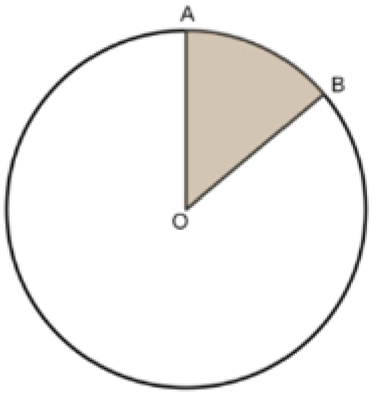
(v) equal

Equal arcs subtend equal segments and sectors.

(vi) radii

A sector is the region enclosed by an arc and two radii as shown below.

OAB is a sector.

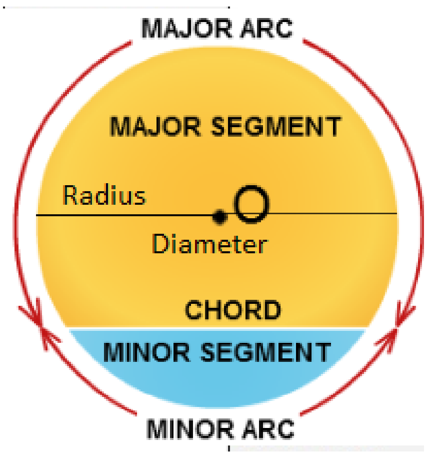


(vii) Greater than radius

3. Question

With the help of scale and pencil compass let us draw a circle and indicate centre, chord, diameter, radius, major arc, minor arc on it

Answer



4. Question

Let us write true or false:

i. The circle is a plane figure.

ii. The segment is a plane region.

iii. The Sector is a plane region.

iv. The chord is a line segment

v. The arc is a line segment.

vi. There are finite number of chords of same length in a circle

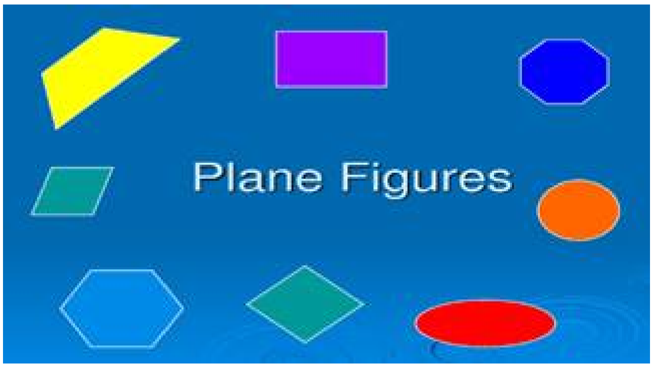
vii. One and only one circle can be drawn by taking a fixed point as its centre.

viii. The lengths of the radii of two congruent circles are equal.

Answer

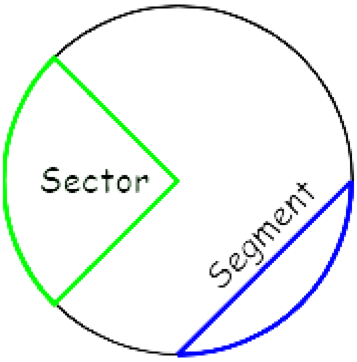
(i) True

Plane Figure, A figure drawn on a 2-D plane.



(ii) True

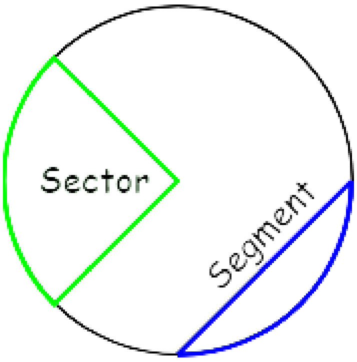
The *segment* of a *circle* is the *region* bounded by a chord and the arc subtended by the chord.



As you can see in the figure the segment is a plane figure.

(iii) True

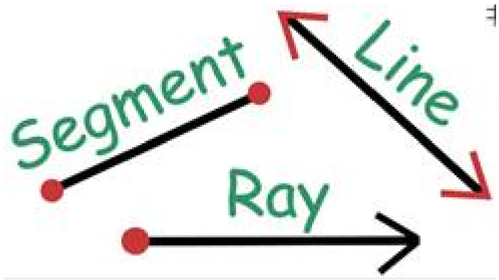
A *sector* is the part of a *circle* enclosed by two radii of a *circle* and their intercepted arc.

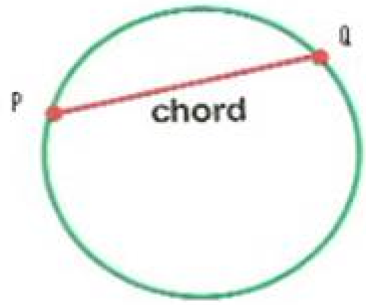


As you can see in the figure the Sector is a plane figure.

(iv) True

A line segment is a distance between two points.

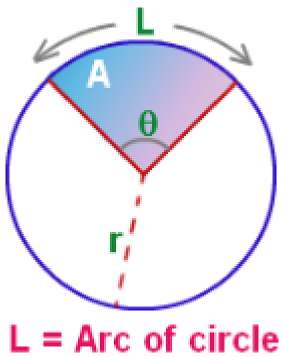




A chord is a distance between two points on the circumference.

(v) False

Arc is a segment of differential Curve.

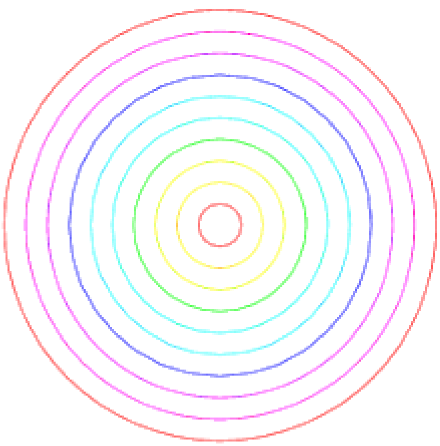


(vi) False

There are infinite number of Chords of same length.

(vii) False

Concentric circles are *circles* with a common center. As you can see in this image We can draw infinite circles.



(viii) True

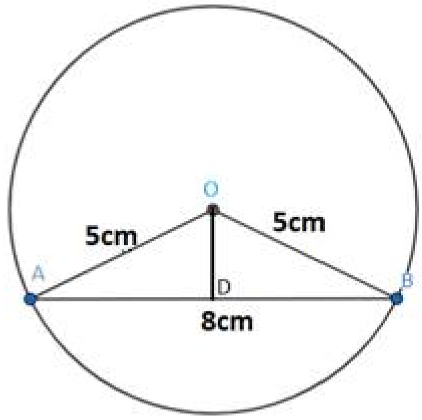
Two circles are congruent if they have the same size. The size can be measured as the radius, diameter or circumference. They can overlap

Let us Work Out 3.2

1. Question

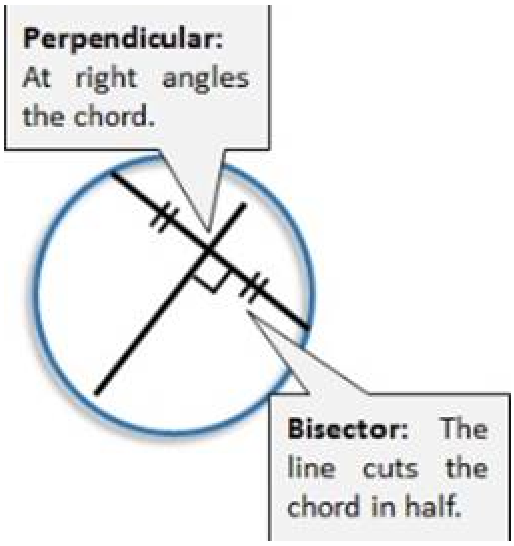
The length of a radius of a circle with its centre O is 5 cm. and the length of its chord AB is 8 cm. Let us write by calculating, the distance of the chord AB from the centre O.

Answer



Given radius = 5cm, AB = 5cm

Perpendicular from the center of the circle to any Chord bisects it in two line segments



So, AD = BD

In $\triangle$ODB

OB = 5cm

BD = 4cm

Using Pythagoras Theorem

base^2 + Height^2 = Hypotenuse^2

$\Rightarrow OD^2 + DB^2 = OB^2$

$\Rightarrow OD^2 = 25-16$

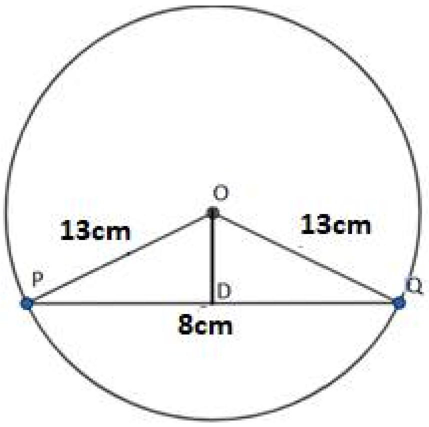
$\Rightarrow OD^2 = 9$

$\Rightarrow OD = 3cm$

2. Question

The length of a diameter of a circle with its centre at O is 26 cm. The distance of the chord PQ from the point O is 5 cm. Let us write by calculating, the length of the chord PQ.

Answer



Given Length of the diameter = 26cm, Distance of Chord PQ From center = 5cm

Radius $= \frac{\text{Diameter}}{2}$

Radius = 13cm

In ΔO PD , Using Pythagoras Theorem

Hypotenuse^2 = base^2 + Height^2

$\Rightarrow OP^2 = OD^2 + PD^2$

$\Rightarrow 13^2 = 5^2 + PD^2$

$\Rightarrow PD^2 = 169-25$

$\Rightarrow PD^2 = 144$

$\Rightarrow PD = 12cm$

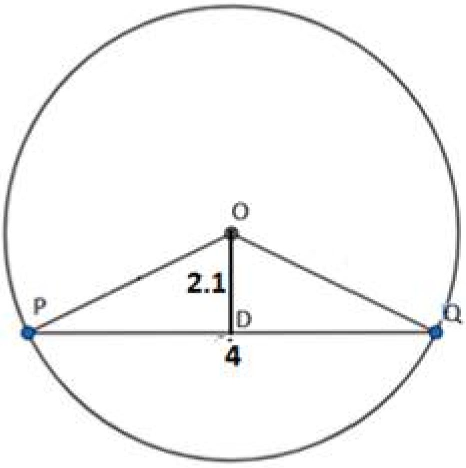
$\Rightarrow PQ = 2 \times PD$

$\Rightarrow PQ = 24cm$

3. Question

The length of a chord PQ of a circle with its centre O is 4 cm. and the distance of PQ from the point O is 2.1 cm. Let us write by calculating, the length of its diameter.

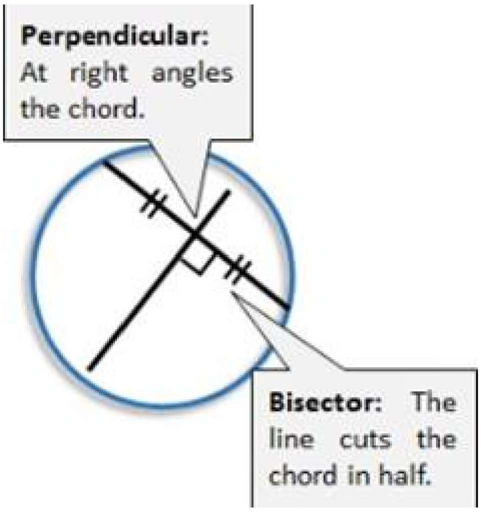
Answer



Given PQ = 4cm, Distance from the Center is 2,1cm

$PQ = 2 \times PD$

Perpendicular from the center of the circle to any Chord bisects it in two line segments



$\Rightarrow PD = \frac{PQ}{2}$

$\Rightarrow PD = 2cm$

In ΔO PD , Using Pythagoras Theorem

$\Rightarrow OD^2 + PD^2 = OP^2$

$\Rightarrow OP^2 = 2.1^2 + 2^2$

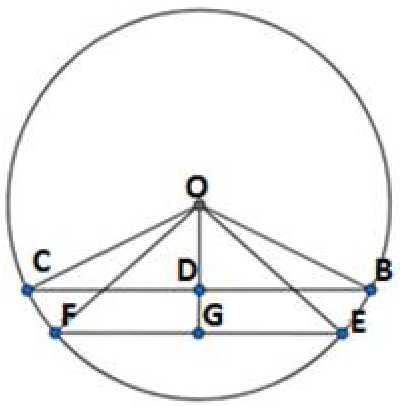
⇒ $OP^2 = 8.41$

⇒ OP = 2.9cm

4. Question

The lengths of two chords of a circle with its centre O are 6 cm. and 8 cm. If the distance of smaller chord from centre is 4 cm, then let us write by calculating, the distance of other chord from the centre.

Answer

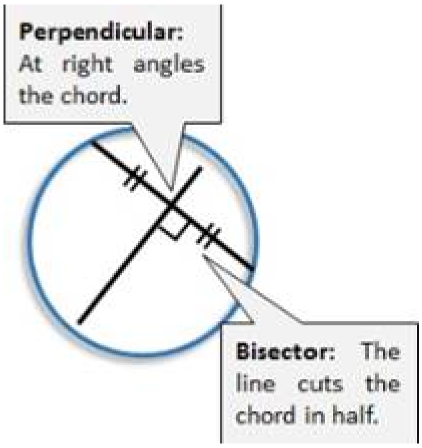


Let Smaller Chord is EF, the other chord is BC, distance from the centre o to smaller chord is OG.

⇒ BC = 8cm

⇒ $CD = \frac{BC}{2}$

Perpendicular from the center of the circle to any Chord bisects it in two line segments



⇒ CD = 4cm

⇒ EF = 6cm

⇒ $EG = \frac{EF}{2}$

⇒ EG = 3cm

⇒ OG = 4cm

In △OEG, Using Pythagoras Theorem

⇒ $OE^2 = OG^2 + EG^2$

⇒ $OE^2 = 16 + 9$

⇒ $OE^2 = 25$

⇒ OE = 5cm

In △OCD,Using Pythagoras Theorem

⇒ $OC^2 = CD^2 + OD^2$

⇒ $5^2 = 4^2 + OD^2$

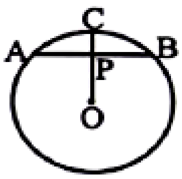
⇒ $25 = 16 + OD^2$

⇒ $OD^2 = 9$

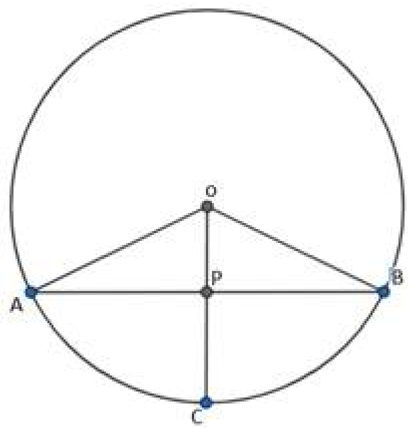
⇒ OD = 3cm

5. Question

If the length of a chord of a circle is 48 cm and the distance of it from the centre is 7 cm. then let us write by calculating, the length of radius of the circle.



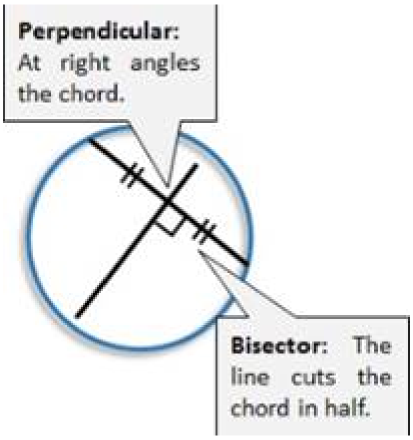
Answer



Given AB = 48cm, OP = 7cm

$AP = \frac{AB}{2}$

Perpendicular from the center of the circle to any Chord bisects it in two line segments



$\Rightarrow$ AP = 24cm

In $\triangle$OAP, Using Pythagoras Theorem

$\Rightarrow OA^2 = AP^2 + OP^2$

$\Rightarrow OA^2 = 24^2 + 7^2$

$\Rightarrow OA^2 = 576 + 49$

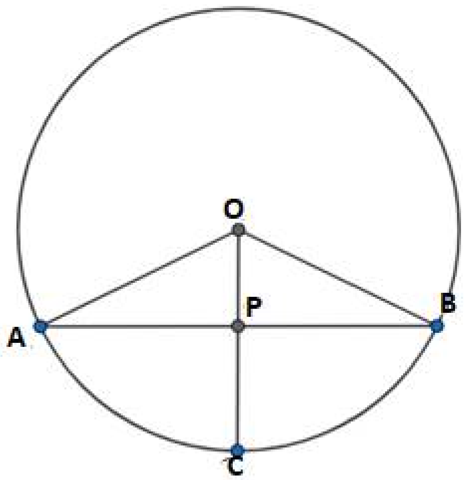
$\Rightarrow OA^2 = 625$

$\Rightarrow$ OA = 25cm

6. Question

In the circle of adjoining figure with its center at O, OP $\perp$ AB; if AB = 6 cm. and PC = 2cm, then let us write by calculating, the length of radius of the circle.

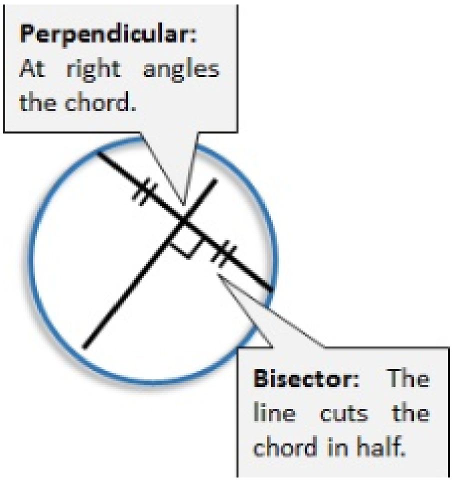
Answer



Given, AB = 6cm, PC = 2cm

AP $= \frac{AB}{2}$

Perpendicular from the center of the circle to any Chord bisects it in two line segments



$\Rightarrow AP = \frac{6}{2}$

$\Rightarrow AP = 3cm$

$\Rightarrow OA = OC$

$\Rightarrow OA = OP + CP$ ……….(1)

In $\triangle OAP$, Using Pythagoras Theorem

$OA^2 = OP^2 + AP^2$

$\Rightarrow (OP + PC)^2 = OP^2 + 9$ (from eq.(1))

$\Rightarrow OP^2 + PC^2 + 2(OP)(PC) = OP^2 + 9$

$\Rightarrow PC^2 + 2(OP)(PC) = 9$

$\Rightarrow 4 + 2(OP)(2) = 9$

$\Rightarrow 2(OP)(2) = 5$

$\Rightarrow OP = \frac{5}{4}$

Radius = OC = OP + PC

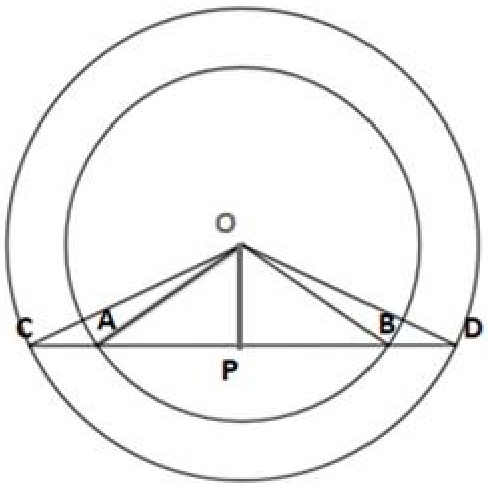
$\Rightarrow OC = 2 + \frac{5}{4}$

$\Rightarrow OC = 2.25cm$

7. Question

A straight line intersects one of the two concentric circles at the points A and B and the other at the point C and D. I prove with reason that AC = DB.

Answer



Given, Concentric Circles, CD and AB chords.

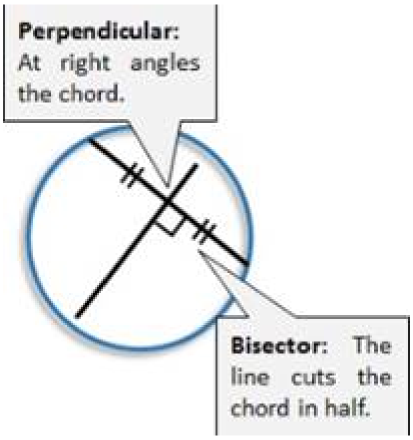
To prove: AC = DB

Construction: OP is a perpendicular bisector of CD and AB.

In ΔOAP and ΔOBP, We have

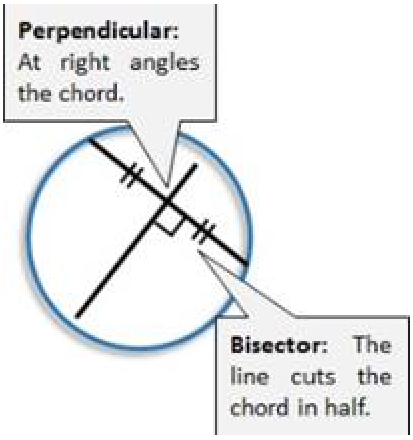
AP = BP

Perpendicular from the center of the circle to any Chord bisects it in two line segments



In ΔOCP and ΔODP

CP = DP



⇒ CA + AP = BP + DB

⇒ CA = DB + BP-AP

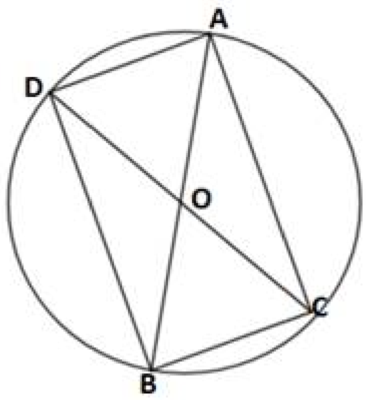
⇒ CA = DB + 0

⇒ **CA = DB**

8. Question

I prove that, the two intersecting chords of any circle cannot bisect each other unless both of them are diameters of the circle.

Answer



Given, AB and CD are Diameters.

To prove: OA = OB, OC = OD

Construction: Point D joined with B, Point A joined with C.

∠DAC = 90^0 (Rectangle)

∠ACB = 90^0 (Rectangle)

∠CBD = 90^0 (Rectangle)

∠BDA = 90^0 (Rectangle)

ACBD is Rectangle, So AD = CB, AD∥ BCand BD = AC,BD∥ AC

In △OCB and △ODA

∠OCB = ∠ODA (Interior angles, BC ∥ AD)

BC = AD (Rectangle)

∠OBC = ∠OAD (Interior angles, BC ∥ AD)

BCA SA Congruency

In △OCB ≅ △ODA

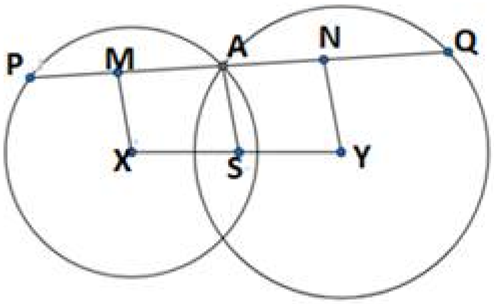
Hence Using CPCT, OA = OB, OD = OC

Hence Proved.

9. Question

The two circles with centers X and Y intersect each other at the points A and B. A is joined with the mid-point 'S' of XY and the perpendicular on SA through the point A is drawn which intersects the two circles at the points P and Q. Let us prove that PA = AQ.

Answer



Given, Two Circles with Centers X and Y intersect each other at point A and B. S is a mid-point of XY, AS is perpendicular to PQ.

To Prove: AP = AQ

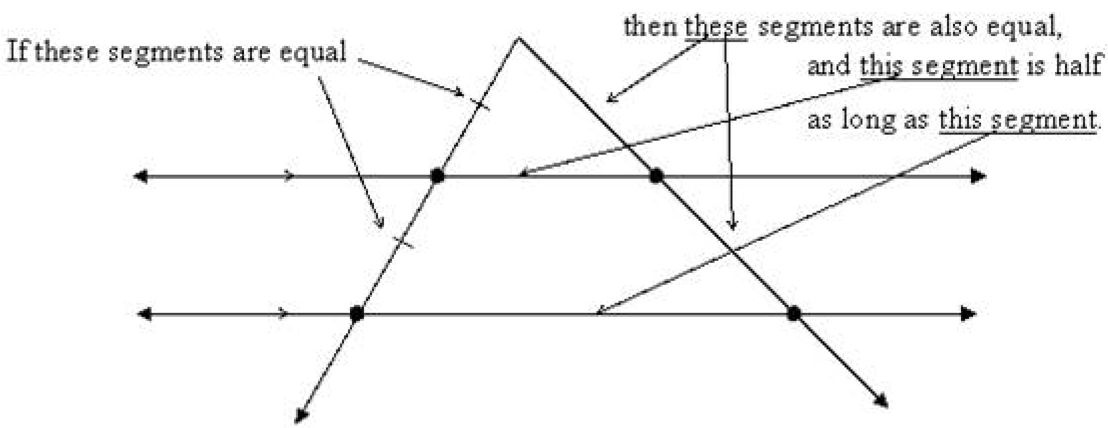
Construction: XM perpendicular to chord PA. And YN perpendicular to chord AQ of respective circles.

Since SA is also perpendicular to PQ (given)

So, all these perpendiculars to the same line are parallel to each other.

SX = SY

AM = AN



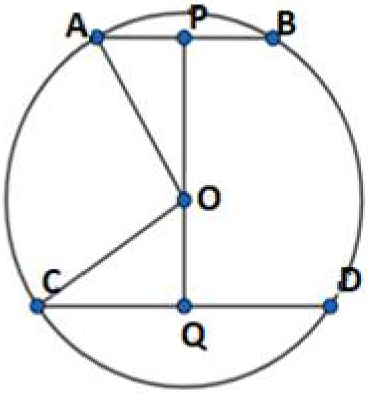
So, $\frac{AP}{2} = \frac{AQ}{2}$

AP = AQ

10. Question

The two parallel chords AB and CD with the lengths of 10 cm and 24 cm in a circle are situated on the opposite sides of the centre. If the distance between two chords AB and CD is 17 cm. then let us write by calculating, the length of the radius of the circle.

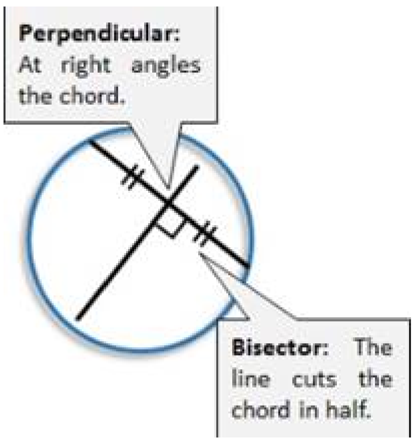
Answer



Given, AB = 10cm, CD = 24cm, PQ = 17cm

$AP = \frac{AB}{2}$

Perpendicular from the center of the circle to any Chord bisects it in two line segments



$AP = \frac{10}{2}$

AP = 5cm

$CQ = \frac{CD}{2}$

$\Rightarrow CQ = \frac{24}{2}$

⇒ CQ = 12cm

In $\triangle$OAP, Using Pythagoras Theorem

$OA^2 = AP^2 + OP^2$

⇒ $OA^2 = 5^2 + OP^2$

⇒ $OA^2 = 25 + OP^2$ ………… (1)

⇒ In △OCQ, Using Pythagoras Theorem

⇒ $OC^2 = CQ^2 + OQ^2$

⇒ $OC^2 = 12^2 + OQ^2$

⇒ $OC^2 = 144 + OQ^2$ ………. (2)

⇒ OC = OA

⇒ $144 + OQ^2 = 25 + OP^2$

⇒ $OP^2 - OQ^2 = 119$

⇒ (OP-OQ)(OP + OQ) = 119 (using $(A^2 + B^2 = (A + B)(A-B))$

⇒ OQ + OP = 17 ………. (3)

⇒ (OP-OQ)17 = 119

⇒ OP-OQ = 119/17

⇒ OP-OQ = 7 …….. (4)

Eq.3 + Eq.4

⇒ 2OP = 24

⇒ OP = 12cm

⇒ OP-OQ = 7

⇒ OQ = OP-7

⇒ OQ = 12-7

⇒ OQ = 5cm

In △OAP, Using Pythagoras Theorem

⇒ $OC^2 = 144 + OQ^2$

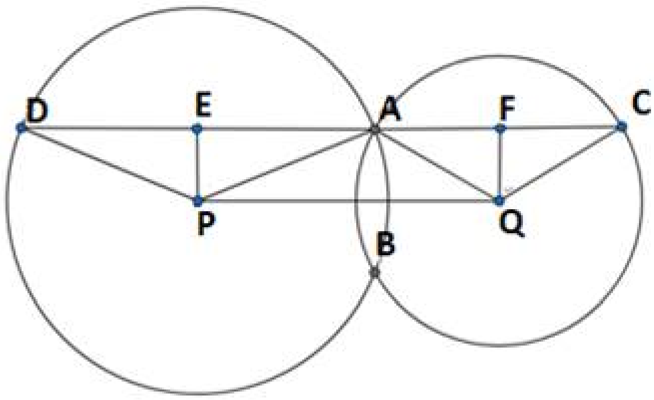
⇒ $OC^2 = 144 + 25$

⇒ OC = 13cm

11. Question

The centers of two circles are P and Q; they intersect at the points A and B. The straight line parallel to the line-segment PQ through the point A intersects the two circles at the points C and D. I prove that, CD = 2PQ.

Answer



Given, CD is parallel to PQ, Two Circles with centers P and Q intersect at point A and B.

To Prove: 2PQ = CD

Construction: PE and QE are the Perpendiculars on the Chord CD From centers P and Q respectively.

⇒ DE = EA

⇒ CF = AF

⇒ DE + EA + AF + FC = DC

⇒ 2EA + 2AF = DC

⇒ 2(EA + AF) = DC ……..(1)

PQ and CD are parallel lines and PE and QE are the perpendiculars.

So, ∠AEP = 90^0,∠AFQ = 90^0,∠EPQ = 90^0,∠AFQ = 90^0

EPQF is a rectangle

So, EF = PQ ……… (2)

⇒ 2(EA + AF) = DC

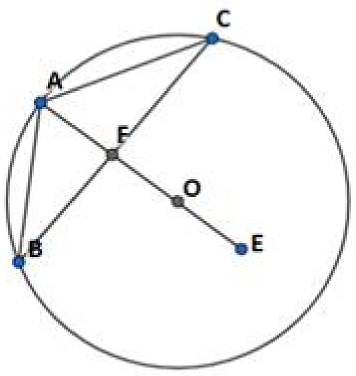
⇒ 2EF = DC

⇒ **2PQ = DC**

12. Question

The two chords AB and AC of a circle are equal. I prove that, the bisector of ∠BAC passes through the centre.

Answer



Given, AC = AB, ∠BAF = ∠CAF

To prove: FB = FC

In △ABF and △CAF

AC = AB (Given)

∠BAF = ∠CAF (Given)

AF = AF (Common)

△ABF ≅ △CAF

∠BFA = ∠CFA (CPCT)

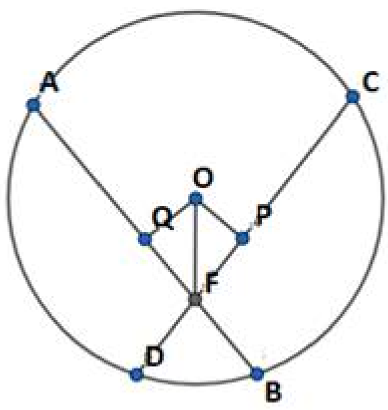
FB = FC (CPCT)

AE is a perpendicular bisector of chord BC, So, It has to pass through the center of the circle.

13. Question

If the angle-bisector of two intersecting chords of a circle passes through its centre, then let me prove that the two chords are equal.

Answer



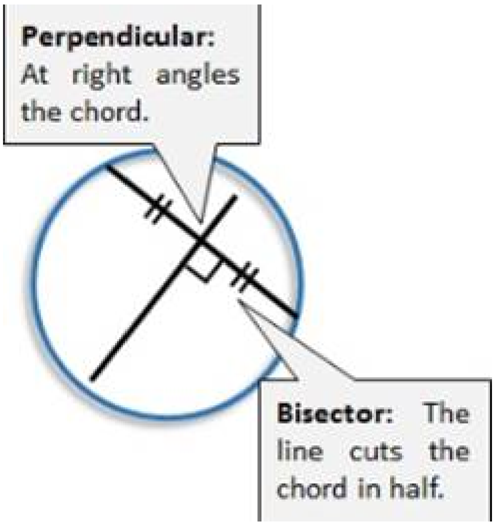
Given, OF is angle bisector of ∠AFC.

Construction: OQ ⊥ AB and OP ⊥ CD

In ΔOFQ and $\Delta OFP \angle OFQ = \angle OFP$ (given) $OF = OF$ (Common) $\angle OQF = \angle OPF$ (Construction)

AAS Congruency. $\Delta OPR \cong \Delta OPQ$.

$\therefore$ OR = OQ (C.P.C.T)

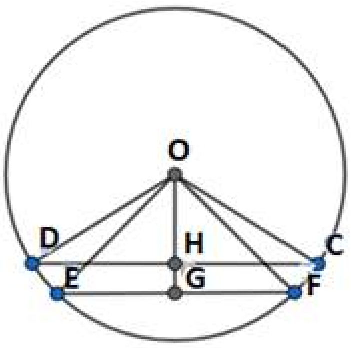


Hence AB = CD

14. Question

I prove that, among two chords of a circle the length of the nearer to centre is greater than the length of the other.

Answer



To Prove: DH>EG

Construction: B is the center of the circle, CD and EF are the chords, BH and BG are the perpendicular bisector of CD and EF respectively.

Given BH<BG As it given, CD is nearer to center than EF.

In ΔBDH

$\Rightarrow BD^2 = BH^2 + DH^2$ …………… (1)

In ΔBEG

$\Rightarrow BE^2 = BG^2 + EG^2$ ………… (2)

⇒ BD = BE

From Eq1 and Eq2

⇒ $BH^2 + DH^2 = BG^2 + EG^2$

⇒ BH<BG

⇒ $BH^2 < BG^2$

⇒ $BH^2 - BG^2 < 0$

From Eq.3

⇒ $BH^2 + DH^2 = BG^2 + EG^2$

⇒ $BH^2 - BG^2 = EG^2 - DH^2$

⇒ $EG^2 - DH^2 < 0$

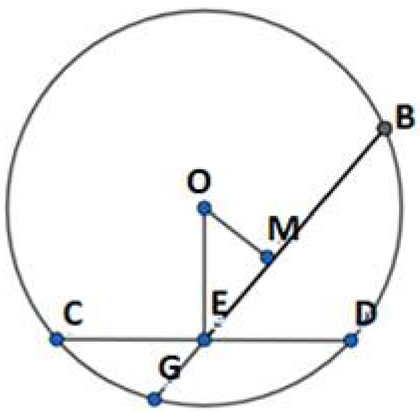
⇒ $EG^2 < DH^2$

⇒ EG<DH

15. Question

Let us write by proving the chord with the least length through any point in a circle.

Answer



To Prove CD<BG

Let, Center is O, Two chords are BG and CD, OM and OE are perpendicular bisector of BG and CD respectively.

In △OME,

⇒ OE>OM

In △OMB, Using Pythagoras Theorem

⇒ $OB^2 = OM^2 + BM^2$ .... (1)

In △OCE

⇒ $OC^2 = OE^2 + CE^2$ .... (2)

⇒ OC = OB

⇒ $OM^2 + BM^2 = OE^2 + CE^2$

⇒ OE>OM

So, BM>CE

⇒ BG>CD

As Chord goes near, its length increases.

16 A1. Question

The lengths of two chords of a circle with centre O are equal. If ∠AOB = 60°, then the value of ∠COD is

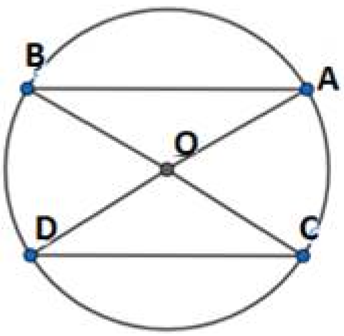
A. 40^o

B. 30^o

C. 60^o

D. 90^o

Answer



In △AOB and △COD

AB = CD

OA = OC = OB = OD

SSS Congruency.

So each and every angles and sides should be equal.

∠COD = 60^0

16 A2. Question

The length of a radius of a circle is 13 cm. and the length of a chord of a circle is 10 cm, the distance of the chord from the centre of the circle is

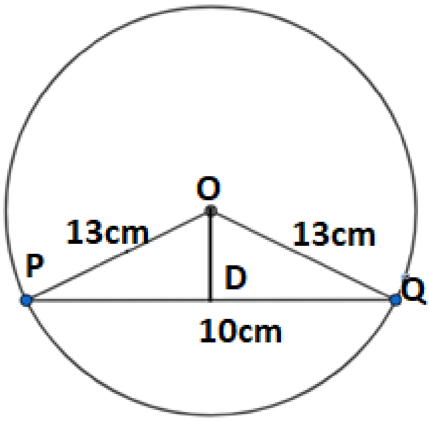
A. 12.5 cm

B. 12 cm

C. $\sqrt{69}$ cm.

D. 24 cm

Answer



Given radius = 13cm, PQ = 10cm

$\Rightarrow PD = \frac{PQ}{2}$

$\Rightarrow PD = \frac{10}{2}$

$\Rightarrow$ PD = 5cm

Using Pythagrus theorem in ΔO PD

$\Rightarrow OP^2 = OD^2 + PD^2$

$\Rightarrow 13^2 = OD^2 + 5^2$

$\Rightarrow OD^2 = 169-25$

$\Rightarrow OD^2 = 144$

$\Rightarrow$ OD = 12cm

16 A3. Question

Ab and CD are two equal chords of a circle with its centre O. If the distance of the chord AB from the point O is 4 cm. Then the distance of the chord from the centre O of the circle is

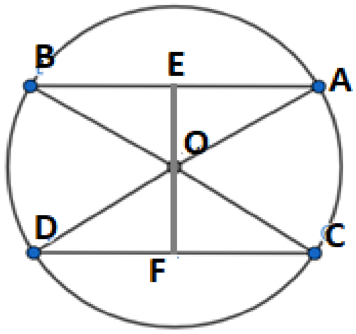
A. 2 cm

B. 4 cm

C. 6 cm

D. 8 cm

Answer



In ΔAOB and ΔCOD

AB = CD

OA = OC = OB = OD

SSS Congruency.

So each and every angles and sides should be equal.

$\angle$OAB = $\angle$OCD = $\angle$OBA = $\angle$ODC

$\Rightarrow$ EA $= AB/2$

$\Rightarrow$ FC $= \frac{CD}{2}$

$\Rightarrow$ AB = CD

So, EA = FC

In ΔAOE and ΔCOE

$\Rightarrow$ EA = FC

$\Rightarrow$ $\angle$EAO = $\angle$FCO

$\Rightarrow$ OA = OC

SAS Congruency.

So, By CPCT OE = OF = 4cm

16 A4. Question

The length of each of two parallel chord is 16 cm. If the length of the radius of the circle is 10 cm, then the distance between two chords is

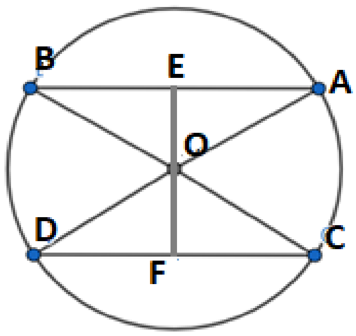
A. 12 cm

B. 16 cm

C. 20 cm

D. 5 cm

Answer



Let Parallel Chords are AB and CD. OE and OF are perpendicular bisector of AB and CD respectively.

Given, OB = OA = OD = OC = 10cm, AB = CD = 16cm

$\Rightarrow$ DF = $\frac{CD}{2}$

$\Rightarrow$ DF = $\frac{16}{2}$

$\Rightarrow$ DF = 8cm

$\Rightarrow$ EB = $\frac{AB}{2}$

$\Rightarrow$ EB = $\frac{16}{2}$

$\Rightarrow$ EB = 8cm

In $\triangle$AOE

$\Rightarrow OA^2 = AE^2 + OE^2$

$\Rightarrow OE^2 = OA^2 - AE^2$

$\Rightarrow OE^2 = 10^2 - 8^2$

$\Rightarrow OE^2 = 100 - 64$

$\Rightarrow OE^2 = 36$

⇒ OE = 6cm

As we can see from the previous question both the triangles are congruent.

So, OE = OF

Hence Distance between parallel lines = OE + OF = 6 + 6 = 12cm

16 A5. Question

The centre of two concentric circle is O; a straight line intersects a circle at the points A and B and other circle at the points C and D. If AC = 5 cm. then the length of BD is

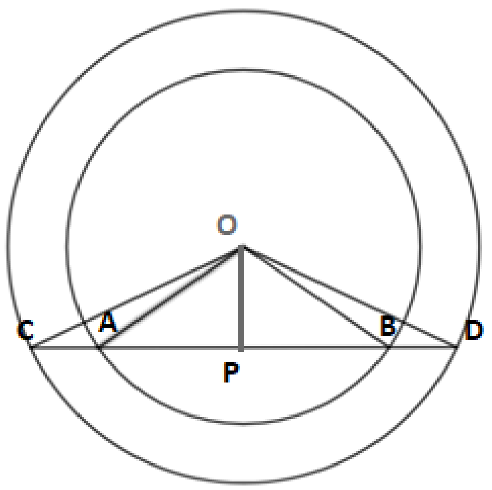
A. 2.5 cm

B. 5 cm

C. 10 cm

D. none of these

Answer



In ∆OAP and ∆OBP

⇒ AP = BP

⇒ OA = OB

⇒ OP = OP

In ∆OCP and ∆ODP

⇒ CP = DP

⇒ CA + AP = BP + DB

⇒ CA = DB + BP-AP

⇒ CA = DB + 0

⇒ CA = DB

So, BD = 5cm

16 B. Question

Let us write True/False:

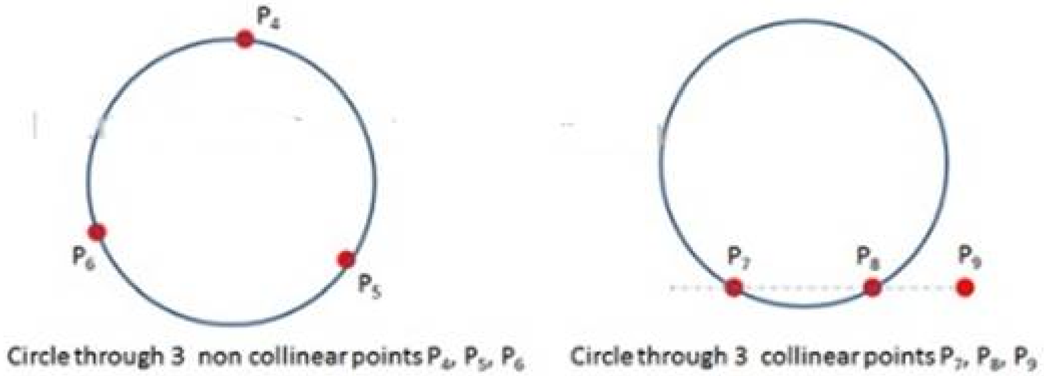
i. Only one circle can be drawn through three collinear points

ii. The two circles ABCDA and ABCEA are same circle.

iii. If two chord AB and AC of a circle with its centre O are situated on the Opposite side of the radius OA, then ∠OAB=∠OAC.

Answer

(i) False

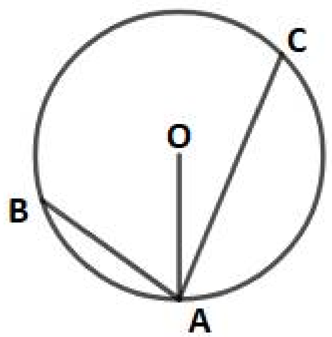


There is no Circle passes through the 3 Collinear points.

(ii) True

Only one Circle Passes through three Collinear Points.

(iii) False



You can see in this image, AB and AC are Situated on the Opposite side of the radius OA, Angles OAB and OCA are not Equal. It holds only for Equal chords.

16 C. Question

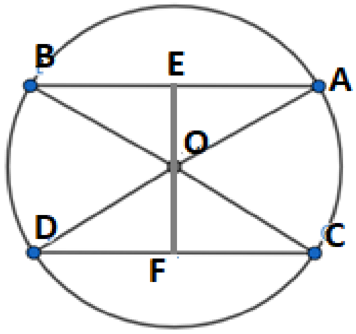
Let us fill in the blanks:

i. If the ratio of two chords PQ and RS of a circle with its centre O is 1 : 1, then ∠POQ: ∠ROS=____________

ii. The perpendicular bisector of any chord of a circle is ________ of that circle.

Answer

(i) 1:1



In △AOB and △COD

AB = CD

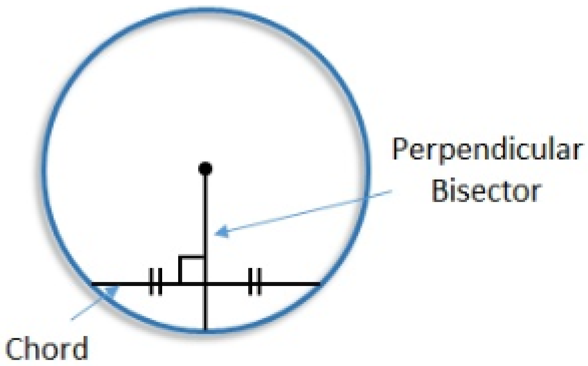
OA = OC = OB = OD

SSS Congruency.

So each and every angles and sides should be equal.

∠OAB = ∠OCD = ∠OBA = ∠ODC

(ii) Passes through the origin of that Circle.

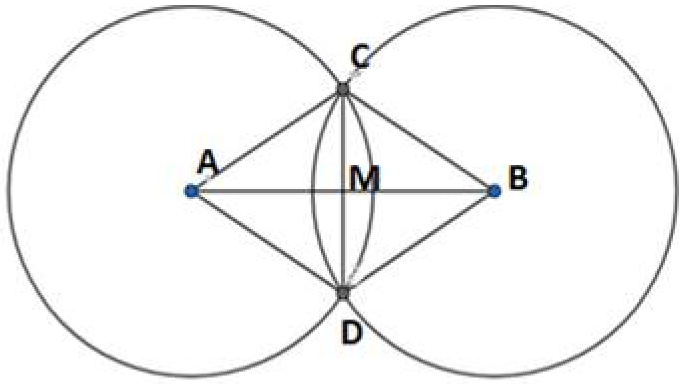


As you can see in this image The perpendicular bisector passes through the centre.

17 A. Question

Two equal circles of radius 10 cm. intersect each other and the length of their common chord is 12 cm. Let us determine the distance between the two centers of two circle

Answer



Let Center of the Circles are A and B. CD is a common Chord of the circle. AB is the perpendicular bisector of the chord CD.

If AB is a perpendicular bisector of CD then it should passes through both the centers.

So, AB is the distance that we need to calculate.

Given, AC = 10cm, CD = 12cm

$\Rightarrow CM = \frac{CD}{2}$

$\Rightarrow CM = \frac{12}{2}$

$\Rightarrow$ CM = 6cm

In ΔACM

$\Rightarrow AC^2 = AM^2 + CM^2$

$\Rightarrow AM^2 = AC^2 - CM^2$

$\Rightarrow AM^2 = 100-36$

$\Rightarrow AM^2 = 64$

$\Rightarrow$ AM = 8cm

In ΔBCM

$\Rightarrow BC^2 = BM^2 + CM^2$

$\Rightarrow BM^2 = BC^2 - CM^2$

$\Rightarrow BM^2 = 100-36$

$BM^2 = 64$

BM = 8cm

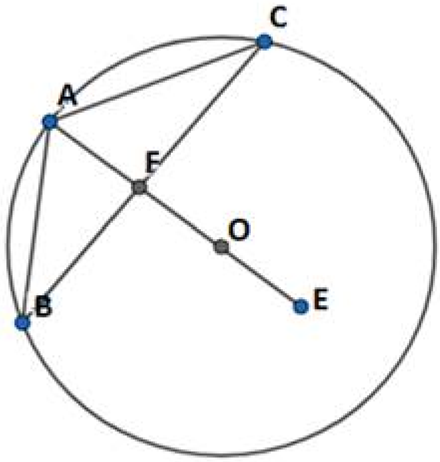
AB = 16cm

17 B. Question

AB and AC are two equal chords of a circle having the radius of 5 cm. The centre of the circle

is situated at the outside of the triangle ABC. If AB = AC = 6 cm. then let us calculate the length of the chord BC.

Answer



Given, AC = AB, ∠BAF = ∠CAF, AB and AC are two equal Chords of a circle, therefore the centre of the circle lies on the bisector of ∠BAC.

In ΔABF and ΔCAF

⇒ AC = AB

⇒ ∠BAF = ∠CAF

⇒ AF = AF

⇒ ΔABF ≅ ΔCAF

⇒ ∠BFA = ∠CFA

⇒ FB = FC

AE is a perpendicular bisector of chord BC.

In ΔABF, by Pythagoras theorem,

⇒ $AB^2 = AF^2 + BF^2$

⇒ $BF^2 = 6^2 - AF^2$ ....(1)

In ΔOBF

⇒ $OB^2 = OF^2 + BF^2$

$\Rightarrow 5^2 = (5 - AF)^2 + BF^2$

$\Rightarrow BF^2 = 25 - (5 - AP)^2$ ....(2)

Equating (1) and (2), we get

$\Rightarrow 6^2 - AF^2 = 25 - (5 - AF)^2$

$\Rightarrow 11 - AF^2 = -25 - AF^2 + 10AF$

$\Rightarrow 36 = 10AF$

$\Rightarrow AF = 3.6$ cm

Putting AF in (1), we get

$\Rightarrow BF^2 = 6^2 - (3.6)^2 = 23.04$

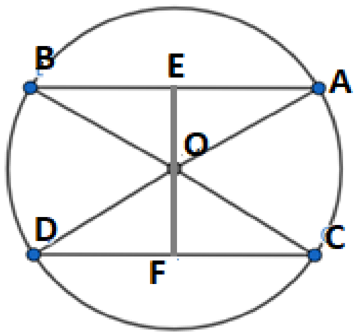
$\Rightarrow BF = 4.8$ cm

$\Rightarrow BC = 2BF = 2 \times 4.8 = 9.6$ cm

17 C. Question

The lengths of two chords AB and CD of a circle with its centre O are equal. If $\angle AOB = 60°$ and CD = 6 cm. then let us calculate the length of the radius of the circle.

Answer



In $\triangle AOB$ and $\triangle COD$

$\Rightarrow AB = CD$

$\Rightarrow OA = OC = OB = OD$

SSS Congruency.

So each and every angles and sides should be equal.

AB = 6cm

$\Rightarrow \angle COD = 60^0$

$\Rightarrow AE = \frac{AB}{2}$

$\Rightarrow AE = \frac{6}{2}$

⇒ AE = 3cm

In $\triangle$AOE and $\triangle$BOE

⇒ OA = OB

⇒ OE = OE

⇒ AE = BE

Hence Using SSS congruency

$\Rightarrow \triangle AOE \cong \triangle BOE$

$\Rightarrow \angle AOE = \angle BOE$

$\Rightarrow \angle AOE = 30^0$

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypetenuse}}$$

$\Rightarrow \text{Sin}30 = \frac{AE}{OA}$

$\Rightarrow \frac{1}{2} = \frac{AE}{OA}$

⇒ OA = 2AE

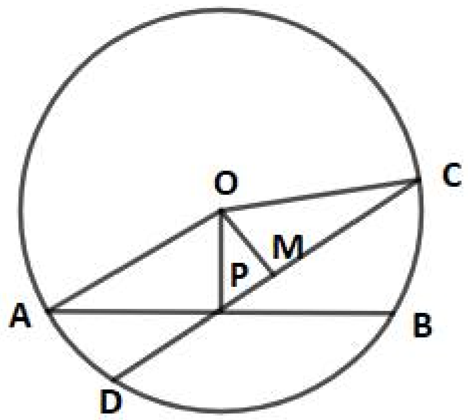
⇒ OA = 2(3)

⇒ OA = 6cm

17 D. Question

P is any point in a circle with its centre O. If the length of the radius is 5 cm. and OP = 3 cm., then let us determine the least of the chord passing through the point P.

Answer



Given: OP = 3cm, Radius = 5cm

Let OA, OC radius of the circle, OM is perpendicular Bisector Passes through the centre O of Chord CD. OP is a Perpendicular Bisector of Chord AB passes through the Centre O.

In ΔOPM

$\angle$OMP = 90^0

Using, Pythagoras Theorem

$\Rightarrow OP^2 = OM^2 + PM^2$

$\Rightarrow OP > OM$

We have proved in Question 14, That Nearer chord is greater than the other.

It shows us that

AB<CD

Hence The Least chord passes through point P is AB, OP is Perpendicular bisector of the chord.

In ΔOPA

Using, Pythagoras Theorem

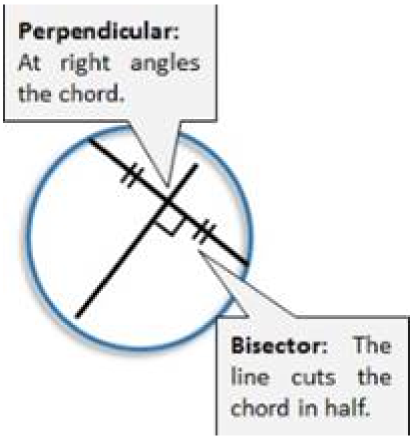
$\Rightarrow OA^2 = OP^2 + AP^2$

$\Rightarrow 5^2 = 3^2 + AP^2$

$\Rightarrow AP^2 = 25-9$

$\Rightarrow AP^2 = 16$

$\Rightarrow AP = 4cm$



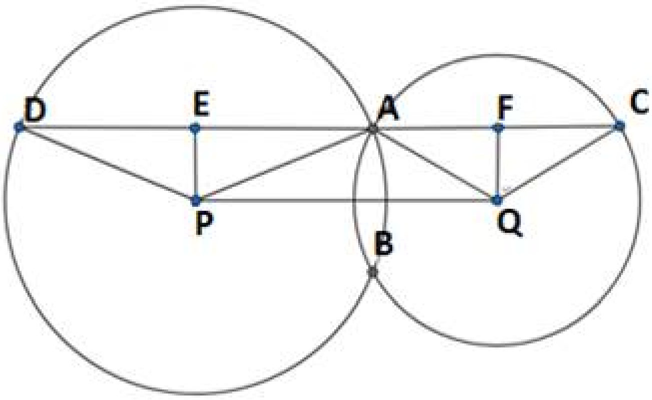
Using above Theorem

AB = 8cm

17 E. Question

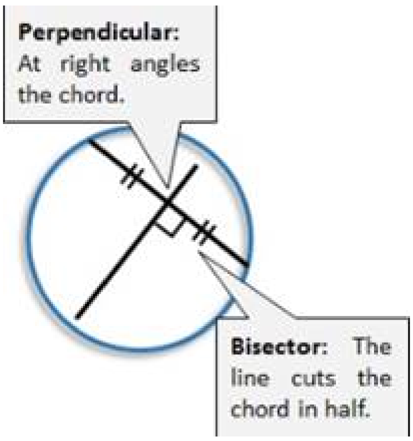
The two circles with their centres at P and Q intersect each other at the points A and B. Through the point A, a straight line parallel to PQ intersects the two circles at the points C and D respectively. If PQ = 5 cm., then let us determine the length of CD.

Answer



Given: PQ = 5cm, CD is parallel to PQ, Two Circles with centers P and Q intersect at point A and B.

Construction: EF and FQ are Perpendicular bisector drawn from P and Q Respectively.



The Perpendicular from The Centre to the chord, bisects the chord.

⇒ DE = EA

⇒ CF = AF

⇒ DE + EA + AF + FC = DC

⇒ 2EA + 2AF = DC

⇒ 2(EA + AF) = DC ………..(1)

$EF \parallel PQ$ (Given)

$\angle PEA = 90^o$ (Construction)

$\angle QFA = 90^o$ (Construction)

Using Interior Angle Theorem,

$\angle PEA + \angle BPE = 180$

$\angle BPE = 90^o$

PQFE is a rectangle.

So, EF = PQ …………… (2)

⇒ 2(EA + AF) = DC

⇒ 2EF = DC

⇒ **2PQ = CD**

CD = 10cm